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Abstract

A lowest unique bid auction allocates a good to the agent who submits the lowest bid that is not matched by any other bid. This peculiar auction format is becoming increasingly popular over the Internet. We show that when all the bidders are rational such a selling mechanism can lead to positive profits only if there is a large mismatch between the auctioneer's and the bidders' valuation. On the contrary, the auction becomes highly lucrative if at least some bidders are myopic. In this second case, we analyze the key role played by the existence of some private signals that the seller sends to the bidders about the status of their bids. Data about actual auctions confirm the profitability of the mechanism and the limited rationality of the bidders.

JEL Classification: D44, C72.

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1 Introduction

A new wave of websites is intriguing consumers over the Internet. These websites sell goods of considerable value (electronic equipment, watches, holidays, and even cars and houses) through quite a peculiar auction mechanism: the winner is the bidder who submits

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the lowest *unique* offer, i.e., the lowest offer that is not matched by any other bid. Such a mechanism is commonly called a lowest unique bid auction (LUBA) and leads to impressively low selling prices; one of these websites reports that an iPod (value 200 Euros) has been sold for 0.25 Euros, a Sony Playstation 3 (400 €) for 0.81 €, and a new Volkswagen Beetle Cabriolet (32,000 €) for 32.83 €. These are not exceptions. As a rule of thumb, goods are usually sold for a price that is 0.1-0.3% of the market value.

Websites offering LUBAs first appeared in Scandinavian countries in early 2006. Since then, they rapidly developed in many other European countries (France, Germany, Holland, Italy, Spain, and the UK). Word of mouth is fast, and this auction format is gaining increasing media attention. Some people say LUBAs are a game of strategy; some say they are just a lottery, but some suspect they are a plain scam. In this paper, we contribute to this debate by studying this selling mechanism from a game theoretic point of view.

Let us introduce in more detail the functioning of a LUBA. As a first step, agents must register to one of these websites and transfer an amount of money of their choice to a personal deposit. Users can then browse through the items on sale and submit as many bids as they want on the items in which they are interested. Bids are expressed in cents and are private. Every time that a user places a bid, a fixed amount of money (typically 2 Euros) is deducted from his deposit. The auctioneer justifies this cost as a price for a (compulsory) "packet of information" that he sends to the bidder. In fact, as soon as a bid is submitted, the user receives one of the three following messages: 1) Your bid is currently the unique lowest bid; 2) Your bid is unique but is not the lowest; or 3) Your bid is not unique. During the bidding period, which usually lasts for a few days, users can at any time log in to their account in order to check the current status of their bids, to add new ones, or to refill the deposit. Once the auction closes, the object is sold to the bidder who submitted the lowest unique bid. For instance, if agents A and B offer 1 cent, C offers 2 cents, A and D offer 3 cents, and E offers 6 cents, then the object is sold to C for a price of 2 cents.

This allocation mechanism is, therefore, considerably different with respect to tradi-

tional auction formats.[1] In particular, it is the requirement about the uniqueness of the winning bid that represents a novelty. On one hand, this requirement undermines key objectives that lie at the core of standard auction theory like, for instance, the efficiency of the final allocation. On the other hand, it adds some new strategic elements. In fact, from a strategic point of view, a LUBA is more similar to other well-known games than to a standard auction. Given that agents want to outguess the rivals, the game has something in common with the Guessing Game (Nagel, 1995). There is an important difference though. In the Guessing Game, the pattern of best responses follows a unique direction. This does not happen in a lowest unique bid auction. In fact, a player that expects all the opponents to bid 1 cent maximizes his payoff by bidding 2 cents. But if the player expects all the opponents to bid 2 cents, then he should switch back and bid 1 cent. Therefore, the game is not dominance solvable. On the other hand, some other features of the game (the possibility of multiple bidding, a fixed cost for each bid, and instantaneous knowledge of the bids' status) makes it similar to a War of Attrition (Maynard Smith, 1974). But the closest relative of the lowest unique bid auction is the Dollar Auction Game (Shubik, 1971). This is a public auction in which the prize (say, one dollar) is won by the highest bidder, but both he and the second highest bidder must pay their bids. When participants are not fully rational, this game can lead to some paradoxical results that highly reward the auctioneer. We will see that something analogous can easily happen in the case of LUBAs.

Apart from these classical contributions, there are also some very recent papers that explicitly study various versions of unique bid auctions. Houba *et al.* (2009) and Rapoport *et al.* (2009) analyze the equilibria of a LUBA in which bidders submit a unique bid, there is a non-negative bidding fee, and the winner pays his bid. Both papers find that in the symmetric mixed equilibrium, bidders randomize with decreasing probabilities over a support that comprises the lowest possible bid and is made of consecutive numbers.[2] Östling *et al.* (2009) obtain a similar result for what they call a LUPI (Lowest Unique

[1] See Klemperer (1999) or Krishna (2002) for detailed reviews of standard auction theory.

[2] Rapoport *et al.* (2009) also analyze HUBAs, i.e., unique bid auctions in which the winner is the bidder who submits the highest unmatched offer. Such a mechanism is also studied by Raviv and Virag (2009).

Positive Integer) game in which players can again submit a single bid, but there are no bidding fees, and the winner does not have to pay his bid. The peculiarity of this study is that the number of participants is unknown and is assumed to follow a Poisson distribution. Finally, Eichberger and Vinogradov (2008) analyze a LUBA (that they call LUPA, i.e., Least Unmatched Price Auction) where bidders can submit multiple costly bids, and the winner must pay his winning bid. Given that no information about other bidders' behavior is released during the auction, they model the game as a simultaneous game. For some special ranges of the parameters, they show the existence of a unique Nash equilibrium in which agents mix over bidding strings that comprise the minimum allowed bid and are made of consecutive numbers. In addition to the theoretical analysis, the papers by Houba *et al.* (2009) and by Rapoport *et al.* (2009) propose some algorithms for computing the symmetric mixed strategy equilibrium. The papers by Östling *et al.* (2009) and Eichberger and Vinogradov (2008) have instead an empirical part, which is based on field and/or experimental data. Theoretical predictions find some empirical evidence at the aggregate level but a much lower one at the individual level.

With respect to this ongoing literature, our paper differs in a number of ways. The main novelty is the analysis of the role played by the signals that the bidders receive about the status of their submitted bids. We study how these signals influence the bidding strategies, and we show them to be a key element of the mechanism, especially for what concerns out of equilibrium play. Second, we explicitly model bidders' decisions about how much to invest in the auction (i.e., how many bids to submit). We frame the problem as a rent-seeking game, and we study how the optimal level of investment is influenced by the parameters of the game. Finally, by modeling LUBAs as a sequential game that captures the actions of both the bidders and the auctioneer, we focus on the profitability of the mechanism. We show that if agents are rational then the expected profits of the auctioneer can be positive only if his valuation of the good is (much) lower than the valuation of the bidders. This would imply that websites offering LUBAs should not proliferate the way they do. We then adopt a more behavioral approach and show how a LUBA can become highly profitable when at least some of the bidders lack the necessary commitment to

stick to equilibrium strategies. The profitability of this selling mechanism and the limited rationality of the bidders find an empirical confirmation in the analysis of a dataset that collects information about actual LUBAs.

The remainder of the paper is organized as follows: Section 2 formalizes the strategic situation and characterizes its equilibria under the assumption of perfect rationality of the players. Section 3 investigates what happens when some of the bidders are boundedly rational. Section 4 examines a dataset, which collects detailed information about 100 LUBAs. Section 5 concludes.

