FINAL EXAMINATION

December 20, 2007

You have three hours to complete the exam. Answer all questions and write clearly. The points for each problem are shown in parentheses (total of 160 points).

1. DEFINITIONS (25 points)

Briefly (in a maximum of three sentences) define the following (each definition carries 5 points):

i) Pareto Optimal Allocation

Answer

For a production economy with *I* agents and *J* firms, a (feasible) allocation $\left(x^*, y^*\right)$ is Pareto optimal if there exist no other (feasible) allocation (x, y) which makes at least one individual better off without someone else being made worse off. That is, there does not exist (x, y) such that

(a) It is feasible: $\sum_{i\in I} x_i = \sum_{j\in J} y_j$

(b) The allocated consumption vector x_i is preferred to x_i^* at least weakly for all agents *i* in the economy and strictly for at least one of them.

For an exchange economy with *N* goods and *H* agents, a Pareto optimal allocation can be defined as above but also as the solution to the following problem:

$\max_x U_i(x)$

$s.t.$ for all $j \in H - \{i\}$

$U_j(x) \geq \bar{u}_j$

where $x = (x_1, x_2, ..x_N)$ an array of *N*-dimensional vectors representing the consumption bundle of each agent within the feasibility set.

ii) Constant Returns to Scale

Answer

Usually, used to characterize production functions but, generally, it refers to a function which is homogenous of degree 1. A production function $F(y_1,..., y_K)$ exhibits constant returns to scale iff $F(ay_1,..., ay_K) = aF(y_1,..., y_K)$ for all $a \geq 0$. In other words, scaling up all inputs by some factor results in the scaling of output by the same factor.

For a production set Y, it exhibits constant returns to scale if it is a convex cone. That is, if $ay \in Y$ for any $y \in Y$ and any $a \geq 0$.

iii) Convex Production Set

Answer

The production set Y is convex if for any production vectors $y^1, y^2 \in Y$, their convex combination $ay^1 + (1-a)y^2 \in Y$ for any $a \in [0,1]$.

iv) Marginal Rate of Technical Substitution

Answer

This is defined with respect to differentiable production functions $F(\cdot)$ corresponding to production sets Y that satisfy $\exists y^0 \in Y : F(y^0) = 0$. Then, for any two goods l, k the marginal rate of technical transformation of good *l* for good *k* is given by $MRT_{lk}(y^0) = \frac{\partial F(y^0)/\partial y_l}{\partial F(y^0)/\partial y_k}$. It is measures the rate of increase in the (net) output of good *k* when the firm decreases the (net) output of good *l*.

v) The First Fundamental Theorem of Welfare Economics

Answer

If preferences are locally non-satiated and (x^*, y^*, p^*) is a price equilibrium with transfers, the allocation (x^*, y^*) is Pareto optimal. In particular, any Walrasian equilibrium is Pareto optimal.

2. TRUE/FALSE/UNCERTAIN (30 points)

Define the terms in *italics* and briefly (in a maximum of three sentences) explain whether the statements are false/true/uncertain. 4 points will be awarded for each definition and 6 points for each explanation.

i) Consider a competitive pure exchange economy with finite numbers N and I of commodities and individuals, respectively. Although each agent has preferences over all N commodities, the commodities themselves could be grouped into two types: food and non-food goods. In this case, *Walras' law* implies that, if the total market value of the excess demand for all types of food commodities is zero, then the total market value of the excess demand for all non-food commodities must be zero.

Answer

Walras' law states that the total market value of the sum of the aggregate excess demands must be zero. In other words, in an economy with *N* commodities:

$$\sum_{n=1}^{N} p_n z_n = \sum_{n=1}^{N} p_n (e_n - x_n) = 0$$

where $x_n = \sum_{i=1}^{I} x_n^i$ is the aggregate (across all agents) demand for good *n* and

$e_n = \sum_{i=1}^{I} e_n^i$ is the aggregate (across all agents) endowment of good *n*

Now let us order the *N* commodities such that the first *k* comprise the food-group. By Walras' Law, $\sum_{n=1}^{k} p_n z_n = -\sum_{n=k+1}^{N} p_n z_n$ while, by the question, we are given that the left-hand side of this relation is zero. The statement is TRUE.

ii) In the range of output levels where the ***marginal cost*** curve is rising, the corresponding average cost curve must also be rising.

