

**Collegio Carlo Alberto**



The Time-Varying Risk of Macroeconomic  
Disasters

Roberto Marfè

Julien Penasse

No. 463  
July 2016

**Carlo Alberto Notebooks**

[www.carloalberto.org/research/working-papers](http://www.carloalberto.org/research/working-papers)

# The Time-Varying Risk of Macroeconomic Disasters\*

Roberto Marfè<sup>†</sup>      Julien Penasse<sup>‡</sup>

May 25, 2016

## Abstract

While time-varying disasters can explain many characteristics of financial markets, their quantitative assessment is still missing. We propose a latent variable approach to estimate the time-varying probability of a macroeconomic disaster, using a dataset of 42 countries over more than 100 years. We find that disaster risk is volatile and persistent, strongly correlates with the dividend yield, and forecasts stock returns. A state-of-the-art model calibrated with our disaster risk estimates generates a large and volatile equity premium and a low risk free rate under standard assumptions. This evidence supports the idea that investors' fear of disasters drives equity premium dynamics.

*JEL:* E44, G12, G17.

*Keywords:* rare disasters, equity premium, return predictability, state-space model.

---

\*We would like to thank seminar participants at the Luxembourg School of Finance and Columbia University for valuable comments and suggestions. We thank Robert Barro, José Ursúa, Asaf Manela, and Alan Moreira for making their data publicly available.

<sup>†</sup>Collegio Carlo Alberto, Via Real Collegio, 30, 10024 Moncalieri (Torino), Italy, Telephone: +39 (011) 670-5229; roberto.marfe@carloalberto.org; <http://robertomarfe.altervista.org/>.

<sup>‡</sup>Luxembourg School of Finance, 4 rue Albert Borschette, 1246 Luxembourg, Luxembourg; +352 466644 5824; julien.penasse@uni.lu; <http://jpenasse.wordpress.com>.

Many puzzles in macro-finance arise from the inability of standard models to quantitatively reconcile asset returns and macroeconomic risks. A classic example is the equity premium puzzle: average stock returns are far too high to be explained by the observed risk in consumption (Mehra and Prescott, 1985). Another famous example is that stock returns are predictable by valuation ratios such as the dividend yield (Campbell and Shiller, 1988b). Likewise, the volatility in the stock market is too great to represent forecasts of future dividends (Shiller, 1981).

A possible reconciliation is to assume that financial markets compensate the risk of infrequent but severe macroeconomic disasters. Using international data from the last 100 years, Barro (2006) documents a number of episodes with striking declines in consumption growth. Such disasters occur with an annual frequency of about 3% and are strong enough to generate a sizable equity premium, as hypothesized by Rietz (1988). A first generation of models assumes that disasters arise with a constant probability (see, e.g. Barro (2009); Barro and Jin (2011); Nakamura et al. (2013)). The next generation of papers shows that letting disasters vary over time can generate excess volatility and predictability (Gabaix, 2012; Gourio, 2012; Wachter, 2013; Hasler and Marfe, 2016), and solve puzzles pertaining to the bond and option markets (Gabaix, 2012; Gourio, 2013), and currencies such as the forward premium puzzle (Farhi and Gabaix, 2016).<sup>1</sup> Variable disasters also show promises in connecting macroeconomic aggregates with asset prices in production economies (Gabaix, 2011; Gourio, 2012; Kilic and Wachter, 2015).

In spite of its theoretical successes, the empirical evidence on rare disasters remains scarce. The strong limitation of the rare disaster hypothesis is that, by definition, rare disasters are difficult to measure. Extant models of time-varying disasters entirely rely on calibrations that are designed to match asset prices. The empirical literature has only provided indirect evidence that macroeconomic disasters really matter for asset prices (e.g. Berkman et al. (2011); Manela and Moreira (2015)). Quoting Gabaix (2012):

*A question very high on the empirical agenda is to find ‘objective’ measures*

---

<sup>1</sup>Another strand of the literature maintains a constant probability of a disasters but assumes that the representative agent learns about the model parameters such as this probability, e.g. Weitzman (2007), Koulovatianos and Wieland (2011), Du and Elkamhi (2012), Johannes et al. (2016).

*of disaster risk that ideally do not come from asset prices. [...] A fully structural empirical analysis has yet to be carried out.*

This paper precisely tackles this question, by estimating an econometric model where the risk of a disaster varies over time. We exclusively rely on macroeconomic data to estimate disaster risk. We use the fitted disaster probability to calibrate a state-of-the-art model and evaluate its asset pricing properties.

Formally, this paper proposes a latent variable approach to infer the next-year probability of a macroeconomic disaster. As in Barro (2006) and subsequent literature, we use an international panel to gauge the frequency and size of disasters in the United States. Unlike Barro, however, we relax the assumption that the probability of a disaster is constant over time. We instead treat the number of disasters occurring over a given year as a *signal* (which includes noise) of the true, unobserved, disaster probability. Under the assumption that this probability follows an exogenously specified time-series process, we infer the time-varying probability with a (non-Gaussian) Kalman filter.

We embed our estimated probability of time-varying disaster in the context of a state-of-the-art asset pricing model similar to Wachter (2013). Namely, we consider a discrete-time economy populated by a representative agent with recursive preferences, and where asset prices are derived from the dynamics of aggregate consumption. Consumption growth is subject to rare disasters driven by a time-varying probability. The model is designed in the exponential-affine class and, hence, can be solved with standard techniques and provides simple closed form solutions. We use this model as a simple laboratory to assess both the qualitative and quantitative effects of our measure of time-varying disaster probability, our single state variable. The predictions of the model span a number of asset pricing quantities, such as the risk-free rate, the equity premium, the return volatility and the dividend yield.

We find that disaster risk is time-varying, persistent, and is a good predictor of future disasters — not only at the aggregate level, but also country per country. For example, although the United States experienced only four disasters, we find that our filtered predictability forecasts these disasters significantly with a slope of about one. In addition,

the probability strongly correlates (both in levels and differences) with aggregate valuation ratios, such as the stock market dividend yield. This implies that our estimated probability effectively captures U.S. disaster risk, but also the time-series dynamics of the equity premium.<sup>2</sup> Notably, the disaster probability predicts returns in the post-war data. This suggests that disaster concerns drive the equity premium even when disasters do not actually take place, as in the post-war U.S. economy.

While we only use macroeconomic data to estimate our model, we find that its deep parameter are close to Wachter (2013)'s, which were calibrated to fit key asset pricing moments. Consequently, the model can generate a large and volatile equity premium together with a low risk free rate under conservative preferences (unit elasticity of intertemporal substitution and relative risk aversion of two). Time-varying disaster risk shapes the variation of return volatility and risk premium, but it is also the main source of their level. Model-implied predictability is consistent with the intertemporal relation between disaster risk and excess returns and with the lack of consumption growth predictability in actual data.

This paper contributes to the rapidly growing literature on disaster risk. Our approach complements the former analyses by Barro (2006), Barro and Jin (2011) and Nakamura et al. (2013) who compute the (constant) probability of a disaster using international data. In its spirit, our paper is also related to Berkman et al. (2011), who proxy disaster risk using the number and severity of international political crises. Consistently with our results, their measure of political disaster risk correlates with the U.S. dividend yield. They also find that disaster risk is contemporaneously related to stock returns (although it doesn't forecast returns). Another natural venue to estimate disaster risk is to examine option prices. Backus et al. (2011) extract market crash risk premia from index options, but assumes a constant disaster probability. Seo and Wachter (2015), Gao and Song (2015), Kelly and Jiang (2014), Farhi et al. (2015), and Siriwardane (2015) assume time-varying disaster risk and infer tail risk premia from option prices and from

---

<sup>2</sup>van Binsbergen and Koijen (2010) provide evidence that the dividend yield effectively captures future returns, and much less expected dividend growth.

the cross section of stock returns. [Manela and Moreira \(2015\)](#) constructs a proxy for the VIX using front-page articles of the *Wall Street Journal* that extends back to 1890. Their measure is positively correlated with ours, and predicts returns in postwar data. Interestingly, they find that a significant share of the variation of implied risk premia can be traced back to rare events such as wars and tax-policy policy changes.

A limitation of these prior approaches is that they are indirect. For example, they are — by construction — unable to pin down the average probability of a disaster. Consequently, these papers mostly focus on the relation between asset returns and *changes* in disaster risk. The general tenet of this literature is that stocks that load highly on disaster risk tend to earn higher returns, suggesting that disaster risk is a priced factor in the cross section of stock returns. Estimates based on asset prices come with the extra cost that they represent the *risk-neutral* probability of a disaster.

Our paper differs from the prior literature in both its focus and its methodology. Our focus is on the time-series (rather than cross-sectional) properties of disaster risk, and its relation with macroeconomic aggregates. Our methodology extracts the physical probability of a macroeconomic disaster, rather than the risk neutral probability of a market crash. This enables us to verify whether these time series properties can generate a sizeable and volatile equity premium — we find that this is the case. We also offer empirical support to the idea that the slow decline in the dividend yield can be captured by a persistent decrease in macroeconomic risk, as previously noted in [Lettau et al. \(2008\)](#).

The remainder of this paper is organized as follows. Section [I](#) describes our asset pricing model of time-varying rare disasters. Section [II](#) discusses the data and our estimation strategy. Section [III](#) studies the time-series properties of disaster risk. Section [IV](#) assesses the asset-pricing implications of our model. Section [V](#) concludes.

## I. A Model of Time-Varying Disasters

This section presents a simple equilibrium asset pricing model with time-varying disaster risk. The model represents a discrete-time version of [Wachter \(2013\)](#) and belongs to the

discrete-time affine class proposed by [Drechsler and Yaron \(2011\)](#). We first describe the economy and the dynamics of aggregate consumption and, then, we derive equilibrium asset prices.

### A. *The Economy*

We consider a pure-exchange economy à la [Lucas \(1978\)](#) populated by a representative investor with recursive preferences ([Epstein and Zin, 1989](#)). We focus on the case of elasticity of intertemporal substitution equal to one for the sake of tractability. Similarly to [Collin-Dufresne et al. \(2016\)](#), it is useful to normalize utility  $V$  by consumption level  $C$  such that the log value function  $vc_t \equiv \log V_t/C_t$  is given by

$$vc_t = \frac{\delta}{1-\gamma} \log \mathbb{E}_t [e^{(1-\gamma)(\Delta c_{t+1} + vc_{t+1})}]. \quad (1)$$

where  $\delta$  is the time discount factor,  $\gamma \neq 1$  is the relative risk aversion and  $\Delta c_{t+1} \equiv \log C_{t+1}/C_t$  is consumption growth. The dynamics of the latter evolve according to:

$$\Delta c_t = \mu + \sigma \epsilon_t + v_t \quad (2)$$

where  $\mu$  and  $\sigma$  are constants,  $\epsilon_t$  is a standard normal random variable and  $v_t$  is a disaster term. We model  $v_t$  as a compound-Poisson shock:  $v_t = J_t \mathbf{1}_{N_t \neq N_{t-1}}$ .  $N_t$  is a Poisson counting process and the disaster event ( $N_t \neq N_{t-1}$ ) has size described by a random variable  $J_t$  with time-invariant distribution. The latter is governed by the moment generating function  $\varphi(u) = \mathbb{E}[e^{uJ_t}]$ . The intensity of disaster arrivals is time-varying and modeled by a discretized square-root process  $\pi_t$ :

$$\pi_t - \bar{\pi} = \rho(\pi_{t-1} - \bar{\pi}) + \nu \sqrt{\pi_{t-1}} u_t \quad (3)$$

where  $\bar{\pi} > 0$ ,  $0 < \rho < 1$  and  $\nu > 0$  are constants and  $u_t$  is a standard normal random variable uncorrelated with  $\epsilon_t$ .

These dynamics have been shown to be useful to capture a number of asset pricing

facts (Wachter, 2013). Our model corresponds to prior models with constant disaster probability (e.g. Barro (2006); Barro and Jin (2011)) when  $\pi_t$  is time invariant. As in Barro (2006), we treat the contraction proportion,  $J_t$ , as a random variable with time-invariant distribution. The latter is later estimated from international consumption data. We keep the model simple and do not account for either unfolding disasters or post-disaster recovery (Gourio (2008), Nakamura et al. (2013), Hasler and Marfe (2016)). This is because the focus is on the time-series relationship between disaster intensity and equilibrium asset prices, which is qualitatively unaffected by the specific shape of the disaster path.

