The study estimated the von Neumann-Morgenstern utility index for the representative consumer in this area to be

$$U(w) = 1935.1(w + 0.6849)^{0.37}$$

The average consumer has monthly income of $1699.4. The monthly probability of telephone line trouble was estimated to be 0.5% and the typical repair charge is $55. What is the maximum amount this consumer would be willing to pay per month for an IWM contract?

QUESTION 3

Consider an individual who earns $2000 in income. Income is taxed at 20%. This individual cab underreport his income to the IRS and pay taxes only on the amount he reports, but should he be audited, the IRS will impose a surcharge of 200% on the unpaid taxes; that is, he will have 60% tax on any unreported income if he is audited.[1]
This taxpayer's von Neumann-Morgenstern utility index is
$U(Y) = \ln Y$ where Y is income.

(a) On a graph showing income if he is audited on the horizontal axis and income if he is not audited on the vertical axis, indicate this individual's bundle of contingent claims if he reports all of his income, his bundle of contingent claims if he reports no income, and his budget constraint. Find the equation of his budget constraint for contingent claims to income if he is audited and if he is not audited. Over what range of contingent claims does this equation describe his options? Interpret the value of the slope.

(b) Calculate his optimal bundle of contingent claims. How much income will he report to the IRS? How much will he not report?

(c) Suppose the IRS increases the penalty for underreporting income. How will he adjust his optimal bundle of contingent claims and the amount of income he reports to the IRS?

[1] IRS stands for "Internal Revenue Service", the US tax authority.

PRACTICE PROBLEM SET

Suggested Solutions

QUESTION 1

(a) The aim of this question is to demonstrate the use of the indifference curves – budget line diagram in calculating the optimal bundles of contingent claims. The farmer maximizes expected utility by choosing the bundle of contingent claims on his budget constraint for which his MRS equals the slope of the budget line.

He starts from the endowment point of having $700 when it is sunny and $0 when it rains. This point is feasible for him and, consequently, on his budget line; it is shown by point A in Fig. I.

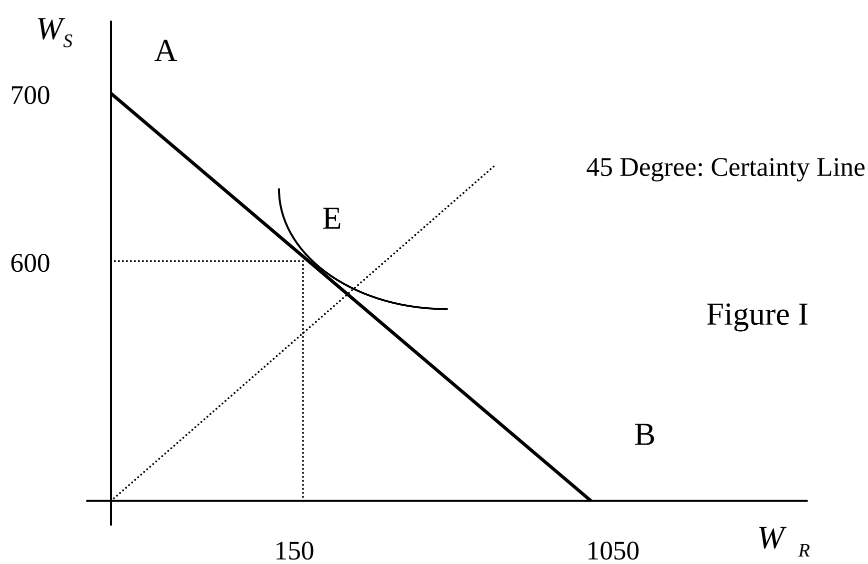

Figure I

This farmer's budget line is the line through A that has slope $-\frac{0.4}{0.6} = -\frac{2}{3}$. Starting from A, for each dollar of coverage that the farmer buys, his income falls by $0.40 if it is sunny (this is the premium per dollar of coverage that he has to pay to the insurance company) but rises by $0.60 in the rainy state (the net receipts from the insurance company: $1 of coverage minus the $0.4 of premium).

This transfer of income from the sunny to the rainy state can take place for the entire region of combinations up to the point where the farmer is left with no income at all in the sunny state in order to get $\frac{3}{2}\times 700 = \$1050$ of net income if it rains. This is point B in Fig. I. Since we know a point on the budget line (say A) and its slope, we can derive its equation:

$$W_s - 700 = -\frac{2}{3}(W_R - 0) \Leftrightarrow W_S = -\frac{2}{3}W_R + 700$$

As a parenthetical note, let us derive the slope of the budget constraint in the general case where the cost of insurance is \$*c* per \$1 of coverage when it rains and the farmer starts out with income W_S^0 in the sunny state and W_R^0 when it rains. His initial endowment point is (W_S^0, W_R^0). Consider that he chooses to buy \$*I* of coverage; he will have to pay to the insurance company a premium of \$*cI* in both states of the world.

