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Liquidity and Economic Fluctuations

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# Liquidity and Economic Fluctuations\*

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## Abstract

This paper shows that private information may be crucial in explaining the relationship between liquidity, investment and economic fluctuations. First, it defines liquidity in a way that is clearly connected to investment and output. Second, it models economies where privately informed entrepreneurs issue debt to fund their investment opportunities and identifies a theoretically based, empirically usable, and macroeconomic relevant measure of liquidity of the economy: the cross-firm dispersion in debt yields. Finally, it rationalizes one novel stylized fact regarding the US corporate bond market: the positive relationship between the proposed measure of liquidity - the cross-firm dispersion in the "yields to maturity" on newly issued publicly traded debt - and subsequent aggregate economic activity.

JEL codes: E2, E3, G14 Keywords: Liquidity, private information, robust pooling equilibrium, bond yield

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# 1 Introduction

During financial and economic crises, such as the very recent one, there are frequent concerns regarding the level of liquidity available to the corporate sector. In practise, though, it is difficult to assess the severity of these concerns because it is not obvious what is a macroeconomically meaningful notion of liquidity and how it could be measured. We address this issue in three steps. First, we propose a definition of liquidity that is clearly connected to investment and output and, therefore, is useful to macroeconomics. Second, we present an empirical measure of liquidity that is consistent with this definition. Finally, we show that the predictions of our model regarding the relationship between liquidity and economic fluctuations are consistent with U.S. data and help rationalizing some not previously observed stylized facts.

*Liquidity* is defined here as the ease of translating the future values of assets into current market prices.<sup>1</sup> This is not a trivial issue because, when firms issue securities to fund their investment, they generally have private information regarding the quality of the opportunities at hand.<sup>2</sup> We show that this simple informational friction may be responsible for the illiquidity of the economy, formally defined by the fact that available securities are mispriced, i.e. their prices differ from the discounted value of the payments they entail. The mispricing of securities used to finance investment is important since it causes misallocation of resources and suboptimal equilibria in the real economy.

A recent and active area of research, starting with the works of Kiyotaki and Moore (2001a, 2001b), Caballero and Krishnamurthy (2001, 2002) and Krishnamurthy (2003), has delivered insightful findings on the relationship between liquidity and investment in economies with exogenously given illiquid assets. The key theoretical contribution of our paper to this growing literature is to show that the mispricing that characterizes the lack of liquidity in the economy is an equilibrium result and should not be implicitly assumed. In particular, we argue that the securities used to

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<sup>1</sup>This definition follows most closely Eisfeldt (2004) and Holmstrom and Tirole (2001): "liquidity [...] does not [only] refer to the ease with which assets can be resold, but rather to [...] the value of financial instruments used to transport wealth across time [...]."

<sup>2</sup>Johnson (2004) stresses that the finance literature highlights at least three distinct sources for (il)liquidity: search costs, inventory risks and asymmetric information. Here, following an established macroeconomic tradition (Diamond and Dybvig (1983), Caballero and Krishnamurthy (2001) (2002), Krishnamurthy (2003)) I focus on private information.

finance investment, and assets in general, are not liquid or illiquid *per se* but their liquidity is determined in equilibrium by the optimizing behavior of firms, which respond to the presence of asymmetric information and the state of economic fundamentals. Lack of liquidity is therefore an equilibrium result that may—not must—appear. We then elicit the interesting implications that can be derived by studying how the liquidity of assets results from the change in entrepreneur's optimizing behavior in front of shifting economic fundamentals.

By viewing the liquidity of assets as an equilibrium outcome, we propose a setup with the clear advantage of linking together liquidity, investment and economic fluctuations. Focusing on the relationship between liquidity in the corporate bond market and aggregate fluctuations, our model predicts that liquidity, investment, and the variance of leverage decisions across firms should all be positively correlated with economic growth. We then compare these theoretical predictions with data regarding the US corporate bond market and, consistently with the model's set up, we interpret the cross-firm dispersion in the prices—"yields to maturity"—of newly issued publicly traded debt as a measure of liquidity. With our theoretical framework in mind, we highlight a novel stylized fact in the US corporate bond market: the positive relationship between the cross-firm dispersion in the "yields to maturity" on newly issued publicly traded debt - i.e. liquidity in our perspective - and subsequent aggregate economic activity. This observation contributes to the literature not only because it provides an empirically usable and macroeconomically relevant measure of liquidity, but also because it observes and rationalizes a novel correlation between aggregate variables in the US economy.

Going in more details, we consider economies where *entrepreneurs* finance their investment by issuing bonds, subject to the possibility of default. There are two distinctive features of this set of economies. The first is that entrepreneurs have private information about the productivity of the investment opportunity at hand. The second is that entrepreneurs choose how many bonds they issue, i.e. their leverage, hence selecting the size of their investment and the probability and extent of default on their bonds.<sup>3</sup> This matches what happens in the real world where firms actively manage their leverage, as Leland and Pyle (1977) and Myers and Majluf (1984) pointed out in a related context.

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<sup>3</sup>This study does not explicitly model financial intermediation by banks. The analysis could be easily applied to loan contracts.

Consumers lend funds to entrepreneurs by purchasing bonds in competitive markets but they can only distinguish one firm from the other on the basis of its leverage, i.e. the number of bonds a firm issues. Capital markets are consistently organized so that all bonds issued by firms undertaking the same leverage decision are traded in the same "market". The price of a given bond thus reflects the information consumers extract in equilibrium about the mix of good and bad entrepreneurs undertaking that specific leverage level. Entrepreneurs simultaneously decide how many bonds they issue, comparing the bond price with their expected payments across different leverage levels.

In the absence of informational asymmetry, high productivity entrepreneurs have a higher return on their investment and so their size is also larger than low productivity entrepreneurs. When asymmetric information is introduced, productive entrepreneurs react by distorting their leverage decision in an attempt to signal their quality to consumers. In order to distinguish themselves, high productivity entrepreneurs increase their leverage, raising the number of bonds they issue. Low quality entrepreneurs then face the crucial trade-off. They can decide to mimic the leverage decision of better quality entrepreneurs so that their bonds will be overpriced (positive *price effect*), but they also bear the cost of distorting their leverage decision (negative *leverage effect*) and increasing their probability of default. Otherwise, they may decide to go their separate way, avoid to distort their leverage and receive a fair price for their bonds. This trade-off is at the basis of the type of equilibrium—pooling or separating—that may emerge in the bond market of this economy.

Our main theoretical contribution is to show that this trade-off endogenously shifts making the type of equilibrium depend on aggregate productivity. We find that, when the state of aggregate productivity is low so that the probability that investment opportunities succeed falls, illiquid pooling equilibria emerge. The intuition is that good and bad technologies are very different during bad times because the realization of the bad contingency is relatively worse for the volatile technology of worse entrepreneurs. Thus the "price effect" is stronger than the "leverage effect" pushing low quality entrepreneurs to mimic better entrepreneurs and, thus, all entrepreneurs issue the same number of bonds. The economy is illiquid in a pooling equilibrium because, although bonds are correctly priced on average, each of them is individually mispriced, i.e. it has a price that differs from the discounted value of its future payments. In fact, better entrepreneurs find it relatively more difficult to borrow funds: bonds issued by high (low) quality entrepreneurs are underpriced

(overpriced). This generates a misallocation of investment that lowers aggregate production. Lack of liquidity is interpreted here not only as the fact that good investment opportunities face hard terms when applying for credit, but also as the fact that credit providers have a hard time in identifying the most worthy projects and so funds are diverted towards less productive investment. This interpretation of liquidity is economically meaningful and already present in the literature (Eisfeldt (2004) and Kurlat (2009)).

In contrast, when the economy picks up, liquid separating equilibria arise. When the probability that investment opportunities succeed increases, the "distance" between high and low productivity entrepreneurs reduces. Therefore, the subsidization in bonds' prices—the price effect—becomes less attractive for low quality entrepreneurs. As the state of the economy improves, these entrepreneurs prefer to abandon the pooling equilibrium to avoid the distortion in their leverage decision. When entrepreneurs with different technologies issue different amounts of bonds and undertake different leverage decisions, the separating equilibrium that appears in the bond market restores liquidity: private information is fully revealed and consumers identify the quality of each bond—i.e. its expected delivery— simply by observing leverage decisions. Therefore, bond prices reflect the actual present value of future payments for each bond.

The fact that liquidity is procyclical in this setup may seem counterintuitive: it appears natural to argue that, when the economic outlook turns grim, high quality firms will want to avoid the additional liquidity cost connected with subsidizing low quality bonds in the pooling equilibrium. This intuition, however, is not correct if one accepts the crucial trade-off stressed here: it is the relatively stronger incentive of bad quality entrepreneurs to mimic better entrepreneurs when productivity is low to determine the failure of the good entrepreneur's signalling effort.

In order to obtain this novel result we overcome a well known problem in the literature on adverse selection. There is a general consensus that, for economies with asymmetric information, pooling equilibria are either not existent or difficult to sustain because they are sensitive to agents' beliefs about off-equilibrium. This would be a problem in the present context because pooling equilibria are central to the analysis of liquidity. We show that under fairly general conditions, illiquid pooling equilibria are indeed robust to optimistic off-equilibrium expectations, *provided* that bonds' payments are positively related to the productivity of the underlying firm as it is

assumed here. By adapting to the framework of a production economy the definition of equilibrium in Dubey and Geanakoplos (2003), we show that, differently from Dubey and Geanakoplos (2003) and Martin (2008), also pooling equilibria, and thus illiquidity, are robust.<sup>4</sup>

The model has one main prediction that find supporting evidence in the US bond market. The theory predicts that when the economy faces poor economic prospects, an illiquid pooling equilibrium emerges. In this equilibrium, firms undertake *similar* leverages and so the prices of bonds—the yields—issued by different entrepreneurs converge. I identify cross-firm bond yields dispersion as the measure of the economy-wide liquidity: the smaller the dispersion, the *less* liquid is the economy. When the economy accelerates and a liquid separating equilibrium arises, firms instead differentiate themselves by undertaking *different* leverages, and bond yields diverge accordingly, showing an improvement in the economy-wide liquidity. Thus the theory predicts that the cross sectional variance in bond yields issued by different firms in a given quarter—the proposed measure of economy-wide liquidity- should increase in anticipation of an improvement in economic activity.

