Pricing Network Effects: Competition

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Abstract

This paper studies the practice of influencer marketing in oligopoly markets and its effect on market efficiency. We develop a duopoly model in which firms sell horizontally differentiated products. Consumers are influenced by other consumers' choices, and some consumers are more influential than others. Firms' influencer marketing strategy involves discovering the influence of a subset of consumers and price discriminating based on this information.

In equilibrium, firms subsidize consumers whose influence is above average and charge premia to below average influential consumers; the equilibrium premia/discounts depend on the strength of network effects and the level of information that firms have on consumers' influence. From a normative perspective, we show that influencer marketing leads to inefficient consumer-product matches. Firms' investments in discovering consumers' networks are strategic complements, leading to a race for information acquisition that erodes total surplus and firms' profits but increases consumer surplus.

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1 Introduction

Digital marketing is an important component of many firms' strategy, and a key activity of digital marketing is influencer marketing.[1] This entails gathering and using data on users' social media activity to identify influencers of a specific market segment and then orienting marketing activities around them. Information about influencers is collected directly by firms; by social media platforms such as Facebook, Twitter, and Google+; and by consultancy firms such as Klout.com and Ammo Marketing, which specialize in mapping customers' influence. Firms' most common activity after acquiring this information is to price discriminate across consumers, offering discounts to customers depending on their influence. At the extreme, customers with sufficiently high influence are offered free products. We provide examples of these practices in the next section.

The rationale behind influencer marketing is that, for many products, buyers' actions are positively influenced by other's consumption, and that certain consumers command more influence than others. For products such as software, mobile phones, video game consoles, communication apps and the like, these peer-effects are technological in nature: consumers need to adopt technologies compatible with those of their peers in order to have effective interaction. But peer-effects in consumption are more widespread: they are important for social and entertainment goods, such as movies, books, and music, which allow consumers to share experiences and impressions; and, As Kuhn et al. (2011) show empirically, peer-effects are also important for conspicuous goods—e.g., luxury durable goods such as cars.

We formalize these considerations in a model in which two firms offer horizontally differentiated products that exhibit network effects: consumers' valuations of the products depend on their peers' consumption. Firms simultaneously choose a price scheme: a price offer to each consumer. Consumers observe the price scheme and decide which product to adopt/buy, taking into account their idiosyncratic preferences, the offers received, and the expected network effects.

Network effects result from the interaction with other consumers and produce positive externalities. Empirical analysis shows that there is heterogeneity in the level of influence across consumers and that the strength of network effects and the dispersion of influence differ across product markets.[2] Our formulation allows for different strengths of network effects and

[1] An industry survey from 2015 suggests that 75% of marketers are using influencer marketing—see http://www.adweek.com/socialtimes/report-75-of-marketers-are-using-influencer-marketing/628211.

[2] There is evidence that the strength of network effects is higher for products with higher "conspicuousness" and for products that are "consumed publicly"—e.g., Bearden and Etzel (1982) and Kuhn et al. (2011). Leskovec et al. (2007) studies how recommendations lead to adoption (influence) in four product cathegories: DVDs, book, music

for heterogeneity in the level of influence across consumers. Moreover, the ability of firms to leverage network effects, by price discriminating across consumers, depends on the information that firms have about consumers' level of influence.[3] Firms know the distribution of influence, but they have information about the level of influence of only a subset of consumers–the *targeted consumers*. We start with the case in which the information that firms have is exogenous. In the second part of the paper, we study information acquisition.

Our aim is to understand how firms use information on consumers' influence to design profitable pricing strategies, as well as firms' incentives to acquire such information. We also study the inefficiencies that influencer marketing can introduce. In our companion paper, Fainmesser and Galeotti (2016), we address these questions in a monopolistic environment, whereas here, we focus on an oligopoly market. The importance of understanding the competitive environment is highlighted in recent observations from the marketing literature, indicating that network effects are especially pronounced in product categories with competing technological standards—e.g., Van den Bulte and Stremersch (2004). Moreover, the most successful influencer marketing campaigns have been conducted for product categories with few competing firms.

Our first result is a characterization of equilibrium prices. A firm's strategy specifies a uniform price offer to non-targeted consumers, and a price offer to targeted consumers that depends on their level of influence. In equilibrium, firms subsidize consumers whose influence is above average and charge a premium to those whose influence is below-average. The magnitudes of discounts and premia depend on the strength of network effects and the level of information that firms have. The premium-discount feature of the equilibrium pricing strategy is reminiscent of the monopoly case. Fixing the strategy of the competing firm, a firm best responds by giving a discount to highly influential consumers in order to attract them to their network. In this way, the firm leverages consumers' expectations of network effects in its favor and extracts surplus from less-influential consumers.

Our second result is a characterization of inefficiencies, consumer surplus, and profits. The presence of consumers who are targeted by one firm but not by the other, together with the fact that targeted consumers are offered a price that is declining in their influence, introduces the possibility of misallocation. For example, with positive probability, an above-average

and video. They report high recommendation and moderate influence for DVDs; moderate recommendation and high influence for books and music; and very low recommendation and influence for videos.

[3] Our model of influence is static. Influence occurs via the rational expectation of others' consumptions. By abstracting entirely from dynamic aspects, we obtain a very parsimonious, and yet rich, model of influence. Dynamic aspects are natural when we think of processes of influence, and embedding a dynamic model in this marketing setting is challenging. This is a fascinating topic that we hope to work on in the future.

influential consumer is not targeted by her preferred firm X and, thus, receives an offer for product X at the average market price; but she may be targeted by the competing firm Y and, because of her influence, receive a price discount. When the price differential is large enough, the consumer will buy product Y.

The loss in surplus associated with this misallocation is passed on to firms, whereas consumer surplus and inefficiencies increase hand-in-hand. Indeed, consider a consumer who is targeted by one firm and not the other. If she is influential, she can choose the targeted– hence discounted– product, whereas if her influence is low, she can choose the non-targeted product and avoid paying the premium. Understanding the possible inefficiencies that influencer marketing introduces is important in view of the current debate about informational privacy, which is motivated by the unprecedented abilities of private and state organizations to collect personal data; see Froomkin (2000).[4] Our analysis points out that influencer marketing through price discrimination reduces total surplus when competing firms access partial information about influence. However, if the focus is on consumer surplus, then it is beneficial to allow firms to have some information about consumers' influence.

As the discussion on misallocation suggests, inefficiency and consumer surplus increase and firms' profits decline in the dispersion of the prices offered to consumers with different influence levels. The dispersion of equilibrium prices is larger when firms have greater incentives to leverage network effects, which is the case when network effects are strong and when the level of influence is dispersed. These results highlight the distinctive effect of competition in the design of influencer marketing strategies. The basic intuition in the marketing literature (see Krackhardt, 1996) is that great structural differentiation across consumers is necessary for exploiting network effects. This is a sound intuition in a monopoly setting. Broadly speaking, greater heterogeneity in influence across consumers allows a monopoly to leverage network effects more effectively, and this increases the monopoly's revenue without altering the cost of the marketing campaign—see, e.g., Fainmesser and Galeotti (2016) and Galeotti and Goyal (2006). But this is different under competition. Greater heterogeneity in influence across consumers triggers a race between competing firms to shift network effects in their favor, with the result that many consumers get to choose between the targeted price of one firm and the non-targeted price of the other. This reduces the effective price to consumers and erodes firms'

[4] There are many examples of influencer marketing practices that led to class-action lawsuits. As a concrete example, Beacon was launched in 2007 and formed part of Facebook's advertisement system, with the aim of improving targeted advertisement. Beacon reported to Facebook on its members' activities on third-party sites that also participated with Beacon, and this occurred without the knowledge of Facebook users. This service became the target of a class-action lawsuit and it was shut down in September 2009.

profits.

In the final part of the paper, we study firms' incentive to acquire information. We assume that firms, in addition to choosing their pricing strategy, also choose how much information to acquire, at a cost, on consumers' influence. We first establish that at the Nash equilibrium pricing, information acquisitions by the two firms are strategic complements. We then derive a clear comparative statics result that indicates that an increase in the strength of network effects, greater dispersion of influence across consumers, or more-efficient technologies for information acquisition lead firms to acquire more information, which erodes firms' profits and benefits consumer surplus.

Our paper relates to the classic literature on network effects and network industries initiated in the 1980s by Farrell and Saloner (1985) and Katz and Shapiro (1985). We depart from this literature by focusing on how competing firms can exploit information on consumers' heterogeneity in influence to leverage network effects in their favor, and on the resulting effects for market outcomes.

The literature on optimal marketing strategies in the presence of network effects is a recent and active field of research in economics, marketing, and computer science. Few papers have analyzed monopoly pricing for products with network effects; these include Hartline et al. (2008), Arthur et al. (2009), Bloch and Quérou (2013), Candogan et al. (2012), and Fainmesser and Galeotti (2016).[5] We share the underlying motivation of these papers; the current's paper focus on competition is a natural and important step forward in this research agenda. It is worth noting that the premium-discount feature of optimal pricing in the monopoly setting is preserved when competition is introduced. However, competition creates new forces in the way that firms adapt their marketing strategies to changes in the environment, and this creates very distinctive comparative statics results, and different insights into market efficiency.

Models of price competition in the presence of network effects have also been studied in the literature on two-sided markets. Here, competing platforms price to attract different types of consumers who benefit by belonging to the same platform—e.g., Armstrong (2006), Caillaud and Jullien (2003), Ambrus and Argenziano (2009), and Jullien (2011). Most of these papers focus on uniform pricing. Julien (2011) studies the incentives of a leading platform to price discriminate across users in order to prevent a follower platform from entering the market. A particular interest of these papers is to understand the role of price competition in allowing

[5] A complementary literature focuses on optimal advertising and seeding strategies (distributing, initially, products for free to key consumers), given an exogenous process of product diffusion—e.g., Galeotti and Goyal (2009), Campbell (2013), Goyal and Kearns (2012), Domingos and Richardson (2001), and Kempe et al. (2003).

consumers to coordinate on certain platforms. Hence, the focus and method of analysis in these papers are complementary to our work.

A more recent literature has focused on price competition among firms that try to attract consumers connected in a network. Banerji and Dutta (2009) and Aoyagi (2013) consider uniform pricing and study when firms can segment the market. Chen, Zenou, and Zhou (2016) focuses on price discrimination.[6] In all these papers, the pattern of network effects across consumers is modeled by a deterministic network, and firms and consumers have perfect information about the architecture of the network. Modeling network effects using a deterministic graph and the assumption that its structure is commonly known to the agents adds substantial complexity to the analysis. More importantly, this approach is not suitable for addressing how the use of different levels of information on network effects shapes consumer surplus and inefficiencies. It also does not address questions about firms' incentives to acquire information about network effects. Our work, therefore, complements Banerji and Dutta (2009), Aoyagi (2013), and Chen, Zenou, and Zhou (2016) by developing a model that can address these questions.

We summarize some key aspects of influencer marketing in the remainder of this introduction. Section 2 describes the formal model, and Section 3 characterizes equilibrium pricing and discusses inefficiencies, profits and consumer surplus. Section 4 extends the analysis to include firms' information acquisition decisions. Section 5 concludes.

1.1 On influencer marketing

A common view among practitioners is that influencer marketing is about leveraging consumption externalities and that these effects are strong for certain product categories, such as products with competing standards, social and entertainment goods, and conspicuous consumption goods.[7] Depending on the product, different mechanisms are behind the resulting consumption externalities. However, regardless of the precise underlying mechanism, by incentivizing the adoption of a product by certain consumers– the influencers– a firm increases the demand of other consumers. We briefly discuss practical examples of influencer marketing in different industries.

[6] Two other papers study price competition in a model in which consumers are connected in a network. Galeotti (2010) studies price competition among firms selling identical products, in a context in which consumers share information about firms' prices. Katona (2015) also considers a homogeneous product market, in which firms both try to persuade consumers to transmit information to other consumers and also price discriminate across consumers based on their network location.

[7] Jay Baer, a prominent American marketing consultant and the author of *The New York Times* bestselling book *Youtility*, when referring to influencer marketing, pointed out: "True influence drives action, not just awareness."

Klout.com measures influence by using data points from users' activity in a variety of online social platforms, such as Twitter, Facebook, Google+, LinkedIn, YouTube, Instagram, and Wikipedia. The Klout score is aimed at measuring the extent to which an individual transmits information to others and the extent to which an individual's actions affect others' behavior. The primary business model for Klout involves companies paying for Klout Perks campaigns. In a typical such campaign, a company offers discounts, special pricing, products, packages and privileges to Klout users with a high Klout score. Klout users who receive Perks are under no obligation to take action to sponsor/advertise these products. Just three years after its launch, Klout announced that its users had claimed more than one million Perks across more than 400 campaigns.

Technology: Microsoft, Sony and Samsung. Microsoft, Sony and Samsung effectively conduct influencer campaigns via Klout. In 2011, Microsoft increased its marketing efforts to promote the Windows Phone 7.5. As part of this effort, Microsoft partnered with Klout, offering a free phone to users, who are influential about Microsoft and technology. In addition to a Windows phone, the campaign offered an invitation to an exclusive Microsoft party, where the company would show off its new phones and provide other entertainment. Recently, highly influential Klout users received a free Sony NEX 3N camera and Sony Action Cam. Influencers in New York City were invited to the Samsung Smart TV Launch party, which included a performance by Flo Rida and an appearance by Kate Upton.

Social and entertainment goods. In 2010, Disney rolled out a Klout Perks campaign for the launch of its movie "Tangled." Individuals with high Klout scores received invitations to early screenings, as well as goodies such as the sound-track cd, dolls, etc., delivered as an 'influencer kit.' Universal Studios is another client of Klout Perks.

Conspicuous consumption goods. In March 2012, luxury fashion shopping site Gilt Groupe Inc. announced an exclusive partnership with Klout. Gilt Groupe offered product pricing discounts proportional to the consumer's Klout score: "Through this first-of-its-kind partnership, Gilt members and Klout users can use their influence to receive a percentage off of their Gilt.com purchase that matches their Klout Score; for example, if your Klout Score is 81-100, you could receive up to 100 percent off of your purchase."[8] More generally, practitioners in the fashion industry consider influencer marketing an effective way to create a "street style" and has become a huge part of creating buzz around designer label.[9]

[8] see https://www.gilt.com/company/press/gilt-groupe-inc-and-klout-partner-offer-first-its-kind-reward-users

[9] Network effects in fashion can result when influencers use their creativity to demonstrate how clothing articles

Hospitality industries such as airlines and hotels use similar strategies. The Palms Hotel and Casino in Las Vegas introduced the Klout Klub as part of its Social Rewards Program, allowing customers with a high Klout Score to access additional amenities during their stay. ReviewPro, an online reputation and social media management solution provider for the B2B hotel sector, has integrated Klout scores into its product to help hoteliers make smarter decisions related to interacting with their guests.

1888 Hotel in Sydney offers one free night to guests with more than 10,000 followers on Instagram. The luxury hotel Stockholm's Nordic Light offers booking discounts to customers based on their followers on Facebook—e.g., a five-percent discount for customers with 500-1000 Facebook friends; a ten-percent discount for those with more than 1,000 friends; and customers with over 100,000 followers on Facebook or Instagram can stay at the hotel completely free. Other luxury hotels, such as The Ritz Carlton, Triumph Hotels, and Gurney's Montauk, have adopted similar campaigns.

In the airline industry, following the success of a Klout Perk campaign by Virgin America Airlines, it is now common for airline companies to offer promotions, VIP status and inflight services to high Klout score fliers.

2 Model

There are two competing firms, X and Y, that produce, at no cost, horizontally differentiated (indivisible) products, X and Y, and a unit measure of consumers $N = [0, 1]$, each with unit demand. Consumers base their decision on which product to buy on their idiosyncratic preferences, the prices that firms charge, and the consumption of their peers–network effects.

In the first stage of the game, firms choose simultaneously how to price their product, $\mathbf{p}^X = \{p_i^X\}_{i \in N}$ and $\mathbf{p}^Y = \{p_i^Y\}_{i \in N}$. Firms' ability to assign different prices to different consumers depends on their information about the patterns of network effects. This information, for now, is taken as exogenous; we study information acquisition in the second part of the paper. In the second stage, consumers observe firms' pricing and choose, simultaneously, which product to buy, taking into account the resulting network effects.

