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Resuscitating Businessman Risk: A Rationale for Familiarity-Based Portfolios*

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Abstract

This paper studies two frequently observed portfolio behaviors that are seemingly inconsistent with rational portfolio choice. The first is the tendency of workers and entrepreneurs to hold their company's stock. The second is the propensity of workers to limit their equity holdings through time. The explanation offered here for both of these behaviors lies in the option to switch jobs when one's company does poorly. This is equivalent to holding put options on one's own company stock and call options on the other company's stock, where both options must be exercised at the same time. Given these initial undiversified implicit financial holdings, workers need to allocate a relatively large share of their regular financial assets to their own company's stock and a relatively small share to the stock of their alternative employment simply to restore overall portfolio balance. I find that, under certain conditions, workers optimally hold almost 40% of their financial wealth in their company's stock.

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1 Introduction

The concept of *businessman risk* holds that among those with similar financial wealth, young businessmen should invest considerably more in risky assets than older widows. Since Samuelson's (1969) questioning of its validity, many others have extensively explored it and much debated whether to reject it or save it.¹

In parallel, Markowitz (1952) and Sharpe (1964)'s economic prescriptions have been challenged by the evidence that individuals seemingly deviate from the principles of portfolio diversification and invest in the "familiar." Allocation strategies driven by the investor's geographical or professional proximity to a particular stock are generally conceptualized in the term *familiarity*, "the tendency of households' portfolios to be concentrated, of employees [...] to own their employers' stocks in their retirement accounts and [...] of home country bias in the international arena."² For example, Coca-Cola employees allocate to company stock 76% of their discretionary contributions to the plan assets (Benartzi, 2001) and the seemingly financially savvy employees of J.P. Morgan invest 19% of their 401(k) plan money in Morgan stock (Huberman, 2001).

Curcuro, Heaton, Lucas and Moore (2004) exhaustively summarize the evidence on household portfolio composition and re-examine some of the theories that have been proposed to account for portfolio heterogeneity across individuals. Within the traditional utility maximizing framework, labor and entrepreneurial income, transaction costs, borrowing constraints and other life-cycle considerations seem to explain some aspects of the observed cross-sectional variation in portfolio holdings. However, the lack of diversification in some unconstrained individual portfolios remains a challenge for quantitative theories.

This paper studies the impact of labor flexibility on optimal life-cycle portfolio decisions, particularly the ability to change industries or firms within industries. The model addresses both the tendency of workers to hold their company's stock and their propensity to limit equity holdings through time.

Prior theoretical models have investigated optimal portfolio decisions under the assumption of a lifetime employment at the same employer.³ These models hold that because a person already has a significant risk exposure to the company she works for through labor/human capital, she should hold less of the company stock in her financial portfolio than an investor who does not work for the company. However, there is considerable empirical evidence that investors do not hedge and invest in stocks correlated to their nonfinancial income.⁴

¹For instance, Brown (1990, p. 905) concludes "The middle-aged by comparison have a high tolerance for risk and a low marginal propensity to consume relative to the young; the middle-aged with significant savings is willing to take on the businessman's risk." Similarly, Malkiel (2007, p. 101) writes "According to one well-known theory, the bigger the swings – relative to the market as a whole – in an individual company's stock prices, the greater the risk. [...] A nonswinger gets the *Good Housekeeping* seal for "widows and orphans." [...] On the other hand a "flyer" is a businessman's risk."

²Huberman, 2001, p. 659.

³Specifically, the standard life-cycle portfolio literature ignores the *endogenous* decision to terminate employment.

