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Voting as a Credible Threat.

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Abstract

We offer a rationale for elections that take place in the shadow of power. Factions unhappy with policy can threaten violence. But when they lack common knowledge about (i) one another's rationality, and (ii) their chances of victory at arms, mutual overconfidence can precipitate civil war. We argue that elections can clarify the likely consequences of violence, and so facilitate peaceful resolution. Our theory is based on the recognition that both voting and fighting are intrinsically *correlated* actions: individuals who undertake the individually irrational act of voting are unusually prone the individually irrational act of voluntary combat.

Introduction

A crucial feature of democracy is the acceptance of loss in the electoral arena. Those who are dissatisfied with the outcome typically retain some recourse to other forms of conflict, including, in extreme cases, civil war. Democracies survive by not pushing their losers over the threshold of armed rebellion. Of course, most elections do not revolve around such significant issues, but occasionally democracies must face momentous choices that inflame passions

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and divide the public. When such choices must be made elites will bargain towards a solution "in the shadow of power" (Powell, 1999), keeping in sight the importance of not precipitating violent unrest, or in extreme cases, even civil war. If the relative strengths of the factions in a potential civil war are common knowledge, then the intrinsic costliness of civil war will typically create a feasible set of possible compromises. In contrast, false optimism about the outcome of a potential civil war can provoke such a conflict.

Our point of departure for this paper is a conflict over policy by factions that may attempt to use civil war to impose their policy if they are unable to achieve an acceptable negotiated outcome[1] If the factions' have inconsistent beliefs about victory in the case of armed conflict, and in particular, if there is an excess of optimism (Johnson, 2004) there may be no compromise that can simultaneously satisfy both factions, and threats may be translated into actions. We show how elections might solve this problem by creating a convergence of expectations about the outcome of a conflict.

Our argument will apply if voting and violent struggle are correlated activities. In that case, the election is a sort of bloodless civil war in which the strengths of the two factions are, at least partially, revealed[2]. By dispelling factions' mutual optimism an election can make a negotiated settlement easier to reach, expanding the set of compromises acceptable to both.

We discuss several mechanisms by which voting in an election and fighting in a civil war might be connected. Both electoral and armed conflict require partisans to take individually costly actions that have infinitesimal effects on the outcome, whether this involves turning out to the polls to cast a ballot that is all but certain not to be decisive or exposing oneself to danger in combat. Elections reveal the number of people on each side willing to incur

[1]Thus, our view of the political process shares with Weber and (Schmitt 1988, 1996), the recognition that politics centers on fundamental conflicts between opposing factions.

[2]In contrast with a real civil war, which frequently produces a definite winner along with all the rubble, we argue that elections tend to produce common expectations about the probability of each side winning–the factions remain uncertain, but they share the same uncertainty.

palpable costs to gain only a trivial advantage for their side, and so provide a signal about the relative number of combatants each side could draw on in case of civil war[3].

Provided the election does provide a public and informative signal to both sides about the threats each can make credibly, it can facilitate a negotiated solution that stops short of violence.

Przeworski and Sprague (1986) acknowledge a potential parallel between voting and fighting "at the barricades", likening ballots to "paper stones" in the hands of workers. However, their focus is on explaining the electoral stragegies adopted by Socialist parties in various countries. From the standpoint of our model, Socialists' difficulty in obtaining electoral majorities probably curbed their optimism about the prospects for victory at the barricades, and so may have prevented a series of armed civil conflicts.

Our argument differs from the rationale for holding elections offered by Weber (1982), and others after him, and made more formally by (Ticchi and Vindigni, 2003a, 2003b) that expanding the suffrage is a way to pay the public for its military service. Instead, the key feature of our analysis is the role of elections in credibly signaling information about the likely consequences of a civil war fought to resolve the issues at stake in the election. In doing so, the election returns facilitate bargaining among factions, and so make civil war less likely.

Our analysis also departs from that of several other scholars. Ellman and Wantchekon (2000) analyze how potential social unrest influences political competition and policy outcomes in unconsolidated democracies, and show that threat of explosion of social unrest may in some circumstances induce the implementation of a relatively moderate policy. However, their result

[3] A simple census of people who identify with a faction might include many passive individuals unwilling to sustain the sort of costly individual action in exchange for negligible returns that is needed for both voting and fighting.

emerges as the outcome of spatial political competition, very different from the learning process on which our own model is based.

Taking the analysis of the rule of law by Weingast (1997) as his point of departure, Fearon (2006) offers an informational rationale for elections as a means for citizens to coordinate their collective action against a ruler violating the "social contract", rather than as a means for competing factions within society to learn their relative strength should social institutions break down.

Przeworski (1991) models competitive elections as exogenously random events, rather than as endogenous indicators about the likely outcome of conflict. The political parties in his model care about holding office–and as long as the chances of eventually getting into power outweigh the prospect of initiating an armed rebellion, out of power parties in Przeworski's model bide their time.

The paper proceeds as follows. In the next section we present a simple model of bargaining on the eve of a potential civil war. This model indicates the importance of the parameters of potential conflict being common knowledge. Section 2 sets out several models that encompass both voting and fighting, and for each model establishes the link between the two activities. In section 3 we show for each of our models how elections can dispel mutual optimism and thereby allow factions to peacefully compromise on policy, while a brief discussion of our results follows in section 4.

1 A Model of Pre-Conflict Bargaining

Our "baseline" model of pre-conflict bargaining closely resembles the model of bargaining in the shadow of conflict developed by Powell (1999) chapter 3. Suppose that society must choose the location of a policy variable $x \in \mathbb{R}$. In the first instance this policy choice is delegated to two faction leaders. The

leader of faction L has a preferred policy of x_L, while the faction R leader prefers a policy of $x_R > x_L$. The utility derived from a policy outcome x for and individual with a preferred policy outcome of θ is:

$$U(x|\theta) = -d(x, \theta) \tag{1}$$

where $d(a, b)$ is an increasing function of the distance between a and b[4].

The leaders bargain over the outcomes, making alternating offers. The leader of faction L has an instantaneous discount rate of r_L, while the instantaneous discount rate for the faction R leader is r_R. The length of the bargaining interval, Δ, is taken to be arbitrarily small.

At each bargaining interval, the leader can accept the offer that the other leader has made, or he can make a counter offer, or he can opt for civil war. If the civil war option is chosen, the winning faction pays a cost C from fighting the war, and implements it's preferred policy $x^W \in \{x_L, x_R\}$. The losing side must endure the policy chosen by the winning side, and it incurs a cost $D \geq C$.

Up to this point, our model of elite bargaining has been isomorphic to that of Powell (1999), however, we incorporate uncertainty in a slightly different manner; whereas Powell models bargainers as being uncertain about one another's costs of bargaining, we treat them as being uncertain about the probability of winning an armed conflict. As we discuss below, this means that rationality is not "common knowledge" between the two factions.

The probability faction L wins if civil war breaks out is p_L, while the victory probability for R is $p_R = 1 - p_L$. The prior beliefs about p_L for the leadership of faction $\beta \in \{L, R\}$ are characterized by the density $\pi_\beta(p_L)$ which has support everywhere on the interval $[0, 1]$. We assume that the leadership of faction L know that the prior beliefs of faction R are characterized by $\pi_R(p_L)$

[4] That is, $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $d(a, b) = d(b, a) > 0, \forall a \neq b$, $d(a, b) > d(a', b') \Rightarrow |a - b| > |a' - b'|$ and $d(a, a) = 0, \forall a$.

and likewise that faction R knows that faction L has beliefs characterized by $\pi_{\mathsf{L}}(\mathfrak{p}_{\mathsf{L}})$. We nevertheless allow that $\pi_{\mathsf{L}}(\mathfrak{p}_{\mathsf{L}}) \neq \pi_{\mathsf{R}}(\mathfrak{p}_{\mathsf{L}})$.