Finally, the aggregate dividend paid by the equity claim is modeled in a very parsimonious way as in Abel (1999), Campbell (2003) and Wachter (2013):  $\Delta d_{t+1} = \phi \Delta c_{t+1}$ . Although simplistic, for  $\phi > 1$  dividends fall by more than consumption in the event of a disaster, consistently with the U.S. data (Longstaff and Piazzesi, 2004).

## B. Equilibrium

In order to solve for the prices, we express the stochastic discount factor in terms of the investor's value function, which is affine in the disaster intensity  $\pi_t$ . Then, we solve for the return on the equity claim via the investor's Euler equation up to the usual Campbell and Shiller (1988a)'s log-linearization. The stochastic discount factor is given by

$$M_{t+1} = \underbrace{\delta e^{-\gamma \Delta c_{t+1}}}_{\text{discounting of expected utility}} \times \underbrace{\frac{e^{-(1-\gamma)v c_{t+1}}}{\mathbb{E}_t [e^{(1-\gamma)(\Delta c_{t+1} + v c_{t+1})}]}_{\text{discounting of continuation utility}}, \quad (4)$$

(Collin-Dufresne et al., 2016) and the value function satisfies

$$v c_t = v_0 + v_\pi \pi_t, \quad (5)$$

where

$$v_0 = \frac{\delta}{1-\delta}(\mu + 2\bar{\pi}(1-\rho)v_\pi - \gamma\sigma^2/2),$$

$$v_\pi = \frac{(1-\delta)(1-\gamma) + \sqrt{(1-\gamma)^2(1+\delta(\delta+2\delta\sigma^2-2)) - 2\delta\sigma^2\varphi(1-\gamma)}}{\delta(1-\gamma)^2\sigma^2}.$$

Note that the stochastic discount factor variance (i.e. the price of risk in the economy) is increasing in disaster intensity:

$$\text{var}_t(\log M_{t+1}) = \gamma^2\sigma^2 + (\gamma-1)^2v_\pi^2\nu^2\pi_t + \gamma^2\mathbb{E}_t[J_{t+1}^2]\pi_t$$

where  $\mathbb{E}_t[J_{t+1}^2]$  is given by the second order derivative of the moment generating function evaluated at zero.

The stochastic discount factor implies the following results. First, investor's wealth is proportional to consumption:  $W_t = C_t \frac{\delta}{1-\delta}$ , and the log return on wealth satisfies  $r_{c,t+1} = -\log \delta + \Delta c_{t+1}$ , given that the elasticity of intertemporal substitution equal to one. Second, the log risk-free rate,  $r_{f,t} = -\log \mathbb{E}_t[M_{t+1}]$ , is affine in the disaster intensity:

$$r_{f,t} = -\log \delta + \mu - \gamma\sigma^2 + \pi_t(\varphi(1-\gamma) - \varphi(-\gamma)). \quad (6)$$

The risk-free rate is stationary and linearly decreasing with disaster intensity  $\pi$ . The reason is that an increase in the probability of a disaster yields an increase in consumption risk, which the investor is willing to hedge with risk-free investments. This increase in the risk-free asset holding implies an increase in the price of the risk-free asset and thus a decrease in the risk-free rate. This effect increases in magnitude with relative risk aversion.

To solve for the equity claim price, we log-linearize returns around the unconditional mean of the log price-dividend ratio  $pd \equiv \mathbb{E}[pd_t]$  with  $pd_t \equiv \log P_t/D_t$ :

$$r_{d,t+1} = \log(e^{pd_{t+1}} + 1) - pd_t + \Delta d_{t+1} \approx k_0 + k_1 pd_{t+1} - pd_t + \Delta d_{t+1},$$

where the endogenous constants  $k_0$  and  $k_1$  satisfy

$$k_0 = -k_1 \log(k_1) - (1 - k_1) \log(1 - k_1) \quad \text{and} \quad k_1 = e^{pd}/(1 + e^{pd}).$$

Campbell et al. (1997) and Bansal et al. (2012) have documented the high accuracy of such a log-linearization, which we assume exact hereafter. Using the Euler equation  $1 = \mathbb{E}_t[M_{t+1}e^{r_{d,t+1}}]$ , we recover that the log price-dividend ratio is affine in the disaster intensity:

$$pd_t = p_0 + p_\pi \pi_t, \tag{7}$$

where

$$p_0 = \log(k_1) - \log(1 - k_1) - p_\pi \bar{\pi},$$

$$p_\pi = \frac{1}{k_1^2 \nu^2} \left( \Omega - \sqrt{\Omega^2 + 2k_1^2 \nu^2 (\varphi(1 - \gamma) - \varphi(\phi - \gamma))} \right),$$

with  $\Omega = 1 - (1 - \gamma)k_1^2 \nu^2 v_\pi \rho$ .

The price-dividend ratio is stationary and decreases (resp. increases) with the disaster intensity  $\pi_t$  when  $\gamma$  is larger (resp. smaller) than one. This reflects the preference for the early (late) resolution of uncertainty about time-variation in  $\pi_t$ . An increase in disaster intensity increases the likelihood that disasters will affect future consumption. Under preference for the early resolution of uncertainty ( $\gamma > 1$ ), the investor is worried about both current disaster risk as well as uncertainty in future disaster risk. Thus, prices are low relative to dividends when  $\pi_t$  is high and vice-versa. Note that the substitution and income effect offset each other when the elasticity intertemporal of substitution is equal to one, as we assume here.

The log equity premium is given by

$$\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} = \underbrace{\gamma \phi \sigma^2}_{\text{non-disaster risk}} + \underbrace{(\gamma - 1)k_1 p_\pi v_\pi \nu^2 \pi_t}_{\text{disaster intensity risk}} + \underbrace{(\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1)\pi_t}_{\text{disaster size risk}}. \tag{8}$$

The return variance equals

$$\text{var}_t(r_{d,t+1}) = \underbrace{\phi^2 \sigma^2}_{\text{non-disaster risk}} + \underbrace{k_1^2 p_\pi^2 \nu^2 \pi_t}_{\text{disaster intensity risk}} + \underbrace{\phi^2 \mathbb{E}_t(J_{t+1}^2)}_{\text{disaster size risk}} \pi_t. \quad (9)$$

Both the equity premium and the return variance are given by three terms. The first term concerns non-disaster risk and gives rise to the usual values in a [Lucas \(1978\)](#) economy. Variation in disaster intensity gives rise to the second component of the equity premium and return variance. This term is increasing with current disaster intensity as well as its persistence and volatility, whereas it disappears for  $\gamma = 1$ . The third term is associated with the disaster event and increases with disaster size variance and current disaster intensity.

## II. Empirical Implementation

This section first presents the international data set we use. Then, we explain how we measure macroeconomic disasters and comment on the resulting international disasters in the 20th century. Third, the section describes how we estimate the latent time-varying disaster probability.

### A. Data

We use the dataset on long-term consumer expenditures compiled by Robert Barro and José Ursúa, and described in [Barro and Ursúa \(2010\)](#). The dataset spans two centuries, but only contains a handful of countries in the early years. Modeling a time-varying disaster probability requires a rather large number of countries each year to obtain meaningful estimates. We therefore focus on the 1900-2009 period, which is the original time frame of [Barro \(2006\)](#). This leaves us with a minimum of 20 countries in 1900. The data comprises 25 OECD countries, 14 countries from Latin America and Asia, Egypt, Russia and South Africa. In Austria, Singapore and Malaysia, some observations are missing during at least one of the world wars. In these cases, we use a cubic spline to interpolate

missing observations. This choice is conservative because it tends to smooth consumption during these troubled periods and conducts us, in Singapore and Malaysia, to treat the years around World War II as non-disasters.

### *B. Measuring Rare Disasters*

Central to this paper is the measurement of macroeconomic disasters. We choose to estimate disaster risk by pooling international data. In this dimension, we strictly follow the prior literature pioneered by Barro (2006). Specifically, we assume that the probability and size parameters are equal across countries, and are relevant to characterize the behavior of the U.S. asset markets. These are of course strong assumptions. For example, the occurrence of a macroeconomic disaster in Argentina conducts us to assume that the risk of a disaster has increased *in the United States*. A disaster occurring in the United States would be given the same weight. We stick to this approach for its simplicity, and because it avoids the temptation to data snoop across various specifications. In addition, this approach brings the benefit that the model can pool information across countries. In particular, we can focus on the number of disasters each year, instead of country-level probabilities. From an econometric viewpoint, we have little statistical power to reject the assumption that conditional disaster probabilities differ across countries, given the scarcity of historical data. We provide evidence that our approach is well-suited for the United States in Section III.C.

An alternative interpretation of our measure is that it captures the risk of a world disaster, rather than the risk of a disaster in the United States. We still view our approach as a reasonable approximation under that interpretation, given both the size of the U.S. economy and the size of the U.S. stock market. Under the assumption that markets are perfectly integrated, investors who share the same preference parameters hold the world aggregate index and consume the world output. In principle, we could test our model using world consumption and a world stock index. We do not take this path because this would add another level of complexity and measurement error to our analysis. In addition, note that the probability of a U.S. disaster conditional on a world disaster is

likely to be very high. [Nakamura et al. \(2013\)](#) distinguish between domestic and world disasters and find that this conditional probability is above 60%.

Estimating our econometric model requires a quantitative definition of macroeconomic disasters. The standard Rietz-Barro approach is to measure disasters as peak-to-trough declines in consumption or GDP in excess of 10% or 15% in cross-section of countries. [Barro \(2006\)](#), for example, considers a panel of 35 countries and documents 60 episodes of GDP contractions greater than 15% in the 20th century. The rationale for using peak-to-trough measures is that, historically, disasters such as wars put countries into regimes of distress for several years. While fairly intuitive, this choice has a number of shortcomings. An important critique, in particular, is that it posits a model where disasters occur instantly and that the disasters have no time units (see e.g. [Constantinides \(2008\)](#), [Donaldson and Mehra \(2008\)](#), [Julliard and Ghosh \(2012\)](#)). Another concern is that a fixed threshold may overestimate the number of disasters in economies where consumption volatility is large. For example, in our sample, the annual volatility of Argentina’s consumption growth is 7.9%, which is about twice the volatility of U.S. consumption growth.

We treat disasters as deviations from normality. We start by normalizing international consumption series such that their means and volatilities match U.S. first moments. We then work with normalized series to measure the frequency and severity of macroeconomic disasters. Normalizing consumption amounts to assuming a country-specific threshold, so that a disaster in Argentina is a priori as unusual and as severe as a disaster in the United States. A disaster is defined as an *annual* drop in consumption, in excess of what could have usually been generated by a normal distribution. We choose two standard deviations as our threshold to detect deviations from normality. Formally, a country enters a disaster state if

$$\Delta c_t < \text{mean}(\Delta c_t) - 2 \times \text{stdev}(\Delta c_t) \approx -4.7\%. \quad (10)$$

Eq. (10) essentially filters out disasters from consumption innovations, so that the left

tail of consumption growth is approximately normal.<sup>3</sup>

Figure 1 illustrates our reasoning by showing the cross-country distributions of annual consumption growth, with and without disasters. The solid line represents the raw distribution of consumption growth. It is heavily skewed to the left, which is the main motivation to incorporate disasters in our model. The dashed line gives the kernel distribution which excludes disasters. We see that this line approximately coincides with the third (dotted) line corresponding to the gaussian approximation. Our specification leaves a small skew in the right tail of consumption growth distribution excluding disasters, which reflects our choice to concentrate on disasters and to leave apart rare macroeconomic booms. Such booms matter little for asset prices when risk aversion is relatively large, as we assume here.

We denote by  $\tilde{N}_t$  the number of countries entering in a disaster at time  $t$ . (We use a tilde to distinguish  $N_t$ , the disaster count variable in a given country and used in Eq. (2), from  $\tilde{N}_t$ , the total number of disasters across countries.) In constructing the count variable  $\tilde{N}_t$ , we account for the fact that our panel is unbalanced. We first divide the number of disasters at time  $t$  by the number of countries for which data is available in  $t$  and multiply it by 42. We then round this quantity to the nearest integer.

A two-standard deviations threshold produces disasters which are about as rare as in the peak-to-through approach with a 10% threshold. For example, Barro and Jin (2011) report an average 3.8% disaster frequency, while we obtain  $\sum_t \tilde{N}_t = 181$  disasters, which corresponds to a 3.9% disaster frequency.<sup>4</sup> Our approach differs in that it generates shorter and smaller disasters. Disasters last for one year by assumption and we find an average size of 7.7%. In comparison, Barro and Jin (2011) report a typical size of 20% and an average duration of 3.5 years. Tsai and Wachter (2015) note that assuming large disasters or multiple small declines in consumption is relatively innocuous, provided that the representative agent prefers early resolution of uncertainty. Nakamura et al. (2013) model long-lasting disasters followed by partial recoveries and find that such a model

---

<sup>3</sup>Note that we assume that agents know the first moments of U.S. consumption growth.