⁴"One trend that almost everyone applauds is the attempt to prevent employees from investing too heavily in company stock, in some cases by restricting the amount of company stock that employees can buy," said

The explanation offered here for both of these behaviors lies in the *option* to switch jobs when one's company does poorly. The term *option* is telling because the ability to change jobs represents the implicit holding of a financial option, namely a put option written on one's current company. The intuition is that this option puts a "floor," or put protection, on the human capital exposure to the current company employer and, because it is an exchange option, it also creates a long exposure of that human capital to the companies or industries to which the worker could move but does not currently work for. To see this, assume a worker's wage is perfectly correlated with her employer's stock. In this case, her human capital represents an implicit employer stock holding, but of a special kind. If the value of her employer's stock falls, she can avoid a loss on her implicit employer stock holding simply by changing jobs. Consequently, her human capital constitutes the holdings not just of her employer's stock, but also of put options on that stock, albeit rather exotic put options, called *spread* options. The return on these implicit spread options is higher both when one's own company does poorly and when one's alternative company of employment does well. Consequently, holding these spreads is very similar to holding call options on the other company's stock and put options on one's own company, where both options must be exercised at the same time. Given these initial undiversified implicit financial holdings, workers need to allocate a relatively large share of their regular financial assets to their own company's stock and a relatively small share to the stock of their alternative employment simply to restore overall portfolio balance. Although this effect does not structurally create a hedging demand for company's stock, it is, as shown here, a factor of potentially major import for assessing the suitability of workers' financial decisions.

I consider the case of an option to switch between a relatively safe and a relatively risky job. I find that when workers start their career in the safer job, they optimally hold a positive amount in their company's stock (under some conditions almost 40% of their financial wealth). However, if workers start out in the riskier job, they short their company's stock – just as in the absence of job-switching options – but they short less. Thanks to mobility costs, the extent to which the ability to switch jobs constitutes a valuable spread option varies with age. As one approaches retirement, the spread option's value goes to zero leaving workers more exposed to their company's performance. This leads them to diversify away from their own company's equity holdings and, indeed, from equity holdings in general – a result that recalls Bodie, Merton, Samuelson's (1992) prescription to reduce equities with age, but that arises from a different source namely the implicit reduction through time in the amount of equity insurance provided by the job-switching put.

The structure of this paper is as follows. Section 2 describes the basic model and derives the solution of the optimal consumption–portfolio–job–regime plan. The baseline calibration is presented in Section 3. Section 4 gives the simulation results for the benchmark calibration and explores the effects of changes in various parameter values. Section 5 compares the model's predictions to the data. Section 6 concludes. Proofs and some details on the implementation of the numerical algorithm are collected in Appendix A. Appendix B illustrates how the model can be extended.

Stephen P. Utkus of Vanguard's Center for Retirement Research.

(Source: <http://www.washingtonpost.com/wp-dyn/content/article/2007/10/20/AR2007102000141.html>)

2 The Model

2.1 The investment opportunity set

This Section describes the operating markets available to the individual for trading in financial assets, her preferences over the life-cycle and affordability constraints.

I posit a financial market comprised of a riskless asset and two risky dividend-paying assets. The dynamics of the riskless security satisfy

$$dB_t = r_f B_t dt, \quad B_0 > 0 \text{ given}, \quad (1)$$

in which $r_f (> 0)$ is the instantaneous market rate of interest, and is constant over time. The two risky security prices, $S (= S_i, S_j)$, follow the two-dimensional Itô process

$$dS_t = S_t [(\mu_S - \delta_S) dt + \sigma_S dz_t], \quad t \in [0, T], \quad S_0 > 0 \text{ given}, \quad (2)$$

where δ_S is a vector of dividend yields, μ_S is a vector of instantaneous expected rates of return and σ_S is a two-by-two diagonal matrix of instantaneous volatility coefficients. z_t is a Brownian Motion process defined in $\mathbb{R}^2$ and T is the individual's finite fixed planning horizon. The implied market price of risk is denoted by $\theta \equiv \sigma_S^{-1}(\mu_S - r_f \mathbf{1}_2)$, where $\mathbf{1}_2$ is the two-dimensional unit vector, and the state-price density process is denoted by $\xi_t \equiv e^{\{-r_f t - \frac{\theta' \theta}{2} t - \theta' z_t\}}$.

Financial markets are complete and frictionless. Assets are traded continuously in the absence of transaction costs and both borrowing and short-selling are allowed without restrictions.

2.2 The investor's preferences

Let $c_t \geq 0$ denote the individual's rate of consumption at time t . Preferences are posited to be an instantaneous time-separable power utility function over consumption

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad t \in [0, T], \quad (3)$$

in which $\gamma \geq 1$ is the constant coefficient of relative risk aversion – the elasticity of intertemporal substitution of consumption is $\frac{1}{\gamma}$.⁵

2.3 The labor income process

This Subsection presents the labor income process and the cost of switching careers.

