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A Rational Theory of “Irrational Exuberance”*

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Abstract

The arrival of new, unfamiliar, investment opportunities—e.g., internet commerce, emerging markets, novel financial instruments—is often associated with large, “exuberant,” movements in asset prices and real investment. While irrational explanations of these phenomena abound, in this paper we show that the dispersion of information that is likely to surround these new, unfamiliar investment opportunities may help explain these phenomena within an otherwise canonical, fully rational, neoclassical model of the interaction of financial markets and the real economy. On the positive front, we identify a mechanism that amplifies the response of the economy to noise (correlated, but rational, errors in assessments of the fundamentals), while at the same time formalizing the idea that “inflated prices” and “exuberant investment” may feed one another during these episodes. This mechanism rests on the property that, when information is dispersed and only then, financial markets look at aggregate investment as a signal of the underlying fundamentals. On the normative front, we document that this amplification is a symptom of constrained inefficiency: there exist policy interventions that can improve welfare without requiring the government either to have superior information than the market or to centralize the information that is dispersed in the economy.

Keywords: mispricing, heterogeneous information, complementarity, volatility, inefficiency, beauty contest.

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1 Introduction

Episodes of large joint movements in asset prices and aggregate investment, such as the internet boom of the late 90s, pose a number of positive and normative questions. Do these movements simply reflect the arrival of news about the future profitability of physical (and intangible) capital? Or do they reflect an excessive response to temporary waves of “optimism” and “pessimism,” appropriately defined? If so, is there a role for government intervention?

The mere magnitude of these episodes makes it hard to reconcile them with the predictions of conventional, rational-expectations, representative-agent models.¹ A simple alternative is to attribute the observed fluctuations to purely irrational shifts in beliefs. Within the economics profession, this approach is considered erratic by some, but is favored by advocates of behavioral economics.² Outside the economics profession, related ideas simply dominate every single account of market behavior by the public media and various market pundits.

We will not go in that direction here. This is partly a matter of preferred methodology. We are particularly uncomfortable with policy prescriptions that rely on the presumption that policy makers, or some market pundits, have superior ability than the market mechanism in evaluating the profitability of the available investment opportunities. But it is also because our curiosity, as theoretical economists, is far from appeased by explanations that attribute the observed fluctuations to irrational forces.

We are particularly intrigued by the observation that episodes of large, “exuberant,” movements in asset prices and investment activity often coincide with the arrival of new, unfamiliar investment opportunities—whether this is internet commerce, new markets in India and China, or perhaps novel financial instruments. The challenge for us, then, is to understand what may lie beneath this coincidence.

We are equally intrigued by the informal argument, made by various observers, that financial markets have a de-stabilizing role during these episodes, as the agents in charge of real investment decisions are “overly” concerned about the short-run valuation of their capital in financial markets. But we are also aware that his argument can be reduced to mere non-sense within a conventional neoclassical framework a la Lucas (1978): asset prices then reflect merely the market’s common expectation of economic fundamentals, so that it is simply irrelevant whether agents look at prices or fundamentals when making investment decisions. The challenge for us, then, is to make sense of this argument within a rational framework, without the intervention of exotic forces such as irrational traders and the like, while at the same time explaining why this argument may be particularly relevant around the arrival of new investment opportunities.

¹Of course, not everybody will agree with this sentence; but this feeling is widespread even if not precisely defined.

²For a recent example, see Akerlof and Shiller (2009).

These considerations define the scope of our theoretical contribution. We start by adhering to the axiom of rationality and, more narrowly, to a neoclassical benchmark that guarantees that the interaction of financial markets and the real economy is first-best efficient during normal times. We then seek to understand what could make this otherwise flawless interaction go “wrong” at times associated with the arrival of new, unfamiliar, investment opportunities.

The answer we propose to this question rests on the introduction of *heterogeneous information* in an otherwise frictionless, competitive, rational-expectations model of the interaction between financial markets and the real economy. This modeling choice is grounded on two considerations. The one is empirical: we find it appealing to assume that information is particularly dispersed during the episodes of interest, due to the unfamiliarity of the new investment opportunities, the lack of relevant historical data, the diverse assessments that different agents may have at early stages of investment and financial trading, and the impossibility of previous social learning. The other is theoretical: when information is heterogeneous, and only then, a rational agent can face uncertainty about the expectations of other rational agents over and above the one he faces about the fundamentals, thus allowing us to break apart the agents’ uncertainty about prices and investment levels from their uncertainty about underlying profitability.³

Our contribution is then to show that the heterogeneity of information may cause the otherwise flawless interaction of financial markets with the real economy to go wrong in two ways. First, it may amplify the response of the economy to correlated, but rational, errors in the agents’ assessments of profitability, thus helping explain why the episodes of interest may look to an outside observer as too “exuberant”, or hard to reconcile with fundamentals. Second, it can introduce a particular type of inefficiency in real investment decisions, thus providing a formal, normative, content to the argument that investors are “overly” concerned with short-run behavior of asset prices. We then discuss how our results open the door to certain policy interventions, without ever presuming that the government “knows better” than the market.

Preview of model and results. We consider a model in which a large number of “entrepreneurs” gets the option to invest in a new technology. Each must make his decision based on imperfect, and heterogeneous, information about the profitability of this new technology. At a later date, but before the complete resolution of uncertainty, each entrepreneur may have to (or get the option to) sell his capital in a competitive financial market. The financial market, in turn, is populated by a large number of “traders”. These traders are also imperfectly informed, but get to observe aggregate investment before trading occurs.

At the core of this model is a two-way interaction between financial markets and the real economy, or “Wall street” and “Main Street”. On the one hand, entrepreneurs base their initial

³Within a business-cycle context, the theoretical point that the heterogeneity of information introduces a distinct source of uncertainty about economic activity is emphasized by Angeletos and La’O (2008, 2009).

investment decisions partly on their expectations about the price at which they may sell their capital; this captures more broadly the idea that the incentives of those in charge of real investment decisions depend in part on their expectations of the short-run behavior of asset prices. On the other hand, traders look at aggregate investment to learn about profitability; this captures more broadly the idea that financial market participants follow closely the release of macroeconomic and sectoral data, looking for clues about underlying fundamentals.

The first direction of this interaction identifies a pecuniary externality: part of the return to investment for one group of agents (the entrepreneurs) is the price at which they can sell this capital to another group of agents (the traders). The second direction identifies an informational spillover: the collective behavior of the former group at the time of real investment impacts the information that is available to the latter group at the time of financial trades. As standard in competitive frameworks, the pecuniary externality is not itself the source of any inefficiency in our model: the possibility of trade among the two groups only improves welfare. Furthermore, the informational spillover itself is also beneficial: the transmission of information from one group to another facilitates more efficient investment and financial trade. Our contribution is to show that, when information is dispersed, the interaction of these two forces leads to very distinct positive and normative implications.

On the positive side, we show that this interaction amplifies the response of real investment and asset prices to “common noise shocks” relative to “fundamental shocks”. The latter are defined as shocks to the underlying profitability of the new technology. The former are defined as correlated errors in the entrepreneurs’ expectations of this profitability, and introduce in our model a source of “non-fundamental movements” in investment and asset prices. The existence of these shocks is taken for granted, as in any model with uncertain fundamentals; our contribution is to study how the aforementioned interaction impacts the propagation of these shocks when information about these shocks is dispersed.

Towards this goal, we compare the response of the economy under two scenarios. The benchmark scenario assumes either that trading does not take place or that information about the aforementioned shocks is common at the trading stage, thus ruling out at least one of the two directions of the aforementioned interaction. Under this scenario, real investment and asset prices are driven merely by expectations of the fundamentals, much alike in any canonical neoclassical model. The alternative scenario—when trading takes place under dispersed information—identifies the case of interest.

Thus consider this case and, towards a contradiction, suppose that the entrepreneurs’ investment decisions depend only on their expectations of the fundamentals, as in the benchmark scenario. The realized level of investment will then reveal the entrepreneurs’ average opinion about

the fundamentals. This in turn will serve, in the eyes of the traders, as a informative signal of the underlying fundamentals, contributing towards a higher price in the financial market. This signal is noisy: any given agent—whether a trader or an entrepreneur—need not be able to tell whether higher aggregate investment is caused by a positive shock to fundamentals or by a positive correlated error in the entrepreneurs’ opinion of the fundamentals. However, relative to the typical trader, the typical entrepreneur is bound to have some additional information about the noise in this signal; This is because the origin of this noise is the entrepreneurs’ own opinions and actions.

This asymmetry is crucial, for it implies that the (rational) pricing errors that occur in the financial market are partly predictable in the eyes of the typical entrepreneur. In particular, whenever a common noise shock occurs, each entrepreneur will expect the average opinion of the other entrepreneurs—and hence aggregate investment—to increase more than his own opinion. But then the entrepreneur will also expect the financial market to overprice his capital. This in turn creates an incentive for the entrepreneur to invest more than what warranted from his expectation of the fundamentals.

This argument proves that, when other entrepreneurs invest on the basis of their expectations of the fundamentals, the individual entrepreneur has an incentive to deviate from this strategy, reacting more heavily to correlated sources of information. But as all entrepreneurs have such an incentive, the equilibrium ends up featuring a heightened sensitivity to correlated errors in the entrepreneurs’ expectations, explaining our key positive result.

Turning to the normative side, the question of interest is whether this amplification effect is also a symptom of inefficiency. To address this question, we consider a constrained-efficiency notion similar to that proposed in Angeletos and Pavan (2007, 2009). Namely, we consider the problem faced by a planner who has no informational advantage vis-a-vis the market—either in the form of additional information or in the form of the power to centralize the information that is dispersed in the economy—but who has full power on the incentives of the agents. We then show that such a planner would dictate to the entrepreneurs to ignore the expected mispricing in the stock market and base their investment decisions merely on their expectations of the fundamentals. This is because any gain that the entrepreneurs can make by exploiting such a mis-pricing is only a private rent—a zero-sum transfer from one agent to another, which creates a wedge between the private and social return to investment.

We conclude that our results open the door to policy interventions even if the government is restricted to base its policies only on information that is in the public domain. We then examine two types of policies that satisfy this restriction. The one identifies interventions that are aimed at stabilizing asset prices and that take place “during the fact,” while information is dispersed. The other identifies a tax on capital holdings that is collected only “after the fact,” once uncertainty

has been resolved, and is allowed to depend both on a measure of realized profitability and on a measure of realized aggregate investment (alternatively, on financial prices). Both types of policies can improve welfare, but the latter is superior on normative grounds.

Discussion. Which policies are more appealing on practical grounds is left to the reader to decide. The key point we wish to make here is that these policies do not require the government to centralize the information that is dispersed in the economy, nor do they presume that the government has any “rationality” or “intelligence” superiority over the private sector. They rest only on the fact that the dispersion of information introduces a wedge between private and social incentives.

The origin of both this wedge and the amplification effect in our model is the informational role of aggregate investment: entrepreneurs overreact to noise shocks only because they expect the market to misinterpret their “exuberance” as a signal of high profitability. This may help explain how “exuberant” investment activity and “inflated” asset prices may feed one another, not because of the workings of irrational forces, but rather as a consequence of the heterogeneity of information. Interestingly, these effects occur in our model without any of the entrepreneurs being “strategic,” in the sense that all agents in our model are infinitesimal and take prices and aggregate outcomes as exogenous to their own choices. Thus, despite they share the same flavor, our results are fundamentally distinct from those in the finance literature that, in the tradition of Kyle (1986), focuses on how big informed players can manipulate asset prices. In particular, no individual entrepreneur in our model is concerned with how his own actions may affect prices, for he simply takes prices as given. Furthermore, if the other entrepreneurs did not act on their information, then aggregate investment would not serve as a signal of the fundamentals, in which case there would be no pricing error for the individual entrepreneur to exploit. Therefore, it is only the “invisible hand” of the market mechanism that leads the entrepreneurs to a collective behavior that permits them to capitalize on their dispersed, but correlated, information.

We formalize this last idea by establishing an isomorphism between the general equilibrium of our competitive economy and the Perfect Bayesian equilibrium of a certain game with a continuum of players. This game obtains by “reducing out” the equilibrium behavior of the financial market and focusing on the real side of the model, namely on the behavior of the entrepreneurs at the moment of their investment decisions. The signaling role of aggregate investment then translates in a particular form of strategic complementarity: because a higher aggregate level of investment sends a positive signal to the financial market, leading to higher asset prices, the incentive of each individual entrepreneur to invest increases with his own expectation of aggregate investment.

Both our positive and normative results can then be re-interpreted in the light of this isomorphism. On the positive side, a heightened response to correlated errors is a robust property of

games with strategic complementarity. This insight was first highlighted by Morris and Shin (2002) within the context of an abstract game that was meant to capture Keynes's (1936) "beauty contest" parable for financial markets. The main idea in Keynes's argument appears to be that professional investors care more about predicting one another's beliefs, so as to predict prices, rather than the true fundamentals, and that this somehow leads to "undesirable" volatility.⁴ However, as emphasized by Angeletos and Pavan (2006, 2007), the presence of strategic complementarity in games à la Morris and Shin (2002) does not necessarily imply inefficiency. For that, it is necessary to have a wedge between the private and the social benefits of coordination. The presence of such a wedge is only *assumed* in Morris and Shin (2002). In this respect, that earlier work has not advanced our understanding of how the "beauty-contest" character of financial markets could be the source of inefficiency. In contrast, the current paper provides a fully micro-founded rationale for this wedge and identifies a very specific information externality as the origin of this wedge.

Finally, it is worth noting that, by focusing on the heterogeneity of opinions regarding the profitability of new investment opportunities and a certain form of "mispricing" in financial markets, our paper shares a certain flavor with the recent literature that focuses on speculative trading due to heterogeneous priors (e.g., Scheinkman and Xiong, 2003; Gilchrist, Himmelberg, and Huberman, 2005; Panageas, 2005). However, by imposing homogeneous information, this prior literature has ruled out the informational externality that is the source of both the amplification and the inefficiency in our model. Furthermore, this literature fails to provide a rational for policy intervention that would not require an "intelligence" superiority on the side of the government. Debreu long ago established Welfare Theorems that allow for subjective (agent-specific) probabilities, thus guaranteeing that the type of speculative trades this recent literature focus on are not by themselves a symptom of inefficiency, no matter how volatile they might be. Justifying government intervention would then require one to argue that the priors of market participants are "irrational," which is what we have sought to avoid as a matter of principle.⁵

The rest of the paper is organized as follows. Section 2 introduces the baseline model. Section 3 characterizes the equilibrium and derives the positive implications of the model. Section 4 characterizes the constrained efficient allocation and contrasts it to the equilibrium. Section 5 discusses policy implications. Section 6 considers a number of possible extensions. Section 7 concludes. All proofs are in the Appendix.

⁴See also Allen, Morris and Shin (2005), Bacchetta and Wincoop (2005) and Cespa and Vives (2007) for explicit formalizations of asset pricing models that highlight the *positive* role of higher-order beliefs.

⁵By this we do not wish to dismiss models with heterogeneous priors—such models can have a number of novel, and intriguing, predictions that may be useful for explaining real-world phenomena. We only seek to separate the predictions that rest on interpreting this subjectiveness as a form of "irrationality."

2 The baseline model

To fix ideas, we consider an environment in which heterogeneously informed agents choose how much to invest in a “new technology” with uncertain returns. After investment has taken place, but before uncertainty is resolved, agents trade financial claims on the returns of the installed capital. At this point, the observation of aggregate investment partially reveals the information that was dispersed in the population during the investment stage.

Timing, actions, and information. There are four periods, $t \in \{0, 1, 2, 3\}$, and two types of agents: “entrepreneurs,” who first get the option to invest in the new technology, and “traders,” who can subsequently purchase claims on the installed capital of the entrepreneurs. Each type is of measure $1/2$; we index entrepreneurs by $i \in [0, 1/2]$ and traders by $i \in (1/2, 1]$.

At $t = 0$, nature draws a random variable θ from a Normal distribution with mean $\mu > 0$ and variance $1/\pi_\theta$ (i.e., π_θ is the precision of the prior). This random variable represents the exogenous productivity of the new technology and is unknown to all agents.

At $t = 1$, the “real sector” of the economy operates: each entrepreneur decides how much to invest in the new technology. Let k_i denote the investment of entrepreneur i . The cost of this investment is $k_i^2/2$ and is incurred within the period. When choosing investment, entrepreneurs have access to various sources of information (signals) that are not directly available to the traders. Some of these signals may have mostly idiosyncratic noise, while others may have mostly common noise (correlated errors). To simplify, we assume that entrepreneurs observe two signals. The first one has only idiosyncratic noise and is given by $x_i = \theta + \xi_i$, where ξ_i is Gaussian noise, independently and identically distributed across agents, independent of θ , with variance $1/\pi_x$ (i.e., π_x is the precision of the idiosyncratic signal). The second has only common noise and is given by $y = \theta + \varepsilon$, where ε is Gaussian noise, common across agents, independent of θ and of $\{\xi_i\}_{i \in [0, 1/2]}$, with variance $1/\pi_y$ (i.e., π_y is the precision of the common signal). The more general case where all signals have both idiosyncratic and common errors is examined in the Supplementary Material.