We now formalize the ingredients of our model: network effects, consumers' preferences,

and accessories can be worn beyond the way that the designers' catalogs show, thus increasing the consumers' utility from these items. An additional channel is the halo of prestige that a glamorous celebrity gives a fashion item or brand that she wears, again increasing the utility that other consumers derive from owning the item. In the fashion example, one might think that network effects are non-monotonic because congestion effects may, at some point, become important. Our model abstracts away from such effects, but the method and analysis we develop can be extended to incorporate such considerations.

and firms' information about network effects.

Network Effects. We parametrize network effects by assigning each consumer with a level of susceptibility, k, and a level of influence, $l \in D = \{0, 1, ..., l^{max}\}$. Each consumer draws, after making her consumption decision, k others, and a consumer with a level of influence l is drawn by l others.[10] $H : D \rightarrow [0, 1]$ denotes the probability distribution of the level of influence across consumers. Consistency requires that the level of susceptibility k equals the average level of influence– $k = \sum_l H(l)\, l$. Hereafter, we refer to k as the average level of influence. We assume that the different draws of a consumer, as well as the draws of different consumers, are independent of each other. Now suppose that consumer i samples consumer j. Then, Bayes' rule implies that the conditional probability that consumer j is sampled by $l \geq 1$ consumers, is $\overline{H}(l) = \frac{1}{k} H(l) l$.

Consumer preferences. We model product differentiation à la Hotelling with linear transport costs: firm Y is located at 0 and firm X at 1 of a unit interval, and consumers are distributed uniformly on the interval. Consumer i's idiosyncratic preference is then captured by her location $\xi_i \in [0, 1]$; ξ_i is private information to consumer i. With transport cost $\tau \geq 1$, consumer i has an idiosyncratic preference for product X equal to $(\xi_i - 1)\,\tau$, and her respective valuation of product Y is $-\xi_i \tau$.

Consumers also experience network effects. In particular, if consumer i adopts product $J \in \{X, Y\}$, her utility increases by a constant value γ for each of her sampled consumers who has adopted product J; the parameter γ measures the strength of network effects. Overall, the utility to consumer i by adopting product X, given that $m \leq k$ out of k sampled consumers have also adopted X, is

$$u_i(X, m; \xi_i, p_i^X) = (\xi_i - 1)\,\tau - p_i^X + \gamma m,$$

and the utility of buying the alternative product Y is $u_i(Y, m; \xi_i, p_i^Y) = -\xi_i \tau - p_i^Y + (k - m)\gamma$.

A consumer's strategy specifies the probability of adopting product X and product Y, given her level of influence and location and the price schedules offered by the two firms. Let $x_j(l, \xi_j, \mathbf{p}^X, \mathbf{p}^Y)$ (resp. $1 - x_j(l, \xi_j, \mathbf{p}^X, \mathbf{p}^Y)$) be the probability that consumer j with level of influence l and location ξ_j adopts good X (resp. good Y); let $\mathbf{x}_{-i}$ summarize the adoption strategies of all consumers other than i. Since different draws of a consumer, as well as the

[10] More precisely, following Fainmesser and Galeotti (2016) and Galeotti and Goyal (2009), each consumer draws k other consumers according to an atomless weighted-uniform distribution on the unit interval, where the weights are determined by the level of influence of the sampled consumers. Therefore, the unconditional probability distribution function assigns to a consumer with level of influence l a density for being sampled that is l times higher than it assigns for a consumer with level of influence 1.

draws of different consumers, are independent of each other, we have that the expected utility to consumer i by adopting good X is

$$U_i(X, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) = (\xi_i - 1)\,\tau - p_i^X + \gamma k A^X(\mathbf{x}_{-i}),$$

where $A^X(\mathbf{x}_{-i})$ is the expected probability that any of i's sampled consumers will adopt product X. Similarly, $U_i(Y, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) = -\xi_i\tau - p_i^Y + \gamma k A^Y(\mathbf{x}_{-i})$. Therefore, consumer i adopts product X whenever

$$U_i(X, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) > U_i(Y, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) \Longleftrightarrow \theta_i - \mathsf{p}_i + \gamma k \mathsf{A}(\mathbf{x}_{-i}) \geq 0, \tag{1}$$

where $\theta_i = (2\xi_i - 1)\,\tau$, $\mathsf{p}_i = p_i^X - p_i^Y$; and $\mathsf{A}(\mathbf{x}_{-i}) = A^X(\mathbf{x}_{-i}) - A^Y(\mathbf{x}_{-i})$. Otherwise, if the inequality does not hold, she adopts Y.

Given $\mathsf{A}(\mathbf{x}_{-i})$ and θ_i, consumer i's choice depends on her level of influence only via the difference in prices that firm X and firm Y charge her, p_i. Therefore, for any pricing scheme for which p_i depends, at most, on i's level of influence, we can write the probability that a consumer located at ξ and with level of influence l adopts product X as $x(l;\xi)$ and $x(l) = \mathbb{E}_\xi x(l,\xi)$. We obtain that

$$\mathsf{A}(\mathbf{x}_{-i}) = \sum_{l\in D} \overline{H}(l)[2x(l) - 1].$$

Before studying competition between firms, we analyze the consumption equilibrium induced by the consumers' decisions in the second stage of the game, given any pricing scheme chosen by the two firms. The following result establishes conditions under which there exists a unique consumption equilibrium and characterizes equilibrium consumption levels. Let $\overline{\mathsf{p}}(l)$ be the difference between the price offers that an individual with degree of influence l expects to receive from firm X and firm Y. It follows that $\overline{\mathsf{p}} = \sum \overline{H}(l)\overline{\mathsf{p}}(l)$ is the difference between the expected price that a randomly selected consumer's neighbor observes from firm X and firm Y.

Condition A: A pricing profile $(\mathbf{p}^X, \mathbf{p}^Y)$ satisfies *condition A* whenever $\frac{1}{2}\left(1 - \frac{1}{\tau}\mathsf{p}_i - \frac{\gamma k}{\tau(\tau-\gamma k)}\overline{\mathsf{p}}\right) \in (0,1)$, for all $i \in N$.

Proposition 1. *Suppose that $\gamma k/\tau < 1$. Then, for any price schedules $\mathbf{p}^X$ and $\mathbf{p}^Y$, there exists a unique equilibrium of the consumption stage. Moreover, if $(\mathbf{p}^X, \mathbf{p}^Y)$ satisfies condition*

A, then the equilibrium probability that a randomly drawn consumer i purchases product X is

$$D_i^X(\mathbf{p}^X, \mathbf{p}^Y) = \frac{1}{2}\left(1 - \frac{1}{\tau}\mathsf{p}_i - \frac{\gamma k}{\tau(\tau - \gamma k)}\overline{\mathsf{p}}\right). \tag{2}$$

Uniqueness of equilibrium in the consumption stage is obtained when consumption externalities are not too strong and/or when products are sufficiently differentiated. The proof, developed in Online Appendix A, uses conditions derived in games with strategic complementarities to ensure existence and uniqueness. In particular, a low average level of influence, weak network effects, and/or high product differentiation impose an upper bound on the level of strategic complementarities in consumers' adoption decision, and this ensures existence and uniqueness of equilibrium. In equilibrium, the probability that consumer i buys the product from firm X is higher the lower is the price that X offers relative to Y—i.e, the lower is p_i. Moreover, $\overline{\mathsf{p}}$ describes the impact of network effects on the probability of adoption. In particular, a negative $\overline{\mathsf{p}} < 0$ means that consumer i expects that her peers will receive, on average, a better price offer from firm X than from firm Y, and so consumer i expects to enjoy greater network effects, should she adopt product X.

For the remainder of the paper, we assume that consumers behave according to the unique interior equilibrium described in Proposition 1 whenever possible, and, therefore, consumption decisions are consistent with (2).

Firms' information and Firms' profits. The ability of a firm to design targeted pricing strategies depends on the information that the firm has about the level of influence of the different consumers. We assume that each firm draws a fraction $w \in [0, 1]$ of consumers and learns about their level of influence. These samplings are random and independent of each other. Therefore, the distribution of influence of non-sampled consumers is the same as the distribution of influence in the population; furthermore, the probability that a consumer is sampled by firm X is independent of the sampling realization of the competing firm Y.

The strategy of firm J is to offer a price p^J to the non-targeted consumers and to offer the product at price $p^J(l)$ to targeted consumers with level of influence l. Hence, w subsumes firms' ability to design targeted pricing strategies. When $w = 0$, firms set a uniform price to all consumers; when $w = 1$, firms price discriminate perfectly. For a given $\mathbf{p}^J$, let $\hat{p}^J(l) = wp_l^J + (1 - w)p^J$ be the expected price offered by firm J to a random consumer with influence l, and let $\hat{p}^J = \sum_l H(l)\hat{p}^J(l)$. Whenever $\mathbf{p}^X$ and $\mathbf{p}^Y$ satisfy condition A, the expected profit of firm X is:

$$\Pi^X(\mathbf{p}^X, \mathbf{p}^Y) = \frac{(1-w)}{2} p^X \left(1 - \frac{1}{\tau}\mathsf{p} - \frac{\gamma k}{\tau(\tau - \gamma k)}\overline{\mathsf{p}}\right) + \frac{w}{2} \sum_l H(l) p^X(l) \left(1 - \frac{1}{\tau}\mathsf{p}(l) - \frac{\gamma k}{\tau(\tau - \gamma k)}\overline{\mathsf{p}}\right),$$

where, $\mathsf{p} = p^X - \hat{p}^Y$, $\mathsf{p}(l) = p^X(l) - \hat{p}^Y(l)$, and

$$\overline{\mathsf{p}} = w \sum \overline{H}(l)[p^X(l) - p^Y(l)] + (1-w)[p^X - p^Y].$$

In the first stage, firms anticipate consumers' consumption decisions and choose their prices simultaneously. We focus on Nash equilibrium of this induced pricing game. The following assumption guarantees that the pricing game has a unique interior equilibrium for every level of information of the firms, w. We maintain Assumption 1 hereafter.

Assumption 1. $\gamma l^{\max} < 1/2$.

2.1 Comments on assumptions and interpretation of the model

The consumption stage builds on Galeotti et al. (2010) and allows for two interpretations. One interpretation is that network effects are local and that when consumers make their consumption decisions, they take into account these local effects, given some residual uncertainty about their future interactions. This is relevant, for example, for technological products with competing standards– in particular, for communication- and interaction- driven products. For these products, the benefits of adopting the same product are materialized via local interaction. The other interpretation is that network effects are global. In this case, each consumer cares about a weighted average consumption of the entire population of consumers, where the weight to the consumption of a particular consumer depends on her level of influence. This interpretation is a better fit for applications in which the network effects arise by conforming to a trend or to social status considerations, as is typical for conspicuous consumption goods. We have developed a formal connection between these two interpretations in our earlier work—see online Appendix I of Fainmesser and Galeotti (2016).

We model product differentiation using a standard Hotelling setting with linear transportation costs. This is appealing because a consumer's the demand for a firm is linear in the consumption decisions of other consumers, which allows us to derive closed-form solutions of equilibrium pricing, profits, and consumer surplus. More generally, we could consider a setting in which consumer i's idiosyncratic preference for firm J is given by θ_i^J, drawn from a distribution F. In this case, consumer i adopts product X whenever

$$U_i(X, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) > U_i(Y, \mathbf{x}_{-i}, \mathbf{p}^X, \mathbf{p}^Y) \Longleftrightarrow \theta_i - \mathsf{p}_i + \gamma k \mathsf{A}(\mathbf{x}_{-i}) \geq 0, \tag{3}$$

where $\theta_i = \theta_i^X - \theta_i^Y$. Let G be the c.d.f of θ_i and let g be the respective density; suppose, for illustration, that firms have information about the level of influence of all consumers, so that each consumer with influence l receives offers $(p^X(l), p^Y(l))$. The probability that a consumer with influence l will buy product X is:

$$x(l) = 1 - G(\mathsf{p}(l) - \gamma k \mathsf{A}(\mathbf{x}_{-i})),$$

and consistency requires that $\mathsf{A}(\mathbf{x}_{-i}) = \sum_l \overline{H}(l)[2x(l) - 1]$; that is

$$\mathsf{A}(\mathbf{x}_{-i}) = 1 - 2 \sum_l \overline{H}(l) G(\mathsf{p}(l) - \gamma k \mathsf{A}(\mathbf{x}_{-i})). \tag{4}$$

Notably, if θ_i is uniformly distributed between $[-1, 1]$, we are back in the Hotelling model. Two key properties of our model are at the core of our analysis. One property is that an increase in the price that firm X offers to consumers with influence l decreases the expectation that each consumer has about her peer consumption of product X—i.e., $\mathsf{A}(\mathbf{x}_{-i})$ declines. The second property is that, per-consumer, this effect is stronger when firm X increases the price for highly influential consumers. As we shall see, these two properties imply that a firm will prefer, in equilibrium, to decrease the price charged to highly influential consumers and extract surplus from the less-influential ones; this equilibrium property is, in turn, behind the main results we obtain about profits and consumer surplus. We now show that these two properties extend to the more general specification. By using the implicit function theorem for the consistency condition (4), and recalling that $\overline{H}(s)/H(s) = s/k$, we have that

$$\frac{d\mathsf{A}(\mathbf{x}_{-i})}{dp^X(s)} / H(s) = -\frac{2(s/k) g(\mathsf{p}(s) - \gamma k \mathsf{A}(\mathbf{x}_{-i}))}{1 + 2\gamma k \sum_l \overline{H}(l) g(\mathsf{p}(l) - \gamma k \mathsf{A}(\mathbf{x}_{-i}))}.$$

For γ sufficiently low, we have that $\frac{d\mathsf{A}(\mathbf{x}_{-i})}{dp^X(s)}/H(s) < 0$, and, since the nominator is linearly increasing in s, the effect of an increase in the price of $p^X(s)$ lowers the (per-consumer) expected network effects in favor of firm X less than a similar increase of $p^X(s+1)$.

We have assumed that a consumer adopting, say, A, does not benefit by interacting with a consumer that has adopted product B. This is an assumption about product incompatibility. In the case of technological products, it makes sense to allow for partial compatibility. In our setting, assume that if consumer i adopts product $J \in \{X, Y\}$, her utility increases by a constant value $\overline{\gamma}$ for each of her sampled consumers who has adopted product J, and by a constant value $\underline{\gamma} \in [0, \overline{\gamma}]$ for each of her sampled consumers who has adopted the other product. As a result, we can write the utility to consumer i from adopting product X, given that $m \leq k$ out of k sampled consumers have also adopted X as

$$u_i(X, m; \xi_i, p_i^X) = (\xi_i - 1)\,\tau - p_i^X + \gamma m + \underline{\gamma} k,$$

where $\gamma = \overline{\gamma} - \underline{\gamma} \in [0, \overline{\gamma}]$ measures the degree of compatibility of the two products; higher γ means less-compatible products. All of our results carry over to this specification and to this new interpretation of the parameter γ.[11]

Finally, a firm learns about the level of consumers' influence in a way that is independent of the consumers' idiosyncratic preferences and of the learning of the competing firm. This technology mimics the properties of the advertising technology employed in seminal oligopolistic models of informative advertising—e.g., Butters (1977) and Grossman and Shapiro (1984). Firms, in reality, may try to develop technologies that allow them to learn of highly influential consumers.[12] How feasible these technologies are and how firms learn, in practice, about levels of influence is not well documented. Hence, we view our technology as a neutral starting point in order to understand how information about network effects interact with strategic pricing.

3 Equilibrium and welfare

We describe the property of equilibrium pricing and then comment on welfare, profits and consumer surplus.

3.1 Equilibrium pricing

Our first result characterizes Nash equilibrium prices.

[11] All the proofs, with the exception of the proof of Proposition 3, extend with no change. The proof of Proposition 3 needs to be modified in order to take into account the case in which an increase in γ is due to a decline in $\underline{\gamma}$. The details are available upon request from the authors.

[12] For example, a firm could sample some consumers, and then learn the level of influence of the consumers that influence the sampled consumers. This method allows firms to learn about consumers that, on average, have higher influence. In fact the average influence of a randomly selected consumer's neighbor is higher than the average influence of a randomly selected consumer, as observed by Krackhardt (1996), and as captured in our model by the fact that $\sum \overline{H}(l)l > \sum H(l)l$.

