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Solvency requirement in a unisex mortality model

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Abstract

Following the EU Gender Directive, that obliges insurance companies to charge the same premium to policyholders of different genders, we address the issue of calculating solvency capital requirements (SCRs) for pure endowments and annuities issued to mixed portfolios. The main theoretical result is that, if the unisex fairness principle is adopted for the unisex premium, the SCR of the mixed portfolio calculated at issuing time assuming unisex survivorship is greater than the sum of the SCRs of the gender-based subportfolios. Numerical results show that for pure endowments the gap between the two is negligible, but for lifetime annuity the gap can be as high as 3-4%. We also analyze some conservative pricing procedures that deviate from the unisex fairness principle, and find that they lead to SCRs that are lower than the sum of the gender-based SCRs, because the policyholders are overcharged at issuing time.

Keywords. SCR, life insurance pricing, unisex tariff, unisex fairness principle, life table, risk margin, Gender Directive, gender discrimination.

JEL codes. C1, C13, C18, C38, J11

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1 Introduction and motivation

There have been two major changes for insurers operating in the life insurance markets in the European Union since the start of the new millennium.

The first major change is the regulation on gender discrimination, also well known as the EU Gender Directive (Aseervatham et al., 2016).[1] This norm establishes that insurance products must be offered at the same price for men and women. Responding to this change, there appears some academic literature addressing the unisex insurance (pricing) practice. For instance, Guillen (2012) indicates that gender information shall be taken into consideration when analyzing the insurance companies' data and risk, despite the ban on the gender discrimination on price. Ornelas and Guillen (2013) compare Mexican unisex life tables that are used for insurance purposes with those of the general population. Sass and Seifried (2014) analyze the effects of mandatory unisex tariffs on the optimal insurance demand. Schmeiser et al. (2014) discuss unisex insurance pricing also from the regulator's perspective and Thiery and Van Schoubroeck (2006) deal with the legal aspects of fairness and equality in actuarial risk selection. Chen and Vigna (2017) show how insurance companies should price a portfolio of policies issued to males and females of the same age if they want to respect actuarial fairness at the portfolio level, and they introduce the unisex mortality intensity that is in accordance with the fairness principle.

The second major change in the European context is that capital requirements are now highly regulated with the implementation of the Solvency II directive. The magnitude of the Solvency Capital Requirement (SCR) is of high relevance, because it restricts the financial capacity of a company.

In the present paper, we investigate the implications of the adoption of the unisex fairness principle on the solvency capital requirement, particularly the initial SCR. Taking pure

[1]In the remaining of the paper we will refer to the EU Gender Directive, however, the results apply to similar no-gender discrimination rules anywhere in the world. Indeed, the analysis of gender equalization is relevant not only to the European context. In 1978, the United States Supreme Court first prohibited gender-based divisions in insurance in the case City of Los Angeles. In 1983, the courts banned gender-based insurance distinctions for Tax Deferred Annuity and Deferred Compensation Plans in Arizona. Insurance companies have opposed any legislation that restrict their ability to use gender-based distinctions in developing insurance classifications, rates and coverages, but fighting discrimination is on the agenda of social movements all over the world.

endowments and life annuities as examples, we compute the initial SCR per policy relying on a single "unisex" portfolio and satisfying initial actuarial fairness principle (c.f. Chen and Vigna (2017)); and compare it with the weighted average of the gender-based per policy SCRs. For these life insurance contracts, we show that the SCR calculated at issuing time assuming unisex survivorship is greater than the weighted average of the gender-based SCRs. Moreover, we analyze how the gap between the capital requirements calculated under the two approaches depends on the size of the shock on mortality, on the compositional balance between men and women, and on the type of life insurance products.

An illustration is presented with the mortality experience for the 1950 cohort of men and women in the United Kingdom. The numerical analysis justifies our main theoretical result that the initial SCR relying on a single "unisex" portfolio is higher than the weighted one. However, the difference in SCR between these two approaches is negligible for pure endowments, and it becomes more substantial for life annuities products. Despite its very stylized nature, our model suggests that insurers should perform internal actuarial analysis with survival tables that distinguish between men and women. To the best of their interests, this leads to a more accurate risk analysis and, interestingly, under some conditions, to a smaller solvency capital compared to the case where information on sex is deleted from their files and no specific analysis by gender-group is done.

In the paper, we also discuss some approaches to computing the unisex tariffs and the SCRs used in practice. Apparently, insurers sometimes deviate from the unisex fairness principle. They either use the price of the riskier gender for all policyholders, or they use a weighted mix of the gender-based survival rates and add an extra-loading to it. When such practices are adopted, insurers are overcharging policyholders. Due to the excessive premiums, they need a smaller capital requirement than what the adoption of the unisex fair premium would imply. Some policy-oriented recommendations are given in the conclusions.

The remainder of the paper is as follows. Section 2 introduces the unisex fairness principle and its implications on the fair premium. Section 3 presents the main results on SCR. Section 4 introduces the stochastic mortality model. Section 5 shows numerical applications. Section 6 shows the consequences of some alternative practices. The last section concludes.

2 Unisex fairness principle and implications

In this section, we report the main results in Chen and Vigna (2017), recalling the definition of unisex fairness principle.

Suppose that the insurance company issues a portfolio of identical policies to m males and n females with same age x. This portfolio will be called a *mixed* portfolio in the following. Since the EU Gender Directive, the price to be charged is the same for males and females, say P^u. Assume that the fair price for the males *before* the EU Gender Directive was P^m, calculated according to the males' mortality table

$$[p_x^m, p_{x+1}^m, ..., p_{\omega-1}^m], \tag{1}$$

where ω is the maximal allowed age, and that the fair price for the females *before* the EU Gender Directive was P^f, calculated according to the females' mortality table

$$[p_x^f, p_{x+1}^f, ..., p_{\omega-1}^f]. \tag{2}$$

Before the EU Gender Directive, the mixed portfolio consisted of two subportfolios, the first one with m males and price P^m, the second one with n females and price P^f. The total amount of premia collected before the EU Gender Directive was

$$m \cdot P^m + n \cdot P^f. \tag{3}$$

After the EU Gender Directive, in order to respect actuarial fairness at the global portfolio level, the insurer should collect the amount in (3), therefore

$$(m+n) \cdot P^u = m \cdot P^m + n \cdot P^f.$$

This is formalized in the definition of unisex fairness principle and unisex fair premium:

Definition 2.1 (Unisex fairness principle and unisex fair premium)**.** *For a given portfolio of m male policyholders and n female policyholders, whose fair premiums are P^m and P^f respectively, we say that the unisex tariff P^u is calculated according to the* unisex fairness principle *if*

$$P^u = \gamma \cdot P^m + (1-\gamma) \cdot P^f, \tag{4}$$

where

$$\gamma = \frac{m}{m+n} \in [0,1]. \tag{5}$$

The actuarial fairness for the mixed portfolio can be achieved only by charging the unisex fair premium, whose amount depends only on the fair prices for males and females and on the proportion of each gender in the mixed portfolio. Obviously, when there are no females in the portfolio, $\gamma = 1$ and $P^u = P^m$; when there are no males in the portfolio, $\gamma = 0$ and $P^u = P^f$. Similarly, if $P^m = P^f$, then $P^u = P^m = P^f$ independent of γ.