At $t = 2$, the “financial market” operates: some entrepreneurs sell their installed capital to the traders. In particular, we assume that each entrepreneur is hit by a “liquidity shock” with probability $\lambda \in (0, 1)$. Liquidity shocks are i.i.d. across agents, so λ is also the fraction of entrepreneurs hit by the shock. Entrepreneurs hit by the shock are forced to sell all their capital to the traders. For simplicity, entrepreneurs not hit by the shock are not allowed to trade any claims on installed capital.⁶ The financial market is competitive and p denotes the price of one unit of installed capital. When the traders meet the entrepreneurs hit by liquidity shocks in the financial market, they observe the aggregate level of investment from period 1, $K = \int_0^1 k_i di$. They can then

⁶We relax this assumption in Section 6.

use this observation to update their beliefs about θ .⁷

Finally, at $t = 3$, θ is publicly revealed, each unit of capital gives a cash flow of θ to its owner, and this cash flow is consumed.

Payoffs. All agents are risk neutral and the discount rate is zero. Payoffs are thus given by $u_i = c_{i1} + c_{i2} + c_{i3}$, where c_{it} denotes agent i 's consumption in period t . First, consider an entrepreneur. If he is not hit by the liquidity shock his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, 0, \theta k_i)$, so that his payoff is $u_i = -k_i^2/2 + \theta k_i$. If he is hit by the shock, he sells all his capital at the price p and his consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (-k_i^2/2, pk_i, 0)$, so that his payoff is $u_i = -k_i^2/2 + pk_i$. Next, consider a trader and let q_i denote the units of installed capital he purchases in period 2. His consumption stream is $(c_{i1}, c_{i2}, c_{i3}) = (0, -pq_i, \theta q_i)$, so that his payoff is $u_i = (\theta - p)q_i$.

Remarks. The two essential ingredients of the model are the following: (i) information about the profitability of the new technology is dispersed, and agents have access to multiple sources of information with different degrees of correlation in the errors; (ii) there is some common “noise” that prevents aggregate investment from perfectly revealing the entrepreneurs’ information to the traders, so that the dispersion of information does not completely vanish by the time agents trade in the financial market.

The specific information structure we have assumed is a convenient way to capture these two properties. In particular, the role of the common signal y is to introduce correlated errors in the entrepreneurs’ assessments of profitability in stage 1, thereby adding noise to the inference problem that the traders face in stage 2: in equilibrium, aggregate investment will move both with the fundamental θ and with the common error ε , ensuring that aggregate investment reveals θ only imperfectly. As mentioned above, in the Supplementary Material we dispense with the common signal y and instead consider the case where entrepreneurs observe multiple private signals, all of which have both idiosyncratic and common errors. We also consider a variant that introduces unobserved common shocks to the entrepreneurs’ cost of investment as an alternative source of noise in aggregate investment. In both cases, our main positive and normative results (Corollaries 1 and 2) remain intact, highlighting that the key for our results is the existence of different sources of information with different correlation in their noise, not the specific form of it.

A similar remark applies to other simplifying modeling choices. For example, we could have allowed the entrepreneurs that are not hit by a liquidity shock to participate in the financial market; we could further have allowed all entrepreneurs to observe a noisy signal of aggregate investment, or a noisy price signal, at the time they make their investment decisions.⁸ What is essential for

⁷Letting the traders observe the entire cross-sectional distribution of investments does not affect the results. This is because, in equilibrium, this distribution is Normal with known variance; it then follows that the mean investment contains as much information as the entire cross-sectional distribution.

⁸See Section 6 for these extensions.

our results is only that the dispersion of information is present both at the investment and at the trading stage.

Also note the “liquidity shock” need not be taken too literally. Its presence captures the more general idea that when an agent makes an investment decision, be him a start-up entrepreneur or the manager of a public company, he cares about the market valuation of his investment at some point in the life of the project. A start-up entrepreneur may worry about the price at which he will be able to do a future IPO; a corporate manager may be concerned about the price at which the company will be able to issue new shares. In what follows, we interpret λ broadly as a measure of the sensitivity of the firms’ investment decisions to forecasts of future equity prices.⁹

Finally, note that there are no production spillovers and no direct payoff externalities: both the initial cost $k_i^2/2$ and the return on capital θk_i are independent of the investment decisions of other agents. The strategic complementarity among the entrepreneurs that will be identified in Section 3.3 originates purely in the informational externality generated by the information that aggregate investment conveys to financial markets.

2.1 Benchmark with no informational frictions

Before we proceed, it is useful to examine what happens when the dispersion of information vanishes at the time of trading in the financial market. That is, suppose that all the information that was dispersed in period 1 (namely, the signals $\{x_i\}_{i \in [0,1/2]}$ and y) becomes common knowledge in period 2. The fundamental θ is then perfectly revealed and the financial market clears if and only if $p = \theta$. It follows that the expected payoff of an entrepreneur who receives the signals x and y in period 1 is $\mathbb{E}[\tilde{\theta}|x, y]k - k^2/2$. This implies that his optimal investment is:

$$k(x, y) = \mathbb{E}[\tilde{\theta}|x, y] = \delta_0\mu + \delta_1x + \delta_2y,$$

where

$$\delta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_1 = \frac{\pi_x}{\pi_\theta + \pi_x + \pi_y}, \quad \delta_2 = \frac{\pi_y}{\pi_\theta + \pi_x + \pi_y}. \quad (1)$$

As mentioned in the introduction, this benchmark captures the idea that, in the absence of information externalities, whether investment is driven by the entrepreneurs’ expectations of profitability or by their expectations of the financial price for their installed capital is inconsequential. In either case, equilibrium investment is driven solely by first-order expectations regarding the fundamental and is independent of the intensity of the entrepreneurs’ concern about financial prices—measured by λ . This result does not require θ to be perfectly known in period 2 and it applies more generally,

⁹See Baker, Stein and Wurgler (2003) for complementary evidence that the sensitivity of corporate investment to stock prices is higher in sectors with tighter financing constraints (which here can be interpreted as higher λ).

as long as the asymmetry of information about θ vanishes in period 2. To clarify this point, consider an arbitrary information structure. Let $\mathcal{I}_{i,t}$ denote the information of agent i in period t . Impose that no agent has private information about θ in period 2 so that $\mathbb{E}[\tilde{\theta}|\mathcal{I}_{i,2}] = \mathbb{E}[\tilde{\theta}|\mathcal{I}_2]$ for all i . From market clearing, we then have that $p = \mathbb{E}[\tilde{\theta}|\mathcal{I}_2]$. From the law of iterated expectations, we then have that $\mathbb{E}[\tilde{p}|\mathcal{I}_{i,1}] = \mathbb{E}[\mathbb{E}[\tilde{\theta}|\mathcal{I}_2]|\mathcal{I}_{i,1}] = \mathbb{E}[\tilde{\theta}|\mathcal{I}_{i,1}]$ for all i . It follows that every entrepreneur chooses $k_i = \mathbb{E}[\tilde{\theta}|\mathcal{I}_{i,1}]$.

In this benchmark case, the responses of individual investment to the signals x and y , purely reflect their relative precision, captured by the coefficients δ_1 and δ_2 in (1). In turn, these responses determine the response of aggregate investment to the aggregate shocks θ and to the common noise shock ε , as $K = \delta_0 + (\delta_1 + \delta_2)\theta + \delta_2\varepsilon$. From now on, we will refer to this benchmark case as the case with “no informational frictions.”

3 Equilibrium

Consider now the environment with informational frictions described in Section 2. Individual investment is described by the function $k : \mathbb{R}^2 \rightarrow \mathbb{R}$, where $k(x, y)$ denotes the investment made by an entrepreneur with information (x, y) . Aggregate investment is then a function of the aggregate shocks θ and ε :

$$K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y|\theta, \varepsilon), \quad (2)$$

where $\Phi(x, y|\theta, \varepsilon)$ denotes the cumulative distribution function of x and y given θ and ε . Since traders observe aggregate investment and are risk neutral, the unique market-clearing price is $p = \mathbb{E}[\tilde{\theta}|K]$, where the latter denotes the expectation of θ given the observed level of investment K .¹⁰ Since K is determined by (θ, ε) , p is also a function of (θ, ε) .

Definition 1 A (symmetric rational expectation) equilibrium is an investment strategy $k(x, y)$ and a price function $p(\theta, \varepsilon)$ that satisfy the following conditions:

(i) for all (x, y) ,

$$k(x, y) \in \arg \max_k \mathbb{E} \left[(1 - \lambda) \tilde{\theta} k + \lambda p(\tilde{\theta}, \tilde{\varepsilon}) k - k^2/2 \mid x, y \right];$$

(ii) for all (θ, ε) ,

$$p(\theta, \varepsilon) = \mathbb{E}[\tilde{\theta} \mid K(\theta, \varepsilon)],$$

where $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y|\theta, \varepsilon)$.

¹⁰Since the price is only a function of K and K is publicly observed, the price itself does not reveal any additional information. Therefore, we can omit conditioning on p . The case where p conveys additional information is examined in Section 6.

Condition (i) requires that the entrepreneurs' investment strategy be individually rational, taking as given the equilibrium price function. Condition (ii) requires that the equilibrium price be consistent with market clearing and rational expectations on the traders' side, taking as given the strategy of the entrepreneurs.

As it is often the case in the literature, for tractability, we restrict attention to linear equilibria, so as to guarantee that the information structure remains Gaussian.¹¹

Definition 2 *A linear equilibrium is an equilibrium where the investment strategy $k(x, y)$ is linear. That is, there exist $\beta_0, \beta_1, \beta_2 \in \mathbb{R}$ such that, for all (x, y) ,*

$$k(x, y) = \beta_0 + \beta_1 x + \beta_2 y. \quad (3)$$

It is natural to focus on the case where investment is increasing in both the idiosyncratic signal x and the common signal y , in which β_1 and β_2 are positive. Below, we will first prove that an equilibrium with this property always exists and is unique for λ small enough. Next, we will examine the response of such an equilibrium to fundamental and common noise shocks. Lastly, we will show how certain properties of this equilibrium can also be conveniently understood by fixing the price function and then looking at the interaction among the entrepreneurs as a coordination game with strategic complementarities.

3.1 Equilibrium characterization

When individual investment follows (3), aggregate investment is given by

$$K(\theta, \varepsilon) = \beta_0 + (\beta_1 + \beta_2)\theta + \beta_2\varepsilon, \quad (4)$$

and is driven by two shocks: a *fundamental shock* θ and a *common noise shock* ε . The response of individual investment to the two signals x and y —captured by the coefficients β_1 and β_2 —determines the response of aggregate investment to the two shocks θ and ε : the response to θ is given by the sum $\beta_1 + \beta_2$, while the response to ε is simply equal to β_2 . Throughout the paper, we will be interested in characterizing the relative response of aggregate investment to these two shocks, that is, the ratio

$$\varphi \equiv \frac{\beta_2}{\beta_1 + \beta_2},$$

which is related one-to-one to the relative response of individual investment to the two signals x and y , that is, the ratio β_2/β_1 .

¹¹A linear equilibrium can be defined as an equilibrium where the price function is linear or as one where the investment strategy is linear. Since one implies the other, the two definitions are equivalent.

Next consider how the price in the financial market responds to the (rational) expectation that aggregate investment is given by (4). Note that, when $\beta_1 + \beta_2 \neq 0$, observing K is informationally equivalent to observing a signal

$$z \equiv \frac{K - \beta_0}{\beta_1 + \beta_2} = \theta + \varphi\varepsilon, \quad (5)$$

about θ which is a Gaussian with precision

$$\pi_z \equiv \pi_y/\varphi^2 \quad (6)$$

Bayesian updating then implies that the traders' expectation of θ given the observation of K is a weighted average of the prior and of the signal z :

$$\mathbb{E} [\tilde{\theta}|K] = \mathbb{E} [\tilde{\theta}|z] = \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu + \frac{\pi_z}{\pi_\theta + \pi_z} z. \quad (7)$$

Market clearing, together with rational expectations, then implies that the price of capital in the financial market must be given by

$$p(\theta, \varepsilon) = \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu + \frac{\pi_z}{\pi_\theta + \pi_z} (\theta + \varphi\varepsilon). \quad (8)$$

When aggregate investment responds positively to both fundamental and common noise shocks, so does the price; in fact, because traders cannot distinguish between increases in investment driven by fundamentals θ from those driven by common noise shocks ε , in equilibrium, the market-clearing price must necessarily respond (positively) to both θ and ε .

We are now ready to analyze the best response of an individual entrepreneur, who expects all other entrepreneurs to follow the strategy in (3) and, by implication, expects the price in the financial market to be given by (8). Optimality requires that

$$k(x, y) = \mathbb{E}[(1 - \lambda)\tilde{\theta} + \lambda p(\tilde{\theta}, \tilde{\varepsilon}) | x, y]. \quad (9)$$

Substituting the price function (8) into (9) and solving for the expectations, then gives investment as a linear function of x and y . Matching the coefficients in this function with the coefficients $(\beta_0, \beta_1, \beta_2)$ in (3), defines a simple fixed point problem. Finding a solution to this problem gives a linear equilibrium. This fixed point problem captures the essence of the two-way feedback between the real and the financial sector of our model. On the one hand, the responses β_1 and β_2 of individual investment to the two signals x and y determines the relative response $\varphi = \beta_2/(\beta_1 + \beta_2)$ of aggregate investment to the two shocks θ and ε and thus the precision $\pi_z \equiv \pi_y/\varphi^2$ of the signal that K transmits to the traders. This precision in turn determines the response of the price $p(\theta, \varepsilon)$

to the two aggregate shocks θ and ε . On the other hand, the price response to the two aggregate shocks determines the entrepreneurs' behavior, as it determines the stochastic properties of p and the entrepreneurs' ability to forecast it. As we shall see in a moment, this forecasting problem plays a crucial role for our positive results. Before turning to these results, we establish existence and uniqueness.

Proposition 1 *There always exists a linear equilibrium in which investment increases with both signals, that is, $\beta_1, \beta_2 > 0$. Furthermore, there exists a cutoff $\bar{\lambda}$, such that if $\lambda < \bar{\lambda}$ this is the unique linear equilibrium.*

3.2 Impact of fundamental and common noise shocks

Having characterized an equilibrium and established existence, we now turn to our main positive results. To this purpose, it is useful to rewrite the entrepreneurs' investment strategy as

$$k(x, y) = \mathbb{E} \left[\tilde{\theta} + \lambda \left(p(\tilde{\theta}, \tilde{\varepsilon}) - \tilde{\theta} \right) \mid x, y \right] = \mathbb{E} \left[\tilde{\theta} + \lambda \left(\mathbb{E} [\tilde{\theta} \mid \tilde{K}] - \tilde{\theta} \right) \mid x, y \right] \quad (10)$$

That entrepreneurs base their investment decisions both on the expectation of the fundamental θ of *and* on the expectation of the differential ($p - \theta$) should not surprise. This property holds in any environment where entrepreneurs have the option to sell their capital in a financial market. In particular, this property also applies to the benchmark model without informational frictions considered above. What distinguishes this environment from that benchmark is that the entrepreneurs possess information that permits them to predict not only the fundamental θ but also the traders' forecast error $\mathbb{E} [\tilde{\theta} \mid \tilde{K}] - \tilde{\theta}$. This error is driven by the common noise ε in the entrepreneurs' information which increases p independently of θ . It follows that, in equilibrium, each entrepreneur will adjust his response to the signals x and y so as to reflect his forecast not only θ , but also of the common error ε . To understand this, note that, when it comes to forecasting θ , what distinguishes the two sources of information x and y is simply their precisions π_x and π_y . When instead it comes to forecasting the common error ε , the signal y —which contains information on both θ and ε —becomes a better predictor than the idiosyncratic signal x —which only contains information on θ . This suggests that an individual entrepreneur who expects all other entrepreneurs to follow the strategy in (3) will give relatively more weight to the common signal y than what he would have done in the absence of informational frictions—that is, as compared to the case where his problem reduces to forecasting θ . This property is inherited in equilibrium, as shown in the following Proposition.

Proposition 2 *In any linear equilibrium in which investment responds positively to all sources of information (i.e. $\beta_1, \beta_2 > 0$), individual investment $k(x, y)$ responds less to the idiosyncratic signal*

x and more to the common signal *y*, relative to the benchmark with no informational frictions:

$$\beta_1 = \left(1 - \lambda\varphi \frac{\pi_y + \varphi\pi_\theta}{\pi_y + \varphi^2\pi_\theta}\right) \delta_1 \quad (11)$$

$$\beta_2 = \left(1 + \lambda\varphi \frac{\pi_x + (1 - \varphi)\pi_\theta}{\pi_y + \varphi^2\pi_\theta}\right) \delta_2 \quad (12)$$

where δ_1 and δ_2 are as defined in (1) and where $\varphi \equiv \beta_2/(\beta_1 + \beta_2)$. Furthermore, aggregate investment $K(\theta, \varepsilon)$ responds more to common noise shocks ε and less to fundamental shocks θ , again relative to the benchmark with no informational frictions:

$$\beta_2 > \delta_2 \text{ and } \beta_1 + \beta_2 < \delta_1 + \delta_2.$$

Proposition 2) illustrates the amplification mechanism generated by the interaction between real and financial decisions with dispersed information. In Section 6, we will show that this amplification mechanism is quite general. However, we will also see that the more robust positive prediction is about the *relative* responses to the two shocks $\varphi = \beta_2/(\beta_1 + \beta_2)$, rather than about the absolute responses β_2 and $(\beta_1 + \beta_2)$. Therefore, below we state the main positive prediction of the paper in the following form.

