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Abstract

This paper studies a general equilibrium model with an investor controlled firm. Shareholders can vote on the firm's production plan in an assembly. Prior to that they may trade shares on the stock market. Since stock market trades determine the distribution of votes, trading is strategic. There is always an equilibrium, where share trades lead to owners deciding for competitive behavior, but there may also be equilibria, where monoplistic behavior prevails.

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JEL Classification Numbers: D21, D43, D51, G32, G34.

1 Introduction

Economic agents are affected by the activity of firms in two ways. First, as investors they receive dividends derived from the firms' revenues. Second, as consumers they are affected by externalities generated by firms. Those may be externalities in the narrow sense (pollution, innovation, etc.) or simply the fact that firms' production decisions affect market prices.

Under perfect competition agents are not aware of how firms' production activities affect prices, because they take prices as given. Therefore, in this case all shareholders unanimously want the firms to maximize profits. But under imperfectly competitive conditions agents, who understand how firms affect prices, will also understand how they themselves are affected by the firms' decisions. This leads to a failure of shareholder unanimity and constitutes the simplest case, where economic agents hold heterogenous views about what firms ought to do.

In such cases *control* over firms becomes an issue. In modern industrialized societies investors' control over firms is institutionalized through property and control rights. While a variety of securities separate property from control rights (e.g. bonds or preferred stocks), the core institution of *equity* combines them. This creates an industrial democracy distinct from political democracy in two ways. First, a firm's equity owners have a direct saying on the company's operation *in proportion* to their property rights, through voting in shareholder assemblies ("one-share-one-vote"). Second, and again unlike political democracy, voting shares can be *traded* at the stock market.

Despite a wide recognition that *empirically* control commands value (e.g. Zingales, 1994; Rydqvist, 1996; Modigliani and Perotti, 1997; Nenova, 2003) little seems to be known about the interaction of these two aspects in theory. The literature on takeover bids has focussed on a partial equilibrium framework, where, first, the control-threshold and the identity of the bidder are exogenous and, second, takeovers only affect the value of the firm about which investors otherwise agree (e.g. Grossman and Hart, 1980; Grossman and Hart, 1988; Bagnoli and Lipman, 1988; Harris and Raviv, 1988; Hirshleifer and Titman, 1990; Burkart, Gromb, and Panunzi, 1998). A similar partial equilibrium approach is often used in modelling shareholder voting (e.g. Maug and Yilmaz, 2002).

General equilibrium treatments have concentrated on the objective function of the firm, ignoring shareholder voting and the conflict that the desire to control creates at the stock market (e.g. Milne, 1981; Haller, 1986; Grodal, 1996; Dierker and Grodal, 1996; Dierker and Grodal, 1999). Few papers have tried to integrate shareholder voting and the interaction on the stock market in a general equilibrium framework. The few that we are aware of include Renström and Yalçin (2003), Bejan (2003), and Kelsey and Milne (forthcoming) on imperfect competition, and Drèze (1974), DeMarzo (1993), and Bejan (2004) on incomplete market economies.

This paper studies a general equilibrium model with a monopolist firm that is controlled by investors through shareholder voting. As agents understand how the firm's policy will influence market conditions, there is scope for shareholder disagreement. Thus, the model provides a general equilibrium foundation for the notion of private benefits of control that is frequently used in the literature on corporate governance (e.g. Holmström and Tirole, 1997). Prior to the shareholder assembly investors can trade their shares at the stock market, thereby determining the distribution of votes and the extent of control.

We find that there is always an equilibrium, where after trade at the stock market shareholders decide for the efficient production plan, that is, they make the firm behave competitively. On the other hand, there may also be equilibria, where shareholders decide in favor of monopolistic behavior. The latter tend to be associated with a concentrated ownership structure, while the former are associated with more dispersed ownership. Only when stock market trades are coordinated by some central agency (in a transferable utility model) monopolistic firm behavior disappears. Thus, the model provides a theory of the firm's competitive conduct that is driven by the ownership structure, rather than the market structure.

The model is admittedly stylized in order to focus on the key issues. In a more general model many feedback effects would appear that are shut down in the present model. But it turns out that the interactions are sufficiently complex even in the present model, so that in order to isolate intuition the simplifications are justified.

The rest of the paper is organized according to backwards induction. Section 2 sets out the basic model and analyses consumption decisions in the final period, after the firm's production plan has been decided. Section 3 considers shareholder voting in an assembly which determines the firm's production plan prior to commodity markets. Section 4 provides an equilibrium notion for the initial stage of strategic trading at the stock market, first for tender offers, and then for general control trades. Section 5 concludes and discusses implications. Lengthy proofs are relegated to the Appendix.

2 The Economy

The model encompasses an arbitrary (but finite) number of economic agents and a single firm. Agents come endowed with a composite commodity that is perfectly divisible, and with ownership shares in the firm. They wish to consume the composite commodity and a second commodity, that is exclusively produced by the firm and comes in discrete, indivisible units.

The interaction extends over three stages. At the initial stage agents can trade shares at a stock market. When the stock market closes, ownership shares are "frozen" and a shareholder assembly may be called. This is the second stage: At the assembly shareholders can decide to participate and vote on the firm's production plan under simple majority rule, where one share counts as one vote. Given the outcome of shareholder voting, at the final stage commodity markets open, consumers spend their incomes, and consumption goods prices are determined. This last stage is a standard Cournot-Walras model, as in Gabszewicz and Vial (1972). The three stages are now explained in detail, working backwards from the last to the first.

2.1 Commodity Markets

Consider an economy with n agents, $i \in I = \{1, ..., n\}$, and two goods, $l = 0, 1$. Good $l = 0$ (the composite commodity) is perfectly divisible and can serve both as a consumption good and as a factor of production. Every agent has initially (at the first stage) an endowment $\omega_0 > 0$ with good $l = 0$, i.e. $e_i = (\omega_0, 0)$ for all $i \in I$. Commodity $l = 1$ is indivisible and is produced by the only firm in the economy from good $l = 0$ via a technology that converts $c > 0$ units of good $l = 0$ into one unit of good $l = 1$.[1]

Agents' utilities are quasi-linear in good $l = 0$. Each agent i has a valuation ("willingness to pay") $v_i > 0$ for one unit of commodity $l = 1$ (and valuation zero for more than one unit). Agent i's indirect utility function at income $w_i = w$ and relative price $p = p_1/p_0$ is, therefore,

$$U_i(p, w) = w + \max\{0, v_i - p\} \tag{1}$$

For the sake of tractability it is assumed that there are only two types of agents, those with high valuation $v_i = \overline{v}$ and those with low valuation $v_i = \underline{v}$, where $c \leq \underline{v} < \overline{v} \leq 1$. Let $H = \{i \in I \,|v_i = \overline{v}\}$ and $L = \{i \in I \,|v_i = \underline{v}\}$ and denote by $m = |H|$ the number of agents with high valuation, $1 \leq m < n$. Then, the aggregate demand function for commodity $l = 1$ is a step function that gives market clearing prices

$$p(y) = \begin{cases} \overline{v} & \text{if} \quad 0 \leq y \leq m \\ \underline{v} & \text{if} \quad m < y \leq n \\ 0 & \text{if} \quad n < y \end{cases} \tag{2}$$

when y is the firm's output.

Agents are also shareholders in the firm, with i owning share $\theta_i \geq 0$, where $\sum_{i \in I} \theta_i = 1$. Therefore, at relative price p and output y agent i's wealth w_i is given by $w_i = \omega_i + \theta_i (p - c) y$, where $\omega_i - \theta_i c y$ is agent i's interim endowment with good $l = 0$ and $\theta_i y$ is her interim endowment with good $l = 1$. The part ω_i of i's interim endowment with good $l = 0$ derives from her initial endowment $e_{i0} = \omega_0$ minus i's expenditure for share purchases, or plus i's revenue from share sales at the stock market. Hence, i's indirect utility from the firm's decision y is given by

$$\begin{aligned} V_i(\theta_i, y) &= \omega_i + \theta_i [p(y) - c] y + \max\{0, v_i - p\} \\ &= \begin{cases} \omega_i + \theta_i [\overline{v} - c] y & \text{if} \quad 0 \leq y \leq m \\ \omega_i + \theta_i [\underline{v} - c] y & \text{if} \quad m < y \leq n \text{ and } v_i = \underline{v} \\ \omega_i + \theta_i [\underline{v} - c] y + \overline{v} - \underline{v} & \text{if} \quad m < y \leq n \text{ and } v_i = \overline{v} \\ \omega_i - \theta_i c y + v_i & \text{if} \quad n < y \end{cases} \end{aligned} \tag{3}$$

The maximum of the first line under curly brackets (under the constraint $0 \leq y \leq m$) obtains at $y = m$. The maximum of the second and third line under curly brackets (subject to $m < y \leq n$) obtains at $y = n$ (uniquely so if $c < \underline{v}$). And the last line under curly brackets

[1] Under constant returns to scale it is difficult to see why there is a single firm even though each agent could run his own firm and produce as efficiently. To avoid this, a small but positive fixed cost could be assumed without changing the analysis.

is strictly decreasing in y with a maximum at $y = n + 1$. Denote by $\overline{\pi} = [\overline{v} - c]\, m > 0$ the maximum profit at the high price $p = \overline{v}$ (low output, $y = m$) and by $\underline{\pi} = [\underline{v} - c]\, n \geq 0$ the maximum profit at the low price $p = \underline{v}$ (high output, $y = n$).

The indirect utility function V_i in (3) is clearly not single-peaked (quasi-concave). This is fairly typical for models like this. Even with perfect divisibility and in regular cases, like with Cobb-Douglas preferences, the induced indirect utility function over a firm's output will be strictly convex for low shareholdings and (at least) triple-peaked for higher shareholdings. This is because an agent, who holds almost no shares cares exclusively about the externality, while for an investor, who holds more shares, the profit motive becomes more important. The failure of single-peakedness forbids the application of the median voter theorem, which would otherwise be a feasible theory of control for publicly held corporations.[2] A reordering (as in Roberts, 1977) cannot turn preferences single-peaked either, because "types" are in fact two-dimensional, consisting of valuations v_i and shares θ_i.