We might expect that in the course of learning about the other side's beliefs, the factional leadership would update their own beliefs. The Spanish Republicans of 1936 might have reasoned that if the Nationalists were so confident of victory, then perhaps resistance was indeed doomed. But if rationality is not common knowledge this sort of updating need not lead to converged beliefs. In the case of 1930's Spain, rather than being intimidated by the Nationalists' willingness to fight, many Republicans concluded that their opponents willingness to fight was simply another manifestation of the irrationality that led the Nationalists to prefer fascism. Many Nationalists held a reciprocal contempt for the rationality of the Republicans.

In our abstract framework, the decision to engage in armed hostility amounts to an expensive bet with the other side, something we should not observe if rationality is common knowledge (Aumann, 1976 and Sabenius and Geanakoplos, 1983). We assume that convergence of prior beliefs on the part of the factions does not take place because each faction attributes the deviation between its own prior beliefs and those of the other faction to delusional thinking on the part of the other side. One way this might arise would involve the factions selecting their leaders in a process that rewarded overconfidence[5]. Johnson (2004) argues that systematic overconfidence may confer advantages upon leaders who hold them. Regardless of the mechanism that produces this, our model takes as given that the faction leaders start out with potentially divergent beliefs.

Integrating over the beliefs of the leader of faction L we find that the expected probability that she prevails in case of war is:

[5]The strategic selection of agents on the basis of their *preferences* is analyzed by Rogoff (1985) and Fershtman and Judd (1987), here we suggest that leaders may also be selected on the basis of a tendency to hold biased *beliefs*

$$\hat{\mathfrak{p}}_L^{\mathfrak{L}} = \int_0^1 \mathfrak{p}_L \pi_L(\mathfrak{p}_L) d\mathfrak{p}_L$$

While the faction R leader expects the victory probability for faction L to be:

$$\hat{\mathfrak{p}}_L^{\mathfrak{R}} = \int_0^1 \mathfrak{p}_L \pi_R(\mathfrak{p}_L) d\mathfrak{p}_L$$

The "individual rationality" constraint that must be satisfied if faction L does not resort to civil war is that it's utility from accepting the peaceful settlement x exceeds the expected utility of war:

$$d(x, x_L) \leq \hat{\mathfrak{p}}_L^{\mathfrak{L}} C + (1 - \hat{\mathfrak{p}}_L^{\mathfrak{L}}) (d(x_R, x_L) + D) \tag{2}$$

Let x_L^* be the rightmost value for x that satisfies the inequality (2).

Likewise, the individual rationality constraint that must be satisfied if faction R does not wage war is:

$$d(x, x_R) \leq \hat{\mathfrak{p}}_L^{\mathfrak{R}} (d(x_R, x_L) + D) + (1 - \hat{\mathfrak{p}}_L^{\mathfrak{R}}) C \tag{3}$$

Let x_R^* be the leftmost value for x satisfying inequality (3).

The threat of civil war guarantees that any peaceful settlement, represented by a policy x^*, must simultaneously satisfy (2) and (3), that is, it must satisfy:

$$x_R^* \leq x^* \leq x_L^* \tag{4}$$

Let $\mathfrak{S}$ denote the set of offers satisfying condition (4). This will correspond to the interval $[x_R^*, x_L^*]$ when $x_L \geq x_R$, otherwise it will be the empty set.

If both sides share the same beliefs about the outcome of a potential armed conflict then civil war will be avoidable. To be precise: given that civil war is costly, and that losing one is no less costly than winning, and given that

neither side is risk loving, the set of possible peaceful bargains will not be empty. We formalize this result as Claim 1.

Claim 1: If $d(x+y, x)$ is convex in y, so that neither side is risk loving with respect to policy outcomes, and if losing a war is not intrinsically advantageous, so that $D \geq C \geq 0$, then if both sides share the same beliefs about p_L, so that $E_L\{\mathfrak{p}_L\} = E_R\{\mathfrak{p}_L\}$, it follows that the set of possible peaceful bargains $\mathfrak{S}$ is nonempty, and corresponds to the interval $[x_R^*, x_L^*]$.

Proof: See Appendix A.

However, if both sides do not share common beliefs about a civil war, the situation becomes more dangerous. Notice that the righthand side of equation (2) is decreasing[6] in $\hat{\mathfrak{p}}_L^{\mathfrak{L}}$, that is, the more optimistic faction L becomes about it's prospects in a civil war, the less they will be willing to compromise.

Claim 2a: The more likely party L believes victory will be, the farther to the left will be the righmost policy compromise it is willing to tolerate rather than fight: $\frac{\partial x_L^*}{\partial \mathfrak{p}_L^{\mathfrak{L}}} < 0$

Proof: See Appendix A.

Likewise, the more pessimistic party R is about its prospects in a civil war, the less closely the peaceful settlement must approximate x_R to satisfy equation (3).

Claim 2b: The more likely party R believes victory will be for faction L, the farther to the left will be the leftmost policy compromise it is willing to tolerate rather than fight: $\frac{\partial x_R^*}{\partial \mathfrak{p}_L^{\mathfrak{R}}} < 0$

Proof: See Appendix A.

If both factions' leaders are optimistic, the bargaining set will shrink; with enough optimism, it may be empty, with the result being civil war. Johnson (2004) provides evidence from case studies that overoptimism, triggered by positive illusions was an important cause of international war during

[6] Substituting $\hat{\mathfrak{p}}_L^{\mathfrak{L}}$ into equation (2) and differentiating we have: $C - d(x_R, x_L) - D$ which is guaranteed to be less than zero as $C \leq D$ while $d(x_R, x_L)$ is positive.

the twentieth century (including World War I and the Vietnam War), while Gilbert (2006) argues that "In August 1914 the empires of Europe embarked on a war that each of them believed would be swift and victorious.".

While an outbreak of mutual pessimism would have a pacific effect, this would require both sides to genuinely hold negatively biased beliefs. Moreover, if one party appointed a leader who was known to be pessimistic, the other side's best response would be to choose an optimistic leader to gain a bargaining advantage. However, if $\mathfrak{p}_L$ could be set equal to $\mathfrak{p}_R$, so that both sides shared the same beliefs about the outcome of a civil war, both factions could avoid the possibility that $\mathfrak{S} = \emptyset$, and hence avoid civil war. In fact, it will often be enough for $E_L\{\mathfrak{p}_L\}$ and $E_R\{\mathfrak{p}_L\}$ to be merely close to one another. But given the enormous benefits of mutual deception[7] how are such commonly held beliefs to be attained? Our answer, which we begin to set forth in the next section, is that fair elections can provide such information.

2 Citizenship, Voting, and Civil War

The so-called "paradox" of voting that juxtaposes the minuscule probability an individual vote influences the result of an election (Barzel and Silberberg, 1973, Beck, 1975, Margolis, 1977, Chamberlin and Rothschild, 1981, Gelman, King and Boscardin 1998) against the small but real costs of casting a ballot[8]. Here we consider three possible explanations for this, and we extend each to encompass the decision to fight if the policy dispute erupts into civil war.

[7] Consider first that once it knows the other sides true beliefs, each side would like to pretend to be somewhat more optimistic than it really is. Secondly, there is the matter of "secret weapons". For example, in order to make the advantages of the T-34 tank with its special gear system credible to the Germans, the Russians would have had to share the design, which the Germans could have then copied. Nor was this concern misplaced: almost soon as the Germans faced the T-34 during their invasion of the USSR they went assiduously to work incorporating its design advantages into their own tanks. Many other examples in the same vein could be cited.