Corollary 1 (Main positive prediction) *Informational frictions increase the impact of common noise shocks relative to fundamental shocks.*

Put it slightly differently, informational frictions amplify *non-fundamental* volatility relative to *fundamental* volatility; that is, they reduce the R-square of a regression of aggregate investment on expected profits. Unlike in the case with no informational frictions, the entrepreneurs' concern for financial prices (captured by λ) is crucial in determining the equilibrium behavior of investment and asset prices. Absent this concern (if $\lambda = 0$) investment is only driven by expected profitability, as shown by (11) and (12), and there is no amplification result. As we will show below, increasing λ , that is, strengthening the entrepreneurs' concern about asset prices, increases the amplification effect.

3.3 (Information-driven) complementarity

The literature on market microstructure emphasizes that market participants may bias their investment decisions in an attempt to influence the beliefs of other participants (e.g., as in Kyle's seminal paper). This type of strategic behavior is not present here, as entrepreneurs are atomistic and the financial market is Walrasian. However, the entrepreneurs *as a group* can influence the beliefs of the financial market. This induces a bias in their behavior: they rely more on sources of

information with common noise since they know that their coordinated actions will affect market beliefs. To better capture this intuition, it is useful to look at the problem from a different angle.

Substituting (5) into (7), we can rewrite the traders' expectation of $\tilde{\theta}$ given K as

$$\mathbb{E} [\tilde{\theta}|K] = \gamma_0 + \gamma_1 K, \quad (13)$$

where

$$\gamma_0 \equiv \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu - \frac{\pi_z}{\pi_\theta + \pi_z} \frac{\beta_0}{\beta_1 + \beta_2} \quad \text{and} \quad \gamma_1 \equiv \frac{\pi_z}{\pi_\theta + \pi_z} \frac{1}{\beta_1 + \beta_2}. \quad (14)$$

The equilibrium price (8) can then be rewritten as

$$p(\theta, \varepsilon) = \gamma_0 + \gamma_1 K(\theta, \varepsilon), \quad (15)$$

Replacing (15) into condition (??), we can then rewrite the individual best response as

$$k(x, y) = \mathbb{E}[(1 - \alpha)\kappa(\tilde{\theta}) + \alpha K(\tilde{\theta}, \tilde{\varepsilon}) | x, y], \quad (16)$$

where $\alpha \equiv \lambda\gamma_1$ and $\kappa(\theta) \equiv ((1 - \lambda)\theta + \lambda\gamma_0) / (1 - \alpha)$. Condition (16) can be read as the best response function of an individual entrepreneur in the “reduced-form” coordination game among the entrepreneurs that is obtained by fixing the price function as in (15): it describes the optimal investment of an individual entrepreneur as a function of his expectation about the fundamental θ and aggregate investment K . As high aggregate investment is “good news” for profitability—a higher K raises the traders' expectation of θ —financial prices increase with K , that is, γ_1 is positive. This induces strategic complementarity in investment decisions. The coefficient α measures the degree of strategic complementarity in this game: the higher α , the higher the slope of the best response of individual investment to aggregate investment, that is, the higher the incentive of entrepreneurs to align their investment choices. The function $\kappa(\theta)$, on the other hand, captures the impact of the fundamental on the individual return of investment for given K , normalized by $1 - \alpha$.¹²

This best response function is formally equivalent to those arising in the linear-quadratic games in Morris and Shin (2002) and Angeletos and Pavan (2007a). However, there are two important differences. First, in those games the degree of strategic complementarity α is an exogenous parameter, while here it is endogenously determined as an integral part of the equilibrium. Second, in those games the degree of complementarity is independent of the information structure, while here it originates in the dispersion of information. In fact, the complementarity in our setup is

¹²This normalization serves two purposes. First, it identifies $\kappa(\theta)$ with the complete-information equilibrium level of investment in the game among the entrepreneurs, for a given price function. Second, it ensures that the unconditional mean of investment is given by $\mathbb{E}[k(x, y)] = \mathbb{E}[\kappa(\theta)]$.

solely due to the informational content of aggregate investment. How much information aggregate investment conveys about θ determines γ_1 , and γ_1 pins down the degree of complementarity α . Absent informational frictions, aggregate investment provides no information to the traders, prices are independent of K , and the complementarity disappears.

This interpretation provides an alternative way of deriving our main positive result, as illustrated in the following lemma. The lemma shows that the degree of strategic complementarity α fully captures the forces that determine the relative response of individual investment to idiosyncratic and common signals.

Lemma 1 *In any linear equilibrium,*

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha}. \quad (17)$$

Therefore, in any equilibrium with $\beta_1, \beta_2 > 0$, the sensitivity of individual investment to the common signal y relative to the idiosyncratic signal x is higher the higher the equilibrium degree of complementarity α .

Let us provide some intuition for this result. Expected aggregate investment is equal to

$$\mathbb{E}[K(\tilde{\theta}, \tilde{\varepsilon})|x, y] = \beta_0 + \beta_1 \mathbb{E}[\tilde{\theta}|x, y] + \beta_2 y.$$

The private signal x helps predict aggregate investment only through $\mathbb{E}[\tilde{\theta}|x, y]$, while the common signal y helps predict aggregate investment both through $\mathbb{E}[\tilde{\theta}|x, y]$ and directly through its effect on the term $\beta_2 y$. Therefore, relative to how much the two signals help predict the fundamental, the common signal y is a better predictor of aggregate investment than the private signal x . Recall that a higher α means a stronger incentive for an individual entrepreneur to align his investment choice with that of other entrepreneurs. It follows that when α is higher entrepreneurs find it optimal to rely more heavily on the common signal y relative to the private signal x , for it is the former that best helps them align their choice with the choice of others.

3.4 Comparative statics

We conclude this section by showing that increasing λ , that is, strengthening the entrepreneurs' concern about asset prices, increases the amplification effect of Corollary 1. Specifically, increasing λ increases $\varphi = \beta_2/(\beta_1 + \beta_2)$, the relative response of investment to common noise shocks relative to fundamental shocks. To conduct our comparative static exercise, we assume that, if there are multiple equilibria, the agents select the equilibrium with the smallest positive value of φ , that is, the equilibrium least responsive to common noise shocks.

Proposition 3 *In any linear equilibrium, the sensitivity of investment to common noise shocks relative to fundamental shocks is increasing in λ .*

Inspecting (12) shows how the information provided by aggregate investment to financial markets can magnify the effect of λ on the equilibrium response of investment to common noise shocks. To see this, start by considering an economy where entrepreneurs do not care about financial prices, i.e., $\lambda = 0$ and $\varphi = \delta_2 / (\delta_1 + \delta_2)$. As a simple partial equilibrium exercise, suppose a single entrepreneur with $\lambda > 0$ joins this economy. Since the entrepreneur is atomistic, equilibrium aggregate investment and prices are unchanged, and so is φ . Nonetheless, (12) shows that the stronger this entrepreneur's concern for asset prices, the more his individual behavior will be biased in favor of the common signal y . Next, suppose that all entrepreneurs start caring about financial market prices, that is, all entrepreneurs are now characterized by a positive λ . Relative to the partial equilibrium exercise above, now φ is endogenously determined, and the coefficient β_2 in (12) changes with both λ and φ . Two additional forces are at work in general equilibrium. First, as all entrepreneurs respond more to y , aggregate investment becomes more sensitive to the common shock ε . Second, the very fact that aggregate investment is more sensitive to ε makes K a noisier signal of profitability, so the financial market price become less responsive to it. The first effect tends to make the price more sensitive to ε , the second to make it less sensitive. When the price becomes more responsive to ε , this further increases the entrepreneurs' reliance on the common signal y . Therefore, when the first effect dominates, increasing λ has a magnified effect on β_2 , through the general equilibrium adjustment in the information structure.¹³ Numerical examples show that indeed the first effect can dominate, so that even a moderate concern for financial market prices can determine a sizeable amplification of expectational shocks.

The argument above highlights the potential destabilizing effect the two-way feedback between real and financial activity identified in the paper. With some parameter configurations, this feedback can be so strong that it generates multiple equilibria. In this case, different values of φ correspond to different equilibria. In equilibria with larger φ , the entrepreneurs' stronger relative response to y is self-sustained: as they respond relatively more to y , they make asset prices more sensitive to common noise shocks, which in turn justifies their stronger response to the common signal y .

Proposition 4 *There is an open set $S \subset \mathbb{R}^4$ such that if $(\lambda, \pi_\theta, \pi_x, \pi_y) \in S$ there are multiple linear equilibria.*

Notice that multiplicity arises here solely from an informational externality rather than from the more familiar payoff effects featured in coordination models of crises like, for example, Diamond and

¹³Formally, these two effects determine whether the expression $\varphi(\pi_x + (1 - \varphi)\pi_\theta) / (\pi_y + \varphi^2\pi_\theta)$ in (12) is increasing or decreasing in φ .

Dybvig (1983) and Obstfeld (1996). Clearly, multiplicity could induce additional non-fundamental volatility in both real investment and financial prices. However, the possibility of multiple equilibria is not central to our analysis, so for the rest of the paper we will focus on the case where λ is small so that the equilibrium is unique.

4 Constrained efficiency

The analysis so far has focused on the positive properties of the equilibrium. We now study its normative properties by examining whether there is an allocation that, given the underlying information structure, leads to higher welfare.

The question of interest here is whether society can do better, relative to equilibrium, by having the agents use their available information in a different way—not whether society can do better by giving the agents more information. We thus adopt the same constrained efficiency concept as in Angeletos and Pavan (2007a,b): we consider the allocation that maximizes ex-ante welfare subject to the sole constraint that the choice of each agent must depend only on the primitive information available to that agent. In other words, we let the planner dictate how agents use their available information, but we do not let the planner transfer information from one agent to another. In so doing, we momentarily disregard incentive constraints; in the next section we will identify policies that implement the efficient allocation as an equilibrium.

Note that the payments in the financial market represent pure transfers between the entrepreneurs and the traders and therefore do not affect ex-ante utility.¹⁴ We can then focus on the investment strategy and define the efficient allocation as follows.

Definition 3 *The efficient allocation is a strategy $k(x, y)$ that maximizes ex-ante utility*

$$\mathbb{E}u = \int \left\{ \frac{1}{2} \int \left[(1 - \lambda) \theta k(x, y) - \frac{1}{2} k(x, y)^2 \right] d\Phi(x, y|\theta, \varepsilon) + \frac{1}{2} \theta \lambda K(\theta, \varepsilon) \right\} d\Psi(\theta, \varepsilon) \quad (18)$$

with $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y|\theta, \varepsilon)$.

Condition (18) gives ex-ante utility for an arbitrary strategy. The first term is the payoff of an entrepreneur with information (x, y) ; the second term is the payoff of a trader when aggregate

¹⁴By ex-ante utility, we mean prior to the realization of any random variable, including those that determine whether an agent will be an entrepreneur or a trader. However, note that, because utility is transferable, any strategy $k(x, y)$ that maximizes ex-ante utility also maximizes any weighted average of the expected utility of an entrepreneur and of a trader. By the same token, any strategy $k(x, y)$ that improves upon the equilibrium in terms of ex-ante utility can yield a Pareto improvement. It suffices, for example, that the planner dictates $k(x, y)$ at date 1 to all entrepreneurs and then lets entrepreneurs and traders trade at the price $p(\theta, \varepsilon) = \mathbb{E}[\theta|K(\theta, \varepsilon)]$, where $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y|\theta, \varepsilon)$. This way, the traders' expected utility remains equal to zero, as in the competitive equilibrium, and the increase in social surplus entirely goes to the entrepreneurs.

investment is $K(\theta, \varepsilon)$; finally Ψ denotes the cumulative distribution function of the joint distribution of (θ, ε) . Note that the transfer of capital from the entrepreneurs that are hit by the liquidity shock to the traders does not affect the return to investment. It follows that (18) can be rewritten compactly as¹⁵

$$\mathbb{E}u = \frac{1}{2}\mathbb{E}[V(k(\tilde{x}, \tilde{y}), \tilde{\theta})] = \frac{1}{2}\mathbb{E}[\mathbb{E}[V(k(\tilde{x}, \tilde{y}), \tilde{\theta}) | \tilde{x}, \tilde{y}]],$$

where $V(k, \theta) \equiv \theta k - (1/2)k^2$. From the society's viewpoint, λ is irrelevant and it is as if the entrepreneurs' payoff was $V(k, \theta)$. It follows immediately that a strategy $k(x, y)$ is efficient if and only if it maximizes $\mathbb{E}[V(\tilde{k}, \tilde{\theta}) | x, y]$ for almost all x and y , leading to the following proposition.

Proposition 5 *The efficient investment strategy is given by*

$$k(x, y) = \mathbb{E}[\tilde{\theta} | x, y] = \delta_0\mu + \delta_1x + \delta_2y,$$

where δ_0 , δ_1 and δ_2 are given in (1).

Note that the efficient strategy coincides with the equilibrium strategy in the benchmark with no informational frictions. Therefore, our positive results admit a normative interpretation.¹⁶

Corollary 2 (Main normative prediction) *In the presence of informational frictions, the impact of common noise shocks relative to fundamental shocks is inefficiently high.*

5 Policy implications

Having identified a potential source of inefficiency, we now analyze the effect of different policies. First, we consider interventions “during the fact,” by which we mean interventions in the financial market at $t = 2$, when uncertainty about θ has not been resolved yet. Next, we consider interventions “after the fact,” by which we mean policies contingent on information that becomes public at $t = 3$, after uncertainty about θ has been resolved. In both cases, we give the government no informational advantage over the private sector. At the end of this section, we consider situations where the government can also directly affect the sources of information available to the agents.

5.1 Interventions “during the fact”: price stabilization

We start by considering policies aimed at reducing asset price volatility. In particular, suppose the government imposes a proportional tax τ on financial trades at date 2. This tax is paid by the

¹⁵ Just substitute the expression for $K(\theta, \varepsilon)$ in (18).

¹⁶ Corollary 2 presumes that the equilibrium is unique. When there are multiple equilibria, the result holds for any equilibrium in which $\beta_1, \beta_2 > 0$. Since the efficient allocation satisfies $\beta_1, \beta_2 > 0$, this also ensures that no equilibrium is efficient.

Figure 1: The impact of price-stabilization policies.

traders who purchase capital from entrepreneurs in that period. The tax rate τ is contingent on the price, which is publicly observed. For simplicity, it takes the linear form:

$$\tau(p) = \tau_0 + \tau_1 p, \quad (19)$$

where (τ_0, τ_1) are scalars. Tax revenues are rebated as a lump sum transfer.

The equilibrium price in the financial market is now given by $p = \mathbb{E}[\tilde{\theta}|K] - \tau(p)$, which yields

$$p = \frac{1}{1 + \tau_1} \left(\mathbb{E}[\tilde{\theta}|K] - \tau_0 \right). \quad (20)$$

If the tax is procyclical, i.e., if $\tau_1 > 0$, its effect is to dampen the response of asset prices to the traders' expectation of θ , and thereby to the information contained in aggregate investment. In turn, this dampens the price response to the common noise ε and reduces the relative bias towards the common signal y in the entrepreneurs' best response (9). At the aggregate level, this tends to make investment relatively less responsive to common noise shocks. However, notice that there is a countervailing effect: as entrepreneurs assign less weight to y , K becomes a more precise signal of fundamentals, making prices more responsive to K and thus to ε . This tends to make individual investment relatively more responsive to y and aggregate investment relatively more responsive to ε . However, the following proposition shows that the first effect always dominates and the total effect of a procyclical tax is to reduce the sensitivity of investment to noise shocks.

Proposition 6 *If the linear equilibrium is unique or if one selects the linear equilibrium with the lowest φ , the sensitivity of investment to common noise shocks relative to fundamental shocks is decreasing in τ_1 .*

Increasing the procyclicality of the tax rate, that is, increasing τ_1 , thus reduces the relative impact of common noise shocks. However, by reducing the overall sensitivity of prices to all sources of variation in investment, a higher τ_1 also reduces the impact of fundamental shocks. As argued in the previous section, in the absence of policy intervention, investment is excessively sensitive to common noise shocks and insufficiently sensitive to fundamental shocks. It follows that the welfare consequences of the tax are ambiguous: reducing the impact of common noise shocks improves efficiency, reducing the impact of fundamental shocks has the opposite effect.

These intuitions are illustrated in Figure 1 where for each value of τ_1 , the value of τ_0 is chosen optimally to maximize welfare. The top panel depicts the difference in welfare under the stabilization policy considered here and under the constrained efficient allocation; the bottom panels

depict the sensitivity to common noise shocks β_2 and to fundamental shocks $\beta_1 + \beta_2$.¹⁷ The figure is drawn for a baseline set of parameters: $\pi_\theta = \pi_x = \pi_y = 1$ and $\lambda = 0.5$. However, its qualitative features are robust across a wide set of parametrizations. In particular, we have randomly drawn 10,000 parameter vectors $(\pi_\theta, \pi_x, \pi_y), \lambda$ from $\mathbb{R}_+^3 \times (0, 1)$. For each such vector, we have found that the optimal τ_1 is positive and it induces a lower β_2 and a lower $\beta_1 + \beta_2$ as compared to the equilibrium without policy, reflecting the trade-off discussed above.