Answer

The *marginal cost* is the rate of change of the total costs with respect to output:

$$MC = \frac{\partial}{\partial q} TC(q)$$

The average cost is rising if and only if the marginal cost exceeds the average cost since

$$\frac{d}{dq} ATC(q) = \frac{MC(q)q - TC(q)}{q^2} = \frac{MC(q) - ATC(q)}{q}$$

The statement is FALSE.

iii) A profit-maximizing monopolist will never produce a quantity that corresponds to the range where the market demand curve is ***elastic***.

Answer

Let $p = D(q)$ be the market demand curve facing the monopolist. The profit maximizing condition will be given by $MR(q) = MC(q) \Leftrightarrow D'(q)q + D(q) = MC(q)$.

From the definition of the elasticity, however, we get $\varepsilon(q) = \frac{dq(p)}{dp}\frac{p}{q(p)} = \frac{p(q)}{qD'(q)}$.

Hence, the optimal output choice of the monopolist must satisfy

$$\frac{p_m}{\varepsilon(q_m)} + p_m = MC(q_m) \Leftrightarrow p_m = \frac{\varepsilon(q_m)MC(q_m)}{1 + \varepsilon(q_m)}$$

Since $p_m \geq 0, \varepsilon(q_m) \leq 0$, it must be $\varepsilon(q_m) \leq -1$. As long as he is facing a positive marginal cost of production, a monopolist will produce only at the elastic portion of the market demand curve. The statement is FALSE.

PROBLEMS

3. (20 points)

Consider a profit-maximizing monopolist with a constant \$*c* marginal cost production technology and facing a constant-elasticity market-demand curve (i.e. the price-elasticity of demand is the same constant ε throughout the curve).

i) Derive the equilibrium market price and quantity as functions of the parameters *c* and ε (you will need to derive the general functional form of a constant-elasticity demand curve).

ii) The government introduces a specific tax of \$*t* per unit on the monopolist. Derive the new equilibrium market price and quantity and the tax revenue that the government will be collecting as functions of the parameters c, ε, and *t*.

Answer

(i) From the last question above, we know that the price the monopolist wants to charge in order to maximize profits will be given by $p_m = \frac{\varepsilon c}{1+\varepsilon}$. The corresponding quantity will be defined by the inverse of the market demand, $q_m = D^{-1}(p_m)$. To this end, we must derive first the functional form of a constant-elasticity demand function. We have

$$\frac{\frac{d}{dp}D^{-1}(p)}{D^{-1}(p)} = \frac{\varepsilon}{p} \Leftrightarrow \frac{d}{dp}\left(\ln D^{-1}(p)\right) = \frac{\varepsilon}{p} \Leftrightarrow \ln D^{-1}(p) = \varepsilon \ln p + b \Leftrightarrow D^{-1}(p) = Bp^{\varepsilon}$$

with $B = e^b$ a constant. Thus, $q_m = B\left(\frac{\varepsilon c}{1+\varepsilon}\right)^{\varepsilon}$.

(ii) A tax of \$*t* per unit, simply raises the constant marginal cost to $c' = c + t$. The new monopolist price will be $p'_m = \frac{\varepsilon c'}{1+\varepsilon} = \frac{\varepsilon(c'+t)}{1+\varepsilon}$.

The tax results in a price increase of $\Delta p_m = \left(\frac{\varepsilon}{1+\varepsilon}\right)t$ in absolute terms (equivalently, $\frac{\Delta p_m}{p_m} = \frac{t}{c}$ in percentage terms). The new equilibrium quantity will be given by $q'_m = \left(\frac{\varepsilon(c+t)}{1+\varepsilon}\right)^{\varepsilon}$ while the tax revenue is $T = tq'_m$.