- In the sunny state, his income will be

  $W_S = W_S^0 - cI$ (I)

  Compared with his income in the same state but when he doesn't buy any insurance, his income has now changed by

  $dW_S = -cI$

- In the case where it rains. He now gets \$I from the insurance company but he still has to pay \$*cI* in premium. His income will now be

  $W_R = W_R^0 + (1-c)I$ (II)

  Hence, compared with his income in the same state but when he doesn't get insured, his income has now changed by

  $dW_R = (1-c)I$

Therefore, starting from the endowment point (W_S^0, W_R^0), by entering in the insurance contract, this individual can transfer income between the two states of the world at the rate

$$\frac{dW_S}{dW_R} = -\frac{c}{1-c} \quad .$$

This is the slope of his budget line. We know also that this line passes through the endowment point (W_S^0, W_R^0). Therefore, using the standard form of the equation of a straight line when its slope and a point on it are known, we can derive the equation of the budget constraint for this problem:

$$W_S - W_S^0 = -\frac{c}{1-c}(W_R - W_R^0) \Leftrightarrow$$
$$W_S = -\frac{c}{1-c}W_R + (W_S^0 + \frac{c}{1-c}W_R^0) \qquad \text{(III)}$$

The optimal bundle of contingent claims is that bundle on his budget line for which his MRS is equal to the slope of the budget line. Such a point is defined by two equations.

First, the slope of the indifference curve (in expected utility terms) between the two states

$$MRS = -\frac{\pi_R}{1-\pi_R}\frac{MU(W_R)}{MU(W_S)} = -\frac{0.25}{0.75}\frac{\frac{1}{2\sqrt{W_R}}}{\frac{1}{2\sqrt{W_S}}} = -\frac{1}{3}\sqrt{\frac{W_S}{W_R}}$$

must equal the slope of the budget line

$$MRS = -\frac{c}{1-c} \Leftrightarrow \frac{\pi_R}{1-\pi_R}\frac{MU(W_R)}{MU(W_S)} = \frac{c}{1-c} \Leftrightarrow \frac{1}{3}\sqrt{\frac{W_S}{W_R}} = \frac{2}{3} \Leftrightarrow \qquad (1)$$
$$W_S = 4W_R$$

Second, the optimal point must be, of course, feasible; it needs to be on the budget line

$$W_S = -\frac{2}{3}W_R + 700 \qquad (2)$$

Solving these two equations gives the optimal bundle of contingent claims

$$(W_S, W_R) = (600,150)$$

Note that, from either of equations (I) and (II) above, we get

(I): *600=700-0.4I* Or (II): *150=0.6I*

Hence, the amount *I* of coverage bought is \$250 and he pays a premium of \$100 for it[1].

[1] Which can be readily seen without solving neither of equations (I) or (II). In the sunny state he has now \$600. This means that he must be paying \$100 in insurance premium, which is 0.40 times the coverage he requires from the insurance company. Hence, the coverage must be \$250.

(b)[2] If the cost of insurance goes up, the budget line will become steeper (the slope becomes more negative)[3]. Purchasing $1 of insurance coverage will reduce his income if it is sunny by a larger amount and increase his income by a smaller amount if it is rainy. The new budget line still goes through the endowment point A (because he still has the option of not buying any insurance)[4]. We can analyze the change in the optimal bundle of contingent claims by considering the substitution and income effects following this change in the relative "price" of getting coverage in the state of rain.

The *substitution effect* dictates that we must find a new bundle for which the MRS is equal to the new slope of the budget line – this is a bundle where the MRS is now higher than before. Applying the law of diminishing marginal utility of income (in general – irrespectively of the two states), this means that we need to find a bundle where the MU of income in the rainy state will be higher than at E while the MU of income in the sunny state will be lower. Intuitively, because the budget line is now steeper, income in the rainy state has become relatively more expensive to obtain through the insurance contract (in terms of forgone income in the sunny state). Since we always want to substitute away from the good which has become relatively more expensive, income in the rainy state must fall and income in the sunny state must rise. Substitution effect: $W_R \downarrow, W_S \uparrow$

[2] Note that the mathematically rigorous way to solve parts (a) and (b) would be to solve the following constrained problem:

$$\max_{W_R, W_S} E[U] = \pi U(W_R) + (1-\pi)U(W_S)$$

subject to the budget constraint: $W_S - 700 = -\frac{c}{1-c}(W_R - 0)$.