A legitimate question one can have is: How should unisex survival probabilities look like in order to produce a unisex price that is fair? The answer depends on the insurance product issued. We shall address this issue separately for the two important life insurance products: pure endowment and lifetime annuity.

Pure endowment

The fair prices of a pure endowment insurance contract with a duration T a unitary payment issued to a male and a female aged x are, respectively,

$$P^m =_T E_x^m =_T p_x^m \, e^{-rT} \qquad \text{and} \qquad P^f =_T E_x^f =_T p_x^f \, e^{-rT},$$

where r is the risk-free rate and e^{-rT} is the financial discount factor from T to 0. According to (4), we have:

$$P^u =_T E_x^u =_T p_x^u \, e^{-rT} \tag{6}$$

where

$${}_T p_x^u = \gamma {}_T p_x^m + (1-\gamma) {}_T p_x^f. \tag{7}$$

The interpretation of (6)-(7) is rather important. For the pure endowment, the unisex fair premium is equal to the fair premium issued to a policyholder whose T-years survival probability is a weighted average of the T-years survival probabilities of males and females, the weights being the proportions of males and females in the portfolio.

Lifetime annuity

The fair prices of a continuous unitary lifetime annuity issued to a male and a female aged x are, respectively,[2]

$$P^m = a_x^m = \int_0^{\omega-x} {}_tp_x^m e^{-rt} dt \qquad \text{and} \qquad P^f = a_x^f = \int_0^{\omega-x} {}_tp_x^f \, e^{-rt} dt.$$

According to (4), we have:

$$P^u = a_x^u = \int_0^{\omega-x} {}_tp_x^u \, e^{-rt} dt \tag{8}$$

where

$${}_tp_x^u = \gamma {}_tp_x^m + (1-\gamma) {}_tp_x^f \qquad \text{for all } t \leq \omega - x. \tag{9}$$

For the lifetime annuity, the unisex fair premium is equal to the fair premium issued to a policyholder whose t-years survival probability is a weighted average of the t-year survival probabilities of males and females for all $t \leq \omega - x$, the weights being the proportions of males and females in the portfolio.

3 General results on SCR

3.1 Standard calculation of SCR

In order to see how the unisex actuarial fairness impacts on the solvency capital requirement, we consider both the pure endowment and the lifetime annuity products introduced in the last section.

Generally, the amount of regulatory capital required by Solvency II standards is consistent

[2]This product is similar to immediate lifetime annuity product in Milevsky and Salisbury (2015). Unlike Milevsky and Salisbury (2015) where the optimal annuity payoff is determined, we are more interested in the solvency capital requirement related to these products.

with a Value-at-Risk assessment at a 99.5% confidence interval on a one year time horizon, see also EIOPA (2014). In Olivieri and Pitacco (2009) and Börger (2010), there are several definitions for the capital charge for the longevity risk. Following them, we choose to define the initial solvency capital requirement for one single policy as

$$\mathrm{SCR}^i(0) = BEL^{i,shock}(0) - BEL^i(0),\ i = m, f, u, \tag{10}$$

where $BEL^{i,shock}(0)$ is best estimate liability value at time 0 under longevity shock, and $BEL^i(0)$ the best estimate liability value used in the net premium charging. According to Solvency II, insurers are required to assume that a longevity shock will reduce the annual death probabilities by 20%. Returning to the pure endowments and life annuity products considered in the previous section, we obtain for pure endowments:

$$BEL^i(0) = P^i_{end}(0) = S^i(0,T)e^{-rT},\ i = m, f, u, \tag{11}$$

where $S^i(0,T)$ is the survival probability from 0 to T for a policyholder of gender i. For the continuous life annuity products:

$$BEL^i(0) = P^i_{ann}(0) = \int_0^{\omega - x} S^i(0,s)e^{-rs}ds,\ i = m, f, u, \tag{12}$$

where $S^i(0,s)$ is the survival probability from 0 to s for a policyholder of gender i.

In the following, we need to find a way of introducing the longevity shock in order to obtain the shocked survival probabilities $S^{i,shock}(0,T)$, $i = m, f, u$, where the superscript *shock* stands for longevity shock. Hereby we follow the approach of Lin and Cox (2005) and, for a given cohort x, the survival probabilities (for all $t > 0$) are simultaneously shocked. More specifically, we assume

$$S^{i,shock}(0,T) = (S^i(0,T))^{1-\epsilon}, \quad i = m, f, u, \tag{13}$$

where $\epsilon \in [0,1]$ is a constant.

3.1.1 SCR for pure endowment and lifetime annuity

Applying the definition of SCR per policy in (10) and the shocked survival probability as in (13) to the pure endowment, we obtain

$$\begin{aligned}\mathrm{SCR}^i_{end}(\epsilon, 0, T) &= \left[S^{i,shock}(0,T) - S^i(0,T)\right] e^{-rT} \\ &= \left[(S^i(0,T))^{1-\epsilon} - S^i(0,T)\right] e^{-rT},\ i = m, f, u. \end{aligned} \tag{14}$$

Applying the definition of SCR in (10) and the shocked survival probability as in (13) to the annuity, we obtain

$$\begin{aligned}\mathrm{SCR}^i_{ann}(\epsilon, 0) &= \int_0^{\omega - x} \left[S^{i,shock}(0,s) - S^i(0,s)\right] e^{-rs} ds \\ &= \int_0^{\omega - x} \left[(S^i(0,s))^{1-\epsilon} - S^i(0,s)\right] e^{-rs} ds,\ i = m, f, u. \end{aligned} \tag{15}$$

We notice from (14) that, for a fixed interest rate $r > 0$, the SCR for the pure endowment with duration T is a function of two variables: the shock $\epsilon \in [0,1]$, and the duration T. Similarly, we see from (15) that the SCR for the lifetime annuity is a function of the shock $\epsilon \in [0,1]$. In the following, we will need to use the dependence of SCR on the different variables, and it is therefore important to highlight it. However, the complete notation with the dependence of $SCR(\cdot)$ on two variables is heavy and often unnecessary. For notational convenience, in the rest of the paper, we will sometimes adopt the following simplified notation:

$$SCR^i_{end-T}(\epsilon) := SCR^i_{end}(\epsilon, 0, T) \tag{16}$$

$$SCR^i_{ann}(\epsilon) := SCR^i_{ann}(\epsilon, 0) \tag{17}$$

$$SCR^i_{end}(\epsilon, \tau) := SCR^i_{end}(\epsilon, 0, \tau) \tag{18}$$

In other words, we suppress 0 from the arguments of $SCR(\cdot)$ and leave $SCR(\cdot)$ as a function of ϵ (and possibly τ) only, see (16), (18), (17); when the duration of the pure endowment T does not change, we just report it in the subscript, see (16); when the duration of the pure endowment τ does change, we leave it as an argument of $SCR(\cdot)$, see (18).

