MATHEMATICS FOR FINANCE

TOPICS OF THE COURSE

Part I: Review of calculus

Deterministic calculus

- Derivability, differentiability and Taylor's formula for multivariate real functions
- Quadratic forms and their signature
- Maxima and minima for multivariate real functions
- Classical optimization (unconstrained and constrained) for multivariate real functions

Stochastic calculus

- Stochastic differential equations, stochastic processes and Wiener processes
- Ito's lemma
- Applications in finance

Part II: Basics of Finance

Term structure of interest rates

- Zero rates and yields to maturity, forward rates, bootstrapping
- Duration and immunization
- Convexity
- Duration as a volatility index

Risk-neutral pricing techniques

- Fundamental theorem of asset pricing (finite-time and discrete-state case) and martingale measures
- Binomial trees and replicating portfolios
- Continuous case: the geometric Brownian motion
- Cameron-Martin-Girsanov theorem

Expected utility

- Expected utility and certainty equivalent
- Risk-aversion
- Popular utility functions

Part III: Dynamic analysis

Fundamentals of ODE's

- ODE's and systems of ODE's, Cauchy problems
- Differential equations with separable variables
- First-order linear differential equations (homogeneous and non-homogeneous case)
- First-order linear differential systems (homogeneous and non-homogeneous case, constant coefficients)
- Second-order linear differential equations (homogeneous and non-homogeneous case, constant coefficients)
- Applications in finance

Fundamentals of PDE's

- Classification of PDE's: parabolic, hyperbolic and elliptic
- Initial and boundary conditions
- The method of separation of variables
- The heat equation

• The Black-Scholes equation
Fundamentals of dynamic programming
• The Bellman's principle
• Applications in finance

READING LIST:

Part I

• Chiang, A., "Fundamental Methods of Mathematical Economics", New York,McGraw-Hill, 1984.
• Neftci, S., "An Introduction to the Mathematics of Financial Derivatives", San Diego, Academic Press, 1996.

Part II

• Baxter, M. and Rennie, A., "Financial Calculus", Cambridge, Cambridge University Press, 1996.
• Duffie, D., "Dynamic Asset Pricing Theory", Princeton, Princeton University Press, 1996.
• Hull, J. C., "Option, Futures, and Other Derivatives", New Jersey, Pearson Education, 2003.

Part III

• Chiang, A., "Fundamental Methods of Mathematical Economics", New York,McGraw-Hill, 1984.
• Neftci, S., "An Introduction to the Mathematics of Financial Derivatives", San Diego, Academic Press, 1996.
• Wilmott, P., Howison, S. and Dewyne, J., "The Mathematics of Financial Derivatives", Cambridge, Cambridge University Press, 1995