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Geographical diversification and longevity risk mitigation in annuity portfolios*

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Abstract

This paper provides a method for assessing the regulatory and systemic risk relief deriving from a foreign expansion by a life-insurance company. We build a parsimonious continuous-time model for longevity risk, that captures the dependence across different ages in domestic versus foreign populations. We provide two measures of the diversification effects of expanding an annuity portfolio toward a foreign population. The reduction in the risk margin, computed à la Solvency II, provides a measure of the benefit in the tail risk. The Diversification Index provides an assessment of the average diversification benefit of combining different populations in one portfolio. We calibrate the model to portray the case of a UK annuity portfolio expanding internationally towards Italian policyholders. Our application shows that the longevity risk diversification benefits of an international expansion are sizable, in particular when interest rates are low.

Keywords: geographical diversification; life insurance; risk management; multi-population mortality; longevity risk modeling, Solvency II capital requirements.

JEL classification: F23, F65, G15, G22.

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1 Introduction

In the last twenty years, insurance companies have been expanding internationally, via subsidiaries operating in different countries or via cross-border mergers and acquisitions. From the middle of the Nineties, in Europe, due to the creation of a common regulatory framework, and in the US as well, cross-border expansions and M & A operations in the insurance sector trended upwards (see Ma and Pope, 2003). Cummins et al. (1999) argues that geographical diversification was a primary determinant of mergers and acquisitions in the US insurance industry in the Nineties, because geographically diversified firms were more likely to be the target of acquisitions. The largest insurers and reinsurers are indeed multinational companies, with subsidiaries and branches located in several countries. Schoenmaker and Sass (2016) reports that in Europe, as of 2012, the share of cross-border activity in the insurance sector is higher than in the banking one, and that the degree of internationalization of the 25 largest European insurers increased over the period 2000-2012, despite the financial crisis. The OECD[1], as of 2014, reports that on average around 30% of the gross premiums of life insurance companies in 20 European countries, United States and Japan comes from foreign-based controlled undertakings. Reinsurance companies have traditionally been more geographically diversified than insurance companies (Cummins and Xie, 2008), because their portfolios are more easily disconnected from the geographical localization of their branches.

The question that naturally arises is whether the international diversification trend leads to gains for insurers and reinsurers. The empirical literature on the role of internationalization on profitability and efficiency in the insurance industry is still scant, especially as concerns life insurers. Outreville (2012) and Elango et al. (2008) find that, in the reinsurance industry and in the US property-liability one, respectively, the level of international diversification positively affects performance, and that the relation is non-linear. On the contrary, focusing on the life insurance industry, Biener et al. (2015) shows that the costs of coordination and organization of complex international structures may offset the potential benefits from internationalization, resulting in a negative relationship between performance and internationalization. However, Biener et al. (2016), considering the Swiss insurers and reinsurers, finds that life insurers with higher levels of internationalization appear more efficient than their peers.

An interesting, but, up to our knowledge, so far overlooked effect of internationalization for life insurers and reinsurers is longevity risk pooling. Longevity risk is the risk of experiencing losses, due to unexpected fluctuations in mortality rates. It affects annuity providers in particular when, as occurred in the last decades, policyholders' longevity exceeds the expectations. Why is international diversification of life insurance portfolios beneficial in terms of risk? Because, even if – in expectation – longevity has been steadily increasing on a worldwide scale, idiosyncratic longevity risks of different populations are different and may be non-perfectly correlated across countries. As a result, pooling portfolios of policies written on the lives of different populations allows to diversify longevity risk. The present paper aims at filling a gap in the literature by providing two measures of the diversification effects deriving from

[1]See the OECD Statistics website (https://stats.oecd.org), in particular the Insurance Indicator *Market share of foreign companies in the domestic market* and *Market share of branches/agencies of foreign undertakings in the domestic market.*

longevity risk pooling across populations. As an example, we consider an annuity provider, who can decide to expand her portfolio by selling policies to members of a population different from the one to which she is currently exposed. Our goal is to quantify the diversification benefit deriving from such an expansion, relative to an expansion of the portfolio not involving internationalization.

To this end, we first introduce a novel parsimonious model for the joint mortality dynamics of policyholders in different countries, which extends the model presented in De Rosa et al. (2017) to a multi-population setting. We set ourselves in the continuous-time framework, which has gained increasing popularity, alongside the more traditional discrete-time one, because of its analytical tractability. The model we propose is a stochastic, continuous-time multi-population extension of the deterministic Gompertz mortality law, a benchmark in the classical modelling of mortality arrival rates. It allows to compute survival probabilities and hedge ratios in closed form, differently from models à la Lee-Carter. At the same time, it allows a rich description of the mortality dynamics of multiple populations, and generations within them. In the continuous-time framework, some models cope with the longevity risk of two cohorts or populations (Dahl et al., 2008), or several cohorts within one population (Blackburn and Sherris, 2013; Jevtić et al., 2013). Up to our knowledge, only Sherris et al. (2018) has attempted to combine the description of the mortality intensities of multiple populations and generations together in a continuous-time setting. Their model is driven by three independent Brownian motions (risk factors) and entails Gaussian intensities. We assume as many dependent risk factors as domestic generations and an idiosyncratic source that drives the mortality intensity of the foreign population. Thus, we are able to capture the correlation structure of different generations within and across populations accurately, while preserving a good level of parsimony. Our intensities follow square-root processes, and therefore can not become negative.

Building on our model, we provide two measures for the effects of geographical diversification on the longevity risk of the portfolio. The first one is the change in the risk margin à la Solvency II. To compute it, we define the value of the portfolio of annuities as the sum of the actuarial value of the policies (best estimate) and of a risk margin, i.e. an amount that the insurer has to set aside to cover up for the unhedgeable risks, defined as the value at risk (VaR) of the unexpected loss in the portfolio value at a certain confidence level. If the risk margin decreases, after an international expansion, its change is a dollar-based measure of the benefits of diversification, since it measures the capital requirement relief for the insurer. The second measure, that we call "Diversification Index" exploits instead the features of our model, by measuring the diversification obtained via the international expansion as a weighted average of the correlations across populations. The two measures, analyzed together, provide two complementary views to assess the risk effects of the international expansion. While the risk margin reduction offers a measure of the mitigation of the "tail" risk, because it represents the loss in a worst-case scenario occurring with a low probability, the diversification index provides a synthetic indicator of how dissimilar from the initial portfolio the portfolio obtained after the expansion is.

In a numerical application, that portrays the situation of a UK annuity provider that can expand to Italy, we first assess that the model is able to fit well the observed mortality rates of individuals aged 65-75 in the two populations, while capturing, using the Gaussian mapping technique, the imperfect correlations observed across ages

and populations. Based on our model estimates, we then compute our international diversification measures for different portfolio expansions. We show that the risk margin can be as high as 3% as a proportion of the actuarial value, in the case of a foreign expansion, targeted to those cohorts in the Italian population who have low covariance with the initial annuity portfolio. We also highlight that longevity risk mitigation effects are more sizable when the interest rate – a flat term structure, for simplicity – is lower. The expansion can be performed, at a practical level, by starting foreign branches, acquiring foreign undertakings or, as shown in an appendix, through the use of longevity derivatives.

The paper unfolds as follows. Section 2 describes the set up and the problem of the insurer. Section 3 presents the longevity risk model. Section 4 defines the two international diversification measures we propose. Section 5 provides a calibrated application, describing our model calibration procedure, computes the diversification measures of various portfolio choices and provides sensitivity analysis to relevant parameters. Appendix A details the Gaussian mapping technique used to estimate the correlation structure, while Appendix B compares the two ways of achieve the international diversification: a physical one, in which a foreign affiliate is opened, and a synthetic one, through a longevity swap.

2 Set up

We consider a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$, endowed with the usual properties, where $\mathcal{F}$ is the filtration containing the information regarding all the relevant variables and $\mathbb{P}$ is the historical probability measure. In this probability space, the mortality intensities of individuals are described as stochastic processes, and longevity risk, i.e. the risk of unexpected fluctuations in the likelihood of deaths of individuals, arises. In what follows, we will consider longevity risk as the only source of risk in our setup.