[8] One could posit that the act of voting itself is somehow enjoyable (Riker and Ordeshook, 1968), but this "explanation" has a somewhat circular flavor. Moreover, if one's impact on the result was not important, and if the mere act of casting a ballot was enjoyable to so many people, why not stage non-binding elections every day?

We demonstrate that each of these models is consistent with our central argument about the information contained in elections.

2.1 Models of Voting

Finkel, Miller and Opp (1989) note that given the contrast between the small but palpable costs of voting and the negligible impact of an individual's vote on the election outcome, it is irrational to cast a ballot. Never the less, they argue that we might observe mobilization if individuals acted upon their sense of duty rather than with the direct aim of changing the election outcome. It is easy to extend their argument to encompass people taking the costly action of fighting, despite their infinitesimal impact on the outcome of a war. Quattrone and Tversky (1986) describe what they call "magical thinking" leading people to mistake actions that are diagnostic of an outcome, such as casting a vote for the winning side, for actions that have a causal impact. In such circumstances magical thinkers might vote or even fight. Kahneman (1982) notes that individuals tend to overestimate the probabilities of very unlikely events. Such an overestimate could lead them to mistakenly believe that they are likely to be decisive in an election, or a war.

Here we show that these various sources of motivation lead to very similar formalization of the decision to act. Suppose individuals' preference settings for x are uniformly distributed along the interval $[0, 1]$. Suppose that policy preferences are captured by a parameter θ that differs among individuals, and that a type θ individual has Euclidean policy preferences:

$$U(x|\theta) = -d(x, \theta)$$

where we defined the function d in the discussion surrounding equation (1).

First, we shall consider the motives people might have to take costly action to vote for one of a set of competing alternatives even when their individual

votes are extremely unlikely to be pivotal to the outcome. Next, we will develop a model of why citizen combatants might incur even greater costs associated with taking the initiative in combat.

A Sense of Duty

Finkel, Miller and Opp (1989) notes that we might be able to explain voting behavior if people were motivated to participate in elections by a strong sense of duty. Suppose that an individual's sense of duty is proportional to the importance of an election for the welfare of his reference group, which we assume shares[9] his preferred outcome of θ. For a type θ individual, the stakes in a given election are given by:

$$S = d(x_L, \theta) - d(x_R, \theta) \tag{5}$$

Let's suppose that one's sense of duty is proportional to the group's interest in the outcome[10]:

$$D = mS \tag{6}$$

where m is a positive constant. We can then expect that one will do one's civic duty whenever the sense of duty looms larger than the costs, that is, whenever $D > c$.

If we let $\dot{x}$ denote the midpoint between the positions of the two parties:

$$\dot{x} = \frac{x_L + x_R}{2} \tag{7}$$

it is immediately obvious that if $\theta \leq \dot{x}$, then voting for R is a strictly dominated strategy, while likewise, if $\theta \geq \dot{x}$ then it is a dominated strategy to vote for L.

[9] Indeed, we may expect that the group is the *source* of the individual's policy preference.

[10] We might reasonably expect that civic education, family child rearing practices, and social norms have all evolved to make one's sense of duty to perform some socially desirable activity roughly proportional to it's value to society, for example saving a drowning child is a more important civic duty than not crossing against the light when there is no traffic.

Substituting this condition into equations (6) and (5), and letting χ denote the ratio of the cost of acting c to the parameter m from equation (6) we see that an individual will vote for party L provided:

$$\theta < \dot{x} \qquad \text{and} \qquad \chi < d(x_R, \theta) - d(x_L, \theta)$$

whereas an individual for whom:

$$\theta > \dot{x} \qquad \text{and} \qquad \chi < d(x_L, \theta) - d(x_R, \theta)$$

will act out of a sense of duty to vote for party R.

If we let the density of individuals with preference parameters (θ, χ) be given by $g_D(\theta, \chi|\xi)$, where ξ is unknown, we see that the probability that an individual votes for party L is given by:

$$\zeta_L^D(\xi, x_L, x_R) = \int_0^{\dot{x}} \int_0^{d(x_R,\theta)-d(x_L,\theta)} g_D(\theta, \chi|\xi) d\chi d\theta$$

similar calculations reveal that the corresponding probability that an individual votes for party R is:

$$\zeta_R^D(\xi, x_L, x_R) = p_R^D = \int_{\dot{x}}^1 \int_0^{d(x_L,\theta)-d(x_R,\theta)} g_D(\theta, \chi|\xi) d\chi d\theta$$

Magical Thinking

Quattrone and Tversky (1986) use the term "magical thinking" to describe people's tendency to confuse actions that are diagnostic of an outcome with actions that have a causal impact. In a political setting, an individual's vote is diagnostic of how other, similar, individuals might vote, this is in fact the basis for political polling. Someone given to magical thinking may falsely extrapolate from the diagnostic value of their vote to the belief that their vote actually has an appreciable impact on the election result. We can

model this formally by assuming that there is a parameter μ that measures resistance to magical thinking[11]. Let μ and the policy preference parameter θ be distributed in the population according to $g_M(\mu, \theta|\xi)$ where ξ is an unknown parameter.

An individual may be expected to vote for the candidate of the party on the left if:

$$\theta < \dot{x} \qquad \text{and} \qquad \mu < d(x_R, \theta) - d(x_L, \theta)$$

whereas an individual for whom:

$$\theta > \dot{x} \qquad \text{and} \qquad \mu < d(x_L, \theta) - d(x_R, \theta)$$

may be expected to vote for the party of the right.

Putting this together, we see that the probability that an individual votes for party L is given by:

$$\zeta_L^M(\xi, x_L, x_R) = p_L^M = \int_0^{\dot{x}} \int_0^{d(x_R,\theta)-d(x_L,\theta)} g_M(\mu, \theta|\xi) d\mu d\theta$$

similar calculations reveal that the corresponding condition for an individual to vote for party R is:

$$\zeta_R^M(\xi, x_L, x_R) = p_R^M = \int_{\dot{x}}^1 \int_0^{d(x_L,\theta)-d(x_R,\theta)} g_M(\mu, \theta|\xi) d\mu d\theta$$

The Illusion of Being Pivotal

Whereas we posit that the probability that an individual is actually decisive for provision of the public good, *e.g.* by casting the decisive vote, or making the difference between victory and defeat in war, is ϵ, where ϵ is a infinitesimally small and positive, it may be that an individual exaggerates the extent to which he is pivotal [12]. Suppose that an individual's assessed probability

[11] We would expect this to be greater if the costs of taking the diagnostic action are higher!

[12] This might be thought of as a special case of the general tendency of individuals to overestimate the probability of rare events such as airline crashes and terrorist attacks (Kahneman, 1982).

that he is pivotal is decisive for the provision of the public good in question is $\delta > 0$. Let $g^*(\delta, \theta|\xi)$ denote the distribution of δ and θ. Notice that this distribution depends on the unknown parameter ξ.