⁵The individual is not permitted to bequeath wealth to her descendants at death: an extension of the current model specification could introduce a bequest motive component to Eq. (3) to avoid complete wealth dissaving by the end of the individual's life. This would be equivalent to lengthening the individual's planning horizon.

I explicitly allow for labor mobility by providing the individual with a real option to switch jobs during her working years. Consequently, along with optimal consumption and portfolio decisions, the individual must select the optimal time for a change in her career. The option to leave one employment for another is especially valuable when the evolution of prices and wages in financial and labor markets is uncertain.

The finite-lived individual of this model finances consumption and investments by earning a per period salary, along with earnings on accumulated financial assets. The vector of wage flows offered in each of the two jobs, current and potential, $w_t \equiv [w_{i,t}, w_{j,t}]'$, has dynamics given by

$$dw_t = w_t [\alpha dt + \sigma dz_t], \quad t \in [0, \tau_2], \quad w_0 > 0 \text{ given.}^{6,7} \quad (4)$$

The instantaneous expected wage growth rates are collected in the two-dimensional vector $\alpha > 0$, while σ is a two-by-two invertible matrix of wage volatility coefficients, whose entry $\sigma_{i,j}$ denotes the volatility of wage i with respect to the j^{th} dimension of $z_t \in \mathbb{R}^2$. Retirement is modeled as an irreversible labor income state beginning at time τ_2 , $\tau_2 \in [0, T]$. Notice that the present analysis reflects job market turnovers only partially since the individual's decision is confined to just two jobs and the job-switching option can be exercised at most once prior to the retirement date.

The modeling structure of Eqs. (2) and (4) posits perfect correlation between labor income innovations and stock returns in each job. Appendix B demonstrates how an economy with less than perfectly correlated labor and financial markets can be transformed to restore the canonical model specification in this section of the paper.

Letting $\tau_1 \in S_{0, \tau_2 - \Delta}$ be the optimal time of a job change in the set of feasible stopping times $S_{0, \tau_2 - \Delta}$, and $\Delta \geq 0$ represent a deterministic transition phase at the onset of a new job, Eq. (5) characterizes the static budget constraint of an individual leaving employment i for employment j . Individual *total wealth* is comprising both *financial wealth* – with initial value $W_0 > 0$ and current value W_t , $t \in (0, T]$, echoing past saving and investing – and *human capital wealth* – the present value of future labor income.⁸ The lifetime budget constraint can be written as

$$E \left[\int_0^T \xi_t c_t dt \right] \leq W_0 + \sup_{\tau_1 \in S_{0, \tau_2 - \Delta}} E \left[\int_0^{\tau_1} \xi_t w_{i,t} dt + \int_{\tau_1 + \Delta}^{\tau_2} \xi_t w_{j,t} dt - \xi_{\tau_1} f \right]^+, \quad (5)$$

where E denotes the time-zero expectation operator that encompasses the probability distribution of all states of the world over the individual's planning horizon. $f > 0$ is a fixed cost, e.g., the payment of school tuitions, which the individual faces as she leaves the

⁶The wage distribution of Eq. (4) is non-stationary. An alternative specification with wage distributions independent of wage rates is offered by Van Der Berg (1992).

⁷This assumption is not supported within a competitive labor market in which wages are determined by the marginal product of labor. Here, starting one's career in a relatively safe or a relatively risky job could be deemed the mere outcome of a random draw.

⁸Bodie, Merton, and Samuelson, 1992, p. 428.

workforce to acquire additional skills necessary to her new career.^{9,10} The left-hand side quantifies the present value of consumption affordable to the individual, conditional on total lifetime resources consisting of initial financial wealth and labor earnings, the right-hand side.

The Subsection to follow focuses on the individual's security holdings in relation to her implicit human capital risk exposures. This analysis disentangles mean-variance, job-specific and hedging components that form the optimal investment policies. Malliavin calculus methods are implemented in the derivation of the policy equations.