2.2 Bliss Points

If a shareholder had to choose the firm's production plan, she would face a trade-off: As a recipient of dividends she wants the firm to realize high profits; as a consumer she wants high levels of output so as to lower the consumption good price. The first aspect is more important for large blockholders, while high-valuation types with small shareholdings are more sensitive to the second. Since high levels of output (low price) may be inconsistent with high profits, a conflict of interest between shareholders may arise for a certain parameter range. In particular, low-valuation types and large blockholders may prefer to constrain output, while high-valuation types with few shares will wish to expand output so as to lower prices. In quantitative terms the optimal production plans from the individual agents' viewpoints are summarized in the first result.

Proposition 1 *Given the share θ_i owned by agent $i \in I$ and her type $v_i \in \{\underline{v}, \overline{v}\}$, her most preferred production plan is given by the following table:*

$\overline{\pi} > \underline{\pi}$	$0 \leq \theta_i < \alpha$	$\alpha \leq \theta_i < \beta$	$\beta \leq \theta_i \leq \gamma$	$\gamma < \theta_i$
$v_i = \underline{v}$	$y = n + 1$	$y = m$	$y = m$	$y = m$
$v_i = \overline{v}$	$y = n + 1$	$y = n + 1$	$y = n$	$y = m$
$\overline{\pi} \leq \underline{\pi}$	$0 \leq \theta_i < \beta$		$\beta \leq \theta_i$	
$v_i = \underline{v}$	$y = n + 1$		$y = n$	
$v_i = \overline{v}$	$y = n + 1$		$y = n$	

where

$$\alpha = \frac{\underline{v}}{\overline{v}m + c\,(n - m + 1)},\ \beta = \frac{\underline{v}}{\underline{v}n + c},\ \textit{and}\ \gamma = \frac{\overline{v} - \underline{v}}{\overline{\pi} - \underline{\pi}},$$

with $\alpha < 1/m$, $\beta < 1/n$, $\overline{\pi} > \underline{\pi} \Rightarrow \gamma \leq 1/m$, and $\overline{\pi} > \underline{\pi} \Leftrightarrow \alpha < \beta$.

[2] Some authors apply the median voter theorem in analoguous contexts nevertheless, e.g. Renström and Yalçin (2003). Similarly, the use of first-order conditions, as in Kelsey and Milne (forthcoming), is not always justified in models of this class.

Proof: See Appendix.

For the parameter constellation $\overline{\pi} \leq \underline{\pi}$ the bliss points of shareholders depend only on their holdings θ_i, but not on their types. All shareholders with shares below β ($< 1/n$) favor a zero price ($y = n + 1$), and all shareholders with shares above β favor the low price $p = \underline{v}$ ($y = n$). No shareholder will ever want a high price. With this parameter constellation, therefore, the choice would only be between giving away the firm's produce for free and serving all potential customers. This is why we regard this case as less interesting. Instead, we will concentrate on the upper part of the table.

There, $\overline{\pi} > \underline{\pi}$ obtains and a conflict between shareholders with equal shares, but different types, can arise (for shares that are at least $\alpha < 1/m$). With shares between α and β low-valuation types favor a high price, but high-valuation types prefer a zero price. With shares between β and γ low-valuation types continue to prefer a high price, but high-valuation types favor a low price. Only with shares above γ the two types again agree on the high price. Since γ can be large (as compared to α), when $\overline{\pi} - \underline{\pi} > 0$ becomes small compared to $\overline{v} - \underline{v}$, the region of potential conflict can be wide. (The coefficient γ measure the relative importance of the externality versus the profit increment.) Therefore, we henceforth concentrate on the case $\overline{\pi} > \underline{\pi}$.

Moreover, it will be *assumed* that the alternatives of the firm are restricted to $y = m$ (low output, high price) and $y = n$ (high output, low price). This is for three reasons. First, a zero price entails negative profits. If this were an equilibrium outcome, rational investors, foreseeing that the firm will lose money, would not invest into shares in the first place—so, the firm would not come into existence in equilibrium. Second, only investors with very small shares (below $\beta < 1/n$) will favor a zero price. Most likely, a shareholder assembly, where excessive output is proposed, can never be won. Third, a binary decision of the firm avoids the preference aggregation problems that are familiar from social choice theory (see also Section 3). In summary, we assume the following parameter restrictions.

Assumption: $y \in \{m, n\}$, $\overline{\pi} > \underline{\pi}$, $c < \underline{v}$, and $\omega_0 > \overline{\pi} + \overline{v}$.

The first two assumptions have been explained. The third assumes a generic parameter constellation that avoids ties. The fourth assumes a sufficiently large endowment with the composite commodity to avoid results that are driven by wealth constraints. It effectively means that every investor has, in principle, sufficient resources to buy all stocks at the high firm value $\overline{\pi}$, can even pay a control premium equal to the price difference $\overline{v} - \underline{v}$, and can still afford to buy good $l = 1$ at the low price $p = \underline{v}$. That is, a low-valuation type could buy all shares from a high-valuation type, who originally owns all shares, at the post-trade value of the firm and could compensate the high-valuation seller for the resulting utility loss. Likewise, a high-valuation type could buy all shares from the single low-valuation owner at the pre-trade value of the firm, thus compensating the seller for the profit loss, and could still afford to buy good $l = 1$ even at the high price $p = \overline{v}$. This assumption avoids that nonnegativity constraints bind on the consumption of the composite commodity $l = 0$ in the share trades to be considered below.

These are restrictive assumptions applied to an already restrictive model, of course. Yet, in general models of this type, without quasi-linear preferences, the market clearing price p

will depend on the whole share distribution, and so will bliss points. Moreover, bliss points will vary with individual shareholdings even locally. In the present model bliss points do depend on the investor's shareholdings, when considered over the whole range. But locally they are constant. These simplifications make the model tractable.

Among the two possible output choices of the firm, $y = m$ and $y = n$, the efficient one is $y = n$. For, by quasi-linearity of utility there exists a (welfare) representative consumer with indirect utility function

$$V(y) = \sum_{i=1}^{n} V_i(\theta_i, y) = n\omega_0 + [p(y) - c]\, y + m \max\{0, \overline{v} - p(y)\} \tag{4}$$

because $\sum_{i \in I} \omega_i = n\omega_0 - q \sum_{i \in I} (\theta_i - \vartheta_i) = n\omega_0$ by stock market clearing, where ϑ_i is i's initial share and q the stock market price of shares. Since $V(n) - V(m) = (n - m)(\underline{v} - c) > 0$, it follows that $y = n$ is welfare maximizing. Henceforth we refer to $y = n$ as the *efficient* production plan and to $y = m$ as the *monopolistic* production plan.

2.3 Budget Constraints

In principle there are two possibilities to account for budget constraints. One is a sequence of two budget constraints, one for the stock market and one for commodity markets. This would imply a market incompleteness, as agents would not be permitted to take credit, and it would require the specification of a third commodity against which shares are traded.

The other possibility, adopted here, is a single budget constraint that extends over all three stages of the model. For each $i \in I$ this is of the form

$$\omega_0 + \theta_i \pi + q\vartheta_i \geq p z_1 + q\theta_i \tag{5}$$

where $z_1 \in \{0, 1\}$ is the purchasing decision of the indivisible commodity $l = 1$, q is the stock market price, ϑ_i is i's initial endowment with shares, and both π and p depend on the firm's output y.

Since first-date transactions at the stock market enter this budget constraint, it has to hold for all production plans of the firm. On the other hand, a single budget constraint fits better with general equilibrium than a sequence of two budget constraints would. Moreover, a single budget constraint allows agents to take credit on account of their future income. It thus allows for debt financed share acquisitions. Such debt financed bootstrap acquisitions have been important in the US takeover wave of the 1980ties (see Müller and Panunzi, 2004).

The single budget constraint implies for agent $i \in I$ that

$$q(\theta_i - \vartheta_i) \leq \begin{cases} \omega_0 + \theta_i \underline{\pi} - \underline{v} & \text{if} \quad y = n \\ \omega_0 + \theta_i \overline{\pi} - \overline{v} & \text{if} \quad y = m \text{ and } v_i = \overline{v} \\ \omega_0 + \theta_i \overline{\pi} & \text{if} \quad y = m \text{ and } v_i = \underline{v} \end{cases} \tag{6}$$

Since $\overline{\pi} > \underline{\pi} > 0$ and $\omega_0 > \overline{v} > \underline{v}$ were assumed, the right hand side of this budget constraint is always at least as larger as $\omega_0 - \overline{v} > \overline{\pi} > 0$. Therefore, at any stock price, which does not exceed the highest possible firm value, every investor can afford to buy all stocks.

3 Shareholder Voting

Shareholders' bliss points determine their voting behavior on the firm's production plan in a shareholder assembly. By the assumption that $y \in \{m, n\}$ the vote among shareholders is a *binary* decision. This avoids the well-known preference aggregation problems, like Arrow's (1963) impossibility, which would otherwise surface in the model with three alternatives.

Example 1 *As a concrete example consider a case with three shareholders, $n = 3$, and three alternatives, $y \in \{m, n, n+1\}$. For parameters choose $\overline{v} = 1$, $\underline{v} = 1/2$, and $c = 1/4$, so that $\overline{\pi} = 3/2$ and $\underline{\pi} = 3/4$, $\alpha = 1/5$, $\beta = 2/7$, $\gamma = 2/3$, and $\eta = \overline{v}/\left(\overline{\pi} + c\,(n+1)\right) = 2/5$. Now assume that $L = \{1\}$, $H = \{2, 3\}$, and $\theta_1 = \theta_2 = 0.27$, so that $\theta_3 = 0.46$.*

Since the low-valuation agent 1 holds a share θ_1 with $\alpha = 0.2 < \theta_1 = 0.27 < \beta = 0.2857$, she ranks the monopolistic output $y = m$ above excessive output $y = n + 1$ and the latter above the efficient output $y = n$. The small high-valuation shareholder owns $\theta_2 = 0.27 < \beta = 0.2857$ and, therefore, ranks excessive output $y = n+1$ above the efficient output $y = n$ and the latter above monopolistic output $y = m$. The large high-valuation investor holds a share with $\eta = 0.4 < \theta_3 < \gamma = 0.6667$. Consequently, she ranks the efficient output $y = n$ above the monopolistic output $y = m$ and the latter above excessive output $y = n + 1$.