We consider an Annuity Provider, or Life-Insurer, based in a certain country (that we call *Domestic*), having a portfolio of deferred annuities written on different cohorts belonging to the Domestic Population. Let $\mathcal{X} = \{x_1, \ldots, x_m\}$ be the set of annuitants' ages at time zero, and let n_i, for $i = 1, \ldots, m$, be the number of annuities sold to people aged x_i. When an annuity is sold at time zero, the annuitant pays an initial premium. We compute the actuarial value of the liabilities net of that premium. After signing the contract, the annuitant will receive a series of fixed annual instalments R, starting from the year-end of his 65-th birthday if $x_i < 65$, or immediately if $x_i \geq 65$, until his death, that may happen at most when he reaches a final age ω, at which he will die with probability 1.

2.1 Portfolio value

In Europe, the life-insurance business falls under the Solvency II regulation, that requires insurers to value their liabilities at market value and set aside VaR-based risk margins with respect to the sources of risk that affect these valuations. These risk margins are amounts prudentially set aside by the insurer, meant as financial covers for the unhedgeable risk that the insurer bears. We consider, then, that the overall value $\Pi^0(t)$ of the liability portfolio of a life insurer at time t is the sum of two components: the Actuarial Value $AV_{\Pi^0}(t)$, which is the sum of the actuarial values of each individual

contract $N_i(t)$ and represents a best estimate of the liabilities of the insurer, and the Risk Margin $RM_{\Pi^0}(t)$ of the portfolio itself. In formulas, we have that:

$$\Pi^0(t) = AV_{\Pi^0}(t) + RM_{\Pi^0}(t) = \sum_{i=1}^{m} n_i N_i(t) + RM_{\Pi^0}(t). \tag{1}$$

We now detail further the assumptions we make to compute the two components. The actuarial value of the contract is its fair premium. To compute it, we first define the number of years before the individual reaches age 65 as $\tau = max(65 - x_i, 0)$. If $\tau > 0$, then the contract is a deferred annuity, while if $\tau = 0$ then the contract is an immediate annuity. Because we consider no risk source other then longevity risk, the actuarial value of an annuity can be expressed as

$$N_i(t) = D(t, t+\tau)S_i(t, t+\tau)\left[R \sum_{u=1}^{\omega-t-\tau} D(t+\tau, t+\tau+u)S_i(t+\tau, t+\tau+u)\right], \tag{2}$$

where $D(t, s)$, $s \geq t$ denotes the deterministic financial discount factor, $D(t, s) = e^{-r(s-t)}, r \in \mathbb{R}$ and $S_i(t, \cdot)$ is the time-t survival probability curve of the individual aged x_i at time t.

We define the portfolio risk margin $RM_{\Pi^0}(t)$ as the discounted Value-at-Risk, at a given confidence level $\alpha \in (0, 1)$, of the unexpected portfolio's future actuarial value at a given time horizon T:

$$RM_{\Pi^0}(t) = D(t, t+T) \cdot VaR_\alpha\big(AV_{\Pi^0}(t+T) - \mathbb{E}_t[AV_{\Pi^0}(t+T)]\big), \tag{3}$$

$$= D(t, t+T) \cdot inf\{l \in \mathbb{R}^+ : \mathbb{P}(AV_{\Pi^0}(t+T) - \mathbb{E}_t[AV_{\Pi^0}(t+T)] > l) < 1 - \alpha\}, \tag{4}$$

where $\mathbb{P}(\cdot)$ denotes the probability of the event that the future actuarial value exceeds its time-t expected value by more than l.

2.2 Portfolio Expansion

In our setup, we consider the case in which the Insurer wants to expand the size of her annuity portfolio and can choose between two alternative strategies. The first one is just sell new contracts to her own Domestic population. In this case, we denote with n_i' the number of new contracts sold to individuals aged x_i, with Π^D the portfolio composed of just these new annuities, and with Π^1 the portfolio after the expansion, composed of the old and the new contracts. The actuarial value of the new portfolio is simply

$$AV_{\Pi^D}(t) = \sum_{i=1}^{m} n_i' N_i(t), \tag{5}$$

and

$$AV_{\Pi^1}(t) = AV_{\Pi^0}(t) + AV_{\Pi^D}(t). \tag{6}$$

The value of the total portfolio Π^1 is the sum of the actuarial value of the old portfolio, the actuarial value of the new portfolio and the risk margin of the total portfolio:

$$\Pi^1(t) = AV_{\Pi^1}(t) + RM_{\Pi^1}(t) = AV_{\Pi^0}(t) + AV_{\Pi^D}(t) + RM_{\Pi^1}(t). \tag{7}$$

The second possible strategy is to acquire a new portfolio of annuities Π^F, written on a foreign population. We assume that, for each age x_i, the number of annuities written on people aged x_i in the foreign population is n'_F. The actuarial value of portfolio Π^F is

$$AV_{\Pi^F}(t) = \sum_{i=1}^{m} n'_F N_i^F(t). \tag{8}$$

We denote with Π^2 the portfolio obtained after the expansion towards the foreign country. The actuarial value of such portfolio is

$$AV_{\Pi^2}(t) = AV_{\Pi^0}(t) + AV_{\Pi^F}(t) \tag{9}$$

and its overall value is

$$\Pi^2(t) = AV_{\Pi^2}(t) + RM_{\Pi^2}(t) = AV_{\Pi^0}(t) + AV_{\Pi^F}(t) + RM_{\Pi^2}(t). \tag{10}$$

Notice that the original portfolio and the one obtained after the expansion do not have the same actuarial value, neither when the expansion is domestic nor foreign. The risk margin of the two portfolios is different as well.

Our aim is to measure the effects of the two alternative strategies on the longevity risk profile of the insurer. To this end, in the next sections we introduce a novel longevity risk model and two measures of the diversification effects.

3 Longevity Risk Modeling

We now turn to the description of the source of uncertainty that affects the value of the Insurer's portfolio: the risk of longevity, i.e. the risk that her policyholders live longer than expected. We set ourselves in the well-established continuous-time stochastic mortality setting initiated by Milevsky and Promislow (2001) that models the death of individuals as a Cox process. The time to death of an individual belonging to cohort x_i is the first jump time of a Poisson process with stochastic intensity. This intensity is indeed the force of mortality of the individual. When we consider different populations, and different cohorts within each population, it is reasonable to assume that their mortality intensities processes will be different, even though they may be (even closely) related one another. In this section, we propose a novel, parsimonious model to describe the evolution of the mortality intensities of several cohorts in two different populations. The parsimony of our approach stems from making the intensity of one population (the "foreign" one) a linear combination of the other, benchmark, population's intensity (the "domestic" one) and of an idiosyncratic risk factor. This makes the whole correlation structure across populations dependent on the weight of the linear combination.

To preserve tractability, allowing for closed form expressions for the survival probabilities, but at the same time ensuring non-negativity of the intensities, we adopt stochastic processes belonging to the affine family, of the Cox et al. (1985) type. These models have been used in single-country longevity modeling by Dahl et al. (2008) and Luciano et al. (2012).

Let us consider two populations, each containing m different cohorts. The first population is called the *Domestic population* and the second one is called the *Foreign*

population. A given cohort i, with $i = 1, \ldots, m$, belonging to one of the two populations, is identified by the (common) initial age x_i at time zero. The set $\mathcal{X}$ of initial ages is common to the two populations.

Domestic Population

The mortality intensity of each cohort x_i, for $i = 1, \ldots, m$, belonging to the Domestic population is denoted with λ_i^d, and follows a non-mean reverting CIR process:

$$d\lambda_i^d(t) = (a_i + b_i\lambda_i^d(t))dt + \sigma_i\sqrt{\lambda_i^d(t)}dW_i(t), \tag{11}$$

where $a_i, b_i, \sigma_i,\ \lambda_i^d(0) \in \mathbb{R}^{++}$ are strictly positive real constants and the W_i's are instantaneously correlated standard Brownian Motions: $dW_i(t)dW_j(t) = \rho_{ij}dt$ with $i, j \in \{1, \ldots, m\}$. As a consequence, the mortality intensities of two different cohorts belonging to the Domestic Population are instantaneously correlated, as soon as $\rho_{i,j} \neq 0$.