3.2 General results

This section is the mathematical core of the paper. We prove that for the pure endowment and the annuity the adoption of the unisex fairness principle implies that the standard capital requirement at issuing time of the mixed portfolio calculated with the unisex survival probability is greater than or equal to the weighted sum of the standard capital requirements of the two gender-based subportfolios. Here we consider the standard capital requirement as calculated at time 0; accordingly, and following the notation of Section 3.1.1, the SCR is modeled as a function of ϵ (and possibly τ) only.

Proposition 3.1. *Assume that a portfolio of $m+n$ pure endowments with duration T and sum assured $M=1$ issued to m males and n females aged x is priced according to the unisex fairness principle. Assume that the standard capital requirement at time 0 is calculated according to*

$$SCR^{i}_{end-T}(\epsilon) = e^{-rT}\left[\left(S^{i}_{x}(T)\right)^{1-\epsilon} - S^{i}_{x}(T)\right] \qquad \textit{for} \quad i = u, m, f, \tag{19}$$

where $S^{i}_{x}(T)$ is the T-years pre-shock survival probability for age x and gender $i=u,m,f$, and $\epsilon \in [0,1]$. Then, the SCR at time 0 for a unisex policyholder is greater than or equal to the weighted average of the SCRs at time 0 for male and female:

$$SCR^{u}_{end-T}(\epsilon) \geq \gamma SCR^{m}_{end-T}(\epsilon) + (1-\gamma) SCR^{f}_{end-T}(\epsilon) \qquad \forall\, \epsilon \in [0,1] \tag{20}$$

where

$$\gamma = \frac{m}{m+n} \in [0,1]$$

is the proportion of males in the portfolio. Assuming that $S^{m}_{x}(T) \neq S^{f}_{x}(T)$, then the inequality in (20) is strict if and only if $\epsilon \in (0,1)$ and $\gamma \in (0,1)$.

Proof. Let us define the function $h(\cdot)$:

$$h(\epsilon) = SCR^{u}_{end-T}(\epsilon) - \left(\gamma SCR^{m}_{end-T}(\epsilon) + (1-\gamma) SCR^{f}_{end-T}(\epsilon)\right). \tag{21}$$

Claim (20) is equivalent to the non-negativity of the function $h(\epsilon)$. From (19) and (21) (for notational convenience, in the following we will write S^i in the place of $S_x^i(T)$) we have:

$$h(\epsilon)e^{rT} = (S^u)^{1-\epsilon} - S^u - \left(\gamma\,(S^m)^{1-\epsilon} - \gamma S^m + (1-\gamma)\left(S^f\right)^{1-\epsilon} - (1-\gamma)S^f\right).$$

Due to the unisex fairness principle, the relationship (7) holds true:

$$S^u = \gamma S^m + (1-\gamma)S^f. \tag{22}$$

By simplifying, we have

$$h(\epsilon)e^{rT} = (S^u)^{1-\epsilon} - \left(\gamma\,(S^m)^{1-\epsilon} + (1-\gamma)\left(S^f\right)^{1-\epsilon}\right). \tag{23}$$

Let us define the function $f(\cdot)$:

$$f(x) = x^{1-\epsilon}$$

Then, due to (22), (23) becomes:

$$h(\epsilon)e^{rT} = f\left(\gamma S^m + (1-\gamma)S^f\right) - \gamma f(S^m) - (1-\gamma)f(S^f) \geq 0$$

where the inequality results from applying the Jensen's inequality to the concave function $f(x)$ for $\epsilon \in [0,1]$. Hence, the claim (20) is proven. If $\gamma = 0$ (or $\gamma = 1$) there are no males (or females) in the mixed portfolio, and the inequality becomes an equality. If $\epsilon = 0$, then $SCR^i_{end-T} = 0$ for all $i = u, m, f$ and the equality holds; if $\epsilon = 1$, the function $f(x)$ is linear and the equality holds. If $\epsilon \in (0,1)$ and $\gamma \in (0,1)$, then the function $f(x)$ is strictly concave and the inequality in (20) is strict. □

The same result holds as a corollary also for the annuity case. In the following corollary we use the fact that the annuity is the union of pure endowments with different durations, and adopt the notation introduced in (18).

Corollary 3.2. *Assume that a portfolio of $m+n$ lifetime annuities with a unitary payment issued to m males and n females aged x is priced according to the unisex fairness principle.*

Assume that the standard capital requirement at time 0 is calculated according to

$$SCR^i_{ann}(\epsilon) = \int_0^{\omega-x} \left[\left(S^i_x(\tau)\right)^{1-\epsilon} - S^i_x(\tau) \right] e^{-r\tau} d\tau = \int_0^{\omega-x} SCR^i_{end}(\epsilon;\tau) d\tau \qquad \text{for} \quad i = u, m, f, \tag{24}$$

where $S^i_x(t)$ is the t-years pre-shock survival probability for age x and gender $i = u, m, f$, $SCR^i_{end}(\epsilon;\tau)$ is the standard capital requirement at time 0 for a pure endowment duration τ, and $\epsilon \in [0,1]$. Then, the SCR at time 0 for a unisex policyholder is greater than or equal to the volume-related weighted average of the SCRs at time 0 for male and female:

$$SCR^u_{ann}(\epsilon) \geq \gamma SCR^m_{ann}(\epsilon) + (1-\gamma) SCR^f_{ann}(\epsilon) \qquad \forall\, \epsilon \in [0,1] \tag{25}$$

where

$$\gamma = \frac{m}{m+n} \in [0,1]$$

is the proportion of males in the portfolio. Assuming that $S^m_x(\tau) \neq S^f_x(\tau)$ for all τ, then the inequality in (25) *is strict if and only if $\epsilon \in (0,1)$ and $\gamma \in (0,1)$.*

In the Corollary we set $c(t) = 1$ for simplicity, but the extension to the general payment $c(t)$ is straightforward.