Suppose, for the moment, that voting in a shareholder assembly is costless and all shareholders participate. When excessive output $y = n + 1$ (zero price) runs against efficient output $y = n$ (low price), the former wins with the support of the low-valuation type $i = 1$ and the small high-valuation investor $i = 2$ (as $\theta_1 + \theta_2 = 0.54$) against $i = 3$. When the efficient output $y = n$ (low price) runs against monopolistic output $y = m$ (high price), the former wins with the support of the two high-valuation shareholders $i = 2$ and $i = 3$ (as $\theta_2 + \theta_3 = 0.73$) against the low-valuation type $i = 1$. Finally, when monopolistic output $y = m$ (high price) runs against excessive output $y = n + 1$ (zero price), the former wins with the support of the low-valuation investor $i = 1$ and the large high-valuation shareholder $i = 3$ (as $\theta_1 + \theta_3 = 0.73$) against the small high-valuation type $i = 2$. Thus, the social preference ordering induced by simple majority voting exhibits a Condorcet cycle.

The assumption of only two feasible production plans finesses this problem. Furthermore, we adopt a *strategic* model of shareholder voting with small participation costs, instead of a normative social choice approach. Strategic voting models with participation costs were pioneered by Palfrey and Rosenthal (1983, 1985) for political elections, and adapted to shareholder voting by Ritzberger (2005).

3.1 Shareholder Assembly

After the stock market closes, shares are frozen and a shareholder assembly is called. The assembly works as follows. Every shareholder ($i \in I$ with $\theta_i > 0$) decides whether or not to participate and, if she does, how to cast her vote for one of the two alternatives $y = n$ and $y = m$. Yet, participation in the assembly carries a small privately born cost—similar to a "lexicographic" preference to get one's preferred alternative without participation. The decision in the assembly is taken with simple majority of represented votes, where one share counts for one vote. When ties occur in the voting, a coin is flipped to determine the outcome.[3]

The assumption of small participation costs implies that an agent will participate in the assembly if and only if her probability of being pivotal times the associated benefit is at least as large as the participation cost. And once an agent participates, she casts her vote for her preferred production plan, because participation with voting against one's preference is dominated.

For given share distribution $\theta = (\theta_1, ..., \theta_n) \in \Theta = \left\{\theta \in \mathbf{R}_+^n \left| \sum_{i=1}^n \theta_i = 1\right.\right\}$ denote the set of shareholders supporting the efficient production plan by $H(\theta) = \{i \in H \,|\theta_i < \gamma\}$ and the set of supporters of monopolistic output by $L(\theta) = L \cup \{i \in H \,|\theta_i > \gamma\}$ (see Proposition 1). High-valuation shareholders $i \in H$ with $\theta_i = \gamma$ are indifferent with respect to the firm's production plan and, therefore, will never participate in the assembly. Henceforth, shareholders $i \in L(\theta)$ will be referred to as *financial owners*, because their incentives are dominated by the profit motive. Shareholders $i \in H(\theta)$ will be called *stakeholders*, because their incentives are predominantly governed by the externality. Note that the distribution between stakeholders and financial owners is endogenous. The following 'dominant shareholder theorem' characterizes the pure strategy equilibria of the voting game.

Proposition 2 *For sufficiently small participation costs there exists a pure strategy equilibrium in the voting game if and only if one of the following three conditions holds:*

(a) there is $i \in H(\theta)$ such that $\theta_i > \theta_j$ for all $j \in L(\theta)$, in which case $y = n$ is adopted with certainty;

(b) there is $j \in L(\theta)$ such that $\theta_j > \theta_i$ for all $i \in H(\theta)$, in which case $y = m$ is adopted with certainty;

(c) $\sum_{i\in H(\theta)} \theta_i = \sum_{j\in L(\theta)} \theta_j$, in which case a coin toss determines the production plan.[4]

Proof: See Appendix.

This result states that, except for a knife-edge case, control of the company rests with a *dominant* shareholder, who owns strictly more shares than any one of the shareholders from the opponent faction does. (The dominant shareholder need not be the largest shareholder, though, provided the latter belongs to the same faction.)

This result is, in fact, quite typical for models of strategic shareholder voting with small participation costs. Even though the tie-breaking rule may make a difference with respect to pure strategy equilibria, the fact that the dominant shareholder fully controls the company is robust. Ritzberger (2005) considers a similar model, but with a "status-quo" that always wins at ties. In that case pure strategy equilibria take a dominant shareholder, who opposes the status-quo. Dorofeenko et al. (2005, Proposition 1) assume that one supporter of the

[3] The coin does not have to be fair. All that is important is that the probability of each of the two outcomes is bounded away from zero.

[4] If preferences were truly lexicographic, the pure equilibria described in (a)-(c) would be the only equilibria. For, at a mixed equilibrium at least one agent will be indifferent between participating and abstaining. But if her probability of being pivotal were zero, she would prefer to abstain; if it were positive, she would prefer to participate—a contradiction.

status-quo (the "president") is committed to participate in the assembly. Still, however, pure strategy equilibria feature a dominant shareholder, who controls the company.

If none of the three conditions from Proposition 2 is met, only mixed equilibria exist for the voting game. This obtains in the exceptional case, where $\max_{i\in H(\theta)} \theta_i = \max_{i\in L(\theta)} \theta_i$ and $\sum_{i\in H(\theta)} \theta_i \neq \sum_{i\in L(\theta)} \theta_i$. There may be more than one mixed equilibrium, though, in this case. Any equilibrium of the overall model will, however, involve a selection[5] of precisely one mixed equilibrium when the conditions from Proposition 2 fail.

To obtain well-defined payoffs for the stock market, we, therefore, fix one such selection throughout. This fixed selection is assumed to be such that it always selects a pure strategy equilibrium, whenever there exists one. (Note that it may be impossible to choose a continuous such selection.) Formally, the fixed selection is summarized by a function $\Lambda : \Theta \to [0,1]$. This function Λ is effectively the composition of the selection from the equilibrium correspondence, that maps share distributions $\theta \in \Theta$ into participation probabilities with the function that takes participation probabilities into the probability $\lambda = \Lambda(\theta) \in [0,1]$ that the assembly adopts the monopolistic production plan $y = m$. According to the assumption on the selection, condition (a) from Proposition 2 implies $\Lambda(\theta) = 0$, condition (b) implies $\Lambda(\theta) = 1$, and condition (c) implies $0 < \Lambda(\theta) < 1$.

3.2 Take-over Constraints

The function Λ captures the outcome of shareholder voting as far as production plans are concerned. At the same time it pins down what is required to alter the production plan by shifts in the share distribution.

In particular, if initially $\Lambda(\theta) = 1$, then Proposition 2 implies that $\max_{j\in L(\theta)} \theta_j > \theta_i$ for all $i \in H(\theta)$. A stock market trade from θ to a new share allocation θ' yields a change in the production plan, that is, $\Lambda(\theta') < 1$, if and only if $\max_{i\in H(\theta')} \theta'_i \geq \theta'_j$ for all $j \in L(\theta')$ holds after trade. Likewise, a trade from a share distribution θ with $\Lambda(\theta) = 0$ to a new allocation θ' induces a change in the production plan if and only if $\max_{i\in L(\theta')} \theta'_i \geq \theta'_j$ for all $j \in H(\theta')$ holds after trade.

The same reasoning applies, of course, when initially (at θ) $0 < \Lambda(\theta) < 1$ holds. In this case $\Lambda(\theta') = 1$ (resp. $\Lambda(\theta') = 0$) obtains after trade if and only if $\max_{j\in L(\theta')} \theta'_j > \theta'_i$ for all $i \in H(\theta')$ (resp. $\max_{i\in H(\theta')} \theta'_i > \theta'_j$ for all $j \in L(\theta')$). In other words, a change (in the probabilities) of the production plan(s) by trading from θ to θ' requires that after trade (at θ') a different type than before trade (at θ) is (one of) the dominant shareholder(s). Otherwise the dominant shareholder would still be of the same type as before trade, which would imply no change in the production plan.

As for notation, let the expected profit of the firm, given a probability $\lambda \in [0,1]$ of the monopolistic production plan, be defined by $\pi(\lambda) = \lambda\overline{\pi} + (1-\lambda)\underline{\pi}$. Substituting $\lambda = \Lambda(\theta)$ to obtain $\pi(\Lambda(\theta))$ gives the profit that agents expect to earn at the share distribution $\theta \in \Theta$ from their equity participation.

[5] A *selection* from a correspondence $F : \Theta \to [0,1]^n$ is any function $f : \Theta \to [0,1]^n$ such that $f(\theta) \in F(\theta)$ for all $\theta \in \Theta$.

4 The Stock Market

The shareholdings that are relevant for the vote on the production plan are those that result from trade at the stock market prior to the shareholder assembly. Modelling the stock market is a nontrivial exercise, though, because share trades imply possibly different outcomes (λ's) in shareholder voting. Trading at the stock market is, therefore, genuinely strategic. In particular, unless no change of λ is implied by stock market activity, how a given net trade is evaluated by an agent depends on what other agents trade. More precisely, it depends on whether or not an agent's net trade is pivotal or not for a target change in the probability λ of the monopolistic production plan.