An individual with characteristic δ believes that the probability faction L wins the election is p_0 if he does not vote, and $p_0 + \delta$ if he votes for L, and $p_0 - \delta$ if he individual casts a ballot for faction R. Likewise, he believes that the probability R wins in each case is simply the complement of the probability that L emerges victorious[13]. Let us suppose further that the cost of voting is given by $c > 0$. Under these circumstances the subjective expected utility for a type (δ, θ) from voting for party L would be:

$$E\{U(L|\delta, \theta)\} = -(p_0 + \delta)d(x_L, \theta) - (1 - p_0 - \delta)d(x_R, \theta) - c \tag{8}$$

Similarly, the subjective expected utility from voting for R would be:

$$E\{U(R|\delta, \theta)\} = -(p_0 - \delta)d(x_L, \theta) - (1 - p_0 + \delta)d(x_R, \theta) - c \tag{9}$$

while the expected utility from not showing up to vote at all would be:

$$E\{U(N|\delta, \theta)\} = -p_0 d(x_L, \theta) - (1 - p_0)d(x_R, \theta) \tag{10}$$

Comparing equations (8), and (10) we see that a type (δ, θ) individual will vote for party L rather than abstain if:

$$\theta < \dot{x} \qquad \text{and} \qquad \gamma < d(x_R, \theta) - d(x_L, \theta) \tag{11}$$

where we let γ measure the cost of voting relative to an individual's over estimate of his impact on the outcome, and we will refer to this parameter as the individual's *inertia*:

$$\gamma = \frac{c}{\delta} \tag{12}$$

[13]Here we assume that individuals do not anticipate that either side will precipitate a civil war.

while a similar comparison of equations (9) and (10) reveals that he will prefer a vote for R to abstention if instead:

$$\theta > \dot{x} \qquad \text{and} \qquad \gamma < d(x_L, \theta) - d(x_R, \theta) \tag{13}$$

while otherwise, he will abstain from voting.

Equations (11) and (13) tell us that for a given level of concern about the outcome of the election, $|d(x_R, \theta) - d(x_L, \theta)|$ only individuals with sufficiently low inertia will vote. Consulting equation (12) we see that inertia will be lower when costs are low, and when an individual more severely over estimates the probability of being decisive.

Given c is the same across individuals, the probability distribution for δ and θ gives rise to the joint distribution for γ and θ: $g_I(\gamma, \theta|\xi)$. where γ and θ are independent. We see that the probability that a randomly selected individual votes for party L is given by:

$$\zeta_L^I(\xi, x_L, x_R) = \int_0^{\dot{x}} \int_0^{d(x_R,\theta)-d(x_L,\theta)} g_I(\gamma, \theta|\xi) d\gamma d\theta$$

similar calculations reveal that the corresponding condition for an individual to vote for party R is:

$$\zeta_R^I(\xi, x_L, x_R) = p_R^I = \int_{\dot{x}}^1 \int_0^{d(x_L,\theta)-d(x_R,\theta)} g_I(\gamma, \theta|\xi) d\gamma d\theta$$

2.2 Elections

While the underlying causal mechanisms differ, each of the three models presented in the preceding section gives rise to a voting probability that depends on the unknown parameter ξ, and on parties' fixed issue positions that we assume they are not able to credibly commit to change during the time frame considered in our model.

Let's index our models by $\alpha \in \{D, M, I\}$ where "Model D" treats voters as motivated by a sens of duty, whereas in "Model M" they are motivated by "magical thinking", and in "Model I" the voters suffer from the illusion that they are more likely to be pivotal than they really are. The probability that in model α an individual votes for party L given that he votes at all is:

$$\tilde{p}_L^\alpha(\xi) = \frac{\zeta_L^\alpha(\xi, x_L, x_R)}{\zeta_L^\alpha(\xi, x_L, x_R) + \zeta_R^\alpha(\xi, x_L, x_R)}$$

Where the dependence of the voting probability on the unchanging party positions x_L and x_R is left implicit.

Let $h_L^\alpha(\xi)$ denote the prior probability density over ξ that characterizes the beliefs of the leader of party L in model α, while $h_R^\alpha(\xi)$ characterizes the prior beliefs of the party R leadership about ξ.

Thus we see that all three of our models lead to a very similar formal structure of prior beliefs about the probability of winning elections that is based on uncertainty about the ξ parameter.

2.3 Models of Combat and Insurrection

Now that we have examined the "paradox of voting", consider the similar contrast between the costly actions needed to produce victory and a negligible individual impact on the result that applies to combatants in a military conflict. There is the widespread belief among military historians that victory can only be achieved "...after the battle has been delivered into the hands of men who move *in imminent danger of death*" (Marshall, 1947) p.208. Moreover, the "danger of death" can be substantial[14]. Yet a paucity of soldiers willing to give the extra measure, placing victory ahead of personal survival, can lead to military defeat (Marshall, 1947) p.211, just as the lack

[14]For example, one man in six who was mobilized into the French Army during the first world war lost his life (with even greater losses concentrated among those who served in the infantry) (Ferguson, 1999) pp.299,364, other major belligerents sustained losses almost as high, and casualty figures for the US civil war are of the same order of magnitude.

of registered voters willing to cast secret ballots for a candidate can lead to electoral defeat.

Military commanders cannot take enthusiasm in battle for granted. Ashworth (1980) shows that combatants in the first world war had substantial leeway in choosing how aggressively to engage the enemy, with a more aggressive stance typically being associated with an elevated probability of being killed (Ashworth, 1980). A soldier's negligible impact on the ultimate outcome of a conflict, combined with the substantial avoidable risks that choosing an aggressive approach to combat entails, create substantial incentives for soldiers in combat to display only "a perfunctory showing of the daily discourtesies of war" (Grifiths (1931) pp. 71-3 quoted in Ashworth (1980) p.15). Indeed, Marshall (1947) p.54 famously asserted that only about 15% of infantrymen actually fire their weapons in combat. Although Glenn (2000) p.135 calls this numerical estimate into serious question, soldiers' discretion about how aggressively to contribute to combat is undoubtedly very substantial. Lynn (1984) argues that it was the commitment of citizen soldiers to fight that allowed the armies of revolutionary France to deploy large numbers of skirmishers operating on their own initiative. Until the British managed to copy this innovation, which requires combat motivated troops, it gave the French army an important advantage (Keegan, 1993). Costa and Kahn (2003) remark that the Civil War caused the death of one every five combatants. Yet, a soldier deserting would have faced only a 40% chance of being caught, and a minuscule risk of execution in that event (Linderman, 1987). Given the negligible impact of each individual enlisted man on the course of the war, desertion would have thus been the rational behavior of a self-interested soldier, yet over 90% of all Union Army soldiers did not do so.

Because of the decentralized control of guerrilla fighters, we believe that combat motivation is at least as important for informal combatants in civil conflicts as it is for soldiers in regular armies.

Some have contended that larger motives fade in combat. One German soldier captured during the Second World War[15] remarked that ideology "begins ten miles behind the front" (Shils and Janowitz, 1948). Yet it is widely recognized "...that belief in a cause is the foundation of the aggressive will in battle." (Marshall, 1947) p.162. Moreover, it appears that the *casus belli* is especially important in motivating fighters in a civil war. McPherson (1997) notes the importance of ending slavery as a motive for US troops, while the resolve of rebel soldiers was strengthened by the conviction that they were defending "liberty", while Costa and Kahn (2003) find that during hte same war, and even after controlling for various other influences, US soldiers from counties that had favored Lincoln in the 1860 election were less liklely to desert or to commit infractions. Dollard (1944) interviewed Republican fighters from the Spanish Civil war and found that their ideologically based "hatred of fascism" was an important motivating factor in combat.