Our paper also contributes to the line of research that links asymmetric information to the notion of liquidity and relates most closely to Eisfeldt (2004) and, more recently, Kurlat (2009). It differs from these works because it highlights the role of endogeneity of financial contracts and leverage decisions in establishing a relationship between liquidity and economic fluctuations and providing an observable measure of liquidity that can be employed in future empirical analysis.<sup>5</sup> In a seminal contribution, Eisfeldt (2004) finds that, as the productivity of technology changes along the business cycle, entrepreneurs have the incentive to raise more funds when productivity is higher. Entrepreneurs finance their new investment by selling claims over the future production of projects *previously initiated*. When the economy faces a high productivity shock, the most productive entrepreneurs tend to sell relatively more claims than the owners of worse technologies. Therefore, when the economy faces a higher productivity shock, the quality of the average traded claim increases, the market price increases and this is interpreted as an increase in liquidity to the corporate sector. This explanation relies on the restriction that we relax in this paper: entrepreneurs

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<sup>4</sup>Please see Martin (2008) and section 3 for details on this issue.

<sup>5</sup>See, for instance, Eisfeldt and Rampini (2006) and Farhi, Golosov, and Tsyvinski (2008) regarding other empirical measures of liquidity.

face a security space made up of only one "contract". They can not use their leverage decision—the choice of how many securities they issue—to signal their private information because this choice can not be observed by financial traders by assumption. This assumption seems both theoretically and empirically questionable as Demarzo and Duffie (1999) and Demarzo (2003) pointed out in the context of security design. Although the relaxation of this restriction delivers substantial complications, we address these complications in this paper and their solution enables us to derive a novel and well defined empirical measure of liquidity, as well as a prediction that seems to find support in the context of US corporate debt market.

The paper is organized as follows. Section 2 lays out the setup and equilibrium definition. Section 3 discusses the equilibrium in economies with symmetric information. Section 4 studies the economies with informational asymmetry while section 5 presents the empirical regularity in the US bond market consistent with the predictions of the model. Section 6 concludes.

## 2 The Economy: Setup

In the discussion that follows the reader may refer to the following figure:

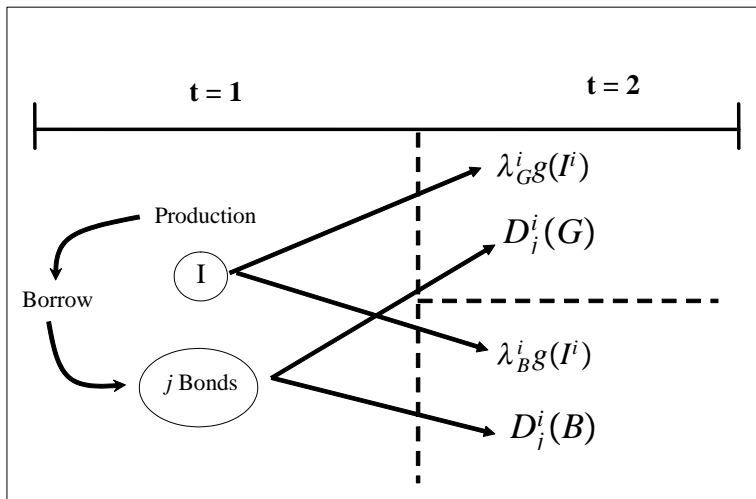


Figure 1

**Time:** the economy lasts two periods:  $t = 1, 2$ . Two contingencies,  $s \in S = \{G, B\}$ , can be realized at  $t = 2$  with probabilities  $\alpha_s = \Pr(s)$ ;

**Commodity Space:** there is a single perishable consumption/investment good in each period.

Let  $c_t^h(s)$  denote the amount consumed by agent  $h$  at time  $t$  in state  $s$ ;

**Agents and Endowment:** there is a continuum of consumers  $h$  uniformly distributed on the interval  $[0, z]$ ,  $z > 1$ . Consumers are endowed with  $w > 0$  units of the consumption good at  $t = 1$  only. There is also measure 1 of entrepreneurs endowed with investment technology  $i = \{H, L\}$  distributed according to:

$$\begin{aligned}\eta &= (\eta(H), \eta(L) = 1 - \eta(H)) \\ 1 &> \eta(H) > 0\end{aligned}$$

and no consumption good endowment.

**Preferences:** in order to abstract from risk, all agents in the economy have identical linear preferences:

$$V[c] = c_1 + [\alpha_G \cdot c_2(G) + (1 - \alpha_G) \cdot c_2(B)] \quad (1)$$

**Technology:** there are 2 types of investment opportunities, labeled by  $i = \{H, L\}$  where  $H$  ( $L$ ) stands for high (low) productivity. They are available at  $t = 1$  and produce stochastic output at  $t = 2$  according to the following structure:

$$\begin{aligned}\lambda_s^i g(I^i) \\ s \in \{G, B\}, i \in \{H, L\} \\ g(0) = 0, g'(\cdot) > 0, g''(\cdot) < 0 \\ \lim_{x \rightarrow 0} g'(x) = +\infty\end{aligned} \quad (2)$$

The production function is neoclassical,  $I^i$  labels the units of consumption good invested in technology  $i$ , and the technological parameters  $\lambda$  satisfy:

$$\begin{aligned}\lambda^H &= [\lambda_s^H = \lambda^H, \forall s] \\ \lambda^L &= \begin{bmatrix} \lambda_G^L = \lambda^L \\ \lambda_B^L = 0 \end{bmatrix} \\ \lambda^L &< \lambda^H \\ E_s(\lambda_s^H) &> E_s(\lambda_s^L)\end{aligned} \quad (3)$$

so that " $H$ " is clearly the better technology since it gives higher expected production with no

variance.<sup>6</sup> Moreover I will write:

$$g'(I) = \frac{d[g(I)]}{d[I]}$$

**Perfect positive correlation** (i.e. aggregate uncertainty) in technologies' payoffs is assumed. This is equivalent to an economy with two aggregate states but where each investment opportunity is subject to a technology-specific state contingent shock.<sup>7</sup>

**Bond Market:** there are many financial contracts, each generated by the entrepreneur's choice of how many bonds she issues. Each financial contract is therefore characterized by a vector  $D_j^i$  of state contingent payments to bondholders,  $D_j^i(s)$ , which depends on the number of bonds issued,  $j$ , and the quality of the technology in the issuing firm,  $i$ . With a little abuse in the definition - but for the sake of simplicity, I will often refer to the financial contract corresponding to the issuance of  $j$  bonds as "security  $j$ ". Securities translate into payments at  $t = 2$  in the following way:

$$D_j^i(s) = \min \left\{ 1, \frac{(1 - \gamma) \lambda_s^i g(I^i)}{j} \right\} \quad (4)$$

The payment vector of a security depends on the default decision of the issuing entrepreneur. Given the chosen security  $j$ , the entrepreneur decides whether to default and surrender part of his production  $(1 - \gamma) \lambda_s^i g(I^i)$ , net of the  $0 < \gamma < 1$  share she can conceal, or to honor her initial promise and pay 1 unit of the consumption good per issued bond. Default is thus strategic in this setup. The assumption that, after default, entrepreneurs can conceal a share of the firm value to creditors is standard and seems realistic. I do not need to assume that this share is large as long as it is different from zero.

Moreover, it is important to stress why focusing on the bond market in general, and corporate debt in particular is appropriate. First, bonds are the largely predominant part of external financing. Bolton and Scharfstein (1996) report that, from 1946 to 1987, 85 percent of total U.S. firms external financing was raised through debt offerings. Second, this assumption is theoretically consistent with the present model: it can be easily shown that bonds (or bank loans for all that matter) are in fact the optimal credit contract here. Adding equity contracts would not change any of the qualitative results of this paper.

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<sup>6</sup>In order for the arguments in the paper to go through, it is only required that the best technology has higher expected production and smaller variance. The specific structure assumed here is for expositional simplicity only.

<sup>7</sup>This assumption simplifies notation and is made without loss of generality.

Finally observe that, since better technologies default less often - for given number of issued bonds-, equation (4) implies that the security's expected payment is an increasing function of the technology's productivity/quality. This is equivalent to assuming that more productive firms issue more valuable securities on average and, at the same time, that the payment linked to a security depends on the economic performance of a firm - its production here. These realistic assumptions will play a central role in establishing a relation between economic fluctuations and liquidity.

**Information Structure:** At  $t = 1$  every entrepreneur is privately informed about the productivity of her investment project. Since the issuer-entrepreneur knows the quality of the technology, she knows the actual payoff/value of the bonds she issued. Thus entrepreneurs are privately informed about the actual quality of the security they are selling. Consumers can only observe the number of bonds that a firm is issuing, not its technological quality.

**Security Holdings and Prices:** the specific financial contract is identified observing the number of bonds, ( $j$ ), issued by the firm. Consumers observe the number of bonds a firm issues - which is public information - but not their qualities, which depends on the firm's technological quality, the entrepreneur's private information. Consumer  $h$  purchases  $a^h(j)$  units of financial contract  $j$  while entrepreneur  $i$  issues security  $j$  (or  $j^i$  when notation requires it) defined by the issuance of number  $j$  of bonds.  $q(j)$  denotes the price of security of type " $j$ ".<sup>8</sup>

**Individual Budget Constraint**  $B^i(q(j))$ ,  $B^h(q(j))$ : since the issue of bonds requires an investment technology that can be offered as collateral, only entrepreneurs can issue securities. Entrepreneur  $i$  budget constraint,  $B^i(q(j))$ , is:

$$\begin{aligned} c_1^i + I^i &\leq q(j)j & t = 1 \\ c_2^i(s) &\leq \lambda_s^i g(I) - D_j^i(s)j & t = 2 \end{aligned}$$

Consumer  $h$  budget constraint,  $B^h(q(j))$ , is:

$$\begin{aligned} c_1^h &\leq w - \sum_j q(j)a^h(j) & t = 1 \\ c_2^h(s) &\leq \sum_j \sum_i D_j^i(s)a^h(j)^h & t = 2 \end{aligned} \tag{5}$$

Securities' payoff depends on their *actual* value. Since consumers are all identical I assume, without

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<sup>8</sup>Technically speaking, we should restrict  $j$  to a grid of finite values to avoid measurability problems. In the discussion that follows we will treat  $j$  as a continuous variable. We could always do so and then define the grid to include the optimal  $j$  we derived. See Dubey *et al.* (2004) for a detailed discussion.

loss of generality, they hold the same portfolio, i.e. the average one. The budget constraint is thus standard in the interpretation.