At a share distribution $\theta \in \Theta$ with an implied probability $\lambda = \Lambda(\theta)$ of monopolistic output shareholder i's expected utility is

$$u_i(\theta_i, \lambda) \equiv \omega_i + \theta_i \pi(\lambda) + (1 - \lambda) \max\{0, v_i - \underline{v}\}$$

where the endowment ω_i may include revenue or expenditure from share sales or purchases that took investor i from initial share holdings to θ_i. Accordingly, the utility difference resulting from a trade from θ_i to θ_i', which implies a change in the probability of monopolistic output from $\lambda = \Lambda(\theta)$ to $\lambda' = \Lambda(\theta')$, is given by

$$\begin{aligned} u_i(\theta_i', \lambda') - u_i(\theta_i, \lambda) - q(\theta_i' - \theta_i) &= \\ (\lambda' - \lambda)\left[\theta_i' \Delta - \max\{0, v_i - \underline{v}\}\right] - (\theta_i' - \theta_i)\left[q - \pi(\lambda)\right] &= \\ (\lambda' - \lambda)\left[\theta_i \Delta - \max\{0, v_i - \underline{v}\}\right] - (\theta_i' - \theta_i)\left[q - \pi(\lambda')\right] & \end{aligned} \tag{7}$$

where $\Delta = \overline{\pi} - \underline{\pi} > 0$. Consequently, $i \in I$ prefers share θ_i' purchased (sold) at price q combined with the probability λ' of monopolistic output to share θ_i and λ if and only if

$$\begin{aligned} u_i(\theta_i', \lambda') - u_i(\theta_i, \lambda_i) &\geq q(\theta_i' - \theta_i) \Leftrightarrow \\ (\lambda' - \lambda_i)\Delta\left[\theta_i - \frac{\max\{0, v_i - \underline{v}\}}{\Delta}\right] &\geq (\theta_i' - \theta_i)\left[q - \pi(\lambda')\right] \Leftrightarrow \qquad (8) \\ (\lambda' - \lambda_i)\Delta\left[\theta_i' - \frac{\max\{0, v_i - \underline{v}\}}{\Delta}\right] &\geq (\theta_i' - \theta_i)\left[q - \pi(\lambda_i)\right] \qquad (9) \end{aligned}$$

where $\lambda_i = \Lambda(\theta_{-i}', \theta_i)$ is the probability of monopolistic output when all others trade to $\theta_{-i}' = (\theta_1', ..., \theta_{i-1}', \theta_{i+1}', ..., \theta_n')$, but i refuses to trade—assuming that this is feasible. If i is pivotal, in the sense that the change from λ to λ' fails, if she refuses to trade, then $\lambda_i = \lambda$. If it succeeds irrespective of whether or not i trades, then $\lambda_i = \lambda'$. Consequently, at $\lambda' = \lambda_i$ shareholder i will want to buy (resp. sell) an arbitrary amount of shares whenever $q < \pi(\lambda) = \pi(\lambda')$ (resp. $q > \pi(\lambda) = \pi(\lambda')$), and at $q = \pi(\lambda) = \pi(\lambda')$ she is indifferent.

To model the full complexity of stock market trades as a non-cooperative game seems intractable. Therefore, we limit the present analysis to a criterion that identifies when a given share distribution $\theta \in \Theta$ will *not* be changed by stock market activity aimed at buying control over the firm.

Methodologically this bears some resemblance to solution concepts for games, as it declares something—in this case some share distribution $\theta \in \Theta$—a "solution" if it is immune against a certain class of "deviations"—in this case stock market trades. But it is not a non-cooperative solution for a game, since it remains silent on the "off equilibrium" moves that bring about the equilibrium distribution. (Technically, there is no game form that underlies this "solution.") It is more like looking for a point, where a long process of trades will come to a halt in the sense that no further trades for control purposes will occur.

Two such criteria will be studied. For both the object is a share distribution $\theta \in \Theta$, and for both the class of allowed "deviations" will be stock market (net) trades at a *single* price. The two criteria differ with respect to how trades are initiated. The first criterion will consider tender offers that are unilaterally proposed by an agent. The second criterion will allow for coordinated trades that are initiated by a benevolent "auctioneer" or "market maker."

4.1 Tender Offers

Tender offers are taken to be offers by an agent to buy a specified quantity of shares at a specified price with the goal to alter the chances of the firm's production plans in shareholder voting. For an initial share distribution $\theta \in \Theta$ to qualify as an "equilibrium" distribution it is necessary that no agent can make a tender offer which is successful in the sense that other agents will voluntarily supply the shares required for the offer to succeed.

Because effective net trades have to be feasible, the following definition of a rationing rule is needed. For $x > 0$ define $\Omega_x = \left\{ \xi \in \mathbf{R}_-^{n-1} \middle| x + \sum_{i=1}^{n-1} \xi_i \leq 0 \right\}$ as the net supply vectors that at least fulfill the demand x and $\partial\Omega_x = \left\{ \xi \in \mathbf{R}_-^{n-1} \middle| x + \sum_{i=1}^{n-1} \xi_i = 0 \right\}$ as those that precisely meet x. A (supply side) *rationing rule* is a function $r_x : \Omega_x \to \partial\Omega_x$ such that $\xi \leq r_x(\xi)$ for all $\xi \in \Omega_x$.[6] Denote by $\mathcal{R}(x)$ the set of all such rationing rules r_x for a given $x > 0$.

Definition 1 *A* ***tender offer*** *by $i \in I$ is a pair $(q, x_i) \in \mathbf{R}_{++}^2$ consisting of a bid price $q > 0$ and a quantity $x_i > 0$ of shares demanded by i.*

The ***game induced*** *by a tender offer (q, x_i) by i at the share distribution $\theta \in \Theta$ is the n-player normal form game $\Gamma_\theta(q, x_i) = (S, \upsilon)$, where $S = \times_{j=1}^n S_j$ with $S_j = [-\theta_j, 0]$ for all $j \neq i$ and $S_i = \mathcal{R}(x_i)$, and $\upsilon = (\upsilon_1, ..., \upsilon_n) : S \to \mathbf{R}^n$ is given by*

$$\begin{aligned} \upsilon_j(s) = {} & (\theta_j + r_j(s_{-i}))\,\pi(\Lambda(\theta + r(s_{-i}))) \\ & + [1 - \Lambda(\theta + r(s_{-i}))]\max\{0, v_j - \underline{v}\} - q r_j(s_{-i}) \end{aligned} \tag{10}$$

for all $s \in S$ and all $j \in I$, where $r_{-i} = s_i \in \mathcal{R}(x_i) = S_i$ and $r_i(s_{-i}) = x_i$ if $x_i + \sum_{j \neq i} s_j \leq 0$ (that is, if $s_{-i} \in \Omega_{x_i}$) and $r(s_{-i}) = 0 \in \mathbf{R}^n$ if $x_i + \sum_{j \neq i} s_j > 0$ (that is, if $s_{-i} \notin \Omega_{x_i}$), $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$, and $r_{-i}(\cdot) = (r_1(\cdot), ..., r_{i-1}(\cdot), r_{i+1}(\cdot), ..., r_n(\cdot))$.

That the agent, who made the offer, can choose a rationing rule in the game induced by the tender offer expresses her discretion to trade with whom she wishes. Even though regulation in many countries requires the bidder to offer the same price to all potential

[6] Notation for vector inequalities is standard: $x \geq y$ means $x_i \geq y_i$ for all i, $x > y$ means $x \geq y$ but not $x = y$, and $x \gg y$ means $x_i > y_i$ for all i.

sellers, it cannot rule out this discretion. That strategy sets of suppliers are bounded (by $-\theta_i$) from below excludes short sales. After all, negative shares would have no meaning in a shareholder assembly, nor would shares above 1. A tender offer (q, x_i) that is not met by sufficient supply, that is, $x_i + \sum_{j \neq i} s_j > 0$, fails and all trades are cancelled.

Definition 2 *A tender offer* (q, x_i) *by* $i \in I$ *is* ***successful at*** $\theta \in \Theta$ *if the game* $\Gamma_\theta(q, x_i)$ *induced by it has a pure strategy Nash equilibrium* $s^\theta \in S$ *such that* $x_i + \sum_{j \neq i} s_j^\theta \leq 0$ *and at least one agent* $j \in I$ *is strictly better off with than without trade, that is,* $v_j(s^\theta) > v_j\left(s_{-j}^\theta, 0\right)$.

For a tender offer to be successful it takes an equilibrium in the game induced by it at which the demand is fulfilled. But the equilibrium needs to be nontrivial in the sense that at least one player has a strict incentive to play the equilibrium. This condition excludes trades at indifference. It also excludes offers that amount to a reshuffling of shares, say, between agents of the same type, without altering (the probabilities of) the production plan(s).

Since Grossman and Hart (1980) it has become customary to assume that traders, who are not pivotal to a trade, will not tender unless they are offered more than the after-trade value of the firm: the "atomistic shareholder model." This is what the definition of a successful tender offer captures.[7] If no trader is pivotal, then sellers will demand a price of at least the after-trade value of the firm. But the bidder, who wishes to buy, will optimally only make an offer with a price not exceeding the after-trade value of the firm. Therefore, when no agent is pivotal, by (7) no trader has a strict incentive to trade—thus excluding this as a successful tender offer.

Likewise, a trade between agents of the same type that implies no change of the probability $\Lambda(\theta)$ does not qualify as a successful tender offer. In fact, a successful tender offer by i at θ implies that $\Lambda(\theta) \neq \Lambda\left(\theta + \left(s_i^\theta\left(s_{-i}^\theta\right), x_i\right)\right)$. For, suppose that equality would hold. Then by the argument above trade can only take place at $q = \pi(\Lambda(\theta))$. But, under the assumption that $\Lambda(\theta) = \Lambda\left(\theta + r\left(s_{-i}^\theta\right)\right)$, (7) implies that all players are indifferent among all their strategies in $\Gamma_\theta(\pi(\Lambda(\theta)), x_i)$.

The following is the first equilibrium criterion that requires a share distribution which is not vulnerable to successful tender offers.

Definition 3 *A share distribution* $\theta \in \Theta$ *is* ***uncontestable*** *if there exists no successful tender offer at* θ*. It* ***supports*** *the efficient (resp. monopolistic) production plan if* $\Lambda(\theta) = 0$ *(resp.* $\Lambda(\theta) = 1$*).*

Uncontestability demands that the share allocation will not change due to any tender offer by a bidder, who attempts to take over in order to alter the production plan. In this sense an uncontestable share distribution is stable and constitutes an equilibrium notion. A share distribution that is not uncontestable is called *contestable*.

[7] Note, however, that Bagnoli and Lipman (1988) have shown that the argument by Grossman and Hart (1980) does not survive in a fully specified game model.

4.1.1 Existence

To prove logical consistency of uncontestability, the first result characterizes when a share allocation that supports efficiency is contestable, under the hypothesis that all financial owners together own no more shares than the dominant stakeholder.