2 The game and its equilibria

We introduce and analyze a sequential game that captures some of the key features of a lowest unique bid auction. The game spans over $T+2$ periods with $t \in \{-1, 0, 1, ..., T\}$ and has $(N+1)$ risk-neutral players: an auctioneer (a) and $N \geq 2$ symmetric potential buyers. We assume that N is known. At period $t = -1$ the auctioneer, whose outside option is $u_a = 0$, can decide to auction a certain good through a LUBA. We indicate with V_a the value of the good for the auctioneer and with V the homogeneous valuation of any potential buyer $i \in N$. We assume that $V_a \leq V$.[3] If a opens the auction he credibly commits to sell the good to the buyer who offers the lowest positive bid that is not matched by any other bid. The N buyers must then solve two distinct and subsequent problems. In the first one, which takes place at $t = 0$ and which we label the "investment decision", each agent decides the maximum amount that he is willing to invest in the game. Given that each bid costs $c \in [1, V-1]$, this amount determines the number of bids that the agent is willing to submit throughout the game. In the second problem, which we label the "bidding phase", each bidder must decide where and when to place these bids. The bidding phase starts at $t = 1$ (the opening of the auction) and ends at $t = T$ (the closing of the auction) where T is common knowledge. At any period $t \in \{1, ..., T\}$ each player i plays $x_i^t \in \{\phi\} \cup \{1, ..., \infty\}$. Action $x_i^t = \phi$ indicates that agent i does not bid at period

[3] V can be interpreted as the retail price of the good. The assumption $V_a \leq V$ captures the fact that the auctioneer may pay the good less than its retail price because of quantity discount and/or marketing reasons.

t. Action $x_i^t \neq \phi$ indicates that agent i submits at time t the bid $x_i^t \in \{1, ..., \infty\}$. As soon as a bid $x_i^t \neq \phi$ has been placed, player i is charged c and receives from the auctioneer a truthful and private signal $\sigma^t\left(x_i^t\right) \in \{W, M, L\}$. It is common knowledge that the signals mean the following:

- $\sigma^t\left(x_i^t\right) = W$ indicates that x_i^t is currently the Winning bid (i.e., at time t x_i^t is the lowest unique bid).
- $\sigma^t\left(x_i^t\right) = M$ indicates that x_i^t Might be the winning bid (i.e., at time t x_i^t is unique but it is not the lowest).
- $\sigma^t\left(x_i^t\right) = L$ indicates that x_i^t is a Losing bid (i.e., x_i^t is not unique).

The status of some bids can thus change over time. In particular, the signal $\sigma^t\left(x_i^t\right) = W$ can be updated by $\sigma^s\left(x_i^t\right) = M$ (a bidder j places at time $s \in \{t+1, ..., T\}$ the bid $x_j^s < x_i^t$ such that $\sigma^s\left(x_i^s\right) = W$) or by $\sigma^s\left(x_i^t\right) = L$ (a bidder j bids $x_j^s = x_i^t$). For similar reasons the signal $\sigma^t\left(x_i^t\right) = M$ can be updated by $\sigma^s\left(x_i^t\right) = W$ or by $\sigma^s\left(x_i^t\right) = L$. The signal $\sigma^t\left(x_i^t\right) = L$ cannot be updated as the status of a bid that is not unique cannot change any more. If at period t a unique bid does not exist then a specific tie-breaking rule ensures that a single bid receives the signal W.[4] In other words, there is always a bidder who holds the provisional winning bid and this bidder is unique. Each bidder can check the current status of his own bids at any time and at no cost.

In order to formalize players' payoffs we let $\eta_i^t \in \mathbb{N}$ be the number of bids submitted by agent i up to period t such that η_i^T is the number of bids submitted by i over the course of the entire auction $\left(\text{i.e., the cardinality of the set } \left\{x_i^t \mid x_i^t \neq \phi\right\}_{t=1}^T\right)$. Moreover we use the notation $\hat{x}_i^t$ to indicate the bid that wins the auction. Stressing that all the monetary values $\left(V_a, V, c \text{ and } \left\{x_i^t\right\}_{t=1}^T \text{ for all } i\right)$ are expressed in the same unit (say Euro cents), payoffs take the following form:

$$u_a = \begin{cases} \sum_{i \in N} \eta_i^T c + \hat{x}_i^t - V_a & \text{if } a \text{ opens the LUBA} \\ 0 & \text{otherwise} \end{cases}$$

[4] The rule specifies that if at time $t \in \{1, ..., T\}$ a unique offer does not exist, then the current winner is the bidder that submitted first the lowest bid chosen by the lowest number of agents. We add the further rule that if two or more agents chose this bid simultaneously then the tie is broken randomly.

$$u_i = \begin{cases} V - \eta_i^T c - \hat{x}_i^t & \text{if } \exists \hat{x}_i^t \in \left\{x_i^t\right\}_{t=1}^T \text{ s.t. } \sigma^T\left(\hat{x}_i^t\right) = W \\ -\eta_i^T c & \text{otherwise} \end{cases} \quad \text{for } i \in N$$

Notice that the payoffs of the bidders comprise their outside option of not participating to the auction as $u_i = 0$ when $\eta_i^T = 0$.

We solve the game by backwards induction. Therefore, we first analyze the bidding phase of the game. Then, we study the investment decision of the bidders. Finally, we examine the decision of the auctioneer if to open or not the LUBA.

2.1 The bidding phase

Let $\eta^{\max} \in \mathbb{N}$ be the maximum number of bids that a rational bidder is willing to submit in the LUBA. Section 2.2 will provide a rationale for such a formulation, explicitly derive $\eta^{\max}$ as a function of the parameters of the game and show that $\eta^{\max}$ is symmetric. By now, we take $\eta^{\max}$ as given with $\eta^{\max} \geq 1$. Bidders must then choose when and where to place their bids. In what follows, we investigate these problems.

We differentiate between two cases: $\eta^{\max} = 1$ and $\eta^{\max} > 1$. The second situation is obviously more complex as bidders must condition their behavior on their former bids and on the associated signals. Still, the two cases share some common features. First, in both situations an equilibrium surely exists. In fact, the number of players is finite and so is their strategy space once that strictly dominated bids are eliminated, i.e., $x_i^t \in \{1, ..., V - c\}$ rather than $x_i^t \in \{1, ..., \infty\}$. Indeed, it is easy to notice that equilibria actually abound. In particular, there exist a large number of asymmetric equilibria in pure strategies.[5] However, given the symmetry and the anonymity of the bidders, we restrict our attention to symmetric equilibria. Symmetric equilibria in pure strategies cannot exist: bidders want to outguess the rivals such that for any $N > 2$ a profitable deviation surely exists from any symmetric pure strategy profile. It follows that a symmetric equilibrium must necessarily involve mixed strategies.

As for the timing dimension of the game, we assume $T >> \eta^{\max}$ such that agents have enough periods to use all their available bids if so they wish. This assumption is

[5] For example, if $N = 3$ and $\eta^{\max} = 1$ the profiles $\left\{x_i^1 = 1, x_j^1 = 1, x_k^1 = 2\right\}$ and $\left\{x_i^1 = 1, x_j^1 = 2, x_k^1 = 3\right\}$ are Nash equilibria as there are no (strictly) profitable deviations.

not particularly restrictive given that in actual LUBAs the bidding period lasts for a few days while the time needed to submit a bid amounts to a few seconds. Moreover, the tie-breaking rule (see footnote 4) implies that all those strategies in which bidders delay the submission of their bids are weakly dominated. By invoking a trembling hand argument we disregard these strategies and we focus on the equilibria in which bidders place their bids as soon as possible.