## 2.1 Definition of the Equilibrium

An equilibrium in this economy is defined by consumption allocations  $c = [c_1^h, c_1^i, c_2^h = \{c_2^h(G), c_2^h(B)\}, c_2^i = \{c_2^i(G), c_2^i(B)\}]$ , asset holdings  $a = [j^i, a^h(j)]$ ,  $\forall h, i$ , and asset prices  $q(j)$ ,  $\forall j$ , satisfying the followings:

### 1. Individual Optimum

$$(a^{h,i}, c^{h,i}) \in \arg \max \left\{ \begin{array}{l} V [c^{h,i}] \\ \text{s.t. } [B^{h,i}(q(j))] \end{array} \right\} \text{ at given } q(j), \forall h, i$$

### 2. Market Clearing Conditions

$$\int_0^z a^h(j) dh - \int_0^1 j^i \eta(i) di = 0, \forall j \text{ at } t = 1 \quad (\text{Security Markets})$$

$$\begin{aligned} \int_0^z c_1^h dh + \int_0^1 c_1^i di + \int_0^1 I^i di &= w \int_0^z dh, \quad t = 1 \\ \int_0^z c_2^h(s) dh + \int_0^1 c_2^h(s) di &= \int_0^1 \lambda_s^i g(I^i) di; \quad t = 2, \quad s \in \{G, B\} \end{aligned} \quad (\text{Goods Market})$$

## 2.2 The Bond Market: Payments and Quantity

Each security is priced according to the *expected* payments/consumption it delivers. This expectation is affected in equilibrium by the presence of asymmetric information. The payment implications of choosing how many bonds a firm issues can be conveyed through Figure 2 below.

Given our assumptions and the fact that technology  $H$  is more productive than technology  $L$ , the expected payment of their securities behave as shown in Figure 2 - for an arbitrary constant price  $\bar{q}$  - where  $\bar{j}^i$  denotes the number of assets after which entrepreneur with technology  $i$  wishes

to default on his bonds in all contingencies.<sup>9</sup>

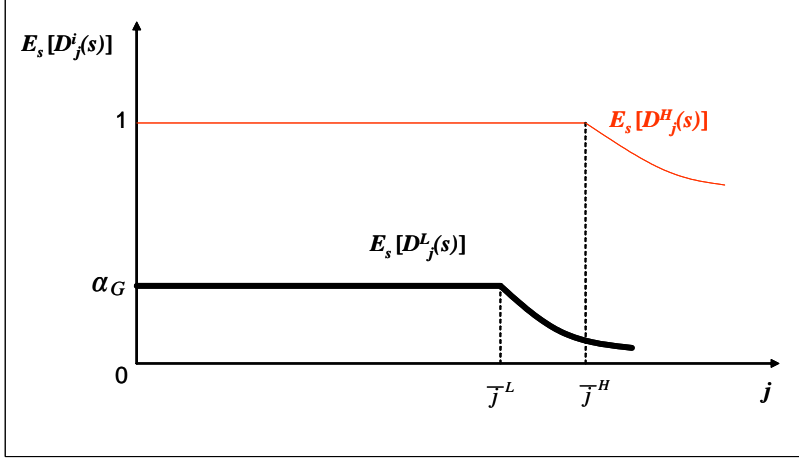


Figure 2

### 3 The Economy with Symmetric Information

It is useful to establish the symmetric information benchmark of this economy, so that we can precisely isolate what is the effect of private information on pricing, investment and liquidity.

#### 3.1 The Equilibrium

In the *symmetric* information benchmark, when firms borrow, bonds' quality is "transparent" (i.e. anyone knows the actual worth of any bond traded in the market). Consumers/savers always know the actual expected payment of the bond they are purchasing. Since agents are risk neutral, a bond's price is a linear function of its payments<sup>10</sup>:

$$q(j, i) = E_s [D_j^i(s)] ; \forall i \quad (6)$$

where  $q(\cdot)$  is indexed by both  $i$  and  $j$  since consumers can observe the quality of the issuing firm. Given our assumptions, access to the credit market is beneficial to entrepreneurs since it allows them to undertake the investment opportunity.

<sup>9</sup>By the definition of bonds  $\bar{j}^i$  solves  $\frac{\lambda^i g(I^i = \bar{q}\bar{j}^i)(1-\gamma)}{\bar{j}^i} = 1$ .

<sup>10</sup>Consumer's utility maximization and standard no arbitrage reasoning delivers so.

Let

$$R^i(j) = \frac{E_s \left[ D_j^i(s) \right]}{q(j, i)}$$

be the bond expected rate of return for security  $j$  issued by firm  $i$ , i.e. the ratio between expected delivery,  $E_s \left[ D_j^i(s) \right]$ , and market price,  $q(j, i)$ . Since the focus here is to measure economy-wide liquidity, I define the following:<sup>11</sup>

**Definition 1** *The **Economy Liquidity Premium** is defined as:*

$$LP = R^H(j^H) - R^L(j^L)$$

*i.e. as the difference between the expected rates of return of the security issued by entrepreneurs  $H$  endowed with better technology,  $j^H$  and of the security issued by entrepreneurs  $L$  with the worse technology,  $j^L$ .*

Moreover:

**Definition 2** *Let*

$$Y^i(j) = \frac{1}{q(j, i)}$$

*be the **Bond Yield** for security  $j$  issued by firm  $i$ , i.e. the ratio between the promised delivery/payment, 1, and its market price,  $q(j, i)$ .*

Given conditions (2) and (4), we have the following profit function  $\pi^i(j)$  :

$$\pi^i(j) = \sum_{s \in S} \alpha_s \cdot \{ [\lambda_s^i g(I^i)] - j D_j^i(s) \} \quad (7)$$

where  $I^i = q(j)j$ . Notice that function (7) is nothing else than the difference between investment proceedings and payments to bondholders. It depends only on those states in which entrepreneur  $i$  produces positive output. The isoprofit curves corresponding to function (7) can be drawn as in

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<sup>11</sup>This definition is equivalent to Eisfeldt (2004).

the following figure:

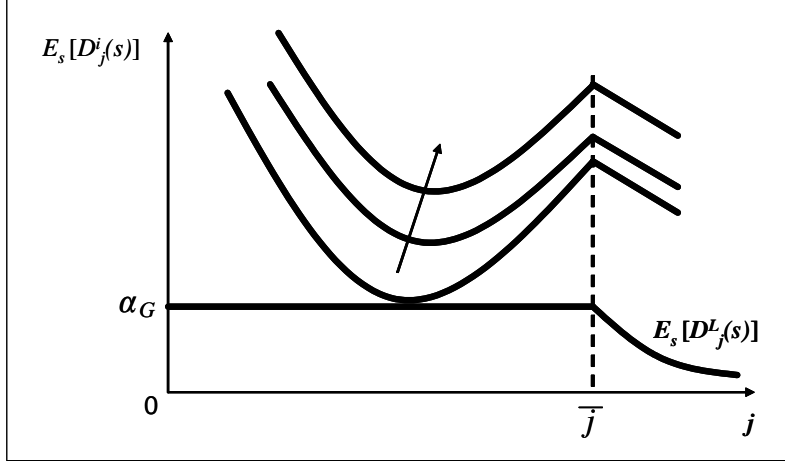


Figure 3

where

$$\bar{j}^i : \frac{\lambda^i g(I^i = q(\bar{j}^i)\bar{j}^i)(1 - \gamma)}{\bar{j}^i} = 1$$

is, as before, the number of bonds after which entrepreneur  $i$  starts to default. There are two observations to be made regarding the isoprofit. First, perhaps not surprisingly, given security  $j$  profits increase in  $q(j)$ , the price paid by bondholders to entrepreneurs to purchase a bond. This can be checked in Figure 3 by the fact that higher isoprofits in the diagram are also associated with higher profits. Second, any isoprofit is U-shaped if the firm issues a number of bonds  $j \leq \bar{j}^i$ , it is kinked at  $j = \bar{j}^i$  and decreasing after  $\bar{j}^i$ . The U-shape in the first part of the isoprofit is the result of the fact that the technology is strictly concave. At any given price, there is a unique optimal  $j^*$  that maximizes profit at that price. Whenever you move the leverage decision  $j$  of the firm away from  $j^*$ , the only way to keep profits at the same level is to increase the price. This can only be done though up to  $\bar{j}^i$ , where the isoprofit is kinked because this is the number of bonds at which the entrepreneur is indifferent between defaulting and paying back his creditor. For any  $j > \bar{j}^i$ , the entrepreneur would rather default and keep share  $\gamma$  of his production  $\lambda^i g(I = q(j)j)$ . Since  $g(q(j)j)$  is increasing in  $j$ , the only way to keep profit constant is by decreasing the bond price: thus the isoprofit must be downward sloping in this region.

Entrepreneurs maximize expected profits  $\pi^i(j)$ . Exploiting (7) and taking the FOC with respect

to the number of bonds being issued,  $j$ :<sup>12</sup>

$$\left[ \sum_{s \in S^i} (\alpha_s \lambda_s^i) + \gamma \sum_{s \in S \setminus S^i} (\alpha_s \lambda_s^i) \right] \cdot g'(I^i) = \frac{\sum_{s \in S^i} \alpha_s}{q(j, i) + j \frac{\partial q(j, i)}{\partial j}} \quad (8)$$

where  $S^i = \left\{ s \in \{G, B\} \mid D_j^i(s) = 1 \right\}$  denotes the contingency/ies where firm  $i$  does *not* default and  $S \setminus S^i$  denotes its complement. The interpretation is standard: securities are issued until the level of investment,  $I^i$ , equalizes the marginal benefit of investing - marginal productivity - to its marginal cost. The marginal cost is the ratio between the payment to lenders in no default states,  $\sum_{s \in S^i} \alpha_s$ , and the benefit of issuing an additional bond,  $q(j, i)$ , *plus* the effect that the marginal bond has on the price of all infra-marginal bonds,  $j \frac{\partial q(j, i)}{\partial j}$ .

Given the technological assumptions in (3), when security  $j \leq \bar{j}^H$  is chosen, entrepreneurs endowed with better technology  $H$  never default while the ones with worse technology  $L$  default with positive probability, i.e.  $\{s = G\} = S^L \subset S^H$ . A direct implication of our assumptions and condition (8) are:

**Proposition 3** *In the equilibrium of the economy with symmetric information the level of investment undertaken by an entrepreneur increases with technological quality, i.e.  $(I^H)^* > (I^L)^*$ .*

**Proof.** Appendix. ■

Using condition (8), I can provide a simple graphical illustration of proposition 3 in Figure 4. The profit maximizing security is chosen so that isoprofit and expected payments are tangent. By no arbitrage condition, different technologies must guarantee the same expected rate of return on the bonds they issue:

$$R^L(j_L^*) = \frac{E_s \left[ D_{j_L^*}^L(s) \right]}{q(j_L^*, L)} = \frac{E_s \left[ D_{j_H^*}^H(s) \right]}{q(j_H^*, H)} = R^H(j_H^*)$$

$j_i^*$  is the security issued by entrepreneur  $i$

and so all bond prices  $q(j_i^*, i)$  correctly reflect the fundamental value of each and every bond present in the economy,  $E_s \left[ D_{j_i^*}^i(s) \right]$ , while no adverse selection in investment takes place. I summarize the main results by the following proposition given without proof:

**Proposition 4** *In the economy with no private information, the following hold:*

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<sup>12</sup>Appendix.