Proposition 3 *A share distribution* $\theta \in \Theta$ *with* $\Lambda(\theta) = 0$ *and* $\sum_{i \in L(\theta)} \theta_i \leq \max_{i \in H(\theta)} \theta_i$ *is contestable if and only if* $\max_{i \in L} \theta_i + \max_{i \in H(\theta)} \theta_i > \gamma$.

Proof: See Appendix.

The intuition for this result is straightforward. When the characterizing condition is met, the largest financial owner can "bribe" the currently dominant high-valuation type by offering a premium over the after-trade value of the firm that compensates the high-valuation seller for the lost utility from the externality. This premium is affordable for the low-valuation bidder, because by $1/m \leq \gamma < 1$ (as implied by the characterizing condition) there remain enough high-valuation types that can be exploited after the take-over.

An important consequence of Proposition 3 is an existence result for uncontestable share distributions. For, let $\theta^* \in \Theta$ be given by $\theta_i^* = 1/m \leq \gamma$ for all $i \in H$ and $\theta_i^* = 0$ for all $i \in L$. Then $H(\theta^*) = H$, $L(\theta^*) = L$, and $\max_{i \in L} \theta_i^* + \max_{i \in H} \theta_i^* = 1/m \leq \gamma$. Therefore, the "only if"-part of Proposition 3 implies that θ^* is uncontestable.

Corollary 1 *There exists an uncontestable share distribution* $\theta^* \in \Theta$ *and it supports the efficient production plan,* $\Lambda(\theta^*) = 0$.

Intuitively, this result states that shares sufficiently dispersed among high-valuation types, and small enough stakes among low-valuation types, will guarantee uncontestability of the efficient production plan. In this sense competitive behavior goes along with dispersed ownership in the firm.

While Corollary 1 establishes existence, uniqueness remains open even with respect to the production plan supported by an uncontestable share distribution. This is to be studied next.

4.1.2 Multiplicity

Proposition 3 relies on the hypothesis that all financial owners together do not own more shares than the largest stakeholder. If they did, the efficient production plan would be contestable. This is so, because in that case the largest low-valuation type could buy all the shares held by financial owners and take over. This "pooling" of shares benefits all financial owners, as it achieves their preferred production plan, so that they do not even require a control premium in order to sell. Therefore, any share distribution that supports efficiency must be such that all financial owners together own no more than the dominant stakeholder.

Proposition 4 *Any share distribution* $\theta \in \Theta$ *with* $\Lambda(\theta) = 0$ *and* $\sum_{i \in L(\theta)} \theta_i > \max_{i \in H(\theta)} \theta_i$ *is contestable.*

Proof: See Appendix.

Intuitively, when all financial owners together own more than the largest stakeholder, they have an incentive to "pool" their shares to increase the chances of monopolistic output in shareholder voting. This does not involve a control premium, that is, the bid price does not exceed the after-trade value of the firm. This is so, because financial owners only tender an amount just enough to take over and benefit through their retained shares from the increased after-trade value of the firm. Take-overs by financial owners, therefore, involve a concentration of ownership.

That the efficient production plan is contestable does not necessarily imply, though, that the monopolistic production plan is uncontestable. But it may be for a particular parameter range.

To establish the possibility of multiple uncontestable share distributions, first observe that by Proposition 4 what counts for contestability of efficient output is the *sum* of shares held by financial owners. If this sum is large enough, they can pool sufficiently many shares in the hands of *one* financial owner to take over. If the resulting block of shares is big enough, it may be immune to counter-attacks, provided the externality is not too important as compared to the profit increment Δ (that is, γ is not too large). This is what the next result captures.

Proposition 5 *If $\gamma < 1/2$, any share distribution $\theta \in \Theta$ with $\max_{k \in L(\theta)} \theta_k + \min_{j \in H(\theta)} \theta_j > 2\gamma$ is uncontestable and supports the monopolistic production plan, $\Lambda(\theta) = 1$.*

Proof: See Appendix.

Let $k \in L(\theta)$ be such that $\theta_k > 2\gamma - \min_{j \in H(\theta)} \theta_j$. Then $\theta_j < \gamma$ for all $j \in H(\theta)$ implies by Proposition 2(b) that $\Lambda(\theta) = 1$, as $\theta_k > \gamma > \theta_j$ for all $j \in H(\theta)$. Suppose that $i \in H(\theta)$ can make a tender offer (q, x_i) such that after trade a probability $\lambda' < 1$ of monopolistic output obtains, if the offer is successful.

That $\lambda' < 1$ implies that $\theta_i + x_i \leq \gamma$, because otherwise i would also prefer monopolistic output by Proposition 1. If the offer is successful, then $\lambda' < 1$ also implies $\theta_i + x_i = \theta'_i \geq \theta'_k$ by Proposition 2(b) and the choice of the selection. Combining these two inequalities yields $\theta'_k \leq \gamma$. Since $x_i \geq \theta_k - \theta'_k$, it follows that $x_i \geq \theta_k - \gamma$. But then $\theta_i + x_i \geq \theta_i + \theta_k - \gamma > \theta_i + \gamma - \min_{j \in H(\theta)} \theta_j$ (where the last inequality follows from the hypothesis) implies $\theta_i + x_i = \theta'_i > \gamma$, a contradiction.

End of Proof.

The intuition for this result is based on preference reversal. A stakeholder, who attempts to buy control, must after trade own more shares than the originally dominant financial owner. But when the latter controls as much as she does according to the hypothesis of Proposition 5, this is impossible. For, by purchasing such a big quantity the buyer changes her preferences so as to also prefer monopolistic output. (That such a preference reversal can occur requires $\gamma < 1/2$.)

The high share of the dominant financial owner shareholder constitutes a "barrier" against tender offers by stakeholder. Therefore, monopolistic behavior is associated with concentrated ownership in the firm. A partial converse is that perfectly concentrated ownership implies monopolistic behavior, even if γ exceeds 1/2, but not 1.

Corollary 2 *If $\gamma < 1$, then any share distribution $\theta \in \Theta$, where one agents owns all shares, $\theta_i = 1$ for some $i \in I$, is uncontestable and supports the monopolistic production plan, $\Lambda(\theta) = 1$.*

Proof: See Appendix.

If $\gamma < 1/2$, then $\theta_i = 1$ implies that $i \in L(\theta)$, irrespective of whether or not $i \in L$. The desired result then follows from Proposition 5.

If $1/2 \leq \gamma < 1$, again $\theta_i = 1$ implies that $i \in L(\theta)$. Suppose some $j \in H = H(\theta)$ can make a tender offer (q, x_j) such that after trade $\lambda' < 1$ obtains. By hypothesis j can only buy from $i \in L(\theta)$. Therefore, (8) implies $\Delta\gamma \geq x_j(q - \pi(\lambda'))$ for $j \in H$ and $\Delta \leq x_j(q - \pi(\lambda'))$ for $i \in L(\theta)$, because $\theta'_i = 1 - x_j$. But these two inequalities combine to the contradiction $\Delta\gamma \geq \Delta \Rightarrow \gamma \geq 1$. **End of Proof.**

This result is like a mirror image of Corollary 1, where a dispersed ownership structure supports the efficient production plan. Here, one agent owning all shares supports the monopolistic production plan.

For completeness it needs to be said, however, that an uncontestable share distribution with full concentration of ownership may also support the efficient production plan. This will be the case, when the externality is sufficiently important compared to the profit increment Δ, that is, when $\overline{v} - \underline{v} > \Delta = \overline{\pi} - \underline{\pi} > 0$.

Corollary 3 *If $\gamma > 1$, any share distribution $\theta \in \Theta$ with $\theta_i = 1$ for some $i \in H$ is uncontestable.*

Proof: See Appendix.

Let $i \in H$ be such that $\theta_i = 1$, so that $\Lambda(\theta) = 0$ by Proposition 2(a). If a low-valuation type $j \in L$ were to stage a tender offer, she would have to buy from i. Thus, (8) implies $0 \geq x_j(q - \pi(\lambda'))$ for $j \in L$, where $\lambda' > 0$ if the offer succeeds. Therefore, $q \leq \pi(\lambda')$. On the other hand, (8) for $i \in H$ implies $\lambda'\Delta(\gamma - 1) \leq x_j(q - \pi(\lambda'))$, because $\theta'_i = 1 - x_j$. But the last inequality combined with $q \leq \pi(\lambda')$ and $\lambda' > 0$ implies the contradiction $\gamma \leq 1$.**End of Proof.**

As Proposition 5 and Corollary 2 show, when γ is not too large, control by financial owners is robust against take-over attempts under concentrated ownership. For, if stakeholders attempt a take-over, they reverse their preferences, when they hold such high shares in the company. This can only change when takeover attempts by high-valuation types are coordinated so that each of them buys only very little, but collectively they buy enough to unseat the controlling financial owner. This observation motivates the next, stronger criterion.

4.2 Coordinated Action

The scope of the uncontestability criterion is limited to decentralized trading, where one agent takes the initiative and makes a tender offer. More coordination is conceivable, though. As an extreme case imagine an auctioneer or a market-maker, who suggests net trades to agents until all gains from trade are exhausted. This can potentially overcome the coordination problem of stakeholders in trading towards a share distribution that supports the efficient production plan.

Definition 4 *An* ***offer at*** $\theta \in \Theta$ *is a pair* (q, x) *consisting of a share price* $q > 0$ *and a net trade vector* $x \in \mathbf{R}^n$ *such that* $\theta + x \in \Theta$.

The ***game induced*** *by an offer* (q, x) *at* θ *is the n-player normal form game* $\hat{\Gamma}_\theta (q, x) = \left(\hat{S}, \hat{v}\right)$, *where* $\hat{S} = \times_{i \in I} \hat{S}_i$, $\hat{S}_i = \{\min\{0, x_i\}, \max\{0, x_i\}\}$ *for all* $i \in I$, *and* $\hat{v} = (\hat{v}_1, ..., \hat{v}_n) : \hat{S} \to \mathbf{R}^n$ *with*

$$\begin{aligned} \hat{v}_i (s) = \phi_i (s) \pi (\Lambda (\phi (s))) - q (\phi_i (s) - \theta_i) \\ + [1 - \Lambda (\phi (s))] \max\{0, v_i - \underline{v}\} \end{aligned} \tag{11}$$

for all $s \in S$ *and all* $i \in I$, *where* $\phi (x) = \theta + x \in \Theta$ *and* $\phi (s) = \theta \in \Theta$ *for* $s \neq x$.