2.4 Optimal consumption, portfolio and job-regime plans

The solution methodology for optimal policies is based on Cox and Huang (1989), Detemple, Garcia, and Rindisbacher (2003) and Nualart (2006). Stochastic integration permits to re-express Eq. (5) in the more compact form

$$E \left[\int_0^T \xi_t c_t dt \right] \leq W_0 + w_{i,0} \frac{e^{\{\eta_i \tau_2\}} - 1}{\eta_i} + \underbrace{\sup_{\tau_1 \in S_{0, \tau_2 - \Delta}} E [\xi_{\tau_1} (HC_{j, \tau_1 + \Delta, \tau_2} - HC_{i, \tau_1, \tau_2} - f)^+]}_{\equiv V_0(w_i, w_j, \tau_1)}, \quad (6)$$

where $HC_{(\cdot), t, v} \equiv w_{(\cdot), \tau_1} \frac{e^{\{\eta_{(\cdot)}(v - \tau_1)\}} - e^{\{\eta_{(\cdot)}(t - \tau_1)\}}}{\eta_{(\cdot)}}$, in which $\eta_{(\cdot)}$ is the algebraic sum $\alpha_{(\cdot)} - r_f - (\sigma_{(\cdot), i} \theta_i + \sigma_{(\cdot), j} \theta_j)$, $i \neq j$. Although the functional form taken by $HC_{(\cdot), t, v}$ is rather complicated, the idea behind its definition is intuitive: $HC_{(\cdot), t, v}$ quantifies the individual's human capital wealth – the present value of her future labor earnings – while employed in $(\cdot) = i, j$ over the time interval $[t, v]$.

The right-hand side of Eq. (6) disentangles the individual's initial financial wealth, her discounted stream of earnings conditional on maintaining job i till retirement and the value of having the flexibility to engage in the alternative job, j . The last term has the structure of a contingent claim written on the spread between the present value of payments received from jobs j and i , whose cost upon exercise amounts to f . By inspection, this is the payoff structure of an American *spread* option.¹¹

Because labor supply does not enter the individual's preferences, Fisher's Separation Theorem obtains and the solution of the optimal consumption–portfolio–job-regime program for the individual can be described as taking place in two steps. The first is to select the optimal timing for exercising the life-cycle option so as to maximize lifetime wealth. The

⁹Note that, for points in time sufficiently close to τ_2 , it is optimal to reject all job change opportunities since insufficient time remains to recover tuition expenses. This reasoning justifies feasible stopping times in the closed interval $[0, \tau_2 - \Delta]$.

¹⁰This is only one instance of mobility costs: housing, pension claims, fringe benefits, psychological and social costs, and age are at the core of a vast literature examining impediments to labor mobility.

¹¹A call spread option is a contingent claim written on the spread between two prices, S_j and S_i , with payoff $(\max(S_j - S_i, 0) - K)^+ = (S_j - S_i - K)^+$ upon exercise. When the exercise price K is null the spread option becomes an option to exchange asset i for asset j . Detemple (2006), pp. 121-124, provides key properties of a spread option exercise region.

second is to use financial markets to allocate wealth so as to achieve optimal consumption. The former optimization is embodied in the condition

$$V_0(w_i, w_j, \tau_1) \equiv \sup_{\tau_1 \in S_0, \tau_2 - \Delta} E \left[\xi_{\tau_1} (HC_{j, \tau_1 + \Delta, \tau_2} - HC_{i, \tau_1, \tau_2} - f)^+ \right],$$

which was previously stated in Eq. (6). The latter optimization is stated as follows

$$\begin{aligned} \max_{c_t \geq 0} E \left[\int_0^T e^{\{-\beta t\}} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right], \quad t \in [0, T] \\ \text{s.t.} \quad E \left[\int_0^T \xi_t c_t dt \right] \leq W_0 + w_{i,0} \frac{e^{\{\eta_i \tau_2\}} - 1}{\eta_i} + V_0(w_i, w_j, \tau_1), \end{aligned} \quad (7)$$

in which $\beta > 0$ is the constant subjective discount factor. Solving the unconstrained analog to Eq. (7) yields the optimal contingent consumption allocation