Foreign Population

The mortality intensity of cohort x_i belonging to the Foreign population is denoted with λ_i^f, and is given by the convex combination of the mortality intensity of the corresponding cohort belonging to the Domestic population λ_i^d and an idiosyncratic component λ', which affects the Foreign population only and that depends on the initial age x_i in a deterministic way, i.e.

$$\lambda_i^f(t) = \delta_i\lambda_i^d(t) + (1 - \delta_i)\lambda'(t; x_i), \tag{12}$$

where

$$d\lambda'(t; x_i) = (a' + b'd\lambda'(t; x_i))dt + \sigma'\sqrt{d\lambda'(t; x_i)}dW'(t), \tag{13}$$

with $\delta_i \in [0, 1]$.[2] The functions a', b' and σ' are positive constants, while W' is a standard Brownian Motion, that is assumed to be independent of W_i for each $i = 1, \ldots, N$.

Intuitively, the idiosyncratic risk source W' is population-specific, in the sense that it is common to all the cohorts of the Foreign population. Nonetheless, each foreign cohort x_i has a specific sensitivity to the idiosyncratic component $\lambda'(t; x_i)$, that is given by the parameter δ_i, which is, instead, cohort-specific. The mortality intensities of two different cohorts of the Foreign population are correlated, and the correlation between λ_i^f and λ_j^f depends both on the correlation between λ_i^d and λ_j^d and on the weights δ_i and δ_j. Moreover, thanks to the presence of the idiosyncratic component λ' affecting the Foreign population, our model allows to account for the non-perfect correlation between cohorts across the two populations. The correlation structure among the different cohorts of the two populations will be derived in Section 5.2.

From (11) we have that the survival probability of generation x_i in the Domestic population is given by:

$$S_i^d(t, T) = A_i^d(t, T)e^{-B_i^d(t,T)\lambda_i^d(t)}, \tag{14}$$

[2] In principle, linear affine coefficients a', b' and σ' could be chosen.

where

$$A_i^d(t,T) = \left(\frac{2\gamma_i e^{\frac{1}{2}(\gamma_i - b_i)(T-t)}}{(\gamma_i - b_i)\big(e^{\gamma_i(T-t)} - 1\big) + 2\gamma_i} \right)^{\frac{2a_i}{\sigma_i^2}}, \tag{15}$$

$$B_i^d(t,T) = \frac{2\big(e^{\gamma_i(T-t)} - 1\big)}{(\gamma_i - b_i)\big(e^{\gamma_i(T-t)} - 1\big) + 2\gamma_i}, \tag{16}$$

with $\gamma_i = \sqrt{b_i^2 + 2\sigma_i^2}$. Similarly, for the Foreign population we have:

$$S_i^f(t,T) = A_i^d(t,T)A'(t,T)e^{-B_i^d(t,T)\delta_i\lambda_i^d(t) - B'(t,T)(1-\delta_i)\lambda_i'(t)}, \tag{17}$$

where

$$A'(t,T) = \left(\frac{2\gamma' e^{\frac{1}{2}(\gamma' - b')(T-t)}}{(\gamma' - b')\big(e^{\gamma'(T-t)} - 1\big) + 2\gamma'} \right)^{\frac{2a'}{(\sigma')^2}}, \tag{18}$$

$$B'(t,T) = \frac{2\big(e^{\gamma'(T-t)} - 1\big)}{(\gamma' - b')\big(e^{\gamma'(T-t)} - 1\big) + 2\gamma'}, \tag{19}$$

with $\gamma' = \sqrt{(b')^2 + 2(\sigma')^2}$.

The time-t survival probability curves of the two populations are thus both available in closed form, and depend on the parameters of the model.

4 Measuring the longevity risk effects of geographical diversification

4.1 Percentage Risk Margin change

To be able to compare the effects of the two expansions, we consider a normalized quantity, i.e. the ratio of the risk margin and the actuarial value of a portfolio Π, which we call percentage risk margin:

$$\%RM_\Pi = \frac{RM_\Pi(t)}{AV_\Pi(t)}. \tag{20}$$

A lower percentage risk margin is due to a lower loss in the worst-case scenario, relative to portfolio value. Hence, it can be beneficial for the company in two respects. First, it indicates a mitigation in the risk connected to adverse scenarios. In this sense, the risk margin can be considered as a measure of the systemic risk that the company may generate, by triggering losses that will hit its creditors. Second, it represents a capital requirement reduction, which frees up resources. Because the risk margin can be interpreted as both a capital requirement and a measure of the loss the company can generate – at a given level of confidence – among its creditors, it is then conceivable that minimization of the percentage risk margin aligns the interests of both the insurance company and its regulators. In what follows we take the point of view of the insurer, taking for granted the alignment of her interest with the ones of the regulator.

4.2 Similarity/Diversification index

Building up on the longevity model described in the previous section, we propose another synthetic measure to describe the similarity/dissimilarity between the annuity portfolios written on two populations, that we define as *Similarity* and *Diversification* index. Let n_i^d be the number of annuities written on cohort x_i belonging to the domestic population, n_i^f the number of annuities written on cohort x_i belonging to the foreign population, $n_i = n_i^d + n_i^f$ and m the number of generations in the initial, domestic portfolio. Then the Diversification Index (DI) is equal to:

$$DI = \frac{1}{m} \sum_{i=1}^{m} \frac{n_i^f (1 - \delta_i)}{n_i}, \tag{21}$$

and the Similarity Index (SI)[3] is:

$$SI = 1 - DI. \tag{22}$$

The Diversification Index represents a weighted average of the dissimilarities between the same cohorts in different populations[4], present in both the initial portfolio and in the portfolio after the expansion. Dissimilarities are captured by the complement to 1 of δ_i, the generation-specific parameter that captures the degree of correlation between the same generation of the different populations. The weights, n_i/n, are given, for each cohort in the initial portfolio, by the number of annuities in the foreign population (after the expansion) relative to the total number of annuities written on that cohort in both populations. We average the weighted dissimilarities across all the m cohorts of the domestic population initially present in the annuity portfolio.

Our proposed indicator has some properties, that we highlight. First, $0 \leq DI \leq 1$. If $\delta_i = 1$ for every i, i.e. the two portfolios are written on perfectly correlated populations, then, obviously, $SI = 1$ and $DI = 0$. On the other hand, if $\delta_i = 0$, for every i, which means that the intensities of the foreign population are independent of the risk factor of the domestic, the DI does not go to 1 independently of the portfolio composition. If $n_i^f \to \infty$ and n_i^d remains constant, then $SI \to 0$, $DI \to 1$. This happens because the longevity risk of the foreign population is completely idiosyncratic and therefore diversification is reaped only enlarging the foreign portfolio as much as possible. This shows that the Diversification Index appropriately reflects both the properties of the intensity correlation structure and the portfolio mix chosen by the underwriter.

5 Application

In this section, we calibrate our proposed model and try to quantify the diversification gains deriving from an international expansion towards Italy of an initially UK-based

[3]Since we are only averaging over the generations belonging to the domestic portfolio, the Similarity Index defined in equation (22) should be interpreted as a synthetic measure of the similarity with respect to the domestic population. If instead m is defined as the number of generations in the foreign portfolio, the resulting measure should be interpreted as the similarity with respect to the foreign portfolio.

[4]Indeed, by construction, the Diversification Index does not take into account the diversification benefit across different generations in the two populations.

annuity portfolio. The situation we consider is that of a UK annuity provider who has the option of expanding her business either in her home country or abroad, selling additional policies to Italian policyholders. In practice, this expansion can be performed by creating an Italian branch or acquiring an Italian undertaking. As an alternative, the geographical diversification can be obtained through the use of longevity derivatives (see Blake et al., 2006 for instance), which allow the insurer to gain some exposure to the mortality development of a different population. We explore this case in Appendix A.

