Problem Set 2

1. Player 1 has an action set $X = [0,1]$, while player 2 has action set $Y = [0,1]$. The payoff to player 1 as a function of actions $x \in X$ and $y \in Y$ is given by:

$$U(x,y) = 1 + x - x^2 - \tfrac{1}{2}y.$$

For player 2 the payoff reads as follows (also a function of x and y):

$$V(x,y) = -(x-y)^2.$$

a) Find the NE in the game where both players choose actions x and y simultaneously.
b) Consider the extensive form game where player 1 chooses his action first. Player 2 observes player 1's choice and next chooses his own action. Solve for a subgame perfect equilibrium of this game.
c) Explain why player 1 has an incentive to choose differently in (b) compared to (a).

2. An amount of money accumulates, so that in period t its size is £ $2t$ where $t \in \{1, 2, ...T\}$. T is the last period of the game. In each period, two players must simultaneously decide whether or not to *claim* the money back. If only one player does so, then he obtains the entire sum of money accumulated to that date, the other player gets zero and the game ends. If both players claim, they split the accumulated sum equally. If neither of the players claims, the game proceeds to the next period, except for the final period T when both players obtain £ T. Each person cares only about the sum of money he gets. Formulate this situation as an extensive game and find its subgame perfect equilibria. (Hint: start by considering $T = 1$ and $T = 2$, and then try to generalise to any $T > 2$).

3. The members of a hierarchical group of n lions face a prey. If lion 1 does not eat the prey, they prey runs away and the game ends. If instead lion 1 eats the prey, it gains weight and becomes slow; as a result, lion 2 can eat lion 1. If lion 2 does not eat lion 1, the game ends. If lion 2 decides to eat lion 1, then lion 2 puts on weight and becomes slow too; as a result, lion 2 may be eaten by lion 3, and so on and so forth for lion $3, 4, ...n$. Each lion prefers to eat than to remain hungry. However, each lion prefers to be hungry than to be eaten. Find the outcomes of the game for any generic number n of lions by backwards induction.

4. **Bargaining game with concern for fairness:** Players 1 and 2 engage in a two-period bargaining game (à la Rubinstein) to split a "pie" of size 1. In $t = 0$ player 1 makes the offer and next in $t = 1$ player 2 makes the offer. If no agreement is reached players get 0 payoff (the pie is chucked and no player gets anything). Player 2 exhibits standard preferences with discount factor $\delta \in (0,1)$, that is: $u_2(y_0, 0) = y_0$ and $u_2(y_1, 1) = \delta y_1$, where y_t is the part of the pie that goes to player 2 in period t. Player 1 instead not only cares about his pie consumption and the time of consumption, but is also somehow concerned about the *fairness* of the agreement. In particular, player 1 does not like to feel that player 2 is trying to get a larger share of the pie than that player 1 attempted to obtain in $t = 0$. To be precise, preferences of player 1 are given by (both players have the same discount factor):

$$u_1(x_0, 0) = x_0 \qquad \text{and} \qquad u_1(x_1, 1) = \delta x_1 - \max\{(y_1 - x_0), 0\};$$

where x_t is the part of the pie that goes to player 1 in period t.
a) Find the outcome of the bargaining game that is delivered by the subgame perfect equilibrium.
b) Compare the outcome of this game with the one obtained in a standard two-period Rubinstein game (with $\delta_1 = \delta_2 = \delta$). Comment on the differences.

5. **Repeated Game with infinite horizon:** (Gibbons, Problem 2.15.) Suppose there are n firms in a Cournot oligopoly. Demand is given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + ... + q_n$. Consider the infinitely repeated game on this stage game. What is the lowest value of δ such that the firms can use trigger strategies to sustain the monopoly output level as a subgame perfect equilibrium. How does the answer vary with n, and why?