Setting up the Lagrangean and solving the problem is just a straightforward application. With respect to part (b), we are interested in how the bundle of optimal contingent income claims (W_S, W_R) changes with the cost of insurance *c*. One can, therefore, calculate the following derivatives: $\frac{\partial W_S}{\partial c}, \frac{\partial W_R}{\partial c}$, the signs of which dictate the directions of change for the <u>sum</u> of income and substitution effects.

[3] To see this, consider that the slope changes positively with the price of insurance c:

$$\frac{d}{dc}\frac{c}{1-c} = \frac{1}{(1-c)^2}$$

[4] An example of a new budget line is shown as AC in Fig II.

The *income effect*, however, does not operate always in one direction like the substitution effect does. Here, the higher cost of coverage means that the individual is worse off (the budget line has tilted inwards, defining a smaller feasibility set than before) – we have a negative income effect. As contingent claims must be normal goods, this means that income must fall in both the rainy and the sunny state of the world.

Income effect: $W_R \downarrow, W_S \downarrow$.

Therefore, overall, we know that income in the rainy state will fall but we cannot be sure about what is to happen in the sunny state because for the latter state the two effects work in opposite directions. Note that, here, we cannot determine whether or not he will buy more insurance coverage than he used to at E. Income in the rainy state might be lower than at E due to the farmer buying less coverage than before but it could also be due to him buying the same amount (or even a higher one) which, however, is not enough to offset the income he looses from having to pay more on insurance premium. Without knowing what happens to income in the sunny state also, we cannot determine if he buys more or less coverage than before. Two opposite, such cases are shown in Fig. II.

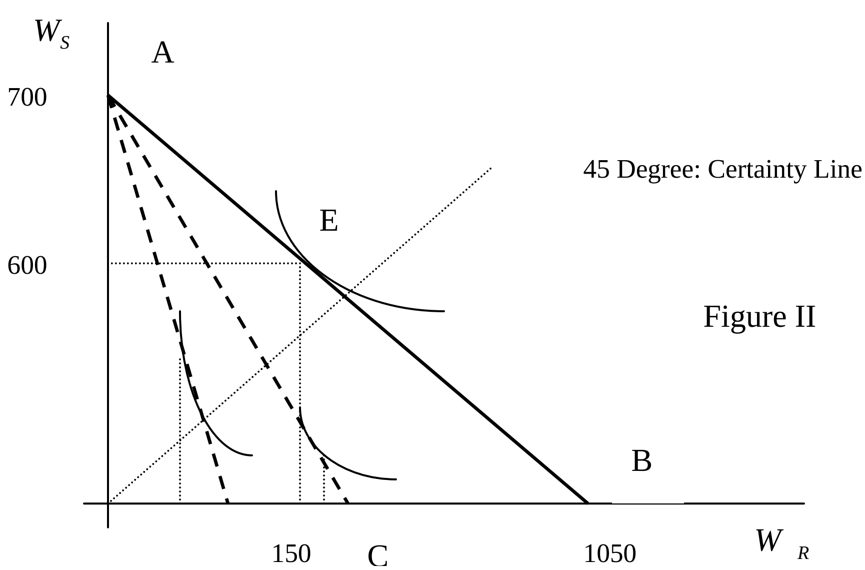

Figure II

(c) The Arrow-Pratt coefficients are measures of the curvature of the von- Neumann Morgenstern utility function. They measure one's risk-aversion (the higher an Arrow-Pratt coefficient, the more risk averse an individual is).

Absolute Measure of Risk Aversion

$$R_A = -\frac{U''(W)}{U'(W)} \Rightarrow R_A = -\frac{-\frac{1}{4\sqrt{W^3}}}{\frac{1}{2\sqrt{W}}} = \frac{1}{2W}$$. This will be, in this example, 0.005 at a wealth level of $100.

Relative Measure of Risk Aversion

$R_R = WR_A = -\frac{WU''(W)}{U'(W)}$. This will be 5, in this case.

(d) Consider a gamble that gives a final level of wealth of W_1 with probability π and one of W_2 with probability $1-\pi$. Expected utility from this gamble

$$E\left[U(\tilde{W})\right] = \pi U(W_1) + (1-\pi)U(W_2)$$

The certainty equivalent (CE) is that amount of wealth that this individual requires *with certainty* in order to be indifferent between receiving it and taking the gamble. In other words, it is the certain amount of wealth that this individual would regard as equivalent (in utility terms) to the gamble itself. This amount is given by:

$$E\left[U(\tilde{W})\right] = U(CE) \Leftrightarrow \pi U(W_1) + (1-\pi)U(W_2) = U(CE)$$

In our example

$$0.25U(700) + 0.75U(0) = U(CE) \Leftrightarrow 0.25\sqrt{700} = \sqrt{CE} \Leftrightarrow CE = 43.75$$