5.1 Mortality intensities estimation

To calibrate our model, we proceed in two steps. First, we calibrate the parameters of the two intensity processes, of the domestic and of the foreign population respectively. Then, in a second step, we calibrate the correlation parameters ρ_{ij}. We calibrate the parameters of the mortality model to the generations of UK and Italian males whose age, at 31/12/2012, is between 64 and 74, that is, the cohorts born between 1937 and 1947. We consider thus 11 different cohorts present in the initial portfolio: $x_i = 65, \ldots, 75$. We use the 1-year×1-year cohort death rates data provided by the Human Mortality Database and recover, using the 20 observations from 1993 and 2012[5] the observed conditional survival probabilities, for each cohort, for the individuals alive in 1993. The estimation of the parameters is performed minimizing the Rooted Mean Squared Error (RMSE) between the observed and the model-implied survival probabilities. Tables 1 and 2 report the calibrated parameters for the two populations, while Figures 1 and 2 report the actual and fitted survival probabilities and the calibration errors, respectively. The model, although parsimonious, is able to capture well the survival probability curves of the two populations, for all the cohorts considered.

Table 1. Domestic Population (UK) calibration results.

Age	a	b	σ	λ_0	RMSE
65	$2.7878 \cdot 10^{-5}$	0.0723	0.0075	0.0116	0.00035
66	$6.5423 \cdot 10^{-5}$	0.0652	0.0059	0.0124	0.00028
67	$1.8424 \cdot 10^{-5}$	0.0740	0.0080	0.0135	0.00035
68	$5.3144 \cdot 10^{-5}$	0.0685	0.0084	0.0160	0.00043
69	$1.2500 \cdot 10^{-4}$	0.0589	0.0091	0.0164	0.00039
70	$8.4734 \cdot 10^{-5}$	0.0646	0.0108	0.0189	0.00056
71	$7.1323 \cdot 10^{-5}$	0.0667	0.0106	0.0212	0.00038
72	$4.1759 \cdot 10^{-5}$	0.0688	0.0073	0.0239	0.00040
73	$2.2984 \cdot 10^{-5}$	0.0689	0.0066	0.0262	0.00063
74	$9.6036 \cdot 10^{-5}$	0.0663	0.0131	0.0282	0.00040
75	$3.3898 \cdot 10^{-5}$	0.0684	0.0077	0.0316	0.00049

[5]These correspond to the last 20 observations available to date for the Italian males. However, since the UK dataset is updated until 31/12/2013, we have excluded the last available observation for the UK cohorts.

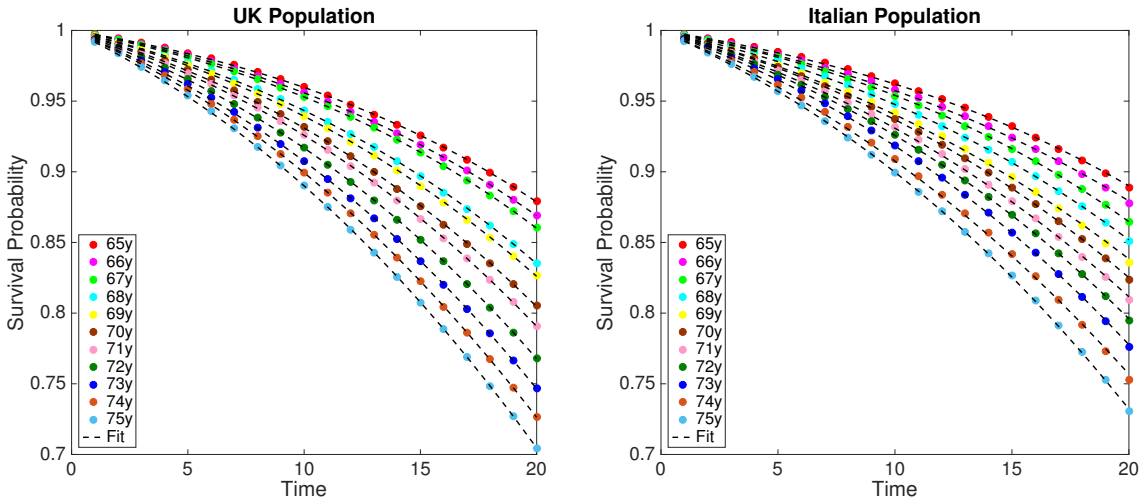

Figure 1. Observed and theoretical survival probabilities. The left panel shows the observed vs. fitted survival probabilities for the Foreign population, while the right reports the figures for the Domestic population.

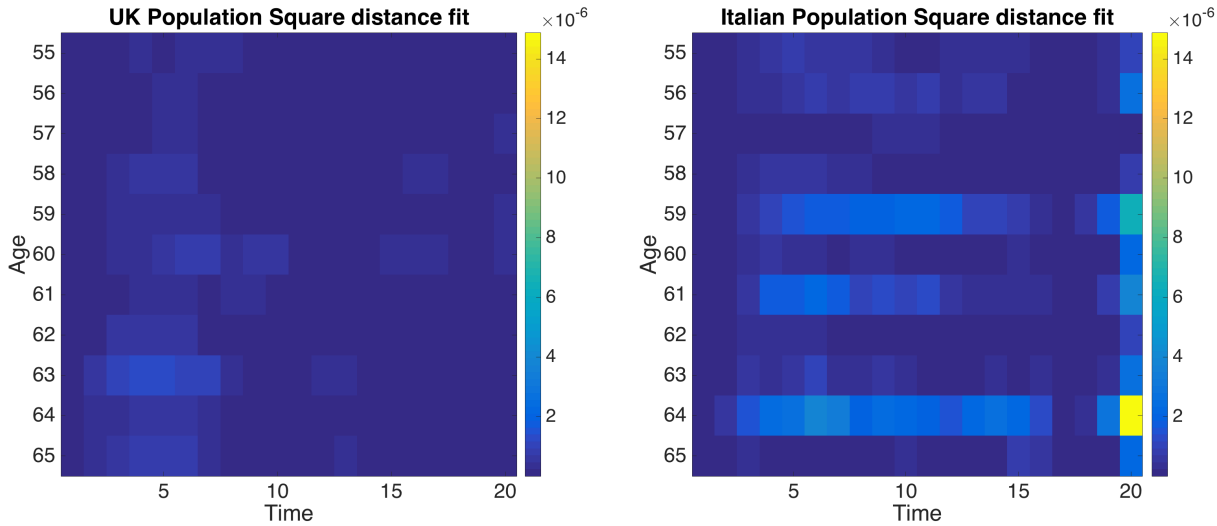

Figure 2. Calibration errors.

Table 2. Foreign Population (IT) calibration results.

Age	a'	b'	σ'	δ	RMSE	λ'_0
65				0.8071	0.00060	0.0075
66				0.8036	0.00073	0.0127
67				0.9348	0.00031	0.0190
68				0.8074	0.00045	0.0115
69				0.7893	0.00120	0.0163
70	$5.8458 \cdot 10^{-5}$	$4.2841 \cdot 10^{-11}$	$1.1464 \cdot 10^{-7}$	0.8119	0.00053	0.0141
71				0.7903	0.00099	0.0124
72				0.8006	0.00039	0.0092
73				0.8106	0.00064	0.0115
74				0.7622	0.00160	0.0209
75				0.8470	0.00053	0.0182

5.2 Correlation matrix estimation

After having estimated the cohort-specific parameters of the two populations, we turn to the estimation of their correlation structure. Having chosen a non-Gaussian process for the mortality intensities of the cohorts, we are not able to derive a formula for their correlations in closed form. However, to estimate correlations, we can apply the Gaussian Mapping technique, which has been used extensively in the pricing of Credit Default Swaps (see Brigo and Mercurio, 2001). Such technique allows to obtain a closed-form approximation of the correlations between the intensities of the different cohorts, in turn permitting the direct estimate of the correlation parameters ρ_{ij}. Technically, it consists in mapping a CIR process into a Vasicek process that is as close as possible to the original one, i.e. returning the same survival probability. Since we are able to compute analytically the correlations between each λ_i^d and λ_j^d, with $i, j = 1, \ldots, N$ in the mapped Vasicek process, we can then retrieve our desired parameters in closed-form.
Starting from the CIR process (11) describing the mortality intensity of cohort x_i belonging to the domestic population, we consider a Vasicek process driven by the same Brownian Motion $W_i(t)$, having the same drift and the same initial point:

$$d\lambda_i^V(t) = (a_i + b_i\lambda_i^V(t))dt + \sigma_i^V dW_i(t), \quad \lambda_i^V(0) = \lambda_i^d(0). \tag{23}$$