Proposition 2. *There exists a unique Nash equilibrium of the pricing stage. In equilibrium firms price symmetrically: the equilibrium price that firm X and firm Y charge to their non-targeted consumers is $p = \tau - \gamma k$. The equilibrium price that firm X and firm Y charge to their targeted consumers with influence $l \in \{1, ..., l^{\max}\}$ is*

$$p(l) \quad = \quad p + \frac{\gamma}{2-w}[k-l].$$

Proposition 2 shows that targeted consumers receive a price offer that declines with their level of influence. Targeted consumers with below-average influence—i.e. $l < k$– receive a price premium, whereas the above-average influential consumers receive a price discount. To gather some intuitions, we inspect how marginal changes in $p^X(l)$ alter firm X's profits. For a given strategy of firm Y, we obtain

$$\frac{\partial \Pi^X}{\partial p^X(l)} = \underbrace{\frac{w}{2}H(l)\left[1 - \frac{\gamma k}{\tau(\tau-\gamma k)}\overline{\mathsf{p}} - \frac{1}{\tau}p^X(l) + \frac{1}{\tau}\hat{p}^Y(l)\right]}_{\text{price-margin effect}} - \underbrace{\frac{w}{2\tau}H(l)p^X(l)}_{\text{classical demand effect}} - \underbrace{\frac{\hat{p}^X}{2}\frac{\gamma k}{\tau(\tau-\gamma k)}\frac{\partial \overline{\mathsf{p}}}{\partial p^X(l)}}_{\text{shift in network effects}}. \tag{5}$$

The first term is the increase in the price margin that firm X obtains from targeted customers with influence l; the second term is the decline of firm X's demand of targeted consumers with influence l as a consequence of the price increase. The third term reflects the decline in firm X aggregate demand due to a shift of network effects in favor of the competing firm. In fact, an increase in $p^X(l)$ decreases the expectation that consumers will adopt product X (because $\overline{\mathsf{p}}$ increases), and, therefore, the consumers who were indifferent between adopting X or Y will now strictly prefer to adopt product Y. If firm X increases the price for highly influential consumers, then the decrease in firm X's demand due to network effects is large, because their consumption decision has a large impact on the demand of other consumers. So, in equilibrium, the price of firm X is always declining in the influence of the targeted consumer.[13]

Figure 1 illustrates how equilibrium prices change when we increase γ and/or k, and when we increase w. First, when the average level of influence increases or the strength of network effects are stronger, firms have greater incentives to leverage network effects. This makes price

[13]Simple investigations of the above expressions also show that, as is standard in pricing games, firms' prices are strategic complements. However, from the viewpoint of a firm, the prices that the firm offers to targeted and to non-targeted consumers are strategic substitutes. By decreasing the price offered to non-targeted consumers, network effects for firm X becomes stronger, and, in turn, consumers are less price-sensitive, which allows firm X to increase the price charged to targeted consumers.

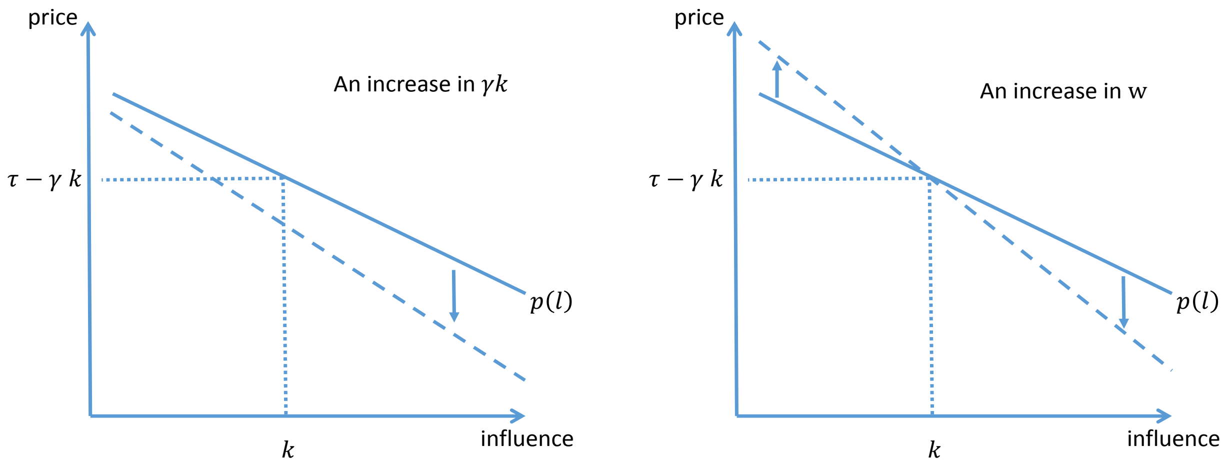

Figure 1 – The effect of γk and w on equilibrium price schedules.

competition fiercer, and more so for highly influential consumers. As a result, price offers decline for all consumers, and the decline is increasing in the level of influence.[14] Formally, taking the derivative of expression 5 with respect to γk and evaluating it at equilibrium, we observe that the marginal profit of firm X with respect to $p^X(l)$ is decreasing in γ by a factor proportional to $\partial \overline{\mathsf{p}}/\partial p^X(l)$, which, conditional on the number of consumers with given levels of influence, is increasing in l.

Second, when both firms are able to target more consumers, their pricing strategy becomes more aggressive for highly influential consumers, who receive higher discounts, and most of the surplus is extracted from non-influential consumers. This price-targeting effect is related to the change in competition that the two firms face for targeted consumers. Consider firm Y targeting a consumer with above-average influence $l > k$. An increase in w means that now it is more likely that the consumer is also targeted by firm X, and so the expected price that the consumer receives from firm X has declined. This implies that to compete with firm X, firm Y must increase the discount offered to the consumer. The opposite effect is present when firm Y targets a consumer with a below-average level of influence.

3.2 Profits, consumer surplus, and inefficiencies

The presence of consumers who are targeted by one firm but not the other, coupled with the fact that targeted consumers are offered a price that is declining with their level of influence,

[14]Note, in fact, that $p_X(l) = \tau - \gamma\frac{1-w}{2-w}k - \frac{\gamma l}{2-w}$.

introduces the possibility of misallocation: some consumers located closer to, say, firm X, may purchase the object from the other firm Y. The (loss from) misallocation equals

$$\begin{aligned} M(w) &= 2w(1-w)\sum_{l=1}^{k} H(l)\Pr\left[\theta \in \left[0, \frac{p(l)-p}{\tau}\right]\right] E\left[\theta | \theta \in \left[0, \frac{p(l)-p}{\tau}\right]\right] + \\ &+ 2w(1-w)\sum_{l=k+1}^{l^{\max}} H(l)\Pr\left[\theta \in \left[\frac{p(l)-p}{\tau}, 0\right]\right] E\left[\theta | \theta \in \left[\frac{p(l)-p}{\tau}, 0\right]\right]. \end{aligned} \tag{6}$$

Consumers who are either targeted by both firms or by neither, receive the same price offer for the two products and are exposed to identical network effects for the two products. Subsequently, such consumers always purchase the product that is closer to their idiosyncratic taste. Consider, now, a consumer who is targeted by one firm and not by the other (there is a mass of $2w(1-w)$ of such consumers). For concreteness, consider a consumer who is targeted by firm X. Two events lead to misallocation. First, the consumer favors product X, $\theta > 0$, but her level of influence is low, and so the price offer that she receives from firm X, $p(l)$, is higher than the one that she receives from firm Y, p. Second, the consumer favors product Y, $\theta < 0$, but her level of influence is high, and so she receives a much better price offer from firm X than from the competing firm Y.

The above discussion suggests that overall misallocation depends on how dispersed the prices charged to consumers with different influence levels are. Indeed, we can rewrite (6) as follows:

$$\begin{aligned} M(w) &= \frac{1}{2}\frac{w(1-w)}{\tau^2}\sum_{l=1}^{l^{\max}} H(l)[p(l)-p]^2 \\ &= \frac{1}{2}\frac{w(1-w)}{\tau^2}\sigma^2_{p(l)}, \end{aligned} \tag{7}$$

where we have denoted by $\sigma^2_{p(l)}$ the variance of $[p(1), ..., p(l^{\max})]$. Using Proposition 2, we have that $\sigma^2_{p(l)} = \frac{\gamma^2\sigma^2_H}{(2-w)^2}$, where $\sigma^2_H = \sum_l H(l)[l-k]^2$ is the dispersion of influence across consumers. Hence, misallocation is governed by two forces. First, misallocation is high when the number of consumers for whom a price differential exists is large. These consumers are targeted by one firm and not the other, and their mass is proportional to $w(1-w)$. Second, misallocation is high when consumers' choices are likely to be impacted by the price differentials they face. This is the case if price differentials are expected to be large—i.e., when prices are very dispersed– as captured by a large $\sigma^2_{p(l)}$, or when products are not very differentiated—i.e., when τ is small.

Misallocation is driven by *targeting mismatch* between the firms, and is therefore inherently linked to equilibrium profits and consumer surplus. The equilibrium profit of firm X is

$$\Pi^X = \frac{(1-w)p}{2} + \frac{w}{2} \sum H(l)p(l)\left[1 - p(l) + wp(l) + (1-w)p\right].$$

The first term represents the expected revenue of firm X from its non-targeted consumers. Firm X's expected demand from those consumers is just $1/2$, as firm Y either target them and offer an expected price that is equal to firm X's price; or firm Y does not target these consumers, in which case firm Y will offer them the same price that firm X offers.

The second term is firm X's expected revenue from its targeted consumers. The expected demand from a targeted consumer with influence l is higher than $1/2$ if, and only if, the price that firm X offers, $p(l)$, is lower than the expected price that firm Y offers, which is $wp(l) + (1-w)p$. This happens if, and only if, the targeted consumer has above-average influence.

As this discussion suggests, firms' revenue is, therefore, connected to the dispersion of prices. Indeed, using the fact $\sum H(l)p(l) = p$, and after some manipulation, we can rewrite the equilibrium firm X's profits as

$$\Pi^X = \frac{p}{2} - \tau M(w).$$

Clearly, $\Pi^X = \Pi^Y$; consumer surplus is then derived by using the following identity:

$$CS(w) + 2\Pi^X(w) = W^{\max} - M(w) \Rightarrow CS(w) = W^{\max} - p + M(w)(2\tau - 1),$$

where $W^{\max}$ is the maximal total surplus that can be generated by welfare matching of products to consumers.[15] To summarize:

Corollary 1. *In equilibrium, the level of misallocation is* $M(w) = \frac{1}{2}\frac{w(1-w)}{\tau^2}\sigma^2_{p(l)}$*, where* $\sigma^2_{p(l)} = \frac{\gamma^2}{(2-w)^2}\sigma^2_H$*; equilibrium firms' profits are* $\Pi(w) = \frac{p}{2} - \tau M(w)$*; and equilibrium consumers' surplus is* $CS(w) = W^{\max} - p + M(w)(2\tau - 1)$*.*

Proposition 3. *An increase in the dispersion of influence,* σ^2_H*, an increase in the strength of network effects* γ*, and a decrease in the degree of product differentiation,* τ*, lead to an increase in misallocation, a decrease in firms' profits and an increase in consumer surplus.*

An increase in the dispersion of influence leads to no change in the equilibrium price

[15] To be specific, $W^{\max} = -\frac{1}{4}\tau + \frac{1}{2}k\gamma$, and then $M(w) \in \left[0, \frac{\tau}{2}\right]$ because the minimal welfare without information costs equals $-\frac{3}{4}\tau + \frac{1}{2}k\gamma$, which corresponds to the case in which each consumer is matched with her less-preferred good.

schedule, but it does increase the dispersion of the price offers across consumers with different levels of influence because now there are more consumers with very high and very low influence levels. As a consequence, the level of misallocation rises, firms' profits decline and consumer surplus increases. An increase in the strength of network effects makes the price schedule offered to targeted consumers steeper. Hence, consumers with below-average influence receive higher price offers, and consumers with above-average influence receive lower price offers. Again, since this increases the dispersion of prices, the level of misallocation rises, and firms' profits decline. Consumer surplus increases because firms price more aggressively, and so consumers get, on average, better deals; and because the increase in γ leads to a shift up in consumers' utility.

3.3 How much data should firms be allowed to use in their pricing strategies?

A regulator may ban the use of data on individuals' levels of influence, may allow the use of such data, or may choose a more nuanced policy that includes taxes or subsidies for the use of such data. The following result provides guidance to regulators:

Proposition 4. *An increase in the information that firms have about network effects, w, increases misallocation, decreases firms' profits, and increases consumer surplus if $w < 2/3$; otherwise, the reverse holds.*

The first observation is that aggregate welfare and consumer surplus move in opposite directions. A second observation is the inherent non-monotonicity of both in the information available. We find that welfare is maximized when $w \in \{0, 1\}$; therefore, unless the information that firms have is very precise, to maximize welfare, a regulator may want to ban the use of data on individuals' levels of influence (or the collection of such data). If, on the other hand, the regulator seeks to maximize consumer surplus, she may need a more nuanced policy incentivizing firms to collect an intermediate level of data.[16] In search of advice for such regulators, the next section presents an enriched model in which firms' information levels are determined endogenously, and in which a regulator may be able to increase or decrease the information acquisition costs for firms.

[16] These conclusions are derived using the Hotelling model and, thus, assuming full consumers' participation. If the market is not fully covered, the conclusions of Proposition 3 and Proposition 4 regarding total surplus and profits will be attenuated by the fact that, for example, more information to firms, may increase network effects and, thus, consumers' participation.

Another consideration that is pertinent to a regulator's decision is the effect of the availability of information on consumers' influence on firms' entry decision.[17] To illustrate, consider the following stylized scenario: two firms, X and Y, decide whether to enter a new market, and suppose that they make their entry decisions sequentially; that is, first firm X makes its decision and then firm Y. Suppose, further, that there is a fixed cost of entering the market, say E. We then note that Proposition 3 makes the observation that if both firms enter, and for any $w > 0$, an increase in the dispersion of the influence levels and an increase in the strength of network effects lead to a decrease in firms' profits. This observation stands in sharp contrast to the monopoly case (see Fainmesser and Galeotti (2016)), in which an increase in the dispersion of the influence levels and or in the strength of network effects increases monopoly profits. This suggests, that if firms are allowed to price discriminate based on influence, and if $w > 0$, then an increase in the dispersion of the levels of influence across consumers and an increase in the strength of network effects increase the range of entry costs E, for which the unique subgame equilibrium of the entry game has only firm X enter the market. This effect disappears when firms are not allowed to discriminate based on consumers' influence levels, that is, if $w = 0$. Therefore, our analysis suggests that as long as competition can be maintained, a regulator may want to consider banning the use of information on individuals' influence levels in markets in which such levels are very dispersed, and products exhibit strong network effects.

4 Endogenous information acquisition

We now enrich the model to study firms' incentives to acquire information on network effects. In the first stage, each firm $J \in \{X, Y\}$ chooses how many consumers to sample, w^J and the pricing schedule, $\mathbf{p}^J$. Firms take these actions simultaneously. We postulate that the cost to a firm to choose w is quadratic, $C(w) = \alpha w^2/2$. In the second stage, consumers observe firms' choice and make their consumption decision as in the benchmark model. An equilibrium is an investment decision (w^X, w^Y) and a pricing strategy profile $(\mathbf{p}^X, \mathbf{p}^Y)$ such that $(w^X, \mathbf{p}^X)$ maximizes firm X's profits given $(w^Y, \mathbf{p}^Y)$ (and vice-versa), and given that consumers behave according to the equilibrium characterized in Proposition 1.

[17] This has come to light in recent public debates about the extent to which law should facilitate informational privacy, including, for example, a debate about whether antitrust authorities should refocus their present investigation of Google on how Google's control of large data sets about consumers' behavior may entrench monopoly power and harm consumers' welfare– see Newman (2013).

4.1 On the strategic nature of information on network effects

We first investigate the strategic relation between the incentives of firm X and firm Y to invest in information about the influence of consumers. That is, given equilibrium pricing, are the marginal profits of firm X with respect to w^X increasing or decreasing in the level of firm Y's information? To answer this question, we first characterize the Nash equilibrium prices for arbitrary levels of information across the two firms.