These numerical results, which span the entire parameter space, suggest that the optimal policy always involves a strictly positive degree of price stabilization—i.e., $\tau_1 > 0$, even though we do not have an analytical proof for this claim. However, full price stabilization—i.e., $\tau_1 \rightarrow \infty$ and $-\tau_0/(1 + \tau_1) \rightarrow \bar{p}$ —is never optimal. In this limit case, equilibrium investment reduces to $k(x, y) = (1 - \lambda) \mathbb{E}[\tilde{\theta}|x, y] + \lambda \bar{p}$. As the price ceases to respond to K , uncertainty disappears, and the entrepreneurs are no longer concerned about forecasting the price movements driven by the common noise. By implication, the relative sensitivity of investment to common noise shocks $\beta_2/(\beta_1 + \beta_2)$ is at its efficient level. However, the levels of β_1 and β_2 are not efficient, and the sensitivity of investment to both shocks is λ times lower than at the efficient allocation. At this point, a marginal increase in the relative sensitivity implies only a second-order welfare loss, while a marginal increase in the overall sensitivity implies a first-order welfare gain. It follows that it is never optimal to fully stabilize the price.

Proposition 7 *A tax that stabilizes prices can increase welfare. However, a tax that completely eliminates price volatility is never optimal.*

5.2 Interventions “after the fact”: corrective taxation

Suppose now that the government imposes a proportional tax τ on capital holdings in period 3. The tax is paid by all agents holding capital in that period: entrepreneurs not hit by the liquidity shock and traders who purchased capital in period 2.¹⁸ The advantage of introducing a tax in period 3 is that the tax rate τ can now be made contingent on all the information publicly available in that period, including both the price p in the financial market and the realized fundamental θ . We focus on linear tax schemes of the form

$$\tau(p, \theta) = \tau_0 + \tau_1 p + \tau_2 \theta, \quad (21)$$

¹⁷Note that τ_0 affects the unconditional average of $k(x, y)$, but has no effect on the sensitivity of investment to the signals x and y , i.e., on β_1 and β_2 . We henceforth concentrate on τ_1 .

¹⁸That these taxes are paid by all capital holders is not essential. What is important is that they are paid by the entrepreneurs with positive probability: in fact, it is easy to see that a tax that is paid only by the traders never improves upon the type of policies considered above, even if it is made contingent also on information about θ available at $t = 3$.

where (τ_0, τ_1, τ_2) are scalars. Tax revenues are rebated in a lump-sum fashion.

Entrepreneurs expect to pay the tax with probability $1 - \lambda$ in period 3. Therefore, their best response (10) now takes the form

$$k(x, y) = \mathbb{E}[\tilde{\theta} | x, y] + \lambda \mathbb{E}[\tilde{p} - \tilde{\theta} | x, y] - (1 - \lambda) \mathbb{E}[\tau(\tilde{p}, \tilde{\theta}) | x, y]. \quad (22)$$

Consider the linear tax

$$\tau(p, \theta) = \frac{\lambda}{1 - \lambda} (p - \theta),$$

that is, set $\tau_0 = 0$, $\tau_1 = \lambda / (1 - \lambda)$ and $\tau_2 = -\lambda / (1 - \lambda)$. It is immediate from (22) that this tax implements an equilibrium where $k(x, y) = \mathbb{E}[\tilde{\theta} | x, y]$, achieving efficiency. Under the optimal tax scheme the last two terms in (22) cancel out: the tax exactly offsets the entrepreneurs' response to the expected pricing error on the financial market. Since, as argued in Section 3, this response is behind the entrepreneurs' relative bias in favor of the common signal y , and, as argued in Section 4, this bias is the source of inefficiency, the tax is sufficient to achieve an efficient equilibrium. We have then proved the following proposition.

Proposition 8 *There exists a linear tax scheme that implements the efficient allocation as an equilibrium.*

Under the optimal tax scheme the equilibrium price is simply $p = \mathbb{E}[\tilde{\theta}|K]$ and the traders' expected tax, at the time of trading in the financial market, is zero, i.e. $\mathbb{E}[\tau(\tilde{p}, \tilde{\theta})|K] = 0$. Therefore, the tax can be rewritten as $\tau = (\lambda/1 - \lambda) (\mathbb{E}[\tilde{\theta}|K] - \theta)$. Whenever the government recognizes in period 3 that the financial market was overestimating the returns of capital in period 2, it hits the capital holders with a higher tax rate. The objective of this policy is not to punish financial market participants for their mistakes, but to provide the right incentives to the entrepreneurs in charge of the initial investment decision, who have ex ante private information on the financial market's pricing error.

Relative to the simple stabilization policy discussed in Section 5.1, now the government has an extra instrument available: the elasticity τ_2 of the tax to the realized fundamental. With two instruments available, the government can thus set τ_1 to induce the optimal relative sensitivity to common noise shocks $\beta_2/(\beta_1 + \beta_2)$ while, at the same time, adjusting τ_2 to obtain the optimal absolute sensitivities.¹⁹

Although this result does not require any informational advantage on the government's side, it assumes that the government observes perfectly the fundamental θ and the agents' capital holdings

¹⁹The optimal τ_0 is then chosen to induce the optimal level of unconditional average investment. Similar tax schemes implement the efficient investment strategy in all the extensions considered in Section 6.

at the time taxes are collected. However, the result extends to situations where these quantities are imperfectly observed. In particular, suppose that in stage 3 the government only observes $\theta^o = \theta + \epsilon$ and $q_i^o = q_i + \eta_i$ for each i , where q_i is the capital holding of agent i and ϵ and η_i are measurement errors, possibly correlated with one another, but independent of θ and of the agents' information in period 2. Consider a proportional tax on the observed capital holding q_i^o of the form: $\tau(p, \theta^o) = \tau_0 + \tau_1 p + \tau_2 \theta^o$. It is then easy to check that there continues to exist a unique set of coefficients (τ_0, τ_1, τ_2) that implement the efficient allocation as an equilibrium and that these coefficients continue to satisfy $\tau_1 < 0 < \tau_2$.

To recap, the key insight here is that the government can use the contingency of the tax rate on public information revealed at stage 3 to achieve efficiency in the decentralized use of information in stage 1. Although this information becomes available only after all investment decisions are sunk, by promising a specific policy response to this information the government can manipulate how entrepreneurs' use their available sources of information when making their investment decisions, even if it cannot directly monitor these sources of information.²⁰

5.3 Optimal release of information

We now turn to policies that affect the information available to the agents. This seems relevant given the role of the government in collecting (and releasing) macroeconomic data.

To capture this role, suppose that in stage 2 traders can only observe average investment with noise, that is, they observe

$$K^o = K + \eta,$$

where η is aggregate measurement error, which is a random variable, independent of all other shocks, with mean zero and variance $1/\pi_\eta$.²¹ Suppose now that the government can affect the precision π_η of the macroeconomic data available to financial traders. By changing π_η the government determines the weight that traders assign to \tilde{K} when estimating future profitability. This is another channel by which the government is able to affect the degree of strategic complementarity in investment decisions.

Indeed, the choice of π_η is formally equivalent to the choice of τ_1 in the setup with a tax on financial transactions (Section 5.1). For each value π_η of the precision of the signal about aggregate activity, there is a value τ_1 of the tax elasticity that induces the same equilibrium strategy, and vice versa. To see this, note that in any linear equilibrium, the observation of $\tilde{K} = \beta_0 + (\beta_1 + \beta_2)\theta + \beta_2\varepsilon + \eta$

²⁰These intuitions, and the implementation result in Proposition 8, build on Angeletos and Pavan (2007b), which considers optimal policy within a general class of economies with dispersed information on correlated values. See also Lorenzoni (2009) for an application of this approach to optimal monetary policy.

²¹The equilibrium characterization for this case is a straightforward extension of the baseline case.

is informationally equivalent to the observation of a signal

$$z \equiv \frac{\tilde{K} - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2} \varepsilon + \frac{1}{\beta_1 + \beta_2} \eta,$$

with precision

$$\pi_z = \left(\left(\frac{\beta_2}{\beta_1 + \beta_2} \right)^2 \pi_y^{-1} + \left(\frac{1}{\beta_1 + \beta_2} \right)^2 \pi_\eta^{-1} \right)^{-1}.$$

The equilibrium price is then given by $p(\theta, y, \eta) = \gamma_0 + \gamma_1 [K(\theta, y) + \eta]$, with γ_0 and γ_1 given by (14), and hence the degree of strategic complementarity remains equal to $\alpha \equiv \lambda \gamma_1$, as in the baseline model. By changing the value of π_η , the government can then directly manipulate γ_1 and thus the degree of strategic complementarity perceived by the entrepreneurs.²²

We conclude that the choice of π_η is subject to the same trade-offs emphasized for the choice of τ_1 : decreasing π_η reduces the relative response of investment to expectational shocks, but it also reduces its response to fundamental shocks. The results of Section 5.1 then imply that an intermediate degree of release of macroeconomic data may be optimal even when the cost of collecting such data is zero.²³

Finally, one could consider policies which affect directly the agents' information regarding the fundamental θ . In particular, the government can collect some information about θ in period 1 and decide whether to disclose this information to the entrepreneurs, or to both the entrepreneurs and the traders. In the first case, the policy corresponds to an increase in the precision of the signal y in the baseline model. Although entrepreneurs have a more precise estimate of the fundamental, this information is not shared with the traders. Therefore, this policy could exacerbate the asymmetry of information and could magnify the feed-back effects between investment and asset prices, with possible negative consequences on social welfare.

In the second case, instead, the policy corresponds to an increase in the precision of the common prior in the baseline model. This policy is socially beneficial for two reasons: first, it improves the quality of the information available to the entrepreneurs and hence it permits them to better align their decisions to the fundamental. Second, it reduces the reliance of financial markets on the endogenous signal K in their estimate of the fundamental. This second effect tends to reduce the degree of strategic complementarity in investment decisions, and hence also the discrepancy between equilibrium and efficient allocations. Both effects then contribute to higher welfare.

²²See the Supplementary Material for the proof of this claim.

²³Note, however, that this holds only as long as the equilibrium is inefficient. If, instead, the policies considered in Section 5.2 are in place, guaranteeing that the equilibrium is efficient, then a higher π_η is always welfare improving.

6 Extensions

Our analysis has identified a mechanism through which the dispersion of information about new technologies, markets, or financial instruments, induces complementarity in real investment choices, amplification of non-fundamental disturbances, and inefficiency of market outcomes, all at once. In the baseline model, we have made a number of assumptions to illustrate this mechanism in the simplest possible way. In particular, we have assumed that the traders' demand for installed capital is perfectly elastic, that entrepreneurs who are not hit by the liquidity shock do not trade in the financial market, and that the traders' valuation of the asset coincides with that of the entrepreneurs. In this section, we relax each of these assumptions. The purpose of the exercise is twofold: to enrich the analysis by incorporating some of the elements missing in the baseline model and at the same time showing how the key insights extend to these more general settings.

We first extend the model to allow for the traders' demand for capital to be downward sloping. This extension is interesting because it introduces a potential source of strategic substitutability in the entrepreneurs' investment decisions: when aggregate investment is higher, the supply of installed capital in the financial market is also higher, putting a downward pressure on asset prices and lowering the ex-ante incentive to invest.

In a second extension, we allow entrepreneurs not hit by the liquidity shock to participate in the financial market. This extension is interesting for two reasons: first, it allows for some of the entrepreneurs' information to be aggregated in the financial market; second, it introduces a non-trivial allocative role for prices.

Although some interesting differences arise, the key positive and normative predictions of the paper (Corollaries 1 and 2) remain valid in both extensions: as long as the dispersion of information does not completely vanish in the financial market, the signaling effect of aggregate investment continues to be the source of amplification and inefficiency in the response of the equilibrium to common sources of noise.

Finally, we consider a variant that introduces shocks to the financial-market valuation of the installed capital. This variant brings the paper closer to the recent literature on “mispricing” and “bubbly” asset prices. It also helps clarify that the details of the information structure assumed in the baseline model are not essential: any source of common noise in the information that aggregate investment conveys about the underlying fundamentals opens the door to amplification and inefficiency.

6.1 The supply-side effect of capital: a source of strategic substitutability

We modify the benchmark model as follows. The net payoff of trader i , who buys q_i units of capital at the price p , is now given by

$$u_i = (\theta - p) q_i - \frac{1}{2\phi} q_i^2, \quad (23)$$

where ϕ is a positive scalar. The difference with the benchmark model is the presence of the last term in (23), which represents a transaction cost associated to the purchase of q_i units of capital. A convex transaction cost ensures a finite price elasticity for the traders' demand, which is now given by $q(p, K) = \phi (\mathbb{E}[\tilde{\theta}|K] - p)$. The parameter ϕ captures the price elasticity of this demand function; the baseline model corresponds to the special case where the demand is infinitely elastic, i.e. $\phi \rightarrow \infty$.²⁴ As in the baseline model, in any linear equilibrium, the traders' expectation of $\tilde{\theta}$ given K continues to be given by $\mathbb{E}[\tilde{\theta}|K] = \gamma_0 + \gamma_1 K$, with the coefficients γ_0 and γ_1 as in (14). However, unlike in the benchmark model, the equilibrium price no longer coincides with $\mathbb{E}[\tilde{\theta}|K]$. Market clearing now requires that $q(p, K) = \lambda K$, so that the equilibrium price is now given by

$$p = \mathbb{E}[\tilde{\theta}|K] - \frac{\lambda}{\phi} K = \gamma_0 + \left(\gamma_1 - \frac{\lambda}{\phi} \right) K. \quad (24)$$

It follows that aggregate investment has two opposing effects on the price of installed capital, p . On the one hand, it raises the traders' expectation of $\tilde{\theta}$, thereby pushing the price up, as in the baseline model. On the other hand, it raises the supply of capital, thereby pulling the price down. The strength of these two effects determines whether investment choices are strategic complements or substitutes.

Proposition 9 (i) *In any linear equilibrium, the investment strategy satisfies*

$$k(x, y) = \mathbb{E} \left[(1 - \alpha) \kappa(\tilde{\theta}) + \alpha K(\tilde{\theta}, \tilde{\varepsilon}) \mid x, y \right], \quad (25)$$

with $\alpha \equiv \lambda\gamma_1 - \lambda^2/\phi$ and $\kappa(\theta) \equiv \frac{(1-\lambda)\theta + \lambda\gamma_0}{1 - \lambda\gamma_1 + \lambda^2/\phi}$.

(ii) *When λ is small enough there exists a unique linear equilibrium and is such that investment increases with θ , and γ_1 is positive.*

The degree of complementarity α in investment decisions is now the sum of two terms. The first term $\lambda\gamma_1$ captures the, by now familiar, informational effect of investment on asset prices documented in the baseline model. The second term, $-\lambda^2/\phi$, captures the simple supply-side effect

²⁴A more familiar way of introducing a finitely elastic demand function is to assume risk aversion. The alternative we use here captures the same key positive and normative properties—namely, demands are finitely elastic and individual payoffs are concave in own portfolio positions—but has the advantage of keeping the analysis tractable by making the elasticity of demands invariant to the level of uncertainty.

that emerges once the demand for the asset is finitely elastic. If either the information contained in aggregate investment is sufficiently poor (low γ_1) or the price elasticity of demand is sufficiently low (low ϕ), investment choices become strategic substitutes ($\alpha < 0$). However, the question of interest here is not whether investment choices are strategic complements or substitutes, but how the positive and normative properties of the equilibrium are affected by the dispersion of information. In this respect, the implications that emerge in this extension are essentially the same as in the baseline model.

First, consider the positive properties of the equilibrium. Lemma 1 clearly extends to the modified model: equilibrium investment satisfies $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$ with

$$\frac{\beta_2}{\beta_1} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha}. \quad (26)$$

Provided that investment increases with both signals, then aggregate investment is necessarily good news for θ (i.e., $\gamma_1 > 0$), in which case Proposition 9 implies that $\alpha > -\lambda^2/\phi$. In contrast, when there are no informational frictions, the equilibrium price is $p = \theta - (\lambda/\phi)K$. The private return to investment is then equal to $\theta + \lambda(p - \theta) = \theta - (\lambda^2/\phi)K$ and $\alpha = -\lambda^2/\phi$. As in the baseline model, the dispersion of information thus increases the value of α . This in turn amplifies the impact of common noise relative to that of fundamental shocks, even if α happens to be negative. To see this more clearly, note that when the dispersion of information does not vanish at the trading stage, the discrepancy between the market-clearing price p and the fundamental θ (i.e., the price error) is given by $p - \theta = [\mathbb{E}[\tilde{\theta}|K] - \theta] - (\lambda/\phi)K$. The private return to investment is then given by $\theta + \lambda(p - \theta) = \theta + \lambda[\mathbb{E}[\tilde{\theta}|K] - \theta] - (\lambda^2/\phi)K$. The difference relative to the case without information frictions is thus the term $[\mathbb{E}[\tilde{\theta}|K] - \theta]$ which captures the error in the traders' forecast about θ . The fact that y is a better predictor of $[\mathbb{E}[\tilde{\theta}|K] - \theta]$ than x then induces each entrepreneur to respond relatively more to y than to x when making his investment decision. As in the baseline model, the dispersion of information thus increases the impact of common noise shocks relative to fundamental shocks. We conclude that Corollary 1, which summarizes the key positive predictions of the model, extends to this environment.