Note that the CE is less than the expected monetary outcome of the gamble

$$E\left[\tilde{W}\right] = \pi W_1 + (1-\pi)W_2 \Rightarrow E\left[\tilde{W}\right] = 0.25(700) + 0.75(0) = 175$$

Which is what we expect since this individual is risk averse at all levels of wealth (his utility of wealth function is everywhere strictly concave) – see Fig III below.[5]

[5] The CE being smaller (larger) than the expected monetary outcome of a gamble (for any lottery) is an alternative definition of the agent being risk averse (loving). Similarly, the CE being equal to the expected monetary outcome defines risk neutrality.

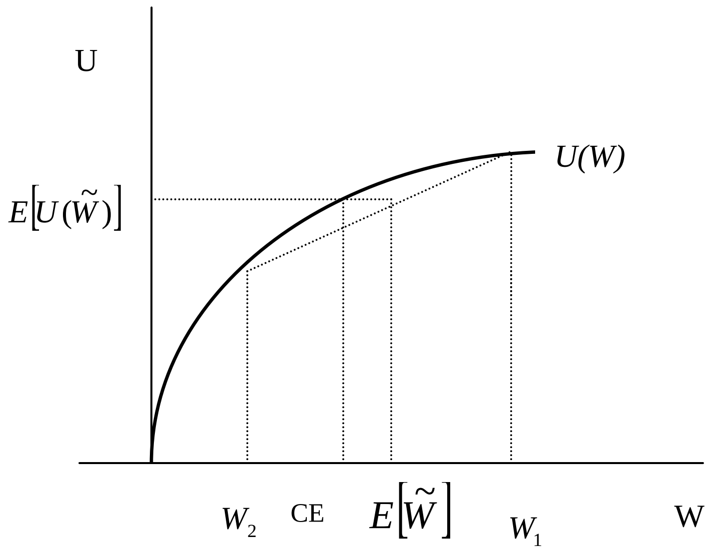

Figure III

The <u>probability premium</u> (PM) asks a related but different question. Given a fixed amount of money, say *W*, and a positive number ε, the probability premium is the excess winning probability over a fair bet that makes the individual indifferent between the *certain* outcome W and a gamble between the two outcomes $W+\varepsilon$ and $W-\varepsilon$.

If we are to create a new gamble with excess winning probability PM over a fair bet around W, we need a bet that offers $\frac{1}{2}+\rho$ probability of wealth $W+\varepsilon$ and probability $\frac{1}{2}-\rho$ of wealth $W-\varepsilon$.[6] We require

$$E[U(\tilde{W})] = U(W) \Leftrightarrow \frac{1}{2}+\rho \;\; U(W+\varepsilon) + \frac{1}{2}-\rho \;\; U(W-\varepsilon) = U(W)$$

Unfortunately, the question doesn't specify the levels of neither *W* nor ε. To show you how this is to be done, take the level of W to be the mean wealth level of the given gamble 175 and let us take $\varepsilon = 131.25$.[7]

$$(0.5+\rho)\sqrt{175+131.25} + (0.5-\varepsilon)\sqrt{175-131.25} = \sqrt{175} \Leftrightarrow$$
$$(0.5+\rho)\sqrt{306.25} + (0.5-\rho)\sqrt{43.75} = \sqrt{175} \Leftrightarrow \rho = 0.108$$

---

[6] You should verify that this lottery gives indeed an expected monetary payoff of *W*.

[7] This is the difference between the expected monetary outcome of the given gamble 175 and its CE which was calculated to be 43.75.

In other words, I need to increase the probability of winning by 10.8% (over the 50% of a fair bet around the wealth level of \$175) in order to make this agent indifferent between taking the bet (get \$306.25 with probability 60.8% and get \$43.75 with probability 39.2%) or keeping \$175 with certainty – see Fig. IV.

Note that the probability premium is positive. This has to be since we know that this agent is risk averse[8].