Yet if soldiers' voluntary efforts are essential to victory, and their motive in fighting is to contribute to the victory of their cause then we are left with a seemingly paradoxical exertion of unrequited effort: the substantial costs of taking an aggressive stance in combat are in stark contrast with the negligible contribution of one individual to the collective outcome. Each of the three arguments we made in conjunction with voting has its parallel here: individuals may act out of a sense of duty, they may engage in "magical thinking", or they might overestimate their probability of having a decisive impact on the war[16]. Our agnostic approach toward these competing explanations is to show that if any of them applies to both the decision to participate in an

[15] Even Shils and Janowitz, prominent skeptics about the role of ideology and patriotism in motivating combat troops, note that their sample of German POWs contained few professional NCOs and fewer fanatical Nazi junior officers: both groups spurned surrender and fought effectively.

[16] Given the enormous difference between victory and defeat, even a small probability of being decisive, say on the order of one in ten thousand, might be enough to motivate a rational individual to fight, however for most soldiers in most wars the actual probability of being *individually* decisive is yet many of orders of magnitude smaller than even one in ten thousand.

election and to the much more costly decision to fight, then voting behavior provides a forecast of combat behavior.

We recognize that fighting is much more costly than voting, so that far more people may be willing to vote for a cause than are willing to risk their lives for it. Never the less, provided the disposition to engage in these two behaviors stand in proportion to one another, the election outcome can still provide information useful to forecast the relative number of willing combatants each side could draw on if conflict erupted into civil war.

Here, to simplify the exposition, we treat the decision of whether to participate aggressively in combat as binary[17]. We formulate several models of combat participation along the lines of the models of voting we presented in the previous section.

A Sense of Duty

Suppose now that individuals fight out of a sense of civic duty, As we argued in our discussion of duty and voting, let's suppose that individuals' civic duty to fight is proportional to what is at stake in the contest, while we can likewise assume that the level of resistance to performing one's civic duty to fight for the cause in which one believes is greater than an individual's resistance to voting, with the reticence to fight for faction α given by $\phi_\alpha > 1$ times the resistance to voting, *e.g.* people become more "socially autistic" when called upon to risk their lives than they are when they are merely expected to show up at their neighborhood polling station[18].

Thus, we can expect an individual to fight for faction L if:

[17]Both electoral politics and combat offer more than an "all or nothing" choice of how enthusiastically to participate: citizens can actively campaign and aid in voter turnout efforts, while soldiers have a whole range of choices in how much extra risk to take when engaging the enemy.

[18]Notice that we allow ϕ_L to differ from ϕ_R. Because of differences in arms and tactics, it may be that the extra cost of fighting for faction L differs from the cost of fighting for faction R. To see why this might be so, consider that during the US Civil War captured Union soldiers faced even more ghastly conditions in rebel prisons than did captured rebel soldiers languishing in Union camps.

$$\theta < \dot{x} \qquad \text{and} \qquad \chi < \frac{1}{\phi_L}\Big(d(x_R,\theta) - d(x_L,\theta)\Big)$$

whereas an individual for whom:

$$\theta > \dot{x} \qquad \text{and} \qquad \chi < \frac{1}{\phi_R}\Big(d(x_L,\theta) - d(x_R,\theta)\Big)$$

will fight for faction R.

Recalling our discussion of voting and a sense of duty, the density of individuals with preference parameters (θ, χ) is $g_D(\theta, \chi|\xi)$, where ξ is unknown, so that the probability that an individual is willing to fight for party L is given by:

$$\int_0^{\dot{x}} \int_0^{\frac{1}{\phi_L}(d(x_R,\theta)-d(x_L,\theta))} g_D(\theta, \chi|\xi) d\chi d\theta = \kappa_L^D(\xi, x_L, x_R)$$

similar calculations reveal that the corresponding probability that an individual is prepared to fight for party R is:

$$\int_{\dot{x}}^{1} \int_0^{\frac{1}{\phi_R}(d(x_L,\theta)-d(x_R,\theta)} g_D(\theta, \chi|\xi) d\chi d\theta = \kappa_R^D(\xi, x_L, x_R)$$

Magical Thinking

Suppose now that the "magical thinking" proposed by Quattrone and Tversky (1986) also motivates individuals to become belligerents in a civil war. Let us suppose that people's resistance to fighting for faction $\alpha \in \{L, R\}$ in a civil war is greater than their resistance to voting by a factor of $\omega_\alpha > 1$. Then, we see that an individual may be motivated to fight for faction L if:

$$\theta < \dot{x} \qquad \text{and} \qquad \mu < \frac{1}{\omega_L}\Big(d(x_R,\theta) - d(x_L,\theta)\Big)$$

whereas an individual for whom:

$$\theta > \dot{x} \qquad \text{and} \qquad \mu < \frac{1}{\omega_R}\Big(d(x_L,\theta) - d(x_R,\theta)\Big)$$

may be expected to fight for the faction on the right.

Again paralleling our discussion of voting, the density of individuals in the magical thinking model with preference parameters (θ, μ) is $g_M(\theta, \mu|\xi)$, where ξ is unknown, so that the probability that an individual is willing to fight for party L is given by:

$$\int_0^{\dot{x}} \int_0^{\frac{1}{\omega_L}(d(x_R,\theta)-d(x_L,\theta))} g_M(\theta, \mu|\xi) d\mu d\theta = \kappa_L^M(\xi, x_L, x_R)$$

similar calculations reveal that the corresponding probability that an individual is prepared to fight for party R is:

$$\int_{\dot{x}}^{1} \int_0^{\frac{1}{\omega_R}(d(x_L,\theta)-d(x_R,\theta))} g_D(\theta, \mu|\xi) d\mu d\theta = \kappa_R^M(\xi, x_L, x_R)$$

Combat and the Illusion of being Pivotal

Suppose that an individual has the illusion of being pivotal to the outcome of an armed conflict. This gives rise to a situation that is similar to the effects of this kind of illusion on the voting decision. Let's suppose that an individual's assessed probability that he is pivotal to the outcome of a war is $\rho'\delta$, while the true, infinitesimal, probability ϵ^* is effectively equal to 0. Likewise, suppose that the cost of fighting, $C_\beta = \rho''_\beta c$, where we expect that $\rho''_\beta > 1$, fighting for one's cause is considerably more costly than merely casting a vote for it[19].

[19] Not only are the immediate costs of fighting higher than those of voting, there may be long term entailments even for those who escape uninjured from the conflict itself. Angrist (1998) shows that many US soldiers who volunteered for service in the years immediately *after* Vietnam experienced reduced lifetime earnings, suggesting that absent a sense of duty it would have been misguided for them to have enlisted. But see Ticchi and Vindigni (2003a) and Acemoglu and Robinson (2005) who argue that credible promises of post war benefits make C_β, the net "non-policy" costs of fighting, negative. However, we note that a heavy reliance on the draft would seem to be redundant if the expected returns to enthusiastic fighting were universally perceived as positive.

If we let $\rho_L \equiv \frac{\rho''_L}{\rho'_L}$, calculations parallel to those we undertook for the case of voting show that a type (δ, θ) individual will fight for faction L provided:

$$\theta < \dot{x} \qquad \text{and} \qquad \gamma < \frac{1}{\rho_L}[d(x_R, \theta) - d(x_L, \theta)]$$

where $\dot{x}$ is as defined earlier in equation (7), and γ is the individual's inertia with respect to voting. An individual will take up armes for R if instead:

$$\theta > \dot{x} \qquad \text{and} \qquad \gamma < \frac{1}{\rho_R}[d(x_L, \theta) - d(x_R, \theta)]$$

while otherwise, he will attempt to remain neutral in the fighting.