The instantaneous volatility coefficient σ_i^V of (23) is then determined by making the two processes as close as possible. Here, having fixed a maturity T, by close we mean that the two processes return the same survival probability:

$$S_i^d(t,T) = S_i^V(t,T;\sigma_i^V). \tag{24}$$

Then, we approximate the correlation between $\lambda_i^d(t)$ and $\lambda_j^d(t)$ by the correlation between $\lambda_i^V(t)$ and $\lambda_j^V(t)$:

$$Corr_0(\lambda_i^d(t), \lambda_j^d(t)) \approx Corr_0(\lambda_i^V(t), \lambda_j^V(t)), \tag{25}$$

since this last correlation can be computed analytically. Each pair-wise correlation is a function of the parameters b_i and b_j of the mapped Vasicek process and of ρ_{ij}:

$$
\begin{aligned}
Corr_0(\lambda_i^V(t), \lambda_j^V(t)) &= \frac{Cov_0(\lambda_i^V(t), \lambda_j^V(t))}{\sqrt{Var_0\left[\lambda_i^V(t)\right] Var_0\left[\lambda_j^V(t)\right]}} \\
&= \frac{2\rho_{ij}}{b_i + b_j} \cdot \frac{e^{(b_i+b_j)t} - 1}{\sqrt{\frac{(e^{2b_i t}-1)(e^{2b_j t}-1)}{b_i b_j}}} \qquad (26)
\end{aligned}
$$

To estimate the correlation parameters, first the parameters of the process described by 23 are recovered. Then, using the central mortality rates data available in the UK life tables[6], we estimate the instantaneous correlations ρ_{ij} between $d\lambda_i$ and $d\lambda_j$ by inverting the approximated correlation expression (26). To compute the correlations between the 11 cohorts involved, we start from the central mortality rates in 1958 of the people aged between 1 and 11, and we follow the diagonal of the life table until we reach the central mortality rates of the people aged between 65 and 75 in 2012. The central mortality rates table constructed this way has dimension 55×11 and allows to estimate the correlation coefficients which we report in Table 3. Correlations are close to 1, but they tend to decrease with the distance between the initial ages of the two considered cohorts. This behavior aligns with the intuition that the changes leading to longevity improvements (such as healthy habits or medical advancements) have different impact on different generations and that cohorts effect are at play. Table 4 reports the correlations across the two populations. Also in this case, the correlations appearing in the diagonals are the highest, and they tend to decrease along the rows and column dimensions, indicating the presence of common cohort effects across populations. Figure 3 shows the covariances between the different UK cohorts, and between the UK and Italian cohorts. The two generations with lowest covariance are the 66 year-old UK and 66 years old Italian cohorts.

Table 3. Instantaneous correlation matrix UK population.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
65	1										
66	0.9990	1									
67	0.9983	0.9992	1								
68	0.9983	0.9988	0.9989	1							
69	0.9973	0.9985	0.9988	0.9993	1						
70	0.9969	0.9979	0.9983	0.9989	0.9995	1					
71	0.9972	0.9977	0.9983	0.9987	0.9988	0.9986	1				
72	0.9964	0.9970	0.9977	0.9984	0.9987	0.9986	0.9994	1			
73	0.9962	0.9970	0.9976	0.9985	0.9988	0.9989	0.9992	0.9997	1		
74	0.9959	0.9967	0.9974	0.9983	0.9989	0.9991	0.9991	0.9995	0.9996	1	
75	0.9957	0.9960	0.9964	0.9974	0.9978	0.9981	0.9990	0.9996	0.9994	0.9995	1

5.3 Evaluating the diversification gains in terms of risk margin

Because the oldest cohort considered in our application is 75, and we assume a maximum life span of $\omega = 105$ years, we fix the time horizon of our simulations to 30 years. Consistent with this choice, we consider a constant interest rate of 2%, matching the 30-year risk-free-rate indicated by EIOPA for the calculation of technical provisions.

[6]Source: Human Mortality Database.

Table 4. Correlation between populations. Rows are UK generations, columns are Italian generations.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
65	0.9821	0.9815	0.9803	0.9807	0.9803	0.9797	0.9798	0.9790	0.9789	0.9786	0.9785
66	0.9815	0.9830	0.9817	0.9817	0.9820	0.9812	0.9809	0.9800	0.9801	0.9799	0.9793
67	0.9803	0.9817	0.9819	0.9812	0.9817	0.9810	0.9809	0.9802	0.9802	0.9801	0.9795
68	0.9807	0.9817	0.9812	0.9827	0.9826	0.9820	0.9816	0.9813	0.9815	0.9813	0.9805
69	0.9803	0.9820	0.9817	0.9826	0.9839	0.9831	0.9823	0.9822	0.9824	0.9825	0.9814
70	0.9797	0.9812	0.9810	0.9820	0.9831	0.9834	0.9819	0.9819	0.9822	0.9823	0.9815
71	0.9798	0.9809	0.9809	0.9816	0.9823	0.9819	0.9831	0.9825	0.9824	0.9823	0.9823
72	0.9790	0.9800	0.9802	0.9813	0.9822	0.9819	0.9825	0.9830	0.9828	0.9826	0.9827
73	0.9789	0.9801	0.9802	0.9815	0.9824	0.9822	0.9824	0.9828	0.9832	0.9829	0.9826
74	0.9786	0.9799	0.9801	0.9813	0.9825	0.9823	0.9823	0.9826	0.9829	0.9832	0.9827
75	0.9785	0.9793	0.9795	0.9805	0.9814	0.9815	0.9823	0.9827	0.9826	0.9827	0.9833

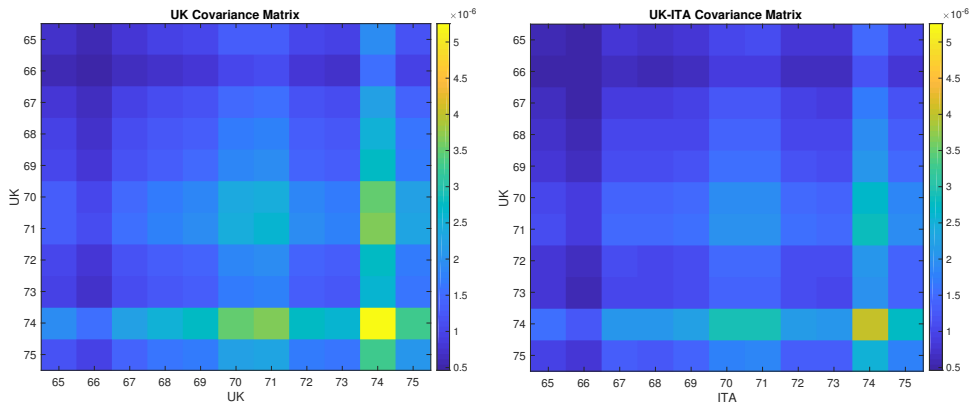

Figure 3. Left Panel: Covariance matrix between UK generations. Right Panel: Covariance matrix between Italian and UK generations.

The choice of a constant interest rate term structure allows us to isolate and capture any possible added benefit specifically due to the geographical diversification of an annuity portfolio. The time horizon at which the Risk Margin is computed is 15 years. This choice is justified because we want to focus on the medium-long term benefits of geographical diversification. Consistently with the Solvency II regulation, we select a confidence level $\alpha = 99.5\%$ when calculating the Risk Margin associated to the portfolio.

Initial Portfolio

We consider a UK Insurer with an initial portfolio Π^0, made of 1000 contracts sold to males whose age, at 31/12/2012, is between 65 and 75. The distribution of contracts among ages reflects the proportions of individuals aged between 65 and 75 in the UK national population. Since in the general UK population 69 years old constitute 11.00% of all the people aged between 65 and 75, the domestic portfolio contains 110 contracts sold to 69 years old (see Table 5).