2.1.1 The case with $\eta^{\max} = 1$

The case with $\eta^{\max} = 1$ is analogous to a LUBA in which the rules of the auction specify that each player can submit a single bid. This situation has been carefully investigated by Houba *et al.* (2009) and Rapoport *et al.* (2009). In line with their findings, the following proposition describes some features of the equilibrium distribution.

Proposition 1 *In the symmetric equilibrium of the LUBA with $\eta^{\max} = 1$, each bidder chooses x_i^1 according to the distribution p such that:*
(i) p has support $S(p) = \{1, ..., K\}$ with $K \leq V - c$.
(ii) $p(x)$ is strictly decreasing in x.

Proof. Assume that there exists an equilibrium in which $p(k) = 0$ for some $k \in \{1, ..., K\}$ but $p(\lambda) > 0$ for $\lambda > k$. Then pure strategy $x_i^1 = \lambda$ would be strictly dominated by strategy $x_i^1 = k$. This implies that λ should not be played in the mixed equilibrium, a negation of the initial assumption. Therefore, the support of the distribution comprises 1 and has no gaps. The fact that $K \leq V - c$ follows from elimination of strictly dominated bids. As for the second point, assume that in equilibrium the probability distribution is non strictly decreasing and there exists at least a $\lambda \in S(p)$ for which $p(\lambda) \geq p(k)$ with $k \in \{1, ..., \lambda - 1\}$. Then $E\left(u_i | x_i^1 = k\right) > E\left(u_i | x_i^1 = \lambda\right)$. In fact, either $x_i^1 = k$ is more likely to be unique than $x_i^1 = \lambda$ (the case with $p(\lambda) > p(k)$) or $x_i^1 = \lambda$ and $x_i^1 = k$ are equally likely to be unique (the case with $p(\lambda) = p(k)$). But in both cases $x_i^1 = k$ is more likely to result into the *lowest* unique bid simply because $k < \lambda$. Moreover with $x_i^1 = k$ the price that the bidder must pay if he wins is lower than with $x_i^1 = \lambda$. But

$E\left(u_i|x_i^1=k\right) > E\left(u_i|x_i^1=\lambda\right)$ contradicts the fact that both k and λ are in the support of $p(x)$. For this to be the case $E\left(u_i|x_i^1=k\right) = E\left(u_i|x_i^1=\lambda\right)$ must hold which requires $p(\lambda) < p(k)$ in order to balance the advantages of bidding on k. By setting $\lambda = k+1$ and $k \in \{1,...,K-1\}$ this result must hold for any pair of consecutive numbers in $S(p)$. It follows that $p(x)$ is strictly decreasing in $x \in \{1,...,K\}$. ■

In the symmetric equilibrium all the bidders mix according to p. It follows that every player is equally likely to win. Signals do not matter in this context: each bidder receives the signal $\sigma^1(x_i^1) \in \{W, M, L\}$ but, given $\eta^{\max} = 1$, he does not have any additional bid to submit. Therefore $x_i^t = \phi$ for any i and any $t \in \{2,...,T\}$ and $\sigma^T(x_i^1) = \sigma^1(x_i^1)$ for any i.

2.1.2 The case with $\eta^{\max} > 1$

If $\eta^{\max} > 1$, bidders can submit multiple bids. Because of the tie-breaking rule, every bidder submits his first bid at $t = 1$. And given that bids are costly, agents place their first bid x_i^1 in the optimal way, i.e., by using the probability distribution that characterizes the equilibrium when $\eta^{\max} = 1$ (see Proposition 1). We now label this distribution p^1 where the superscript indicates that this is the distribution from which agents draw their first bid. Bidders then receive the signal $\sigma^1(x_i^1) \in \{W, M, L\}$ and can decide if to submit additional bids. Who will do so? The following two lemmas answer this question.

Lemma 1 *For any $t \geq 1$, there exist $N-1$ bidders for which $\sigma^t\left(x_i^r\right) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^t$.*

Proof. The rules of the game ensures that, for any $t \geq 1$ and any possible distribution of bids, there is a unique bidder j who submitted the unique bid $\hat{x}_j^r \in \left\{x_j^r\right\}_{r=1}^t$ for which $\sigma^t\left(\hat{x}_j^r\right) = W$. It follows that the remaining $N-1$ bidders do not hold the current winning bid, i.e., $\sigma^t\left(x_i^r\right) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^t$ and for any $i \neq j$. ■

Lemma 2 *For any $t \geq 1$ and any bidder i, $x_i^t \neq \phi$ if and only if $\eta_i^{t-1} < \eta^{\max}$ and $\sigma^{t-1}\left(x_i^r\right) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^{t-1}$. Otherwise $x_i^t = \phi$.*

Proof. $\eta^{\max}$ is the maximum number of bids that a rational bidder is willing to submit. It is derived (see section 2.2) as the optimal solution to the investment decision agents face at $t = 0$. It follows tautologically that a player who does not hold the current winning bid keeps on submitting bids until $\eta_i^t = \eta^{\max}$. On the other hand, a bidder who holds the bid $\hat{x}_i^r \in \{x_i^r\}_{r=1}^{t-1}$ for which $\sigma^{t-1}(\hat{x}_i^r) = W$ does not submit additional bids given that, conditional on $\sigma^T(\hat{x}_i^r) = W$, his payoff is higher with $\eta_i^T = \eta_i^{t-1} < \eta^{\max}$. And if the signal $\sigma^{t-1}(\hat{x}_i^r) = W$ is updated by $\sigma^{t-1+k}(\hat{x}_i^r) \neq W$ then the agent still has the option to submit the remaining $\eta^{\max} - \eta_i^{t-1}$ bids given the assumption $T >> \eta^{\max}$. ■

Therefore, every agent that does not hold the current winning offer and that did not reach the upper bound $\eta_i^{t-1} = \eta^{\max}$ keeps submitting additional bids. These subsequent bids are clearly not independent. In fact, not only a rational agent will not submit the same bid more than once but he will also condition his bidding strategy on the signals he receives from the auctioneer. Proposition 2 describes how a rational player updates the probability distribution p_i^t from which he draws x_i^t. Notice the subscript i attached to this distribution. This indicates that, while $p_i^1 = p^1$ for any i, subsequent distributions may differ across bidders. In equilibrium, bidders with an identical history of bids and signals use identical distributions while bidders who submitted different bids and/or received different signals randomize according to different distributions.