1. equilibrium investment  $(I^i)^*$  is increasing in the quality of technology:  $(I^H)^* > (I^L)^*$ ;
2.  $LP = 0$ , i.e. the expected rate of return is equalized across technologies;
3. bond yields decrease in the quality of technology:  $Y^H(j_H^*) < Y^L(j_L^*)$ .
4. the equilibrium is pareto optimal.

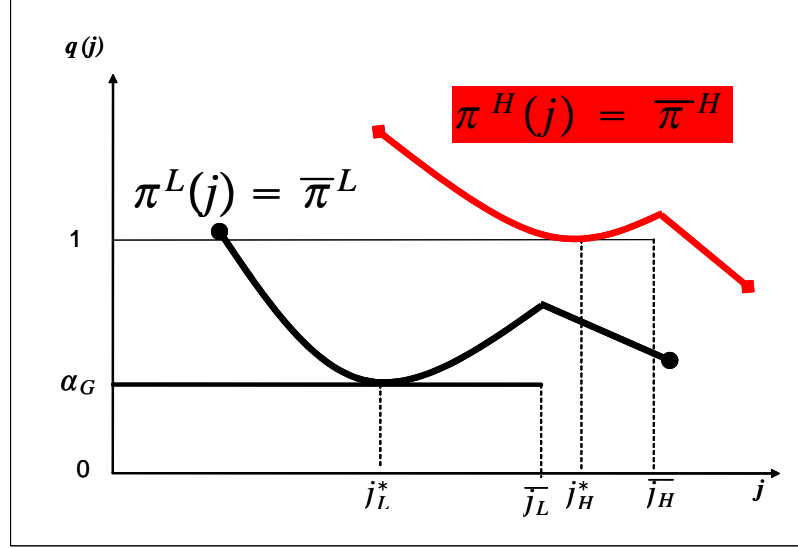


Figure 4

## 4 The Economy with Asymmetric Information

In this case it is necessary to derive the equilibrium of the economy by backward induction. At  $t = 2$ , production realizes, financial contracts are settled and bond payments made. At settlement date, entrepreneurs either pay their contractual obligations, one unit of consumption per bond, or they surrender share  $(1 - \gamma)$  of their production. Consumers holding bonds receive payments corresponding to the quality and kind of securities they own. Everyone consumes his net wealth.

At  $t = 1$  entrepreneurs raise funds through capital markets by issuing bonds. Private information plays a crucial role:

**Axiom 5 (Private Information)** *At  $t = 1$  the entrepreneur issuing bonds is privately informed about their actual payments.*

Assumption (5) is a direct consequence of the assumption that entrepreneurs are privately

informed about the quality of the investment technology they own. The buyer of a security can *only* observe the number,  $j$ , of bonds issued by a firm - and so the security type - not their quality. The bond market is thus segmented into specific security market where all firms issuing the same number of bonds are traded. In each market, consumers form expectations about the value of the average bond. This expectation is the crucial determinant of asset prices,  $q(j)$ , and so security prices are indexed by  $j$  only.

Entrepreneurs take into account the role of private information when they decide which security/how many bonds they issue. The central question becomes how much separation across technologies, if any, will be present in equilibrium, i.e. whether the equilibrium in the security space is going to be pooling or separating. I turn to this issue now.

## 4.1 Equilibrium Liquidity

### 4.1.1 Equilibrium Securities: the Basic Trade-Off

In equilibrium, each security is priced according to the expected consumption it delivers. Because of asymmetric information, buyers do not know the actual delivery of the bond they are buying and so they must form expectations about its average quality/payment. Since consumers maximize (1) under (5), security prices are:

$$q(j) = E_i [E_s [D_j^i(s)] | j^H, j^L] \quad (9)$$

Since buyers can *only* observe  $j$ , the number of bonds a firm issues, bond prices are indexed by  $j$  only in asymmetric information economy. Equation (9) formalize rational expectations in this framework: the equilibrium price must be equal to the expected delivery of the *average* bond traded in the market for security  $j$ . The average bond quality in turn depends on the relative shares of good and bad entrepreneurs issuing a given security, i.e. undertaking the same leverage decision  $j$ .

Since the ratio between good ( $H$ ) and bad ( $L$ ) entrepreneurs may change across different securities, there are incentives for entrepreneurs to distort their choices attempting to distinguish from/mimic others through their leverage decisions. In general, the easier it is to reproduce the behavior of the most productive entrepreneurs, the more depressed the market price,  $q(j)$ , is and

the higher the discount charged to bonds issued by entrepreneurs  $H$ .

Productive entrepreneurs, anticipating this “shading”, may decide to distort their security choice  $j$ . This distortion implies a trade-off for entrepreneurs with bad technology  $L$ : on the one hand, they unambiguously benefit from a subsidized market price (a positive *price effect*) if they mimic better quality entrepreneurs; on the other hand, they also have to mimic the distorted security decision of entrepreneur  $H$  (a negative *leverage effect*).

The buyer of a bond thus may face a “Lemons market” problem: he knows that entrepreneurs  $L$  issue bonds with actual delivery below the market price, if they pool together with more productive entrepreneurs. The equilibrium market price of all traded securities issued by both high and low productivity entrepreneurs implicitly imposes a premium charged upon type  $H$  entrepreneurs. A pooling equilibrium may nonetheless survive, even if the most productive entrepreneurs are paying a premium - a liquidity premium- because the leverage/security choice distortion necessary for entrepreneurs  $H$  to differentiate themselves may be too costly. Since all agents are risk neutral, the liquidity premium resulting here is due *only* to asymmetric information. In the following sections I will show that the kind of equilibrium appearing in the bond market closely depends on aggregate conditions of the economy. But before doing so I need to discuss in details the relevant notion of equilibrium for this set of economies.

#### 4.1.2 Equilibrium Securities: Off-Equilibrium Pricing and Robustness of the Equilibrium<sup>13</sup>

To characterize the equilibrium of this economy one needs to be careful in defining pricing over all securities/leverage levels  $j$ . When we couple competitive markets and private information, rational expectations alone provide no guidance in the determination of prices for securities that are not actively issued and traded in equilibrium. This is important because, in order to choose the optimal leverage, entrepreneurs must be able to compare prices for all possible securities they may issue. This may lead to a paradox: every agent may expect the price of all off-equilibrium securities to be zero, simply because no one is trading them. But these "pessimistic" off-equilibrium expectations would make it possible to support almost any allocation as an equilibrium. In order to get around

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<sup>13</sup>This subsection addresses few technical issues within the present context in the definition of the equilibrium pricing functional. The uninterested reader may skip to section 4.2 without impairing his understanding of the main results.

this problem, I apply the methodology proposed by Dubey and Geanakoplos (2002) in the context of the insurance model *a la* Rothschild and Stiglitz (1976). They address this feature by imposing a tremble “on the market”: they introduce an external agent of positive measure forced to issue all securities, even the ones that would otherwise be absent, and this allows them to pin down off-equilibrium prices.

In this paper I introduce in the economy an *external agent* of measure  $\varepsilon = \{\varepsilon_j\}_{j \in J}$  issuing every off-equilibrium security, say  $j'$ , as if he were an entrepreneur of type  $H$  endowed with the better technology. The price of off-equilibrium security  $j'$  would then be equal to  $E_s \left[ D_{j'}^H \right]$ , if only the external agent were to issue it. In this way the external agent allows to pin down security prices for all securities. I denote the economy where the external agent is introduced as the " $\varepsilon$ -economy".

In order to determine the equilibrium of the  $\varepsilon$ -economy, one has to check how entrepreneurs react to the introduction of the external agent. In particular, we have to check whether entrepreneurs find it profitable to issue the same security that was part of the equilibrium we want to support or some wish to deviate to another security, once the external agent is introduced. In practice I am asking each agent whether an entrepreneur would "change his mind", once the external agent enters the economy. Taking into account the optimizing behavior of all agents, one can, by rational expectations, compute security prices in the given  $\varepsilon$ -economy - i.e. the equilibrium prices of the economy,  $q_\varepsilon(j) \forall j$ , where the external agent is forced to issue all securities and entrepreneurs  $H$  and  $L$  optimally choose  $(j^H)^{\varepsilon_j}$  and  $(j^L)^{\varepsilon_j}$ . In the same spirit of (9), we have:

$$q_\varepsilon(j) = \frac{[\mu(H, j)^{\varepsilon_j} + \varepsilon_j] E_s \left[ D_j^H(s) \right] + \mu(L, j)^{\varepsilon_j} E_s \left[ D_j^L(s) \right]}{[\mu(H, j)^{\varepsilon_j} + \varepsilon_j] + \mu(L, j)^{\varepsilon_j}}$$

where  $\mu(i, j)^{\varepsilon_j}$  labels the measure of  $i$ -type entrepreneur issuing security  $j$  if the external agent has measure  $\varepsilon_j$ . Thus the equilibrium of the  $\varepsilon$ -economy is defined:

**Definition 6** *An equilibrium of the  $\varepsilon$ -economy is defined by consumption allocations  $c_\varepsilon^h = [c_{1\varepsilon}^h, c_{2\varepsilon}^h = \{c_{2\varepsilon}^h(G), c_{2\varepsilon}^h(B)\}]$ , asset holdings  $a_\varepsilon^h(j)$  and issuances  $j_\varepsilon^i, \forall h$ , asset prices  $q_\varepsilon$  satisfying:*

### 1. Individual Optimum

$$\left( a_\varepsilon^h, c_\varepsilon^h \right) \in \arg \max \left\{ \begin{array}{l} V^h \left[ c_\varepsilon^h \right] \\ s.t. \quad \left[ B^h \left( q_\varepsilon \right) \right] \end{array} \right\} \text{ at given } q_\varepsilon, \forall h$$

## 2. Market Clearing Conditions

$$\int_0^z a_\varepsilon^h(j)dh - \int_0^1 j_\varepsilon^i \mu(i, j)^{\varepsilon_j} di - \varepsilon_j = 0, \quad \forall j \text{ at } t = 1 \quad (\text{Security Markets})$$

$$\begin{aligned} \int_0^z c_{1\varepsilon}^h dh + \int_0^1 c_{1\varepsilon}^i di + \int_0^1 I_\varepsilon^i di &= w \int_0^z dh, \quad t = 1 \\ \int_0^z c_{2\varepsilon}^h(s)dh + \int_0^1 c_{2\varepsilon}^h(s)di &= \int_0^1 \lambda_s^i g(I^i) di; \quad t = 2, \quad s \in \{G, B\} \end{aligned} \quad (\text{Goods Market})$$

To check whether an allocation is sustained as an equilibrium defined in Section 2.1, one has to control that it is the limit a sequence of  $\varepsilon$ -economies where the measure of the external agent goes to zero, i.e.  $\varepsilon_j(n) \rightarrow 0, \forall j$  for  $n \rightarrow +\infty$ . If, as the external agent gets smaller and smaller, more and more entrepreneurs leave off-equilibrium securities and return back to issue the security/ies of the equilibrium allocation one wants to support, we will say that the original security/ies is/are an equilibrium surviving the ‘‘tremble’’. This way of defining the equilibrium is very close, *mutatis mutandis*, to the notion of trembling-hand perfect equilibrium introduced by Selten (1975) and the interested reader may refer to Dubey and Geanakoplos (2002) for a more thorough discussion.