The initial Actuarial Value $AV_{\Pi^0}(0)$ of the portfolio is:

$$AV_{\Pi^0}(0) = 1.4104 \cdot 10^4, \tag{27}$$

Table 5. Domestic portfolio composition.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
% Range	9.68%	9.79%	9.98%	10.34%	11.00%	10.10%	10.10%	8.43%	8.10%	7.36%	6.56%
Π^0	97	98	100	103	110	101	86	84	81	74	66

Table 6. Foreign portfolio composition.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
% Range	10.31%	10.73%	10.48%	10.39%	8.01%	8.18%	8.26%	8.09%	8.09%	8.86%	8.60%
Π^F	103	107	105	104	80	82	83	81	81	89	86

while the Risk Margin computed at time 0 is

$$RM_{\Pi^0}(0) = 1.1838 \cdot 10^3. \tag{28}$$

Hence, the initial portfolio value is

$$\Pi^0(0) = AV_{\Pi^0}(0) + RM_{\Pi^0}(0) = 1.5288 \cdot 10^4. \tag{29}$$

The Risk Margin accounts for 8.39% of the initial portfolio Actuarial Value.
Portfolio Π^F is exposed to the foreign population only, distributed among ages according to Table 6, useful for comparison. As we did for the initial portfolio, we assume that the policyholders' distribution reflects the proportion of individuals belonging to each generation between 65 and 75 in the Italian population (see Table 6). Figure 4 shows the different percentage of individuals per cohort in the UK and Italian population. In this case, the risk margin is 7.15%. One could guess that by expanding towards Italy, the UK underwriter could, at most, reduce his risk margin to this level. However, we will show later on that, thanks to the diversification effect, the risk margin of the underwriter can be even lower.

Domestic Expansion

With a Domestic Expansion, we assume that the Insurer doubles the size of her annuity portfolio, selling additional policies to her domestic population, i.e. the UK population. The new portfolio Π^1 is therefore composed of 2000 contracts and is obtained by simply doubling the number of contracts for each generation. Hence,

$$AV_{\Pi^1}(0) = 2.8208 \cdot 10^4, \tag{30}$$
$$RM_{\Pi^1}(0) = 2.3676 \cdot 10^3, \tag{31}$$
$$\Pi^1(0) = 3.0576 \cdot 10^4. \tag{32}$$

The Risk Margin proportion relative to actuarial value is unaffected by the size of the portfolio, and still accounts for 8.39% of the Actuarial Value of the Domestically Expanded portfolio. In this case, the diversification index between Π^0 and $\Pi^1 - \Pi^0$ is 0, as no diversification gain can be obtained. However, some diversification gains could be obtained through a domestic expansion, in case the new portfolio had a different

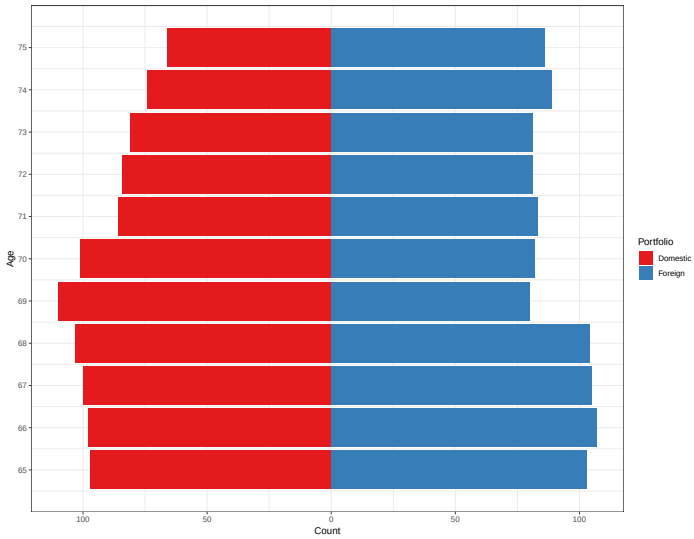

Figure 4. Domestic and Foreign Portfolio Composition.

composition, in terms of policyholders' ages, than the initial one.

Foreign Expansion

In case of a Foreign Expansion, we assume that the Insurer doubles the number of policies in its annuity portfolio by selling contracts written on policyholders belonging to the Foreign population. The new portfolio Π^2 is, therefore, composed of 1000 contracts sold to the UK population composing the initial portfolio (distributed as described in Table 5) and of 1000 contracts written on the Italian population, $\Pi^2 = \Pi^0 + \Pi^F$. It has the following actuarial value and risk margin:

$$AV_{\Pi^2}(0) = 2.8745 \cdot 10^4, \tag{33}$$

$$RM_{\Pi^2}(0) = 2.2308 \cdot 10^3, \tag{34}$$

As a consequence,

$$\Pi^2(0) = 3.0976 \cdot 10^4. \tag{35}$$

For this portfolio, the Risk Margin accounts for 7.76% of the Actuarial Value, reduced, as expected, by 0.53 percentage points relative to the one of the initial portfolio. The diversification index, consistently, increases to 0.0924.

The diversification gain provided by the Foreign portfolio just described can be further exploited. We then explore alternative portfolios and summarize the results in terms of actuarial values, risk margins and total values in Table 7.

Portfolio Π^3 represents a more aggressive foreign expansion, where the number of policies sold to each generation of foreign policyholders is twice the number of policies in Π^F. Tilting the portfolio towards the foreign population has the effect of both decreasing the percentage risk margin (7.56%) and increasing the diversification index (0.1231). However, it is evident that, at most, by increasing the exposure to the Italian population, the risk margin can not be lower than 7.15%. This suggests to optimize the portfolio mix using not only the diversification across populations, but also across generations.

The portfolio Π^1_{opt} is obtained diversifying within the UK population. Its composition is optimized to obtain the minimum risk margin achievable, under the constraint that the number of new contracts is 1000. It can then be considered as the maximally diversified portfolio, in the absence of geographical diversification. This maximum diversification is obtained by selling 1000 annuities to the UK 66 years old, whose mortality intensity process shows the minimum covariance with the other UK cohorts (see left panel of Figure 3). Note that the percentage risk margin of this portfolio is 6.61%, which is already lower than 7.15%. Being entirely composed of UK annuitants, this portfolio has a null Diversification Index.

Similarly, Π^2_{opt} is obtained allowing for geographical diversification and optimizing the composition of the foreign portfolio. The risk margin is 6.12%, obtained by selling the whole 1000 contracts to the Italian males aged 66. As for Π^1_{opt} its composition is driven by the covariance matrix between the two populations. In that matrix, one can observe that Italian 66-years old males have the lowest covariance with all the cohorts of UK insureds (see right panel of Figure 3). The percentage risk margin of portfolio Π^2_{opt} is the lowest among the portfolios we have considered. The DI of this last portfolio is small compared to the DIs of the other portfolios involving an international expansion, being 0.0163. It is small because the expansion is performed by concentrating the sales of policies in the foreign population in one generation. This confirms that the Diversification Index provides a non-dollar measure of diversification which "averages" the contributions of different generations and penalization any concentration in a particular one, even though the latter is justified by a strategy which aims at minimizing the risk margin reduction. This is why we presented both.