Proposition 2 *For any $t > 1$, a bidder i for which $x_i^t \neq \phi$ chooses x_i^t according to p_i^t where p_i^t is such that:*
(i) $S(p_i^t) = \left\{ \left\{ 1, ..., \min \left\{ \{x_i^r - 1 | \sigma^{t-1}(x_i^r) = M\}_{r=1}^{t-1} \cup \{K\} \right\} \right\} \setminus \{x_i^r | \sigma^{t-1}(x_i^r) = L\}_{r=1}^{t-1} \right\}$
(ii) $p_i^t(x)$ *is strictly decreasing in* x *for* $x \in S(p_i^t)$
(iii) p_i^t *is derived from* p_i^{t-1} *according to Bayes's rule.*

Proof. The fact that a player must exclude from the support the bids that he already submitted and whose associated signal is $\sigma^{t-1}(x_i^r) = L$ is obvious. Similarly, the upper bound of the support must be updated with the predecessor of the smallest bid whose associated signal is $\sigma^{t-1}(x_i^r) = M$. In fact such a signal implies that the current winning

bid lies somewhere between 1 and $x_i^r - 1$. The bidder must bid in this interval in order to either match the current winning bid (hoping to get $\sigma^t(x_i^r) = W$) or to find a new lowest unique bid. The proof that $p_i^t(x)$ is strictly decreasing over $S(p_i^t)$ is analogous to the proof of Proposition 2. The fact that players update the probability distribution according to Bayes's rule directly derives from the assumption of rational behavior. ■

2.2 The investment decision

Bidders accumulate sunk costs at rate $c > 0$ for every bid they submit. Before the beginning of the bidding phase (i.e., at $t = 0$), a rational bidder must then set an upper bound on the amount of money he is willing to invest in the game.[6] This immediately determines the maximum number of bids the agent can submit. We indicate this number with $\eta_i^{\max} \in \mathbb{N}$. The return on the investment $\eta_i^{\max} c$ is uncertain given that, as we saw, the outcome of a LUBA is non deterministic. The bidder must then trade-off the probability of winning the LUBA with the losses he suffers in case he does not win. The agent optimally solves this trade-off by maximizing his expected utility $E_0(u_i)$. By rearranging the payoffs introduced in section 2, the agent's problem can be expressed as:

$$\max_{\eta_i^{\max}} E_0(u_i) = \left(V - \hat{x}_i^t\right) P_i - \eta_i^{\max} c \tag{1}$$

where P_i is the probability that at period $t = T$ bidder i holds the bid $\hat{x}_i^t \in \left\{x_i^t\right\}_{t=1}^T$ such that $\sigma^T\left(\hat{x}_i^t\right) = W$. Given that in the symmetric equilibrium (see section 2.1), each bidder i chooses where to place his bids $x_i^t \neq \phi$ according to a symmetric mixed strategy, it follows that all the players are ex-ante equally likely to win if they all submit the same number of bids. But it is also true that a bidder who submits more bids than his opponents has better chances to win the LUBA. In other words, P_i depends on the relative levels of investment of the players i. More formally, $P_i = P_i(\omega_1, ...\omega_N)$ where $\omega_i = \eta_i^{\max} c$ is the investment (or "effort") chosen by agent i.

Now let $\omega_i \in \mathbb{R}^+$ and $\hat{x}_i^t \to 0$. The first assumption transforms the problem from a

[6] In particular, the agent must perform some sort of worst-case scenario analysis and ask himself: "In case I receive the signal $\sigma^t(x_i^r) \neq W$ for every bid x_i^r that I submit, when shall I stop?"

discrete one to a continuous one such that calculus techniques can be applied. The second assumption implies that at $t = 0$ agents do not consider that they will also have to pay $\hat{x}_i^t$ in case they win. We already mentioned that $\hat{x}_i^t$ is negligible with respect to V (around $0.1 - 0.3\%$) and thus unlikely to really affect the investment decision at $t = 0$.[7] With these two assumptions, (1) is strategically equivalent to:

$$\max_{\omega_i} E_0(u_i) = VP_i(\omega_1, ...\omega_N) - \omega_i \tag{2}$$

This last formulation expresses the investment decision of a LUBA as a symmetric rent-seeking game, i.e., a probabilistic contest in which players compete for a prize by expending costly resources.[8] To find the optimal solution to the agent's problem we still need to specify a functional form for the success function $P_i(\omega_1, ...\omega_N)$. Given all the possible histories of bids and signals that agents can get, a precise characterization of such a function appears to be a daunting task. We thus look for a tractable approximation that may satisfy 5 fundamental properties.

P1) $P_i = 0$ if $\omega_i = 0$

P2) $P_i = 1$ if $\sum_{j \neq i} \omega_j = 0$ and $\omega_i > 0$

P3) $P_i = \frac{1}{N}$ if $\omega_i = \omega_j$ for all j

P4) $\frac{\partial P_i}{\partial \omega_i} > 0$, $\frac{\partial P_i}{\partial \omega_j} < 0$

P5) $\frac{\partial^2 P_i}{\partial \omega_i^2} > 0$, $\frac{\partial^2 P_i}{\partial \omega_j^2} < 0$

The first three properties define the limits of P_i and impose symmetry. P4 captures the fact that in a LUBA an agent who invests more (i.e., submit more bids) has a higher probability of winning. P5 requires increasing returns to scale for the marginal bid. In equilibrium in fact (see Proposition 2), the support of the distribution from which a bidder draws the bid x_i^t shrinks as a result of the signals $\left\{\sigma^{t-1}(x_i^r)\right\}_{r=1}^{t-1} \in \{M, L\}$ associated with the agent's previous bids. In particular, while the signal L eliminates from the support a

[7] A similar approach has been used by Raviv and Virag (2009) for what concerns HUBAs.

[8] Rent-seeking games are used to model a wide spectrum of phenomena that involve political lobbying, investment in R&D activities, lotteries. See Tullock (1980), Baye *et al.* (1994), Kooreman and Schoonbeek (1997) and Baye and Hoppe (2003).

unique value, the signal M eliminates an entire string of values. Therefore, the signal M increases more than proportionally the probability of finding the lowest unique bid. And given that the probability of submitting a bid that receives the signal M increases with the number of bids, it follows that the probability of winning a LUBA increases more than proportionally with the "effort" the agent exerts.

A success function that satisfies all the 5 properties is the famous Tullock function (Tullock, 1980): $P_i = \frac{\omega_i^R}{\omega_i^R + \sum_{j \neq i} \omega_j^R}$. The parameter R captures the returns to scale that the investment ω_i has on the probability of winning. In order to capture the increasing returns to scale postulated by P5 we set $R > 1$.[9] With this specification, (2) becomes:

$$\max_{\omega_i} E_0(u_i) = \frac{\omega_i^R}{\omega_i^R + \sum_{j \neq i} \omega_j^R} V - \omega_i \tag{3}$$

The following proposition solves (3) by trivially generalizing the analysis of Baye *et al.* (1994) from the 2 players case to the N players case.

Proposition 3 *Let $\omega \in \mathbb{R}^+$ and $\hat{x}_i^t \to 0$, then the investment decision of a lowest unique bid auction with N bidders has solution $\omega = \frac{N-1}{N^2} VR$ with $R \in \left(1, \frac{N}{N-1}\right)$.*

Proof. In the appendix. ■

In line with what intuition suggests, ω is increasing in V and R and decreasing in N. The optimal ω uniquely determines the maximum number of bids that an agent is willing to submit. In fact, introducing the "floor" operator $\lfloor \cdot \rfloor$ such that $\lfloor z \rfloor$ maps the real number z into the integer n with $n \leq z < n + 1$, we can state the following lemma.

Lemma 3 *In the symmetric LUBA, each bidder submits up to $\eta^{\max}$ bids with $\eta^{\max} = \left\lfloor \frac{\omega}{c} \right\rfloor = \left\lfloor \frac{N-1}{N^2 c} VR \right\rfloor$.*

[9] If $R = 1$ the problem becomes a standard (Tullock) lottery in which the probability of winning linearly increases with the investment. On the other hand, as $R \to \infty$ the game approaches an all-pay auction in which the agent that invests more wins for sure. Both specifications are clearly unsuitable to model the success function of a LUBA.