<sup>4</sup>We report the list of disasters in the Online Appendix, Table A.I.

still generates a sizable equity premium when the representative agent exhibits [Epstein and Zin \(1989\)](#) preferences.<sup>5</sup> Since our focus is on time-varying risk premia and excess volatility, we assume short-lived disasters for simplicity and tractability.

Before moving on to the estimation of our model, it is interesting to ask whether our sample of international countries is representative of disaster risk in the United States. Counting disasters, we find that the United States experienced 4 disasters over the sample period, corresponding to a 3.6% disaster probability. The average U.S. disaster size is 7.0%. Both are close to their cross-country averages, which is probably unsurprising because we normalize international consumption growth rates to match the first two moments of U.S. consumption growth.

### C. Estimation

We next describe our statistical model.  $\tilde{N}_t$  follows a time-varying Poisson distribution:

$$p(\tilde{N}_t|\lambda_t) = \exp \left[ \tilde{N}_t \lambda_t - \exp(\lambda_t) - \log(\tilde{N}_t!) \right], \quad (11)$$

where  $\lambda_t$  is the model's state variable. Eq. (11) implies that the expected number of disasters over one period is  $\exp(\lambda_t)$ . A disaster probability is thus defined implicitly as  $\pi_t^* = \exp(\lambda_t)/N$ . We assume that  $\lambda_t$  follows an AR(1):

$$\lambda_{t+1} - \bar{\lambda} = \rho^*(\lambda_t - \bar{\lambda}) + \nu^* e_t. \quad (12)$$

Equations (11) and (12) define together a state-space system. The former equation is observed and constitutes our measurement equation. The latter equation is unobserved and will be estimated using the Kalman filter. Ideally,  $\pi_t$  would replace  $\lambda_t$  as state variable, and the law of motion for  $\pi_t$  would be given by (3). From an econometric perspective, however, the exponential AR specification is more convenient because it ensures that  $\pi_t^*$  remains positive. By contrast, at annual frequency, the discretized square-

---

<sup>5</sup>[Hasler and Marfe \(2016\)](#) find that assuming recoveries helps generating a downward-sloping term-structures of equity risk premia, as in the data.

root process has a non-negligible probability to generate negative values.

The model is fairly simple and consists of only three parameters  $\psi = (\bar{\lambda}, \rho^*, \nu^*)$ . However, analytical methods are not feasible for this kind of non-Gaussian observations and we must rely on simulation techniques. In particular, we adopt the method developed in [Durbin and Koopman \(1997\)](#), which consists in approximating the non-Gaussian model by a traditional linear Gaussian model. The true likelihood can then be computed by adjusting the likelihood of the approximating Gaussian model (which is available in closed form). The adjustment term corrects for the departure of the true likelihood from the Gaussian likelihood and is obtained by simulation.

Formally, the likelihood is defined as  $\mathcal{L}(\psi) = p(\tilde{N}|\psi)$  and corresponds to the joint density of the model defined by (11) and (12). We have:

$$\begin{aligned} \mathcal{L}(\psi) = p(\tilde{N}|\psi) &= \int_{\lambda} p(\tilde{N}, \lambda|\psi) d\lambda \\ &= \int_{\lambda} p(\tilde{N}|\lambda, \psi) p(\lambda|\psi) d\lambda \end{aligned} \quad (13)$$

In the first line of Eq. (13), we write the likelihood as the joint distribution of the observed disasters  $\tilde{N}$  and the unobserved state variables  $\lambda$ , given  $\psi$ . In the second line, the joint distribution of  $\tilde{N}$  and  $\lambda$  is expressed as the conditional density of  $\tilde{N}$  given  $\lambda$  multiplied by the marginal density of  $\lambda$ .

A closed form solution for integral (13) is not available if  $p(\tilde{N}_t, \lambda_t|\psi)$  follows a Poisson distribution. In principle, we could simulate trajectories of  $\lambda$  from the density  $p(\lambda|\psi)$  and evaluate (13) by the sample mean of the corresponding values of  $p(\tilde{N}|\lambda, \psi)$ . Rather, we rely on importance sampling technique (e.g. [Ripley \(1987\)](#)), which as been shown to be much more efficient in this context. Importance sampling consists in choosing a density  $g(\lambda|\tilde{N}, \psi)$  that is as close to  $p(\lambda|\tilde{N}, \psi)$  as possible. One can then evaluate the likelihood (13) while making an appropriate adjustment to correct for the difference between the true density and its approximation. First, observe that provided  $g(\lambda|\tilde{N}, \psi)$  is positive

everywhere, (13) can be rewritten as

$$\begin{aligned}\mathcal{L}(\psi) &= \int_{\lambda} p(\tilde{N}|\lambda, \psi) \frac{p(\lambda|\psi)}{g(\lambda|\tilde{N}, \psi)} g(\lambda|\tilde{N}, \psi) d\lambda \\ &= E_g \left[ p(\tilde{N}|\lambda, \psi) \frac{p(\lambda|\psi)}{g(\lambda|\tilde{N}, \psi)} \right],\end{aligned}\tag{14}$$

where  $E_g$  denotes expectation with respect to the ‘‘importance density’’  $g(\lambda|\tilde{N}, \psi)$ . While  $g(\lambda|\tilde{N}, \psi)$  can a priori be any conditional density, we choose a gaussian distribution parametrized to be as closed as possible to  $p(\lambda|\tilde{N}, \psi)$  in order to achieve efficiency. We use the method based on mode estimation described in Chapter 10 of [Durbin and Koopman \(2012\)](#) to select this importance density.

Next, note that although the conditional densities differ between the exact and approximate models, the *marginal* densities need to be the same in both models. Hence  $g(\lambda|\tilde{N}, \psi) \neq p(\lambda|\tilde{N}, \psi)$ , but  $g(\lambda|\psi) = p(\lambda|\psi)$ . This lets us rewrite the importance density as

$$g(\lambda|\tilde{N}, \psi) = \frac{g(\tilde{N}, \lambda|\psi)}{g(\tilde{N}|\psi)} = \frac{g(\tilde{N}|\lambda, \psi)g(\lambda|\psi)}{g(\tilde{N}|\psi)} = \frac{g(\tilde{N}|\lambda, \psi)p(\lambda|\psi)}{g(\tilde{N}|\psi)}.$$

In turn, this implies that

$$\frac{p(\lambda|\psi)}{g(\lambda|\tilde{N}, \psi)} = \frac{g(\tilde{N}|\psi)}{g(\tilde{N}|\lambda, \psi)}.\tag{15}$$

Observe that  $g(\tilde{N}|\psi)$  is the likelihood function of the model under the approximating Gaussian distribution  $g$ . We thus write  $\mathcal{L}_g(\psi) = g(\tilde{N}|\psi)$ . Substituting Eq. (15) into Eq. (14) yields

$$\mathcal{L}(\psi) = E_g \left[ p(\tilde{N}|\lambda, \psi) \frac{\mathcal{L}_g(\psi)}{g(\lambda|\tilde{N}, \psi)} \right] = \mathcal{L}_g(\psi) E_g \left[ \frac{p(\tilde{N}|\lambda, \psi)}{g(\tilde{N}|\lambda, \psi)} \right].\tag{16}$$

Eq. (16) expresses the non-Gaussian likelihood  $\mathcal{L}(\psi)$  as an adjustment to the linear Gaussian likelihood  $\mathcal{L}_g(\psi)$ , which is easily computed with the Kalman filter. This expression presents the advantage that it only requires simulation of the second term, instead of the likelihood itself.

To summarize, estimates of the vector of parameters  $\psi = (\bar{\lambda}, \rho^*, \nu^*)$  and of the latent  $\lambda_t$  are obtained by maximizing the log of (16), which is given as the sum of a standard gaussian loglikelihood and a correction term. Once the system has been estimated, we compute  $\pi_t^* = \exp(\lambda_t)/N$ . The final step recovers  $\pi_t$ ,  $\rho$ , and  $\nu$  by running the OLS regression

$$\frac{\pi_t^* - \bar{\pi}}{\sqrt{\pi_{t-1}^*}} = \rho \frac{\pi_{t-1}^* - \bar{\pi}}{\sqrt{\pi_{t-1}^*}} + \nu u_t^*, \quad (17)$$

where  $\bar{\pi}$  is taken to equal the sample average of disasters in the data. We find that the correlation between the filtered series  $\pi_t^*$  and the series for  $\pi_t$  obtained in the second stage is above 0.99.<sup>6</sup>

### III. Time-Series Properties of Disaster Risk

This section investigates the contemporaneous and intertemporal relation between  $\pi_t$  and many aggregate variables.

#### A. Disaster risk and macroeconomic variables

We first study the time-series relation between  $\pi_t$  and macro-financial variables. Table I presents the correlations between our estimated time series with variables that are traditionally considered as proxy for risk premia, and measures of macroeconomic risk and uncertainty.

**Dividend yield** The first variable we consider is naturally the dividend yield, which we obtain from Global Financial Data. A large literature documents the ability of the dividend yield to forecast future returns, and its inability to forecast future cash flows (Cochrane, 1992, 2008). Our model predicts a perfect correlation between disaster risk and the dividend yield and we find a 0.58 correlation.

Figure 3 illustrates the strong connection between the two series. For example, prices

---

<sup>6</sup>See Figure A.II in the Online Appendix.

dropped (the dividend yield rose) during the Knickerbocker Crisis in 1907. This drop in prices comes a year after a spike in disaster risk, corresponding to crises in Argentina and Brazil. Both series surge again during the two World Wars and the Great Depression. The correlation remains remarkable in the postwar period. Notably the series increase in the aftermath of WWII, which corresponds to the Korean War (Korea experienced three disasters between 1949 and 1951). Disaster risk and the dividend yield are relatively high and volatile between 1973 (corresponding to the Chilean coup d'état) and 1987. The sample stops in 2009, which corresponds to the Icelandic financial crisis.<sup>7</sup>

We observe at least two sequences during which increases in disaster risk are not associated with meaningful changes in the dividend yield. First,  $\pi_t$  surges in 1960 after the occurrence of three macroeconomic disasters in China (in 1958-60). These disasters correspond to the Great Leap Forward, a catastrophic episode of China's history, but whose consequences were mostly confined to China. This illustrates the inevitable limit of our empirical strategy. We treat all disasters the same way — irrespectively of whether these disasters are likely to be informative of U.S. disaster risk. As emphasized earlier, we privilege an Occam's razor approach that follows Barro (2006) to a more sophisticated one that would suffer from the risk of data snooping. The second episode of clear disconnect between the two series corresponds to the 1997 Asian financial crisis, with disasters in Indonesia, Korea and Malaysia. Contrary to the earlier sequence, the Asian financial crisis *had* systemic implications, suggesting that the risk of a disaster had increased in the United States. The non-response of U.S. stock prices in 1997 and following years constitutes a genuine anomaly, which is traditionally associated to the Tech 'Bubble'.

International disasters tend to cluster and one may thus wonder whether our results would be affected if we relaxed the assumption that disasters are independent conditional on  $\pi_t$ . If we assumed that international disasters are correlated, large realizations of  $\tilde{N}_t$  could result from an increase in disaster risk or a bad state of the world where many

---

<sup>7</sup>Interestingly, outside Island, the year 2009 is not associated to contractions strong enough to qualify as macroeconomic disasters. For example, consumption contracted by -6.22% in Spain in 2009, which after rescaling to match U.S. first moments corresponds to a -1.36% drop, which is above our -4.7% threshold.

disasters occurred. This would conduct us to increase our estimate of  $\pi_t$ , but less that if we assumed independence. Symmetrically, with correlated disasters, low realizations of the measurement variable  $\tilde{N}_t$  would lead us to decrease our estimate of  $\pi_t$ , but less than when disasters are independent. It follows that  $\pi_t$  would be lower in the prewar sample, and higher in the postwar sample, increasing the correlation between  $\pi_t$  and the dividend yield. We thus conclude that our assumption of independent disasters is “conservative”, in that we are likely to underestimate the correlation with the dividend yield.

We also learn from Table I about correlations in differences: shocks to  $\pi_t$  and shocks to the dividend yield are significantly positively correlated. The correlation with changes in dividend growth rates (not tabulated) is insignificant. This implies that disaster risk shocks are related either to returns or to cash flows, but through a nonlinear relation (i.e. disaster risk shocks correlates with cash flows volatility, but not with cash flows expected growth — as we comment below).