The definition of an offer at θ allows for the trivial case $x = 0 \in \mathbf{R}^n$ for generality. It will soon be shown that this does not do any harm. Short sales are excluded, though, as $\theta + x \geq 0$. Another restriction incorporated in the definition of an offer is that all investors trade at the same price q. And their market operations are coordinated, that is, the structure of trades is not decided individually, but elicited by the auctioneer.

Intuitively, an offer is a trade proposal to the agents that each one, who is affected ($x_i \neq 0$), can either accept ($s_i = x_i$) or reject ($s_i = 0$). If all accept, it is implemented, otherwise there is no trade. Clearly, this is a stylized way to represent a stock market as it requires *consensus* among affected agents. But, because all possible offers are considered and they cannot be influenced by the agents, it captures the best that a centralized market can achieve. In this sense such centralized offers constitute an interesting benchmark case.

If trade were allowed when some, but not all affected agents accept, markets would not clear and quantity rationing would be required. Yet, if an offer is accepted by some affected agents, but not by others, and the trades among the accepting agents are consistent with market clearing (possibly after some rationing), then there is an alternative offer that is accepted by all affected agents. In this sense there is no loss of generality involved with this mechanism, save for the implicit coordination.

Definition 5 *A* ***control trade at*** $\theta \in \Theta$ *is an offer* (q, x) *such that, in the game* $\hat{\Gamma}_\theta (q, x)$ *induced by the offer* (q, x), *the strategy profile* $s = x \in \hat{S}$ *constitutes a Nash equilibrium, where at least one affected agent* i *is strictly better off with trading than without, that is,* $\hat{v}_i (x) > \hat{v}_i (x_{-i}, 0)$.

A control trade is an offer that is successful in equilibrium. But again it is so in a nontrivial way. The condition that at least one player has a strict incentive to play the equilibrium excludes trades at indifference. In particular, it excludes trivial offers with $x = 0 \in \mathbf{R}^n$. The following is the stronger equilibrium criterion that is used to analyze coordinated market interaction.

Definition 6 *A share distribution* $\theta \in \Theta$ *is* ***universally uncontestable*** *if there exists no control trade at* θ.

The same share distribution as in Corollary 1 establishes existence of universally uncontestable share distributions. But, because the criterion is stronger than uncontestability, the production plan supported by a universally uncontestable share distribution is now unique: It is the efficient production plan.

Proposition 6 *There exists a universally uncontestable share distribution* $\theta^* \in \Theta$*, and every universally uncontestable share distribution supports the efficient production plan.*

Proof: See Appendix.

The intuition for this result is as follows. By quasi-linear preferences this is a transferable utility model. Therefore, appropriate transfers among the agents can always achieve efficiency. Since there are only two types, the market maker can find such a transfer scheme that is equivalent to net trades at a single price. This is, in particular, the logic underlying uniqueness of the supported production plan (part (b) of the proof).

This argument reveals that, unlike Corollary 1, the conclusion from Proposition 6 may be peculiar to the present model. While existence of an uncontestable share distribution that supports efficient production appears to hold quite generally, that *only* efficient production plans are supported by (universally) uncontestable share distributions depends crucially on a transferable utility framework—and, of course, on the presence of a coordinating auctioneer. Nevertheless, the power of universal uncontestability in a transferable utility model can serve as a useful benchmark for more general models of corporate control and stock market trade.

5 Conclusions

The present paper studies the interaction between two core institutions of industrial democracy: shareholder voting and stock market trade. In the model, economic agents can trade voting stock before the firm's production plan is decided by a vote among shareholders (under the one-share-one-vote rule). Even though this leads to a highly strategic interaction at the stock market, we identify surprising support for competitive behavior and efficiency.

A share distribution is *uncontestable* if it cannot be changed by a unilateral tender offer—an equilibrium criterion about share allocations. It is shown that there always exists an uncontestable share distribution that supports the efficient production plan. If the equilibrium criterion is extended so as to exclude all possible multilateral control trades ("universally uncontestable"), then *only* efficiency can be supported. Though this conclusion depends on the transferable utility framework employed here, it shows that share trading prior to decisions within the firm tends to push the allocation closer to where gains from trade are exhausted. And when gains from trade are exhausted, efficiency obtains.

Without the coordination implicit in universal uncontestability the picture is a bit more ambiguous. Even though there is always an uncontestable share distribution that supports the efficient production plan, the latter appears fragile. This is because the types supporting efficiency are endangered by preference reversal when their stakes in the company become too high. Therefore, supporting efficiency takes dispersed shares. But dispersed distributions are vulnerable to “bribing” some supporters of efficient production on account of exploiting the rest.

This opens the possibility of uncontestable distributions that support monopolistic behavior, that is, multiplicity of uncontestable share distributions. If shares are sufficiently concentrated in the hands of a type, who supports monopolistic behavior, such a distribution may be uncontestable. This is because a takeover attempt by a supporter of efficient production may be thwarted by preference reversal, as she needs to accumulate too many shares.

These observations combine to testable implications of the theory. According to the present model, competitive behavior (the efficient production plan) should be associated with dispersed share ownership. Reciprocally, concentrated ownership tends to come with monopolistic behavior. An empirical investigation of these relations is a topic for further research.

In terms of policy implications the results are encouraging for shareholder activism. As the model illustrates, a tighter grip of shareholders on their company can reduce the monopoly distortion. Voting trusts among supporters of the efficient production plan, for instance, could help maintaining dispersed ownership, without leaving control to supporters of monopolistic output. Thus, the present model could be used to provide a rationale for syndication.

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Appendix

Proof of Proposition 1. A shareholder with low valuation $v_i = \underline{v}$ (weakly) prefers the high price $p = \overline{v}$ $(y = m)$ to the low price $p = \underline{v}$ $(y = n)$ if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \theta_i \left[\underline{v} - c\right] n \Leftrightarrow \overline{\pi} \geq \underline{\pi}$$

that is, if and only if the maximum profit at the high price is at least as large as the maximum profit at the low price. Such a shareholder prefers the high price $(y = m)$ to a zero price $(y = n + 1)$ if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \underline{v} - \theta_i c \left(n + 1\right) \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\overline{\pi} + c\left(n + 1\right)} = \frac{\underline{v}}{\overline{v}m + c\left(n - m + 1\right)} \equiv \alpha$$

that is, if and only if her share is larger than a threshold α. Finally, the low price $p = \underline{v}$ $(y = n)$ is preferred by such a shareholder to a zero price $(y = n + 1)$ if and only if

$$\theta_i \left[\underline{v} - c\right] n \geq \underline{v} - \theta_i c \left(n + 1\right) \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\underline{\pi} + c\left(n + 1\right)} = \frac{\underline{v}}{\underline{v}n + c} \equiv \beta$$

Note that $\beta < 1/n$ and $\beta > \alpha$ if and only if $\overline{\pi} > \underline{\pi}$.

That is, low-valuation types with shares below $\min\{\alpha, \beta\}$ favor a zero price $(y = n+1)$. With shares above this threshold they favor the high (resp. low) price if $\overline{\pi} > \underline{\pi}$ (resp. $\overline{\pi} \leq \underline{\pi}$). In other words, low-valuation types with sufficiently many shares $(\theta_i \geq \min\{\alpha, \beta\})$ favor the alternative that yields higher profits.

A shareholders with high valuation $v_i = \overline{v}$ prefers a low price $p = \underline{v}$ $(y = n)$ to a high price $p = \overline{v}$ $(y = m)$ if and only if

$$\theta_i \left[\overline{v} - c\right] m \leq \theta_i \left[\underline{v} - c\right] n + \overline{v} - \underline{v} \Leftrightarrow \theta_i \leq \frac{\overline{v} - \underline{v}}{\overline{\pi} - \underline{\pi}} \equiv \gamma$$

provided $\overline{\pi} > \underline{\pi}$. (Under this condition the threshold γ satisfies $\gamma \geq 1/m$, because it is strictly decreasing in c and $c \leq \underline{v}$.) If $\overline{\pi} \leq \underline{\pi}$, then she *always* prefers the low price $p = \underline{v}$ $(y = n)$ to the high price $p = \overline{v}$ $(y = m)$. She prefers a low price $p = \underline{v}$ $(y = n)$ to a zero price $(y = n + 1)$ if and only if

$$\theta_i \left[\underline{v} - c\right] n + \overline{v} - \underline{v} \geq \overline{v} - \theta_i cn \Leftrightarrow \theta_i \geq \frac{\underline{v}}{\underline{\pi} + c\left(n + 1\right)} = \beta$$

that is, if her share is above the threshold β. Finally, she prefers a high price $p = \overline{v}$ $(y = m)$ to a zero price $(y = n + 1)$ if and only if

$$\theta_i \left[\overline{v} - c\right] m \geq \overline{v} - \theta_i cn \Leftrightarrow \theta_i \geq \frac{\overline{v}}{\overline{\pi} + c\left(n + 1\right)} \equiv \eta$$

i.e., if her share is at least as large as a threshold η. Note that $n > m$ and $\overline{v} > c$ imply $\beta < \eta$. Moreover, because $\overline{v} > c$, the inequality $\eta < \gamma$ holds if and only if $\overline{\pi} > \underline{\pi}$.

That is, high-valuation types with shares below β favor a zero price. With shares between β and η they favor a low price. If $\overline{\pi} \leq \underline{\pi}$, high-valuation types with shares above η continue to favor a low price, because in this case the threshold γ is negative, $\gamma < 0 < \eta$. If $\overline{\pi} > \underline{\pi}$, then $\eta < \gamma$, but high-valuation types with shares between η and γ still favor a low price. With shares above γ and $\overline{\pi} > \underline{\pi}$ they prefer a high price.