Proof. Using (24), it yields

$$SCR^u_{ann}(\epsilon) - \left(\gamma SCR^m_{ann}(\epsilon) + (1-\gamma) SCR^f_{ann}(\epsilon)\right) = \tag{26}$$

$$\int_0^{\omega-x} \left[SCR^u_{end}(\epsilon;\tau) - \gamma SCR^m_{end}(\epsilon;\tau) - (1-\gamma) SCR^f_{end}(\epsilon;\tau) \right] d\tau \geq\ 0, \tag{27}$$

where the inequality is due to the fact that, by Proposition 3.1, the integrand function is positive. The other claims follow easily. □

Remark 1. Notice that the results of Proposition 3.1 and Corollary 3.2 hold only at time 0, or issuing time. In general, they do not hold at time $t > 0$. Indeed, the unisex fairness principle holds only at issuing time. The reason behind it is that we have used the **initial** unisex fairness principle in our derivation. This fairness principle can be violated in a future time $t > 0$.

Remark 2. Proposition 3.1 and Corollary 3.2 remark the impact of the EU Gender Directive on the value of SCR at issuing time. Indeed, *before* the EU Gender Directive, the SCR was calculated separately on the two subportfolios and the aggregate SCR was

$$mSCR^m + nSCR^f = (m+n)SCR^{weighted}$$

where

$$SCR^{weighted} = \gamma SCR^m + (1-\gamma)SCR^f. \tag{28}$$

After the EU Gender Directive, the insurer can still calculate the SCR in the old way with the two subportfolios. But if, instead, he prefers to calculate the SCR considering a single mixed portfolio with $m+n$ unisex policyholders (maybe because of fiscal incentives),[3] he should calculate $(m+n)SCR^u$. Proposition 3.1 and Corollary 3.2 compare the SCR at issuing time calculated with the two different procedures.

In Section 4 we introduce a stochastic mortality model, and in Section 5 we calibrate it and calculate the difference between the unisex SCR, SCR^u, and the weighted sum of the gender-based SCRs as in (28). This illustrates the practical implications of the results just proven.

4 The stochastic mortality model

In this section we review the stochastic unisex mortality model introduced in Chen and Vigna (2017). Let us introduce a complete filtered probability space $(\Omega, \mathfrak{F}, \mathbb{P})$ and a filtration $\mathcal{F}_t$ of sub-σ-algebras representing the state of information at time $0 \le t \le T$. An insurance company manages a mixed portfolio with m male policyholders and n females policyholders with the same age x. For notational convenience, throughout this section we omit the dependence on x of the mortality processes. We describe the stochastic force of mortality of each gender as an affine process λ. In other words, the time of death is modelled as the first jump time of a doubly stochastic process with intensity λ (see, Biffis, 2005; Dahl, 2004; Duffie et al., 2000; Luciano and Vigna, 2008; Milevsky and Promislow, 2001). In particular, the

[3]In some countries like Denmark insurers receive fiscal incentives if they merge the two subportfolios.

stochastic mortality intensities of the two genders are described by two different Ornstein-Uhlenbeck processes (OU processes) with positive drift and no mean reversion:

$$
\begin{aligned}
d\lambda^m(t) =& \mu_m \lambda^m(t)dt + \sigma_m dW^m(t), \\
d\lambda^f(t) =& \mu_f \lambda^f(t)dt + \sigma_f dW^f(t),
\end{aligned} \tag{29}
$$

where $\mu_i > 0$ and $\sigma_i > 0$ for $i = m, f$, and W_m and W_f are two standard Brownian motions under the real world measure $\mathbb{P}$, correlated with a correlation coefficient ρ. The OU process for the mortality intensity is a natural stochastic generalization of the Gompertz law for the force of mortality and is introduced by Luciano and Vigna (2008), where the conditions for biological reasonableness are also analysed. The survival probability function of males and females can be expressed in closed-form (see, Luciano and Vigna, 2008):

$$
\begin{aligned}
S^i(t,T) =& E\left[\exp\left\{-\int_t^T \lambda^i(u)du\right\}\Big|\mathcal{F}_t\right] = \exp\left\{\alpha_i(\tau) + \beta_i(\tau)\lambda^i(t)\right\},\ i = f, m \\
\alpha_i(\tau) =& \frac{\sigma_i^2}{2\mu_i^2}\tau - \frac{\sigma_i^2}{\mu_i^3}e^{\mu_i\tau} + \frac{\sigma_i^2}{4\mu_i^3}e^{2\mu_i\tau} + \frac{3\sigma_i^2}{4\mu_i^3},\ i = f, m \\
\beta_i(\tau) =& \frac{1}{\mu_i}(1 - e^{\mu_i\tau}),\ i = f, m
\end{aligned} \tag{30}
$$

where $\tau := T - t$.

Chen and Vigna (2017) model the mortality intensity of a representative unisex policyholder of the mixed portfolio as a weighted average of the males' and females' mortality intensities, and provide the following definition:

Definition 4.1 (Unisex mortality intensity). *For a mixed portfolio of male and female policyholders, whose stochastic mortality intensities are λ^m and λ^f respectively, we define the* ξ-driven unisex mortality intensity *by mixing the male and female intensities with the weight $\xi \in [0, 1]$:*

$$
\lambda_\xi^u(t) = \xi\lambda^m(t) + (1 - \xi)\lambda^f(t). \tag{31}
$$

Chen and Vigna (2017) find the survival probability of a unisex policyholder whose mortality intensity is given by (31) (for simplicity, in the following the subscript ξ in the functions α, β_1 and β_2 is omitted):

Proposition 4.2. *Conditional on t, the survival probability for the remaining time $\tau = T - t$ related to the mixed mortality intensity λ_ξ^u in (31) is given by*

$$\begin{aligned}
S^u(t,T) =& E\left[\exp\left\{-\int_t^T \lambda_\xi^u(s)ds\right\}\Big|\mathcal{F}_t\right] \\
=& E\left[\exp\left\{-\int_t^T \xi\lambda^m(s)ds - \int_t^T (1-\xi)\lambda^f(s)ds\right\}\Big|\mathcal{F}_t\right] \\
=& \exp\Big\{\alpha_u(\tau) + \beta_{1,u}(\tau)\lambda^m(t) + \beta_{2,u}(\tau)\lambda^f(t)\Big\},
\end{aligned} \tag{32}$$

with

$$\beta_{1,u}(\tau) = \frac{\xi}{\mu_m}(1-e^{\mu_m\tau}), \tag{33}$$

$$\beta_{2,u}(\tau) = \frac{1-\xi}{\mu_f}(1-e^{\mu_f\tau}), \tag{34}$$

and

$$\begin{aligned}
\alpha_u(\tau) =& \frac{\sigma_m^2\xi^2}{4\mu_m^3}\left[(e^{\mu_m\tau}-2)^2 + 2\mu_m\tau - 1\right] + \frac{\sigma_f^2(\xi-1)^2}{4\mu_f^3}\left[(e^{\mu_f\tau}-2)^2 + 2\mu_f\tau - 1\right] \\
&- \frac{\rho\sigma_m\sigma_f\xi(\xi-1)}{\mu_m^2\mu_f^2(\mu_m+\mu_f)}\Big\{\mu_m^2(1-e^{\mu_f\tau}) + \mu_f^2(1-e^{\mu_m\tau}) + \mu_m\mu_f\left[(1-e^{\mu_m\tau})(1-e^{\mu_f\tau}) + (\mu_m+\mu_f)\tau\right]\Big\}
\end{aligned} \tag{35}$$