$$c_t^* = (\lambda^*)^{\frac{-1}{\gamma}} e^{\{\frac{-\beta}{\gamma} t\}} \xi_t^{\frac{-1}{\gamma}}, \quad (8)$$

where λ^* , defined below, is the constant multiplier attached to the static budget constraint of Eq. (7)

$$\begin{aligned} (\lambda^*)^{\frac{-1}{\gamma}} &\equiv \mathcal{X} \frac{W_0 + w_{i,0} \frac{e^{\{\eta_i \tau_2\}} - 1}{\eta_i} + V_0(w_i, w_j, \tau_1)}{1 - e^{\{-\mathcal{X}T\}}}, \\ \mathcal{X} &\equiv \frac{\gamma - 1}{\gamma} \left(r_f + \frac{\beta}{\gamma - 1} + \frac{\theta_i^2 + \theta_j^2}{2\gamma} \right). \end{aligned} \quad (9)$$

The static budget constraint also serves as a statement of the individual's desired risk exposure for the sustainment of her lifelong consumption

$$\underbrace{\xi_t c_t^* \frac{1 - e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}}}_{\text{desired risk}} = \underbrace{\xi_t W_t}_{\text{financial risk}} + \underbrace{\xi_t w_{i,t} \frac{e^{\{\eta_i(\tau_2-t)\}} - 1}{\eta_i} + \xi_t V_t(w_i, w_j, \tau_1)}_{\text{endowed risk}}. \quad (10)$$

The right-hand side of Eq. (10) distinguishes between financial risk linked to the funds invested at time zero, whose performance is formalized in Eq. (11), and endowed risk implicit in both current and future wage payments. The optimal portfolio policy guarantees the match of the individual's financial risk exposure to her *net* desired risk exposure, an instance of personal liability-driven investing.

$$dW_t = \pi_{i,t} \sigma_{S_i} (\theta_i dt + dz_{i,t}) + \pi_{j,t} \sigma_{S_j} (\theta_j dt + dz_{j,t}) + W_t r_f dt + w_{(\cdot),t} dt - c_t dt, \quad W_0 \text{ given.}^{12} \quad (11)$$

¹²For simplicity, I elect to ignore retirement income from Social Security. Although this additional wealth source may profoundly impact a worker's portfolio allocation, its effect on the optimal policies is indeterminate. On the one hand, it may increase risk tolerance and contribute to the demand for both current and alternative employer equity; on the other hand, it may induce lower retirement savings.

The optimal holding of risky security $(\cdot)$, either i or j , in dollars, is denoted by $\pi_{(\cdot),t}$ and it amounts to the right-hand side of Eq. (12).

$$\pi_{(\cdot),t} \equiv \frac{\sigma_{S(\cdot)}^{-1} \theta_{(\cdot)} (1 - e^{\{-\mathcal{X}(T-t)\}})}{\gamma \mathcal{X}} c_t^* - \frac{\sigma_{S(\cdot)}^{-1} \sigma_{i,(\cdot)} (e^{\{\eta_i(\tau_2-t)\}} - 1)}{\eta_i} w_{i,t} - \sigma_{S(\cdot)}^{-1} \mathfrak{D}_{(\cdot),t} [V_t(w_i, w_j, \tau_1)]^{.13} \quad (12)$$

The solution is the algebraic sum of three components. The first component replicates the standard mean-variance efficient portfolio and measures the investment required to support future consumption.¹⁴ The second component resembles Bodie *et al.* (1992)'s dollar-value equivalent exposure to the risky asset through labor/human capital. The third component is the novelty of the model: it corresponds to the option's delta in Bermin (2003, p. 77, Eq. 4.1) and quantifies the impact of the worker's option on the optimal portfolio policy. The magnitude, the sign and the intertemporal variations of all three components for the benchmark calibration are reported in Section 4.