Proof. Each bid costs c. It follows that $\frac{\omega}{c}$ is the number of bids an agent would submit if these were perfectly divisible. But the number of bids must be an integer such that $\eta^{\max} = \left\lfloor \frac{\omega}{c} \right\rfloor$. This is the maximum number of bids an agent is willing to submit: bidders' payoff is decreasing in η_i^T such that, for any given outcome of the game, an agent is strictly better off with $\eta_i^T < \eta^{\max}$. ■

The integer $\eta^{\max}$ is a weakly increasing function of ω such that the maximum number of bids that an agent is willing to submit in a LUBA weakly increases with the agent's valuation V and the returns to scale R and weakly decreases with the number of participants N and the bidding fee c. Lemma 3 implies that $\eta^{\max} \geq 1$, such that all the bidders enter for sure, whenever $V \geq \frac{N^2 c}{(N-1)R}$. In other words, the value of the auctioned good must be high enough to compensate for the number of participants and the cost of the bidding fee.[10] If $\eta^{\max} = 1$ the analysis of section 2.1.1 applies. If $\eta^{\max} > 1$, section 2.1.2 is the relevant one.

2.3 The auctioneer's decision

From the point of view of the auctioneer, the decision of opening the LUBA depends on the expected profits that the mechanism can raise. We show these expected profits to be bounded below. Therefore, the auctioneer certainly opens the auction whenever this bound is positive. Proposition 4 formalizes this result while Example 1 explicitly solves an hypothetical LUBA.

Proposition 4 *The auctioneer surely opens the LUBA if* $V_a < ((N-1)\eta^{\max} + 1)\, c + 1$.

Proof. The auctioneer's outside option is 0. In case he opens the LUBA his profits are given by $u_a = \sum_{i \in N} \eta_i^T c + \hat{x}_i^t - V_a$. Because of Lemma 2, in equilibrium the $N-1$ losing bidders submit $\eta^{\max}$ bids while the winning bidder submits at least one bid. Moreover the lowest possible winning bid is 1 cent. It follows that u_a is bounded below by $u_a^{\min} =$

[10] Despite a totally different modelling strategy this result is in line with Houba *et al.* (2009) and Rapoport *et al.* (2009) that also show that full entry does not occur if N or c are too high or V is too low.

$((N-1)\eta^{\max}+1)\,c+1-V_a$ which is strictly positive for any $V_a < ((N-1)\eta^{\max}+1)\,c+1$.

■

Example 1 *Consider a LUBA for an item for which* $V = 10,000$ *(i.e., 100 €),* $c = 200$ *and* $N = 10$ *such that* $R \in \left(1, \frac{10}{9}\right)$. *These parameters imply* $\eta^{\max} = \left\lfloor \frac{N-1}{N^2 c} VR \right\rfloor = 4$ *for any* R. *Auctioneer's profits are bounded below by* $u_a^{\min} = ((36+1) * 200) + 1 - V_a$ *such that* $u_a^{\min} > 0$ *for any* $V_a < 7,401$. *It follows that the auctioneer certainly opens the LUBA if he pays the good no more than* 74% *of its retail price.*

It is interesting to compare auctioneer's profits with the profits that the mechanism would raise if signals were not available. Profits with signals are given by $u_a \in \left\{\left((N-1)\eta^{\max} + \eta_i^T\right) c + \hat{x}_i^t - V_a\right\}_{\eta_i^T=1}^{\eta^{\max}}$ where η_i^T indicates the number of bids submitted by the winning bidder. Profits without signals would amount to $u_a(nosignals) = N\eta^{\max} c + \hat{x}_i^t - V_a$ because, with no feedbacks and in line with Lemma 3, all the N bidders would submit their $\eta^{\max}$ available bids. Therefore, for any given $\hat{x}_i^t$, profits with signals are (weakly) dominated by profits without signals. This consideration leads to question why websites that organize LUBAs implement the mechanism with signals. Two are the possible answers: either the auctioneer adopts a sub-optimal behavior or the bidders do not play the game as equilibrium analysis indicates. Given that the first option seems unlikely, we now turn to analyze the second possibility.

3 The game with (some) boundedly rational bidders

The previous section showed that a lowest unique bid auction can be profitable for the seller even when all the bidders are rational and play the equilibrium strategies. Still, a necessary condition for ensuring positive profits is the existence of a (possibly large) mismatch between the retail price of the good (V) and the auctioneer's valuation (V_a). This finding, while interesting, hardly rationalizes what we observe in reality, namely the continuous opening of websites that organize LUBAs. On the contrary, this trend suggests that the business is much more profitable than what equilibrium analysis indicates. In this

section we relax the assumption of full rationality and we show how LUBAs can become highly profitable when some bidders lack the necessary commitment to stick to equilibrium strategies.[11] We show how these agents can get stuck into a costly war of attrition and how this mechanism is triggered and amplified by the existence of the signals.

Given any $\eta^{\max} \geq 1$, let the LUBA proceed according to equilibrium analysis. Lemmas 1 and 2 imply that the auction reaches a certain period t^* in which $N-1$ bidders have used all their available bids ($\eta_i^{t^*} = \eta^{\max}$) and none of them holds the winning bid. Therefore $u_i^{t^*} = -\eta^{\max} c$. A rational bidder who committed to $\eta_i^T \leq \eta^{\max}$ (see Lemma 3) accepts this loss. In other words, he plays $x_i^t = \phi$ for any $t \in \{t^* + 1, ..., T\}$ such that $u_i^T = -\eta^{\max} c$. However, a boundedly rational bidder may be tempted to submit additional bids hoping to eventually win the auction and turn the sunk costs into a positive payoff. We start by better defining what we mean by boundedly rational behavior in the context of a LUBA.

Definition 1 *Bidder i is boundedly rational if:*
(i) whenever $\sigma^t(x_i^r) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^t$, he holds the probability weighting function $w_i^t(q_i^t) > q_i^t \geq 0$ where $q_i^t = q_i^t(x_i^{t+1})$ is the probability that an additional bid $x_i^{t+1} \neq \phi$ placed according to Proposition 2 will lead to the signal $\sigma^{t+1}(\hat{x}_i^r) = W$ for some $\hat{x}_i^r \in \{x_i^r\}_{r=1}^{t+1}$;
(ii) he is myopic and believes that $u_i^{t+1} = u_i^T$;
(iii) he lacks the commitment to stop at $\eta_i^T = \eta^{\max}$.

At any time t, and out of the many possible distributions of actual bids, there are certainly cases in which bidder i can conquer the winning bid by submitting an additional offer. For example, if all the bidders bid 1 at $t = 1$ then $x_i^2 = 2$ will receive the signal $\sigma^2(x_i^2) = W$. Therefore, the event of winning the auction with an extra bid has an objective probability $q_i^t \geq 0$. However, for realistic values of N and $\eta^{\max}$, this probability, when positive, is certainly small. In line with prospect theory (Kanheman and Tversky, 1979) and the empirical evidence about probability weighting functions (see Prelec, 1998,

[11] A similar approach is adopted by Malmendier and Szeidl (2008) that show how the presence of a minority of overbidding behavioral agents disproportionally inflates profits in the case of standard auctions.

and references within), a boundedly rational bidder overestimates this probability, i.e., $w_i^t\left(q_i^t\right) > q_i^t$. Many are the well-known behavioral biases that can shape such a subjective probability assessment: loss-aversion, over-optimism, wishful thinking, bidding fever. The fact that $w_i^t\left(q_i^t\right) > q_i^t$ when q_i^t is small generates a classical pattern of risk attitudes, namely risk-seeking for small probability gains and large probability losses. This in turn rationalizes widespread phenomena like the purchase of lottery tickets or disproportionate betting on longshots. In the context of a LUBA this same pattern can lead to excessive bidding as the following proposition shows.