Proof. Proof can be found in Chen and Vigna (2017). □

Finally, among the infinitely many possible weights $\xi \in [0,1]$ of the family (31), Chen and Vigna (2017) identify as the *correct* one the weight ξ^* that generates the fair unisex premium. Noting that the fair premium of a life insurance product is a function $\Pi(\cdot)$ of the mortality intensity of the insured:

$$P^i = \Pi(\lambda^i) \qquad \text{for} \quad i = u, m, f,$$

the identification of the correct weight ξ^* can be formalized by the following definition:

Definition 4.3 (Fair unisex mortality intensity)**.** *For a given portfolio of m male policyholders and n female policyholders, whose fair gender-based premiums are $P^m = \Pi(\lambda^m)$ and*

$P^f = \Pi(\lambda^f)$ *respectively, we say that* $\lambda^u_{\xi^*}$ *is a* fair unisex mortality intensity *if the corresponding unisex premium*

$$P^u = \Pi(\lambda^u_{\xi^*}) = \Pi(\xi^* \lambda^m + (1 - \xi^*)\lambda^f)$$

is fair, i.e., it satisfies the unisex fairness principle (4)*:*

$$\Pi(\xi^* \lambda^m + (1 - \xi^*)\lambda^f) = \gamma \cdot \Pi(\lambda^m) + (1 - \gamma) \cdot \Pi(\lambda^f), \tag{36}$$

where $\gamma = m/(m + n)$*, and* ξ^* *is called the* fair mortality mixing parameter*.*

The survival probabilities $S^i(0, T)$ $(i = u, m, f)$ as in equations (30) and (32) are deterministic functions and will be used in Section 5 to calculate the SCR at time 0 for different life insurance products.

5 Numerical application

5.1 Calibration of UK cohort born in 1950

In this section, we calibrate the mortality model presented in Section 4 and, in the presence of the unisex fairness principle, we investigate the magnitude of calibration error between the SCR calculated with the fair unisex mortality intensity and the weighted average of the SCRs of the two subportfolios of males and females. For the calibration of the gender-based mortality intensities, we take data from the Human Mortality Database (Last-modified: 06-May-2013),[4] and consider the males and females born in 1950 in UK, initial age 35. We adopt the same calibration procedure used in Chen and Vigna (2017), and refer the interested reader to the mentioned paper for details. Table 1 reports the calibrated values.

[4]See University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany) (2002).

Table 1: Calibrated values and errors for males and females of cohort 1950 (initial age 35).

	Female	Male
$\lambda_x(0)$	0.00075028	0.00112463
μ_x	0.08001563	0.08171875
σ_x	0.00010305	0.00011789
Calibration Error	0.00000006	0.00000007

5.2 Fair ξ^* and SCR

In this section, we focus on three products (i) pure endowment 20 years, (ii) pure endowment 30 years and (iii) life annuity, which are sold to men and women. We assume that the age at inception of the policy is 35 for the pure endowments and 65 for the annuity. Assuming an interest rate $r = 0.03$ and $\rho = 0.95$,[5] we have calculated ξ^* with portfolio gender composition γ ranging from $\gamma = 0.10$ to $\gamma = 0.90$. The fair ξ^* values are reported in Table 2.

Table 2: Fair ξ^* for pure endowment (PE) and lifetime annuity with parameters: $\rho = 0.95$, $r = 0.03$, generation born in 1950, initial age 35 and the maximal allowed age $\omega = 110$.

$\gamma = \frac{m}{m+n}$	PE, $T = 20$	PE, $T = 30$	Lifetime annuity
0.10	0.0991	0.0976	0.0836
0.25	0.2481	0.2445	0.2154
0.50	0.4974	0.4932	0.4527
0.75	0.7481	0.7449	0.7137
0.90	0.8991	0.8975	0.8823

Then, we have calculated the unisex SCR and the weighted average of SCRs of males and females for all the products with a variety of shocks ϵ and a variety of portfolio gender compositions γ. In particular, using the notation of Section 3.1.1, for

- products $z = end\text{-}20, end\text{-}30, ann$
- shocks $\epsilon = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$.
- portfolio gender-compositions $\gamma = 0, 0.1, 0.25, 0.5, 0.75, 0.9, 1$,

[5] Chen and Vigna (2017) make sensitivity analysis with respect to ρ and find that results are almost insensitive to changes in ρ.

we have calculated:

- $SCR_z^u(\epsilon)$
- $SCR_z^{weighted}(\epsilon) = \gamma SCR_z^m(\epsilon) + (1-\gamma)SCR_z^f(\epsilon)$
- their absolute difference $SCR_z^u(\epsilon) - SCR_z^{weighted}(\epsilon)$
- their relative difference $(SCR_z^u(\epsilon) - SCR_z^{weighted}(\epsilon))/SCR_z^u(\epsilon)$

The main results are displayed in Tables 3 and 4.

Table 3: SCR estimation for unisex $SCR_z^u(\epsilon)$ (left) and weighted sub portfolios $SCR_z^{weighted}(\epsilon)$ (right), for varying proportion (γ) male/female and shock (ϵ).