This absence of correlation between disaster risk shocks and dividend growth is useful to understand the connection between disaster risk and the dividend yield. In the usual terminology of asset pricing, disaster risk shocks can affect prices through either the discount rates channel, or the cash flow news channel, or both. However, the nonlinear relation between disaster risk and cash flow news does not imply a counterfactual predictability of cash flows by prices — a shortcoming of many asset pricing models (see [Beeler and Campbell \(2012\)](#)). In Section III.B, we show that our estimated measure of disaster risk empirically supports such a disconnect between expected growth and prices. This pattern obtains in our model (see Section IV), as in [Wachter \(2013\)](#).

**Measures of macroeconomic risk and uncertainty** We next explore whether  $\pi_t$  is related to the short rate and interest rate spreads (term and default). These variables have been shown to predict stock returns and are traditionally perceived as proxy for macroeconomic risk and risk aversion.<sup>8</sup> We find that disaster risk is positively associated with corporate default spread. As noted by [Backus et al. \(2011\)](#), the price of high quality

---

<sup>8</sup>See e.g. [Fama and Schwert \(1977\)](#), [Keim and Stambaugh \(1986\)](#) and [Campbell \(1987\)](#) for early evidence on return predictability by interest rate and spreads.

debt is a good proxy for the risk neutral probability of rare disasters, it is therefore unsurprising to find a positive relation with our (physical) disaster probability.

Recent models of disaster risk with production economies suggest a link between disaster risk and business cycles. For example, [Gourio \(2012\)](#) proposes a model in which an increase in disaster risk depresses investment and output. In [Kilic and Wachter \(2015\)](#), firms hire less when disaster risk is high. Their model generates fluctuations in unemployment that are disconnected from labor productivity but well correlated with stock market returns. [Table I](#) provides evidence for such a channel. The correlations between changes in  $\pi_t$  and changes in investment is -0.23 and the correlation with changes in unemployment is 0.21. We also find a strong connection with financial leverage. [Table I](#) shows a 0.30 correlation in level and 0.24 correlation in difference. This is in line with evidence in [Gourinchas and Obstfeld \(2012\)](#) that many financial crises were preceded by important buildup in financial leverage.

We also see that periods of high disaster risk correspond to times of higher recession risk and higher macroeconomic uncertainty. In particular, increase in disaster risk tend to come with stronger recession concerns, as proxied by the “Anxious Index” from the Survey of Professional Forecasters. This confirms our earlier observation that international disasters capture meaningful information on U.S. disaster risk.  $\pi_t$  is also strongly related with the news implied volatility (NVIX), a text-based measure of uncertainty constructed in [Manela and Moreira \(2015\)](#) from front-page articles in the *Wall Street Journal*.

Finally, we find an intriguing relation between disaster risk and U.S. macroeconomic volatility. We consider two definitions of consumption and dividend volatility. The first is estimated with a AR(1)-GARCH(1,1) model. We construct the second as the log of 5-year sum of the absolute value of the residual from an AR(1) regression of the growth rates. These two forms are suggested by [Bansal et al. \(2005\)](#). For both definitions of macroeconomic volatility, [Table I](#) indicates a positive correlation in level for both consumption and dividends, but a *negative* or insignificant correlation in difference. The level correlation is of the expected sign ([Drechsler and Yaron, 2011](#)). Periods of relatively large consumption and dividend volatility in the United States also correspond to periods

of increased international disaster risk. The negative correlation in difference is puzzling because it suggests that positive consumption volatility shocks come along with decreases in disaster risk. Correlations between leads and lags are also (insignificantly) negative. Visual inspection of both series reveals a remarkable decrease in consumption volatility and disaster risk over the postwar period.<sup>9</sup> This secular decrease is captured by the positive correlation in level. It echoes earlier findings in [Lettau et al. \(2008\)](#), who associate the period of “great moderation” to the declining equity premium. Yet both series are very different. Consumption volatility tends to increase slower than disaster risk, which helps generating a negative volatility. For example, volatility rose gradually after WWI, and reaches its peak after 1925, while disaster risk has already turned to its minimum. This suggests that disaster risk captures a dimension of risk that is empirically very different from persistent changes in consumption volatility.<sup>10</sup>

### *B. Predicting disasters, consumption, and stock returns*

The central assumption of our model is that disaster risk is predictable. Changes in the forecasts of future disasters induce fluctuations in expected future cash flows, but also on the required premium to hold stocks. The second effect dominates such that stock prices appear cheap when disaster risk is high, inducing predictable returns. This is particularly true in the absence of disasters, when all variation in the dividend yield and expected returns appears unrelated to future cash flows. We thus ask whether  $\pi_t$  predicts the future number of disasters in our international dataset, and whether  $\pi_t$  forecasts future consumption growth and excess returns. In addition, because in our model the only source of dividend yield variability is predictable disaster risks, we repeat this forecasting exercise with the dividend yield.

We undertake this exercise in [Table II](#). We regress disasters, consumption growth, and excess stock returns, measured over horizons of 1, 3, 5, and 10 years, onto  $\pi_t$  and the

---

<sup>9</sup>We plot macroeconomic volatility in the Online Appendix, [Figures A.III-A.IV](#).

<sup>10</sup>We refer the reader to [Bansal et al. \(2014\)](#) for an investigation on the role of time-varying macroeconomic volatility on asset prices and macroeconomic fluctuations.

log dividend yield. We report slope estimates,  $t$ -statistics and  $R^2$  statistics for the full sample (1900-2009), and for both for the prewar (1900-1945) and postwar (1946-2009) periods. Inference is conducted using [Hodrick \(1992\)](#) standard errors, which explicitly account for the moving average structure that overlap introduces into error terms.

The top part of [Table II](#) shows the results for predicting disasters. At the left, we report results for the full sample, and then turn to the prewar and postwar samples. Both variables predict future disasters positively, with coefficients increasing with the horizon.  $\pi_t$  forecasts future disasters with a 1-year slope approximately equal to one, as one would expect. At a 5-year horizon, the  $R^2$  statistic is 24% when forecasting with  $\pi_t$  and 16% when forecasting with the log dividend yield. In subsamples, the forecasting abilities vary, but overall the coefficients are positive and increase with the forecasting horizon.

We next turn to the forecastability of U.S. consumption growth.<sup>11</sup> The previous literature has emphasized that the dividend yield has only limited forecasting power of future consumption growth (e.g. [Beeler and Campbell \(2012\)](#)). Our results for the full sample are in line with prior evidence. Both the dividend yield and disaster risk forecast consumption growth negatively, but predictability is limited to the one-year horizon. Slope estimates decrease in absolute value as the forecasting horizon increases. [Table II](#) also indicates that most of consumption growth predictability is attributable to the prewar sample. Consumption growth predictability vanishes in the postwar sample. This echoes findings in [Chen \(2009\)](#) that dividend growth was strongly predictable in the prewar years but essentially disappeared after WWII.

The bottom part of [Table II](#) reports results for predicting excess returns. In the postwar sample — which does not include U.S. disasters — the disaster probability and the log dividend yield both forecast future excess returns positively. The  $R^2$  statistics are largely positive and increasing with the forecasting horizon, ranging from 0.09 to 0.32 for disaster risk (0.10 to 0.42 for the log dividend yield). Turning to the prewar sample, predictability weakens both for disaster risk and the dividend yield. At the

---

<sup>11</sup>Consumption growth rates and dividend growth rates are highly correlated, and results with dividend growth rates (not shown) are qualitatively similar.

one-year horizon, neither slope coefficients are significant and the slope associated to disaster probability is negative. At the ten-year horizon, both variables forecast returns with large  $R^2$ , but the statistical evidence remains fragile. The weak evidence of prewar return predictability by the dividend yield was previously documented by [Chen \(2009\)](#), along with the stronger evidence of cash flow predictability that we mentioned earlier.

Interestingly, disaster risk offers a natural interpretation to the weaker return predictability in the prewar sample. The U.S. sample contains 4 disasters, all of which unfolded over the prewar period (1919, 1920, 1929, and 1931). Disasters are positively predicted by  $\pi_t$  and tend to be associated to negative returns. Therefore it is not surprising to find a low or negative slope in the presence of disasters, as we report in [Table II](#). When we include a dummy to control for future disasters (i.e. contemporaneous to returns), we indeed find that the slope coefficient increases (it becomes positive for the full sample). Comfortingly, [Manela and Moreira \(2015\)](#) also find that news implied volatility forecast returns in the postwar sample, but not in the prewar sample. They also interpret their findings in terms of disaster concerns.

### *C. Forecasting disasters in individual countries*

We have shown that the number of international disasters is predictable and that the filtered number of disasters  $\pi_t$  is intricately linked the dividend yield. As such,  $\pi_t$  forecasts returns, which we interpret as evidence of time-varying risk premia. Our favored interpretation is that  $\pi_t$  captures the probability of a future disaster in the United States (or equivalently, a future world disaster, or a disaster in another country). To provide direct evidence for this interpretation, we next regress the domestic number of disasters on  $\pi_t$ , country per country. To do so, we need at least of few disasters observations. We therefore focus on the United States and on other countries with at least 4 disasters (results are qualitatively similar although less precise in countries with less disasters).

[Table III](#) presents the slope coefficients and  $R^2$  statistics for regression of the cumulated number of domestic disasters on lagged disaster risk. As in [Table II](#), we report results for forecasting horizons of 1, 3, 5, and 10 years. Slope coefficients significant at

the 5% level are in bold.

Of crucial interest is the slope coefficient for the United States. Table III reports a 1.1 value at the one-year horizon, a coefficient that is statistically significant. The coefficient is very close to one, which indicates that  $\pi_t$  is well calibrated to U.S. data and moves approximately one to one with future disasters. This comforts us in our interpretation that  $\pi_t$  captures U.S. disaster risk. We observe that the coefficient increases over time and remains significant up to 5 years.  $R^2$  statistics also increase up to 5 years from 0.04 to 0.13.<sup>12</sup> This is in contrast with the results in Table II with respect to U.S. consumption growth. We only found significant evidence of consumption growth predictability on a one-year horizon and estimates were decreasing in absolute value, showing weakening economic significance.

The result are largely consistent in the remaining group of countries. As in the United States, coefficients are mostly close or above one and increase with the forecasting horizon. Out of 17 countries, only two — Argentina and Peru — exhibit (insignificantly) negative slopes at the one-year horizon.

## IV. Asset Pricing Results

This section describes the main model predictions about consumption growth and asset prices. We aim to verify whether our estimated measure of time-varying disaster risk quantitatively supports the asset pricing predictions of Wachter (2013).

Table IV presents the estimated parameters of the consumption process. Consumption dynamics are defined by Eq. (2)-(3) and the latent disaster intensity is estimated by Eq. (11)-(12)-(17). We close the model by specifying that disaster size follows a shifted gamma distribution with moment generating function given by

$$\varphi(u) = e^{-u\theta}(1 + u\beta)^{-\alpha},$$

---

<sup>12</sup>Results based on probit regression of the probability of at least one disaster yield similar results.

such that disasters have support on  $(-\infty, -\theta)$ , and mean and variance equal to  $-(\theta + \alpha\beta)$  and  $\alpha\beta^2$ .

Log consumption growth parameters in ‘normal times’ are taken from U.S. consumption series only, excluding disasters. Point estimates belong to the usual range of values:  $\mu = 2\%$  and  $\sigma = 2.7\%$ . Disaster size and disaster probability parameters are computed by pooling disaster observations across countries. Note that the estimation procedure requires to set the minimum threshold for disaster size that we set to  $\theta = 4.7\%$  (i.e. disasters are drops larger than two standard deviations below the average growth rate). The average size of disasters is about 7.73% with a 3.01% standard deviation. The 90th, 95th and 99th percentiles of the disaster size distribution are about respectively 11.7%, 13.7% and 18.6%. Note that the average disaster size is about one third of the magnitude estimated by [Barro and Jin \(2011\)](#) — see our discussion in Section II.B.

The steady-state disaster probability is  $\bar{\pi} = 3.9\%$ . Disaster risk is both persistent ( $\rho = 0.84$ ) and volatile, with a scale parameter  $\nu$  of about 0.14. These two parameters contribute to the unconditional volatility of disaster risk, which is  $\sqrt{\nu^2\bar{\pi}/(1-\rho^2)} = 5.1\%$ . The standard errors on these parameters, not surprisingly, are quite substantial. Interestingly enough, the point estimates are close to corresponding (continuous-time) parameters in [Wachter \(2013\)](#). [Wachter](#) chooses a more persistent  $\rho = 0.92$  (using our notation), which is set to match the autocorrelation of the price-dividend ratio. The scale parameter is lower than in our setup ( $\nu = 0.067$ ). Altogether, these parameters imply a lower unconditional volatility of disaster risk (2.85%).