Again paralleling our discussion of voting, the density of individuals in the magical thinking model with preference parameters (θ, γ) is $g_I(\theta, \gamma|\xi)$, where ξ is unknown, so that the probability that an individual is willing to fight for party L is given by:

$$\int_0^{\dot{\gamma}_L} \int_0^{\frac{1}{\rho_L}(d(x_R,\theta)-d(x_L,\theta))} g_I(\theta, \gamma_L|\xi) d\mu d\theta = \kappa_D^I(\xi, x_L, x_R)$$

similar calculations reveal that the corresponding probability that an individual is prepared to fight for party R is:

$$\int_{\dot{\gamma}_R}^1 \int_0^{\frac{1}{\rho_R}(d(x_R,\theta)-d(x_L,\theta))} g_I(\theta, \gamma_L|\xi) d\mu d\theta = \kappa_R^I(\xi, x_L, x_R)$$

3 Combat and Insurrection

For each of the three models considered in the preceding section, the decision to fight for party $\alpha \in \{L, R\}$ is similar to the decision to vote for that party, though the higher cost of fighting does entail a greater reluctance, for all partisans.

As in our calculation of the probability that an individual chooses to vote for party L given she casts a ballot, so too we can compute the probability that a combatant chooses to fight for L:

$$\tilde{q}_L^{\alpha}(\xi) = \frac{\kappa_L^{\alpha}(\xi, x_L, x_R)}{\kappa_L^{\alpha}(\xi, x_L, x_R) + \kappa_R^{\alpha}(\xi, x_L, x_R)}$$

Where the dependence of the voting probability on the unchanging party positions x_L and x_R is left implicit.

As in the discussion of voting, the prior beliefs of the party L leadership in model α about ξ are given by $h_L^{\alpha}(\xi)$ –the same population who vote are also potential recruits in a civil war if one should break out. Of course, differences between κ_L^{α} and ζ_L^{α}, which reflect differences between the voting and fighting decisions, will result in differences between the probabilities of voting and of fighting.

While the probability that side L would prevail in a civil war itself depends on the relative number of fighters it can muster, other considerations will matter. These include the likely leanings of the armed forces, and relative access to resources. The disposition of the armed forces has been decisive in many civil wars, such as Spain's struggle in the 1930's, and in many coups in which the advantage of the armed forces was so decisive they faced no more than token opposition. Likewise, miners exporting high value minerals have often bought state of the art weapons useful in winning civil wars, consider the Boers in their battle to expel the British, or the copper miners of northern Chile in that country's 1890 civil war. However, numbers still matter–for example in the US Civil War of the 1860's the wealthy cotton exporting Southern US was ultimately unable to prevail against the superior numbers of the North. Let's suppose that if civil war were to break out, the probability L would win is:

$$p_L(\xi) = f(\tilde{q}_L^{\alpha}(\xi))$$

Notice that this might either exceed or fall short of the probability that one casts a vote for L, what matters for our purposes is that it is related to the number of fighters one can hope to raise, and so to ξ. Because of their mutual dependence on ξ, the number of votes one can garner in an election and the size of the army of volunteer combatants one can raise are related, and hence, election results can be used to make inferences about fighting outcomes:

$$E_L^{\alpha}\{\mathfrak{p}_L\} = \int f(\tilde{q}_L^{\alpha}(\xi))h_L^{\alpha}(\xi)d\xi$$

and:

$$E_R^{\alpha}\{\mathfrak{p}_L\} = \int f(\tilde{q}_L^{\alpha}(\xi))h_R^{\alpha}(\xi)d\xi$$

If there is a fair election before the two parties negotiate then the posterior individual rationality constraint for the party L leadership will be given by:

$$d(x, x_L) \leq E_{L,\text{post}}\{\mathfrak{p}_L^{\alpha}\}C + (1 - E_{L,\text{post}}\{\mathfrak{p}_L^{\alpha}\})\,(d(x_R, x_L) + D)$$

where:

$$E_{L,\text{post}}\{\mathfrak{p}_L^{\alpha}\} = \int f(\tilde{q}_L^{\alpha}(\xi))h_{L,\text{post}}^{\alpha}(\xi)d\xi$$

and:

$$h_{L,\text{post}}^{\alpha}(\xi) = \frac{[\tilde{p}_L^{\alpha}(\xi)]^{N_L}[1 - \tilde{p}_L^{\alpha}(\xi)]^{N-N_L}h_L^{\alpha}(\xi)}{\int[\tilde{p}_L^{\alpha}(z)]^{N_L}[1 - \tilde{p}_L^{\alpha}(z)]^{N-N_L}h_L^{\alpha}(z)dz} \tag{14}$$

While the constraint for the party R leadership will be:

$$d(x, x_L) \leq E_{R,\text{post}}\{\mathfrak{p}_L^{\alpha}\}C + (1 - E_{R,\text{post}}\{\mathfrak{p}_L^{\alpha}\})\,(d(x_R, x_L) + D)$$

where:

$$E_{L,post}\{p_L^{\alpha}\} = \int f(\tilde{q}_L^{\alpha}(\xi))h_{R,post}^{\alpha}(\xi)d\xi$$

and:

$$h_{R,post}^{\alpha}(\xi) = \frac{[\tilde{p}_R^{\alpha}(\xi)]^{N_L}[1-\tilde{p}_R^{\alpha}(\xi)]^{N-N_L}h_L^{\alpha}(\xi)}{\int[\tilde{p}_R^{\alpha}(z)]^{N_L}[1-\tilde{p}_R^{\alpha}(z)]^{N-N_L}h_L^{\alpha}(z)dz}$$

Will the post-election beliefs about civil war fighting capabilities tend to converge? They will do so if both parties put enough weight on the election outcome.

To see this, let $\hat{p} \equiv \frac{N_L}{N}$ denote the relative frequency of L votes in the election, while:

$$\xi_L^{\alpha*} \equiv p_L^{\alpha^{-1}}(\hat{p})$$

that is, $\xi_L^{\alpha*}$ is the value for ξ that gives rise to a probability of $\hat{p}$ that a randomly chosen person votes for party L. We can now show that for a large enough election the posterior beliefs about ξ for both sets of leaders will coincide with $\hat{p}$:

Claim 3: For $\xi \neq \xi_L^{\alpha*}$:

$$\frac{h_{L,post}^{\alpha}(\xi_L^{\alpha*})}{h_{L,post}^{\alpha}(\xi)} \to \infty \text{ and } \frac{h_{R,post}^{\alpha}(\xi_L^{\alpha*})}{h_{R,post}^{\alpha}(\xi)} \to \infty \text{ as } N \to \infty$$

Proof: See Appendix A.

This tells us that for a large enough electorate, and for any given intensity of prior beliefs, the post-election beliefs of both sets of party leaders about ξ will converge to $\xi_L^{\alpha*}$. Given Claim 1 in section 1, shared beliefs about the outcome of a war guarantee that there will exist a set of peaceful solutions that will be preferred by both sides to war.

4 Discussion of the Results

This result is asymptotic, and one can construct examples in which, when the number of voters is fixed, and the faction leaders have just the wrong set of beliefs, elections can shrink the set of potential bargains (see appendix B). The key ingredients of such a counterexample are a faction that is highly attached to its prior belief that it is very likely to win a civil war, a second faction that has a very loose attachment to its belief that it is likely to loose, and an lopsided election in favor of the faction with weakly held pessimistic beliefs. Prior to such an election the settlements that will avoid civil war favor the optimistic faction. After the election the winners, who were loosely attached to their prior beliefs, optimistically revise their beliefs, while the other faction brushes aside the bad electoral news and clings to its strongly held optimistic prior. This combination of circumstances can result in the disappearance of the set of feasible bargains, and so guarantee that civil war erupts. However, we view these circumstances as very unusual in application.