Proposition 5. *For arbitrary (w^X, w^Y), there exists a unique Nash equilibrium of the pricing stage. The equilibrium prices that firm X and firm Y charge to their non-targeted consumers are*

$$\begin{aligned} p^X &= \tau - \gamma k - \frac{k\gamma}{3\tau - 2k\gamma}\overline{\mathsf{p}}, \\ p^Y &= \tau - \gamma k + \frac{k\gamma}{3\tau - 2k\gamma}\overline{\mathsf{p}}. \end{aligned}$$

The equilibrium prices that firm X and firm Y charge to their targeted consumers with influence $l \in \{1, ..., l^{\max}\}$ are

$$\begin{aligned} p^X(l) &= p^X + \frac{\gamma(2p^X + w^Y p^Y)}{(\tau - \gamma k)(4 - w^X w^Y)}[k - l], \\ p^Y(l) &= p^Y + \frac{\gamma(2p^Y + w^X p^X)}{(\tau - \gamma k)(4 - w^X w^Y)}[k - l], \end{aligned}$$

where

$$\overline{\mathsf{p}} = -\frac{2\sigma_H^2\gamma(\tau - \gamma k)(3\tau - 2\gamma k)}{k\left[3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma_H^2(w^X + w^Y - w^X w^Y)\right]}(w^X - w^Y).$$

Proposition 5 extends Proposition 2 to situations in which one firm can target more consumers than the other firm. Figure 2 illustrates equilibrium pricing when firm X has an informational advantage—i.e., when $w^X > w^Y$. As in the symmetric case, firms charge a price that is declining in the level of influence of targeted consumers. However, firms now price differently. A few insights stand out.[18]

First, the most informed firm charges a higher average price, and a higher price to non-targeted consumers. To see why, suppose that we start from an equilibrium in which both firms have the same amount of information and then increase firm X's information. Keeping everything else constant, such an increase leads to a shift in network effects in favor of firm

[18] These insights are captured by a series of results, which are stated, proved and discussed in On-line Appendix B.

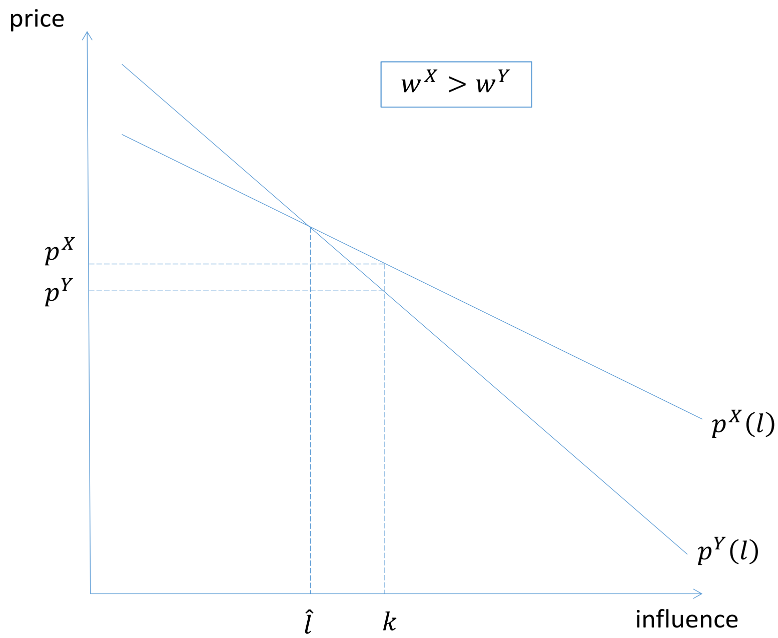

Figure 2 – Equilibrium price schedules when $w^X > w^Y$.

X, and, therefore, firm X's demand increases and firm Y's demand declines. This *adoption-externality* effect, which was muted in the symmetric information case, implies that firm X's demand becomes less elastic and firm Y's demand more elastic, and, therefore, firm X reacts by pricing less aggressively, and firm Y reacts by pricing more aggressively.

Second, relative to the other firm, the more-informed firm also chooses a smaller premium/discount per influence to targeted consumers. This is the result of the same price-targeting effect that led to increased premia/discounts when more information became available in the symmetric case (Proposition 2). Finally, the equilibrium price schedules depend directly on the dispersion of influence across consumers. In particular, if σ_H^2 increases, then the aforementioned adoption-externality effect becomes more pronounced. Subsequently, the most-informed firm prices even less aggressively, and the less-informed firm lowers its prices.

Building on Proposition 5, by taking the cross-partial derivatives of Π^X with respect to w^X and w^Y, and evaluating this expression at the Nash equilibrium pricing for a given (w^X, w^Y), we obtain

$$\begin{aligned}\frac{\partial \Pi^X}{\partial w^X \partial w^Y}|_{Eq.Pricing} &= \frac{1}{2\tau}\left[\sum H(l)p^X(l)p^Y(l) - p^X p^Y\right] \\ &= \frac{1}{2\tau} Cov\left(\{p^X(l)\}_{l\in D}, \{p^Y(l)\}_{l\in D}\right) \\ &= \frac{\gamma^2\sigma_H^2}{2\tau}\frac{(2p^X + w^Y p^Y)(2p^Y + w^X p^X)}{(\tau-\gamma k)^2(4 - w^X w^Y)^2} > 0,\end{aligned} \tag{8}$$

where the second equality follows by using the definition of covariance, and the last equality follows by using the explicit expression of the equilibrium pricing strategy in Proposition 5. Hence:

Proposition 6. *At Nash equilibrium pricing, decisions to invest in information acquisition about network effects are strategic complements.*

An increase in the information acquired by the competing firm increases the marginal profit of acquiring information, and this effect is greater the higher is the covariance of firms' targeted pricing strategies and the less products are differentiated. When firms acquire the same amount of information, their targeted pricing strategies coincide, and, therefore, the extent of strategic complementarities is pinned down by the dispersion of their pricing strategy.

Proposition 6 teaches us that all equilibria with endogenous information must be symmetric. We now characterize the set of (symmetric) equilibria.

4.2 Equilibrium and welfare

By taking the partial derivative of Π^X with respect to w^X and then imposing symmetry $w_X = w_Y = w$ and that pricing strategies are Nash, we obtain

$$\begin{aligned} \frac{\partial \Pi_X}{\partial w_X}|_{w_X=w_Y,Eq.Price} &= \frac{1}{2\tau}\frac{\gamma^2\sigma_H^2}{(2-w)^2} - \alpha w, \\ &= \frac{1}{2\tau}\sigma^2_{p(l)} - \alpha w, \end{aligned}$$

where the second equality follows by recalling the definition of $\sigma^2_{p(l)}$. The marginal benefit $MB(w) \equiv \sigma^2_{p(l)}/2\tau$ of acquiring information for a firm is, therefore, determined by the strength of strategic complementarities in information acquisition. This is positive at $w = 0$ and it is increasing and convex in w, reaching $\frac{\gamma^2\sigma_H^2}{2\tau}$ at $w = 1$. The marginal cost $MC(w) \equiv \alpha w$ is increasing in w and reaches α at $w = 1$.

Denote by w^* the smallest positive solution, if any, to $MB(w^*) = MC(w^*)$. Note that if $\frac{\gamma^2\sigma_H^2}{2\tau\alpha} < 1$, then there is only one solution to $MB(w) = MC(w)$. If $\frac{\gamma^2\sigma_H^2}{2\tau\alpha} \in [1, 32/27]$, there are two positive solutions, and otherwise there is no positive solution. Note, also, that whenever a w^* is well-defined, $w^* \leq 2/3$.

Proposition 7. *The following characterizes Nash equilibria of the game with endogenous information acquisition. Firms price according to Proposition 2, where w is the Nash equilibrium* *investment. In particular:*[19]

1. *If* $\frac{\gamma^2\sigma^2}{2\tau\alpha} > 32/27$*, then there is a unique stable Nash equilibrium, in which firms acquire information on all consumers—i.e.,* $w = 1$*;*
2. *If* $\frac{\gamma^2\sigma^2}{2\tau\alpha} \in (1, 32/27)$*, then there are two stable Nash equilibria. In one equilibrium, firms acquire information on all consumers* $w = 1$*, and in the other equilibrium, firms acquire information on* w^* *consumers;*
3. *If* $\frac{\gamma^2\sigma^2}{2\tau\alpha} < 1$*, then there is a unique stable Nash equilibrium, in which firms acquire information on* w^* *consumers.*

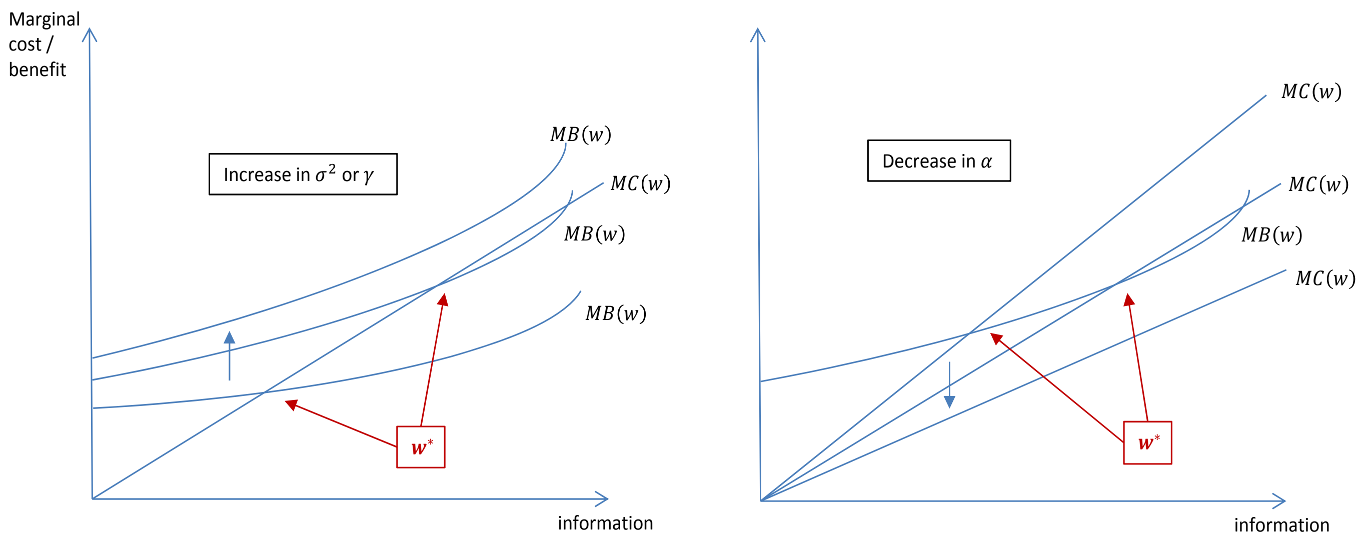

Figure 3 – Information acquisition equilibrium points and comparative statics.

The findings of Proposition 7 are illustrated in Figure 3. If strategic complementarities are sufficiently strong–i.e., $\frac{\gamma^2\sigma^2}{2\tau\alpha} > 32/27$– then $MB(w) > MC(w)$ for all $w \in [0, 1]$, and, therefore, firms fully invest in acquiring information. When strategic complementarities are moderate, there are three symmetric equilibria–two interior equilibria and a corner equilibrium with $w = 1$– but only w^* and the corner $w = 1$ are stable. When strategic complementarities are sufficiently weak, there is a unique equilibrium as defined by w^*.

Despite the multiplicity of equilibria, we obtain the following comparative statics result:

Proposition 8. *Suppose that we start from a symmetric equilibrium* w^* *and consider an increase in the dispersion of influence,* σ_H^2*; an increase in the strength of network effects,* γ*;*

[19]Stable equilibria are defined à la Ellison (2000). In our environment, this can be interpreted as follows: an equilibrium is stable if the basin of attraction of the best response functions has a positive measure.

an increase in the efficiency of information acquisition technology, $\frac{1}{\alpha}$; or a decrease in product differentiation, τ. Then, in the new symmetric stable equilibrium, both firms increase their investments in information.

Information acquisition increases when the marginal benefit of acquiring information increases or the marginal cost of acquiring information declines. The former increases whenever there is a change in the environment that increases the dispersion of the pricing strategy. This occurs when there is higher dispersion of influence across consumers, lower product differentiation, or greater strength of network effects. In particular, if products are highly differentiated, then consumers' adoption choice is not affected much by network effects, and so firms have less interest in gathering information about consumers' degree of influence. Similarly, if the strength of network effects is low–e.g., γ is small– or if there is not much dispersion in influence, then firms cannot use network effects effectively to lock in consumers; thus, they do not spend many resources in acquiring information about network effects.

We conclude by re-examining our welfare analysis in the more general environment, in which information acquisition is endogenous. Note, first, that relative to Section 3.2, the profit of a firm is diminished by the cost of acquiring information, and total inefficiencies in the system are also augmented by this cost—i.e.,

$$T(w) = M(w) + \alpha w^2 \text{ and } \Pi^X = \frac{p}{2} - \tau M(w) - \frac{1}{2}\alpha w^2.$$

We obtain:

Proposition 9. *Suppose that $\frac{\gamma^2\sigma^2}{2\tau\alpha} < 32/27$, and consider the equilibrium in which firms invest w^*. An increase in the dispersion of influence, σ_H^2; an increase in the strength of network effects; an increase in the efficiency of acquiring information, $\frac{1}{\alpha}$; and a decrease in the degree of product differentiation, τ, lead to an increase in misallocation, a decrease in firms' profits, and an increase in consumer surplus.*

Proposition 9 is in line with the results that are obtained when the information that firms have is exogenous—e.g., Proposition 3. In fact, endogenous information acquisition reinforces the effect that we pointed out in Section 3.2. We explain this now.

We begin with the dispersion of influence and first note that holding the amount of information acquired fixed, an increase in the dispersion of influence leads to an increase in misallocation. Considering endogenous information acquisition adds a second effect: it is more profitable for each firm to acquire information on a larger number of consumers, increasing

w^* symmetrically. The increase in w^* leads to an increase in the mass of targeted consumers, as well as in the per-influence price discount. Both further increase the dispersion of equilibrium prices across consumers with different influence, leading to an increase in the targeting mismatch, greater misallocation, lower profits, and higher consumer surplus.

An increase in the strength of network effects or a decrease in the level of product differentiation works through similar channels. First, holding the information available to firms constant, an increase in γ leads to a decrease in the average price level, as well as to an increase in the per-influence discount. That is, increased network effects intensifies price competition, especially for the more influential consumers, and leads to an increase in the targeting mismatch. The result is a first-order decrease in profits. Magnifying this effect is the increase in w^*, which leads to an additional decrease in profits. Consumers, on the other hand, benefit from the increased competition, which leads to lower prices and amplifies the targeting mismatch.

Finally, an improvement in the information acquisition technology generates two countervailing effects. Holding w^* fixed, prices remain constant and firms' profits go up due to the reduced costs of information acquisition. However, making information acquisition cheaper leads to an increase in the amount of information acquired (w^*), which leads to a decrease in firms' profits and an increase in consumer surplus. This latter effect dominates the direct effect of the decreased cost of information acquisition.

5 Discussion

This paper develops a stylized model of competition with network effects in which firms can acquire and use individual-level information on network effects. In our model, consumers are heterogeneous along two dimensions: their idiosyncratic preference for one product versus another, and their influence—i.e., their network location.

There is an established literature on the role of information in consumers' idiosyncratic preferences in price discrimination and competition.[20] For example, Katz (1984) finds that in a monopolistic competition, allowing for price discrimination may lead to a welfare increase or decrease, depending on whether the purchases made by consumers who face price discrimination constitute a small or a large fraction of total purchases. Instead, we focus on the role of information in consumers' network position and influence, which does not enter into consumers' utility function directly, but only via the equilibrium outcome.

[20] We refer to a survey by Stole (2007) for an overview of the literature.

Notably, both the aforementioned literature and the current paper effectively make the simplifying assumption that there is no correlation between a consumer's network position and her idiosyncratic preference. On the other hand, there is now a body of literature studying potential correlations between network position and preferences—examples include work on homophily (for classic references see Kandel (1978) and Currarini, Jackson, and Pin (2009)), as well as work on unobserved heterogeneity in networks dating back to Manski (1993)).

Most ecently, Eliaz and Spiegler (2016) study how a display advertising platform, which knows the network structure, can exploit that correlation to extract profit from firms. In our environment, firms can exploit correlations between network position and preferences, thus introducing a host of new questions regarding information acquisition and multidimensional price discrimination, ranging from the characterization of effective pricing strategies to their effect on market power.

Appendix

Equilibrium pricing

Proof of Proposition 2. Proposition 2 obtains from Proposition 5 when $w^X = w^Y = w$. ■

Proof of Proposition 5. Our strategy of proof is as follows. We first prove that an equilibrium exists and then characterize the equilibrium. Finally, we argue that the equilibrium that we characterized is unique.

