

Final Exam (Economics of Information)

1. Suppose a large number of firms compete in the labour market for workers. Workers are heterogenous in terms of their intrinsic ability $\theta \in [\underline{\theta}, \bar{\theta}]$. Ability is subject to private information (i.e., only each worker knows his own ability). Ability is distributed in the whole population of workers according to the *cumulative* distribution function $F(\theta)$. For simplicity, normalise $\underline{\theta} = 0$ and $\bar{\theta} = 1$, and assume that $F(\theta) = \theta$ for all $\theta \in [0, 1]$; in other words, ability of workers is *uniformly* distributed along the interval $[0, 1]$. Besides working for firms, workers have access to an outside option (say, self-employment) that yields income $r(\theta) : [0, 1] \rightarrow \mathbb{R}$. In particular,

$$r(\theta) : \begin{cases} 0 & \text{for all } 0 \leq \theta \leq \frac{1}{2} \\ \bar{r} & \text{for all } \frac{1}{2} < \theta \leq 1 \end{cases}, \text{ where } 0 < \bar{r} < \frac{1}{2}.$$

- a) Take the specific case when $\bar{r} = \frac{1}{8}$; plot the $E[\theta \mid \theta \in \Theta]$ schedule against w (on the horizontal axis) and find the competitive equilibrium/equilibria therein. Discuss whether the competitive equilibrium (equilibria) is (are) Pareto-efficient. **15 points**
- b) Under the more general assumption that $0 < \bar{r} < \frac{1}{2}$; will this labour market ever deliver multiple competitive equilibria? If your answer to the question is *yes*; does it *always* deliver multiple equilibria, or *only* for a subset of parametric configurations? **15 points**
2. Consider a shop assistant whose utility function is given by: $u(w, e) = \sqrt{w} - e^2$, where w denotes the wage and $e = \{0, 3\}$ stands for the level of exerted effort. The shop assistant has reservation utility $\bar{u} = 21$. The shop owner is risk-neutral. Sales are random, and may take on two different values: 0 or 2500. The probabilities of each level of sales, conditional on effort by the shop assistant, are as follows:

| effort level | sales = 0 | sales = 2500 |
|--------------|-----------|--------------|
| $e = 0$ | 0.6 | 0.4 |
| $e = 3$ | 0.3 | 0.7 |

- a) Write down and solve the optimisation problem for the shop owner when effort is publicly observable (and enforceable). **10 points**
- b) Write down and solve the optimisation problem for the shop owner when effort is unobservable. **10 points**
- c) Compare the shop owner profits levels obtained in a) and b), and provide some *intuition* for the result. What about the shop assistant – Is he better-off in a) or in b)? (provide here too some *intuition* for the result). **10 points**

3. Answer the following true-or-false questions *always* justifying your answer:

a) Take the standard two-type model presented in Macho-Stadler & Perez-Castrillo, section 4.3. There are two types of workers –Good- and Bad-Types– who differ (only) in the probability of success in their job (naturally, the success probability of Good-Types is larger than that of Bad-types). All workers have are risk-averse, with identical Bernoulli utility function. Argue whether the following statement is *true* or *false*, and justify your answer (either a graphic or a rigorous verbal justification are acceptable):

"If in a particular market where the fraction of Bad-Types is $\mu \in (0, 1)$ and their success probability is $p_B \in (0, 1)$ there exists no separating equilibrium *à la Rothschild & Stiglitz*, then in a market with the *same* fraction of Bad-Types, μ , but with success probability equal to $\hat{p}_B < p_B$ there will exist no separating equilibrium *à la Rothschild & Stiglitz* either." **14 points**

b) Take a two-effort levels principal-agent problem. More specifically: effort is unobservable to the principal; output y is observable but random; and the distribution of y conditional on e_H first-order stochastically dominates the distribution of y conditional on e_L . (Naturally, $e_H > e_L$.) Assume the principal is risk-neutral, while the agent is risk-averse and dislikes exerting effort.

"If the principal wishes to induce the agent to exert e_H , then he cannot provide full insurance to the agent conditional on e_H being chosen." **11 points**

c) Consider the signaling model with two types of workers presented in Mas-Colell *et al*, section 13.C. Assume that $\theta_L = 1$ and $\theta_H = 2$. Workers can acquire education before entering the labour market; education does not alter workers' (innate) productivity. Let the cost function for education be: $c(e, \theta) = e^2/\theta$. The utility of a worker of type θ is then given by: $u(w, e | \theta) = w - e^2/\theta$. Assess the following statements:

i) The lowest possible level of education by the types- H in a separating Perfect Bayesian Equilibrium is given by $\tilde{e} = 999.99$. **5 points**

ii) Assume the fraction of types- H is larger than the fraction of types- L . Suppose the government decides to abolish education in the economy. The government declares "we decided to eliminate the education system because this is Pareto-improving". **5 points**

iii) Suppose that education not only inflicts horrendous pain on students, but also increases their productivity as workers in case they are a type- H . More precisely, suppose $\theta_L = 1$ still, but now $\theta_H = 2 + \alpha e$ instead (where $\alpha > 0$). Then, the lowest possible level of education by the types- H in a separating PBE is *strictly smaller* than that in which education has not effects on productivity. (Either a graphic or an algebraic justification are acceptable.) **5 points**