Next, consider the normative properties. Because of the convexity of the transaction costs, it is necessary for efficiency that all traders take the same position in the financial market: $q_i = \lambda K$ for all $i \in (1/2, 1]$. Ex-ante utility then takes the form

$$\begin{aligned} \mathbb{E}u &= \int \left\{ \frac{1}{2} \int [-\frac{1}{2}k(x, y)^2 + (1 - \lambda)\theta k(x, y)] d\Phi(x, y|\theta, \varepsilon) + \frac{1}{2} \left[-\frac{1}{2\phi}[\lambda K(\theta, \varepsilon)]^2 + \theta \lambda K(\theta, \varepsilon) \right] \right\} d\Psi(\theta|2\bar{\tau}) \\ &= \frac{1}{2} \mathbb{E} \left\{ -\frac{\tilde{k}^2}{2} + \tilde{\theta}\tilde{k} - \frac{1}{2\phi}[\lambda\tilde{K}]^2 \right\} \end{aligned}$$

and the efficient investment strategy is the function $k(x, y)$ that maximizes (27).

Proposition 10 *The efficient investment strategy is the unique linear solution to*

$$k(x, y) = \mathbb{E} \left[(1 - \alpha^*) \kappa^* (\tilde{\theta}) + \alpha^* K(\tilde{\theta}, \tilde{\varepsilon}) \mid x, y \right], \quad (28)$$

where $\alpha^* \equiv -\lambda^2/\phi < 0$, $\kappa^*(\theta) \equiv \theta / (1 + \lambda^2/\phi)$, and $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y | \theta, \varepsilon)$.

To understand this result, note that the social return to investment is now given by $(1 - \lambda)\theta + \lambda(\theta - \lambda K/\phi) = \theta - \lambda^2 K/\phi$. The new term, relative to the benchmark model, is $-\lambda^2 K/\phi$ and it reflects the cost associated with transferring λK units of the asset from the entrepreneurs to the traders. If information were complete, efficiency would require that each agent equates his marginal cost of investing to the social return to investment, which would give $k = \theta - \lambda^2 K/\phi$.²⁵ The analogue under incomplete information is that each agent equates the marginal cost to the expected social return:

$$k(x, y) = \mathbb{E} \left[\tilde{\theta} - (\lambda^2/\phi) K(\tilde{\theta}, \tilde{\varepsilon}) \mid x, y \right]. \quad (29)$$

Rearranging this condition gives (28).

The key finding here is that the introduction of downward sloping demands has a symmetric effect on the private and social returns to investment. This is simply because the negative pecuniary externality caused by the higher supply of capital perfectly reflects the social cost associated with having traders absorb this additional capital. As a result, it is only the informational effect that generates a discrepancy between the private and the social return to investment and, by implication, between the equilibrium and the efficient allocation.

As in the benchmark model, this discrepancy manifests itself in the response of equilibrium to common noise and fundamental shocks. Indeed, while equilibrium investment satisfies (26), efficient investment satisfies $k(x, y) = \beta_0^* + \beta_1^* x + \beta_2^* y$ with

$$\frac{\beta_2^*}{\beta_1^*} = \frac{\pi_y}{\pi_x} \frac{1}{1 - \alpha^*}.$$

Because in any equilibrium in which $\beta_1, \beta_2 > 0$ the complementarity in investment decisions satisfies $\alpha^* < \alpha < 1$, the relative sensitivity of the equilibrium strategy to common noise is inefficiently high. We conclude that Corollary 2, which summarizes the key normative predictions of the model, continues to hold.

As common in competitive environments, there are other forms of pecuniary externalities that could induce strategic substitutability in the entrepreneurs' investment decisions, even with a perfectly elastic demand for capital. For example, suppose that, in order to complete their investment,

²⁵Note that under full information the optimal level of investment would be equal to $\kappa^*(\theta)$.

entrepreneurs need to purchase certain inputs whose aggregate supply is imperfectly elastic (e.g., labor or land). Higher aggregate investment then implies higher aggregate demand for these inputs, and hence higher input prices and lower entrepreneurial returns, once again inducing strategic substitutability in the entrepreneurs' investment choices. However, such pecuniary externalities do not, on their own, cause discrepancies between private and social returns. Indeed, it is easy to construct variants of the model that capture such sources of strategic substitutability while retaining the property that the informational effect of aggregate investment is the sole source of amplification and inefficiency, as in the example analyzed here.

6.2 Information aggregation through prices

The analysis so far has imposed that the entrepreneurs who are not hit by the liquidity shock can not access the financial market. Apart from being unrealistic, this assumption rules out the possibility that the price in the financial market aggregates, at least partly, the information that is dispersed among the entrepreneurs. To address this possibility, in this section we extend the analysis by allowing entrepreneurs not hit by the liquidity shock to participate in the financial market.

To guarantee downward sloping demands, we assume that traders and entrepreneurs alike incur a transaction cost for trading in the financial market of the same type considered in the previous section.²⁶ Thus, the payoff of an entrepreneur i who is not hit by a liquidity shock, has invested k_i units in the first period, and trades q_i units in the second period, is given by

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \frac{1}{2\phi}q_i^2 + \theta(k_i + q_i),$$

while the payoff of a trader i is given by (23), as in the previous section.

Because the observation of K in the second period perfectly reveals θ to every entrepreneur,²⁷ their demand for the asset in the second period reduces to $q_E = \phi(\theta - p)$. The demand of the traders, on the other hand, is given by $q_T = \phi(\mathbb{E}[\tilde{\theta}|K, p] - p)$. Note that traders now form their expectation of θ based on K and on the information revealed by the equilibrium price p .²⁸ Because the aggregate demand for the asset is $\frac{1}{2}(1 - \lambda)q_E + \frac{1}{2}q_T$ and the aggregate supply is $\frac{1}{2}\lambda K$, market clearing implies

$$p = \frac{1}{2-\lambda}\mathbb{E}[\tilde{\theta}|K, p] + \frac{1-\lambda}{2-\lambda}\theta - \frac{1}{\phi(2-\lambda)}\lambda K.$$

²⁶We assume that the entrepreneurs hit by the liquidity shock do not pay the transaction cost for the units of the asset that they *have to* sell in the second period; this simplification has no impact on the results.

²⁷This presumes that entrepreneurs use their private information when deciding how much to invest (i.e. $\beta_1 \neq 0$), which is indeed true in equilibrium.

²⁸In the baseline model, as well as in the extension examined in the previous section, we did not condition on the information revealed by the equilibrium price simply because all agents trading voluntary in the financial market had symmetric information.

It follows that the joint observation of K and p perfectly reveals θ to the traders as well. The asymmetry of information thus vanishes and the equilibrium price satisfies $p = \theta - \frac{1}{\phi(2-\lambda)}\lambda K$.

Next note that the private return to investment continues to be given by $\theta + \lambda[p - \theta]$ and is thus equal to $\theta - \frac{1}{\phi(2-\lambda)}\lambda^2 K$. As in the previous section, this is just the social return to investment, adjusted for the fact that the total capital of the entrepreneurs hit by the liquidity shock ($\lambda K/2$) is now equally distributed among the traders and the entrepreneurs not hit by the liquidity shock. Because the private return to investment coincides with the social return, the equilibrium is efficient.

The previous result however hinges on the equilibrium price perfectly revealing θ . To make this clear, in the subsequent analysis we introduce an additional source of noise, which prevents prices from being perfectly revealing. Assume that the cost of trading for the entrepreneurs is subject to a shock ω , that is revealed to each entrepreneur at the time of trading but which is not observed by the traders. In particular, the payoff of an entrepreneur not hit by the liquidity shock is now given by

$$u_i = -\frac{1}{2}k_i^2 - pq_i - \omega q_i - \frac{1}{2\phi}q_i^2 + \theta(k_i + q_i),$$

where ω is assumed to be independent of all other random variables, with $\mathbb{E}[\omega] = 0$ and $Var[\omega] = \sigma_\omega^2 \equiv \pi_\omega^{-1}$.

In what follows, we look at linear rational expectations equilibria; we continue to denote the investment strategy by $k(x, y)$ and we denote by $p(\theta, \varepsilon, \omega)$ the equilibrium price. Because the observation of aggregate investment in the second period continues to reveal θ to the entrepreneurs (but not to the traders), asset demands are given by $q_E = \phi(\theta - \omega - p)$ for the entrepreneurs and $q_T = \phi(\mathbb{E}[\tilde{\theta}|K, p] - p)$ for the traders. Market clearing then implies that the equilibrium price is

$$p = \frac{1}{2-\lambda}\mathbb{E}[\tilde{\theta}|K, p] + \frac{1-\lambda}{2-\lambda}(\theta - \omega) - \frac{1}{\phi(2-\lambda)}\lambda K. \quad (30)$$

Once again, the price is a weighted average of the traders' and of the entrepreneurs' valuation of the asset, net of trading costs. However, because the shock ω is not known to the traders, the price no longer perfectly reveals θ , ensuring that the informational effect of K on the traders' expectation of the fundamental reemerges. In particular, note that the discrepancy between the price and the fundamental (i.e., the price error) is now given by

$$p - \theta = \frac{1}{2-\lambda} \left[\mathbb{E}[\tilde{\theta}|K, p] - \theta \right] - \frac{1-\lambda}{2-\lambda}\omega - \frac{1}{\phi(2-\lambda)}\lambda K \quad (31)$$

Because the entrepreneurs possess no private information about ω at the time they make their investment decisions, their investment strategy must satisfy

$$k(x, y) = \mathbb{E}[\tilde{\theta} + \lambda(\tilde{p} - \tilde{\theta}) | x, y] = \mathbb{E}[\tilde{\theta} + \frac{\lambda}{2-\lambda} \left[\mathbb{E}[\tilde{\theta}|K, \tilde{p}] - \tilde{\theta} \right] - \frac{1}{\phi(2-\lambda)}\lambda^2 \tilde{K} | x, y] \quad (32)$$

As in the previous extension, what tilts investment way from the benchmark without information frictions is not the supply-side effect of K , captured by the last term in the right-hand side of (32) but the fact that entrepreneurs can use their sources of information to predict the traders' forecast error $\mathbb{E}[\tilde{\theta}|K,p] - \theta$.

To see how this in turn affects the use of information, one needs to compute the (linear) rational-expectation equilibrium for this economy. This consists in a linear investment strategy $k(x,y) = \beta_0 + \beta_1 x + \beta_2 y$ together with a price function $p(\theta, \varepsilon, \omega)$ such that, for any (x,y) , $k(x,y)$ solves (32) and, for any $(\theta, \varepsilon, \omega)$, the market clearing price solves (30). Next note that, because observing the price is informationally equivalent to observing a linear signal about θ , in any linear equilibrium, $\mathbb{E}[\tilde{\theta}|K,p]$ is the projection of θ on (K,p) . By (30), it then follows that the market-clearing price p can itself be expressed as a linear combination of (K,θ,ω) . This in turn implies that, for any linear equilibrium, there exist coefficients $(\gamma_0, \gamma_1, \gamma_2, \gamma_3)$ such that

$$\mathbb{E}[\tilde{\theta}|K,p] = \gamma_0 + \gamma_1 K + \gamma_2 \theta + \gamma_3 \omega. \quad (33)$$

Note that γ_1 , which captures the effect of K on the traders' expectation of θ , now combines the information that is directly revealed to the traders by the observation of K with the information that is revealed to them through the observation of the equilibrium price. Replacing (33) into (30) then gives the following characterization result.

Proposition 11 (i) *In any linear equilibrium, the investment strategy satisfies*

$$k(x,y) = \mathbb{E}[(1-\alpha)\kappa(\tilde{\theta}) + \alpha K(\tilde{\theta}, \tilde{\varepsilon}) \mid x, y],$$

where $\alpha = \frac{\lambda}{2-\lambda}\gamma_1 - \frac{\lambda^2}{\phi(2-\lambda)}$ and $\kappa(\theta) = \frac{\lambda\gamma_0+[2(1-\lambda)+\lambda\gamma_2]\theta}{2-\lambda-\lambda\gamma_1+\lambda^2/\phi}$.

(ii) *When λ is small enough, there exists a unique linear equilibrium and is such that investment increases with θ , and γ_1 is positive.*

As in the previous section, α combines an informational effect (captured by $\frac{\lambda}{2-\lambda}\gamma_1$) with a supply-side effect (captured by $-\frac{\lambda^2}{\phi(2-\lambda)}$). The supply-side effect always contributes to strategic substitutability, while the informational effect contributes to strategic complementarity if and only if high investment is good news for θ (i.e. $\gamma_1 > 0$). Once again, the overall effect is ambiguous. However, the role of informational frictions remains the same as before: it increases the relative sensitivity of investment to sources of information with common noise thus increasing the effect of common noise shocks relative to fundamental shocks. Corollary 1 thus continues to hold.

We now turn to the characterization of the efficient allocation for this economy. The efficiency concept we use is the same as in the preceding sections; however, now we need to allow the planner

to mimic the information aggregation that the market achieves through prices. We thus proceed as follows.

First, we define an *allocation* as a collection of strategies $k(x, y)$, $q_E(x, y, K, p, \omega)$ and $q_T(K, p)$, along with a shadow-price function $p(\theta, \varepsilon, \omega)$ with the following interpretation: in the first period, an entrepreneur with signals (x, y) invests $k(x, y)$; in the second period, all agents observe the realizations of aggregate investment $K = K(\theta, \varepsilon)$ and the shadow price $p = p(\theta, \varepsilon, \omega)$; the amount of capital held by an entrepreneur not hit by a liquidity shock (in addition to the one chosen at $t = 1$) is then given by $q_E(x, y, K, p, \omega)$, while the amount of capital held by a trader is given by $q_T(K, p)$.

Next, we say that the allocation is *feasible* if and only if, for all $(\theta, \varepsilon, \omega)$,

$$\lambda K(\theta, \varepsilon) = (1 - \lambda) \int q_E(x, y, K(\theta, \varepsilon), \omega, p(\theta, \varepsilon, \omega)) d\Phi(x, y | \theta, \varepsilon) + q_T(K(\theta, \varepsilon), p(\theta, \varepsilon, \omega)). \quad (34)$$

As with equilibrium, this constraint plays two roles: first, it guarantees that the second-period resource constraint is not violated; second, it defines the technology that is used to generate the endogenous public signal (equivalently, the extent to which information can be aggregated through the shadow price).

Finally, for any given $k(x, y)$, $q_E(x, y, K, p, \omega)$ and $q_T(K, p)$, ex-ante utility can be computed as $\mathbb{E}u = W(k(\cdot), q_E(\cdot), q_T(\cdot))$, where

$$\begin{aligned} W(k(\cdot), q_E(\cdot), q_T(\cdot)) &\equiv \frac{1}{2} \int \int \left\{ -\frac{1}{2} k(x, y)^2 + \lambda p(\theta, \varepsilon, \omega) k(x, y) + (1 - \lambda) \theta k(x, y) + \right. \\ &\quad \left. + (1 - \lambda) R(\theta - \omega, q^E(x, y, K(\theta, \varepsilon), \omega, p(\theta, \varepsilon, \omega))) \right\} d\Phi(x, y | \theta, \varepsilon) d\Psi(\theta, \varepsilon, \omega) \\ &\quad + \frac{1}{2} \int R(\theta, q^T(K(\theta, \varepsilon), p(\theta, \varepsilon, \omega))) d\Psi(\theta, \varepsilon, \omega), \end{aligned}$$

where $R(v, q) \equiv vq - q^2/(2\phi)$ and where $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y | \theta, \varepsilon)$. We then define an efficient allocation as follows.

Definition 4 An efficient allocation is a collection of strategies $k(x, y)$, $q_E(x, y, K, p, \omega)$ and $q_T(K, p)$, along with a shadow price function $p(\theta, \varepsilon, \omega)$, that jointly maximize ex-ante utility, $\mathbb{E}u = W(k, q_E, q_T)$, subject to the feasibility constraint (34).

Because utility is transferable, the shadow price does not affect payoffs directly; its sole function is to provide an endogenous public signal upon which the allocation of the asset in period 2 can be conditioned. The next lemma then characterizes the efficient allocation of the asset.

Lemma 2 *The efficient allocation in the second period satisfies*

$$q_E^*(x, y, K, p, \omega) = \frac{\lambda K}{2 - \lambda} - \frac{\phi\omega}{2 - \lambda} \quad \text{and} \quad q_T^*(K, p) = \frac{\lambda K}{2 - \lambda} - p. \quad (35)$$

along with a shadow price function $p(\theta, \varepsilon, \omega) = -\frac{(1-\lambda)\phi\omega}{2-\lambda}$.