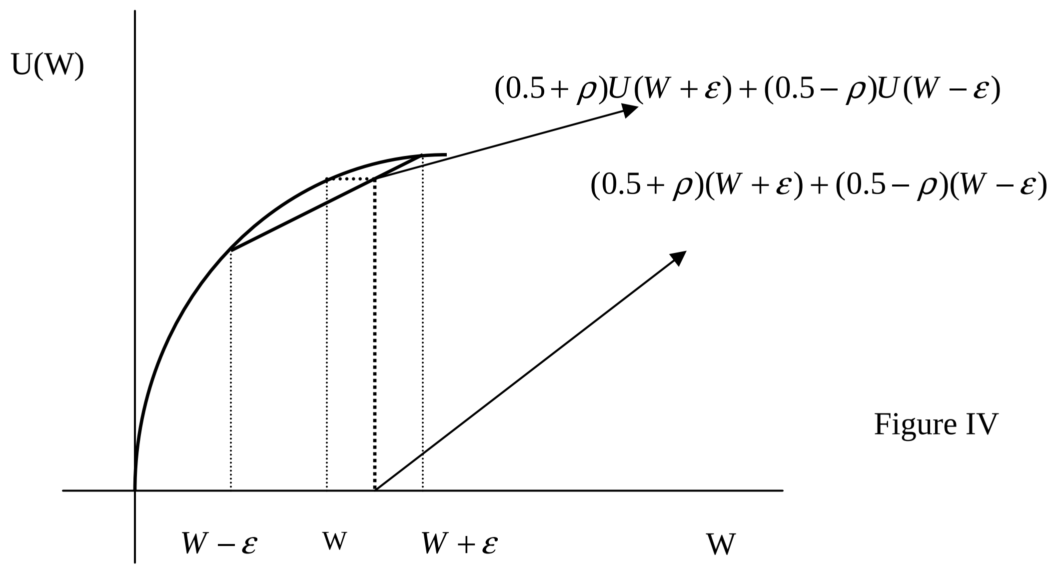

Figure IV

<u>NOTE ON PART (a)</u>

A common mistake is to try and find how much insurance he would buy using the following method. If the farmer doesn't buy any insurance his expected utility is

$$E\left[U^{NI}\right]=\pi_R U(W_R)+(1-\pi_R)U(W_S)=0.25\sqrt{0}+0.75\sqrt{700}=0.75\sqrt{700}$$

Consider now that he purchases \$*S* of coverage. That is he is to receive a total of \$*S* from the insurance if it rains. This can be bought, however, at the cost of paying \$0.4*S* to the insurance company upfront (i.e. in both states of the world). There are two contingencies that he might find himself in

- There is a 25% chance of rain, in which case he gets \$*S* from the insurance company. Therefore, his net income is: $W_R+S-0.4S=0.6S$.

[8] The probability premium being positive (negative) for all W and ε is an alternative definition for being risk averse (risk loving). Similarly, a zero probability premium for all W and ε is an alternative definition for risk neutrality.

- There is a 75% chance of being sunny. In this case, he doesn't receive any payment from the insurance company but he still has to pay the premium. His net income here is: $W_S - 0.4S = 700 - 0.4S$.

He will buy insurance coverage as long as his expected utility from the insurance contract is at least as large as the level that he can get without it (the latter level is his reservation utility).

$$E\left[U^I\right] \geq E\left[U^{NI}\right] \Leftrightarrow 0.25U(0.6S) + 0.75U(700 - 0.4S) \geq 0.75U(W_S) \Leftrightarrow$$

$$0.25\sqrt{0.4S} + 0.75\sqrt{700 - 0.4S} \geq 0.75\sqrt{700} \Leftrightarrow$$

$$0.4S + 9(700 - 0.4S) + 6\sqrt{0.4S(700 - 0.4S)} \geq 9 * 700 \Leftrightarrow$$

$$6\sqrt{0.4S(700 - 0.4S)} \geq 0.4 * 8 * S \Leftrightarrow$$

$$S(700 - 0.4S) \geq 0.16 * \frac{16}{9} * S^2 \Leftrightarrow 0.684S^2 - 700S \leq 0 \Leftrightarrow$$

$$0 \leq S \leq \frac{700}{0.684} = 102273$$

Hence, he will buy at most $1022.73 of coverage in the rainy state. This means that he will have to pay $0.4(1022.73)=$409.09 in insurance premium.

Note, however, that what we are calculating here is NOT the optimal bundle of contingent claims but rather the maximum amount of coverage he would ever buy under this insurance policy. Under this approach, the farmer is <u>not</u> maximizing his expected utility. He is just trying to get to a bundle of contingent claims that is at least as good as the endowment point A which is not necessarily the one that maximizes his expected utility under this insurance policy[9].

QUESTION 2

If the consumer does not purchase the IWM contract, there are two states of the world he might find himself in:

[9] To verify this, calculate his expected utility from the optimal bundle found in part (a). It is higher than the one derived from the bundle (209.91,613.64) which is the one that he gets in this note.

- There is a 0.05% chance of telephone line trouble in which case he will have to pay $55 in repair charges. In this contingency his available income would be $1644.4.
- There is a 99.5% chance that he will experience no telephone line trouble and he will have his entire income of $1699.4 available.