Given consumption parameters, we set the dividends’ leverage parameter to  $\phi = 2.6$ . This implies that dividend growth and consumption growth are perfectly correlated (the empirical correlation is above 60%) and that dividends are more volatile than consumption. Evidence from the Great Depression in the United States suggests that disasters have a much larger impact on dividends than on consumption ([Longstaff and Piazzesi, 2004](#)). [Wachter \(2013\)](#) argues that a value of 2.6 is conservative and captures well the extra-risk of dividends relative to consumption, both in normal times and during disasters.

We complete the calibration by setting the preference parameters  $\delta$  and  $\gamma$  to fit the main asset pricing moments. We find that low values for relative risk aversion (between 2 and 3) suffice to generate a high equity premium in line with the data. For larger values of relative risk aversion, a good match of the equity premium implies an implausible high risk-free rate. Hereafter, we consider the pair ( $\delta = 98.9\%$ ,  $\gamma = 2$ ) as our baseline calibration. Panel A of Table V reports historical asset pricing moments and their model counterparts for several preference settings. The risk-free rate is about 2.6% with 0.4% unconditional volatility. The equity premium is about 6.5% and the return volatility is about 19.6%. Thus, the Sharpe ratio is 32.9%. The dividend yield is about 2.8% with 0.9% unconditional volatility. These values are quite close to actual data.<sup>13</sup>

In Panel B, we study the effect of a change in either the persistence  $\rho$  or the disaster scale  $\nu$ , keeping everything else constant. Raising persistence or scale increases both the equity premium and return volatility in spite of the low risk aversion ( $\gamma = 2$ ). Vice-versa, a decrease in persistence or scale significantly lowers the first two moments of stock returns.

Now the focus turns on the sources of the equity premium. Eq. (8) shows that the equity premium comprises a compensation term for non-disaster risk, which is constant, and two compensation terms respectively for disaster intensity risk and disaster size risk. Both these terms are proportional to the current level of disaster intensity  $\pi_t$  but only the former depends on the characteristics of disaster intensity (i.e.  $\rho$  and  $\nu$ ). The upper panel of Figure 4 displays the fraction of the equity premium associated to these three components as a function of disaster intensity level. In the baseline calibration, we see that the equity premium is largely a compensation for disaster risk intensity. Namely, at the steady-state such a compensation term account for about 92.0% of the equity premium and vary between 90% and 95% over the state space. Instead, the compensation for non-disaster risk and disaster size risk account for about 5.9% and 2.1% respectively. The lower panel of Figure 4 shows a similar decomposition of the return variance: disaster

---

<sup>13</sup>Similar results obtain using the cash-flows parameters estimated from OECD countries only. See Table A.II in the Online Appendix.

intensity risk accounts for about 82.5% of return variation in the steady-state. These results are important because they imply that markets mostly compensate the uncertainty about future disaster risk.

Finally, we investigate the model-implied predictability. We look at the forecastability of future cumulative disasters, consumption growth and excess returns by the current level of disaster intensity. To be consistent with the empirical analysis in Section III.B, we simulate 1000 paths of the economy with 110 yearly observations for each simulation. To avoid generating negative disaster probability, we simulate monthly series and then convert them to annual frequency.<sup>14</sup> Then, we run regressions using the horizon of 1, 3, 5 and 10 years. Table VI reports on the left the average regression estimates for each horizon. We note the following facts. Disaster intensity forecasts the number of realized disasters: coefficients are positive and explanatory power is moderate. In particular, the regression results are close to the corresponding quantities for U.S. actual data in Table III. Disaster intensity does not forecast consumption growth. This result is not surprising given the assumed consumption dynamics of Eq. (2). However, this result conforms with actual data as documented in the middle panel of Table II. Disaster intensity largely predicts excess returns with a positive slope and explanatory power increasing with the horizon. While the large explanatory power is due to the simplicity of the model assumptions, the model captures the positive intertemporal relationship between disaster intensity and excess returns that we find in actual data. Model results are indeed close to U.S. postwar data reported in the lower panel of Table II.

Results on the center and the right of Table VI allow for a more accurate comparison between model-implied predictability and the empirical evidence. On the left we report predictability results from simulations of the economy on a sample of 46 years in which four disasters take place. On the right we consider the case of 64-year economy in which no disasters take place. These two scenarios are then comparable with respectively the prewar and postwar U.S. experience. In both cases we find that disaster intensity does

---

<sup>14</sup>Simulations of  $\pi_t$  at monthly frequency (i.e.  $\pi_{t+\Delta} = (1 - \rho^\Delta)\bar{\pi} + \rho^\Delta\pi_t + \nu\sqrt{\pi_t\Delta}u_t$  for  $\Delta = 1/12$ ) imply only a negligible probability of negative realizations (lower than 0.1%), which we replace by a small positive threshold.

not forecast consumption growth but forecasts excess returns. However, we observe that return predictability is larger in absence of realized disasters. This is consistent with the empirical evidence in the U.S. since our measure of disaster risk largely predicts stock returns in postwar data. Moreover, disaster intensity forecasts the number of disasters in the simulated economy with four realized disasters. Explanatory power is larger than what we find in U.S. data in Table III, which is expected because we are simulating 46 instead of 110 years of data.

Does our estimated measure of time-varying disaster risk support the main idea of Wachter (2013)? The analysis of the model predictions in this section offers several insights. First, the baseline model calibration leads to a good fit of the key unconditional moments of asset prices. This result is not surprising given that the parameter values needed in Wachter (2013) to match asset prices are similar to our estimated parameters for disaster intensity. Second, the equity premium and return variance decompositions in Figure 4 suggest that disaster intensity is a main driver of equity returns. In particular, time-varying disaster intensity not only generates excess volatility but is also responsible for the high level of the equity premium and its time-variation. The latter is consistent with the empirical finding in Table I that our measure of disaster risk strongly correlates with the dividend yield (i.e. a proxy for the conditional equity premium (van Binsbergen and Koijen, 2010)). Third, disaster intensity generates the disconnect between time-variation in consumption growth and equity returns that we observe in actual data (i.e. dividend yield predicts returns but cannot forecast consumption growth), but fails to obtain in many asset pricing models. Overall, our measure of disaster intensity, estimated from macroeconomic data, provides a quantitative assessment of Wachter (2013) and strongly supports that the time-varying nature of disaster risk is key to asset prices.

## V. Conclusion

The rare disaster model is one of the few leading workhorses of empirical asset pricing. The model bases on the idea that equity markets compensate investors' fear of rare but

large macroeconomic events. While this framework and its extensions can potentially explain many characteristics of financial markets, a robust estimation and quantitative assessment of the model is lacking. We use a latent variable approach to formally estimate the time-varying expected probability of a macroeconomic disaster using a large dataset for 42 countries over more than 100 years. Our results provide strong evidence in support of the rare disaster hypothesis. Disaster risk is volatile, persistent and strongly correlates with the U.S. dividend yield. Our model generates a large and volatile equity premium with a coefficient of relative risk aversion of two and an elasticity of intertemporal substitution of one.

## Appendix: Solving the Model

Recall that the dynamics of consumption belong to the affine class and are given by:

$$\begin{aligned}\Delta c_t &= \mu + \sigma \epsilon_t + v_t, \\ \pi_t &= (1 - \rho)\bar{\pi} + \rho\pi_{t-1} + \nu\sqrt{\pi_{t-1}}u_t.\end{aligned}$$

Therefore, the following expectation has exponential affine solution ([Drechsler and Yaron, 2011](#)):

$$\mathbb{E}_t[e^{u_1\Delta c_{t+1} + u_2\pi_{t+1}}] = e^{g_0(u) + g_1(u)'[\Delta c_t, \pi_t]}, \quad u = (u_1, u_2)' \in \mathbb{R}^2,$$

where

$$\begin{aligned}g_0(u) &= \left(\mu - \frac{\sigma^2}{2}\right)u_1 + \bar{\pi}(1 - \rho)u_2 + \frac{1}{2}\sigma^2u_1^2 \\ g_1(u) &= \left[0, \frac{1}{2}\nu^2u_2^2 + \rho u_2 + \varphi(u_1) - 1\right]'. \end{aligned}$$

The representative agent has recursive utility of the form:

$$V_t = \left[ (1 - \delta)C_t^{1-1/\psi} + \delta \left( \mathbb{E}_t[V_{t+1}^{1-\gamma}] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}.$$

Normalized utility obtains taking the limit for  $\psi \rightarrow 1$ , dividing by  $C_t$ , rearranging and taking the logarithm:

$$vc_t = \frac{\beta}{1-\gamma} \log \left( \mathbb{E}_t \left[ e^{(1-\gamma)(\Delta c_{t+1} + vc_{t+1})} \right] \right).$$

We guess an affine form for  $vc_t$ :

$$vc_t = v_0 + v_c \Delta c_t + v_\pi \pi_t,$$

we substitute to get

$$v_0 + v_c \Delta c_t + v_\pi \pi_t = \frac{\delta}{1-\gamma} \log \left( \mathbb{E}_t \left[ e^{(1-\gamma)(\Delta c_{t+1} + v_0 + v_c \Delta c_{t+1} + v_\pi \pi_{t+1})} \right] \right)$$

and compute the expectation on the right hand side:

$$v_0 + v_c \Delta c_t + v_\pi \pi_t = \frac{\delta}{1-\gamma} \left( \begin{array}{l} (1-\gamma)v_0 + g_0([(1-\gamma)(1+v_c), (1-\gamma)(v_\pi)]') \\ + g_1([(1-\gamma)(1+v_c), (1-\gamma)v_\pi]')' [\Delta c_t, \pi_t]' \end{array} \right).$$

Finally we solve for the coefficients:

$$\begin{aligned} v_c &: v_c = 0, \\ v_\pi &: v_\pi = v_\pi \delta + \delta(1-\gamma)v_\pi^2 \nu^2 / 2 + \frac{\delta}{1-\gamma}(\varphi(1-\gamma) - 1), \\ v_0 &: v_0 = v_0 \delta + \mu \delta + v_\pi \delta(1-\rho)\bar{\pi} + \delta(1-\gamma)v_\pi^2 \nu^2 / 2, \end{aligned}$$

where we take the negative root to solve for  $v_\pi$ .

In order to solve for asset prices, we first solve for the stochastic discount factor:

$$\begin{aligned} M_{t+1} &= \delta \frac{e^{-\gamma \Delta c_{t+1} + (1-\gamma)vc_{t+1}}}{\mathbb{E}_t[e^{(1-\gamma)(\Delta c_{t+1} + vc_{t+1})}]} \\ &= \delta e^{-\gamma \Delta c_{t+1} + (1-\gamma)vc_{t+1} - (1-\gamma)v_0 - g_0([(1-\gamma), (1-\gamma)v_\pi]') - g_1([(1-\gamma), (1-\gamma)v_\pi]')' [\Delta c_t, \pi_t]'}. \end{aligned}$$

Since the risk-free rate satisfies  $e^{-r_{f,t}} = \mathbb{E}_t[M_{t+1}]$ , we obtain

$$\begin{aligned} e^{-r_{f,t}} &= \delta e^{(1-\gamma)v_0 + g_0([(1-\gamma), (1-\gamma)v_\pi]') + g_1([(1-\gamma), (1-\gamma)v_\pi]')' [\Delta c_t, \pi_t]'} \\ &\quad \times e^{-(1-\gamma)v_0 - g_0([(1-\gamma), (1-\gamma)v_\pi]') - g_1([(1-\gamma), (1-\gamma)v_\pi]')' [\Delta c_t, \pi_t]'}. \end{aligned}$$

and, hence,

$$r_{f,t} = -\log \delta + \mu - \gamma\sigma^2 + \pi_t(\varphi(1-\gamma) - \varphi(-\gamma)).$$