What is important to point out here is the fact that, in principle, the larger is the set of expected payments of the external agent for which an equilibrium survives, the more robust this equilibrium is. For the sake of simplicity, we have explicitly considered only the external agent most likely to break any equilibrium, i.e. the one behaving as a good quality entrepreneur. This choice is due to the fact that, if an equilibrium survives this ‘‘optimistic’’ tremble, it survives any other tremble where the external agent behaves as an entrepreneur of lower quality with positive probability. In fact, if an entrepreneur does not change his equilibrium security under the prospects of being pooled in a different security with a very good external agent, he will certainly not leave if he risks being pooled with an external agent of worse quality. This discussion can be formally summarized through the following definition:

**Definition 7** *An equilibrium is robust if it is the limit, for  $\varepsilon_j(n) \xrightarrow{n \rightarrow +\infty} 0, \forall j$ , of a sequence of  $\varepsilon$ -economies in which the external agent of measure  $\varepsilon_j(n)$  issues bonds paying  $E_s [D_j^H(s)]$  in each off-equilibrium security.*

Reinterpreting the notion of *pool* in Dubey and Geanakoplos (2002) as *security* here, I prove that, differently from them, pooling equilibria are surprisingly robust to off-equilibrium beliefs. In

the following section I use a constructive approach to identify the conditions under which robust *pooling* equilibria arise.

## 4.2 Liquidity Premium and the State of the Economy

### 4.2.1 Illiquid Pooling Equilibria

In an equilibrium where entrepreneurs endowed with different technologies issue different securities  $j^i$ , i.e. in a separating equilibrium, the expected rate of return on every and each bond is  $R^i(j^i) = 1$ ,  $\forall i, j$ , and the "economy liquidity premium" of definition (1)  $LP = 0$ , while bond yields diverge:  $Y^H(j^H) < Y^L(j^L)$ . Instead, if in equilibrium all entrepreneurs issue the same security  $j^*$ , i.e. if the equilibrium in the bond market is pooling, it must be that:

$$\begin{aligned} R^L(j^*) &> 1 > R^H(j^*) \\ &\Leftrightarrow \\ LP &> 0 \end{aligned} \tag{10}$$

while bond yields of different firms coincide:

$$Y^H(j^*) = Y^L(j^*)$$

since all firms issue bonds at the same price  $q(j^*)$ . Therefore, I say the pooling equilibrium to be *illiquid* while the separating equilibrium is defined *liquid*. Analogously, while bond yields diverge in the liquid separating equilibrium, they converge in the illiquid pooling equilibrium.

Because of rational expectations, equation (9) ensures that security prices reflect the average quality of bonds traded in the market for a single security  $j$ . The better a technology is, the higher is the expected interest rate it pays in a pooling equilibrium. Therefore, in equilibrium, good technology entrepreneurs borrow at unfavorable terms and are adversely selected. Lack of liquidity does not only refer to the fact that good investment opportunities face hard terms when applying for credit but also to the fact that credit providers have a hard time in identifying the most worthy projects and so part of the funds are diverted towards less productive investment.

Equation (8) states the reference criterion by which the entrepreneur chooses the optimal security to issue. The reader is referred to equation (8) for the relevant interpretation. Here it suffices

to remind that entrepreneurs equalize the marginal benefit of investing and the marginal cost of financing. Since equation (8) is also relevant in the case of asymmetric information, it is convenient to rearrange it into the following:

$$= \left\{ \begin{array}{l} \frac{\partial q(j)}{\partial j} \Big|_{\pi^i(j)=\bar{\pi}} = \frac{\partial q^i(j)}{\partial j} = \\ \frac{\sum_{s \in S^i} \alpha_s}{jg'(I^i) \left[ \sum_{s \in S^i} (\alpha_s \lambda_s^i) + \gamma \sum_{s \in S \setminus S^i} (\alpha_s \lambda_s^i) \right]} - \frac{q(j)}{j} \text{ if } j \leq \bar{j}^i \\ -\frac{q(j)}{j} \text{ if } j > \bar{j}^i \end{array} \right. \quad (11)$$

which gives the slope of the isoprofit curve for entrepreneur  $i$ . Given our technological assumption (3) and profit function (7), it is immediate to show that, for a given couple  $(j, q(j))$  of security type and price, we have the following relationship between the slopes of the isoprofit curve for different technologies:

$$\frac{\partial q^H(j)}{\partial j} < \frac{\partial q^L(j)}{\partial j} \text{ if } j \leq \bar{j}^H \quad (12)$$

Given any pooling price  $q(j^*)$  one can compute the number of bonds,  $\bar{j}^L$ , after which low productivity firms default. Inequality (12) implies that, for any security  $j \leq \bar{j}^H$ , the price fall sufficient to keep profits constant as the number of bonds decreases is greater for entrepreneurs endowed with worse technology  $L$  (see Figure 5). This property is sufficient to deliver the following theorem:

**Theorem 8 (Pooling Equilibrium in Slow Growth Economies)** *When the probability that the good contingency ( $s = G$ ) realizes is sufficiently small,  $0 \leq \alpha_G \leq \alpha_G^*$ , there is a unique robust pooling equilibrium where both types of firms issue security  $j^* = \bar{j}^L$  at price*

$$q(j^* = \bar{j}^L) = \eta(H) + (1 - \eta(H))\alpha_G$$

*These economies are illiquid since they display a positive liquidity premium*

$$LP > 0$$

*and bond yields are equalized across different firms*

$$Y^H(j^*) = Y^L(j^*)$$

**Proof.** Appendix. ■

The theorem above is central and it is important to provide an intuition for the reason why the proposed setup delivers it. I refer the interested reader to the appendix for its formal proof. In Figure 5 the two isoprofits (solid lines) for entrepreneurs  $H$  and  $L$  going through the pooling

equilibrium  $(\bar{j}^L, q(\bar{j}^L))$  are drawn, with the addition of a dashed line representing the isoprofit of entrepreneur  $L$  if she chose the profit maximizing security at the "fair" price  $\alpha_G$ . Entrepreneur  $H$  wishes to differentiate himself to avoid subsidizing entrepreneurs  $L$ . Attempting to do so, the most profit he can achieve is obtained by choosing security  $\bar{j}^L$  where he commits to issue more bonds that would be optimal for him to do at the given price  $q(\bar{j}^L)$ .<sup>14</sup> Figure 5 shows that this is the optimal leverage decision of entrepreneurs  $H$ : if they were to issue security,  $j \neq \bar{j}^L$ , entailing a lower or higher number of bonds, they would require a price strictly higher than the one sufficient to attract entrepreneurs  $L$  (see inequality (12)). This means that more low quality entrepreneurs would issue this new security and the price would go below what makes good entrepreneurs indifferent with issuing the original security,  $\bar{j}^L$ .

Entrepreneurs  $L$  may find it profitable to mimic entrepreneurs  $H$ , although this requires a distortion in their issuance decision. Entrepreneurs  $L$  undertake the costly action of distorting their security choice toward  $\bar{j}^L$  only if this gives them higher profits than what they would earn if they chose their security independently and were recognized by the market for what they are: low productivity entrepreneurs issuing bonds with high probability of default. This can be illustrated in figure 5 where the isoprofit tangent to the delivery of low quality entrepreneurs (the dashed line) lies *below* the isoprofit of entrepreneur  $L$  going through the pooling equilibrium.

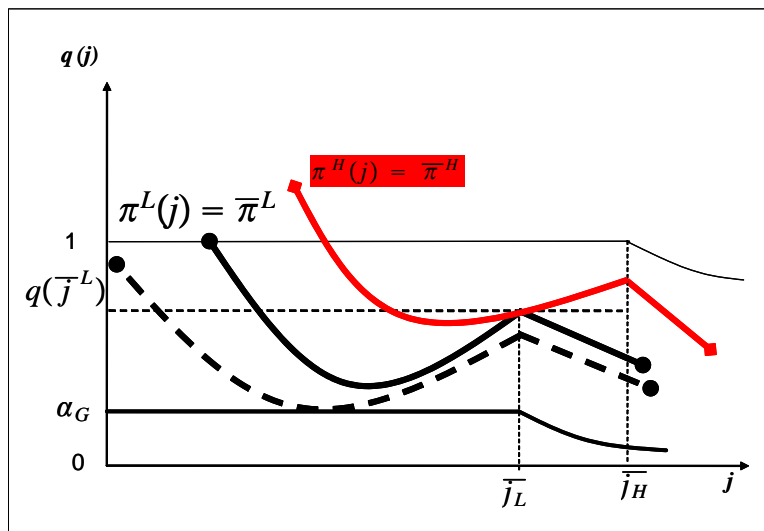


Figure 5

<sup>14</sup>Notice that it is irrelevant to choose security  $j > \bar{j}^H$  for entrepreneurs  $H$  because profits  $\pi^H(j > \bar{j}^H) = 0$ .