The boundaries of the set of feasible policies revealed by elections in our model apply accross the spectrum of political institutions capable of supporting fair elections. Our model even offers a rationale for why elections may be useful in non-democratic societies where, by definition, the power of the government in office is not legally constrained by elections. Specifically, the government may chose to have elections in order to assess the relative strength of its opponent(s), even if the electoral outcome was not by itself decisive. Consider for example the 1988 Mexican presidential election. It is widely believed that opposition candidate Cuauhtémoc Cárdenas actually received more ballots only to have the candidate of the long-ruling PRI party, Carlos Salinas fraudulently declared the winner. Even though the PRI remained in power, that election marked a turning point, with policies allowing more transparency and freer elections following over the ensuing years. The

interpretation of events suggested by our model is that all concerned became aware of the magnitude of discontent with the PRI, and so the set of feasible policies that would avoid civil war shifted in favor of the opposition. While the PRI remained in power it chose from among its remaining options the bargain that was most favorable to itself, but it was constrained to cede ground by the credible signal sent by the election–failure to change carried a real risk of civil war.

Central to our model is that mass participation in social unrest, in this paper we focus on civil war, can be decisive. During some historical periods massive numbers of armed civilians with relatively little training have determined the result, while at other times the battlefield has been dominated by professional soldiers. In particular, the invention of muskets, especially with the relatively easy to use flint-lock mechanism, made it possible for a recruit with comparatively little training to fight more or less on par with professionally trained soldiers. This became a decisive consideration during the US war for independence and the French revolution, and military technology continued to minimize the advantage of professional soldiers into the twentieth century.

Our discussion has focused on civil war, but we note that the basic structure of the argument may be extended in several directions, both towards other types of domestic threats, and also into the arena of international conflict. A general strike can bring an industrialized society to a standstill–the lost output from a protracted strike can have an impact similar to the wealth and output lost during an armed conflict. If the losers from a policy can credibly threaten a massive strike, their views cannot be lightly ignored in the policy making process. If elections signal the size of a strike one can bring about they may be useful in avoiding the need for a walk-out. Other signals besides elections, such mass public marches, can demonstrate a faction's

potential for mischief as well. Likewise, our argument may be usefully generalizable to international conflict situations, with voluntary high turnout elections potentially adding to the bargaining credibility of democracies[20].

Conclusion

In this paper we develop a model that illustrates the way in which fair elections, by revealing the relative fighting strength of the factions, create a shared set of expectations about the consequences of a potential civil war. If beliefs converge sufficiently there will exist a set of compromises that both factions will prefer *ex ante* to civil war.

The key ingredient to our model is that the willingness to vote in electoral competition signals a willingness to fight if conflict emerges. In our analysis we identify several behavioral models that can produce such a correlation, but what is essential is that the election provide a useful signal about fighting intentions.

A Proofs

Proof of Claim 1: The result is an immediate consequence of Lemma 1 and Lemma 2.

Lemma 1: If $d(x+y,x)$ is convex in y, so that both sides are risk averse about policy outcomes, and if war fighting is costly, so that $D \geq C > 0$, then if both sides share the same beliefs about p_L, so that $E_L\{p_L\} = E_R\{p_L\} = p_L^0$, it follows that $x_L^* > p_L^0 x_L + (1-p_L^0)x_R$.

Proof of Lemma 1: By convexity of $d(x+y,x)$ in y we know that :

$$\begin{aligned} d(p_L^0 x_L + (1-p_L^0)x_R, x_L) &\leq p_L^0 d(x_L,x_L) + (1-p_L^0)d(x_R,x_L) = (1-p_L^0)d(x_R,x_L) \\ &< (1-p_L^0)d(x_R,x_L) + p_L^0 C + (1-p_L^0)D = d(x_L^*,x_L) \quad (15) \end{aligned}$$

[20]Notice that this argument is different from that of Fearon (1994) who invokes audience costs.

But $d_1(y, x_L)(y - x_L) > 0$ for $y \neq x_L$, hence, $x_L^* > p_L x_L + (1 - p_L)x_R$ $\square$.

Lemma 2: If $d(x + y, x)$ is convex in y, so that both sides are risk averse about policy outcomes, and if war fighting is costly, so that $D \geq C > 0$, then if both sides share the same beliefs about p_L, so that $E_L\{p_L\} = E_R\{p_L\}$, it follows that $x_R^* < p_L x_L + (1 - p_L)x_R$.

Proof of Lemma 2: By convexity of $d(x + y, x)$ in y we know that :

$$\begin{aligned} d(p_L^0 x_L + (1 - p_L^0)x_R, x_R) &\leq p_L^0 d(x_L, x_R) + (1 - p_L^0)d(x_R, x_R) = p_L^0 d(x_L, x_R) \\ &< p_L^0 d(x_R, x_L) + p_L^0 C + (1 - p_L^0)D = d(x_R^*, x_R) \quad (16) \end{aligned}$$

But $d_1(y, x_R)(y - x_R) > 0$ for $y \neq x_R$, hence, $x_R^* < p_L^0 x_L + (1 - p_L^0)x_R$ $\square$.

Proof of Claim 2a: Noting that[21] $x_L^* > x_L$, so that $d_1(x_L^*, x_L) > 0$, *e.g.* rightward movement of policy from x_L^* increases its distance from x_L, we can implicitly differentiate equation (2) with respect to p_L^ℓ to obtain:

$$\frac{\partial x_L^*}{\partial p_L^\ell} = \frac{C - D - d(x_R, x_L)}{d_1(x_L^*, x_L)} < 0$$

because $C \leq D$, $d(x_R, x_L) > 0$ and $d_1(x_L^*, x_L) > 0$. $\square$

Proof of Claim 2b: Noting that[22] $x_R^* < x_R$, so that $d_1(x_R^*, x_R) < 0$, *e.g.* rightward movement of policy from x_L^* reduces its distance from x_L, we can implicitly differentiate equation (2) with respect to p_L^ℓ to obtain:

$$\frac{\partial x_R^*}{\partial p_L^\ell} = \frac{D + d(x_R, x_L) - C}{d_1(x_L^*, x_L)} < 0$$

because $D \geq C$, $d(x_R, x_L) > 0$ and $d_1(x_R^*, x_R) < 0$. $\square$

Proof of Claim 3: Notice that if we substitute from equation (14) we have:

[21] If $x_L^* = x_L$ then faction L is unwilling to compromise at all.
[22] If $x_R^* = x_R$ then faction R is unwilling to compromise at all.

$$\frac{h^{\alpha}_{L,post}(\xi^{\alpha*}_L)}{h^{\alpha}_{L,post}(\xi)} = \frac{[\tilde{p}^{\alpha}_L(\xi^{\alpha*}_L)]^{N_L}[1-\tilde{p}^{\alpha}_L(\xi^{\alpha*}_L)]^{N-N_L}h^{\alpha}_L(\xi^{\alpha*}_L)}{[\tilde{p}^{\alpha}_L(\xi)]^{N_L}[1-\tilde{p}^{\alpha}_L(\xi)]^{N-N_L}h^{\alpha}_L(\xi)}$$

$$= \frac{\left([\tilde{p}^{\alpha}_L(\xi^{\alpha*}_L)]^{\hat{p}}[1-\tilde{p}^{\alpha}_L(\xi^{\alpha*}_L)]^{1-\hat{p}}\right)^N h^{\alpha}_L(\xi^{\alpha*}_L)}{\left([^{\alpha}_L(\xi)]^{\hat{p}}[1-\tilde{p}^{\alpha}_L(\xi)]^{1-\hat{p}}\right)^N h^{\alpha}_L(\xi)}$$

$$= \frac{\left([\hat{p}]^{\hat{p}}[1-\hat{p}]^{1-\hat{p}}\right)^N h^{\alpha}_L(\xi^{\alpha*}_L)}{\left([\tilde{p}^{\alpha}_L(\xi)]^{\hat{p}}[1-\tilde{p}^{\alpha}_L(\xi)]^{1-\hat{p}}\right)^N h^{\alpha}_L(\xi)}$$

$$= \left(\frac{[\hat{p}]^{\hat{p}}[1-\hat{p}]^{1-\hat{p}}}{[\tilde{p}^{\alpha}_L(\xi)]^{\hat{p}}[1-\tilde{p}^{\alpha}_L(\xi)]^{1-\hat{p}}}\right)^N \frac{h^{\alpha}_L(\xi^{\alpha*}_L)}{h^{\alpha}_L(\xi)}$$