Finally, the change in the monopolist's profits is given by

$$\Delta\pi_m = q'_m\left(p'_m - (c+t)\right) - \left(q_m(p_m - c)\right) = q'_m p'_m - q_m p_m - \left(c\Delta q_m + q'_m t\right)$$

4. (25 points)

In Economia, the qwerty industry is perfectly competitive. Each qwerty firm has the following long run total cost function for its annual output q: $TC = q^3 - 6q^2 + 10q$.

The market demand for qwerties is $Q = 100 - 10P$ where Q is the annual industry sales and P is the market price.

i) Calculate the long run equilibrium price per qwerty, industry output, output per firm and the number of operating firms.

ii) The minister of industry of Economia decides to reduce the number of firms in the qwerty industry by 1/3 and sells the appropriate number of licenses in a competitive market. The number of firms is thereby fixed at this new level, but licensed firms continue to operate perfectly competitively and a new equilibrium is achieved. Without explicitly calculating numerical answers, derive the equations for the new equilibrium price per qwerty, the industry output, the output per firm, and the annual price of a license.

Answer

(i) In the long run equilibrium of this perfectly competitive industry each individual firm must be making zero profits, producing at the minimum point of its average total cost curve. The average total cost is $ATC(q) = q^2 - 6q + 10$. This is a convex function and its minimum occurs at:

$$\frac{dATC}{dq} = 0 \Leftrightarrow 2q - 6 = 0 \Leftrightarrow q = 3$$

The minimum average cost is $ATC(3) = 1$. At the long run equilibrium, the market price will be 1 and total industry output 90 units. Since each individual firm's output is 3 units, there will be 30 firms operating in the market.

(ii) The minister's decision leaves only 20 firms operating in the new setting. Since the market remains perfectly competitive, each firm continues to take the market price as given and to believe that it can sell an unlimited quantity at this price and zero at any price exceeding the market one even by an infinitesimal amount. Thus, the individual firm will be producing according to the following profit maximizing condition:

$$MR = MC \Leftrightarrow p = 3q^2 - 12q + 10$$

$$\Leftrightarrow 3q^2 - 12q + (10 - p) = 0$$

$$\Leftrightarrow q = 2 \pm \frac{\sqrt{144 - 12(10 - p)}}{6} = 2 \pm \frac{\sqrt{12 - (10 - p)}}{\sqrt{3}} = 2 \pm \sqrt{\frac{2 + p}{3}}$$

Keeping both roots, for the time being, the industry output will be:[1]

$$Q = Nq = 20\left[2 \pm \sqrt{\frac{2+p}{3}}\right] = 40 \pm 20\sqrt{\frac{2+p}{3}}$$

The new equilibrium price is

$$p = 10 - \frac{Q}{10} = 6 \mp 2\sqrt{\frac{2+p}{3}} \Leftrightarrow 3p^2 - 40p + 100 = 0 \Leftrightarrow p = 10 \vee p = \frac{10}{3}$$

For $p = 10$, each firm is producing $q = 2$ and industry output will be $Q = 40$.[2]
The profits of each firm are given by

$$\pi = pq - TC(q) = pq - \left[q^3 - 6q^2 + 10q\right] = -q^3 + 6q^2 - (10 - p)q$$

For $p = 10, q = 2$ we get $\pi = 16$. Since the operating licenses are sold in a competitive market, each license will be bid up to a price equal to the profits that each firm is expecting to make form operating in the closed market. Hence, the annual price of the license will be exactly equal to the profits of each firm (i.e. after having paid for an operating license, the individual firm will be making exactly zero profits in the new equilibrium).

For $p = \frac{10}{3}$, each firm is producing $q = \frac{10}{3} \vee q = \frac{2}{3}$ and industry output will be $Q = \frac{200}{3} \vee Q = \frac{40}{3}$ respectively. The individual profits are given by $\pi = \frac{200}{27}$, for $q = \frac{10}{3}$, and $\pi = \frac{16}{27}$, for $q = \frac{2}{3}$. Even though the larger output leads to higher profits (and should, in principle, be chosen by the firms), after biding for the license to operate in this market the firms are left with zero economic profits anyway. Hence, a market price of 10/3 could in fact be supported here by two possible equilibrium scenarios: each firm is producing 10/3 (2/3) units, industry output is 200/3 (40/3), and the annual rental price of an operating license will be bid up to 200/27 (16/27).