The result may be simply stated: if the state of the economy is sufficiently bad, the probability  $\alpha_G$  that the good state of the economy realizes is sufficiently small. Then low productivity technologies are sufficiently different from high productivity ones, and so low quality entrepreneurs gain substantially by mimicking good quality entrepreneurs. This can be graphically shown through Figure 5: the "height" of the dashed line depends on the probability of the good state,  $\alpha_G$ . Whenever the latter is small enough, the positive "price effect" enjoyed by entrepreneurs  $L$  pooling with entrepreneurs  $H$  - the "subsidy" their bond price receives - dominates the negative "leverage effect" they pay - the distortion in their leverage decision. The theorem above suggests that, although good quality entrepreneurs try their best to differentiate themselves and signal the market their quality, it is really the incentive of bad quality entrepreneurs to mimic them that plays the crucial role.

The fact that a sufficiently small probability of the good state of the economy,  $\alpha_G$ , delivers a pooling equilibrium which is robust in the sense of definition 7, i.e. robust to perturbations of any mix of good and bad entrepreneurs<sup>15</sup>, is of interest in its own right. It is worthwhile to observe that these equilibria are more likely to materialize the higher is the relative measure of productive over unproductive entrepreneurs. The intuition behind the robustness of pooling equilibria in our framework is grounded on one assumption only: good quality entrepreneurs may attempt to distinguish themselves by issuing more bonds and increasing their leverage to push bad entrepreneurs into default, but, since their bonds deliver a higher payment, their total expected delivery increases relatively faster than the one of bad quality securities as the number of bonds increases. Therefore, it is the realistic assumption that bond's expected payments and the technology's quality are positively correlated that turns out to be ultimately responsible for the robustness of the illiquid pooling equilibrium. Stating that pooling equilibria are robust has important macroeconomic implications: it supports the view that not only liquidity shortages may arise in the economy, but it states that imperfect liquidity, generated by asymmetric information, is robust to the different expectations that entrepreneurs may form about the prices of off-equilibrium securities.

Notice finally that the level of investment is constant across technologies in the illiquid pooling

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<sup>15</sup>This results complements Dubey and Geanakoplos (2003) where it is argued that, organizing security trade through "pools", separating equilibria always exist and are the only ones robust to "optimistic" off-equilibrium expectations.

equilibrium, although the rates of return at which good entrepreneurs borrow is larger than the one at which bad entrepreneurs do (see inequality (10)). The difference between the terms of borrowing between the two kinds of entrepreneurs, absent in the symmetric information benchmark, is responsible for the instance of the adverse selection in investment we have here.

#### 4.2.2 Liquid Separating Equilibria

Theorem 8 has an important corollary that concludes the study of the relationship between liquidity and economic fluctuations. I have already highlighted that, if the probability of the good state is sufficiently low, the pooling equilibrium is the unique robust equilibrium. It is thus natural to ask the question of what type of equilibrium emerges when the aggregate productivity in the economy improves and the probability,  $\alpha_G$ , of the good contingency,  $G$ , increases. The answer to this question is provided by the following corollary:

**Corollary 9 (Separating Equilibrium in High Growth Economies)** *When the probability of the good contingency is sufficiently large,  $\alpha_G^* < \alpha_G \leq 1$ , the economy displays a separating equilibrium where different kinds of firms issue different securities,  $j^i$ , at prices*

$$\begin{aligned} q(j^H) &= 1 \\ q(j^L) &= \alpha_G \end{aligned}$$

*These economies are liquid since they display no liquidity premium:*

$$LP = 0$$

*and bond yields are different:*

$$Y^H(j) = 1 < Y^L(j)$$

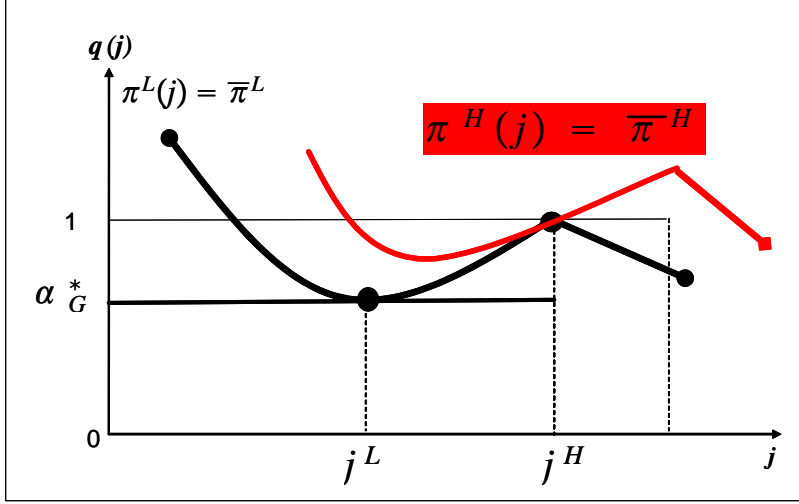


Figure 6

In order to see that a liquid separating equilibrium emerges as the probability of the good state being realized increases, it suffices to argue at the upper bound of the probability distribution over states of the world, i.e. when probability  $\alpha_G$  is equal to 1. In this case, for entrepreneur  $L$  the "price effect" would be equal to zero while the "leverage effect" would still be negative. Thus there would be no advantage for entrepreneurs  $L$  in distorting their leverage choice since they would gain nothing by doing so. A separating equilibrium would naturally arise. But then, arguing by continuity, a separating equilibrium would survive even if the probability of the good state,  $\alpha_G$ , is lowered by an arbitrary small amount. Thus there is a continuum of values of  $\alpha_G \ll 1$  so that the liquid separating equilibrium emerges.

The intuition is that, as  $\alpha_G$  increases, the two technologies become more and more similar. Eventually, entrepreneurs  $L$  prefer to leave the pooling equilibrium, be priced for what they are and avoid the leverage distortion. This equilibrium is incentive compatible when, as illustrated by Figure 6, the security choice of entrepreneurs  $H$ ,  $j^H$ , and the choice of entrepreneurs  $L$ ,  $j^L$ , lie on the same isoprofit of low quality entrepreneur  $L$ .

In conclusion, it is important to point out that in these economies the increase in liquidity as the state of the economy improves is due to the change in the equilibrium security and not to the increase in the average quality of the traded ones, as in Eisfeldt (2004). In the perspective

of this paper, it is the optimizing behavior of entrepreneurs while they choose their leverage that plays a central role in establishing the relationship between the liquidity of the economy and its fluctuations. This dynamic also delivers that the level of aggregate investment is higher when the level of aggregate productivity is higher, i.e.  $\alpha_G \rightarrow 1$ .

## 5 Liquidity and Economic Fluctuations: Evidence in the United States

This paper mainly contributes to the developing line of research in macroeconomics aimed at characterizing and explaining the relationship between economic fluctuations and the liquidity of the economy.<sup>16</sup> Policy analysts seem to agree that market liquidity tends to covary positively with economic growth, even though liquidity is measured differently.<sup>17</sup> The main contribution of this paper has been to introduce a measure of liquidity that is theoretically sound, empirically identifiable and whose empirical behavior is consistent with the predictions of the model.

The main implications of our theory, summarized by Theorem 8 and its corollary is that the proposed measure of liquidity - cross-firm dispersion of newly issued bond yields - should anticipate improvements in aggregate economic performance. Liquidity increases when the economic prospects improve: higher expected growth - captured in the model by increases in  $\alpha_G$  - determines a shift from pooling to separating equilibria, it increases the cross-firm dispersion of bond yields issued in any given quarter and, thus, liquidity. We now turn to the US corporate debt market to provide empirical evidence in support of these theoretical predictions.

In any given quarter in the United States, firms issue a wide array of securities. The vast majority is publicly traded corporate debt, which is the focus of this paper.<sup>18</sup> We collected firm level data through *SDC Platinum* about all issuances of publicly traded, non convertible, private corporate debt in the United States in the period 1970-2005. Eventually, attention was restricted only to those issues by non financial firms where yield, maturity and *Moody's* rating data were

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<sup>16</sup>Eisfeldt (2004), Rampini (2003), Rampini and Eisfeldt (2006), Kurlat (2009).

<sup>17</sup>Measures of liquidity differ on the basis of the macroeconomic or financial focus of the study. In macroeconomics it is typically identified with how accessible credit is, in finance by bid-ask spread, turnover, or market depth.

<sup>18</sup>See Bolton and Scharfstein (1996).

available.<sup>19</sup>

Non Convertible, Publicly Traded, Corporate Debt, 1970q1-2005q4

	Bond Issuances	% (of Total)
Total	20,122	
Non Financial Firms	10,133	50%
Manufacturing	8,100	40%
Services	1,994	9.9%
Agriculture	39	0.1%
Financial Firms	4,931	25%
Firms with NA sector	5,058	25%

Data Source: *SDC Platinum*

Table1

Let us start with considering the liquidity available to non financial firms. Consistently with our model, we empirically measure liquidity in quarter  $t$  by the cross-firm coefficient of variation (CV) of yields of the newly issued bonds in the given quarter. The economy-wide liquidity premium of definition 1 is negatively correlated with the dispersion of bond yields. This means that the larger is the dispersion, the lower is the liquidity premium and the higher thus is liquidity. Figure 7 shows the behavior of liquidity over time. The evidence displayed in this Figure shows that, in the US, (1) the overall level of liquidity has (slightly) increased during the period 1970-2005 and (2) liquidity tends to fall in correspondence of (NBER-dated) economic contractions.<sup>20</sup> In particular, liquidity seems to be a leading variable of the business cycle: typically, it falls in anticipation of a contraction, especially during its first half, while it starts improving towards its end in anticipation

<sup>19</sup>We focus on non financial firms because financial firms issue bonds for a variety of reasons related to financial intermediation and not the undertaking of investment analyzed by our model. Moreover, we adopt the convention that, whenever a company issued more than one type of debt in a given quarter, our dataset imputed only one issue to this company with maturity, ratings and yield equal to the average of the observed issues.

<sup>20</sup>The level of liquidity is higher (below the 1% level of statistical significance) during expansions than during contractions.

of the expansion that follows.