Pure-endowment 20-years				γ								γ			
ϵ	0%	10%	25%	50%	75%	90%	100%	ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.1	0.002	0.002	0.002	0.002	0.003	0.003	0.003
0.2	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.2	0.004	0.004	0.004	0.005	0.005	0.006	0.006
0.3	0.006	0.006	0.007	0.007	0.008	0.009	0.009	0.3	0.006	0.006	0.007	0.007	0.008	0.009	0.009
0.4	0.008	0.008	0.009	0.010	0.011	0.011	0.012	0.4	0.008	0.008	0.009	0.010	0.011	0.011	0.012
0.5	0.010	0.010	0.011	0.012	0.014	0.014	0.015	0.5	0.010	0.010	0.011	0.012	0.014	0.014	0.015
0.6	0.012	0.012	0.013	0.015	0.016	0.017	0.018	0.6	0.012	0.012	0.013	0.015	0.016	0.017	0.018
0.7	0.014	0.015	0.016	0.017	0.019	0.020	0.021	0.7	0.014	0.015	0.016	0.017	0.019	0.020	0.021
0.8	0.016	0.017	0.018	0.020	0.022	0.023	0.024	0.8	0.016	0.017	0.018	0.020	0.022	0.023	0.024
0.9	0.018	0.019	0.020	0.023	0.025	0.026	0.027	0.9	0.018	0.019	0.020	0.023	0.025	0.026	0.027
1.0	0.020	0.021	0.023	0.025	0.028	0.029	0.030	1.0	0.020	0.021	0.023	0.025	0.028	0.029	0.030
Pure-endowment 30-years					γ							γ			
ϵ	0%	10%	25%	50%	75%	90%	100%	ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.1	0.003	0.004	0.004	0.004	0.005	0.005	0.005
0.2	0.007	0.007	0.008	0.009	0.010	0.010	0.010	0.2	0.007	0.007	0.008	0.009	0.010	0.010	0.010
0.3	0.011	0.011	0.012	0.013	0.014	0.015	0.016	0.3	0.011	0.011	0.012	0.013	0.014	0.015	0.016
0.4	0.014	0.015	0.016	0.018	0.019	0.020	0.021	0.4	0.014	0.015	0.016	0.018	0.019	0.020	0.021
0.5	0.018	0.019	0.020	0.022	0.024	0.026	0.027	0.5	0.018	0.019	0.020	0.022	0.024	0.026	0.027
0.6	0.021	0.022	0.024	0.027	0.029	0.031	0.032	0.6	0.021	0.022	0.024	0.027	0.029	0.031	0.032
0.7	0.025	0.026	0.028	0.031	0.035	0.036	0.038	0.7	0.025	0.026	0.028	0.031	0.035	0.036	0.038
0.8	0.029	0.030	0.032	0.036	0.040	0.042	0.043	0.8	0.029	0.030	0.032	0.036	0.040	0.042	0.043
0.9	0.033	0.034	0.037	0.041	0.045	0.047	0.049	0.9	0.033	0.034	0.037	0.041	0.045	0.047	0.049
1.0	0.036	0.038	0.041	0.046	0.050	0.053	0.055	1.0	0.036	0.038	0.041	0.046	0.050	0.053	0.055
Annuity					γ							γ			
ϵ	0%	10%	25%	50%	75%	90%	100%	ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.507	0.512	0.519	0.524	5.522	0.516	0.510	0.1	0.507	0.507	0.508	0.509	0.509	0.510	0.510
0.2	1.068	1.080	1.094	1.108	1.105	1.093	1.079	0.2	1.068	1.069	1.071	1.073	1.076	1.078	1.079
0.3	1.695	1.715	1.740	1.754	1.763	1.744	1.721	0.3	1.695	1.698	1.701	1.708	1.714	1.718	1.721
0.4	2.402	2.432	2.047	2.512	2.516	2.492	2.456	0.4	2.402	2.407	2.415	2.429	2.443	2.451	2.456
0.5	3.208	3.252	3.309	3.375	3.391	3.364	3.316	0.5	3.208	3.219	3.235	3.262	3.289	3.305	3.316
0.6	4.139	4.201	4.284	4.387	4.428	4.403	4.343	0.6	4.139	4.160	4.190	4.241	4.292	4.323	4.343
0.7	5.230	5.317	5.435	5.594	5.682	5.673	5.609	0.7	5.230	5.268	5.325	5.420	5.514	5.571	5.609
0.8	6.528	6.648	6.818	7.063	7.236	7.272	7.224	0.8	6.528	6.597	6.702	6.876	7.050	7.155	7.224
0.9	8.095	8.263	8.507	8.887	9.211	9.350	9.377	0.9	8.095	8.223	8.416	8.736	9.056	9.248	9.377
1.0	10.021	10.257	10.611	11.202	11.793	12.147	12.384	1.0	10.021	10.257	10.611	11.202	11.793	12.147	12.384

From Tables 3 and 4 we can observe what follows:

1. As expected from Proposition 3.1 and Corollary 3.2, for all life insurance products, all ϵ and all γ, SCR^u is greater or equal than $SCR^{weighted}$. When ϵ is 0 or 1, and when γ is 0 or 1, there is no difference between SCR^u and $SCR^{weighted}$, and their gap is 0. For the pure endowment products, in all cases the differences are smaller than 0.001 so they cannot be appreciated from Table 3. However, the difference is non negligible for the annuity product.

2. For all ϵ and γ both the absolute gap $SCR^u - SCR^{weighted}$ and the relative gap $(SCR^u - SCR^{weighted})/SCR^u$ are lowest for pure endowment 20 years, slightly higher for pure endowment 30 years, highest for the annuity. In all cases, the relative gap is bigger than the absolute gap, because the denominator SCR^u is lower than one. The order of magnitude of the relative gap is negligible for the pure endowment for both durations, reaching a maximum of 0.25% for pure endowment with 30 years, with $\epsilon = 0.1$ and $\gamma = 0.5$. The order of magnitude of the relative gap is more important for the annuity case, reaching a maximum of 3.35% for $\epsilon = 0.5$ and $\gamma = 0.5$.

3. Dependence on γ of SCR^u and $SCR^{weighted}$. Interestingly, for all products, in most cases when ϵ is fixed, both SCR^u and $SCR^{weighted}$ increase when γ increases. This is apparently counterintuitive, if one thinks that $\gamma = 0$ means a portfolio consisting of females and $\gamma = 1$ means a portfolio consisting of males only. Because females are riskier than males for pure endowment and annuity products, one would expect a greater solvency capital requirement for females than for males. But this can be explained observing that by definition the SCR is the difference between *what insurers should pay* in case of distorted higher survival probabilities and *what insurers have already set aside* in the reserves with the single premium, see (10). Indeed, let us consider, for simplicity the SCR at time 0 for the pure endowment case for gender i:

$$SCR^i_{end-T}(\epsilon) = e^{-rT}\left(S^i(T)^{1-\epsilon} - S^i(T)\right) = e^{-rT}S^i(T)^{1-\epsilon} - \Pi^i$$

   where Π^i is the fair price for gender i. Obviously, the fact that $S^m(T) < S^f(T)$ produces $S^m(T)^{1-\epsilon} < S^f(T)^{1-\epsilon}$, and therefore the amount to be paid in absolute terms

Table 4: Relative difference when comparing unisex vs. weighted approach for SCR, for varying proportion (γ) male/female and shock (ϵ).