Existence. We describe the steps we use to prove existence. The formal statements (e.g., Lemma 1, Lemma 3) and related proofs are reported in online Appendix A.

First, we show that for any firm $J \in \{X, Y\}$, Π^J is concave in $\left(p^J, \left\{p^J\left(l\right)\right\}_{l\in\{0,\ldots,l^{\max}\}}\right)$. This is covered by Lemma 1. Lemma 1 implies that each firm has a unique best response to any actions set by its opponent.

Second, we derive the first-order conditions that capture firms' best responses and show that there exists some $\overline{\mu} > 0$ such that for all $\mu \geq \overline{\mu}$, if a competitor's prices lie in an interval $[-\mu, \mu]$, then a firm would like to set its prices to be strictly inside the interval $[-\mu, \mu]$. This is covered by Lemma 3.

As a result, showing that there exists an equilibrium when firms' strategy spaces are bounded in an interval $[-\mu, \mu]$ for any $\mu > \overline{\mu}$ for some $\overline{\mu} > 0$, will be sufficient to show that an equilibrium exists when firms' strategy spaces are not bounded.

Finally, for the case in which firms' strategy spaces are bounded, we note that our game is a concave 2-person game that satisfies the conditions of Rosen's (1965) Theorem 1. Thus, an equilibrium exists.

Equilibrium characterization. The profit of firm X can be written as follows:

$$\begin{aligned}\Pi^X &= (1-w^X)p^X\left[(1-w^Y)D^X(p^X,p^Y)+w^Y\sum H(l)D^X(p^X,p^Y(l);l)\right]+\\ &+ w^X\sum H(l)p^X(l)\left[(1-w^Y)D^X(p^X(l),p^Y;l)+w^Y D^X(p^X(l),p^Y(l);l)\right].\end{aligned}$$

Substituting the probabilities of adoption from (2), we obtain

$$\begin{aligned}\Pi^X &= \frac{(1-w^X)p^X}{2}\left[1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{1}{\tau}p^X+\frac{1}{\tau}(1-w^Y)p^Y+\frac{w^Y}{\tau}\sum H(l)p^Y(l)\right]+\\ &+ \frac{w^X}{2}\sum H(l)p^X(l)\left[1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{1}{\tau}p^X(l)+\frac{1}{\tau}(1-w^Y)p^Y+\frac{1}{\tau}w^Y p^Y(l)\right].\end{aligned}$$

For what follows, it is convenient to write $\hat{p}^Y=(1-w^Y)p^Y+w^Y\sum H(l)p^Y(l)$ and $\hat{p}^X=(1-w_x)p^X+w^X\sum H(l)p^X(l)$. Note that $\hat{p}^Y$ is the expected price that a randomly selected consumer observes of firm Y. Taking the derivative with respect to p^X, we obtain

$$\begin{aligned}\frac{\partial\Pi^X}{\partial p^X} &= \frac{(1-w^X)}{2}\left[1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{2}{\tau}p^X+\frac{1}{\tau}(1-w^Y)p^Y+\frac{w^Y}{\tau}\sum H(l)p^Y(l)-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}p^X(1-w^X)\right]-\\ &- \frac{w^X(1-w^X)}{2}\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\sum H(l)p^X(l),\end{aligned}$$

where we have used the fact that $\partial\overline{p}/\partial p^X=(1-w_x)$. Imposing the FOC, we obtain that $\frac{\partial\Pi^X}{\partial p^X}=0$ if and only if

$$1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{2}{\tau}p^X+\frac{1}{\tau}\hat{p}^Y-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\hat{p}^X=0. \tag{9}$$

Similarly, taking the derivatives with respect to $p^X(l)$ for an arbitrary l, we obtain

$$\begin{aligned}\frac{\partial\Pi^X}{\partial p^X(l)} &= -\frac{(1-w^X)p^X}{2}\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\frac{\partial\overline{p}}{\partial p^X(l)}-\frac{w^X}{2}\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\sum H(s)p^X(s)\frac{\partial\overline{p}}{\partial p^X(l)}+\\ &+ \frac{w^X}{2}H(l)\left[1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{2}{\tau}p^X(l)+\frac{1}{\tau}(1-w^Y)p^Y+\frac{1}{\tau}w^Y p^Y(l)\right].\end{aligned}$$

Using $\frac{\partial\overline{p}}{\partial p^X(l)}=\overline{H}(l)w_x=H(l)lw_x/k$ and imposing the FOC, we obtain

$$1-\frac{1}{\tau}\frac{\gamma k}{\tau-\gamma k}\overline{p}-\frac{2}{\tau}p^X(l)+\frac{1}{\tau}(1-w^Y)p^Y+\frac{1}{\tau}w^Y p^Y(l)-\frac{1}{\tau}\frac{\gamma l}{\tau-\gamma k}\hat{p}^X=0. \tag{10}$$

The following observations follow from conditions (9) and (10). First, the sum of (10) across l types should be equal to zero, which leads to

$$1 - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\overline{p} - \frac{2}{\tau}\sum H(l)p^X(l) + \frac{1}{\tau}\hat{p}^Y - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\hat{p}^X = 0,$$

and equating with (9) implies that $p^X = \sum H(l)p^X(l)$, and, therefore, $p^X = \hat{p}^X$. The same must hold for Y, and so $p^Y = \sum H(l)p^Y(l)$ and $p^Y = \hat{p}^Y$. Using this fact, we can rewrite (9) as

$$1 - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\overline{p} - \frac{2}{\tau}p^X + \frac{1}{\tau}p^Y - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}p^X = 0.$$

The analogous implication for firm Y is

$$1 + \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\overline{p} - \frac{2}{\tau}p^Y + \frac{1}{\tau}p^X - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}p^Y = 0.$$

Equating the two expressions, we get that $p^X - p^Y = -\frac{2\gamma k}{3\tau - 2\gamma k}\overline{p}$, and using these equations to solve for p^X and p^Y yields

$$p^Y = \tau - \gamma k + \frac{k\gamma}{3\tau - 2k\gamma}\overline{p} \text{ and } p^X = \tau - \gamma k - \frac{k\gamma}{3\tau - 2k\gamma}\overline{p}.$$

We now take condition (10) again and impose that $p^X = \hat{p}^X$

$$1 - \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\overline{p} - \frac{2}{\tau}p^X(l) + \frac{1}{\tau}(1 - w^Y)p^Y + \frac{1}{\tau}w^Y p^Y(l) - \frac{1}{\tau}\frac{\gamma l}{\tau - \gamma k}p^X = 0,$$

and, similarly,

$$1 + \frac{1}{\tau}\frac{\gamma k}{\tau - \gamma k}\overline{p} - \frac{2}{\tau}p^Y(l) + \frac{1}{\tau}(1 - w^X)p^X + \frac{1}{\tau}w^X p^X(l) - \frac{1}{\tau}\frac{\gamma l}{\tau - \gamma k}p^Y = 0.$$

After some algebra, we get

$$\begin{aligned} p^Y(l) &= \tau + \frac{\tau\gamma k\overline{p}}{(\tau - \gamma k)(3\tau - 2\gamma k)} - \frac{\gamma}{4 - w^X w^Y}\left(1 + \frac{\gamma k\overline{p}}{(\tau - \gamma k)(3\tau - 2\gamma k)}\right)[2(l + k) - kw^X w^Y] - \\ &- \frac{\gamma w^X(l - k)}{4 - w^X w^Y}\left(1 - \frac{\gamma k\overline{p}}{(\tau - \gamma k)(3\tau - 2\gamma k)}\right) \end{aligned}$$

Using the expression of p^X and p^Y, note that

$$\frac{p^Y}{\tau - \gamma k} = 1 + \frac{k\gamma}{(\tau - \gamma k)(3\tau - 2\gamma k)}\overline{p} \text{ and } \frac{p^X}{\tau - \gamma k} = 1 - \frac{k\gamma}{(\tau - \gamma k)(3\tau - 2\gamma k)}\overline{p},$$

and substituting these in $p^Y(l)$, we get

$$p^Y(l) = \frac{1}{\tau - \gamma k}\left[\tau p^Y - \frac{\gamma}{4 - w^X w^Y}\left[p^Y[2(l+k) - kw^X w^Y] + w^X(l-k)p^X\right]\right]$$

and the analogous steps for $p^X(l)$ lead to

$$p^X(l) = \frac{1}{\tau - \gamma k}\left[\tau p^X - \frac{\gamma}{4 - w^X w^Y}\left(p^X[2(l+k) - kw^X w^Y] + w^Y(l-k)p^Y\right)\right].$$

From this last formula, making the change $2(l+k) = 4k + 2(l-k)$, and arranging terms, we get the expressions of $p^X(l)$ and $p^Y(l)$ in the proposition. Furthermore, with some additional algebra,

$$\bar{p} = -\frac{2\sigma^2\gamma(\tau - \gamma k)(3\tau - 2\gamma k)(w^X - w^Y)}{k\left[3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2[w^X + w^Y - w^X w^Y]\right]}.$$

To complete the characterization, we prove in Lemma 4 that under the described pricing behavior, condition A holds. The proof of Lemma 4 is relegated to the online Appendix A.

Uniqueness. To establish equilibrium uniqueness, it is sufficient to note that Lemma 3 implies that there exists some $\bar{\mu} > 0$, such that all equilibria must reside inside the interval $(-\bar{\mu}, \bar{\mu})$. Therefore, all equilibria must satisfy all first-order conditions with equality. Our characterization above finds that only one pricing profile satisfies all first-order conditions with equality. ■

Profits, consumer surplus and welfare

Proof of Proposition 3. The comparative statics with respect to σ_H^2 and γ are immediate. An increase in τ decreases $M(w)$. Regarding profit, an increase in τ increases profits by increasing p. It is also easy to check that the derivative of $\tau M(w)$ with respect to τ is $-M(w) < 0$ and, therefore, is negative. So, an increase in τ decreases τM, which increases profits. For consumer surplus, we have that an increase in τ increases p, and so CS goes down. Moreover, the derivative of $M(w)(2\tau - 1)$ with respect to τ is $-2M(w)(\tau - 1)/\tau$, and so is negative because $\tau > 1$. ■

Proof of Proposition 4. The comparative statics with respect to w follows by noticing that the sign of the derivative of $M(w)$ with respect to w is positive if $w < 2/3$ and negative if $w > 2/3$. ■

Endogenous information acquisition

Proof of Proposition 7.

First, an implication of Proposition 7 is that, in equilibrium, firms must acquire the same amount of information. The following characterizes the possible symmetric solutions. It is easy to check that these candidates satisfy the second-order condition.

Part 1: If $\frac{\gamma^2\sigma^2}{2\tau\alpha} > 32/27$, then $MB(w) > MC(w)$ for all positive w, and so there is a unique equilibrium in which $w = 1$. To see this, we rewrite the equilibrium condition as

$$\frac{\gamma^2\sigma_H^2}{2\tau\alpha} = w(2-w)^2 \tag{11}$$

and note that the maximum of $w(2-w)^2$ (which is first increasing, then decreasing and concave) is 32/27.

Part 2: We have already established that for $w \in [0,1]$, $w(2-w)^2$ is concave, first increasing, then decreasing, and has a unique maximum at 32/27. Noting that $w(2-w)^2|_{w=0} = 0$ and $w(2-w)^2|_{w=1} = 1$ establishes that if $\frac{\gamma^2\sigma_H^2}{2\tau\alpha}$ for any $\frac{\gamma^2\sigma^2}{2\tau\alpha} \in (1, 32/27)$, condition (11) holds at exactly two points, say $\underline{w} < \overline{w}$, such that $0 < \underline{w} < \overline{w} < 1$. Therefore, if $\frac{\gamma^2\sigma_H^2}{2\tau\alpha} \in (1, 32/27)$, then there exist $0 < \underline{w} < \overline{w} < 1$ so that $MB(w) = MC(w)$ if, and only if, $w \in \{\underline{w}, \overline{w}\}$. This implies that there are three equilibrium values of w: $\{\underline{w}, \overline{w}, 1\}$. However, only $\underline{w}$ and 1 are stable. To see why, note that for all $w \in (\underline{w}, \overline{w})$, $MB(w) < MC(w)$, and that for all $w \in (0, \underline{w}) \cup (\overline{w}, 1)$, $MB(w) > MC(w)$.

Part 3: Repeating the exercise from Part 2 and recalling that $w(2-w)^2|_{w=1} = 1$ completes the proof that there is a unique interior equilibrium w^* that is stable, and such that, for all $w \in (0, w^*)$, $MB(w) > MC(w)$, and that for all $w \in (w^*, 1)$, $MB(w) < MC(w)$. ■

Proof of Proposition 8 Note that w^* increases with $\frac{\gamma^2\sigma_H^2}{2\tau\alpha}$. Subsequently, an increase in σ_H^2 or in γ or a decrease in α or in τ increase w^*. If there are multiple symmetric equilibria, we note that the new symmetric stable equilibrium must be either at the new w^*, or at $w = 1$, which is, by definition, greater than the original w^*. ■

Proof of Proposition 9 Recall that an increase in the dispersion of influence, an increase in γ, or a decrease in α or in τ lead to an increase in w^*. First, note that

$$M(w^*) = \frac{1}{2\tau^2}w^*(1-w^*)\sigma_{p(l)}^2 = \frac{\alpha}{\tau}w^{*2}(1-w^*),$$

where the last equality follows using the equilibrium condition for w^*. It is immediate to see that $M(w^*)$ is increasing in w^* for all $w^* \leq 2/3$. Therefore, an increase in σ_H^2 increases $M(w^*)$ and, therefore, increases $CS(w^*)$. Since an increase in σ_H^2 leads to an increase in w^* and in $M(w^*)$, it follows that the firm's profit declines with σ_H^2.

Second, the above considerations, together with the fact that p declines with an increase in γ, directly implies the comparative static results with regard to γ (recall, also, that $W^{\max}$ is increasing in γ— see footnote 15). Third, we prove the comparative statics with respect to α. We first show that $M(w^*)$ declines if α increases. To see this, note that

$$\frac{dM(w^*)}{d\alpha} = \frac{1}{\tau}\left[w^{*2}(1-w^*) + \alpha w^* \frac{dw^*}{d\alpha}[2-3w^*]\right]$$

and applying the implicit function theorem to the equilibrium condition $\frac{\gamma^2\sigma^2}{2\tau\alpha} - w^*(2-w^*)^2 = 0$, we obtain that

$$\frac{dw^*}{d\alpha} = -\frac{\gamma^2\sigma^2}{2\tau\alpha^2(2-w^*)(2-3w^*)},$$

and, therefore,

$$\begin{aligned}\frac{dM(w^*)}{d\alpha} &= w^{*2}(1-w^*) - w^* \frac{\gamma^2\sigma^2}{2\tau\alpha(2-w)} \\ &= w^{*2}(1-w^*) - w^{*2}(2-w) < 0,\end{aligned}$$

where the second equality follows by using the equilibrium condition of w^*. The fact that $M(w^*)$ declines with an increase in α, immediately implies that $CS(w^*)$ declines with an increase in α. To see that the profit increases with an increase in α, it is necessary to observe that αw^* decreases with an increase in α. This follows by noticing that an increase in α leads to a decrease in w^* and that, in equilibrium, $\alpha w^* = \frac{1}{2\tau}\frac{\gamma^2\sigma^2}{(2-w^*)^2}$ and the LHS is increasing in w^*. This concludes the proof of the proposition.

Finally, a decrease in τ increases the equilibrium w, and this leads to an increase in $M(w)$, an increase in $T(w)$, and a decrease in the profit (note that p goes down because τ has increased, and w has increased and $\tau M(w)$ has increased because w has increased). With regard to CS, it is possible to show that a decrease in τ leads to an increase in CS (keeping constant $W^{\max}$). To see this,

$$\frac{dCS}{d\tau} = -1 + 2M(w) + (2\tau - 1)\left[\frac{\partial M(w)}{\partial \tau} + \frac{\partial M}{\partial w}\frac{dw}{d\tau}\right]$$

and

$$\begin{aligned}
\frac{\partial M(w)}{\partial \tau} &= -\frac{M(w)}{\tau} \\
\frac{\partial M(w)}{\partial w} &= \frac{w\alpha(2-3w)}{\tau} \\
\frac{dw}{d\tau} &= -\frac{w(2-w)}{\tau(2-3w)}.
\end{aligned}$$

Therefore,

$$\frac{dCS}{d\tau} = -1 + \frac{w^2\alpha}{\tau^2}\left(1 - w - (2\tau - 1)(2 - w)\right) < 0,$$

where the inequality follows because $1 - w - (2\tau - 1)(2 - w) < 1 - w - (2\tau - 1)(1 - w) = 2(1 - \tau)(1 - w) \geq 0$. Hence, a decrease in product differentiation leads to more misallocation, lower profits but higher consumer surplus (keeping constant the $W^{\max}$). ■

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Consumption Stage

Proof of Proposition 1. We first assume that a consumption equilibrium exists and is unique and we characterize it. We then consider an auxiliary game and show that: (a) the set of equilibria of the two games coincides, and (b) the auxiliary game has a unique equilibrium.