To understand this result, suppose for a moment that information were complete in the second period. For any given K , efficiency in the second period would require that all entrepreneurs hold the same q_E and that (q_T, q_E) maximize

$$\left\{ \theta q_T - \frac{1}{2\phi} q_T^2 \right\} + (1 - \lambda) \left\{ (\theta - \omega) q_E - \frac{1}{2\phi} q_E^2 \right\}$$

subject to the feasibility constraint $(1 - \lambda) q_E + q_T = \lambda K$. Clearly, the solution to this problem is $q_E^* = \frac{\lambda K}{2 - \lambda} - \frac{\phi\omega}{2 - \lambda}$ and $q_T^* = \frac{\lambda K}{2 - \lambda} + \frac{(1-\lambda)\phi\omega}{2-\lambda}$. In our environment, information is incomplete and traders do not observe ω directly. However, the same allocation can be induced through the shadow-price in the lemma.²⁹

We now characterize the efficient investment strategy. Using Lemma 2, ex-ante utility reduces to

$$\begin{aligned} \mathbb{E}u &= \int \left\{ \frac{1}{2} \int [-\frac{1}{2}k(x, y)^2 + \theta k(x, y)] d\Phi(x, y | \theta, \varepsilon) - \frac{1}{2\phi(2-\lambda)} [\lambda K(\theta, \varepsilon)]^2 \right\} d\Psi(\theta, \varepsilon) + \frac{(1-\lambda)}{2(2-\lambda)} (36)^3 \\ &= \frac{1}{2} \mathbb{E} \left\{ -\frac{1}{2} \tilde{k}^2 + \tilde{\theta} \tilde{k} - \frac{1}{2\phi(2-\lambda)} (\lambda \tilde{K})^2 \right\} + \frac{(1-\lambda)}{2(2-\lambda)} \phi \sigma_\omega^2. \end{aligned}$$

Except for two minor differences—the smaller weight on $(\lambda K)^2$, which adjusts the cost associated with absorbing the fixed supply λK in the second period for the fact that now this quantity is split across a larger pool of agents, and the last term in (36), which captures how the volatility of ω affects the allocation of capital across entrepreneurs and traders in the second period—ex-ante utility has the same structure as in (27) in the previous section. The following result is then immediate.

Proposition 12 *The efficient investment strategy is the unique linear solution to*

$$k(x, y) = \mathbb{E} \left[(1 - \alpha^*) \kappa^*(\tilde{\theta}) + \alpha^* K(\tilde{\theta}, \tilde{\varepsilon}) \mid x, y \right], \quad (37)$$

where $\alpha^* \equiv -\frac{\lambda^2}{\phi(2-\lambda)}$, $\kappa^*(\theta) \equiv \frac{1}{1-\alpha^*}\theta$, and $K(\theta, \varepsilon) = \int k(x, y) d\Phi(x, y | \theta, \varepsilon)$.

²⁹Note that the proposed shadow price is also the unique market-clearing price given the proposed demand functions. The efficient trades can thus be implemented by inducing these demand functions through an appropriately designed tax system and then letting the agents trade in the market.

Comparing the efficient strategy with the equilibrium one, we have that, once again, as long as investment increases with both signals, so that high investment is good news for profitability, then α remains higher than α^* , in which case the key normative prediction of the paper, as summarized by Corollary 2, continues to hold.

6.3 Financial-market shocks

In the specifications considered so far, entrepreneurs and traders share the same valuation for the installed capital. We now develop a variant of the model in which entrepreneurs and traders have different valuations. In this variant, additional non-fundamental volatility originates from correlated errors in the entrepreneurs' expectations about the traders' valuations; once again, our mechanism amplifies the impact of these errors. This variant thus helps connect our model to the recent work on speculative trading à la Harrison and Kreps (1978).³⁰

We consider the following modification of the baseline model. The traders' utility in period $t = 3$ is given by $(\theta + \omega) k_i$, where ω is a random variable, independent of θ and of any other exogenous random variable in the economy, Normally distributed with mean zero and variance σ_ω^2 . This random variable is a private-value component in the traders' valuation. It can originate from the hedging motive of the traders, from a different discount factor, or from heterogeneous valuations à la Harrison and Kreps (1978). For our purposes, what matters is that the presence of ω in the traders' utility is taken as given by the social planner; that is, the planner respects the preference orderings revealed by the agents' trading decisions. We thus choose a neutral label for ω and simply call it a "financial market shock."

We also modify the entrepreneurs' information set to allow for information regarding ω that affects investment decisions. In particular, the entrepreneurs observe a common signal $w = \omega + \zeta$, where ζ is common noise, independent of any other exogenous random variable in the economy, with variance σ_ζ^2 . The signal w is observed by the entrepreneurs but not by the traders; as in the baseline model, this is a shortcut for introducing correlated errors in the entrepreneurs' expectations regarding the financial-market shock. Finally, to focus on common expectational shocks about ω rather than about θ , we remove the common signal y : the entrepreneurs observe only private signals about θ , $x_i = \theta + \xi_i$, where ξ_i is idiosyncratic noise as in the baseline model.

In this environment, the asset price in period two is given by

$$p = \mathbb{E} [\tilde{\theta}|K, \omega] + \omega.$$

It follows that equilibrium investment choices depend not only on the entrepreneurs' expectations of θ , but also on their expectations of ω : in any linear equilibrium, there then exist coefficients

³⁰See Scheinkman and Xiong (2003), Gilchrist, Himmelberg, and Huberman (2005), and Panageas (2005).

$(\beta_0, \beta_1, \beta_2)$ such that individual investment is given by $k(x, w) = \beta_0 + \beta_1 x + \beta_2 w$ and, by implication, aggregate investment is given by $K(\theta, w) = \beta_0 + \beta_1 \theta + \beta_2 w$. As in the baseline model, the optimality of each entrepreneur's strategy then requires that

$$k(x, w) = \mathbb{E}[\tilde{\theta} + \lambda(\tilde{p} - \tilde{\theta}) \mid x, w] = \mathbb{E}[\tilde{\theta} + \lambda[\mathbb{E}[\tilde{\theta} | \tilde{K}, \tilde{\omega}] - \tilde{\theta} + \tilde{\omega}] \mid x, w] \quad (38)$$

Once again, it is instructive to compare this strategy with the corresponding one in the absence of informational frictions (i.e., when information becomes symmetric at the time of trading). This is given by $k(x, w) = \mathbb{E}[\tilde{\theta} + \lambda\tilde{\omega} \mid x, w]$; in the absence of informational frictions, entrepreneurs simply use their sources of information to forecast the expected return of their investment $(1 - \lambda)\theta + \lambda(\theta + \omega) = \theta + \lambda\omega$ which incorporates the fact that capital has a different return in case it is transferred to the traders. In contrast, when information is incomplete at the trading stage, then entrepreneurs base their investment strategy not only on the expectation of $\theta + \lambda\omega$ but also on their expectation of the traders' forecast error $\mathbb{E}[\tilde{\theta} | \tilde{K}, \tilde{\omega}] - \tilde{\theta}$. As in the baseline model, this in turn leads to an amplification of common noise shocks relative to fundamental shocks which is a source of inefficiency. Following steps similar to the ones in the baseline model in fact leads to the following result.

Proposition 13 *(i) In any equilibrium in which the investment strategy is linear in x and w , there exist a scalar $\alpha > 0$ and a function $\kappa(\theta, \omega)$ such that*

$$k(x, w) = \mathbb{E}[(1 - \alpha)\kappa(\tilde{\theta}, \tilde{\omega}) + \alpha K(\tilde{\theta}, \tilde{w}) \mid x, w].$$

(ii) When λ is small there exists a unique linear equilibrium and is such that investment increases with both θ and w .

(iii) The efficient investment satisfies

$$k(x, w) = \mathbb{E}[\tilde{\theta} + \lambda\tilde{\omega} \mid x, w].$$

(iv) In any linear equilibrium in which investment increases with both θ and w , investment underreacts to θ and overreacts to w (relative to the efficient allocation).

In this economy, entrepreneurs pay too much attention to their signals regarding shocks in the financial market. The reason is essentially the same as in the benchmark model. When traders interpret high investment as good news for θ , financial prices increase with aggregate investment. Because the noise in the entrepreneurs' signals about the financial market shock ω is correlated, these signals are relatively better predictors of aggregate investment than the signals about θ . By implication, entrepreneurs' investment decisions are oversensitive to information about financial

market shocks relative to information about their fundamental valuation θ . Through this channel, an increase in investment that is purely driven by expectations regarding financial market shock is amplified.

Absent informational frictions (i.e., if θ were known at the time of financial trade), the response of investment to θ and ω would be efficient. Since ω can be interpreted as the difference between the traders' and the entrepreneurs' fundamental valuations of the asset, this case is reminiscent of the efficiency results obtained in richer models of “bubbles” based on heterogeneous priors; in particular, Panageas (2006) derives a similar efficiency result for a model that introduces heterogeneous valuations in a q -theory model of investment. The interesting novelty here is that inefficiency arises once we introduce dispersed information. Traders are then uncertain whether high investment is driven by good fundamentals or by the entrepreneurs' expectations of speculative valuations. This uncertainty opens the door to a destabilizing effect of financial prices on real investment, creating inefficiency in the response of investment to different sources of information.

6.4 Other extensions

An important function of stock prices is to guide corporate investment choices by revealing valuable information that is dispersed in the marketplace and not directly available to corporate managers (e.g., Dow and Gorton, 1997; Subrahmanyam and Titman, 1999; Chen, Goldstein and Jiang, 2007). This effect is absent in the preceding analysis, because the entrepreneurs' investment choices are made before the opening of the financial market. However, we can easily incorporate such an effect by letting the entrepreneurs make an additional investment in stage two, after observing the price in the financial market.³¹ Provided that the dispersion of information does not vanish at the time of trading, both the amplification mechanism and the inefficiency we have documented remain. Interestingly, though, an additional information externality emerges: if all agents were to increase their reliance on idiosyncratic sources of information, then the information contained in prices would be more precise, which in turn would improve the efficiency of the investment decisions that follow the observation of these prices. Clearly, this informational externality only reinforces the conclusion that agents rely too much on sources of information with correlated noise, and hence that non-fundamental volatility is inefficiently high.

Throughout the preceding extensions, we have maintained the assumption that traders cannot directly invest in the new technology during the first period. Clearly, our results do not hinge on this assumption. For example, consider the benchmark model and suppose that each trader j chooses first-period real investment k_j at cost $k_j^2/2$ and then trades an additional q_j units in the second period in the financial market. Neither the equilibrium price in the financial market nor the

³¹ Alternatively, we could introduce a financial market in stage 1 or let entrepreneurs observe a noisy signal of K instead of a noisy price signal.

entrepreneurs' choices in the first period are affected; all that happens is that aggregate investment now includes the investment of the traders, which is simply given by $k_T = \mathbb{E}[\tilde{\theta}]$, which does not affect the information structure in the second period. More generally, one could drop the distinction between entrepreneurs and traders altogether and simply talk about differentially informed agents who first make real investment decisions and then trade financial claims on the installed capital.

Next, consider the assumption that a fraction λ of the entrepreneurs is forced to sell their capital in the financial market; this was a modeling device that ensured that the private return to first-period investment depends on (anticipated) second-period financial prices while ensuring tractability. If one were to drop the assumption of risk neutrality, or assume that the second-period transaction costs depend on gross positions, or introduce short-sale constraints in the financial market, then the profits an agent could make in the financial market would depend on how much capital he enters the market with; this in turn would ensure that private returns to first-period investment depend on expectations of future financial prices, even in the absence of any preference-shock.³²

Finally, consider the assumption that profitability is perfectly correlated across entrepreneurs. Clearly, what is essential is only that there is a common component about which agents have dispersed (and correlated) information. For example, we could let the productivity of the new technology for entrepreneur i be $\tilde{\theta}_i = \theta + v_i$, where θ is the common component and v_i is an idiosyncratic component; we could then also let the entrepreneurs' signals be $\tilde{\theta}_i$ (possibly plus some additional noise) instead of θ plus noise. Alternatively, we could introduce common and idiosyncratic shocks to the entrepreneurs' cost of investment during period 1. In this case, unobservable common shocks to the cost of investment would also act as a source of noise in the information that aggregate investment conveys about θ , essentially playing the same role as the correlated errors in the entrepreneurs' signals about θ .³³

7 Conclusion

This paper examined the interaction between real and financial decisions in an economy in which information about underlying profitability is dispersed. By conveying a positive signal about profitability, higher aggregate investment stimulates higher asset prices, which in turn raise the incentives to invest. This creates an endogenous complementarity, making investment decisions sensitive to higher-order expectations. In turn, this can dampen the impact of fundamental shocks and amplify the impact of common expectational shocks. Importantly, all these effects are symptoms

³²Note, however, that these extensions may feature additional deviations from the first best (e.g., short-sale constraints), which may introduce novel effects in addition to the ones we have documented.

³³Such an extension is studied in the Supplementary Material.

of inefficiency.

These effects are likely to be stronger during periods of intense technological change, when the dispersion of information about the potential of the new technologies is particularly high. Our analysis therefore predicts that such periods come hand-in-hand with episodes of high non-fundamental volatility and comovement in investment and asset prices. At some level, this seems consistent with the recent experiences surrounding the internet revolution or the explosion of investment opportunities in China. What looks like irrational exuberance may actually be the amplified, but rational, response to noise in information. While both explanations open the door to policy intervention, the one suggested by our theory is not based on any presumption of “intelligence superiority” on the government’s side.

Our mechanism also represents a likely source of non-fundamental volatility and inefficiency over the business cycle. Indeed, information regarding aggregate supply and demand conditions seems to be widely dispersed in the population, which explains the financial markets’ anxiety preceding the release of key macroeconomic statistics. Extending the analysis to richer business-cycle frameworks is an important direction for future research.

Appendix: Proofs omitted in the main text

Proof of Proposition 1. The proof proceeds in three steps. First, we fill in the details of the equilibrium characterization in the text. Second, we analyze the fixed point problem and prove existence. Finally, we prove uniqueness.

Step 1. Consider the expected pricing error $\mathbb{E} [p(\tilde{\theta}, \tilde{\varepsilon}) - \tilde{\theta} | x, y]$ on the right-hand side of the entrepreneurs' optimality condition, written in the form (10) in the text. Substituting the price (8) in this expression and using $\pi_z = \pi_y / \varphi^2$, $\mathbb{E}[\tilde{\theta}|x, y] = \mu + \delta_1(x - \mu) + \delta_2(y - \mu)$, and $\mathbb{E}[\tilde{\varepsilon}|x, y] = y - \mathbb{E}[\tilde{\theta}|x, y]$ yields, after some algebra,

$$\begin{aligned}\mathbb{E} [p(\tilde{\theta}, \tilde{\varepsilon}) - \tilde{\theta} | x, y] &= \frac{\pi_y}{\pi_y + \varphi^2 \pi_\theta} [(1 - \varphi) \delta_1 (x - \mu) + ((1 - \varphi) \delta_1 + \varphi) (y - \mu)] + \\ &\quad - \delta_1 (x - \mu) - \delta_2 (y - \mu) \\ &= -\varphi \delta_1 \frac{\pi_y + \varphi \pi_\theta}{\pi_y + \varphi^2 \pi_\theta} (x - \mu) + \varphi \delta_2 \frac{\pi_x + (1 - \varphi) \pi_\theta}{\pi_y + \varphi^2 \pi_\theta} (y - \mu).\end{aligned}$$

Substituting this expression on the right-hand side of (10), gives

$$k(x, y) = \mu + \left(1 - \lambda \varphi \frac{\pi_y + \varphi \pi_\theta}{\pi_y + \varphi^2 \pi_\theta}\right) \delta_1 (x - \mu) + \left(1 + \lambda \varphi \frac{\pi_x + (1 - \varphi) \pi_\theta}{\pi_y + \varphi^2 \pi_\theta}\right) \delta_2 (y - \mu).$$

Since in equilibrium the expression on the right-hand side must be equal to $\beta_0 + \beta_1 x + \beta_2 y$ for all x and y , the following conditions must hold:

$$\begin{aligned}\beta_0 &= (1 - \beta_1 - \beta_2) \mu, \\ \beta_1 &= \left(1 - \lambda \varphi \frac{\pi_y + \varphi \pi_\theta}{\pi_y + \varphi^2 \pi_\theta}\right) \delta_1, \\ \beta_2 &= \left(1 + \lambda \varphi \frac{\pi_x + (1 - \varphi) \pi_\theta}{\pi_y + \varphi^2 \pi_\theta}\right) \delta_2.\end{aligned}$$

Step 2. Instead of analyzing the fixed point problem in terms of $\varphi = \beta_2 / (\beta_1 + \beta_2)$ described in the text, it is convenient to analyze the equivalent fixed point problem in terms of $b = \beta_2 / \beta_1$. Since

$$\varphi = \frac{\beta_2}{\beta_1 + \beta_2} = \frac{b}{1 + b},$$

dividing (11) by (12) yields

$$b = \frac{\pi_y + \left(\frac{b}{1+b}\right)^2 \pi_\theta + \lambda \frac{b}{1+b} \left(\pi_x + \frac{1}{1+b} \pi_\theta\right) \delta_2}{\pi_y + \left(\frac{b}{1+b}\right)^2 \pi_\theta - \lambda \frac{b}{1+b} \left(\pi_y + \frac{b}{1+b} \pi_\theta\right) \delta_1}.$$

Some lengthy but straightforward algebra shows that the right-hand side of this expression is equal to the following function of b

$$F(b) \equiv \frac{\delta_2}{\delta_1} \left\{ 1 + \frac{\lambda(1+b)b}{(1-\lambda)(\delta_0+\delta_2)b^2 + (2-\lambda)\delta_2 b + \delta_2} \right\}. \quad (39)$$

So we are looking for a positive fixed point of F . Notice that F is well defined and continuous on \mathbb{R}_+ , with $F(\delta_2/\delta_1) > \delta_2/\delta_1$ and $\lim_{b \rightarrow \infty} F(b) < \infty$. It follows that F has at least one fixed point $b > \delta_2/\delta_1$. Given this value of b , it is easy to show existence by construction. The equilibrium value of β_2 is derived from (12) (substituting $\varphi = b/(1+b)$) and is clearly positive. Then β_1 is equal to β_2/b and is also positive. Finally, β_0 is set equal to $(1 - \beta_1 - \beta_2)\mu$.