5. (25 points)

In a two person, two-commodity economy, denote the two commodities by X and Y and the two consumers by 1 and 2. Each of the two individuals has initial endowments of 1 unit of each good. The preferences of agent 1 are given by the

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[1] The solution $q = 2 - \sqrt{\frac{2+p}{3}}$ remains non-negative as long as the price does not exceed 10.

[2] The condition MR=MC admits here also the solution $q = 0$. This shut-down outcome corresponds to local profit-minimization (recall that MR=MC is only s first-order, necessary condition).

utility function $U_1(x_1, y_1) = \frac{1}{3}\ln x_1 + \frac{2}{3}\ln y_1$. Those of agent 2 are given by the function $U_2(x_2, y_2, x_1) = \ln x_2 + \ln y_2 + \ln(2 - x_1)$.

i) Find the set of Pareto optimal allocations in this economy. Graph this locus in an Edgeworth Box diagram.

ii) Find the Walrasian equilibrium in this economy. Is it Pareto optimal? Explain why or why not.

Answer

(i) The set of Pareto efficient allocations is defined as:

$$\max_{(x_1, y_1, x_2, y_2)} U_1(x_1, y_1)$$
$$s.t.$$
$$U_2(x_2, y_2, x_1) \geq \bar{u}_2$$
$$x_1 + x_2 = 2$$
$$y_1 + y_2 = 2$$

Substituting from the last two constrains, the Lagrangean of this problem is given:

$$L = \frac{1}{3}\ln x_1 + \frac{2}{3}\ln y_1 + \lambda\left[2\ln(2 - x_1) + \ln(2 - y_1) - \bar{u}_2\right]$$

The first-order conditions are

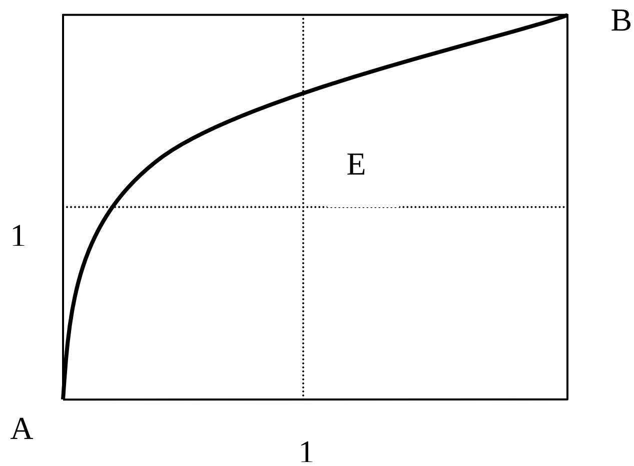
$$\frac{\partial L}{\partial x_1} = 0 \Leftrightarrow MU_x^1 = \frac{2\lambda}{2 - x_1} \Leftrightarrow \frac{1}{3x_1} = \frac{2\lambda}{2 - x_1} \qquad (1)$$

$$\frac{\partial L}{\partial y_1} = 0 \Leftrightarrow MU_y^1 = \frac{\lambda}{2 - y_1} \Leftrightarrow \frac{2}{3y_1} = \frac{\lambda}{2 - y_1} \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} \geq 0 \Leftrightarrow U_2 \geq \bar{u}_2 \quad (3.1) \qquad \lambda \geq 0 \quad (3.2) \qquad \lambda\frac{\partial L}{\partial \lambda} = 0 \quad (3.3)$$

For $x_1, y_1 < 2$, equations (1)-(2) dictate that $\lambda > 0$ and, from the complementary slackness condition (3.3), agent 2 must be getting utility $\bar{u}_2$. From (1) and (2), $\frac{2 - y_1}{2 - x_1} = \frac{y_1}{4x_1} \Leftrightarrow \frac{4x_1}{y_1} - \frac{3x_1}{2} = 1$. This is the equation of the Pareto set measured from the origin of agent 1. It is shown in the Edgeworth box below.[3]