Recall that dividends follow  $\Delta d_t = \phi\Delta c_t$ . The stock return is given by

$$\begin{aligned} r_{d,t+1} &= \log \frac{P_{t+1} + D_{t+1}}{P_t} = \log \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} + \log \frac{D_{t+1}}{D_t} \\ &= \log(e^{pd_{t+1}} + 1) - pd_t + \Delta d_{t+1} \\ &\approx k_0 + k_1 pd_{t+1} - pd_t + \Delta d_{t+1} \end{aligned}$$

for some endogenous constants  $k_0$  and  $k_1$  to be derived later. We guess an affine form for the logarithm of the price-dividend ratio:

$$pd_t = p_0 + p_c\Delta c_t + p_\pi\pi_t.$$

The Euler equation for the stock (i.e. the claim asset on  $D_t$ ) is  $1 = \mathbb{E}_t[M_{t+1}e^{r_{d,t+1}}]$ . We plug in  $M_{t+1}$ , the log-linearized  $r_{d,t+1}$  and the affine guess for  $pd_t$  and  $pd_{t+1}$ :

$$1 = \mathbb{E}_t \left[ \delta e^{-\gamma\Delta c_{t+1} + (1-\gamma)v_{c,t+1} - (1-\gamma)v_0 - g_0([1-\gamma, (1-\gamma)v_\pi]') - g_1([1-\gamma, (1-\gamma)v_\pi]')[\Delta c_t, \pi_t]'} \right. \\ \left. \times e^{k_0 + k_1(p_0 + p_c\Delta c_{t+1} + p_\pi\pi_{t+1}) - (p_0 + p_c\Delta c_t + p_\pi\pi_t) + \phi\Delta c_{t+1}} \right].$$

We rearrange and solve the expectation as follows

$$\begin{aligned} 1 &= \delta e^{-g_0([1-\gamma, (1-\gamma)v_\pi]') - g_1([1-\gamma, (1-\gamma)v_\pi]')[\Delta c_t, \pi_t]'} + k_0 + k_1 p_0 - (p_0 + p_c\Delta c_t + p_\pi\pi_t) \\ &\quad \times \mathbb{E}_t \left[ e^{-\gamma\Delta c_{t+1} + (1-\gamma)v_\pi\pi_{t+1} + k_1(p_c\Delta c_{t+1} + p_\pi\pi_{t+1}) + \phi\Delta c_{t+1}} \right] \\ &= \delta e^{-g_0([1-\gamma, (1-\gamma)v_\pi]') - g_1([1-\gamma, (1-\gamma)v_\pi]')[\Delta c_t, \pi_t]'} + k_0 + k_1 p_0 - (p_0 + p_c\Delta c_t + p_\pi\pi_t) \\ &\quad \times e^{g_0([\phi - \gamma + k_1 p_c, (1-\gamma)v_\pi + k_1 p_\pi]') + g_1([\phi - \gamma + k_1 p_c, (1-\gamma)v_\pi + k_1 p_\pi]')[\Delta c_t, \pi_t]'} \end{aligned}$$

Finally, we solve for the coefficients of the price-dividend ratio and the log-linearization constants:

$$\begin{aligned} k_0 &= -k_1 \log(k_1) - (1 - k_1) \log(1 - k_1) \\ \log k_1 &= \log(1 - k_1) + p_0 + p_c\mathbb{E}[\Delta c_t] + p_\pi\mathbb{E}[\lambda_t]. \end{aligned}$$

The coefficients satisfy

$$\begin{aligned}
p_c &: p_c = 0, \\
p_\pi &: p_\pi = g_1([\phi - \gamma + k_1 p_c, (1 - \gamma)v_\pi + k_1 p_\pi]')[0, 1]' - g_1([1 - \gamma, (1 - \gamma)v_\pi]')[0, 1]', \\
p_0 &: p_0 = k_0 + k_1 p_0 + g_0([\phi - \gamma + k_1 p_c, (1 - \gamma)v_\pi + k_1 p_\pi]') - g_0([1 - \gamma, (1 - \gamma)v_\pi]').
\end{aligned}$$

Note that one has to solve simultaneously for  $k_1$  and  $p_\pi$  (we take the negative root), and then for  $k_0$  and  $p_0$ . It follows that the equity premium is given by

$$\begin{aligned}
\log \mathbb{E}_t[e^{r_{d,t+1}}] - r_{f,t} &= \log(\mathbb{E}_t[e^{r_{d,t+1}}] \mathbb{E}_t[M_{t+1}]) \\
&= \log \mathbb{E}_t[e^{r_{d,t+1}^J}] + \log \mathbb{E}_t[M_{t+1}^J] - \log \mathbb{E}_t[e^{r_{d,t+1}^C} M_{t+1}^C] - \text{cov}_t[r_{d,t+1}^C, m_{t+1}^C] \\
&= [\phi, k_1 p_\pi] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\gamma, (\gamma - 1)v_\pi]' + \pi_t [\varphi(\phi) + \varphi(-\gamma) - \varphi(\phi - \gamma) - 1]
\end{aligned}$$

where superscripts  $C$  and  $J$  denote the normal and non-normal components. The return variance is given by

$$\text{var}_t[r_{d,t+1}] = [\phi, k_1 p_\pi] \begin{bmatrix} \sigma^2 & 0 \\ 0 & \nu^2 \pi_t \end{bmatrix} [\phi, k_1 p_\pi]' + \phi^2 \left( \frac{\partial^2}{\partial u^2} \varphi(u) \Big|_{u=0} \right) \pi_t.$$

## REFERENCES

- Abel, Andrew B., 1999, Risk premia and term premia in general equilibrium, *Journal of Monetary Economics* 43, 3–33.
- Backus, David, Mikhail Chernov, and Ian Martin, 2011, Disasters Implied by Equity Index Options, *Journal of Finance* 66, 1969–2012.
- Baker, Scott R., Nicholas. Bloom, and Steven J. Davis, 2015, Measuring Economic Policy Uncertainty, NBER Working Paper No. 21633.
- Bansal, Ravi, Varoujan Khatchatrian, and Amir Yaron, 2005, Interpretable asset markets?, *European Economic Review* 49, 531–560.
- Bansal, Ravi, Dana Kiku, Ivan Shaliastovich, and Amir Yaron, 2014, Volatility, the Macroeconomy, and Asset Prices, *Journal of Finance* 69, 2471–2511.
- Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An empirical evaluation of the long-run risks model for asset prices, *Critical Finance Review* 1, 183–221.

- Barro, Robert J., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics* 121, 823–866.
- Barro, Robert J., 2009, Rare Disasters, Asset Prices, and Welfare Costs, *American Economic Review* 99, 243–264.
- Barro, Robert J., and Tao Jin, 2011, On the Size Distribution of Macroeconomic Disasters, *Econometrica* 79, 1567–1589.
- Barro, Robert J., and José F. Ursúa, Barro-Ursúa Macroeconomic Data, 2010.
- Beeler, Jason, and John Y. Campbell, 2012, The long-run risks model and aggregate asset prices: an empirical assessment, *Critical Finance Review* 1, 141–182.
- Berkman, Henk, Ben Jacobsen, and John B. Lee, 2011, Time-Varying Rare Disaster Risk and Stock Returns, *Journal of Financial Economics* 101, 313–332.
- Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.
- Campbell, John Y., 2003, Consumption-based asset pricing, in George M. Constantinides, Milton Harris, and Rene M. Stulz, eds., *Handbook of the Economics of Finance*, 803–888 (North-Holland, Amsterdam).
- Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay, 1997, *The econometrics of financial markets* (Princeton University Press).
- Campbell, John Y., and Robert J. Shiller, 1988a, Stock prices, earnings, and expected dividends, *Journal of Finance* 43, 661–676.
- Campbell, John Y., and Robert J. Shiller, 1988b, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.
- Chen, Long, 2009, On the reversal of return and dividend growth predictability: A tale of two periods, *Journal of Financial Economics* 92, 128–151.
- Cochrane, John H., 1992, Explaining the Variance of Price-Dividend Ratios, *Review of Financial Studies* 5, 243–280.
- Cochrane, John H., 2008, The dog that did not bark: A defense of return predictability, *Review of Financial Studies* 21, 1533–1575.
- Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016, Parameter learning in general equilibrium: The asset pricing implications, *American Economic Review* (Forthcoming).
- Constantinides, George M., 2008, Discussion of “Macroeconomic crises since 1870”, *Brookings Papers on Economic Activity* 336–350.
- Donaldson, John, and Rajnish Mehra, 2008, Risk-Based Explanations of the Equity Premium, in Rajnish Mehra, ed., *Handbook of the Equity Risk Premium*, 37–99 (Elsevier, Amsterdam).
- Drechsler, Itamar, and Amir Yaron, 2011, What’s vol got to do with it, *Review of Financial Studies* 24, 1–45.
- Du, Du, and Redouane Elkamhi, 2012, Learning About Rare Economic Disasters and

Asset Prices.

- Durbin, J., and S. Koopman, 1997, Monte Carlo maximum likelihood estimation for non-Gaussian state space models, *Biometrika* 84, 669–684.
- Durbin, James, and Siem Jan Koopman, 2012, *Time Series Analysis by State Space Methods*, second edition (Oxford University Press).
- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns: A Theoretical Framework, *Econometrica* 57, 937–969.
- Fama, Eugene F., and G. William Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics* 5, 115–146.
- Farhi, Emmanuel, Samuel P. Fraiburger, Xavier Gabaix, Romain Ranciere, and Adrien Verdelhan, 2015, Crash Risk in Currency Markets.
- Farhi, Emmanuel, and Xavier Gabaix, 2016, Rare Disasters and Exchange Rates, *Quarterly Journal of Economics* 1–52.
- Gabaix, Xavier, 2011, Disasterization: A simple way to fix the asset pricing properties of macroeconomic models, *American Economic Review* 101, 406–409.
- Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.
- Gao, George P., and Zhaogang Song, 2015, Rare Disaster Concerns Everywhere.
- Gourinchas, Pierre Olivier, and Maurice Obstfeld, 2012, Stories of the twentieth century for the twenty-first, *American Economic Journal: Macroeconomics* 4, 226–265.
- Gourio, Francois, 2008, Disasters and recoveries, *American Economic Review* 98, 68–73.
- Gourio, Francois, 2012, Disaster Risk and Business Cycles, *American Economic Review* 102, 2734–2766.
- Gourio, Francois, 2013, Credit Risk and Disaster Risk, *American Economic Journal: Macroeconomics* 5, 1–34.
- Hasler, Michael, and Roberto Marfe, 2016, Disaster Recovery and the Term Structure of Dividend Strips, *Journal of Financial Economics* (Forthcoming).
- Hodrick, Robert J., 1992, Dividend yields and expected stock returns: Alternative procedures for inference and measurement, *Review of Financial Studies* 5, 357–386.
- Johannes, Michael, Lars Lochstoer, and Yiqun Mou, 2016, Learning About Consumption Dynamics, *Journal of Finance* 71, 551–600.
- Julliard, Christian, and Anisha Ghosh, 2012, Can rare events explain the equity premium puzzle?, *Review of Financial Studies* 25, 3037–3076.
- Keim, Donald B., and Robert F. Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357–390.
- Kelly, Bryan, and Hao Jiang, 2014, Tail Risk and Asset Prices, *Review of Financial Studies* 27, 2841–2871.
- Kilic, Mete, and Jessica A. Wachter, 2015, Risk, Unemployment, and the Stock Market:

- A Rare-Event-Based Explanation of Labor Market Volatility, Working Paper 21575.
- Koulovatianos, Christos, and Volker Wieland, 2011, Asset Pricing under Rational Learning about Rare Disasters, CEPR Discussion Paper No. DP8514.
- Lettau, Martin, Sydney C. Ludvigson, and Jessica A. Wachter, 2008, The Declining Equity Premium: What Role Does Macroeconomic Risk Play?, *Review of Financial Studies* 21, 1653–1687.
- Longstaff, Francis A., and Monika Piazzesi, 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* 74, 401–421.
- Lucas, Robert Jr., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.
- Manela, Asaf, and Alan Moreira, 2015, News Implied Volatility and Disaster Concerns.
- Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of monetary Economics* 15, 145–161.
- Nakamura, Emi, Jón Steinsson, Robert J. Barro, and José F. Ursúa, 2013, Crises and recoveries in an empirical model of consumption disasters, *American Economic Journal: Macroeconomics* 5, 35–74.
- Rietz, Thomas A., 1988, The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.
- Ripley, Brian D., 1987, *Stochastic Simulation* (John Wiley & Sons, Inc., New York).
- Seo, Sang Byung, and Jessica A. Wachter, 2015, Option prices in a model with stochastic disaster risk, NBER Working Paper 19611.
- Shiller, Robert J., 1981, Do stock prices move too much to be justified by subsequent changes in dividends?, *American Economic Review* 71, 421–498.
- Siriwardane, Emil, 2015, The Probability of Rare Disasters : Estimation and Implications, Harvard Business School Working Paper, No. 16-061.
- Tsai, Jerry, and Jessica A. Wachter, 2015, Disaster risk and its implications for asset pricing, *Annual Review of Financial Economics* 7, 219–52.
- van Binsbergen, Jules H., and Ralph S. J. Koijen, 2010, Predictive Regressions: A Present-Value Approach, *Journal of Finance* 65, 1439–1471.
- Wachter, Jessica A., 2013, Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?, *Journal of Finance* 68, 987–1035.
- Weitzman, Martin L., 2007, Subjective Expectations and Asset-Return Puzzles, *American Economic Review* 97, 1102–1130.