Proposition 5 *A boundedly rational bidder i for which $\sigma^t\left(x_i^r\right) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^t$ and $\eta_i^t \geq \eta^{\max}$ plays $x_i^{t+1} \neq \phi$ if $w_i^t\left(q_i^t\right) > \frac{c}{V-\hat{x}_i^r} \simeq \frac{c}{V}$. Moreover if this condition holds at time t then it also holds at time $t+k$ such that $x_i^{t+k+1} \neq \phi$ for any $k \in \{1, ..., T-t-1\}$ whenever $\sigma^{t+k}\left(x_i^r\right) \neq W$ for every element of the set $\{x_i^r\}_{r=1}^{t+k}$.*

Proof. A boundedly rational bidder who does not hold the winning bid and is not committed to $\eta_i^t \leq \eta^{\max}$, submits an additional bid if $E(u_i^{t+1}) > u_i^t$, i.e., $w_i^t\left(q_i^t\right)\left(V - \left(\eta_i^t+1\right)c - \hat{x}_i^r\right) + \left(1 - w_i^t\left(q_i^t\right)\right)\left(-\left(\eta_i^t+1\right)c\right) > -\eta_i^t c$ where $\hat{x}_i^r \in \{x_i^r\}_{r=1}^{t+1}$ is i's eventual winning bid. Solving for $w_i^t\left(q_i^t\right)$, the last condition is verified for any $w_i^t\left(q_i^t\right) > \frac{c}{V-\hat{x}_i^r} \simeq \frac{c}{V}$ given that in practice $\hat{x}_i^r$ is negligible. With this approximation the lower bound for the probability weighting function does not depend on η_i^t and remains constant over time. This means that if the constraint is satisfied at period t, it is also satisfied at any period $t+k$ with $k \in \{1, ..., T-t-1\}$ such that agent i keeps on submitting additional bids until he gets the signal $\sigma^{t+k}\left(\hat{x}_i^r\right) = W$ for some $\hat{x}_i^r \in \{x_i^r\}_{r=1}^{t+k}$. ■

Example 2 *Consider the situation described in Example 1 with $V = 10,000$, $c = 200$, $N = 10$ and $\eta^{\max} = 4$. Assume that there are at least $2 \leq I \leq 10$ boundedly rational bidders (Definition 1). At least $I-1$ of them reach at t^* the situation $\eta_i^{t^*} = 4$ and $\sigma^{t^*}\left(x_i^r\right) \neq W$ for any $x_i^r \in \left\{x_i^1, ..., x_i^{t^*}\right\}$. Proposition 5 states that each one of these bidders submits an additional bid at every $t \in \{t^*+1, ..., T\}$ whenever they do not hold the winning bid and*

their probability weighting function is such that $w_i^{t^*}\left(q_i^{t^*}\right) > \frac{200}{10{,}000-\hat{x}_i^r} \simeq 0.02$. *Notice that the constraint on* $w_i^{t^*}\left(q_i^{t^*}\right)$ *is very low.*

Example 2 implies that the presence of at least two boundedly rational bidders can easily trigger a costly vicious circle in which these players accumulate sunk costs. An upper bound to this process is given either by T (the closing of the auction) or by bidders' budget constraint. Whenever these limits are not binding, this sort of war of attrition can continue even when the costs associated with the number of bids exceed the value of the good on sale.

To see this, let A and B be two boundedly rational players with $w_i^t\left(q_i^t\right) > \frac{c}{V}$ for $i \in \{A, B\}$. In line with Proposition 5, the auction will reach period $\tilde{t}$ in which a bidder, say A, is the current winner such that $u_A^{\tilde{t}} = V - \eta_A^{\tilde{t}} c - \hat{x}_A^r > 0$ with $\hat{x}_A^r \in \{x_A^r\}_1^{\tilde{t}}$ and $u_B^{\tilde{t}} = -\eta_B^{\tilde{t}} c$. Still, one more bid of B can potentially lead to $u_B^{\tilde{t}+1} = V - (\eta_B^{\tilde{t}} + 1)c - \hat{x}_B^r$ with $\hat{x}_B^r \in \{x_B^r\}_1^{\tilde{t}+1}$ but $u_B^{\tilde{t}+1} < 0$. Agent B compares $u_B^{\tilde{t}}$ and $u_B^{\tilde{t}+1}$. Both values are negative. Nevertheless $u_B^{\tilde{t}+1} > u_B^{\tilde{t}}$ such that, given the probability weighting function $w_B^t\left(q_B^t\right) > \frac{c}{V}$, agent B still prefers to play $x_B^{\tilde{t}+1} \neq \phi$ hoping to diminish his own losses. The same argument holds for periods $\tilde{t}+1$, $\tilde{t}+2$, ..., $T-1$. Now assume that before $t = T-1$, agent B conquers the winning bid. Bidder A would then sooner or later find himself in the situation in which B was at period $\tilde{t}$. Therefore, the same logic applies and the mechanism perpetuates itself.

This feature of lowest unique bid auctions is reminiscent of the Dollar Auction Game (Shubik, 1971). The Dollar Auction Game is a public ascending auction where N bidders compete for a dollar. The auction is won by the agent who submits the highest bid but both him and the second highest bidder must pay their bids. Also in this case, the auction is unprofitable for the seller if agents are rational. But if multiple entry occurs, this starts off a bidding war between the two leading bidders such that the winner may end up paying the dollar more than what it is worth. Both in the Dollar Auction Game and in a LUBA, the bidding escalation is detrimental for the bidders but beneficial for the auctioneer. In fact, as Morgan and Krishna (1997) show, war of attritions yield revenues that are superior to standard auction mechanisms.

Going back to the analysis of LUBAs, notice that the assumption of at least two boundedly rational bidders is not sufficient to trigger the bidding escalation. It is in fact the combination of boundedly rational behavior and of the existence of the signals that accomplishes this task. To appreciate the fundamental role that signals play, consider how different the situation would be if agents were not receiving any kind of feedback about the status of their bids. In such a case, each player would hold the legitimate hope to win the auction with one of his $\eta^{\max}$ bids such that the incentives to submit extra bids are much weaker. And when at the closure of the auction the winner is declared, it would be too late for the losers to submit additional offers. In other words, in terms of ambiguity, a LUBA without signals would resemble a traditional lottery. On the other hand, signals make the game more similar to a "scratch and win" lottery. In fact, signals (and in particular the signal L) immediately inform the bidder that some or all of his offers have no chances to win. This clearly encourages overbidding given that an agent that faces potential losses is tempted to submit additional bids in order to catch up.

Quoting what Malmendier and Szeidl (2008) write with regards to standard auction mechanisms "if agents are subject to bidding fever, sellers may instigate this bias using salient messages informing the buyer that he has been outbid". Indeed, the entire signaling mechanism that characterizes LUBAs seems to have been designed with the goal of stimulating emotional responses that may lead to an irrational escalation of commitment. Given that the auctioneer aims to maximize the number of received bids, this obviously comes as no surprise.

4 Empirical analysis

In this section we analyze a dataset that collects information about 100 lowest unique bid auctions that took place in the period February 6th, 2008 - April 6th, 2008. These auctions have been organized by the website bidplaza.it, the leader of the Italian market with more than $1,000,000$ contacts per month.[12] The rules implemented by this auctioneer

[12] At the time the data were collected, bidplaza.it was operating as the italian subsidiary of bidster.com, the world leader in the sector. In November 2008 this partnership broke down and since then both websites independently offer LUBAs in Italy.

are exactly the ones explained in the introduction. In particular the cost associated with each bid was set at 2 Euros in every LUBA. For each auction we know the market value of the item on sale, the winning bid and, most importantly, the complete list of submitted bids. Overall, our dataset collects $100,940$ bids.[13] Unluckily, we do not have information about the number of bidders, how many and which bids each bidder submitted and the signals they received. Nevertheless, the data allow to clearly distinguish some interesting patterns as well as to discriminate between rational versus irrational bidding behavior. Table 1 reports some summary statistics.