The thresholds are ordered as follows: $1/m > \eta > \max\{\alpha, \beta\}$, and $\overline{\pi} > \underline{\pi}$ if and only if $\alpha < \beta$ and $\eta < \gamma$.**End of Proof.**

Proof of Proposition 2. "if:" Let $\sigma_i \in \{0,1\}$ denote the participation probability of shareholder $i \in I$. Assume condition (a) and let $\sigma_i = 1$ for the shareholder $i \in H(\theta)$, who controls more votes than any $j \in L(\theta)$, and $\sigma_k = 0$ for all other shareholders $k \neq i$. Then no shareholder k other than i has an incentive to participate. For, $k \in L(\theta)$ cannot change the outcome (as $\theta_k < \theta_i$) but would have to bear participation costs; and $k \in H(\theta)$ already gets her preferred outcome without spending participation costs. Finally, if $i \in H(\theta)$ were to withdraw, $y = m$ would be adopted with positive probability (by a coin toss). Thus a pure Nash equilibrium has been constructed in which $y = n$ is adopted with certainty. An analogous argument, with $H(\theta)$ and $L(\theta)$ interchanged, establishes the "if"-part under condition (b).

Finally, assume condition (c) and let $\sigma_j = 1$ for all $j \in H(\theta) \cup L(\theta)$. Under this strategy combination both alternatives are adopted with positive probability (by a coin toss). If any participating shareholder would deviate, the preferred alternative of the opponent faction would be adopted with certainty. Hence, this is a Nash equilibrium.

"only if:" Consider a pure strategy Nash equilibrium. If $\sum_{i\in H(\theta)} \sigma_i\theta_i = \sum_{i\in L(\theta)} \sigma_i\theta_i$, then $\sigma_i = 1$ for all $i \in H(\theta) \cup L(\theta)$, because otherwise at least one $i \in H(\theta) \cup L(\theta)$ could improve by participating with certainty. Therefore, $\sum_{i\in H(\theta)} \theta_i = \sum_{i\in L(\theta)} \theta_i$ as required by (c).

Otherwise either $\sum_{i\in H(\theta)} \sigma_i\theta_i > \sum_{i\in L(\theta)} \sigma_i\theta_i$ or $\sum_{i\in H(\theta)} \sigma_i\theta_i < \sum_{i\in L(\theta)} \sigma_i\theta_i$. In the first case equilibrium implies that $\sigma_j = 0$ for all $j \in L(\theta)$ and that $y = n$ is adopted with certainty. If there were more than one $i \in H(\theta)$ with $\sigma_i = 1$, then one of them could withdraw without changing the outcome (as $\sigma_j = 0$ for all $j \in L(\theta)$), but saving participation costs. Thus, by the equilibrium hypothesis there is precisely one $i \in H(\theta)$ such that $\sigma_i = 1$. If there were $j \in L(\theta)$ with $\theta_j \geq \theta_i$, then $j \in L(\theta)$ would have an incentive to participate (as the probability of $y = m$ would jump from zero to a positive value), in contradiction to the equilibrium assumption. Therefore, $\theta_i > \theta_j$ for all $j \in L(\theta)$ which establishes (a). An analogous argument for the second case, where $\sum_{i\in H(\theta)} \sigma_i\theta_i < \sum_{i\in L(\theta)} \sigma_i\theta_i$, establishes (b) and completes the proof of the "only if"-part.

End of Proof.

Proof of Proposition 3. "if:" That $\lambda = \Lambda(\theta) = 0$ implies by Proposition 2(a) that $\gamma > \max_{i\in H(\theta)} \theta_i > \theta_j$ for all $j \in L(\theta)$. Let $k \in \arg\max_{i\in H(\theta)} \theta_i$, $j \in \arg\max_{i\in L} \theta_i$, and $\lambda' = \Lambda(\theta')$, where $\theta' \in \Theta$ satisfies $\theta'_j = \theta_j + \theta_k$, $\theta_k = 0$, and $\theta'_i = \theta_i$ for all $i \in I \setminus \{j,k\}$. Since $\theta_k \geq \theta_i$ for all $i \in H(\theta)$ and $\theta_j > 0$ by the hypothesis, Proposition 2(b) implies, by

$\theta_j' > \theta_i'$ for all $i \in H(\theta)$, that $\lambda' = 1$. Consider an offer by $j \in L$ to buy $x_j = \theta_k$ at a price

$$q = \underline{\pi} + \Delta \frac{\gamma}{\theta_k} > \pi(\lambda') = \overline{\pi}$$

(the latter, because $k \in H(\theta)$ implies $\theta_k < \gamma$). Even though this involves a control premium, it is profitable for $j \in L$, because by (7) and the hypothesis

$$\Delta(\theta_j + x_j) - x_j(q - \underline{\pi}) = \Delta[\theta_j + \theta_k - \gamma] > 0$$

The offer is also affordable for $j \in L$, because $qx_j = \theta_k \underline{\pi} + (\overline{v} - \underline{v})$, so that from (6) in the case $y = m$ and $v_j = \underline{v}$

$$\begin{aligned} qx_j = \theta_k \underline{\pi} + (\overline{v} - \underline{v}) &\leq \omega_0 + (\theta_j + \theta_k)\overline{\pi} \Leftrightarrow \\ \overline{v} - \underline{v} &\leq \omega_0 + \theta_j \overline{\pi} + \theta_k \Delta \end{aligned}$$

follows from the assumption that $\omega_0 > \overline{\pi} + \overline{v}$. For $k \in H(\theta)$ it is not costly to sell, because by (7)

$$-\Delta\gamma - (\theta_k' - \theta_k)(q - \underline{\pi}) = \Delta[\gamma - \gamma] = 0$$

No $i \in H(\theta)$ with $0 < \theta_i < \theta_k$ is willing to sell, though, because by (7) and $x_i \geq -\theta_i$

$$\begin{aligned} \Delta(\theta_i - \gamma) - x_i(q - \pi(\lambda')) &= \Delta\left(\theta_i - \gamma - x_i \frac{\gamma - \theta_k}{\theta_k}\right) \\ \leq \Delta\left(\theta_i - \gamma + \theta_i \frac{\gamma - \theta_k}{\theta_k}\right) &< \Delta(\theta_i - \theta_k) < 0 \end{aligned}$$

High-valuation types $i \in H(\theta)$ with $\theta_i = \theta_k$ are indifferent to trade, so it is optimal for them not to tender. All $i \in L(\theta) \setminus \{j\}$ are willing to sell, because $q > \pi(\lambda')$, irrespective of whether or not they are pivotal. But even if they all sell, they cannot fulfill j's demand, because from $\theta_j > 0$ it follows that $\theta_k \geq \sum_{i \in L(\theta)} \theta_i > \sum_{i \in L(\theta)\setminus\{j\}} \theta_i$, so their combined supply falls short of k's demand. Therefore, they must be rationed in equilibrium: If not, $k \in H(\theta)$ would still be pivotal, but could only sell less than θ_k; selling less than θ_k at q would not be profitable for $k \in H(\theta)$, though. Hence, in equilibrium $j \in L$ decides to ration other low valuation types, but not $k \in H(\theta)$.

"only if:" If at $\lambda = \Lambda(\theta) = 0$ there is a successful tender offer at price q by, say, $j \in L$, then for some $\lambda' > 0$ and $x_j > 0$

$$\begin{aligned} \lambda'\Delta\theta_j - x_j(q - \pi(\lambda')) = \lambda'\Delta(\theta_j + x_j) - x_j(q - \underline{\pi}) \geq 0 \\ \Leftrightarrow \pi(\lambda') + \lambda'\Delta\frac{\theta_j}{x_j} \geq q \end{aligned}$$

By the hypothesis that $\sum_{i \in L(\theta)} \theta_i \leq \max_{i \in H(\theta)} \theta_i$ success ($\lambda' > 0$) implies that at least one $i \in H(\theta)$ must tender some of her shares, that is, there is $i \in H(\theta)$ with $x_i < 0$ such that

$$\begin{aligned} \lambda'\Delta(\theta_i - \gamma) - x_i(q - \pi(\lambda')) = \lambda'\Delta(\theta_i + x_i - \gamma) - x_i(q - \underline{\pi}) \geq 0 \\ \Leftrightarrow \pi(\lambda') + \lambda'\Delta\frac{\gamma - \theta_i}{-x_i} \leq q \end{aligned}$$

which implies $(\gamma - \theta_i) / (-x_i) \leq \theta_j / x_j$, as $\lambda' > 0$ and $\Delta > 0$. Since either for $i \in H(\theta)$ or for $j \in L$ strict inequality must hold at a successful offer, it follows from $x_j > 0$ and $x_i < 0$ that $x_j (\gamma - \theta_i) + x_i \theta_j < 0$. Since $x_j \geq -x_i$ must hold, $\theta_j \geq 0$ and $x_j > 0$ imply $\theta_i + \theta_j > \gamma$. Therefore, $\max_{i \in L} \theta_i + \max_{i \in H(\theta)} \theta_i > \gamma$ as required.

End of Proof.

Proof of Proposition 4. If $\Lambda(\theta) = 0$, then by Proposition 2(a) there is some $i \in H(\theta)$ such that $\theta_i > \theta_j$ for all $j \in L(\theta)$. Fix $k \in \arg\max_{i \in L} \theta_i$ and a positive number $\delta < \sum_{j \in L(\theta)} \theta_j - \max_{i \in H(\theta)} \theta_i$ (which exists by hypothesis). Consider a tender offer (q, x_k) by $k \in L$ to buy quantity $x_k = \max_{i \in H(\theta)} \theta_i - \theta_k + \delta$ at a bid price q that is chosen as follows: If $L(\theta) = L$ (i.e. $H \cap L(\theta) = \emptyset$), then $q \in (\underline{\pi}, \overline{\pi})$; if $H \cap L(\theta) \neq \emptyset$ and

$$\min_{i \in H \cap L(\theta)} \theta_i \geq \frac{\gamma \sum_{j \in L(\theta) \setminus \{k\}} \theta_j}{\sum_{j \in L(\theta)} \theta_j - \max_{i \in H(\theta)} \theta_i - \delta} \tag{12}$$

then $q = \underline{\pi}$; finally, if $H \cap L(\theta) \neq \emptyset$ and the inequality in (12) is reversed ($<$), then

$$q = \underline{\pi} + \left(1 - \frac{\min_{i \in H \cap L(\theta)} \theta_i - \gamma}{x_k \min_{i \in H \cap L(\theta)} \theta_i} \sum_{j \in L(\theta) \setminus \{k\}} \theta_j \right) \Delta \tag{13}$$

(The reverse inequality to (12) implies that the term in brackets in (13) is strictly smaller than 1, hence, $q < \overline{\pi}$.) If this offer succeeds, then $k \in L(\theta)$ will hold more shares than any stakeholder, so that $\lambda' = 1$ will obtain after trade, with $\pi(\lambda') = \overline{\pi} > q$. The latter implies that the offer is affordable for k according to (6), because the price involves no control premium.