**Table I: Correlation of Estimated Disaster Probability with Macro Variables**

	Period	Level	$t$ -stat	Diff.	$t$ -stat
Dividend yield	1899-2009	0.58	5.58	0.28	2.82
Short rate	1899-2009	-0.21	-1.23	0.08	1.01
Term spread	1899-2009	0.01	0.11	-0.08	-1.53
Baa-Aaa spread	1919-2009	0.26	2.42	0.11	1.63
Investment to Capital Ratio	1947-2009	-0.29	-1.66	-0.23	-1.55
Financial leverage	1946-2009	0.30	2.60	0.24	2.22
Unemployment	1948-2009	-0.12	-0.66	0.21	1.24
SPF recession	1968-2009	0.18	1.09	0.37	2.64
Cons. growth GARCH vol.	1899-2009	0.41	3.83	-0.17	-2.34
Cons. growth AR(1) vol.	1899-2009	0.34	3.41	-0.21	-2.59
Div. growth GARCH vol.	1899-2009	0.57	4.80	0.02	0.11
Div. growth AR(1) vol.	1899-2009	0.49	4.28	0.06	0.57
Uncertainty index	1900-2009	0.19	1.70	-0.00	-0.05
NVIX	1899-2009	0.29	3.83	0.13	2.20

This table reports pairwise correlations and associated  $t$ -statistics between  $\pi_t$  and various macroeconomic time series. The first two columns correspond to correlations in levels. The next two columns report correlations where both variables are first-differenced. The dividend yield is obtained from Global Financial Data, the short rate and term spread are downloaded from Robert Shiller's web site, the credit spread and unemployment rate are available from the FRED (St. Louis Fed) database, the Investment to Capital Ratio is downloaded from Ivo Welch's web site, Financial leverage comes from the Financial Accounts of the United States, SPF recession is the Anxious Index from Survey of Professional Forecasters, which is maintained by the Federal Reserve Bank of Philadelphia, Consumption growth GARCH volatility is estimated with a GARCH model using annual U.S. consumption growth data, Consumption growth AR(1) volatility is constructed as the 5-year sum of the absolute value of the residual from an AR(1) regression for consumption growth (see [Bansal et al. \(2005\)](#)), Uncertainty index is the Historical News-Based Policy Index constructed downloaded from [policyuncertainty.com](#) (see [Baker et al. \(2015\)](#)), NVIX is the News implied volatility index created by [Manela and Moreira \(2015\)](#) and obtained from Asaf Manela's web site.

Table II: Long-Horizon Regressions

$K$	Full sample (1900-2009)				Prewar (1900-1945)				Postwar (1946-2009)			
	1	3	5	10	1	3	5	10	1	3	5	10
Disasters												
$\sum_{k=1}^K \tilde{N}_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$												
$\beta$	1.21	2.89	3.59	4.16	1.09	2.57	2.12	-0.87	0.52	1.28	1.01	1.56
$t$ -stat	8.89	4.97	3.06	1.73	4.60	2.59	1.09	-0.30	1.85	1.54	0.78	1.14
$R^2$	0.42	0.36	0.24	0.12	0.32	0.26	0.07	0.01	0.05	0.08	0.03	0.07
$\sum_{k=1}^K \tilde{N}_{t+k} = \alpha + \beta(d_t - p_t) + \epsilon_{t+k}$												
$\beta$	0.05	0.16	0.24	0.47	0.06	0.23	0.16	0.04	0.01	0.04	0.05	0.09
$t$ -stat	3.90	2.96	2.24	1.96	1.32	1.56	0.62	0.09	1.93	1.76	1.52	2.33
$R^2$	0.12	0.18	0.16	0.17	0.04	0.09	0.02	0.00	0.06	0.13	0.14	0.35
Consumption growth												
$\sum_{k=1}^K \Delta c_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$												
$\beta$	-0.22	-0.45	-0.47	-0.28	-0.25	-0.64	-0.64	0.51	-0.02	-0.09	-0.36	-1.59
$t$ -stat	-2.43	-1.56	-1.07	-0.42	-1.72	-1.37	-0.93	0.59	-0.10	-0.14	-0.39	-1.38
$R^2$	0.05	0.05	0.04	0.01	0.06	0.09	0.05	0.03	0.00	0.00	0.01	0.10
$\sum_{k=1}^K \Delta c_{t+k} = \alpha + \beta(d_t - p_t) + \epsilon_{t+k}$												
$\beta$	-0.02	-0.03	-0.03	-0.08	-0.09	-0.08	-0.02	-0.03	-0.01	-0.00	-0.01	-0.03
$t$ -stat	-3.24	-1.34	-0.90	-1.17	-4.04	-1.32	-0.25	-0.26	-1.15	-0.20	-0.34	-0.62
$R^2$	0.09	0.04	0.03	0.07	0.27	0.06	0.00	0.01	0.02	0.00	0.01	0.04
Excess returns												
$\sum_{k=1}^K r_{m,t+k} - r_{f,t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$												
$\beta$	-0.10	0.72	2.46	6.34	-0.47	0.23	1.78	4.42	5.26	12.21	18.87	31.06
$t$ -stat	-0.18	0.50	1.16	2.09	-0.57	0.11	0.62	1.60	2.46	2.57	2.67	2.74
$R^2$	0.00	0.01	0.04	0.17	0.01	0.00	0.02	0.18	0.09	0.19	0.25	0.32
$\sum_{k=1}^K r_{m,t+k} - r_{f,t+k} = \alpha + \beta(d_t - p_t) + \epsilon_{t+k}$												
$\beta$	0.10	0.24	0.40	0.77	0.15	0.47	0.75	0.70	0.12	0.30	0.46	0.98
$t$ -stat	2.23	2.17	2.32	2.67	1.15	1.78	2.23	1.75	2.61	2.46	2.37	2.68
$R^2$	0.04	0.10	0.17	0.28	0.03	0.11	0.19	0.21	0.10	0.22	0.29	0.42

This table presents slope coefficients,  $t$ -statistics, and  $R^2$  statistics for predictive regressions of the international number of disasters, consumption growth, and excess returns on disaster risk and the log dividend yield. The results are displayed for the full sample, as well as subsample covering the prewar and postwar periods. Inference is conducted using Hodrick's (1992) standard error correction for overlapping observations.

Table III: Forecasting disasters in individual countries

$K$	Slope				$R^2$			
	1	3	5	10	1	3	5	10
Argentina	-0.2	-0.7	-1.0	-1.8	0.00	0.01	0.01	0.02
Australia	<b>1.3</b>	<b>3.2</b>	2.0	2.6	0.05	0.11	0.03	0.02
Austria	<b>3.3</b>	<b>9.2</b>	<b>13.3</b>	<b>11.9</b>	0.30	0.48	0.59	0.32
Belgium	<b>1.6</b>	2.1	1.6	2.8	0.08	0.03	0.01	0.02
Canada	<b>1.5</b>	<b>3.6</b>	<b>5.2</b>	4.2	0.06	0.10	0.13	0.03
Chile	0.6	-0.6	-0.5	1.1	0.01	0.00	0.00	0.00
Colombia	<b>2.2</b>	<b>5.6</b>	4.6	6.4	0.09	0.13	0.04	0.04
France	<b>2.5</b>	<b>4.1</b>	3.0	2.7	0.19	0.09	0.03	0.01
Germany	<b>2.2</b>	<b>6.2</b>	<b>7.4</b>	7.8	0.10	0.22	0.15	0.07
Korea	<b>1.8</b>	5.3	8.3	15.2	0.06	0.09	0.12	0.15
Malaysia	<b>3.2</b>	<b>8.5</b>	<b>11.5</b>	10.6	0.12	0.19	0.20	0.11
New Zealand	<b>1.5</b>	<b>3.4</b>	<b>6.2</b>	<b>10.4</b>	0.07	0.13	0.28	0.39
Peru	-0.5	-1.5	-2.8	-4.7	0.01	0.02	0.04	0.06
Portugal	0.5	-0.8	-1.5	0.1	0.00	0.00	0.01	0.00
Russia	<b>2.3</b>	<b>6.9</b>	<b>7.4</b>	5.9	0.16	0.33	0.19	0.06
Taiwan	<b>1.5</b>	<b>4.9</b>	<b>6.7</b>	5.5	0.07	0.17	0.16	0.05
United States	<b>1.1</b>	<b>3.8</b>	<b>5.7</b>	6.7	0.04	0.10	0.13	0.09

This table presents slope coefficients and  $R^2$  statistics for predictive regressions of the domestic number of disasters on disaster risk:

$$\sum_{k=1}^K N_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}.$$

Results are reported for countries with at least 4 disasters over the sample period. Slope coefficient that are significant at the 5% level are indicated in bold. Inference is conducted using [Hodrick's \(1992\)](#) standard error correction for overlapping observations.

**Table IV: Estimation Results**

	Full sample		OECD Countries	
	Estimate	S.E.	Estimate	S.E.
<i>Normal Times:</i>				
Average growth in consumption $\mu$	0.020	0.003	0.020	0.003
Volatility of consumption growth $\sigma$	0.027	0.002	0.027	0.002
<i>Disaster probability:</i>				
Long-term disaster probability $\bar{\pi}$	0.039	0.016	0.037	0.027
Persistence $\rho$	0.835	0.119	0.905	0.148
Volatility parameter $\nu$	0.141	0.063	0.161	0.106
<i>Disaster size:</i>				
Shape parameter $\alpha$	1.010	0.146	1.115	0.203
Scale parameter $\beta$	0.030	0.006	0.030	0.007

This table presents estimates of the time-varying disaster probability model. Log consumption growth evolves according to:

$$\Delta c_t = \mu + \sigma \epsilon_t + v_t,$$

where  $\mu$  and  $\sigma$  are constants,  $\epsilon_t$  is a standard normal random variable, and  $v_t = J_t \mathbf{1}_{N_t \neq N_{t-1}}$ .  $N_t$  follows a Poisson distribution with time-varying intensity  $\pi_t$ :

$$\pi_t - \bar{\pi} = \rho(\pi_{t-1} - \bar{\pi}) + \nu \sqrt{\pi_{t-1}} u_t.$$

Finally  $J_t$  follows a shifted gamma distribution taking a minimum value of  $\theta = 5\%$  and parameters  $\alpha$  and  $\beta$ . The log consumption growth mean  $\mu$  and variance  $\sigma$  are estimated from U.S. data excluding four consumption growth disasters (in 1919, 1920, 1929, and 1931). Disasters are defined according to Eq. (10). The remaining parameters are estimated by maximum likelihood from an international dataset of 42 countries over the period 1900 to 2009. The likelihood for the non-Gaussian state space model is computed by simulation (see Section II.C).