Table 7. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.4104 \cdot 10^4$	$1.1838 \cdot 10^3$	$1.5288 \cdot 10^4$	8.39%	-
Π^F	$1.4641 \cdot 10^4$	$1.0470 \cdot 10^3$	$1.5688 \cdot 10^4$	7.15%	-
Π^1	$2.8208 \cdot 10^4$	$2.3676 \cdot 10^3$	$3.0576 \cdot 10^4$	8.39%	0
Π^2	$2.8745 \cdot 10^4$	$2.2308 \cdot 10^3$	$3.0976 \cdot 10^4$	7.76%	0.0924
Π^3	$4.3387 \cdot 10^4$	$3.2779 \cdot 10^3$	$4.6664 \cdot 10^4$	7.56%	0.1231
Π^1_{opt}	$3.0187 \cdot 10^4$	$1.9950 \cdot 10^3$	$3.2182 \cdot 10^4$	6.61%	0
Π^2_{opt}	$3.0612 \cdot 10^4$	$1.8745 \cdot 10^3$	$3.2487 \cdot 10^4$	6.12%	0.0163

Table 8. Effects of geographical diversification ($r = 0\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.7656 \cdot 10^4$	$1.9408 \cdot 10^3$	$1.9596 \cdot 10^4$	10.99%	-
Π^F	$1.8449 \cdot 10^4$	$1.7252 \cdot 10^3$	$2.0174 \cdot 10^4$	9.35%	-
Π^1	$3.5311 \cdot 10^4$	$3.8815 \cdot 10^3$	$3.9193 \cdot 10^4$	10.99%	0
Π^2	$3.6104 \cdot 10^4$	$3.6660 \cdot 10^3$	$3.9770 \cdot 10^4$	10.15%	0.0924
Π^3	$5.4553 \cdot 10^4$	$5.3912 \cdot 10^3$	$5.99446 \cdot 10^4$	9.88%	0.1231
Π^1_{opt}	$3.8182 \cdot 10^4$	$3.3155 \cdot 10^3$	$4.1497 \cdot 10^4$	8.68%	0
Π^2_{opt}	$3.8841 \cdot 10^4$	$3.1177 \cdot 10^3$	$4.1958 \cdot 10^4$	8.03%	0.0163

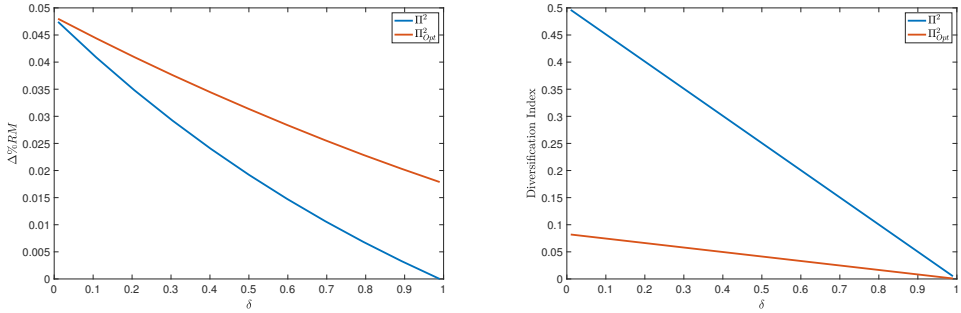

Figure 5. Left Panel: Percentage Risk Margin reduction under different exogeous $\delta_i = \delta$ for every i assumption. Right Panel: Diversification under different exogenous $\delta_i = \delta$ for every i assumption.

5.4 Sensitivity Analysis

Table 8 reports the results for the different portfolios considered in Section 5.3, under the assumption of a zero interest rate, i.e. $r = 0\%$. Under this lower interest rate level, the magnitude of longevity risk is more severe, as expected: the percentage Risk Margins are higher for all portfolios, increasing in the best-case scenario to 8.03%, up from 6.12%. However, diversification is even more valuable, because the reduction from the initial to Π^2_{opt} portfolio reaches 3 percentage points. The Diversification Index, instead, by definition, is not affected by the change in the interest rate, because the weights appearing in (21) are expressed in nominal terms (number of annuities written on a generation) rather than in value terms (value of the annuity portfolios on the different generations, for instance).

We finally assess the impact of the parameter δ_i on portfolio diversification following an expansion. In Section 5.3, we considered two countries, the UK and Italy, that belong to the same continent and share many similar features. As a consequence, also their past mortality dynamics were not so dissimilar. It is reasonable to assume, however, that more different countries may show way lower similarity, and thus lower δ's between cohort intensities. We perform, then, a simulation study where the parameters of the foreign population are set as in Table 2, with the only exception of the parameters δ_i, which we assume to be a constant δ for every generation i. The interest rate is set to $r = 2\%$, as in our base case. We exogenously set δ to a value that ranges from 0 to 1. When δ is 0, the dynamics of the mortality intensities of the domestic and the foreign populations are orthogonal. Thus, the international expansion targets a foreign population whose mortality dynamics is very different from the domestic one. In this case, we expect the maximum level of diversification gains from an international expansion. As δ increases, the correlation between the mortality intensities of the two populations increases as well. When δ is equal to 1, each generation presents the same mortality dynamics in the domestic and foreign population. In this last case, we can expect the lowest level of longevity risk diversification gains from an international expansion strategy. We compute, for each level of δ_i, the DI and the percentage risk margin reduction, for the portfolios Π^2 and Π^2_{opt} described in Section 5.3.

As expected, the highest values of both the percentage risk margin reduction and the Diversification Index, for both portfolios, are achieved when δ_i close to 0. The percentage risk margin reduction is almost 5% in this case, showing that sizable bene-

fits from geographical diversification are possible. Such benefits, measured in terms of either the risk margin reduction or the Diversification Index, decrease as δ approaches 1. The optimal portfolio expansion Π^2_{opt} provides consistently higher risk margin reduction than Π^2, and the gap between the two strategies widens as δ increases (see the left panel of Figure 5). On the contrary, strategy Π^2 shows a higher DI with respect to Π^2_{opt}, for every δ.

The Diversification Index tends to 0 for both portfolios as δ goes to 1. Instead, while the percentage risk margin reduction for Π^2 goes to zero when δ is 1, portfolio expansion Π^2_{opt} offers a diversification benefit relative to the initial portfolio even in that case. This happens because the expansion is targeted in this case to a specific generation. The effects of the international expansion are analogous to those that can be obtained by targeting the domestic expansion to the generation which shows the lowest covariance with the others: Π^2_{opt} reduces the percentage risk margin as much as Π^1_{opt}.

Given the properties of the Diversification Index, and the evidence from this sensitivity analysis, the DI can be an extremely easy-to-handle and useful tool when choosing among competing target foreign populations in an international expansion. The percentage risk margin reduction, being a monetary measure of the diversification gains, is better suited, instead, to select the best strategy when different alternative foreign portfolio compositions can be targeted, once the candidate foreign population has been selected.

6 Conclusions

In this paper, we discussed the benefits of geographically diversified portfolios, due to the non-perfect correlation between the dynamics of the mortality rates of different populations. We have considered the problem of an insurer who has to decide whether to expand his portfolio in the country where it is based or in a foreign country. Some diversification gains can be realized when expanding internationally, due to the mitigation of the exposure to domestic longevity risk. To discuss whether these gains may be sizable in an annuity portfolio, we built a longevity risk model that, while being parsimonious, can capture the non-perfect correlations among the different cohorts of two different populations. We then provided two indicators of the diversification of an international expansion. The percentage risk margin reduction is computed coherently with the Solvency II modeling approach. The Diversification Index is a weighted average which depends on both the portfolio mix and the weight of the idiosyncratic foreign risk factor.

Our application, based on an annuity portfolio written on the UK and the Italian populations, shows that the effects of an international diversification are sizable. Under a 0% interest rate assumption, we showed that an optimally designed expansion can lower the risk margin, relative to the actuarial value of the portfolio, by almost 3 percentage points.

The example in the paper can be considered conservative, since the two populations of UK and Italy present rather similar historical mortality dynamics. The diversification effect is shown to be more relevant, the lower the risk-free interest rate and the lower the correlation between intensities.

The diversification benefits of an international expansion may happen to be counterbalanced by the costs connected to the foreign portfolio acquisition process. These costs, that are - say - the fixed costs of opening a foreign affiliate, or the fees required by the agents involved in the M & A operation, etc., may be substantial. As an alternative to a physical expansion, the insurer may obtain the same diversification benefit operating on the longevity derivatives market. Longevity derivatives, and longevity swaps in particular, are bespoke transactions between (re)insurers and funds or companies, that agree to exchange fixed cash flows and cash flows linked to the survivorship of a particular population (see Blake et al., 2006 for instance). The buyer of the protection provided by a longevity swap transfers the longevity risk linked to a given reference population to the seller, who in turn becomes exposed to such risk. In our case, the insurer can expand internationally by receiving a fixed periodical fee and paying the realized survivorship of the foreign cohorts. Thus, the risk margin reduction benefits of a foreign expansion can be replicated by selling protection through a swap. Even in this case, however, the costs of structuring the agreement and coping with informational asymmetries (Biffis et al., 2016), can substantially reduce the diversification gains. Appendix A shows how to compare the physical and swap-based expansions, when the swap fee is fair, based on their costs.