Without, therefore, the IWM contract his expected utility is

$$E\left[U^{NC}\right] = 0.005U(1644.4) + 0.995U(1699.4) =$$
$$0.005\left[1935.1(1644.4 + 0.6839)^{0.37}\right] + 0.995\left[1935.1(1699.4 + 0.6839)^{0.37}\right] \cong 30335.5$$

Let F the maximum amount that the representative consumer would be willing to pay for the IWM contract[10]. If he buys this contract, his available income would be $(1699.5-F) with certainty. But he would be willing to buy this contract as long as it yields expected utility at least equal to the expected utility from not buying it (the latter is his reservation level of utility).

Thus

$$U(1699.4 - F) \geq 30335.5 \Leftrightarrow 1935.1(1699.4 - F + 0.6839)^{0.37} \geq 30335.5 \Leftrightarrow$$
$$1700.0839 - F \geq \left(\frac{30335.5}{1935.1}\right)^{\frac{1}{0.37}} \Leftrightarrow F \leq \$0.28$$

QUESTION 3

(a) If this individual reports all of his income to the IRS he will have $1600, if he is not audited and $1600 if he is (because the audit will uncover no unreported income in this case). This is shown by point A in Fig V. On the other extreme, consider that he reports no income. Then if he is not audited he has $2000 whereas if he gets audited he will only have $800 (because the audit will uncover $2000 in unreported income and the whole amount will then be taxed at a 60% rate of taxes and surcharges). This is depicted as point B in Fig. V.

This individual's budget constraint is the straight line connecting the two points A and B. Starting from A and *compared to the income at point A*, in each of the two contingencies,

[10] Recall the note to part (a) in question 1. Here, we are concerned with the maximum amount that he would be willing to pay for the contract. Obviously, this would not give a bundle of contingent claims that maximizes his expected utility.

each dollar of unreported income raises net income by $0.20 in the no-audit state (the tax that is not paid by evading) but reduces income by $0.40 if there is an audit (the penalty for evading taxation). Therefore, the slope of the budget line is -0.2/0.4=-1/2.

Of course, this shifting of income from the reported to the unreported category, can take place only up to the point B, where no income at all gets reported.

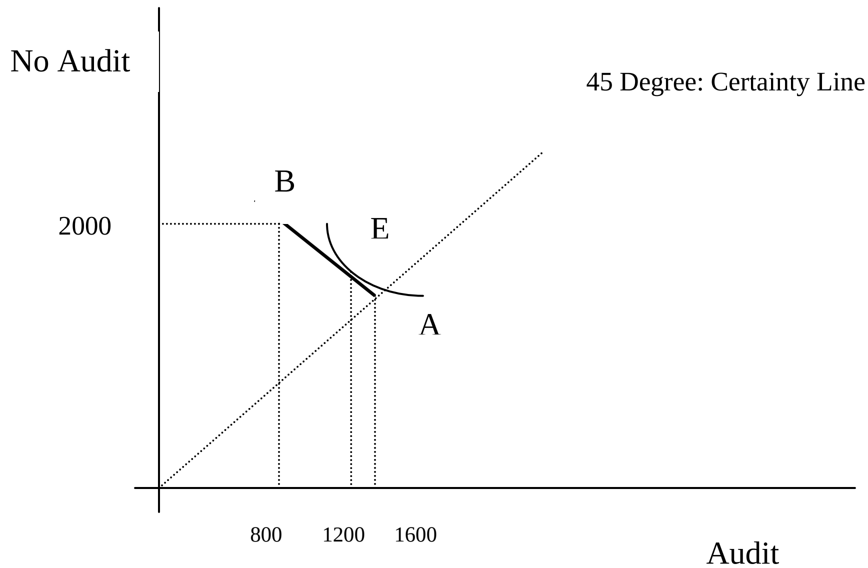

Figure V

Notice that the slope is equal to one over the rate of surcharge because shifting a dollar of income from the reported to the unreported category increases income by the tax to be paid on that dollar in the no audit state of the world, but reduces income in the audit state of the world by the surcharge on that dollar[11].

As a parenthetical note, let us derive the slope of the budget constraint in the general case where the tax rate is *t*, the tax-payer starts out with income *W* and he faces a surcharge *f* on all unreported income if he evades and gets audited.

His initial endowment point A is where he reports everything. In this case, no matter if there is an audit or not he has income *(1-t)W*.

[11] NOT by the tax plus the surcharge. We are examining the effects of transferring a dollar from the reported to the unreported category. Hence, the tax that will be paid on the unreported dollar, in the audit state, would also have been paid if that dollar had remained reported. The tax, in this case, does not cause any difference in overall income when considering shifting income from being reported to being unreported.

Consider that he chooses not to report an amount *S* from his income. This means that he does report *W-S* and, of course, he gets taxed at the rate *t* on this amount.