[3] To plot the Pareto frontier is rather demanding. For $x_1, y_1 > 0$, we can express the relation as follows $8x_1 - 3x_1y_1 = 2y_1$. This gives $8 - 3\left(y_1 + x_1\frac{dy_1}{dx_1}\right) = 2\frac{dy_1}{dx_1} \Leftrightarrow (2 + 3x_1)\frac{dy_1}{dx_1} = 8 - 3y_1$.



(ii) In a Walrasian equilibrium, every agent will be maximizing her utility subject to her budget constraint and the markets for the two goods must clear. Since both agents are non-satiated in both goods while they would never choose to consume nothing of either good, we can only have here interior solutions to the two UMPs. The first order condition for agent $i = 1,2$ is given by: $MRS^{i}_{x_i,y_i} = \frac{p_1}{p_2} = p$. This gives $y_1 = 2px_1$ and $y_2 = px_2$. By the budget constraints, moreover, $px_i + y_i = 1 + p$ or $3px_1 = 1 + p = 2px_2 \Rightarrow x_1 = \frac{2}{3}x_2$. It suffices to make sure that the market for good X clears; $x_1 = 2 - x_2$ dictates that the equilibrium allocations must be $((x_1, y_1),(x_2, y_2)) = \left(\left(\frac{4}{5}, \frac{8p}{5}\right), \left(\frac{6}{5}, \frac{6p}{5}\right)\right)$. From the budget constraint, we can determine the equilibrium price, $\frac{4p}{5} + \frac{8p}{5} = 1 + p \Leftrightarrow p = \frac{5}{7}$.

The Walrasian equilibrium *cannot* be Pareto optimal in this setting. To see this, note that since $x_2 = 2 - x_1$ and $y_2 = 2 - y_1$, we have $MU^2_{x_2} = \frac{1}{x_2} = \frac{1}{2 - x_1}$ and $MU^2_{y_2} = \frac{1}{2 - y_1}$. Consequently, equations (1)-(2) above give $MU^1_{x_1} = 2\lambda MU^2_{x_2}$ and $MU^1_{y_1} = \lambda MU^2_{y_2}$. A Walrasian equilibrium has both agents facing the same price ratio. Since every agent will be optimizing by setting his MRS equal to the common price ratio, the Walrasian optimality condition will be $MRS_1 = MRS_2$. On the

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Moreover, the latter relation gives $(2 + 3x_1)\frac{d^2 y_1}{dx_1^2} = -6\frac{dy_1}{dx_1}$. Since $x_1 \geq 0, y_1 \leq 2$, we have $\frac{dy_1}{dx_1} > 0, \frac{d^2 y_1}{dx_1^2} < 0$.

other hand, the Pareto equations (1)-(2) require $MRS_1 = 2MRS_2$ due to the presence of the externality (the fact that agent 2 cares here about hw much of good X agent 1 consumes). Since the consumption of good X by agent 1 causes disutility to agent 2, it turns out here that Pareto efficiency requires agent 1 to be willing to trade between the two goods at twice the ratio that agent 2 would be willing to trade.

6. **(35 points)** Consider a small country where Goofy Gadgets Inc. is the only firm licensed to produce gadgets. By government decree, gadgets cannot be imported or exported and Goofy gadgets is allowed to operate only two plants. These plants have the following cost functions: $TC_A = q_A^2$ and $TC_B = 4q_B^2$. Demand for gadgets in this country is given by $Q_D = 140 - 2P$.

i) How many gadgets will GGI produce, and how will production be divided between the two plants? What will be the market price of gadgets? How much profit will GGI be making?

ii) Now, suppose that the government relaxes its trade restrictions so that GGI can export gadgets, but importing gadgets is still prohibited. GGI is a large producer relative to the world market and, therefore, has to act as if facing the following demand curve for its exports, $Q_W = 10 - P$. GGI is still a monopolist in its home market. Moreover, because others cannot sell in the domestic market, GGI can charge domestically a different price from the world price. How many gadgets will GGI produce now and how will production be divided amongst the two operating plants? How many gadgets will GGI sell domestically and how many will be exported? What prices will it charge in each market? How much profit will it be making?