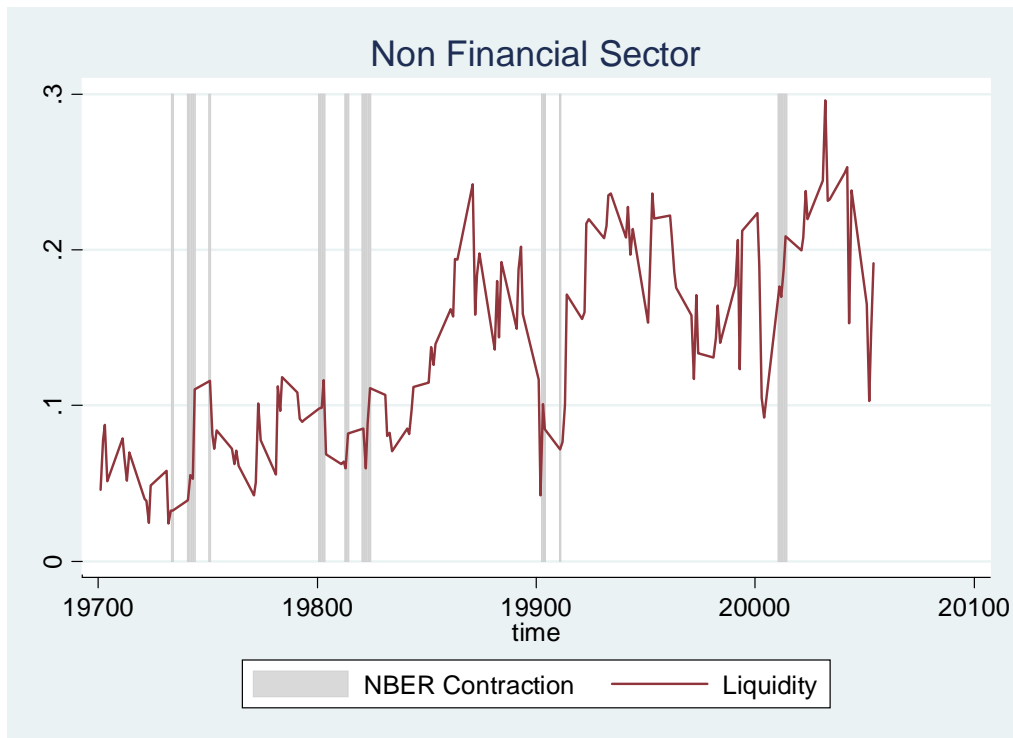


Figure 7

We take the analysis of the empirical evidence one step further by testing whether the level of liquidity within manufacturing firms only is correlated in a statistically significant way with improvements in economic activity, proxied by 1-year ahead changes in industrial production. Figure 8 depicts the behavior of the two time series in the manufacturing sector. Table 2 shows the results of the following simple-minded regression that includes the cross-firm bond dispersion of maturity and ratings:

$$\begin{aligned}
 Liquidity_t &= \beta_0 + \beta_1 \cdot IP_{t+4} + \beta_2 \cdot CV_t(Maturity) + \beta_3 \cdot CV_t(Rating) + \varepsilon_t \\
 \varepsilon_t &\sim N(0, \sigma^2)
 \end{aligned}
 \tag{13}$$

where  $Liquidity_t$  is the cross-firm coefficient of variation of yields in bonds issued in quarter  $t$ ,  $IP_{t+4}$  is the (moving average over quarters  $t + 1$ - $t + 4$ ) change in industrial production over the previous quarter in quarter  $t + 4$ ,  $CV_t(Maturity)$  and  $CV_t(Rating)$  are, respectively, the cross-firm CV of maturities and *Moody's ratings* of newly issued bonds in quarter  $t$ .<sup>21</sup>

<sup>21</sup>We chose *Moody's ratings* because they allowed us to include a larger number of bond issuances in our sample.

<b>Dep. Variable</b>	<b>OLS</b>	<b>Post 1st Oil Shock</b>	<b>Post 2nd Oil Shock</b>
<i>Liquidity<sub>t</sub></i>	1970-2005	1975:3-2005	1983:2-2005
$IP_{t+4}$	0.012** (0.003)	0.015** (0.005)	0.016* (0.007)
$CV_t(Maturity)$	0.228** (0.02)	0.213** (0.003)	0.08 (0.007)
$CV_t(Rating)$	0.051 (0.048)	0.04 (0.053)	0.027 (0.068)
$\beta_0$	-0.031 (0.031)	-0.017 (0.039)	0.083 (0.051)
Observations	144	122	91
R-squared	0.41	0.28	0.06
SE in parentheses		** significant at 1%	* significant at 5%

**Table 2**

Table 2 shows the results of regression (13). This specification is only capturing the positive correlation between liquidity and subsequent economic activity, taking into account the fact that the amount of cross-firm dispersion in bond yields may also depend on the public information available at the time of issuance, such as maturity and *Moody's* rating of issued bonds. The evidence supports the view that the measure of liquidity we propose tends to anticipate improvements in the subsequent (1 year ahead) level of economic activity. This relationship remains statistically significant, controlling for the public information available at the time of issuance, and robustly survives in different time intervals.

The provision of a theoretical explanation linking the future level of economic activity and the dispersion in bonds' yields is, to my knowledge, novel and interesting in its own right. The adopted approach focuses on an additional reason - asymmetric information - to rationalize why different bonds pay different yields even if they are issue at the same time. This focus allows to highlight a

systematic relationship between bond yields cross sectional variance and economic fluctuations.

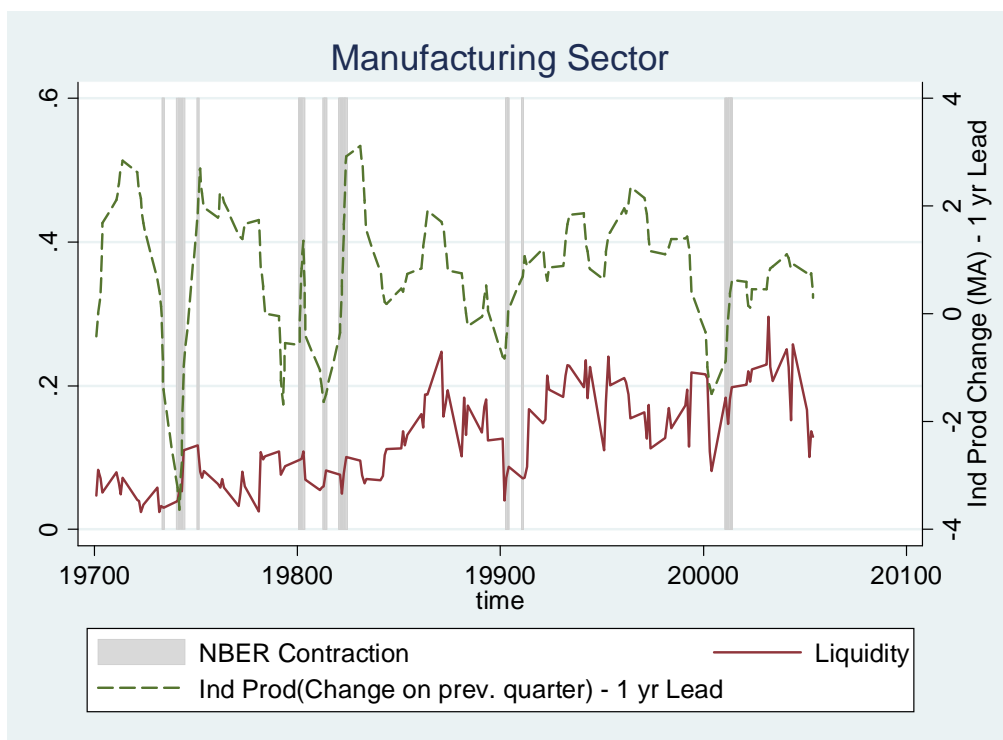


Figure 8

Here I want to stress that, by identifying the cross-firm dispersion in bond yields as a measure of the liquidity of the economy, the provided framework shows how the lack of economy-wide liquidity translates into bonds mispricing, yields concentration and investment misallocation. According to this perspective, it is in anticipation of future adverse economic conditions that good entrepreneurs end up paying disproportionately high interest rates to undertake their investment.

## 6 Concluding Remarks

The main purpose of this work is to propose a theoretical answer to the question of what determines the economy-wide level of liquidity and how we could think about the relationship between liquidity and economic activity. This has been done in a context where firms' leverage is the result of the optimizing behavior by entrepreneurs facing asymmetric information. Under this perspective, the optimal leverage decision drives the relationship between liquidity and the macroeconomy.

We provide a general equilibrium illustration of how the liquidity of the economy can depend on constant informational asymmetry concerning the value of its investment opportunities. The informational asymmetry is responsible of the illiquidity of the economy because it may generate a discrepancy between current market prices and the actual discounted value of bonds issued to finance investment. Then I relate this discrepancy to the suitability of bonds in financing private investment. In this way the equilibrium level of liquidity becomes a crucial factor in the allocation of credit, and thus private investment.

The analysis yields the following main implications. First, it shows that illiquidity is an equilibrium phenomenon, likely to appear in economies with poor economic prospects. Firms take into account the state of the economy when they decide how many bonds they issue, how often these bonds default and how serious the default is. This study shows that firms tend to undertake similar leverage decisions, when economic growth slows down. The positive correlation between payments to bondholders, given leverage, and the underlying technological quality is ultimately responsible for the appearance of robust pooling equilibria. This is not only interesting in its own right, but also because it implies that illiquidity is likely to be a persistent equilibrium outcome.

Second, I relate the theoretical model to an empirical regularity of the US corporate debt market that, to my knowledge, I am the first to highlight: the positive correlation between cross-firm dispersion in bond yields and subsequent economic activity. I provide an explanation for this empirical regularity which is rooted in the effect of asymmetric information on the entrepreneurial optimization problem. This argument is based on the realistic and standard assumption that firms leverage is observable by savers. It is the entrepreneur's choice about how many bonds he issues and the resulting leverage to be ultimately responsible for the level of liquidity of the economy. It is, to my knowledge, the first theoretical explanation that identifies bond yields variance as an empirically useful measure of liquidity and relates it to aggregate economic activity.

Although our technology is neoclassical, the reader may argue that our analysis is based on some restrictive assumptions: let me address this possible concern here. First, the only feature we require for the pooling equilibrium to arise is that the dimension of separation - the leverage choice - must be relatively more expensive for entrepreneurs endowed with the better technology. This is guaranteed whenever better entrepreneurs issue bonds that, *ceteris paribus*, default less often.

And this also seems realistic to assume.

Second, one could argue that there is really no need for the economy to move from pooling to separating equilibria in order to provide a theory of procyclical liquidity. The fact that default rates are typically countercyclical would naturally bring together rates of return in good times and vice versa. This is certainly true, but such a perspective, where only pooling equilibria exists, would have a hard time in explaining why bond yields - not returns - across different firms tend to be closer in bad times than in good times, which is the stylized fact we highlight here.

Third, although we do not allow entrepreneurs to invest internal funds - in fact they have none here, this assumption would not change my results. As Martin (2009) shows, the possibility of good entrepreneurs to invest their own funds or pledge private funds as collateral, allows for pooling equilibria in a general equilibrium setting where lending is channeled through securities and banks.