- In the case where he doesn't get audited, his income will now be

$$W_{NA} = (1-t)(W-S) + S = (1-t)W + tS \qquad \text{(I)}$$

Hence, compared with his income in the same state (i.e. when he doesn't get audited) but when he reports everything, his income has now changed by $dW_{NA} = tS$.

- In the case where an audit takes place. He is now in trouble because the IRS will uncover *S* in unreported income and impose the fine *f* on top of the tax rate on that amount. His income will now be

$$W_{A} = (1-t)(W-S) + (1-t-f)S = (1-t)W - fS \qquad \text{(II)}$$

Hence, compared with his income in the same state (i.e. when he does get audited) but when he reports everything, his income has now changed by $dW_{A} = -fS$.

Therefore, starting from the point A: *((1-t)W,(1-t)W)*, by not reporting some of his income, this individual can transfer income between the two states of the world (audit versus no-audit) at the rate $\frac{dW_{NA}}{dW_{A}} = -\frac{t}{f}$. This is the slope of his budget line.

We know also that the budget line passes through the point A[12]. Therefore, using the standard form of the equation of a straight line when its slope and a point on it are known, we can derive the equation of the budget constraint for this problem:

$$\begin{aligned} W_{NA} - (1-t)W &= -\frac{t}{f}(W_{A} - (1-t)W) \Leftrightarrow \\ W_{NA} &= -\frac{t}{f}W_{A} + (1-t)(1+\frac{t}{f})W \end{aligned} \qquad \text{(III)}$$

Using the parameter values from question 3, the equation of the budget line is:

$$W_{NA} = -0.5W_{A} + 0.8*1.5*2000 \Rightarrow W_{NA} = -0.5W_{A} + 2400$$

This is the equation of the *line* that goes through the points A, B in Fig. V. Note again, however, that the budget line is only the *segment* AB of this line – other points on the line are not economically feasible for our taxpayer.

[12] We could also use the point B here. We know that this point also belongs to the taxpayer's budget line since it is feasible. By not reporting anything he can be at this point B: *(W,(1-t-f)W)*.

(b)[13] The optimal bundle of contingent claims is that bundle on his budget line for which his MRS is equal to the slope of the budget line. Such a point is defined by two equations.

First, the slope of the indifference curve (in expected utility terms) between the two states

$$MRS = \frac{\pi_A}{1-\pi_A}\frac{MU(W_A)}{MU(W_{NA})} = \frac{0.25}{0.75}\frac{\frac{1}{W_A}}{\frac{1}{W_{NA}}} = \frac{1}{3}\frac{W_{NA}}{W_A}$$

must equal the slope of the budget line

$$MRS = -\frac{t}{f} \Leftrightarrow \frac{\pi_A}{1-\pi_A}\frac{MU(W_A)}{MU(W_{NA})} = \frac{t}{f} \Leftrightarrow \frac{1}{3}\frac{W_{NA}}{W_A} = \frac{1}{2} \Leftrightarrow W_{NA} = 1.5W_A \qquad (1)$$

Second, the optimal point must be, of course, feasible. In other words, it needs to be on the budget line

$$W_{NA} = -0.5W_A + 2400 \qquad (2)$$

Solving these two equations gives the optimal bundle of contingent claims

$(W_A, W_{NA}) = (1200{,}1800)$

Note that, from either of equations (I) and (II) above, we get

(I): *1800=1600+0.2S* Or (II): *1200=1600-0.4S*

---

[13] The approach taken in this part essentially uses the two equations one would derive mathematically if he were to solve the constrained problem of choosing the amount of unreported income S so as to maximize expected utility subject to the budget constraint (III). See also footnote (3) of question 1.

$$\max_S E[U] = \pi_A U(W_A) + (1-\pi_A)U(W_{NA})$$

$s.t.$

$$W_{NA} = -\frac{t}{f}W_A + (1-t)(1+\frac{t}{f})W$$

where W_{NA}, W_A are functions of *S*, defined by (I) and (II) above.

The fact that this individual starts already with point A (and B) being feasible clearly establishes that he could only abandon these bundles for some other, *on* his budget line, that makes him better off. Hence, the budget constraint will be satisfied with equality and there is no need in such a problem to set up a Lagrangean. Just substitute for, say, income in the no-audit state from the budget line into the objective (so the objective will now be written as a function of income in the audit state only), substitute next income in the audit state as a function of the unreported income *S* (from equation (II)) and the entire problem gets now transformed to one of unconstrained maximization of one variable *S*. These two steps are, of course, done immediately if one substitutes both W_{NA}, W_A as functions of *S* from equations (I) and (II). Recall that these two equations together entail the budget line.