<u>*Answer*</u>

(i) For any given total output *Q* produced Goofy Gadgets Inc., production will be divided amongst the two operating plants so as to set marginal cost of production equal across the two plants. In other words, the cost of producing the last unit of output supplied in each plant should be the same.[4] The firm will use both plants because dividing production

[4] To see this intuitively, consider that the firm is producing some total output Q, allocating production between the two plants such that $MC_A > MC_B$. Then we can increase profits by keeping total output the same (hence, total revenues are the same) but switching the last unit produced in plant A to being produced

between them is less costly than producing all output in only one plant. Given increasing MC and no fixed costs, it is less costly to produce low MC units in each plant rather than high MC units in only one plant. Moreover, profit maximization requires that the total output supplied in the domestic market for gadgets will be set at such a level where the (common) marginal cost will be equal to the marginal revenue. We have

$$MC_A(q) = 2q_A \qquad MC_B(q) = 8q_B$$

$$MR = \frac{dTR(Q)}{dQ} = \frac{d}{dQ}(pQ) = \frac{d}{dQ}\left(\frac{140-Q}{2}Q\right) = 70 - Q = 70 - (q_A + q_B)$$

$$MC_A = MC_B = MR \Leftrightarrow 2q_A = 8q_B = 70 - (q_A + q_B) \Leftrightarrow q_B = \frac{70}{13}, q_A = \frac{280}{13}$$

GGI sells, therefore, a total of 350/13 gadgets by producing 280/13 in plant A and the remaining 70/13 in plant B. The market price for gadgets will be read from the demand curve: $p = \frac{140-Q}{2} = 70 - \frac{350}{26} = \frac{735}{13}$.

GGI's profit is $\pi = pQ - (TC_A + TC_B) = p(q_A + q_B) - (q_A^2 + 4q_B^2) = \frac{159250}{169}$.

(ii) If the company is to sell in both the domestic and the world markets, it will wish to equate the MRs across the two markets. The intuitive reason for this is identical to the one we made in part (i) regarding the optimal allocation of production across plants. Replace MCs with MRs in that argument to see that, if the marginal revenues are not the same across markets, then we can increase profits simply by switching the last unit supplied in the market with the lower MR to that with highest. Since total production remains the same, total costs are unchanged but total revenues go up by the difference in marginal revenues. And we can keep on doing such reallocations of supply until the two marginal revenues are equal. Thus, we should have

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in plant B. This would reduce total costs by $MC_A - MC_B$, raising profits by the same amount. We can keep raising profits in this way, keeping total output fixed, until the point where production is allocated amongst the two plant according to $MC_A = MC_B$. Mathematically, consider the problem of minimizing total costs $TC_A + TC_B$ for a given total output (i.e. subject to the constraint $q_A + q_B = Q$). The two first order conditions of differentiating the Lagrangean, with respect to q_A, q_B respectively, require that the two marginal costs must be the same at the optimum.

$MR_d = MR_w = MC_A = MC_B \Leftrightarrow 70 - q_d = 10 - 2q_w = 2q_A = 8q_B$ and $q_A + q_B = q_w + q_d$.

Solving this system of 4 equations, we get $q_B = \frac{75}{17}, q_A = \frac{300}{17}, q_w = -\frac{215}{17}, q_d = \frac{590}{17}$.

The fact that the quantity to be sold abroad is negative means that it is clearly not optimal for GGI to sell in both markets. All of you concluded immediately at this point that the optimal must be for the company to keep on selling only on the domestic market as previously. Yet, a complete answer needs to offer a justification for why the possibility of selling only to the world market would not be chosen. The solution we just calculated above is the one satisfying the first-order conditions of selling in both markets. The optimal for GGI, therefore, should be the one that gets us as close as possible to this point while not-violating the non-negativity constraints on the quantities supplied. Since the non-negativity constraint is violated only with respect to q_w, it seems very intuitive that the closest we could get to it is by having $q_w = 0$. However, for a rigorous argument, we ought to actually examine the case $q_w > 0 = q_d$, calculate the corresponding profits and compare them to π_d from part (i).