Fourth, it is important for our main result that, in the event of default, creditors can not seize the entire value of production. This seems a realistic assumption that may be justified with standard arguments - e.g. inappropriability of entrepreneurs' human capital, costly post default screening.

It is probably fruitful to search for a framework that, starting from the macroeconomy, provides asset pricing implications that can help to grasp a better understanding of the liquidity of the economy. We remain in need of a general benchmark, along the lines of Holmstrom and Tirole (1998) and Hellwig and Lorenzoni (2009), to analyze the role of government bonds and many financial arrangements available in reality in determining economy-wide liquidity.

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## 7 Appendix

### 7.1 Entrepreneur's Profit Maximization

Maximizing entrepreneur  $i$ 's utility is equivalent to maximizing her profits  $\pi^i(j)$ . This reduces to:

$$\max_j [\pi^i(j)] = \max_j \left[ \sum_{s \in S^i} \alpha_s \cdot \{ [\lambda_s^i g(I^i)] - j \} \right] \quad (14)$$

where  $S^i = \{s \in \{G, B\} \mid D_j^i(s) = 1\}$  denotes the contingency/ies where firm  $i$  does not default.

Equilibrium investment is:

$$I^i = q(j)j$$

The relevant FOC to  $\max [\pi^i(j)]$  is:

$$\frac{d\pi^i(j)}{dj} = \sum_{s \in S^i} \alpha_s \cdot \left\{ [\lambda_s^i g'(I^i)] \left( q(j) + j \frac{\partial q(j)}{\partial j} \right) - 1 \right\} + \gamma \sum_{s \in S \setminus S^i} \alpha_s \cdot \left\{ [\lambda_s^i g'(I^i)] \left( q(j) + j \frac{\partial q(j)}{\partial j} \right) \right\} = 0 \quad (15)$$

Rearranging (15) we can derive the slope of the isoprofit function:

$$\left. \frac{\partial q(j)}{\partial j} \right|_{\pi^i(j)=\bar{\pi}} = \frac{\sum_{s \in S^i} \alpha_s}{j g'(I^i) \left[ \sum_{s \in S^i} (\alpha_s \lambda_s^i) + \gamma \sum_{s \in S \setminus S^i} (\alpha_s \lambda_s^i) \right]} - \frac{q(j)}{j}$$

which, expressed in greater details, become:

$$\left. \frac{\partial q(j)}{\partial j} \right|_{\pi^i(j)=\bar{\pi}} = \frac{\sum_{s \in S^i} \alpha_s}{j g'(I^i) \left[ \sum_{s \in S^i} (\alpha_s \lambda_s^i) + \gamma \sum_{s \in S \setminus S^i} (\alpha_s \lambda_s^i) \right]} - \frac{q(j)}{j} \text{ if } j \leq \bar{j}^i$$

$$\left. \frac{\partial q(j)}{\partial j} \right|_{\pi^i(j)=\bar{\pi}} = -\frac{q(j)}{j} \text{ if } j > \bar{j}^i$$

**Proof of Proposition 3.** Without loss of generality attention can be restricted to securities  $j \leq \bar{j}^H$ . Thus (6) can be written:

$$q(j, i) = \sum_{s \in S^i} \alpha_s \quad (16)$$

and so we have:

$$q(j, H) = 1$$

$$q(j, L) = \alpha_G$$

which, observing that  $\frac{\partial q(j, i)}{\partial j} = 0$  if  $j \leq \bar{j}^H$ , implies, by (8):

$$E_s [\lambda_s^H g'(I)] = 1$$

$$E_s [\lambda_s^L g'(I)] = \frac{\sum_{s \in S^L} \alpha_s}{q(j, L)} = 1$$

and then

$$g'(I^L) = \frac{1}{\alpha_G \lambda^L}$$

$$g'(I^H) = \frac{1}{\lambda^H}$$

but, since  $E_s (\lambda_s^H) = \lambda^H > E_s (\lambda_s^L) = \alpha_G \lambda^L$  by assumption (3),  $g'(I^H) < g'(I^L)$  and  $I^H > I^L$ . ■

**Proof of Theorem 8.** In order to prove the existence of a pooling equilibrium we must ensure that, at the equilibrium, the relevant local and global incentive compatibility constraints are satisfied for both kinds of entrepreneurs. We prove that the number of bonds  $j^* = \bar{j}^L$  is the pooling

equilibrium of the economy where both kinds of firms issue the same security at price  $q(j^*)$ , which - by rational expectations - is equal to:  $q(\bar{j}^L)$

$$q(j^* = \bar{j}^L) = \eta(H) + (1 - \eta(H))\alpha_G$$

First, consider that the pooling equilibrium is locally incentive compatible for both types of entrepreneurs. In order to prove so, one may study the isoprofit curve, the locus combining  $j$  and  $q(j)$  so that profits remain unchanged. We have already found that:

$$\frac{\partial q^H(j)}{\partial j} < \frac{\partial q^L(j)}{\partial j} \text{ if } j \leq \bar{j}^H$$

Therefore we can use Figure 5 to check that  $\bar{j}^L, q(\bar{j}^L)$  is indeed the pooling equilibrium we are looking for. All we need to ensure is that I am supporting a pooling equilibrium is that there no deviations that are profitable to entrepreneurs  $H$  without being so for entrepreneurs  $L$ . First notice that there no deviation with this property for security  $j < \bar{j}^L$  or  $j > \bar{j}^L$ . In fact, any price for security  $j < \bar{j}^L$  or  $j > \bar{j}^L$  that is above the isoprofit of entrepreneurs  $H$  going through the pooling equilibrium is also above the isoprofit of entrepreneurs  $L$ . Therefore, there is no deviation that can attract entrepreneurs of good quality only. All that remain to check is that entrepreneurs  $L$  would not rather choose a security where they are recognized for what they are. This is equivalent to check that

$$\pi^L(\bar{j}^L) > \pi^L(j^L)$$

that can be worked into to:

$$\lambda^L g \left[ \bar{j}^L q(\bar{j}^L) \right] - \bar{j}^L > \left[ \lambda^L g (j^L q(j^L)) \right] - j^L$$

that is

$$\lambda^L g \left[ \bar{j}^H (\eta(H) + (1 - \eta(H))\alpha_G) \right] - \bar{j}^H > \lambda^L g [j^L \alpha_G] - j^L$$

which is satisfied for  $\alpha_G = 0$ . Thus by continuity, we can argue that there is  $\alpha_G^*$  such that for  $\alpha_G < \alpha_G^*$  the pooling is supported as an incentive compatible equilibrium. ■

I conclude the proof by showing that the pooling equilibrium is robust in the sense that it survives the perturbation where the external agent  $\varepsilon_j(n)$  behaves as high quality ( $L$ ) entrepreneurs

on off-equilibrium securities. The crucial condition for the equilibrium to survive is that the price that makes high productivity entrepreneurs indifferent to the pooling,  $q^H(j)$ , is larger than the price that makes low productivity entrepreneurs indifferent,  $q^L(j)$ . I will present the argument concisely since the structure is very similar to Dubey and Geanakoplos (2002):

**Lemma 10** *When  $0 \leq \alpha_G \leq \alpha_G^*$ , the pooling equilibrium  $j^* = \bar{j}^L$  is robust in the sense of definition 7.*

**Proof.** Pick the pooling equilibrium and consider the only active security  $j^* = \bar{j}^L$ . By equation (9), given the proportions of good and bad quality entrepreneurs  $\eta(L)$  and  $\eta(H)$ ,  $\bar{j}^L$ 's price is determined by the delivery vectors  $D_{\bar{j}^L}^i(s)$ ,  $i = H, L$ . Now focus on off-equilibrium securities,  $j \neq \bar{j}^L$ :

1. consider an external agent entirely characterized by her delivery vector which I assume identical to  $D_j^H(s)$ , the delivery vector of good quality entrepreneurs;
2. for each security  $j \neq \bar{j}^L$ , let  $\varepsilon_j(n)$  be a positive mass of external agent issuing securities in market  $j$ . Given the exogenous mass  $\varepsilon_j(n)$ , find the equilibrium of the  $\varepsilon$ -economy as defined in (6) allowing all agents to reoptimize;
3. finally take  $\varepsilon(n) \xrightarrow{n \rightarrow +\infty} 0$ , i.e. the measure of the external agent to zero. The limit of the sequence,  $\varepsilon(\infty) = 0$ , replicates the original economy. If the equilibrium of the  $\varepsilon(n)$  economy converges to the pooling equilibrium  $j^* = \bar{j}^L$  of the original economy, then we say that the equilibrium survives the external agent "perturbation". If the equilibrium survives this perturbation is robust. Any robust equilibrium survives perturbations defined for delivery rates smaller or equal to the one considered here. Thus the equilibrium survives the perturbation in which the external agent is characterized by any delivery vector between  $D_j^H(s)$  and  $D_j^L(s)$ .

In order to prove the lemma it suffices to show that in the  $\varepsilon(n)$  economy entrepreneurs with low quality technology ( $L$ ) are at least indifferent between issuing  $j^* = \bar{j}^L$  and  $j \neq \bar{j}^L$  while entrepreneurs with high quality technology ( $H$ ) are at most indifferent, if not worse off. This is equivalent to finding that the price that makes " $L$ " entrepreneurs indifferent to the pooling

equilibrium security makes "H" entrepreneurs willing to stick to the pooling, i.e.  $q^L(j) < q^H(j)$ . To this purpose consider the choice of the entrepreneur who is issuing  $j^* = \bar{j}^L$  and is now facing the introduced perturbation on security  $j \neq \bar{j}^L$  in the  $\varepsilon(n)$  economy.

This suffices to support the pooling equilibrium  $j^* = \bar{j}^L$ . This implies that, at the equilibrium prices of the  $\varepsilon$ -economy, low quality technology holders ( $L$ ) are (weakly) better off issuing security  $j \neq \bar{j}^L$  while high quality technology holders are *strictly* worse off by doing so. But this means that, the pooling equilibrium  $\bar{j}^L$  survives the perturbation consisting of an external agent behaving as if he were a good quality entrepreneur on off-equilibrium securities.

Since the deviations of low quality entrepreneurs is triggered by the external agent of measure  $\varepsilon(n)$  to issue  $j \neq \bar{j}^L = j^*$ , the measure of entrepreneurs  $L$  issuing securities different from  $j^* = \bar{j}^L$  converges to zero as  $\varepsilon(n) \rightarrow 0$ . ■