**Table V: Asset Pricing Moments**

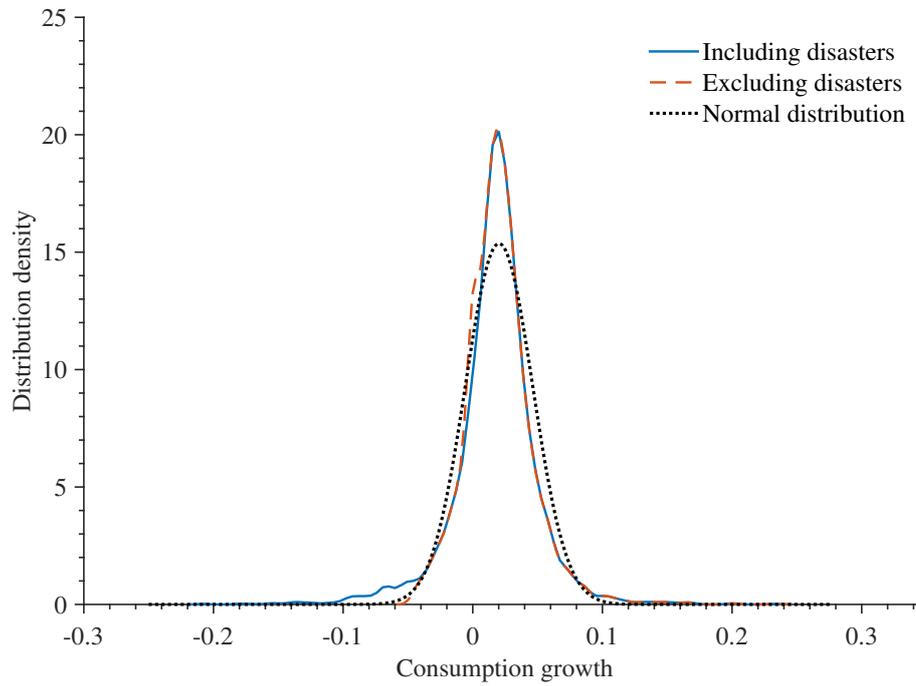
<i>Panel A</i>					
%	Data 1900-2009	Data 1946-2009	$\gamma = 2$ $\delta = 98.9\%$	$\gamma = 3$ $\delta = 98.3\%$	$\gamma = 5$ $\delta = 97.0\%$
Steady-state risk-free rate	1.6	1.4	2.6	3.1	4.2
Risk-free rate unconditional volatility	5.0	3.5	0.4	0.5	0.6
Steady-state equity premium	6.5	6.4	6.5	8.9	9.7
Steady-state return volatility	19.7	17.3	19.6	22.6	22.1
Steady-state Sharpe ratio	32.8	37.2	32.9	39.3	43.8
Steady state-dividend yield	4.2	3.5	2.9	5.3	7.4
Dividend yield unconditional volatility	1.6	1.4	1.0	2.1	2.9
<i>Panel B</i>					
%		$\rho = 0.92$	$\rho = 0.75$	$\nu = 0.18$	$\nu = 0.10$
Steady-state risk-free rate		2.6	2.6	2.6	2.6
Risk-free rate unconditional volatility		0.7	0.3	0.5	0.3
Steady-state equity premium		12.3	3.0	15.4	1.6
Steady-state return volatility		36.5	11.2	36.0	9.5
Steady-state Sharpe ratio		33.8	27.2	42.8	17.3
Steady state-dividend yield		4.1	0.7	7.5	0.1
Dividend yield unconditional volatility		4.0	0.1	4.9	0.0

Panel A reports unconditional moment statistics from S&P 500 real returns and three months U.S. treasury real rates, as well as model-implied steady-state moments using parameters from Table IV, for several preference settings. Panel B reports model-implied steady-state moments using parameters but one from Table IV and preference parameters  $\gamma = 2, \delta = 98.9\%$ .

**Table VI: Model-Implied Long-Horizon Regressions**

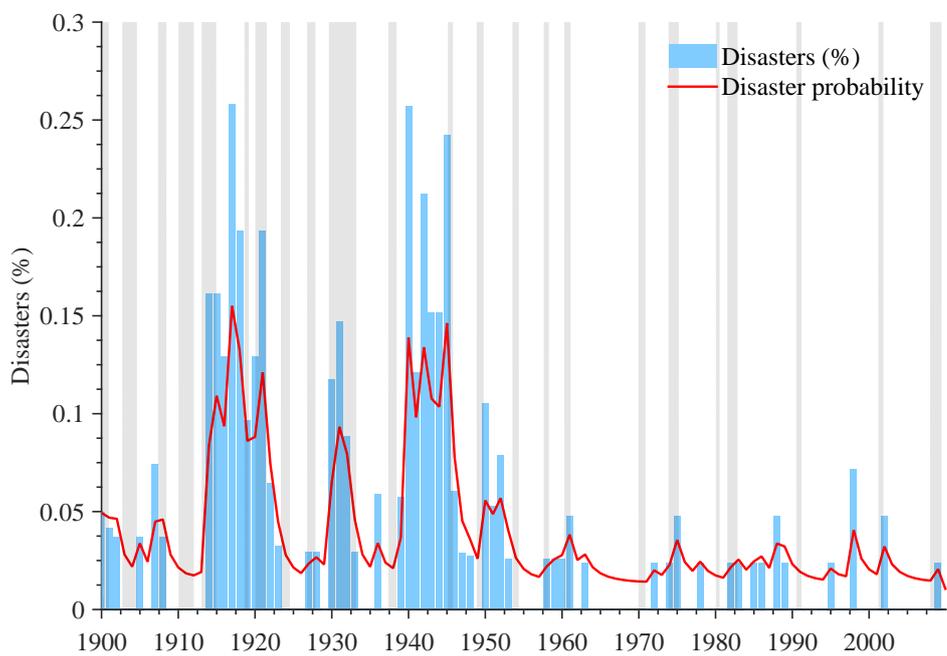
$K$	Unrestricted (110 years)				Four disasters (46 years)				No disasters (64 years)			
	1	3	5	10	1	3	5	10	1	3	5	10
Disasters												
	$\sum_{k=1}^K N_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$											
$\beta$	0.90	2.06	2.68	3.09	1.67	3.72	4.80	4.72	–	–	–	–
$t$ -stat	1.90	2.49	2.47	1.96	1.57	2.03	2.00	1.30	–	–	–	–
$R^2$	0.05	0.08	0.09	0.08	0.08	0.15	0.17	0.15	–	–	–	–
Consumption growth												
	$\sum_{k=1}^K \Delta c_{t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$											
$\beta$	-0.07	-0.16	-0.20	-0.20	-0.13	-0.28	-0.37	-0.36	-0.00	-0.01	-0.01	-0.00
$t$ -stat	-0.81	-1.09	-1.08	-0.78	-0.98	-1.26	-1.26	-0.85	-0.01	-0.06	-0.04	-0.01
$R^2$	0.02	0.04	0.05	0.06	0.05	0.10	0.12	0.13	0.02	0.04	0.05	0.08
Excess returns												
	$\sum_{k=1}^K r_{m,t+k} - r_{f,t+k} = \alpha + \beta\pi_t + \epsilon_{t+k}$											
$\beta$	1.68	3.94	5.31	6.82	1.54	3.75	5.09	6.89	2.08	4.70	6.14	7.58
$t$ -stat	3.73	5.99	7.02	7.50	2.12	3.43	4.00	4.35	3.38	5.40	6.20	6.23
$R^2$	0.11	0.26	0.33	0.38	0.10	0.23	0.31	0.40	0.15	0.33	0.41	0.45

This table presents slope coefficients,  $t$ -statistics, and  $R^2$  statistics for predictive regressions of the domestic number of disasters, consumption growth, and excess returns on disaster intensity. The economy has been simulated 1000 times at monthly frequency with 110 yearly observations for each simulation. Estimates are obtained by pooling regression results from all simulations and reported on the left. Regression results reported on the center are based on 1000 simulations each of 46 years length and four realized disasters. Regression results reported on the right are based on 1000 simulations each of 64 years length and no realized disasters.

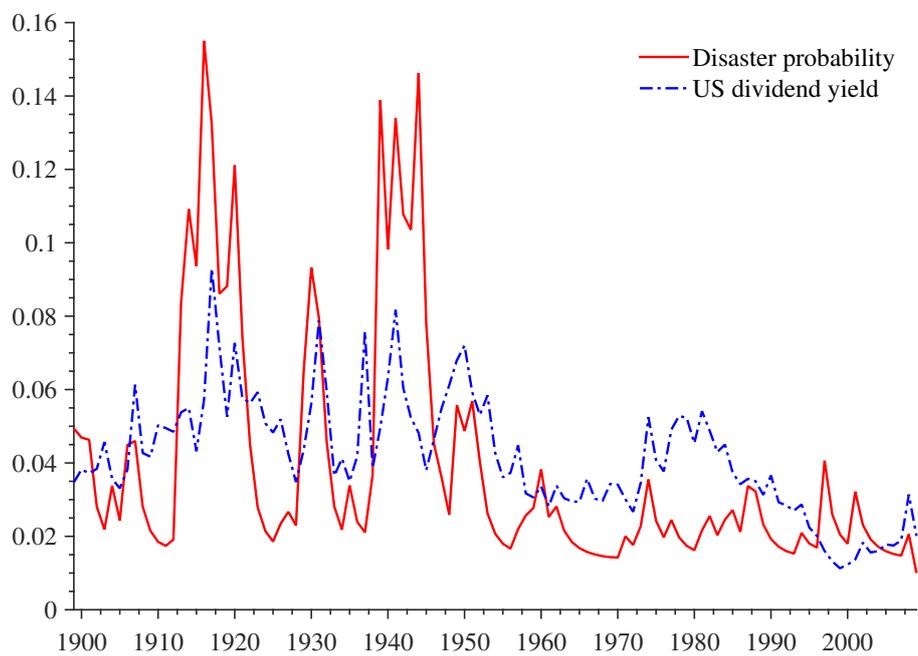


**Figure 1: Consumption growth distribution**

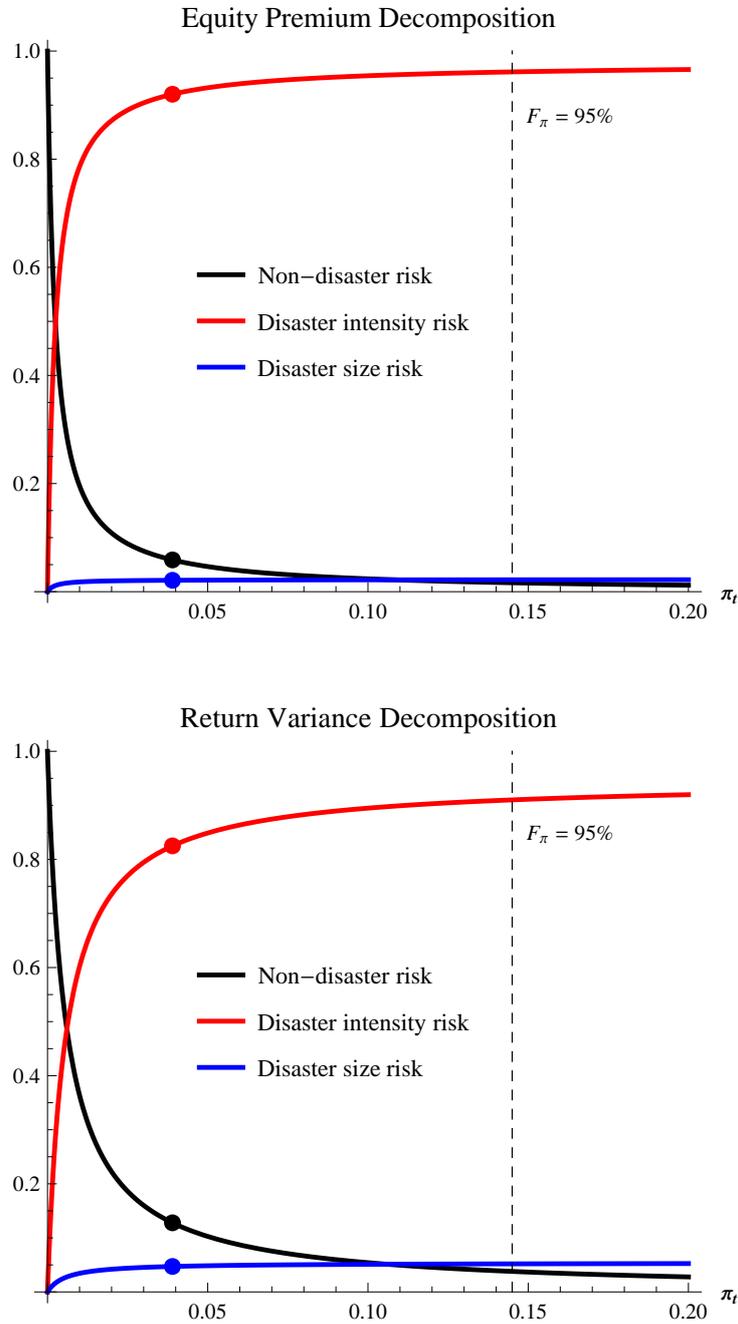
This figure shows the kernel density estimates of annual consumption growth for the true series and the modified series excluding disasters (according to Eq. (10)). The two densities are compared to a normal distribution. The underlying data is constructed by pooling cross-country consumption series, which have been normalized to have the same volatility as U.S. consumption.



**Figure 2: Observed Disasters and Estimated Disaster Probabilities  $\pi_t$**   
 This graph plots the filtered series of expected disasters probabilities  $\pi_t$  and the realized number of disasters using the consumption series. Shaded areas denote NBER recessions.



**Figure 3: Filtered Disaster Probability and the Dividend Yield.** This graph plots the filtered series of expected disasters probabilities  $\pi_t$  and the S&P 500 dividend yield.



**Figure 4: Equity Premium and Return Variance Decomposition.** The upper and lower panels plot the fractions of respectively the equity premium and return variance due to non-disaster risk (black line), disaster intensity risk (red line) and disaster size risk (blue line). Markers denote the disaster intensity steady-state. The vertical dashed line represents the 95th percentile of disaster intensity distribution.