I will actually present a comprehensive analysis. The problem can be stated analytically as follows

$$\max_{q_d, q_w, q_A, q_B} \left(70 - \frac{q_d}{2}\right) q_d + (10 - q_w) q_w - \left(q_A^2 + 4q_B^2\right)$$

$s.t.$

$$q_d + q_w = q_A + q_B$$

$$q_k \geq 0 \quad k \in \{d, w, A, B\}$$

Substituting $q_A = q_d + q_w - q_B$, we can obtain the reduced problem corresponding to the following Lagrangean:

$$L = \left(70 - \frac{q_d}{2}\right) q_d + (10 - q_w) q_w - (q_d + q_w - q_B)^2 - 4q_B^2 + \mu_d q_d + \mu_w q_w + \mu_B q_B$$

with the first-order conditions

$$\frac{\partial L}{\partial q_d} = 70 - q_d - 2q_d + 2q_B - 2q_w + \mu_d = 0 \Leftrightarrow 70 + \mu_d = 3q_d + 2q_w - 2q_B \qquad (1)$$

$$\frac{\partial L}{\partial q_w} = 10 - 2q_w - 2q_w + 2q_B - 2q_d + \mu_w = 0 \Leftrightarrow 10 + \mu_w = 4q_w + 2q_d - 2q_B \qquad (2)$$

$$\frac{\partial L}{\partial q_B} = -2q_B + 2q_d + 2q_w - 8q_B + \mu_B = 0 \Leftrightarrow \mu_B = 10q_B - 2q_d - 2q_w \qquad (3)$$

$$\mu_k \geq 0 \qquad (3.2) \qquad \mu_k q_k = 0 \qquad (3.3) \qquad k \in \{d, w, B\}$$

Before proceeding, observe that, as long as the company does not choose no-production at the optimum ($q_d + q_w > 0$), equation (3) along with the condition $\mu_B \geq 0$ dictate $q_B > 0$.[5]

Starting at an interior solution, $q_k > 0$ (and, thus, $\mu_k = 0$) for $k \in \{d, w, B\}$, we get (by using the three equations above in reverse order) $5q_B = q_d + q_w$, $5 = q_w + 4q_B$, and $70 = 13q_B - q_w$ which give the non-valid solution we have already.

There are two other possible solutions: $q_w = 0 < q_d$ and $q_d = 0 < q_w$. The former one was obtained in part (i) and corresponds to profits $\pi_d = \frac{159250}{169}$.[6] For the latter one, we have $q_k > 0$ (and, thus, $\mu_k = 0$) for $k \in \{w, B\}$. Hence, $5q_B = q_w$ and $5 = 2q_w - q_B$ which give $q_B = \frac{5}{9}, q_w = \frac{25}{9}, q_A = \frac{20}{9}$. GGI would sell, therefore, only to the world market a total of 25/9 gadgets, making 20/9 in plant A and 5/9 in plant B. The market price for gadgets would be read from the world demand curve, $p_w = 10 - q_w = \frac{65}{9}$.

The resulting profits would be $\pi_w = p_w(q_A + q_B) - (q_A^2 + 4q_B^2) = \frac{1125}{81}$.

Since $\pi_d > \pi_w$, GGI will choose to supply only in the domestic market. The answer in this part remains the one we have already given in part (i).

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[5] Since $q_B > 0 = \mu_B$ at any solution of the problem, equation (3) reduces to $5q_B = q_d + q_w$. Observe that, along with the defining condition $q_A = q_d + q_w - q_B$, this dictates that also $q_A > 0$ for q_A cannot be zero unless $q_B = 0$.

[6] The argument made in the preceding footnote establishes, analytically, why GGI producing all of its output in one plant cannot be optimal. Footnote 4 offers an intuitive argument.