Note that for $\xi \neq \xi^{\alpha*}_L$ we have:

$$\hat{p}^{\hat{p}}(1-\hat{p})^{1-\hat{p}} = \tilde{p}^{\alpha}_L(\xi^{\alpha*}_L)^{\hat{p}}(1-\tilde{p}^{\alpha}_L(\xi^{\alpha*}_L))^{1-\hat{p}} > \tilde{p}^{\alpha}_L(\xi)^{\hat{p}}(1-\tilde{p}^{\alpha}_L(\xi))^{1-\hat{p}}$$

where the inequality follows from the well know result that $\hat{p}$ maximizes $p^{\hat{p}}(1-p)^{1-\hat{p}}$, which is easily verified[23].

and so:

$$\frac{\hat{p}^{\hat{p}}(1-\hat{p})^{1-\hat{p}}}{\tilde{p}^{\alpha}_L(\xi)^{\hat{p}}(1-\tilde{p}^{\alpha}_L(\xi))^{1-\hat{p}}} > 1$$

hence

$$\left(\frac{\hat{p}^{\hat{p}}(1-\hat{p})^{1-\hat{p}}}{\tilde{p}^{\alpha}_L(\xi)^{\hat{p}}(1-\tilde{p}^{\alpha}_L(\xi))^{1-\hat{p}}}\right)^N \to \infty \text{ as } N \to \infty$$

and so:

[23]Proof: Let $f(p) = p^{\hat{p}}(1-p)^{1-\hat{p}}$. Differentiating with respect to p we have:

$$f'(p) = \left(\frac{\hat{p}}{p} - \frac{1-\hat{p}}{1-p}\right)p^{\hat{p}}(1-p)^{1-\hat{p}}$$

thus we will have $f'(p) = 0$ if and only if:

$$\frac{\hat{p}}{p} = \frac{1-\hat{p}}{1-p}$$

that is, if and only if $p = \hat{p}$. That this point is maximal follows from the fact that:

$$f''(\hat{p}) = \frac{-1}{\hat{p}(1-\hat{p}}\hat{p}^{\hat{p}}(1-\hat{p})^{1-\hat{p}} < 0 \quad \square$$

$$\frac{h_L^{\alpha}(\xi_L^{\alpha*})}{h_L^{\alpha}(\xi)}\left(\frac{\hat{p}^{\hat{p}}(1-\hat{p})^{1-\hat{p}}}{\tilde{p}_L^{\alpha}(\xi)^{\hat{p}}(1-\tilde{p}_L^{\alpha}(\xi))^{1-\hat{p}}}\right)^N \to \infty \text{ as } N \to \infty \quad \square$$

A parallel argument establishes the result for party R.

B How Elections Can Make War More Likely

While the usual effect of more information is to promote similar beliefs, there are some pathological cases in which some additional information may actually remove the possibility of a negotiated settlement. This will arise when both sides begin believing that the weaker party, without loss of generality let's assume that this is party R, is likely to win. If party R is strongly attached to this belief, while the party that is actually stronger, party L, is not very attached to its prior beliefs, then post election beliefs can create conflict as party L updates to hold substantially more optimistic beliefs, while party R only slightly tones down its overly optimistic priors.

An example serves to illustrate the issues. To keep things very simple, suppose that the probability faction L prevails if there is armed conflict is simply equal to the probability that a randomly chosen invidual votes for party L[24]:

$$\mathfrak{p}(\xi) = p_L^{\alpha}(\xi)$$

and that party L's beliefs about ξ are such that it has a prior density over $p_L(\xi)$ of $\mathtt{beta}(\alpha_L, \beta_L)$, while the prior beliefs of party R about ξ correspond to a prior density of $\mathtt{beta}(\alpha_R, \beta_R)$ over $p_R(\xi)$. suppose further that

$$d(a,b) = |a-b| \text{ while } x_L = -\frac{1}{2} \ , \ x_R = \frac{1}{2} \ , \ C = \frac{1}{8} \text{ and } D = \frac{1}{4}$$

[24] We emphasize that this assumption is made only for the sake of making the example transparent, it is by no means essential to our result, which merely requires that the voting decision is positively correlated with the decision to fight.

Let $\alpha_L = 10^{-6}$ and $\beta_L = 3 \times 10^{-6}$ while $\alpha_R = 10^6$ and $\beta_R = 3 \times 10^6$. So, if there is no election the peaceful settlement interval[25] will be:

$$\mathfrak{S} = [x_R^*, x_L^*] = [\frac{3}{32}, \frac{15}{32}]$$

Now suppose that both leaders observe a free and fair election in which party L garners 1500 votes while party R earns 500. the posterior beliefs for party L will be $\texttt{beta}$ with parameters $\alpha_L^* = 10^{-6} + 1500$ and $\beta_L^* = 3 \times 10^{-6} + 500$, while party R will have posterior beliefs that are $\texttt{beta}$ with $\alpha_R^* = 10^6 + 1500$ and $\beta_R = 3 \times 10^6 + 500$. In this case, the rightmost settlement party L is willing to accept will be:

$$x_{L,\text{post}}^* = -\frac{1}{2} + \frac{\alpha_L^*}{\alpha_L^* + \beta_L^*} C + \frac{\beta_L^*}{\alpha_L^* + \beta_L^*}(D + 1) \approx -\frac{3}{32}$$

while the leftmost settlement party R will accept will be:

$$x_{R,\text{post}}^* = \frac{1}{2} - \frac{\alpha_L^*}{\alpha_L^* + \beta_L^*}(D + 1) - \frac{\beta_L^*}{\alpha_L^* + \beta_L^*} C \approx \frac{3}{32}$$

After the election there is no point which both parties would be simultaneously willing to accept instead of going to war. What has happened is that party L has learned its strength, and so it has abandoned its pessimistic prior expectations, while party R continues to cling to its overly optimistic prior beliefs. This is possible because party R is much more attached to its optimistic beliefs than party L is to its pessimistic ones, hence their beliefs converge towards the sample relative frequency at very different rates. Nevertheless, with a sufficiently large electorate, the evidence would become so

[25] Consulting equation (2) we see that:

$$x_L^* = -\frac{1}{2} + \frac{\alpha_L}{\alpha_L + \beta_L} C + \frac{\beta_L}{\alpha_L + \beta_L}(D + 1) = \frac{15}{32}$$

while substituting our hypothesized settings into equation (3) we have:

$$x_R^* = \frac{1}{2} - \frac{\alpha_R}{\alpha_R + \beta_R}(D + 1) + \frac{\beta_R}{\alpha_R + \beta_R} C = \frac{3}{32}$$

clear that even the stubbornly optimistic leaders of party R would accept party L's greater likelihood of winning, and a peaceful settlement would be possible.

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