Consumption equilibrium characterization. Assume that a consumption equilibrium exists and is unique. We now prove that if $(\mathbf{p}^X, \mathbf{p}^Y)$ satisfies condition A then the equilibrium probability that consumer i purchases product X is given by 2, i.e.,

$$D_i^X(\mathbf{p}^X, \mathbf{p}^Y) = \frac{1}{2}\left(1 - \frac{1}{\tau}\mathsf{p}_i - \frac{\gamma k}{\tau(\tau - \gamma k)}\overline{\mathsf{p}}\right).$$

Consumer i prefers X to Y if, and only if, $\theta_i - \mathsf{p}_i + \gamma k \mathsf{A}(\mathbf{x}_{-i}) > 0$ ($\theta_i > \mathsf{p}_i - \gamma k \mathsf{A}(\mathbf{x}_{-i})$) which happens with probability $x_i = \frac{\tau - \mathsf{p}_i + \gamma k \mathsf{A}(\mathbf{x}_{-i})}{2\tau}$; note that our model implies that θ_i is uniformly distributed in the support $[-\tau, \tau]$. Noting that $A^X(\mathbf{x}_{-i}) = 1 - A^Y(\mathbf{x}_{-i})$ this can be re-written as $x_i = \frac{\tau - \mathsf{p}_i + 2\gamma k A^X(\mathbf{x}_{-i}) - \gamma k}{2\tau}$.

Let $x(l) = E[x_i | l_i = l]$ and $y(l) = E[y_i | l_i = l]$; solving for $\mathsf{A}(\mathbf{x}_{-i})$ we obtain:

$$\begin{aligned}
\mathsf{A}(\mathbf{x}_{-i}) &= \sum_l \overline{H}(l)(x(l) - y(l)) = \sum_l \overline{H}(l)(2x(l) - 1) \\
&= \sum_l \overline{H}(l)\left(E\left[\frac{\tau - \mathsf{p}_i + \gamma k \mathsf{A}(\mathbf{x}_{-i})}{\tau} | l_i = l\right] - 1\right) = -\frac{\overline{\mathsf{p}}}{\tau} + \frac{\gamma k}{\tau}\mathsf{A}(\mathbf{x}_{-i})
\end{aligned}$$

so that

$$\mathsf{A}(\mathbf{x}_{-i}) = -\frac{\overline{\mathsf{p}}}{\tau - \gamma k}$$

and the probability that i buys from firm X is

$$D^X(p_i) = \frac{1}{2}\left(1 - \frac{1}{\tau}\mathsf{p}_i - \frac{\gamma k}{\tau(\tau - \gamma k)}\overline{\mathsf{p}}\right).$$

An auxiliary game. We prove existence and uniqueness by proving existence and uniqueness in an auxiliary game, and then showing that the set of equilibria of the two games coincide.

Consider an economy with the same timeline as in our model with the following two differences:

1. There is only one firm (X) producing a divisible good, and charging linear prices—that

is, consumer i is charged p_i per one unit of the good.

2. Consumers are heterogeneous only with respect to their levels of influence, and consumer i's ex post utility function is:

$$u_i = -\frac{1}{2}x_i^2 - \frac{1}{2}(1-x_i)^2 + \frac{\gamma}{\tau}\sum_{j\in N_j} x_i x_j + \frac{\gamma}{\tau}\sum_{j\in N_j}(1-x_i)(1-x_j) - \frac{1}{\tau}p_i x_i.$$

Recall that this is an auxiliary utility function used only for the purpose of the proof, and that there isn't necessarily any product that will correspond to the function. In this auxiliary game, the expected utility of consumer i is given by

$$U_i = -\frac{1}{2}x_i^2 - \frac{1}{2}(1-x_i)^2 + \frac{\gamma k}{\tau}x_i A^X(x_{-i}) + \frac{\gamma k}{\tau}(1-x_i)(1-A^X(x_{-i})) - \frac{1}{\tau}p_i x_i.$$

Differentiating w.r.t x_i we get

$$\begin{aligned} \frac{\partial U_i}{\partial x_i} &= -2x_i + 1 + \frac{\gamma k}{\tau}\left(2A^X(x_{-i}) - 1\right) - \frac{1}{\tau}p_i \\ \frac{\partial^2 U_i}{(\partial x_i)^2} &= -2 \end{aligned}$$

and the first-order condition yields

$$x_i = \frac{1}{2}\left(1 + \frac{\gamma k}{\tau}\left(2A^x\left(\mathbf{x}_{-i}\right) - 1\right) - \frac{1}{\tau}p_i\right).$$

We note that this is equivalent to the equilibrium conditions in the consumption stage of this paper (with $p_i = p_i^X - p_i^Y$) and thus if we prove that for any price schedule there exists a unique equilibrium in the auxiliary game that satisfies the first-order conditions, then it is also the case that there exists a unique equilibrium satisfying the equilibrium condition of this paper. We next show that for any price schedule there exists a unique equilibrium in the auxiliary game.

Existence. When applied to the auxiliary utlity function, Proposition 1 in Glaeser and Scheinkman (2002) implies that a sufficient condition for existence of equilibrium is $\forall_{p\in\mathbb{R}}\exists_{\overline{x}\geq 0}\forall_{x\leq\overline{x}}\frac{\partial u\left(\overline{x},(x)_{j\neq i}\right)}{\partial x_i} \leq 0$ or $\forall_{p\in\mathbb{R}}\exists_{\overline{x}\geq 0}\forall_{x\leq\overline{x}} - 2\overline{x} + 1 + \frac{\gamma k}{\tau}\left(2A^X\left((x)_{j\neq i}\right) - 1\right) - \frac{1}{\tau}p \leq 0$. To see that this holds when $\gamma k < 1$, note that $-2\overline{x} + 1 + \frac{\gamma k}{\tau}\left(2A^X\left((x)_{j\neq i}\right) - 1\right) = -2\overline{x} + 1 + \frac{\gamma k}{\tau}(x-1) \leq -2\overline{x} + 1 + \frac{\gamma k}{\tau}(\overline{x} - 1)$. Therefore, it is sufficient to show that $\forall_{p\in\mathbb{R}}\exists_{\overline{x}\geq 0}\left(\frac{\gamma k}{\tau} - 2\right)\overline{x} + 1 - \frac{\gamma k}{\tau} - p \leq 0$, which is true for any $\frac{\gamma k}{\tau} < 2$.

Uniqueness. When applied to the auxiliary utlity function, Proposition 3 in Glaeser and Scheinkman (2002) implies that a sufficient condition for uniqueness of equilibrium is $\forall_i \left|\frac{\partial^2 u_i}{(\partial x_i)^2}\right| > \left|\frac{\partial^2 u_i}{\partial x_i \partial A^X(\mathbf{x}_{-i})}\right|$ or $2 > 2\frac{\gamma k}{\tau}$. ■

Equilibrium pricing and consumption

Lemma 1. *For any firm $J \in \{X, Y\}$, Π^J is concave in $\left(p^J, \left\{p^J(l)\right\}_{l\in\{0,\ldots,l^{\max}\}}\right)$.*

Proof of Lemma 1. Let $\mathcal{H}^X$ denote the negative of the Hessian matrix of firm X's profit function with respect to $\left(p^X, \left\{p^X(l)\right\}_{l\in\{0,\ldots,l^{\max}\}}\right)$. To prove that Π^X is concave in $\left(p^X, \left\{p^X(l)\right\}_{l\in\{0,\ldots,l^{\max}\}}\right)$ it is sufficient to show that $\mathcal{H}^X$ is positive definite. For the purposes of this proof, it will be useful to denote $p^X(-1) = p^X$. Thus, we need to prove that Π^X is concave in $\left\{p^X(l)\right\}_{l\in\{-1,0,\ldots,l^{\max}\}}$ (the proof for $J = Y$ is by symmetry), and the negative of the Hessian matrix of firm X's profit function can be written as follows

$$\mathcal{H}^X = \begin{bmatrix} -\frac{\partial^2\Pi^X}{\partial(p^X(-1))^2} & -\frac{\partial^2\Pi^X}{\partial(p^X(-1))\partial(p^X(0))} & \cdots & -\frac{\partial^2\Pi^X}{\partial(p^X(-1))\partial(p^X(l^{\max}))} \\ -\frac{\partial^2\Pi^X}{\partial(p^X(0))\partial(p^X(-1))} & -\frac{\partial^2\Pi^X}{\partial^2(p^X(0))} & \cdots & -\frac{\partial^2\Pi^X}{\partial(p^X(0))\partial(p^X(l^{\max}))} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2\Pi^X}{\partial(p^X(l^{\max}))\partial(p^X(-1))} & -\frac{\partial^2\Pi^X}{\partial(p^X(l^{\max}))\partial(p^X(0))} & \cdots & -\frac{\partial^2\Pi^X}{\partial^2(p^X(l^{\max}))} \end{bmatrix}.$$

Our first step is to prove the following claim:

Claim 1. *There exists $\Gamma \in \mathbb{R}_+$ and $(p_j, b_j, c_j)_{j\in\{-1,0,\ldots,l^{\max}\}} \in \left(\mathbb{R}^3_+\right)^{l^{\max}+2}$ such that for every $s, t \in \{-1, 0, \ldots, l^{\max}\}$,*

$$-\frac{\partial^2\Pi^X}{\partial p^X(s)\,\partial p^X(t)} = \frac{1}{\tau}\left(p_s p_t \Gamma\left[b_t c_s + b_s c_t\right] + 2p_s \mathbf{1}_{\{s=t\}}\right),$$

where $\mathbf{1}_{\{s=t\}}$ is the indicator function for when $s = t$.

Let $\Gamma = \frac{\gamma}{\tau-\gamma k} \in [0, 1)$. To prove the claim note the following:

$$-\frac{\partial^2\Pi^X}{\partial\left(p^X\right)^2} = \frac{1}{\tau}\left(\left(\frac{1-w^X}{2}\right)^2\Gamma\left(2k+2k\right) + 2\frac{1-w^X}{2}\right)$$

$$-\frac{\partial^2\Pi^X}{\partial p^X(s)^2} = \frac{1}{\tau}\left(\left(\frac{w^X H(s)}{2}\right)^2\Gamma\left(2s+2s\right) + 2\frac{w^X H(s)}{2}\right)$$

$$-\frac{\partial^2\Pi^X}{\partial p^X \partial p^X(s)} = \frac{1}{\tau}\left(\frac{w^X H(s)}{2}\frac{\left(1-w^X\right)}{2}\Gamma\left[2k+2s\right]\right)$$

$$-\frac{\partial^2 \Pi^X}{\partial p^X(t)\,\partial p^X(s)} = \frac{1}{\tau}\left(\frac{w^X H(t)}{2}\frac{w^X H(s)}{2}\Gamma(2t+2s)\right)$$

It is then left to note that $(p_j, b_j, c_j)_{j\in\{-1,0,\ldots,l^{\max}\}} \in \left(\mathbb{R}^3_+\right)^{l^{\max}+2}$ can be chosen to be the following:

1. $p_{-1} = \frac{1-w^X}{2}$; and for $s \in \{0,1,\ldots l^{\max}\}$, $p_s = \frac{w^X H(s)}{2}$
2. $b_{-1} = 1$; and for $s \in \{0,1,\ldots l^{\max}\}$, $b_s = 1$
3. $c_{-1} = 2k$; and for $s \in \{0,1,\ldots l^{\max}\}$, $c_s = 2s$

Our second step in proving Lemma 1 is to recall Lemma 2 in the Online Appendix of Fainmesser and Galeotti (2016). That is,

Lemma 2. *(Fainmesser and Galeotti 2016) Let* $\mathcal{G} = \left(\left(p_s p_t \Gamma\left[b_t c_s + b_s c_t\right] + 2p_s \mathbf{1}_{\{s=t\}}\right)\right)_{s,t\in\{-1,0,1,\ldots,l^{\max}\}}$. *Then, the determinant of* $\mathcal{G}$ *is given by*

$$\det(\mathcal{G}) = \left(2_j^{l^{\max}} p_j\right)\left(4 + 4\Gamma\sum_j (b_j c_j p_j) - \Gamma^2 \sum_{i<j}\left(p_i p_j \left[b_j c_i - b_i c_j\right]^2\right)\right)$$

Since for all j, $b_j, c_j, p_j \geq 0$, and since $sign\{\det(\mathcal{G})\} = sign\{K\det(\mathcal{G})\}$ for any $K > 0$, to prove Lemma 1 it is then sufficient to prove that:

$$\Gamma^2 \sum_{i<j}\left(p_i p_j \left[b_j c_i - b_i c_j\right]^2\right) < 4 + 4\Gamma\sum_j (b_j c_j p_j)$$

which is what we prove now in the third and final step in the proof of Lemma 1.

Now note that

$$\Gamma = \gamma\frac{1}{\tau - \gamma k} < \frac{1}{2l^{\max}}2 = \frac{1}{l^{\max}}.$$

and

$$\begin{aligned}
\sum_{i<j}\left(p_i p_j \left[b_j c_i - b_i c_j\right]^2\right) &\leq \sum_i \sum_j \left(p_i p_j \left[b_j c_i - b_i c_j\right]^2\right) \\
&= \sum_i \sum_j \left(p_i p_j \left[c_i - c_j\right]^2\right) \\
&\leq \sum_i \sum_j \left(p_i p_j \left[2l^{\max}\right]^2\right) \\
&= 4\left[l^{\max}\right]^2 \sum_i p_i \sum_j p_j \\
&= 4\left[l^{\max}\right]^2 \frac{1}{4} \\
&= \left[l^{\max}\right]^2
\end{aligned}$$

so we have that

$$\begin{aligned}\Gamma^2 \sum_{i<j} \left(p_i p_j \left[b_j c_i - b_i c_j \right]^2 \right) &< \left(\frac{1}{l^{\max}} \right)^2 \left(l^{\max} \right)^2 \\ &= 1 \\ &< 4 + 4\Gamma \sum_j \left(b_j c_j p_j \right).\end{aligned}$$

This complete the proof of the concavity of Π^X. The proof for Π^Y is equivalent. ■

Lemma 3. *There exists $\overline{\mu} > 0$ such that for all $\mu > \overline{\mu}$ and $J \in \{X, Y\}$, if $p^J, \{p_l^J\}_{l\in\{0,1,\dots,l^{\max}\}} \in [-\mu, \mu]$, then*

$$\arg \max_{p^{-J}, \{p_l^{-J}\}_{l\in\{0,1,\dots,l^{\max}\}}} \Pi^{-J} \in (-\mu, \mu)^{l^{\max}+2}.$$

Proof of Lemma 3. Recall that Firms' prices are strategic complements, whereas different prices of the same firm are strategic substitutes. As a result, to prove the Lemma is it sufficient to show that there exists $\overline{\mu} > 0$ such that for all $\mu > \overline{\mu}$ the following inequalities hold:

$$\varphi_1 = \left(\frac{\partial \Pi^X}{\partial p^X(l)} | p^X, \{p^X(s)\}_{s\neq l} = -\mu, \text{ and } p^Y, \{p^Y(s)\}_s, p^X(l) = \mu \right) < 0$$

$$\varphi_2 = \left(\frac{\partial \Pi^X}{\partial p^X(l)} | p^X, \{p^X(s)\}_{s\neq l} = \mu, \text{ and } p^Y, \{p^Y(s)\}_s, p^X(l) = -\mu \right) > 0$$

$$\varphi_3 = \left(\frac{\partial \Pi^X}{\partial p^X} | \{p^X(s)\}_s = -\mu, \text{ and } p^Y, \{p^Y(s)\}_s, p^X = \mu \right) < 0$$

$$\varphi_4 = \left(\frac{\partial \Pi^X}{\partial p^X} | \{p^X(s)\}_s = \mu, \text{ and } p^Y, \{p^Y(s)\}_s, p^X = -\mu \right) > 0.$$

We begin with φ_1, which can be written as follows

$$\varphi_1 = \frac{w^X H(l)}{2} \left(1 - \frac{1}{\tau}\mu \left[1 + \frac{\gamma k}{\tau - \gamma k} \left[2\frac{l}{k} H(l) w^X - \left(2 + \frac{l}{k} \right) \left(1 - w^X \right) - w^Y - w^X \sum_{s\neq l} \frac{s+l}{k} H(s) \right] \right] \right).$$

Thus, it is sufficient to show that

$$\frac{\gamma k}{\tau - \gamma k} \left[2\frac{l}{k} H(l) w^X - \left(2 + \frac{l}{k} \right) \left(1 - w^X \right) - w^Y - w^X \sum_{s\neq l} \frac{s+l}{k} H(s) \right] > -1.$$

And in fact

$$\frac{\gamma k}{\tau-\gamma k}\left[2\frac{l}{k}H(l)\,w^{X}-\left(2+\frac{l}{k}\right)\left(1-w^{X}\right)-w^{Y}-w^{X}\sum_{s\neq l}\frac{s+l}{k}H(s)\right]$$
$$>\quad \frac{\gamma k}{\tau-\gamma k}\left[-\left(2+\frac{l}{k}\right)-1\right]=-3\frac{\gamma k}{\tau-\gamma k}-\frac{\gamma l}{\tau-\gamma k}>-\frac{3}{4}-\frac{1}{4}=-1$$

where the first inequality holds because $2\frac{l}{k}H(l)\,w^{X}-\left(2+\frac{l}{k}\right)\left(1-w^{X}\right)-w^{Y}-w^{X}\sum_{s\neq l}\frac{s+l}{k}H(s)$ is increasing in w^X and decreasing in w^X, andthe second inequality holds because $\gamma l^{\max}<1/2$.