Pure-endowment 20-years				γ			
ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.00%	0.04%	0.08%	0.09%	0.06%	0.03%	0.00%
0.2	0.00%	0.04%	0.07%	0.08%	0.06%	0.03%	0.00%
0.3	0.00%	0.03%	0.06%	0.07%	0.05%	0.02%	0.00%
0.4	0.00%	0.03%	0.05%	0.06%	0.04%	0.02%	0.00%
0.5	0.00%	0.02%	0.04%	0.05%	0.04%	0.02%	0.00%
0.6	0.00%	0.02%	0.03%	0.04%	0.03%	0.01%	0.00%
0.7	0.00%	0.01%	0.03%	0.03%	0.02%	0.01%	0.00%
0.8	0.00%	0.01%	0.02%	0.02%	0.01%	0.01%	0.00%
0.9	0.00%	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%
1.0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Pure-endowment 30-years				γ			
ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.00%	0.11%	0.21%	0.25%	0.18%	0.08%	0.00%
0.2	0.00%	0.10%	0.19%	0.23%	0.16%	0.07%	0.00%
0.3	0.00%	0.08%	0.17%	0.20%	0.14%	0.06%	0.00%
0.4	0.00%	0.07%	0.14%	0.17%	0.12%	0.05%	0.00%
0.5	0.00%	0.06%	0.12%	0.14%	0.10%	0.05%	0.00%
0.6	0.00%	0.05%	0.10%	0.12%	0.08%	0.04%	0.00%
0.7	0.00%	0.04%	0.07%	0.09%	0.06%	0.03%	0.00%
0.8	0.00%	0.03%	0.05%	0.06%	0.04%	0.02%	0.00%
0.9	0.00%	0.01%	0.02%	0.03%	0.02%	0.01%	0.00%
1.0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Annuity				γ			
ϵ	0%	10%	25%	50%	75%	90%	100%
0.1	0.00%	0.98%	2.09%	2.98%	2.48%	1.31%	0.00%
0.2	0.00%	1.00%	2.15%	3.09%	2.62%	1.41%	0.00%
0.3	0.00%	1.01%	2.19%	3.21%	2.77%	1.52%	0.00%
0.4	0.00%	1.02%	2.23%	3.30%	2.91%	1.63%	0.00%
0.5	0.00%	1.02%	2.23%	3.35%	3.03%	1.74%	0.00%
0.6	0.00%	0.99%	2.18%	3.32%	3.07%	1.82%	0.00%
0.7	0.00%	0.91%	2.03%	3.12%	2.96%	1.80%	0.00%
0.8	0.00%	0.76%	1.70%	2.65%	2.57%	1.61%	0.00%
0.9	0.00%	0.48%	1.08%	1.70%	1.68%	1.08%	0.00%
1.0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

with a distorted survival probability is higher for females than for males; however, in the SCR we have to subtract the fair premium $\Pi^i = e^{-rT}S^i(T)$ that for the females is *higher* than for the males. Thus, if the fair premium for females is higher than the fair premium for males, and if the latter is higher than 0.35 (which happens to be the case for pure endowment 20 or 30 years issued to 35-years old policyholder), then we have:[6]

$$S^f(T)^{1-\epsilon} - S^f(T) < S^m(T)^{1-\epsilon} - S^m(T) \quad \Rightarrow \quad SCR^f_{end-T}(\epsilon) < SCR^m_{end-T}(\epsilon)$$

In all intermediate situations, when $\gamma \in (0,1)$ we have

$$SCR^f_{end-T}(\epsilon) < \gamma SCR^m_{end-T}(\epsilon) + (1-\gamma)SCR^m_{end-T}(\epsilon) < SCR^m_{end-T}(\epsilon)$$

that explains the increasing SCR with γ, both for the unisex case and the weighted case. This explanation holds for the pure endowment only, and in fact for the annuity in some cases SCR^u does not increase with γ.

4. Dependence on γ of absolute and relative gap of SCR^u and $SCR^{weighted}$. When ϵ is fixed, the maximum relative gap is with $\gamma = 0.5$ for all products considered. This is consistent with intuition: the most unfair situation is when the portfolio is perfectly balanced in terms of males and females. As a degenerate case, when there is only one gender in the portfolio, the unisex price is the gender-based fair price, and the EU Gender directive does not impact price and SCR. When there is a majority of one gender, the unisex price is strongly correlated with the gender-based fair price. The worst situation is when there is the same number of males and females.

5. Dependence on ϵ. When γ is fixed, both SCR^u and $SCR^{weighted}$ increase with ϵ, for all products. This is due to the fact that a greater shock to the survival probabilities produces a higher SCR. This has different impact on the absolute and relative gap between SCR^u and $SCR^{weighted}$. For the pure endowment both durations, the absolute

[6] From the mathematical point of view, this can be explained as follows. If $\alpha \in (0,1)$ the function $g(x) = x^\alpha - x$ is decreasing for $x \in (\alpha^{1/(1-\alpha)}, 1)$. We notice that when α ranges in $(0,1)$, $\alpha^{1/(1-\alpha)}$ ranges between 0.08 and 0.35. Given the initial age 35 and the duration $T = 20, 30$ years, we then have $\alpha^{1/(1-\alpha)} < S^m(T) < S^f(T)$, that implies $g(S^m(T)) > g(S^f(T))$ in all cases of pure endowment.

gap remains almost stable with ϵ, and the relative gap decreases with ϵ, the maximum being at $\epsilon = 0.1$. For the annuity both the absolute and relative gap increase with ϵ.

6 Implications of possible alternative practice on unisex SCR

6.1 Two possible ways to deal with EU Gender Directive

Insurance companies do not always seem to adopt the fairness unisex principle in pricing unisex policies. Although officially there is no clear disclosure of pricing procedures, in practice there seem to be two ways to do unisex pricing: prevailing risk and weighted risk.

6.1.1 Prevailing risk: "max-risk procedure"

A possible way to deal with the EU Gender Directive is to consider the mixed portfolio as if it were made only by high-risk policyholders, such as females for the pure endowment or the annuity, and males for the term insurance or the whole life insurance. We are going to call this procedure "max-risk-procedure": it is very conservative and implies charging always the maximum between the two gender-based prices. It is obvious that the price charged with the max-risk-procedure is higher than the unisex fair premium, and, in the considered cases of pure endowment and lifetime annuity, their difference increases with the males' portfolio share γ.

6.1.2 Weighted risk: "weight-load procedure"

Another possible way to deal with the EU Gender Directive is to take all the one-year survival rate of males and females and to mix them with weights that reflect both the portfolio composition γ and the product issued, with an additional loading η on females or males, depending on whether the product covers the risk of survival or the risk of death. We are going to call this procedure "weight-load procedure". In particular, when the product covers against the risk of survival (pure endowment, annuity), more weight is given to the females survival rate and the unisex one-year survival rate is given by

$$p_x^{wl} = (\gamma - \eta)p_x^m + (1 - (\gamma - \eta))p_x^f \tag{37}$$

for some $0 \leq \eta \leq \gamma$.[7] We stress that the weight-load-procedure *is not equivalent* to the unisex fairness principle procedure, even with $\eta = 0$. Indeed, the unisex fairness principle implies that for the pure endowment duration T the T-years unisex survival probability is a weighted average with weights γ and $1-\gamma$ of the males' and females' T-years survival probabilities (see (7)), while for the annuity the k-years unisex survival probabilities are weighted averages with weights γ and $1-\gamma$ of the males' and females' k-years survival probabilities for all $k = 1, ...\omega - x$ (see (9)). It is not difficult to see that these conditions are violated if the survival rates satisfy (37), also with $\eta = 0$. Therefore, the unisex price charged with the weight-load procedure is different from the unisex fair premium, and in the practice it turns out to be generally higher than that.