3 Calibration

Solving the individual's optimization problem in the presence of multiple job choices, as well as stochastic nonfinancial income, is technically complex. The numerical algorithm used here relies on a two-dimensional binomial lattice that extends Broadie and Detemple (1996)'s routine to produce optimal exercise boundaries and portfolio policies. The procedure is summarized in Appendix A. However, the key to obtaining meaningful consumption and portfolio policies is an appropriate model parameter calibration. An issue that I have not addressed so far is the qualification of jobs i and j . The flexibility of the present framework permits to consider employments in alternative firms within the same industry, alternative industries or a specific industry vis-a-vis the rest of the economy. I examine the last instance. As a matter of fact, defining the individual's alternative employment as the set of all other industries in the economy, avoids limiting the job choice to two industries only. Since job-switching options are especially valuable to individuals able to pursue many careers within rather dissimilar industries or companies, this specification appears to be the most appropriate.¹⁵

3.1 Asset return dynamics

The calibration of the dynamics of security j , the market index, is that of Cocco, Gomes, and Maenhout (2005): the mean equity premium is 4% and the standard deviation of innovations

¹³The derivation of $\pi_{(\cdot),t}$ is presented in Appendix A.

¹⁴Note that the mean-variance component is independent of the individual's sector of employment.

¹⁵In addition, this specification allows for an alternative framework where the individual stock i can be priced off of the market index. It suffices it to represent the aggregate (priced) shock in the economy by $z_{j,t}$ and the idiosyncratic (unpriced) shock to S_i by $z_{i,t}$. This environment agrees with the evidence of Storesletten, Telmer and Yaron (2004, 2007).

to the market portfolio is set to its historical value of 15.7%. The risk-free rate of interest is 2%, conforming to the historical average on constant-maturity Treasury Inflation-Protected Securities.¹⁶ The dividend yield is set at 1.64%, the most recent rate available from Shiller's (2000) updated stock market data. I set the parameters of security i , the industry, to match key average statistics on stock returns. I study both lower and higher than market-level volatility industries. In the former calibration, industry volatility is equal to 12.5%, in line with the estimate of Campbell *et al.* (2001); in the latter, it is set to 21%. The last figure is determined using value-weighted annual returns of 49 industry portfolios.¹⁷ Rosenberg and Guy's (1976) industry betas dictate the sizes of the risk premia. The low- (high-)volatility industry stock offers a risk premium equal to 3.2 (5.1)%.¹⁸ Dividend yields are unchanged at 1.64%.

3.2 Preference structure

The discount factor and the risk aversion coefficient defining the individual's preference structure are equal to 0.03 and 6, respectively. I further assume a one-year vocational training spell. The estimated annual expenses of attending school are set to \$9,000, within the range published by ITT Educational Services.^{19,20}

3.3 Labor income dynamics

The expected rate of wage growth over the individual's working years is equal to 2% based on the analysis of McCue (1996, Table 1, p. 182, and Table 3, p. 185). The wage volatility associated with both the low-volatility industry and the market is set at 4.5%. The wage volatility corresponding to the high-volatility industry is 8.5% in accordance with Davis and Willen's (2000) occupational components of wage innovations. Lastly, Bodie, Treussard, and Willen (2006)'s annual earnings for 25-year-old men with a high school diploma dictate my choice of initial wage levels at \$24,199 per year. I estimate initial financial wealth to be \$20,000, circa 80% of the initial salary.²¹ Retirement and death are certain at age 65 and

¹⁶Source: FRED, <http://research.stlouisfed.org/fred2/categories/82>.

¹⁷Source: K. French Data Library, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Among all returns series, I select "Electrical Equipment," industry 22, for which Davis and Willen (2000) provide corresponding wage-volatility values. See Appendix B for more details.

¹⁸Since the correlation between financial and nonfinancial income in industry 22 is equal to 0.45 (< 1), its parameter values need to be transformed as explained in Appendix B.

¹⁹ITT Educational Services, Inc. is a private college system focused on technology-oriented programs of study. According to U.S. Department of Education data, all of the ITT Technical Institutes combined granted the largest percentage (14.7 percent) of the total number of associate and bachelor degrees awarded in the U.S. in electronics and electronics-related programs in the 2000-2001 school year (the latest year for which statistics are available).

²⁰Alternatively, Ruffino and Treussard (2007) model a deterministic transition phase during which the individual need not train at a cost but, instead, earns a reduced salary. All else equal, there are no quantitative differences with the simulations of Section 4, even for temporary losses of 40%.