Variables	average	min	max	st. dev.	sum
Retail price V (€)	274.90	80	450	96.1	$27,490$
Winning bid (€)	0.89	0.01	3.37	0.66	88.66
Number of received bids	$1,009$	119	$2,917$	635	$100,940$
Lower bound on number of bidders ($\underline{N}$)	15.3	5	38	6.45	
Max # of bids under rationality ($\#_k(N_k^*)$)	137.4	40	225	48.04	$13,740$
Estimated profits (€)	$1,239.2$	49	$3,975$	893.1	$123,920$
Estimated profits (% wrt retail price)	441%	19%	$1,082$%	237%	

Table 1. Some summary statistics of the data.

The first three rows of Table 1 report some statistics about the retail price of the auctioned items, the winning bids and the number of received bids. Not surprisingly, there is a positive relationship between the retail price and the number of received bids (Pearson's $r = 0.645$), as well as between the number of received bids and the winning bid ($r = 0.616$).

[13] The dataset, which we manually assembled by retrieving the data from the website bidplaza.it (section "aste chiuse", i.e., expired auctions), is available upon request. This is the list of goods to which the data refer (the notation y (V, k) indicates that good y whose retail price is V has been offered in k different auctions such that $\sum_y k = 100$): Sony Playstation 3 $(400, 10)$, Sony Playstation Portable Slim & Lite $(190, 10)$, Digital Camera IXUS 860IS $(350, 9)$, iPod Shuffle 1 GB $(80, 7)$, iPod Nano 8 GB $(200, 9)$, iPod Touch 16 GB $(400, 8)$, Bose Companion 3 multimedia speaker system $(295, 10)$, Samsung CE 1070TS microwawe oven $(240, 9)$, Nintendo Wii $(250, 10)$, Philips Digital PhotoFrame Wood 10FF2CWO $(250, 4)$, TomTom One V3 Portable GPS Navigation System $(200, 9)$, XBOX 360 Elite $(450, 5)$.

The actual distribution of bids can be used to establish a lower bound for the number of bidders. In fact, by assuming that no agent submitted more than once the same bid in the same auction, the lower bound $\underline{N}$ can be inferred from the frequency of the most frequent bid. This bound ($\underline{N} = 15.3$) is extremely conservative and the actual number of bidders is likely to be much higher than that. Nevertheless we can exclude a rational bidding behavior no matter the real N. Equilibrium analysis (see Lemma 3) indicates in fact that every LUBA $k \in \{1, ..., 100\}$ can raise at most $\#_k$ bids with $\#_k = N_k \eta_k^{\max} = N_k \left\lfloor \frac{N_k - 1}{(N_k)^2 c} V_k R \right\rfloor$ with $R \in \left(1, \frac{N_k}{N_k - 1}\right)$. By letting $R = \frac{N_k}{N_k - 1}$ and substituting the specific values of V_k and c we can thus solve $\#_k$ as a function of N_k for $N_k \in \{\underline{N}_k, ..., \infty\}$ and retrieve $\max \#_k = \#_k (N_k^*)$ (fifth row of Table 1). This is the maximum number of bids auction k could have raised even assuming the most rewarding returns of scale R and the most favorable number of bidders N_k^*.[14] The estimate of $\max \#_k$ falls short of the actual number of received bids by a factor of more than 7 (on average 137.4 vs. $1,009$). Moreover, the fact that $\max \#_k$ is much smaller than the actual number of received bids holds for any single auction k.

As a consequence of the high number of bids, the auctioneer made positive profits in every LUBA. A cautious estimate shows that profits per auction amount on average to the 441% of the retail price.[15] The hypothesis of rational behavior is refuted by these findings. In fact, if bidders were rational and $V_a = V$, auctioneer's profits could be positive only if the winning bid is very large.[16] This is not what we observe in the data where winning bids amount on average to just 0.33% of V.

Some other features of the data also noticeably stand out. First, and once more

[14] More precisely, with $R = \frac{N_k}{N_k - 1}$ the function simplifies to $\#_k(N_k) = N_k \left\lfloor \frac{V_k}{N_k c} \right\rfloor$. This function is maximized by the (possibly multiple) $N_k^* \in \left\{\underline{N}_k, ..., \left\lfloor \frac{V_k}{c} \right\rfloor\right\}$ that minimizes $\left(\frac{V_k}{N_k^* c} - \left\lfloor \frac{V_k}{N_k^* c} \right\rfloor\right)$. At these maxima $\max \#_k = \#_k (N_k^*) = \left\lfloor \frac{V_k}{c} \right\rfloor$.

[15] Despite knowing the market value of the goods, the number of bids received and the unitary cost of 2 Euros per bid, profits cannot be computed with certainty. In fact, the auctioneer offers a welcome bonus such that a user's first deposit of money is doubled. Therefore, some of the bids are virtually for free. We adopt a conservative approach and we assume that a) only 75% of the bids generated actual revenues and b) $V_a = V$ even though the alternative assumption $V_a < V$ is more likely to hold because of quantity discounts and/or marketing reasons.

[16] Profits are bounded above by $u_a^{\max} = N \eta^{\max} c + \hat{x}_i^t - V_a$ with $\hat{x}_i^t \in \{1, ..., V - \eta^{\max}\}$. Given that $\eta^{\max} \leq \frac{N-1}{N^2 c} VR$ we have that $u_a^{\max} \leq \frac{N-1}{N} VR + \hat{x}_i^t - V_a$. If $V_a = V$, $u_a^{\max}$ is certainly negative for any $\hat{x}_i^t < \frac{1}{N} VR$.

contrary to what equilibrium analysis indicates (see propositions 1 and 2), the aggregate frequencies of the bids are not monotonically decreasing. Figure 1 shows the distribution of the $97,225$ bids that picked numbers belonging to the set $\{1, ..., 500\}$. Although a decreasing trend is clearly recognizable, this is not monotonic. A closer look at the data reveals the nature of the spikes that appear in Figure 1: bidders tend to overbid on odd numbers. More precisely, $54,230$ bids (55.8%) are odd while only $42,995$ (44.2%) are even. A normally approximated binomial test shows that this difference is significant at the 1% level. In line with this tendency, 9 out of the 10 most frequent bids are odd.[17]

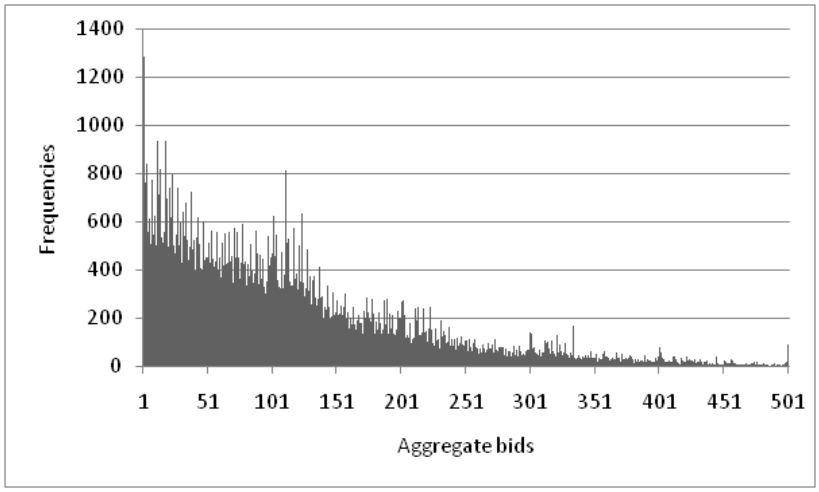

Figure 1: The aggregate pattern of bids.