6.2 Consequences for calculation of SCR

In this section, we compare between the SCR calculated with the two alternative pricing procedures illustrated in Section 6.1 and the fair solvency capital requirement $SCR^{weighted}$ calculated in Section 5.2.

We fix an equal proportion of genders in the portfolio, $\gamma = 0.5$, and a shock on the survival probabilities $\epsilon = 0.5$, and analyse the three products considered in Section 5, namely the pure endowment 20 years, the pure endowment 30 years and the annuity. For the weight-load procedure we set $\eta = 0.1, 0.3, 0.5$. Therefore, due to (37), the results for the max-risk procedure coincide with those of the weight-load procedure in the case $\eta = 0.5$.

Table 5 reports the SCR with the weight-load procedure, SCR^{wl}, and that with the max-risk procedure, SCR^{ml}. It also reports their absolute gap with respect to $SCR^{weighted}$ and their relative gap, for instance, $(SCR^{ml} - SCR^{weighted})/SCR^{ml}$.

We observe that the SCR for the max-risk procedure is almost always lower than the $SCR^{weighted}$. The explanation is equal to that given in comment 3 from Section 5.2 to

[7] When the product covers against the risk of death (term insurance, whole life), more weight is given to the males survival rate and the unisex 1-year survival rate is given by $p_x^{wl} = (\gamma + \eta)p_x^m + (1 - (\gamma + \eta))p_x^f$.

Table 5: $SCR_z^{wl}(\epsilon)$, difference and relative difference to $SCR^{weighted}$ for $\epsilon = 0.5$ and $\gamma = 0.5$ depending of η and type of product.

Pure-endowment 20-years ($SCR^{weighted} = 0.012$)			
		η	
	0.1	0.3	0.5
SCR^{wl}	0.012	0.011	0.010
$SCR^{wl} - SCR^{weighted}$	0.000	-0.001	-0.003
Relative diff.	-4.08%	-13.74%	-25.46%
Pure-endowment 30-years ($SCR^{weighted} = 0.022$)			
		η	
	0.1	0.3	0.5
$SCR_z^{wl}(\epsilon)$	0.021	0.020	0.018
$SCR^{wl} - SCR^{weighted}$	-0.001	-0.003	-0.004
Relative diff.	-3.72%	-13.17%	-24.71%
Annuity ($SCR^{weighted} = 3.268$)			
		η	
	0.1	0.3	0.5
SCR^{wl}	3.285	3.259	3.218
$SCR^{wl} - SCR^{weighted}$	0.017	-0.009	-0.050
Relative diff.	0.52%	-0.28%	-1.55%

explain why the solvency capital requirement for females is lower than that for males. If the insurance company behaves as if there are only females in the mixed portfolio, the price charged to the males of the portfolio is higher than what should be, so less money is needed for the solvency capital requirement, because excessive money has been set aside by the insurance company. This is certainly a safe procedure for the solvency of the company, but the price of this cautious procedure is paid only by the customers.

The weight-load procedure gives an intermediate situation between the max-risk procedure and the weighted procedure. This is due to the fact that the survival probabilities used are not equal to the females' ones as in the max-risk procedure, but they are closer to the females' ones than they should be with just volume-related weights, due to the extra-loading η.

We notice a remarkable difference between the pure endowment and the annuity. For both pure endowments, the difference $SCR^{wl} - SCR^{weighted}$ is always lower than 0 and the relative difference becomes as high as -25% for higher values of η. For the annuity, $SCR^{wl} - SCR^{weighted}$ is positive with low values of η and becomes negative when η increases, but the relative difference is low. This means that there is an intermediate value of η for which the weight-load procedure equals the weighted pre-Gender Directive approach.

7 Discussion and conclusion

Starting from the evidence that the EU Gender Directive obliges issuers of life insurance products to charge the same premium to policyholders of different genders, we address the issue of calculating solvency capital requirements for pure endowments and annuities issued to mixed portfolios. We strongly support the use of the unisex fairness principle in the calculation of the unisex single premium. We analyze the solvency capital requirement in the two possible situations: (i) using the unisex fairness principle; (ii) not using it.

(i) Assuming that the insurer charges the unisex fair premium, we notice that he can calculate the solvency capital requirement in two different ways:

1. first, with the unisex fair survival probability, as if the mixed portfolio was made by $m + n$ homogeneous mixed policyholders with fair unisex survival probabilities; with

this procedure we calculate the quantities $SCRu$;

2. second, mixing the SCR of males and females with volume-related weights, as if the mixed portfolio was made by two subportfolios of m males and n females (in the same way it was done before the EU Gender Directive, see Remark 2); with this procedure we calculate the quantities $SCRweighted$.

We notice that the fair premium charged to the policyholders is the same in the two cases. In Section 3.2 we show that the SCR at issuing time calculated with the fair unisex survival probabilities is higher than or equal to that calculated for the two subportfolios. In other words, if insurers treat the mixed portfolio as a portfolio of homogeneous unisex policyholders with a fair unisex survival probability, they set aside for solvency requirements more money than they would have done pre-EU Gender Directive. We find that the relative gap between SCR calculated in the two different ways is negligible for pure endowments, and it is at most of the order of 3% for annuities. The unisex solvency capital requirement $SCRu$ can be considered a good approximation of the fair solvency capital requirement $SCR^{weighted}$ for pure endowments, and a cautious solvency capital requirement for annuities.

(ii) Assuming that the insurer does not use the unisex fairness principle to calculate the unisex tariff, we find that it is common practice to use survival probabilities closer or equal to those of the females for pure endowments and annuities. We calculate the SCR according to the distorted survival probabilities adopted. We get different results for pure endowments and annuities. For pure endowments, in all cases, the distorted SCR is lower than the $SCR^{weighted}$ that would have been calculated before the EU Gender Directive. The relative difference can be as high as -25%. The reason of this apparently counterintuitive inequality lies in the fact that with distorted survival probabilities the premium charged by insurers is much higher than the unisex fair price. Therefore, more money than needed is set aside in the reserves at the policy inception, and less money than needed is required for the solvency capital requirement. Both a higher unisex tariff and a lower solvency capital requirement seem good news for the insurer. However, the price to be paid for this advantage is paid entirely by the customer, who pays more than needed for the policy. For the annuity the SCR calculated with distorted survival probabilities is lower than the $SCR^{weighted}$ if enough extra-loading is assigned to the females' survival probabilities, while it becomes higher than that for low enough extra-loading on females' survival probabilities. A correct selection of the extra-loading becomes then relevant to insurers.

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