Hence, the amount S of unreported income is \$1000[14].

(c) We have seen that the slope of his budget line is equal to one over the penalty surcharge rate. Therefore, if the IRS increases the penalty for underreporting income, the budget line will become flatter (the slope becomes less negative). The new budget line still goes through the point A (because he still has \$2000 of income and he still gets to keep \$1600 in both states of the world, by reporting everything, no matter what the surcharge is). See Fig. VI.

We can analyze the change in the optimal bundle of contingent claims by considering the substitution and income effects following this change in the relative "prices" of evading and not getting caught versus evading and getting caught (and, consequently, having to pay the penalty).

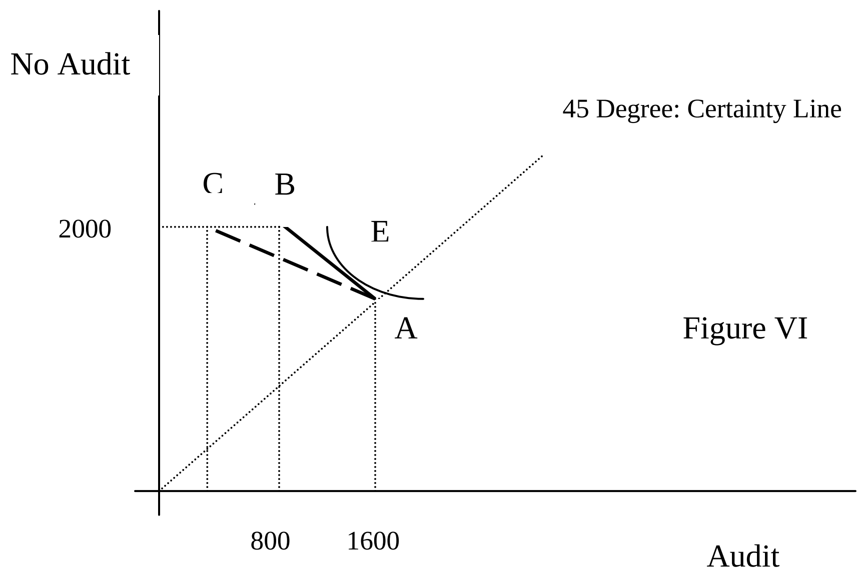

Figure VI

The *substitution effect* dictates that we must find a new bundle for which the MRS is equal to the new slope of the budget line – this is a bundle where the MRS is now lower than before. Applying the law of diminishing marginal utility of income (in general – irrespectively of the audit/no-audit states), this means that we need to find a bundle where the MU of income in the audit state will be lower than at E while the MU of income in

[14] Which can be readily seen without solving neither of equations (I) or (II). In the no-audit case he has \$1800. This means that he must be paying \$200 in tax, which can be his tax bill if he is reporting only \$1000. Hence, he must be not reporting the rest \$1000 of his true income.

the no-audit state will be higher. Thus, income in the audit state must rise and income in the no-audit state must fall. Substitution effect: $W_{NA} \downarrow, W_A \uparrow$

The *income effect*, however, does not operate always in one direction like the substitution effect does. Here, the higher penalty means that the individual is worse off (the budget line has tilted inwards, defining a smaller feasibility set than before) – we have a negative income effect. As contingent claims must be normal goods, this means that income must fall in both the audit and no-audit state of the world[15]. Income effect: $W_{NA} \downarrow, W_A \downarrow$.

Therefore, overall, we know that income in the no-audit state will fall but we cannot be sure about what is to happen in the audit state. Nevertheless, since income in the no-audit state falls for sure, the individual must be paying more taxes (there is no audit in this state, hence the missing income cannot be due to penalty fees), due to reporting more of his income. Concluding, a higher surcharge induces this taxpayer to report a larger amount of his income.

[15] Here is a simple argument for establishing that the contingent claims here must be normal goods. Note, first of all, that the term "contingent claims" refers to income in two different states of the world. But as far as the consumer's preferences go, both states are concerned with the same commodity – income in this example. If income for this person is a normal good, it will be so, therefore, in both states of the world. Since, in this example, the consumer has only one commodity (income) entering in his utility function, this commodity *cannot* be an inferior good. This is the same claim as saying that we cannot have *all* commodities in a utility function being inferior goods. To see the logic behind this, consider all prices falling. In this example, let both the tax rate and the surcharge decrease in equal proportions so that the slope of the budget line remains unchanged. The consumer is facing a positive income effect (his budget line shifts outwards in a parallel to itself way – he must be better off). If income is an inferior good, he must be now choosing to have less income in both states of the world. But this cannot be since it means that he will be at a point strictly in the interior of his budget set – actually he would be at a point even strictly inside his old budget set.