The preference for odd numbers has an intuitive explanation. In a LUBA, players want to submit bids that no one else chooses. Therefore, agents tend to submit bids that they perceive to be original: odd numbers (excluding those whose trailing digit is 5) and, even better, prime numbers. A similar behavior emerges also in the LUBAs studied by Östling *et al.* (2009) and is analogous to the one first described in Crawford and Iriberri (2007) for what concerns Hide and Seek games. Notice that the aggregate result of such a bidding strategy is quite paradoxical as agents end up converging on these peculiar

[17] The complete top ten list, with aggregate frequency in brackets, is the following: 1 (1,287), 11 (936), 17 (936), 3 (841), 13 (822), 111 (813), 23 (798), 7 (777), 2 (766), 27 (741). As a matter of comparison, round numbers like 10, 20 and 100 attracted respectively 506, 498 and 471 bids.

numbers. Indeed, the data show that submitting an odd bid is suboptimal as the large majority of winning bids are even numbers (68 vs. 32, with the difference being significant at 1% level).

Although our model of boundedly rational bidders (Section 3) is silent about this tendency, the bias towards submitting odd bids is another piece of evidence that goes against the hypothesis of perfect rationality. In particular such a bidding behavior seems to be consistent with Level-k analysis (see Stahl and Wilson, 1995, and Costa-Gomes *et al.*, 2001): agents erroneously think to be smarter than the opponents and only perform a limited number of steps of reasoning.[18] Subject to the availability of individual data, we let for future research a more careful formalization and empirical investigation of agents' bidding strategies.

5 Conclusions

The paper introduced and analyzed a peculiar selling mechanism that is becoming increasingly popular over the Internet: lowest unique bid auctions (LUBAs) that allocate valuable goods to the agent who submits the lowest bid that is not matched by any other bid. We showed that if bidders are rational, a LUBA can be profitable for the seller only if his valuation of the good is much lower than the valuation of the potential buyers. But we also showed why in reality this auction format is so successful: boundedly rational bidders may lack the necessary commitment to stick to equilibrium strategies, and thus, they may become locked in a costly war of attrition that highly rewards the auctioneer. In particular, we highlighted how such a mechanism is driven by the existence of the signals the auctioneer sends about the current status of players' bids. It is, therefore, ironic to notice how websites that organize LUBAs overstress, surely a bit in bad faith, the alleged positive role of these signals.[19] While it is clear why they do so (they have to justify the

[18] Notice moreover that the data do not allow to control for the level of experience of the players. The bias in submitting odd bids would probably be even more pronounced if only agents that play the game for the first few times were considered.

[19] For instance, one of these websites claims that "Relying on these signals, using different strategies and different levels of investment, to win the auction becomes a matter of a complex use of various abilities". Another website declares: "The investment, the signals and the bidding strategies make the auction void

fixed cost associated with each bid, and they want to distinguish themselves with respect to pure lotteries and gambling), the paper showed that signals are at best a double-edged weapon.

Lowest unique bid auctions also suffer from other potential problems that should suggest prudence. For instance, they share the technological hitches that characterize on line auctions: problems of connectivity, delays or congestion, possibly due to last minute bidding or "sniping" (see, for instance, Roth and Ockenfels, 2002 for the case of eBay and Amazon auctions). Collusive behaviors are also an important issue. While collusion among bidders seems unlikely due to the secrecy of agents' identities and to problems of coordination, collusion between the auctioneer and a single bidder looks much more easily implementable. Bids are private information, but the auctioneer gets to know them in real time. As such, nothing prevents him from indicating to a third party where to place a winning bid seconds before the auction closes. Obviously, this would turn the auction into a scam. We do not think that LUBAs are scams; the mechanism is too profitable to risk ruining it with such a trick. And indeed, to speak the truth, these websites put quite some effort in trying to build and maintain a reputation for being a trustable and transparent outlet.

To sum up, lowest unique bid auctions are a very smart selling mechanism. On one hand, by giving the possibility to win goods of considerable value for very little money, they share the appeal of lotteries. On the other hand, they give bidders the illusion of being in control of what they do, and they convey the idea that winning is just a matter of being smarter than the others. The combination of these two factors makes the business successful and, in turn, explains the continuous entry in the industry. Entry will surely stimulate competition and lead to better conditions for the players: lower bidding fees, higher welcome bonuses, and lower number of opponents. Nevertheless, the basic mechanism underlying the auction format will remain the same such that the analysis of this paper continues to be valid. We conclude by stressing once more the similarities that lowest unique bid auctions have with other well-known games like the War of Attrition and

of any element of luck and based exclusively on the bidder's ability".

the Dollar Auction Game. It is obviously not a coincidence that these games are used as archetypes for describing situations where irrational behavior leads to an inefficient waste of resources.

6 Appendix

Proof of Proposition 3

Each agent i solves $\max_{\omega_i} E(u_i) = \left(\frac{\omega_i^R}{\omega_i^R+\sum_{j\neq i}\omega_j^R}\right)V - \omega_i$. This leads to the following necessary and sufficient conditions:

$$\frac{\partial E(u_i)}{\partial \omega_i} = \left(\frac{R\omega_i^{R-1}\sum_{j\neq i}\omega_j^R}{\left(\omega_i^R+\sum_{j\neq i}\omega_j^R\right)^2}\right)V - 1 = 0$$

$$\frac{\partial^2 E(u_i)}{\partial \omega_i^2} = \left(\frac{R\omega_i^{R-2}\sum_{j\neq i}\omega_j^R}{\left(\omega_i^R+\sum_{j\neq i}\omega_j^R\right)^3}\left[(R-1)\left(\omega_i^R+\sum_{j\neq i}\omega_j^R\right) - 2R\omega_i^R\right]\right)V < 0$$

Imposing symmetry ($\omega_i = \omega_j = \omega$), these become

$$\frac{\partial E(u_i)}{\partial \omega_i}(\omega_i=\omega_j) = \left(\frac{(N-1)R\omega^{R-1}\omega^R}{(N\omega^R)^2}\right)V - 1 = 0$$

$$\frac{\partial^2 E(u_i)}{\partial \omega_i^2}(\omega_i=\omega_j) = \left(\frac{R\omega^{2R-2}(N-1)}{(N\omega^R)^3}\left[(R-1)\left(N\omega^R\right) - 2R\omega^R\right]\right)V < 0$$

The FOC yields the solution

$$\omega = \frac{N-1}{N^2}VR$$

for which the SOC holds locally for $R < \frac{N}{N-2}$. By substituting the optimal ω within the expected utility, we get that

$$E_0(u_i) = \frac{1}{N}V - \frac{N-1}{N^2}VR$$

which is positive, such that bidders enter the game, for $R < \frac{N}{N-1}$.[20] This upper bound is more restrictive that the constraint identified by the SOC. Therefore, by combining the requirement of increasing returns to scale with this upper bound, we get $R \in \left(1, \frac{N}{N-1}\right)$.

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