To see that the offer succeeds, consider the following net trades in response to the tender offer (q, x_k): Stakeholders do not sell at all, i.e. $\theta'_i = \theta_i$ for all $i \in H(\theta)$, and all financial owners (except k) sell precisely

$$\theta'_i - \theta_i = x_i = \frac{-x_k \theta_i}{\sum_{j \in L(\theta) \setminus \{k\}} \theta_j}$$

for all $i \in L(\theta) \setminus \{k\}$. Since $\sum_{i \in L(\theta) \setminus \{k\}} x_i = -x_k$, each $i \in L(\theta) \setminus \{k\}$ with $\theta_i > 0$ is pivotal for the tender offer to succeed. This implies that if financial owners are better off with selling, then each of them will sell precisely x_i. For, tendering less than $-x_i$ cannot be optimal, because then the offer fails and $q > \underline{\pi} = \pi(\lambda)$. Tendering more cannot be optimal either, because it leaves $\lambda' = 1$ unchanged by rationing, and selling one extra share earns q, but costs $\overline{\pi}$, which is more than q. Therefore, each $i \in L \setminus \{k\}$ sells precisely x_i.

When the tender offer (q, x_k) succeeds, $k \in L$ is better off, because

$$\underline{\pi} + \frac{\theta_k + x_k}{x_k} \Delta = \overline{\pi} + \frac{\theta_k}{x_k} \Delta > q \Rightarrow (\theta_k + x_k) \Delta > x_k (q - \underline{\pi})$$

where the latter is the profitability condition (9) with $\lambda' = 1$, $\lambda_k = \lambda = 0$, and $v_k = \underline{v}$.

Likewise, any $i \in L$ with $\theta_i > 0$ gains by selling, as

$$\theta_i + x_i = \theta_i \frac{\sum_{j \in L(\theta)} \theta_j - \max_{j \in H(\theta)} \theta_j - \delta}{\sum_{j \in L(\theta) \setminus \{k\}} \theta_j} > 0$$
$$\Rightarrow (\theta_i + x_i) \Delta > 0 \geq x_i (q - \underline{\pi})$$

where the latter is again (9) for $\lambda' = 1$, $\lambda_i = \lambda = 0$ (because i is pivotal), and $v_i = \underline{v}$. If $L(\theta) = L$, this completes the proof that all active traders gain.

If $H \cap L(\theta) \neq \emptyset$, consider first the case, where (12) holds, so that $q = \underline{\pi}$. With $l \in \arg\min_{i \in H \cap L(\theta)} \theta_i$ condition (12) is equivalent to $\theta_l + x_l \geq \gamma$, hence, $\theta_i + x_i \geq \gamma$ for all $i \in H \cap L(\theta)$. Therefore, the appropriate version of (9) for $i \in H \cap L(\theta)$ is

$$\Delta (\theta_i + x_i - \gamma) \geq 0 = x_i (q - \underline{\pi})$$

(because i is pivotal) showing that it is optimal for $i \in H \cap L(\theta)$ to sell. If the inequality in (12) is reversed, then q is given by (13), thus, $\underline{\pi} < q < \overline{\pi}$. Denoting again $\theta_l = \min_{i \in H \cap L(\theta)} \theta_i$ the inequality $\theta_i \geq \theta_l$ for all $i \in H \cap L(\theta)$ is equivalent to

$$\underline{\pi} + \frac{\theta_i + x_i - \gamma}{x_i} \Delta = \underline{\pi} + \left(1 - \frac{\theta_i - \gamma}{x_k \theta_i} \sum_{j \in L(\theta) \setminus \{k\}} \theta_j \right) \Delta \leq q$$

which for $x_i < 0$ is in turn equivalent to $\Delta (\theta_i + x_i - \gamma) \geq x_i (q - \underline{\pi})$, which is the appropriate version of (9) for $\lambda' = 1$, $\lambda_i = \lambda = 0$ (because i is pivotal), and $v_i = \overline{v}$. By the previous argument tendering more or less than $-x_i$ is suboptimal, so each $i \in H \cap L(\theta)$ also sells precisely x_i.

No stakeholder $i \in H(\theta)$ would be willing to sell, even if she were pivotal, because

$$\Delta (\theta_i - \gamma) < 0 < (\theta_i' - \theta_i) (q - \overline{\pi})$$

follows from (8) with $\lambda' = 1$, $\lambda_i = \lambda = 0$, and $v_i = \overline{v}$ together with $\theta_i' < \theta_i < \gamma$ and $q < \overline{\pi}$. Given the net trades of financial owners, no stakeholder $i \in H(\theta)$ is pivotal, though, so that $q < \overline{\pi} = \pi(\lambda')$ alone implies that no stakeholder is willing to sell. It follows that $x_k + \sum_{i \in L(\theta) \setminus \{k\}} x_i = 0$ and the tender offer succeeds.

End of Proof.

Proof of Proposition 6. (a) Let $\theta^* \in \Theta$ be given by $\theta_i^* = 1/m$ for all $i \in H$ and $\theta_i^* = 0$ for all $i \in L$. At θ^* the efficient production plan, $\lambda = \Lambda(\theta^*) = 0$, is chosen by the shareholder assembly, because by $c < \underline{v}$ it follows that $\gamma > \theta_i^* = 1/m > \theta_j^* = 0$ for all $i \in H$ and all $j \in L$. Consider an offer (q, x) such that $x_j > 0$ for some $j \in L$. If (q, x) is a control trade, then for $j \in L$ by (7)

$$u_j (x_j, \lambda') - u_j (0, 0) - q x_j = x_j (\pi(\lambda') - q) \geq 0$$

must hold, where $\lambda' = \Lambda(\theta^* + x) > 0$. This implies that $q \leq \pi(\lambda')$. Denote by $H' \subseteq H$ the set of high-valuation sellers, that is $i \in H' \subseteq H \Leftrightarrow x_i < 0$. The set H' must be nonempty, because $\sum_{i=1}^{n} x_i = 0$, $x_j > 0$ for $j \in L$, and $x_k \geq 0$ for all $k \in L$ by $\theta^* + x \geq 0$. But for any $i \in H'$ that $q \leq \pi(\lambda')$ implies from (7) that

$$\begin{aligned} u_i\left(\theta_i^* + x_i, \lambda'\right) - u_i\left(\theta_i^*, 0\right) - q x_i &= \\ \lambda' \Delta \left(\frac{1}{m} - \gamma\right) - x_i\left(q - \pi\left(\lambda'\right)\right) \leq \lambda' \Delta \left(\frac{1}{m} - \gamma\right) &< 0 \end{aligned}$$

that is, no $i \in H'$ is willing to accept the sales offer—a contradiction.

There is the possibility, though, that no $i \in H'$ is pivotal, for instance, if $x_j \geq 2/m$ and all $i \in H'$ tender their shares. Then, given that all others sell, $i \in H'$ sells only if $q \geq \pi(\lambda')$. Therefore, if (q, x) is a control trade, $q = \pi(\lambda')$. But this and the hypothesis that no high-valuation type is pivotal implies that no trader can strictly profit from trade. Hence, this does not qualify as a control trade either.

There remains the possibility that a high-valuation type $j \in H$ buys $x_j > \gamma - 1/m > 0$, so that after trade she prefers to produce monopolistic output. If this is a control trade, then from (7)

$$\begin{aligned} u_j\left(\theta_j^* + x_j, \lambda'\right) - u_j\left(\theta_j^*, 0\right) - q x_j &= \\ \lambda' \Delta \left(\frac{1}{m} - \gamma\right) - x_j\left(q - \pi\left(\lambda'\right)\right) &\geq 0 \end{aligned}$$

implies $q < \pi(\lambda')$. But at $q < \pi(\lambda')$ no high-valuation type $i \in H$ is willing to sell, because

$$\begin{aligned} u_i\left(\theta_i^* + x_i, \lambda'\right) - u_i\left(\theta_i^*, 0\right) - q x_i &= \\ \lambda' \Delta \left(\frac{1}{m} - \gamma\right) - x_i\left(q - \pi\left(\lambda'\right)\right) &< 0 \end{aligned}$$

It follows that there can be neither a control trade with $x_i > 0$ for some $i \in L$, nor one with $x_i > 0$ for some $i \in H$. Since $x \leq 0$ implies $x = 0$ by the definition of an offer, it follows that θ^* is universally uncontestable.

(b) At any $\theta \in \Theta$, where $\lambda = \Lambda(\theta) > 0$, consider the offer (q, x) with $q = \pi(\lambda) + \varepsilon$ for some small ε with $0 < \varepsilon < \lambda\Delta(\gamma m - 1)$, $x_i = -\theta_i$ for all $i \in L$, and $x_i = 1/m - \theta_i$ for all $i \in H$. Then (7) implies for $i \in L$ with $\theta_i > 0$ that

$$u_i\left(\theta_i + x_i, 0\right) - u_i\left(\theta_i, \lambda\right) - q x_i = x_i\left(\pi\left(\lambda'\right) - q\right) = \theta_i \varepsilon > 0$$

and for $i \in H$ that

$$u_i\left(\frac{1}{m}, 0\right) - u_i\left(\theta_i, \lambda\right) - q x_i = \lambda\Delta\left(\gamma - \frac{1}{m}\right) - \left(\frac{1}{m} - \theta_i\right)\varepsilon > 0$$

for ε sufficiently small.

End of Proof.