Step 3. To prove uniqueness, notice that all linear equilibria, irrespective of the sign of β_1 and β_2 , must correspond to a fixed point of the function F defined in (39). The only cases not covered in our characterization are the cases $\beta_1 + \beta_2 = 0$, in which φ is not well defined, and $\beta_1 = 0$, in which b is not well defined, but it is easy to show that there can be no equilibria with either $\beta_1 + \beta_2 = 0$ or $\beta_1 = 0$.

Notice that we can choose $\hat{\lambda} > 0$ so that if λ is in the interval $[0, \hat{\lambda}]$ the expression $(1 - \lambda)(\delta_0 + \delta_2)b^2 + (2 - \lambda)\delta_2 b + \delta_2$ on the right-hand side of (39) is always positive (it is a quadratic function which is positive at $b = 0$ and has negative discriminant $(\delta_2\lambda)^2 - 4\delta_0\delta_2(1 - \lambda)$ if λ is small enough). This implies that the function F is defined and continuously differentiable over the whole real line, with

$$F'(b) = \lambda \frac{\delta_2}{\delta_1} \frac{[\delta_2 - (1 - \lambda)\delta_0]b^2 + 2\delta_2 b + \delta_2}{[(1 - \lambda)(\delta_0 + \delta_2)b^2 + (2 - \lambda)\delta_2 b + \delta_2]^2}.$$

Moreover,

$$\lim_{b \rightarrow -\infty} F(b) = \lim_{b \rightarrow +\infty} F(b) = F_\infty \equiv \frac{\delta_2}{\delta_1} \left[1 + \frac{\lambda}{(1 - \lambda)(\delta_0 + \delta_2)} \right] > \frac{\delta_2}{\delta_1}.$$

We now need to consider two cases. First, suppose $\delta_2 = (1 - \lambda)\delta_0$. Then the function F has a global minimum at $b = -1/2$. In this case, F is bounded above and below, respectively, by $\underline{F} \equiv F(-1/2)$ and $\bar{F} \equiv F_\infty$. Second, suppose $\delta_2 \neq (1 - \lambda)\delta_0$. Then $F'(b)$ has two zeros, at $b = b_1$ and at $b = b_2$, where

$$b_1 \equiv \frac{-\delta_2 - \sqrt{(1 - \lambda)\delta_0\delta_2}}{\delta_2 - (1 - \lambda)\delta_0} \text{ and } b_2 \equiv \frac{-\delta_2 + \sqrt{(1 - \lambda)\delta_0\delta_2}}{\delta_2 - (1 - \lambda)\delta_0}.$$

The function F then has a local maximum at b_1 and a local minimum at b_2 . In this case, the bounds are $\underline{F} \equiv \min\{F_\infty, F(b_2)\}$ and $\bar{F} \equiv \max\{F_\infty, F(b_1)\}$. It is easy to check that in all the cases considered both \underline{F} and \bar{F} converge to δ_2/δ_1 as $\lambda \rightarrow 0$. But then F converges uniformly to δ_2/δ_1 as $\lambda \rightarrow 0$. It follows that for any $\varepsilon > 0$ we can choose $\hat{\lambda}$ so that, whenever $\lambda < \hat{\lambda}$, F has no

fixed point outside the interval $[\delta_2/\delta_1 - \varepsilon, \delta_2/\delta_1 + \varepsilon]$.

With a slight abuse of notation, replace $F(b)$ with $F(b, \lambda)$, making explicit the dependence of F on λ . Notice that $\partial F(b, \lambda)/\partial b$ is continuous at $(\delta_2/\delta_1, 0)$ and $\partial F(\delta_2/\delta_1, 0)/\partial b = 0$. It follows that there exist $\tilde{\varepsilon} > 0$ and $\tilde{\lambda} \in (0, \hat{\lambda}]$ such that $\partial F(b, \lambda)/\partial b < 1$ for all $b \in [\delta_2/\delta_1 - \tilde{\varepsilon}, \delta_2/\delta_1 + \tilde{\varepsilon}]$ and $\lambda \in [0, \tilde{\lambda}]$. Combining these results with the continuity of F , we have that there exist $\bar{\varepsilon} > 0$ and $\bar{\lambda} > 0$ such that, for all $\lambda \in [0, \bar{\lambda}]$, the following are true: for any $b \notin [\delta_2/\delta_1 - \bar{\varepsilon}, \delta_2/\delta_1 + \bar{\varepsilon}]$, $F(b, \lambda) \neq b$; for $b \in [\delta_2/\delta_1 - \bar{\varepsilon}, \delta_2/\delta_1 + \bar{\varepsilon}]$, F is continuous and differentiable in b , with $\partial F(b, \lambda)/\partial b < 1$. It follows that, if $\lambda \leq \bar{\lambda}$, F has at most one fixed point, completing our argument. ■

Proof of Proposition 2. The expressions (11) by (12) were derived in Step 1 of Proposition 1. In any equilibrium with $\beta_1, \beta_2 > 0$, we have $\varphi \in (0, 1)$, and, from (11) by (12), $\beta_1 < \delta_1$ and $\beta_2 > \delta_2$. The last inequality immediately shows that ε is amplified relative to the frictionless benchmark. Moreover, the two inequalities imply

$$\frac{\varphi}{1-\varphi} = \frac{\beta_2}{\beta_1} > \frac{\delta_2}{\delta_1} = \frac{\pi_y}{\pi_x}. \quad (40)$$

Finally, the response of investment to the shock θ is

$$\begin{aligned} \beta_1 + \beta_2 &= \delta_1 + \delta_2 + \lambda \frac{\varphi \pi_y}{\pi_\theta + \pi_x + \pi_y} \frac{\pi_x + (1-\varphi) \pi_\theta}{\pi_y + \varphi^2 \pi_\theta} - \lambda \frac{\varphi \pi_x}{\pi_\theta + \pi_x + \pi_y} \frac{\pi_y + \varphi \pi_\theta}{\pi_y + \varphi^2 \pi_\theta} \\ &= \delta_1 + \delta_2 + \lambda \frac{\varphi \pi_\theta}{\pi_\theta + \pi_x + \pi_y} \frac{(1-\varphi) \pi_y - \varphi \pi_x}{\pi_y + \varphi^2 \pi_\theta} < \delta_1 + \delta_2 \end{aligned}$$

where the last inequality follows from (40). ■

Proof of Lemma 1. Substituting $K = \beta_0 + \beta_1 \theta + \beta_2 y$ and $\kappa = ((1-\lambda)\theta + \lambda \gamma_0) / (1-\alpha)$ into the “best response” (16), yields

$$\begin{aligned} k(x, y) &= \mathbb{E}[(1-\lambda)\tilde{\theta} + \lambda \gamma_0 + \alpha (\beta_0 + \beta_1 \tilde{\theta} + \beta_2 y) | x, y] \\ &= \lambda \gamma_0 + \alpha \beta_0 + (1-\lambda + \alpha \beta_1) \mathbb{E}[\tilde{\theta}|x, y] + \alpha \beta_2 y. \end{aligned}$$

Substitute $\mathbb{E}[\tilde{\theta}|x, y] = \delta_0 + \delta_1 x + \delta_2 y$ on the right-hand side and $k = \beta_0 + \beta_1 x + \beta_2 y$ on the left-hand side. Since the resulting expression has to hold for all x and y , we have

$$\begin{aligned} \beta_1 &= (1-\lambda + \lambda \gamma_1 \beta_1) \delta_1, \\ \beta_2 &= (1-\lambda + \lambda \gamma_1 \beta_1) \delta_2 + \alpha \beta_2. \end{aligned}$$

Rearranging these two and using $\delta_2/\delta_1 = \pi_y/\pi_x$, gives the desired result. ■

Proof of Proposition 3. Consider the function $F(b, \lambda)$ introduced in the proof of Proposition 1, which is continuously differentiable for all $b \geq 0$ and $\lambda \in [0, 1]$. Take any pair $\lambda', \lambda'' \in [0, 1]$ with $\lambda'' > \lambda'$, and let b' be the lowest non-negative solution to $F(b, \lambda') - b = 0$ (which exists from Proposition 1). Then, by definition, $F(b, \lambda') - b > 0$ for all $b \in [0, b')$. Some algebra shows that $\partial F(b, \lambda) / \partial \lambda \geq 0$ if $b \geq 0$, with a strict inequality if $b > 0$. It follows that $\lambda'' > \lambda'$ implies $F(b, \lambda'') - b > 0$ for all $0 \leq b \leq b'$. Therefore the smallest b which solves $F(b, \lambda'') - b = 0$ must be strictly greater than b' . To prove the statement in the proposition, just recall that, for $b \geq 0$, there is a monotone increasing relation between b and φ . ■

Proof of Proposition 4. Consider the function $F(b)$ introduced in the proof of Proposition 1. Take the parameters $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$. With these parameters the function F is continuous over the entire real line, since the quadratic expression $(1 - \lambda)(\delta_0 + \delta_2)b^2 + (2 - \lambda)\delta_2b + \delta_2$ is always positive (it is positive at $b = 0$ and has negative discriminant $(\delta_2\lambda)^2 - 4\delta_0\delta_2(1 - \lambda)$ for our chosen parameters). Moreover, at the point b_2 , also defined in the proof of Proposition 1, we have $F(b_2) < b_2 < 0$. These properties, together with the properties that $F(0) > 0$ and $\lim_{b \rightarrow -\infty} F(b) > 0 > -\infty$, ensure that, in addition to a fixed point in $(\delta_2/\delta_1, +\infty)$, F admits at least one fixed point in $(-\infty, b_2)$ and one in $(b_2, 0)$. Indeed, in this example F admits exactly three fixed points, which are “strict” in the sense that $F(b) - b$ changes sign around them. Because F is continuous in $(b, \delta_1, \delta_2, \lambda)$ in an open neighborhood of $(\delta_1, \delta_2, \lambda) = (.2, .1, .75)$, there necessarily exists an open set $S \subset (0, 1)^3$ such that F admits three fixed points whenever $(\delta_1, \delta_2, \lambda) \in S$. ■

Proof of Proposition 6. Substituting the price (20) in the entrepreneurs’ best response (9), one obtains expressions for β_0 , β_1 and β_2 , as in the proof of Proposition 1. Then, following similar steps, it is possible to show that a linear equilibrium is characterized by a ratio $b = \beta_2/\beta_1$ that satisfies $b = F(b, \tau_1)$ where

$$F(b, \tau_1) \equiv \frac{\delta_2}{\delta_1} \left\{ 1 + \frac{\lambda(1+b)b/(1+\tau_1)}{(1-\lambda)(\delta_0 + \delta_2)b^2 + \left(2(1-\lambda) + \frac{1}{1+\tau_1}\lambda\right)\delta_2b + \left(1-\lambda + \frac{1}{1+\tau_1}\lambda\right)\delta_2} \right\}.$$

Finally, steps similar to those in Proposition 3 show that the equilibrium value of b is monotone decreasing in τ_1 . ■

Proof of Proposition 7. The first claim is proved by the numerical example in the main text. Thus consider the second claim. Given any linear strategy $k(x, y) = \beta_0 + \beta_1x + \beta_2y$, ex-ante utility

is given by

$$\begin{aligned}
2\mathbb{E}u &= \mathbb{E}\left[-\frac{1}{2}k(x, y)^2 + \theta k(x, y)\right] \\
&= -\frac{1}{2}\beta_0^2 + \beta_0(1 - \beta_1 - \beta_2)\mu - \frac{1}{2}\beta_1^2\pi_x^{-1} - \frac{1}{2}(\beta_1 + \beta_2)^2\pi_\theta^{-1} \\
&\quad - \frac{1}{2}\beta_2^2\pi_y^{-1} + (\beta_1 + \beta_2)\sigma_\theta^2 + (\beta_1 + \beta_2)\left[1 - \frac{1}{2}(\beta_1 + \beta_2)\right]\mu^2.
\end{aligned} \tag{41}$$

Now suppose prices are fully stabilized at $p = \bar{p}$. Substituting $p(\theta, y) = \bar{p}$ into the entrepreneurs' best response (??) gives the following coefficients for the equilibrium investment strategy:

$$\beta_0 = (1 - \lambda)\delta_0\mu + \lambda\bar{p}, \quad \beta_1 = (1 - \lambda)\delta_1, \quad \text{and} \quad \beta_2 = (1 - \lambda)\delta_2. \tag{42}$$

Note that \bar{p} affects only the first two terms in (42) through its effect on β_0 . Hence, the maximal welfare that can be achieved with full price stabilization is obtained by choosing \bar{p} so that $\beta_0 = 1 - (1 - \lambda)(\delta_1 + \delta_2)$.

Next, note that for any $a \in (0, 1)$ and any $b \in \mathbb{R}$, there exists a policy $\tau(p) = \tau_0 + \tau_1 p$ that induces an equilibrium in which the investment strategy is given by³⁴

$$\beta_0 = b \quad \beta_1 = \frac{(1 - \lambda)\delta_1}{1 - a\delta_1} \quad \text{and} \quad \beta_2 = \frac{(1 - \lambda + a\beta_1)\delta_2}{1 - a}. \tag{43}$$

To see this, suppose that, given (τ_0, τ_1) , the entrepreneurs follow the linear strategy defined in (43). Then $\mathbb{E}[\theta|K] = \gamma_0 + \gamma_1 K$, where (γ_0, γ_1) are obtained from (43) using the formulas given in (14). The market clearing price is then equal to

$$p = \frac{1}{1 + \tau_1}(\gamma_0 + \gamma_1 K - \tau_0) \tag{44}$$

Replacing (44) and $K(\theta, y) = \beta_0 + \beta_1\theta + \beta_2y$ into (??), we then have that the best response for each entrepreneur consists in following the strategy $k(x, y) = \tilde{\beta}_0 + \tilde{\beta}_1x + \tilde{\beta}_2y$ given by

$$\begin{aligned}
\tilde{\beta}_0 &= \frac{\lambda(\gamma_0 - \tau_0)}{1 + \tau_1} + (1 - \lambda)\delta_0\mu + \tilde{\alpha}[\beta_0 + \beta_1\delta_0\mu] \\
\tilde{\beta}_1 &= (1 - \lambda)\delta_1 + \tilde{\alpha}\beta_1\delta_1 \\
\tilde{\beta}_2 &= (1 - \lambda)\delta_2 + \tilde{\alpha}[\beta_1\delta_2 + \beta_2]
\end{aligned}$$

where $\tilde{\alpha} = \lambda\gamma_1/(1 + \tau_1)$. It is then immediate that there exists a (τ_0, τ_1) such that $\tilde{\beta}_0 = \beta_0, \tilde{\beta}_1 = \beta_1$

³⁴Equivalently, for any $a \in (0, 1)$ and any $\kappa_0 \in \mathbb{R}$, there exists a policy (τ_0, τ_1) that sustains an equilibrium in which the investment strategy satisfies $k(x, y) = \mathbb{E}[(1 - a)\tilde{\kappa}(\theta) + aK(\theta, y) | x, y]$ with $\tilde{\kappa}(\theta) \equiv \left(\frac{1 - \lambda}{1 - a}\right)\theta + \kappa_0$.

and $\tilde{\beta}_2 = \beta_2$ (it suffices to choose τ_1 so that $\tilde{\alpha} = a$ and then adjust τ_0 so that $\tilde{\beta}_0 = b$).