We now turn to φ_2, which can be reduced to:

$$\varphi_2=\frac{w^{X}H(l)}{2}\left[1+\frac{1}{\tau}\mu\left(1-2\frac{\gamma k}{\tau-\gamma k}\right)\right].$$

Thus, it is sufficient to show that

$$\varphi_2=1>2\frac{\gamma k}{\tau-\gamma k}$$

which holds because $\gamma k<1/2$.

We now turn to φ_3, which can be reduced to:

$$\varphi_3=\frac{\left(1-w^{X}\right)}{2}\left[1+\frac{1}{\tau}\mu\left(-1+\frac{\gamma k}{\tau-\gamma k}\left[4w^{X}-1\right]\right)\right].$$

Thus, it is sufficient to show that

$$\frac{\gamma k}{\tau-\gamma k}\left[4w^{X}-1\right]<1$$

and indeed

$$\frac{\gamma k}{\tau-\gamma k}\left[4w^{X}-1\right]\leq 3\frac{\gamma k}{\tau-\gamma k}<\frac{1}{4}$$

where the first inequality holds because $w^X\leq 1$ and the second inequality holds because $\gamma k<1/2$.

Finally, we turn to φ_4, which can be reduced to:

$$\varphi_4=\frac{\left(1-w^{X}\right)}{2}\left[1+\frac{1}{\tau}\mu\left(1-\frac{\gamma k}{\tau-\gamma k}\right)\right].$$

Thus, it is sufficient to show that

$$1-\frac{\gamma k}{\tau-\gamma k}>0$$

which holds because $\gamma k < 1/2$. This completes the proof of Lemma 3. ■

Lemma 4. *Equilibrium prices are such that condition A holds, i.e., for all i, $\frac{1}{2}\left(1-\frac{1}{\tau}\mathsf{p}_i-\frac{\gamma k}{\tau(\tau-\gamma k)}\overline{\mathsf{p}}\right)\in(0,1)$.*

Proof of Lemma 4. We focus, without loss of generality, on the case that $w^X \geq w^Y$, so that $\overline{\mathsf{p}} \leq 0, \mathsf{p} > 0, \mathsf{p}\left(l^{\max}\right) > 0, \mathsf{p}\left(0\right) < 0$. Therefore,

$$\begin{aligned} \max\{p_i\} &= p_X - p_Y\left(l^{\max}\right) > 0 \\ \inf\{p_i\} &= p_X - p_Y\left(0\right) < 0 \end{aligned}$$

and

$$\begin{aligned} \max\left\{\frac{1}{2}\left(1-\frac{1}{\tau}\mathsf{p}_i-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right)\right\} &\leq \frac{1}{2}\left(1-\frac{1}{\tau}\inf\{\mathsf{p}_i\}-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) \\ \min\left\{\frac{1}{2}\left(1-\frac{1}{\tau}\mathsf{p}_i-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right)\right\} &= \frac{1}{2}\left(1-\frac{1}{\tau}\max\{\mathsf{p}_i\}-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) \end{aligned}$$

Part 1: We first show that

$$\frac{1}{2}\left(1-\frac{1}{\tau}\inf\{\mathsf{p}_i\}-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) < 1.$$

Note that,

$$\begin{aligned} \frac{1}{2}\left(1-\frac{1}{\tau}\inf\{\mathsf{p}_i\}-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) &= \frac{1}{2}\left(1-\frac{1}{\tau}\left(p^X-p^Y\left(0\right)\right)-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) \\ &= \frac{1}{2}\left(1+\frac{\gamma(2p^Y+w^Xp^X)}{\tau(\tau-\gamma k)(4-w^Xw^Y)}k+\frac{2k\gamma}{\tau(3\tau-2k\gamma)}\overline{\mathsf{p}}-\frac{\gamma k}{\tau\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) \\ &= \frac{1}{2}\left(1+\frac{\gamma(2p^Y+w^Xp^X)}{\tau(\tau-\gamma k)(4-w^Xw^Y)}k-\frac{\gamma k}{\left(3\tau-2k\gamma\right)\left(\tau-\gamma k\right)}\overline{\mathsf{p}}\right) < 1 \end{aligned}$$

if, and only if,

$$\frac{2p_Y+w^Xp^X}{\tau(4-w^Xw^Y)}-\frac{1}{3\tau-2k\gamma}\overline{\mathsf{p}} < \frac{\tau-\gamma k}{\gamma k}.$$

or, equivalently,

$$\frac{2+w_X}{\tau(4-w_Xw_Y)}+\left(\frac{\left(2-w_X\right)k\gamma}{\tau(4-w_Xw_Y)}-1\right)\frac{1}{\left(3\tau-2k\gamma\right)\left(\tau-\gamma k\right)}\overline{p} < \frac{1}{\gamma k} \tag{12}$$

The first observation is that if $\gamma k \to 0$ then condition 12 because the RHS of 12 tends to $+\infty$. Next, note that the RHS of 12 decreases in γ, whereas the LHS weakly increases in γ.

Therefore, it is sufficient that 12 holds for large γ (recall that $\gamma l^{\max} \le 1/2$). A useful way of rewriting inequality 12 is by substituting $\bar{\mathsf{p}}$ and rearranging:

$$\frac{(2+w^X)\gamma k}{\tau(4-w^Xw^Y)}+\left(\frac{\tau(4-w^Xw^Y)-\left(2-w^X\right)k\gamma}{\tau(4-w^Xw^Y)}\right)\left(\frac{2\sigma^2\gamma^2(w^X-w^Y)}{3\tau(\tau-\gamma k)(4-w^Xw^Y)-2\gamma^2\sigma^2[w^X+w^Y-w^Xw^Y]}\right)<1.$$

This can be verified by using the following bounds. Note that $\frac{w^X-w^Y}{3\tau(\tau-\gamma k)(4-w^Xw^Y)-2\gamma^2\sigma^2[w^X+w^Y-w^Xw^Y]}$ and $\frac{\tau(4-w^Xw^Y)-\left(2-w^X\right)k\gamma}{\tau(4-w^Xw^Y)}$ are decreasing in w^Y and increasing in w^X, and therefore we can find the following upper bounds:

$$\begin{aligned}\frac{2\sigma^2\gamma^2(w^X-w^Y)}{3\tau(\tau-\gamma k)(4-w^Xw^Y)-2\gamma^2\sigma^2[w^X+w^Y-w^Xw^Y]}\le\frac{2\sigma^2\gamma^2}{12\tau(\tau-\gamma k)-2\gamma^2\sigma^2} &< \frac{1}{11}\\ \frac{\tau(4-w^Xw^Y)-\left(2-w^X\right)k\gamma}{\tau(4-w^Xw^Y)} &\le \frac{4\tau-k\gamma}{4\tau}<1\end{aligned}$$

and, in addition, we have that

$$\frac{2+w^X}{\tau(4-w^Xw^Y)}\gamma k<\frac{1}{2}.$$

Part 2: Next we turn to prove that $\frac{1}{2}\left(1-\frac{1}{\tau}\mathsf{p}_i-\frac{\gamma k}{\tau(\tau-\gamma k)}\bar{\mathsf{p}}\right)>0$. We do this by proving that $\frac{1}{2}\left(1-\frac{1}{\tau}\max\{\mathsf{p}_i\}-\frac{\gamma k}{\tau(\tau-\gamma k)}\bar{\mathsf{p}}\right)>0$ or

$$p_X-p_Y\left(l^{\max}\right)+\frac{\gamma k}{\tau-\gamma k}\bar{\mathsf{p}}<\tau.$$

Let $\lambda=\frac{\gamma k}{\tau-\gamma k}$ and let $\beta=\frac{2\gamma k}{3\tau-2\gamma k}$. Because $\gamma k<\frac{1}{2}$ and $\tau\ge 0$ it is the case that $\beta\le\lambda<1$, and

$$p_X-p_Y\left(l^{\max}\right)+\lambda\bar{\mathsf{p}}=\tau-\gamma k-\frac{\beta\bar{p}}{2}-\left(\tau-\gamma k+\frac{\beta\bar{\mathsf{p}}}{2}+\frac{\gamma(2p_Y+w_Xp_X)}{(\tau-\gamma k)(4-w^Xw^Y)}[k-l^{\max}]\right)+\lambda\bar{\mathsf{p}}.$$

Then, letting $\Delta=l^{\max}-k$, we have:

$$\begin{aligned}p^X-p^Y\left(l^{\max}\right)+\lambda\bar{\mathsf{p}} &= (\lambda-\beta)\,\bar{\mathsf{p}}+\frac{\gamma\Delta}{4-w^Xw^Y}\left(2+w^X+\frac{\left(2-w^X\right)\beta\bar{\mathsf{p}}}{2(\tau-\gamma k)}\right)\\ &= \left(\frac{\lambda\tau}{3\tau-2\gamma k}\right)\bar{\mathsf{p}}+\frac{\gamma\Delta}{4-w^Xw^Y}\left(2+w^X+\left(2-w^X\right)\frac{\lambda\bar{\mathsf{p}}}{(3\tau-2\gamma k)}\right)\end{aligned}$$

Recall that $\bar{\mathsf{p}}=-\frac{2\sigma^2\gamma^2(w^X-w^Y)(3\tau-2\gamma k)}{\lambda[3\tau(\tau-\gamma k)(4-w^Xw^Y)-2\gamma^2\sigma^2(w^X+w^Y-w^Xw^Y)]}$. Now let

$$\eta=-\bar{\mathsf{p}}\left(\frac{\lambda}{3\tau-2\gamma k}\right)=\frac{2\sigma^2\gamma^2(w^X-w^Y)}{3\tau(\tau-\gamma k)(4-w^Xw^Y)-2\gamma^2\sigma^2\left(w^X+w^Y-w^Xw^Y\right)}$$

which implies that

$$p^X - p^Y\left(l^{\max}\right) + \lambda\bar{\mathsf{p}} = -\eta\tau + \frac{\gamma\Delta}{4 - w^X w^Y}\left(2 + w^X - \left(2 - w^X\right)\eta\right).$$

Now let $\delta = \frac{2+w^X}{4-w^X w^Y}$ and note that $\delta \leq 1$. Hence,

$$p^X - p^Y\left(l^{\max}\right) + \lambda\bar{\mathsf{p}} = \gamma\Delta\left(\delta - \frac{2 - w^X}{4 - w^X w^Y}\eta\right) - \eta\tau.$$

As a result, it is sufficient to prove that

$$\gamma\Delta\left(\delta - \frac{2 - w^X}{4 - w^X w^Y}\eta\right) < (1 + \eta)\,\tau.$$

Let us expand the expression on the LHS. It becomes:

$$\frac{\gamma\Delta}{4 - w^X w^Y}\frac{3\tau\left(2 + w^X\right)(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2\left[\left(2 + w^X\right)\left(w^X + w^Y - w^X w^Y\right) + \left(2 - w^X\right)\left(w^X - w^Y\right)\right]}{3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2\left(w^X + w^Y - w^X w^Y\right)},$$

and

$$1 + \eta = \frac{3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2\left[\left(2 - w^X\right)w^Y\right]}{3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2\left(w^X + w^Y - w^X w^Y\right)}$$

Observe that $(2 + w^X)(w^X + w^Y - w^X w^Y) + (2 - w^X)(w^X - w^Y) = w^X(4 - w^X w^Y)$. It is therefore sufficient to show that

$$\gamma\Delta\left(3\tau\left(2 + w^X\right)(\tau - \gamma k) - 2\gamma^2\sigma^2 w^X\right) < \left(3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2\left(2 - w^X\right)w^Y\right)\tau.$$

Now,

$$\frac{\partial(L.H.S)}{\partial w^X} = \gamma\Delta\left(3\tau(\tau - \gamma k) - 2\gamma^2\sigma^2\right) > 0$$

and

$$\frac{\partial(R.H.S)}{\partial w^X} = \left(-3\tau(\tau - \gamma k) + 2\gamma^2\sigma^2\right)w^Y\tau < 0$$

where both inequalities hold because $3(1 - \gamma k) > 2\sigma^2\gamma^2$. Moreoever,

$$\frac{\partial(L.H.S)}{\partial w^Y} = 0$$

and

$$\frac{\partial(R.H.S)}{\partial w^Y} = \left(-3\tau(\tau - \gamma k)w^X - 2\gamma^2\sigma^2\left(2 - w^X\right)\right)\tau < 0$$

Therefore it is sufficient to show the inequality is satisfied when $w^X = w^Y = 1$ which it does, given that $\gamma\Delta < 1$ and $\tau \geq 1$ $\blacksquare$

For Online Publication. Online Appendix B: Additional results on pricing and consumption.

We provide additional results on equilibrium pricing and equilibrium consumption, for the case where firms have different levels of information with regard to the level of influence of consumers, i.e., $w^X > w^Y$.

When the two firms have different levels of information, they charge different prices.

Corollary 2. *Suppose that $w^X > w^Y$. Then firm X charges non-targeted consumers a higher price than firm Y. Furthermore, the price premium-discount per influence charged by firm X is lower than the one charged by firm Y. Overall, $p^X(l)$ is higher than $p^Y(l)$ if and only if*

$$4\gamma\sigma_H^2(\tau - \gamma k) > [k - l][3\tau(\tau - \gamma k) - 2\sigma_H^2\gamma^2].$$

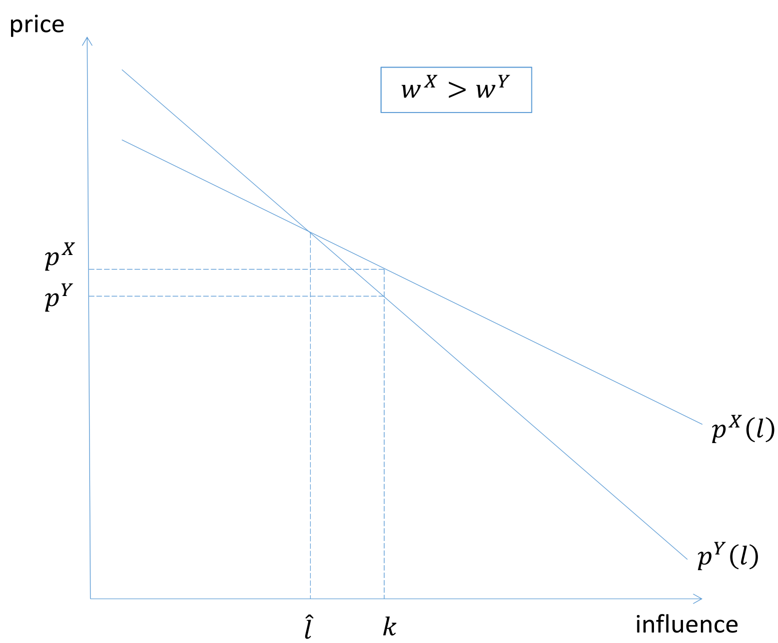

Figure 4 – Equilibrium price schedules when $w^X > w^Y$.