Our paper contributes to the literature on the modeling of longevity risk, by proposing a multi-population model that captures the dependence structure within and across populations. It contributes to the understanding of longevity risk management, by pointing out that, although the same increasing longevity trend is common to the populations of almost every country, geographical diversification may lead to some benefit. Our model is able to quantify such effects.

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Appendix A: Physical versus synthetic expansion

A foreign portfolio expansion like the one we considered with portfolio Π^2_{opt} is in reality very difficult to achieve, because it is unlikely that an insurer can target the sale of annuity contracts only to a specific cohort. Even if the expansion is feasible, it is likely

to entail some cost $C_0 > 0$. Therefore, the real value of the liabilities of the insurer following the optimal foreign portfolio expansion can be rewritten as:

$$\bar{\Pi}^2_{opt} = \Pi^2_{opt} + C_0. \tag{36}$$

On the other hand, the same level of geographical diversification could be synthetically obtained through a longevity swap. Consider our UK life insurer with portfolio Π^0, and assume that she sells a longevity swap written on 1000 individuals aged 66 belonging to the Italian population. Being the seller of the swap, the UK insurer will receive every year, until the maturity of the contract, a fixed amount equal to K and will pay a stochastic amount given by the realized survival rate of the Italian 66 years old males. Let the maturity of the swap be $T = \omega$ and assume independence between mortality and interest rate risk. From the point of view of the seller, the value at time t of the longevity swap is:

$$\begin{aligned} L(t,T) &= 1000 \sum_{\mathcal{T}=t+1}^{T-t} \left[K - S_{66}(t,\mathcal{T})\right] D(t,\mathcal{T}) = \\ &= 1000 \sum_{\mathcal{T}=t+1}^{T-t} \mathbb{E}_t \left[K - \exp\left(-\int_t^{\mathcal{T}} \lambda_{66}(s)ds\right)\right] \mathbb{E}_t \left[\exp\left(-\int_t^{\mathcal{T}} r(u)du\right)\right], \end{aligned} \tag{37}$$

where K is the swap rate, $S_{66}(t,\mathcal{T})$ is the $(t,\mathcal{T})$-Survival probability for a 66 years old Italian male and $D(t,\mathcal{T})$ is the discount factor. If the swap is fairly priced, the swap rate is chosen in such a way that the value of the contract is zero at inception, that is:

$$K = \frac{\sum_{\mathcal{T}=1}^{T-t} \mathbb{E}\left[\exp\left(-\int_0^{\mathcal{T}} \lambda_{66}(s)ds\right)\right] \mathbb{E}\left[\exp\left(-\int_0^{\mathcal{T}} r(u)du\right)\right]}{\sum_{\mathcal{T}=1}^{T-t} \mathbb{E}\left[\exp\left(-\int_0^{\mathcal{T}} r(u)du\right)\right]}.$$

In our calibration, assuming a constant interest rate of 2%, we have that $K = 0.7218$. The actuarial value of $\tilde{\Pi}^2_{opt}$ is then

$$AV_{\tilde{\Pi}^2_{opt}}(t) = AV_{\Pi^2_{opt}}(t),$$

The risk margin of $\tilde{\Pi}^2_{opt}$ is:

$$RM_{\tilde{\Pi}^2_{opt}}(t) = D(t,t+T) \cdot VaR_\alpha\big(AV_{\tilde{\Pi}^2_{opt}}(t+T) - \mathbb{E}_t[AV_{\tilde{\Pi}^2_{opt}}(t+T)]\big) = \tag{38}$$

$$= D(t,t+T) \cdot VaR_\alpha\Big(AV_{\Pi^2_{opt}}(t+T) - \mathbb{E}_t\Big[AV_{\Pi^2_{opt}}(t+T)\Big]\Big) \tag{39}$$

$$= RM_{\Pi^2_{opt}}(t). \tag{40}$$

So, if fairly priced at inception, the longevity swap allows the insurer to achieve the same actuarial value and the same risk margin of a physical sale of annuity contracts to the Italian males. The sales of the longevity swap may entail some initial cost C_0' given, for instance, by the required due diligence actions. Hence, the value of the liability portfolio $\tilde{\Pi}^2_{opt}$ is given by:

$$\tilde{\Pi}^2_{opt} = \Pi^2_{opt} + C_0'. \tag{41}$$

As long as $C_0' < C_0$, the UK insurer will find in the synthetic expansion trough the longevity swap a more attractive solution.

Appendix B: Gaussian Mapping Covariance

A simple application of Itô's Lemma allows us to show that the solution to the SDE (23) is given by:

$$\lambda_i^V(t) = \lambda_i^V(0)e^{b_i t} + \frac{a_i}{b_i}\big(1 - e^{b_i t}\big) + \sigma_i^V \int_0^t e^{b_i(t-s)}\, dW_i(s). \tag{42}$$

Therefore, we have that:

$$\mathbb{E}_0\big[\lambda_i^V(t)\big] = \lambda_i^V(0)e^{b_i t} + \frac{a_i}{b_i}\big(1 - e^{b_i t}\big) \tag{43}$$

$$Var_0\big[\lambda_i^V(t)\big] = \frac{(\sigma_i^V)^2}{2b_i}\sqrt{e^{2b_i t} - 1}. \tag{44}$$

Since $\lambda_i^V(t) - \mathbb{E}_0\big[\lambda_i^V(t)\big] = \sigma_i^V \int_0^t e^{b_i(t-s)}\, dW_i(s)$, the covariance between $\lambda_i^V(t)$ and $\lambda_j^V(t)$ is:

$$\begin{aligned}
Cov_0(\lambda_i^V(t), \lambda_j^V(t)) &= \mathbb{E}_0\left[\sigma_i^V \sigma_j^V \Big(\int_0^t e^{b_i(t-s)}\, dW_i(s)\Big)\Big(\int_0^t e^{b_j(t-s)}\, dW_j(s)\Big)\right] \\
&= \mathbb{E}_0\left[\sigma_i^V \sigma_j^V \rho_{ij} \int_0^t e^{(b_i+b_j)(t-s)}\, ds\right] \\
&= \sigma_i^V \sigma_j^V \rho_{ij} \int_0^t e^{(b_i+b_j)(t-s)}\, ds \\
&= \frac{\sigma_i^V \sigma_j^V \rho_{ij}}{b_i + b_j}\Big(e^{(b_i+b_j)t} - 1\Big).
\end{aligned}$$

Finally, we have:

$$\begin{aligned}
Corr_0(\lambda_i^V(t), \lambda_j^V(t)) &= \frac{Cov_0(\lambda_i^V(t), \lambda_j^V(t))}{\sqrt{Var_0\big[\lambda_i^V(t)\big] Var_0\big[\lambda_j^V(t)\big]}} \\
&= \frac{2\rho_{ij}}{b_i + b_j} \cdot \frac{e^{(b_i+b_j)t} - 1}{\sqrt{\frac{(e^{2b_i t}-1)(e^{2b_j t}-1)}{b_i b_j}}}
\end{aligned} \tag{45}$$

Thanks to the Gaussian Mapping technique we can also compute the conditional correlation between two generations belonging to two different populations. Considering $0 \le u \le t$, the conditional correlation between $\lambda_{x_i}^d(t)$ and $\lambda_{x_j}^f(t)$ is given by:

$$Corr_u\big[\lambda_{x_i}^d(t), \lambda_{x_j}^f(t)\big] = \delta_j \frac{Cov_u(\lambda_{x_i}^d(t), \lambda_{x_j}^d(t))}{\sqrt{Var_u(\lambda_{x_i}^d(t)) \cdot Var_u(\lambda_{x_j}^f(t))}}, \tag{46}$$

where $Cov_u(\lambda^d_{x_i}(t), \lambda^d_{x_j}(t))$ is computed using the Gaussian mapping technique, and

$$Var_u(\lambda^f_{x_j}(t)) = \delta_j^2 Var_u(\lambda^d_{x_j}(t)) + (1-\delta_j)^2 Var_u(\lambda'(t; x_j)). \tag{47}$$