²¹This assumption is in line with Gentry and Hubbard (2000)'s finding. The authors use the 1989 Survey of Consumer Finances to construct household wealth-income ratios by age, education, income and entrepreneurial status.

90, respectively. The full set of parameters is listed in Table 1.

4 Simulation Results

In this Section I describe the model's predictions as they relate to the up-to-date portfolio literature. The set of figures and tables thenceforth illustrates original results on the value of the life-cycle option and on the portfolio policies.

4.1 Maturation of the job-switching option

Figure 1 photographs the maturation of the career option over the state space created by securities i and j .²²

The spread option is very consequential for young workers endowed with rather little financial wealth. 30 years away from retirement the option value is as high as 37 times the worker's initial wealth floor, W_0 .²³ Consequently, young decision-makers, who are likely to recoup mobility costs over the future working years, are more inclined towards exercising their life-cycle option, while the disposition of older workers towards a late change of careers is much depressed by costs. Accordingly, for any given pair (S_i, S_j) , benefits from swapping jobs diminish with working age as the payoff period declines – the option-value surface flattens.²⁴

Additionally, at all ages, the option appreciates (depreciates) in the price of the risky stock that disciplines the alternative (current) industry of employment. This result is a direct consequence of both the labor income processes and the American spread option's payoff structure.²⁵

Finally, observe that, since an option on a portfolio is less valuable than a portfolio of options (Merton, 1973, Theorem 7, p. 48), Figure 1 undervalues the job-switching option. If the individual could choose among many jobs, the spread option would be even more consequential.²⁶

²²Since the career option of Eq. (6) is completely characterized by the individual's human capital wealth, some algebraic manipulations are necessary to specify $V_0(w_i, w_j, \tau_1)$ over the state space created by the two risky assets. The new formula, which enters the numerical algorithm, is derived in Appendix A.

²³Allowing for a modest, though positive, correlation in wage innovations, $\rho_{i,j} = 0.03$, limits the option value to 13 times W_0 . To the opposite, a negative correlation equal to -0.03 enhances the option's value that reaches a maximum of 85 times W_0 . This conforms to the option's function to provide the individual with the opportunity to exchange her labor earnings in industry i with those offered by the rest of the economy. This flexibility is more precious to the worker the less alike her industry and the market are.

²⁴This line of reasoning may motivate steeper slopes in wage profiles during the early years of the individual's career, a reflection of industries' negotiating higher wages for retention purposes. As a matter of fact, Ruffino and Treussard (2007) predict empirically documented concave logarithmic wage patterns resulting from the introduction of career options in a model featuring prototypical constant wage parameters.

²⁵In a discrete-time setting, a small chance of transitioning into an absorbing state where the stock price is zero could cause a worker to lose her retirement savings without any possibility of recovery. In the present continuous-time setting, the job-switching option effectively precludes this scenario by letting the worker seek alternative employment when her company does poorly.

²⁶Valuation formulas for American options on multiple assets can be found in Broadie and Detemple

4.2 Role of the job-switching option in portfolio decisions

To render the optimal investment policies more readily comparable with empirical findings and other available theoretical estimates, I calculate the shares of risky and risk-free investments as functions of accumulated financial wealth. Tables 2, 3, 4 and 5 compile these simulation results.

Table 2 assesses the contribution of job-switching options in determining optimal investment policies. The model's predictions are contrasted with the optimal intertemporal allocations originating from a model without human wealth (column 1) and a model with risky human wealth and without the job-switching option (column 2).

Samuelson (1969) and Merton (1969) demonstrate that, ignoring human wealth, an individual whose preferences are rationalized by a power utility function, and whose investment opportunity set is non-varying, invests in each risky asset a fraction of wealth that only depends on risk aversion and on the asset's excess return moments. Additionally, these shares are constant over time and independent of her financial wealth. The first column of Table 2 reports the results of these calculations using the calibration of Section 3. Since stock i is less volatile than stock j , its return in excess of the risk-free rate per unit of risk is higher than the one provided by the market index. As a result, the share of stock i is larger than the share of stock j and the risk-free investment, which is derived by difference, constitutes the bulk of the individual's assets.²⁷ The composition of the initial portfolio remains unchanged till retirement.