Figure 4 illustrates Corollary 2; we let $\hat{l}$ be such that $4\gamma\sigma_H^2(\tau - \gamma k) = [k - \hat{l}][3\tau(\tau - \gamma k) - 2\sigma_H^2\gamma^2]$. To understand the intuition of Corollary 2 suppose that we start from an equilibrium in which both firms have the same amount of information and let us increase firm X's information. First, keeping everything else constant, such an increase leads to a shift in network effects in favor of firm X and therefore firm X's demand increases and firm Y's demand declines. This *adoption-externality* effect implies that firm X's demand becomes less elastic and firm Y's demand more elastic, and therefore firm X reacts by pricing less aggressively and firm Y more aggressively. Formally, the adoption-externality effect is reflected in the observation that an increase in w^X leads to a decrease in the relative expected price of product X, for a randomly selected neighbor of a consumer. That is:

$$\frac{\partial \overline{\mathsf{p}}}{\partial w^X}|Eq. = \sum_l \overline{H}(l)p^X(l) - p^X < \sum_l H(l)p^X(l) - p^X = 0,$$

where the first inequality follows because the distribution of influence of a randomly selected consumer's neighbor *first order stochastic dominates* the distribution of influence of a randomly selected consumer, and because in equilibrium $p^X(l)$ is a decreasing function of influence.

A second effect is related to the change in competition that the two firms face towards targeted consumers. Firm X now targets more consumers, which implies that, holding all else equal, the average price that highly influential consumers are charged by firm X is lower than the price they are charged by firm Y, and vice versa for the less influential consumers. This *price-targeting effect* implies that to compete with firm X, firm Y must increase the discount it offers to targeted highly influential consumers, and at the same time firm Y can charge higher premia to less influential targeted consumers. Formally, note that an increase in w_X leads to a first-order increase in the price premium-discount per influence charged by firm Y

$$\frac{\partial^2 \Pi^Y}{\partial p^Y(l)\partial w^X}|_{Eq.} = \frac{w^Y}{2\tau}H(l)\left[\underbrace{\frac{\gamma k}{\tau - \gamma k}\left(\sum \overline{H}(s)p^X(s) - p^X\right)}_{\text{adoption-externality effect}} + \underbrace{p^X(l) - p^X}_{\text{price-targeting effect}}\right]$$

The first term captures the adoption-externality effect which, as described above, is negative for firm Y and leads to a shift down of $p^Y(l)$. The second term $p^X(l) - p^X$ is the price-targeting effect. It is positive for below-average influential consumers and negative for above-average consumers. As a result, firm Y increases the premia for low influence consumers and the discounts for high influence consumers.

In the symmetric case captured by Corollary 2, the adoption-externality effect is muted by the symmetry, and the price-targeting effect translates onto the observation that when both firms obtain additional information, the price premia-discounts per influence charged by both firms increase.

We next evaluate how equilibrium pricing depends on the variance of the distribution of influence when firms sample different fractions of consumers. The findings of Proposition 10 are illustrated in Figure 5.

Proposition 10. *Suppose $w^X > w^Y$. If σ_H^2 increases then firm X prices less aggressively–p^X and $p^X(l)$ increase for all l– whereas firm Y prices more aggressively– p^Y and $p^Y(l)$ decrease for all l.*

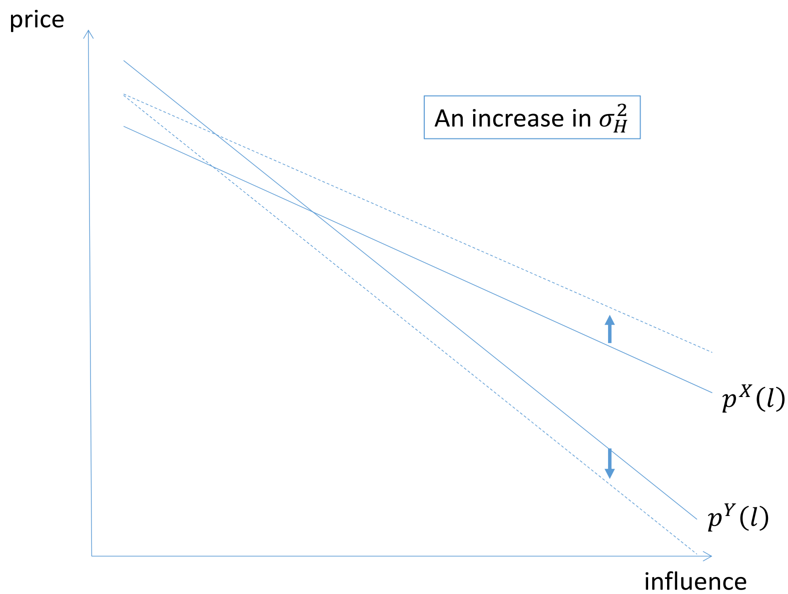

Figure 5 – An increase in the dispersion of influence when $w^X > w^Y$.

That is, an increase in the dispersion of influence amplifies the adoption externality effect. In particular, when the distribution of influence becomes more dispersed then the expected price that a randomly selected neighbor of a consumer observes from firm $J = X, Y$ declines, *ceteris paribus*. To see this note that

$$\sum_l \overline{H}(l)p^X(l) \propto \sum \overline{H}(l)[k-l] \propto k^2 - \sum H(l)l^2 = -\sigma_H^2$$

So, whether $\overline{\mathsf{p}}$ increases or decreases with an increase in σ_H^2 depend on whether this effect is stronger or weaker for firm X relative to firm Y. In turn, this effect for firm J is stronger the larger is w^J and the larger is the price premium/discount. In view of the above result, we have two contrasting effects. On the one hand, since $w^X > w^Y$ the effect is stronger for firm X. On the other hand, because $w^X > w^Y$, the slope of $p^X(l)$ is lower than the one of $p^Y(l)$ and so the effect is stronger for firm Y.

Intuitively, the steeper slope of firm Y's price schedule is a competitive reaction to firm X's informational advantage. The direct effect on firm X's information is, therefore, a first-order one, and, therefore, when $w^X > w^Y$, an increase in σ_H^2 generates higher aggregate demand for firm X. Once this is established, we follow the same intuition we have developed above.

Firm X benefits more from the adoption-externality effect and can price less aggressively as compared to firm Y, who, instead, gains demand only by charging low prices.

Consumption

If firms have the same amount of information, their pricing strategy is the same (see Corollary 2) and, therefore, the demand for product X equals the demand for product Y. In contrast, a firm with more information is more effective in leveraging network effects, and as a consequence faces a higher demand.

Proposition 11. *The expected probability that a consumer with level of influence l purchases product X at the equilibrium price schedules is:*

$$x(l) = \frac{1}{2} + \frac{\gamma\left[\gamma\sigma_H^2 + 3(\tau - \gamma k)(l-k)\right]}{\left[3\tau(\tau-\gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma_H^2(w^X + w^Y - w^X w^Y)\right]}(w^X - w^Y)$$

Suppose $w^X > w^Y$, then whether a consumer with influence l is more likely to buy product X relative to product Y is determined by the sign of $\gamma\sigma_H^2 + 3(\tau-\gamma k)(l-k)$. The first part of this expression, $\gamma\sigma_H^2$, captures the overall increase in demand for product X due to increased network effects attributed to better price targeting. The second part, $3(\tau-\gamma k)(l-k)$, captures the slope of the demand facing by firm X. Firm X can target many consumers, thus, relative to firm Y, there are more influential consumers who receive discounted price offers and more non-influential consumers who receive price offers above average. The result is that there is a threshold $\tilde{l} = k - \frac{\gamma\sigma_H^2}{3(\tau-\gamma k)}$ so that consumers with influence $l > \tilde{l}$ are more likely to purchase product X.

Despite consumers with a lower level of influence are more likely to purchase product Y, the demand of firm X is, in aggregate, larger than the demand of firm Y.

Corollary 3. *Suppose that $w^X > w^Y$. Then, the aggregate demand for product X is larger than the aggregate demand for product Y—i.e., $\sum_l H(l)x(l) > 1/2$. Moreover, an increase in the dispersion of influence, σ_H^2, a decrease in the compatibility of the two products, $1/\gamma$, and a decrease in the degree of product differentiation, τ, increase the aggregate demand for product X.*

An increase in the dispersion of influence and/or an increase in γ allows the firm with more information to increase network effects in its favor, and, in turn, it makes it more likely that consumers purchase its product. Likewise, a decrease in the degree of product differentiation makes network effects more important in determining adoption decisions, and this gives an advantage to the firm with more information on network effects.

Proofs

Proof of Corollary 2. If $w^X > w^Y$ then $\overline{\mathsf{p}} < 0$ which implies that $p^X > p^Y$. Next, we sign $p_X(l) - p_Y(l)$. Note that when $l = k$, $p^X(k) - p^Y(k) = p^X - p^Y > 0$. More generally, $p^X(l) - p^Y(l) > 0$ iff:

$$p^X - p^Y + \frac{\gamma[k-l]}{(\tau - \gamma k)(4 - w^X w^Y)}[2(p^X - p^Y) + w^Y p^Y - w^X p^X] > 0$$

and using the expression for p^X and p^Y the condition is equivalent to

$$-\frac{2k}{3\tau - 2k\gamma}\overline{\mathsf{p}} + -\frac{\gamma k[k-l]\left(4 - w^Y - w^X\right)}{(\tau - \gamma k)(4 - w^X w^Y)\left(3\tau - 2k\gamma\right)}\overline{\mathsf{p}} - \frac{[k-l]\left(w^X - w^Y\right)}{(4 - w^X w^Y)} > 0.$$

Now let $G = 3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma^2[w^X + w^Y - w^X w^Y]$ so that $\overline{\mathsf{p}} = -\frac{2\sigma^2\gamma(\tau - \gamma k)(3\tau - 2\gamma k)(w^X - w^Y)}{kG}$. Then, the above condition becomes

$$\frac{(w^X - w^Y)\gamma}{G}\left[4\gamma\sigma^2(\tau - \gamma k) - [k-l]\left[3\tau(\tau - \gamma k) - 2\gamma^2\sigma^2\right]\right] > 0.$$

Since $w^X > w^Y$ and $G > 0$, this is equivalent to $4\gamma\sigma^2(\tau - \gamma k) - [k-l]\left[3\tau(\tau - \gamma k) - 2\gamma^2\sigma^2\right] > 0$.

To see that the price premium-discount per unit of influence is lower for firm X than for firm Y, note that $\left|p^X(l+1) - p^X(l)\right| < \left|p^Y(l+1) - p^Y(l)\right|$ if and only if $2\left[p^X - p^Y\right] < w^X p^X - w^Y p^Y$, and using the expressions for p^X and p^Y we get that this is equivalent to

$$\frac{(w^X - w^Y)(4 - w^X w^Y)(\tau - \gamma k)}{G}\left[3\tau(\tau - \gamma k) - 2\gamma^2\sigma^2\right] > 0$$

and since $w^X > w^Y$ and $G > 0$, this is equivalent to $3\tau(\tau - \gamma k) - 2\gamma^2\sigma^2 > 0$, which holds for any $\gamma l^{\max} < 1/2$. ■

Proof of Proposition 10. We first prove that if $w^X > w^Y$ then $sign\frac{\partial\left(\frac{1}{\overline{\mathsf{p}}}\right)}{\partial\sigma_H^2} > 0$ (and thus $\frac{\partial\overline{\mathsf{p}}}{\partial\sigma_H^2} < 0$). This follows because

$$\frac{1}{\overline{\mathsf{p}}} = -\frac{3\tau(4 - w^X w^Y)k}{2\sigma_H^2\gamma(3\tau - 2\gamma k)(w^X - w^Y)} + \frac{\gamma k[w^X + w^Y - w^X w^Y]}{(\tau - \gamma k)(3\tau - 2\gamma k)(w^X - w^Y)}$$

which implies that $sign\frac{\partial\left(\frac{1}{\overline{\mathsf{p}}}\right)}{\partial\sigma_H^2} = sign(w_X - w_Y) > 0$, where the inequality follows because

$w^X > w^Y$.

Since p^X declines in $\overline{\mathsf{p}}$ and p^Y increases in $\overline{\mathsf{p}}$ it follows that $\partial p^X/\partial\sigma_H^2 > 0$ and $\partial p^Y/\partial\sigma_H^2 < 0$.

We now study the effect of a change in σ_H^2 on $p^X(l)$ and $p^Y(l)$. First,

$$\frac{\partial p^X(l)}{\partial\sigma_H^2} = \frac{k\gamma}{3\tau - 2k\gamma}\frac{\partial\overline{\mathsf{p}}}{\partial\sigma_H^2}\left(-1 + \frac{\gamma[k-l]}{(\tau-\gamma k)(4-w^Xw^Y)}\left[w^Y - 2\right]\right)$$

and therefore $\frac{\partial p^X(l)}{\partial\sigma_H^2} > 0$ if, and only if,

$$\frac{\partial\overline{\mathsf{p}}}{\partial\sigma^2}\left[-1 + \frac{\gamma[k-l]}{(\tau-\gamma k)(4-w^Xw^Y)}(w^Y-2)\right] > 0.$$

When $w^X > w^Y$ we know that $\frac{\partial\overline{\mathsf{p}}}{\partial\sigma_H^2} < 0$ and therefore $\frac{\partial p^X(l)}{\partial\sigma_H^2} > 0$ if, and only if,

$$\left[1 + \frac{\gamma[k-l]}{(\tau-\gamma k)(4-w^Xw^Y)}(2-w^Y)\right] > 0.$$

This condition holds because:

$$\begin{aligned} & 1 + \frac{\gamma[k-l]}{(\tau-\gamma k)(4-w^Xw^Y)}(2-w^Y) > 1 + \frac{\gamma[k-l^{\max}]}{(\tau-\gamma k)(4-w^Xw^Y)}(2-w^Y) \\ > \quad & 1 - \frac{\gamma l^{\max}}{(\tau-\gamma k)(4-w^Xw^Y)}(2-w^Y) > 1 - \frac{1}{2(4-w^Xw^Y)}2 > 1 - \frac{1}{4} > 0 \end{aligned}$$

The same arguments are used to prove that $p^Y(l)$ declines with σ_H^2. ■

Proof of Proposition 11. The ex-ante probability that a consumer with level of influence l buys from firm X is

$$x(l) = \frac{1}{2} - \frac{1}{2\tau}E[\mathsf{p}|l] - \frac{\gamma k}{2\tau}\frac{\overline{\mathsf{p}}}{\tau-\gamma k},$$

where $E[\mathsf{p}|l] = w^Xp^X(l) + (1-w^X)p^X - w^Yp^Y(l) - (1-w^Y)p^Y$. Using equilibrium pricing we obtain that

$$E[p|l] = -\frac{2k\gamma}{3\tau-2k\gamma}\overline{p} + \frac{2\gamma[k-l](w^X-w^Y)3\tau(\tau-\gamma k)}{G}$$

where we recall that $G = 3\tau(\tau-\gamma k)(4-w^Xw^Y) - 2\gamma^2\sigma_H^2[w^X + w^Y - w^Xw^Y]$. Therefore,

$$x(l) = \frac{1}{2} + \frac{\gamma\left[\gamma\sigma_H^2 + 3(\tau-\gamma k)[l-k]\right]}{\left[3\tau(\tau-\gamma k)(4-w^Xw^Y) - 2\gamma^2\sigma_H^2(w^X+w^Y-w^Xw^Y)\right]}(w^X - w^Y).$$

■

Proof of Corollary 3. The aggregate demand of firm X is therefore

$$\sum H(l)x(l) = \frac{1}{2} + \frac{\gamma^2\sigma_H^2(w^X - w^Y)}{\left[3\tau(\tau - \gamma k)(4 - w^X w^Y) - 2\gamma^2\sigma_H^2(w^X + w^Y - w^X w^Y)\right]} > 1/2.$$

where the inequality follows because $w^X > w^Y$. It is then easy to verify that aggregate demand for product X increases with an increase in σ_H^2, an increase in γ, and a decrease in τ. ■