Now let $b_0 \equiv 1 - (1 - \lambda)(\delta_1 + \delta_2)$ and for any $a \in [0, 1)$ let

$$\tilde{\beta}_1(a) \equiv \frac{(1 - \lambda)\delta_1}{1 - a\delta_1}, \quad \tilde{\beta}_2(a) \equiv \frac{(1 - \lambda + a\beta_1)\delta_2}{1 - a},$$

and

$$\begin{aligned} W(a) &\equiv -\frac{1}{2}b_0^2 + b_0 \left[1 - \tilde{\beta}_1(a) - \tilde{\beta}_2(a) \right] \mu - \frac{1}{2} \left(\tilde{\beta}_1(a) \right)^2 \pi_x^{-1} - \frac{1}{2} \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a) \right)^2 \pi_\theta^{-1} \\ &\quad - \frac{1}{2} \left(\tilde{\beta}_2(a) \right)^2 \pi_y^{-1} + \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a) \right) \pi_\theta^{-1} \\ &\quad + \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a) \right) \left[1 - \frac{1}{2} \left(\tilde{\beta}_1(a) + \tilde{\beta}_2(a) \right) \right] \mu^2 \end{aligned}$$

Note that welfare under full price stabilization is given by $W(0)$, whereas welfare under any policy (τ_0, τ_1) that implements a linear strategy as in (43) with $a \in (0, 1)$ and $b = b_0$ is given by $W(a)$. Next note that W is continuously differentiable over $[0, 1)$. To prove the second claim in the proposition it thus suffices to show that

$$\frac{dW}{da} = \frac{\partial W}{\partial \tilde{\beta}_1} \frac{d\tilde{\beta}_1}{da} + \frac{\partial W}{\partial \tilde{\beta}_2} \frac{d\tilde{\beta}_2}{da} > 0$$

at $a = 0$. First note that

$$\begin{aligned} \frac{\partial W}{\partial \tilde{\beta}_1} &= -b_0\mu - \tilde{\beta}_1\pi_x^{-1} - \left(\tilde{\beta}_1 + \tilde{\beta}_2 \right) \pi_\theta^{-1} + \pi_\theta^{-1} + \left[1 - \left(\tilde{\beta}_1 + \tilde{\beta}_2 \right) \right] \mu^2 \\ \frac{\partial W}{\partial \tilde{\beta}_2} &= -b_0\mu - \tilde{\beta}_2\pi_y^{-1} - \left(\tilde{\beta}_1 + \tilde{\beta}_2 \right) \pi_\theta^{-1} + \pi_\theta^{-1} + \left[1 - \left(\tilde{\beta}_1 + \tilde{\beta}_2 \right) \right] \mu^2 \end{aligned}$$

Using $\tilde{\beta}_1(0) = (1 - \lambda)\delta_1$, $\tilde{\beta}_2(0) = (1 - \lambda)\delta_2$ and $b_0 \equiv 1 - (1 - \lambda)(\delta_1 + \delta_2)$, we thus have that

$$\frac{\partial W}{\partial \tilde{\beta}_1} \Big|_{\tilde{\beta}_1=\tilde{\beta}_1(0)} = \frac{\partial W}{\partial \tilde{\beta}_2} \Big|_{\tilde{\beta}_2=\tilde{\beta}_2(0)} = -b_0\mu + [1 - (1 - \lambda)(\delta_1 + \delta_2)]\mu^2 + \lambda\pi_\theta^{-1} = \lambda\pi_\theta^{-1} > 0.$$

Because $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are both increasing in a , it follows that $dW(0)/da$ is positive, which establishes the result. ■

Proof of Proposition 9. For part (i), it suffices to substitute the price as in (24) into the entrepreneurs' best response (??) and rearranging. Thus consider part (ii). Substituting $\pi_z = [\pi_y(\beta_1 + \beta_2)^2]/\beta_2^2$ into (14) gives

$$\gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z} \frac{1}{\beta_1 + \beta_2} = \frac{(\beta_1 + \beta_2)\pi_y}{\beta_2^2\pi_\theta + (\beta_1 + \beta_2)^2\pi_y} = \frac{(\beta_1 + \beta_2)\delta_2}{\beta_2^2\delta_0 + (\beta_1 + \beta_2)^2\delta_2}.$$

In the limit, as $\lambda \rightarrow 0$, we have that $\beta_0 \rightarrow \delta_0$, $\beta_1 \rightarrow \delta_1$, $\beta_2 \rightarrow \delta_2$, and hence $\gamma_1 \rightarrow \frac{(\delta_1 + \delta_1)\delta_2}{\delta_2^2\delta_0 + (\delta_1 + \delta_1)^2\delta_2} > 0$. By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda})$, $(\beta_1 + \beta_2) > 0$, i.e. investment increases with θ , and $\gamma_1 > 0$, i.e. the traders' expectation of θ increases with K . ■

Proof of Proposition 10. Let $V(k, K, \theta) \equiv -\frac{1}{2}k^2 + \theta k - \frac{\lambda^2}{2\phi}K^2$. From (27), $\mathbb{E}u = \frac{1}{2}\mathbb{E}V(\tilde{k}, \tilde{K}, \tilde{\theta})$. The result then follows from Proposition 3 in Angeletos and Pavan (2007a), noting that $\kappa^*(\theta) \equiv \arg \max_K V(K, K, \theta) = \frac{1}{1+\lambda^2/\phi}\theta$ and $\alpha^* \equiv 1 - \frac{V_{kk} + 2V_{kK} + V_{KK}}{V_{kk}} = V_{KK} = -\lambda^2/\phi$. ■

Proof of Proposition 11. From (30) and (33), the equilibrium price is then

$$p(\theta, \varepsilon, \omega) = P(K(\theta, \varepsilon), \theta, \omega) \equiv \eta_0 + \eta_1 K(\theta, \varepsilon) + \eta_2 \theta + \eta_3 \omega. \quad (45)$$

for some $(\eta_0, \eta_1, \eta_2, \eta_3)$.

Now consider the optimality of the traders' strategies. As in the benchmark model, the information that $K(\theta, \varepsilon)$ reveals about θ is the same as that of a signal

$$z \equiv \frac{K(\theta, \varepsilon) - \beta_0}{\beta_1 + \beta_2} = \theta + \frac{\beta_2}{\beta_1 + \beta_2}\varepsilon$$

whose precision is $\pi_z = \left(\frac{\beta_1 + \beta_2}{\beta_2}\right)^2 \pi_y$, while the information that $p(\theta, \varepsilon, \omega)$ reveals about θ given $K(\theta, \varepsilon)$ is the same as that of a signal

$$s = \frac{1}{\eta_2}[p(\theta, \varepsilon, \omega) - \eta_0 - \eta_1 K(\theta, \varepsilon)] = \theta + \frac{\eta_3}{\eta_2}\omega$$

whose precision is $\pi_s = \left(\frac{\eta_2}{\eta_3}\right)^2 \pi_\omega$. A trader who observes K and p thus believes that θ is normally distributed with mean

$$\begin{aligned} \mathbb{E}[\tilde{\theta} | K(\theta, \varepsilon), p(\theta, \varepsilon, \omega)] &= \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s} \mu_\theta + \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} z + \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} s \\ &= \gamma_0 + \gamma_1 K(\theta, \varepsilon) + \gamma_2 \theta + \gamma_3 \omega \end{aligned}$$

where

$$\gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_z + \pi_s} \mu_\theta - \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} \frac{\beta_0}{\beta_1 + \beta_2} \quad (46)$$

$$\gamma_1 = \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} \frac{1}{\beta_1 + \beta_2} \quad (47)$$

$$\gamma_2 = \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} \quad (48)$$

$$\gamma_3 = \frac{\pi_s}{\pi_\theta + \pi_z + \pi_s} \frac{\eta_3}{\eta_2}. \quad (49)$$

Combining (30) with (45) we then have that

$$\eta_0 = \frac{\gamma_0}{2 - \lambda} \quad (50)$$

$$\eta_1 = \frac{1}{2 - \lambda} \left(\gamma_1 - \frac{\lambda}{\phi} \right) \quad (51)$$

$$\eta_2 = \frac{1}{2 - \lambda} (\gamma_2 + 1 - \lambda) \quad (52)$$

$$\eta_3 = \frac{1}{2 - \lambda} (\gamma_3 - 1 + \lambda). \quad (53)$$

Lastly, consider the optimality of the entrepreneurs' investment strategies. From (32), the strategy $k(x, y) = \beta_0 + \beta_1 x + \beta_2 y$ is individually rational if and only if $(\beta_0, \beta_1, \beta_2)$ satisfy $\beta_0 + \beta_1 x + \beta_2 y = (1 - \lambda) \mathbb{E}[\tilde{\theta}|x, y] + \lambda \mathbb{E}[p(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\omega})|x, y]$ for all (x, y) . That is, $(\beta_0, \beta_1, \beta_2)$ must satisfy the following conditions:

$$\beta_0 = [1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2] \delta_0 \mu_\theta + \lambda \eta_0 + \lambda \eta_1 \beta_0 \quad (54)$$

$$\beta_1 = (1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2) \delta_1 \quad (55)$$

$$\beta_2 = (1 - \lambda + \lambda \eta_1 \beta_1 + \lambda \eta_2) \delta_2 + \lambda \eta_1 \beta_2 \quad (56)$$

A linear equilibrium is thus a solution to (46)-(56).

The existence of a linear equilibrium and its uniqueness for λ small enough can be established following steps similar to those in the baseline model.

To establish part (i) it then suffices to substitute (50)-(53) into (45) and then substitute (45) into

$$k(x, y) = (1 - \lambda) \mathbb{E}[\tilde{\theta}|x, y] + \lambda \mathbb{E}[p(\tilde{\theta}, \tilde{\varepsilon}, \tilde{\omega})|x, y]$$

and rearranging. Thus consider part (ii). Below we prove that λ small enough suffices for $\beta_1 + \beta_2 > 0$, for $\gamma_1 > 0$ and, although not necessary for the result, for $\alpha > 0$.

Substituting $\pi_z \equiv \left(\frac{\beta_1 + \beta_2}{\beta_2} \right)^2 \pi_y$ and $\pi_s = \left(\frac{\eta_2}{\eta_3} \right)^2 \pi_\omega = \pi_\omega$ into (47) gives

$$\begin{aligned} \gamma_1 &= \frac{\pi_z}{\pi_\theta + \pi_z + \pi_s} \frac{1}{\beta_1 + \beta_2} \\ &= \frac{(\beta_1 + \beta_2) \pi_y}{\beta_2^2 \pi_\theta + (\beta_1 + \beta_2)^2 \pi_y + \beta_2^2 \pi_\omega} \\ &= \frac{(\beta_1 + \beta_2) \delta_2}{\beta_2^2 \delta_0 + (\beta_1 + \beta_2)^2 \delta_2 + \frac{\beta_2^2 \pi_\omega}{\pi_\theta + \pi_y + \pi_x}}. \end{aligned}$$

In the limit, as $\lambda \rightarrow 0$, we have that $\beta_0 \rightarrow \delta_0$, $\beta_1 \rightarrow \delta_1$, $\beta_2 \rightarrow \delta_2$, and hence

$$\gamma_1 \rightarrow \frac{(\delta_1 + \delta_1) \delta_2}{\delta_2^2 \delta_0 + (\delta_1 + \delta_1)^2 \delta_2 + \frac{\delta_2^2 \pi_\omega}{\pi_\theta + \pi_y + \pi_x}} > 0.$$

By continuity, then, there exists $\hat{\lambda} > 0$ such that, for all $\lambda \in (0, \hat{\lambda})$, $(\beta_1 + \beta_2) > 0$, $\gamma_1 > 0$ and $\alpha = \frac{\lambda}{2-\lambda} \left(\gamma_1 - \frac{\lambda}{\phi} \right) > 0$. ■

Proof of Proposition 12. Let

$$V(k, K, \theta) \equiv \theta k - \frac{1}{2} k^2 - \frac{\lambda^2}{2\phi(2-\lambda)} K^2.$$

The result then follows for the same argument as in the proof of Proposition 10. ■

Proof of Proposition 13. *Part (i).* In any linear equilibrium, there exist coefficients $(\beta_0, \beta_1, \beta_2)$ such that the investment strategy can be written as

$$k(x, w) = \beta_0 + \beta_1 x + \beta_2 w,$$

implying that aggregate investment satisfies $K(\theta, w) = \beta_0 + \beta_1 \theta + \beta_2 \omega + \beta_2 \zeta$. For the traders, who know ω but do not know either ζ or θ , observing K is then equivalent to observing a Gaussian signal z with precision π_z , where

$$z \equiv \frac{K - \beta_0 - \beta_2 \omega}{\beta_1} = \theta + \frac{\beta_2}{\beta_1} \zeta \quad \text{and} \quad \pi_z \equiv \left(\frac{\beta_1}{\beta_2} \right)^2 \pi_\zeta,$$

with $\pi_\zeta \equiv \sigma_\zeta^{-2}$. It follows that the equilibrium price satisfies

$$p(\theta, \omega, w) = \mathbb{E}[\tilde{\theta}|K, \omega] + \omega = \gamma_0 + \gamma_1 K(\theta, w) + (1 - \gamma_1 \beta_2) \omega, \quad (57)$$

where

$$\gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_z} \mu - \frac{\pi_z}{\pi_\theta + \pi_z} \frac{\beta_0}{\beta_1} \quad \text{and} \quad \gamma_1 = \frac{\pi_z}{\beta_1 (\pi_\theta + \pi_z)} = \frac{\pi_\zeta}{\beta_1 \left(\left(\frac{\beta_2}{\beta_1} \right)^2 \pi_\theta + \pi_\zeta \right)}. \quad (58)$$

Substituting (57) into the entrepreneurs' best response gives

$$\begin{aligned} k(x, w) &= (1 - \lambda) \mathbb{E}[\tilde{\theta}|x, w] + \lambda \mathbb{E}[p(\tilde{\theta}, \tilde{\omega}, \tilde{w})|x, w] \\ &= (1 - \lambda) \mathbb{E}[\tilde{\theta}|x, w] + \lambda \gamma_0 + \lambda \gamma_1 \mathbb{E}[K(\tilde{\theta}, \tilde{w})|x, w] + \lambda (1 - \gamma_1 \beta_2) \mathbb{E}[\tilde{\omega}|x, w] \end{aligned} \quad (59)$$

which can be rewritten as in part (i) of the proposition by letting

$$\alpha \equiv \lambda\gamma_1 \quad \text{and} \quad \kappa(\theta, \omega) \equiv \frac{(1-\lambda)\theta + \lambda\gamma_0 + \lambda(1-\gamma_1\beta_2)\omega}{1-\lambda\gamma_1}.$$

That $\alpha > 0$ is shown in part (ii).

Part (ii). Substituting $K(\theta, w) = \beta_0 + \beta_1\theta + \beta_2w$ into (59) gives

$$k(x, w) = \lambda(\gamma_0 + \gamma_1\beta_0) + (1 - \lambda + \lambda\gamma_1\beta_1)\mathbb{E}[\tilde{\theta}|x, w] + \lambda\gamma_1\beta_2w + \lambda(1 - \gamma_1\beta_2)\mathbb{E}[\tilde{\omega}|x, w]$$

Using the fact that $\mathbb{E}[\tilde{\theta}|x, w] = \mathbb{E}[\tilde{\theta}|x] = \delta_0 + \delta_1x$ and $\mathbb{E}[\tilde{\omega}|x, w] = \mathbb{E}[\tilde{\omega}|w] = \eta w$, where $\delta_0 \equiv \sigma_\theta^{-2}/(\sigma_\theta^{-2} + \sigma_x^{-2})\mu$, $\delta_1 \equiv \sigma_x^{-2}/(\sigma_\theta^{-2} + \sigma_x^{-2})$, and $\eta \equiv \sigma_\zeta^{-2}/(\sigma_\omega^{-2} + \sigma_\zeta^{-2})$, the above reduces to

$$\begin{aligned} k(x, w) &= \lambda(\gamma_0 + \gamma_1\beta_0) + (1 - \lambda + \lambda\gamma_1\beta_1)\delta_0 + (1 - \lambda + \lambda\gamma_1\beta_1)\delta_1x \\ &\quad + \lambda[\eta + (1 - \eta)\gamma_1\beta_2]w \end{aligned}$$

For this strategy to coincide with $k(x, w) = \beta_0 + \beta_1x + \beta_2w$, it is necessary and sufficient that the coefficients $(\beta_0, \beta_1, \beta_2)$ solve the following system:

$$\beta_0 = \lambda(\gamma_0 + \gamma_1\beta_0) + (1 - \lambda + \lambda\gamma_1\beta_1)\delta_0 \tag{60}$$

$$\beta_1 = (1 - \lambda + \lambda\gamma_1\beta_1)\delta_1, \tag{61}$$

$$\beta_2 = \lambda[\eta + (1 - \eta)\gamma_1\beta_2]. \tag{62}$$

By (58),

$$\gamma_1\beta_1 = \frac{\pi_\zeta}{\left(\frac{\beta_2}{\beta_1}\right)^2\pi_\theta + \pi_\zeta} \in (0, 1), \tag{63}$$

which together with (61) guarantees that $\beta_1 \in (0, \delta_1)$. From (61) and (62) we then get

$$\frac{\beta_2}{\beta_1} = \lambda \frac{\eta \left(\left(\frac{\beta_2}{\beta_1} \right)^2 \pi_\theta + \pi_\zeta \right) + (1 - \eta)\pi_\zeta \left(\frac{\beta_2}{\beta_1} \right)}{\delta_1 \left((1 - \lambda) \left(\left(\frac{\beta_2}{\beta_1} \right)^2 \pi_\theta + \pi_\zeta \right) + \lambda\pi_\zeta \right)}$$

or equivalently

$$\frac{\beta_2}{\lambda\beta_1} = F\left(\frac{\beta_2}{\lambda\beta_1}; \lambda\right)$$

where

$$F(b; \lambda) \equiv \frac{\eta}{\delta_1} \left\{ 1 + \lambda \frac{\lambda^2\pi_\theta b^2 + \pi_\zeta b}{(1 - \lambda)\lambda^2\pi_\theta b^2 + \pi_\zeta} \right\}.$$

It is then easy to show that, for λ small enough, F has a unique fixed point and this fixed point is in a neighborhood of

$$\frac{\beta_2}{\lambda\beta_1} = \frac{\eta}{\delta_1}.$$

Along with the fact that $\beta_1 > 0$ always, this guarantees that $\beta_2 > 0$ for λ small enough.

Part (iii). The social planner's problem can be set up as in the baseline model, giving the optimality condition stated in part (iii) of the proposition.

Part (iv). From part (iii) the efficient strategy is given by

$$k(x, w) = \beta_0^* + \beta_1^*x + \beta_2^*w$$

with

$$\beta_0^* = \delta_0, \quad \beta_1^* = \delta_1 \quad \text{and} \quad \beta_2^* = \lambda\eta.$$

We have already shown, in the proof of part (ii), that $\beta_1 < \delta_1 = \beta_1^*$, which means that investment underreacts to θ . Next, note that $\beta_1 > 0$ implies $\gamma_1 > 0$. From (62) it then follows that, in any equilibrium in which $\beta_2 > 0$, it is also the case that

$$\beta_2 = \lambda\eta + \lambda(1 - \eta)\gamma_1\beta_2 > \lambda\eta = \beta_2^*,$$

which means that investment overreacts to w . ■

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