Columns 2 and 3 account for human capital in determining the individual's optimal investment decisions without and with the option to change jobs, respectively. To make a meaningful comparison between the two cases, I choose the initial wage level in the permanent employment model so as to equal the present value of human wealth under both specifications. I find that the individual invests considerably more in her own-industry stock when she is endowed with the option to change jobs. This result, which holds at all ages, reflects the extra "diversification on-the-job" embedded in the career option and it is particularly striking for young investors. After 10 years into her working life an individual who is compelled to keep the same job till retirement should sell short her industry stock. On the contrary, if she had the option to change jobs, she would allocate to her industry stock almost 40% of her financial wealth. Moreover, 5 years away from retirement, this percentage would still be as high as 33%.²⁸ These figures are rather appealing, especially in reference to numerous empirical studies documenting the tendency of investors to pick "familiar" stocks. According to the data, the allocation to employer stock reaches a third of the assets in large retirement-saving plans and about a quarter of employees' discretionary contributions (Benartzi, 2001). Massa and Simonov (2006) further investigate this phenomenon and find that the financial behavior of low-wealth investors, such as those analyzed in this context, is much

(1997).

²⁷Observe that, for the worker with no labor income (column 1, tables 2 and 3), the actual market is the sum of her own- and cross-industry investments. Here, the worker must hold her "own" shock to achieve optimal market exposure.

²⁸Since the benefits from swapping jobs diminish with age (Figure 1), the radical difference in initial portfolio compositions eventually disappears and the exposure to equities narrows as labor income substitutes for risk-free asset holdings.

driven by familiarity. The model’s predictions agree with their, among others’, evidence and shed new light on the importance of building models that specialize human capital wealth along the life-cycle.

Finally, possessing the option to switch jobs reduces the need to borrow at all horizons to retirement: this is a consequence of the greater compensation that the individual receives conditional on her exercising the option.²⁹

All estimates in Table 2 depend on the assumption that employment begins in a low-volatility industry. Table 3 compiles a corresponding set of predictions that obtain when the individual starts her career in a high-volatility industry.

Absent any human wealth, the portfolio composition is tilted more towards stock j than stock i . Indeed, because stock i has a much greater standard deviation than in the calibration of Table 2, its low reward-to-variability ratio makes it less desirable. Furthermore, the fraction of financial wealth allocated to risk-free assets has grown with industry risk from 67% to 71%.

Similarly, higher own-industry volatility increases a worker’s appetite for safe assets. Irrespective of the option, borrowing is never optimal and the individual’s initial conservative positions are gradually adjusted in favor of a more balanced equity/risk-free asset mix (columns 2 and 3). As it is also detectable in Table 2, the final allocation results from continuously increasing one’s position in own-industry stock while reducing cross-industry exposure. The difference between electing a risky initial job and possibly switching to a safer one (Table 3, column 3) and vice versa (Table 2, column 3) is that, in the former case, the extra flexibility supplied by the option moderates, but does not overturn, the basic logic of portfolio construction with a nontraded asset. The intuition behind this effect is as follows. To build a balanced portfolio suitable to her risk preferences, the worker needs to trade away the endowment of her nontraded assets. Without job-switching options, this implies that her share of own-industry stock is negative as long as her wages are relatively risky. With job-switching options, the worker’s human capital constitutes the holdings not just of her employer’s stock, but also of put options on that stock. Because a synthetic put option would involve shorting a fraction, say $\Delta_{S_{i,t}}$, of the underlying stock, the worker would optimally sell $(1 - \Delta_{S_{i,t}})$ shares of stock. Since $(\Delta_{S_{i,t}} < 1)$, her negative position in own-industry stock is lessened but not reversed.

4.3 Unexercised vs. exercised job-switching options

Table 4 displays disentangled investments in securities i and j – the right-hand-side of Eq. (12) – along with the resulting net policies, as they apply to a worker who is employed in the same low-volatility industry till retirement, letting her career option expiring unexercised. This decomposition permits to highlight the main forces at work and to gain intuition on the determinants of portfolio choices.

²⁹If the leverage ability were taken away, then workers could not fully diversify their implicit financial holdings by investing a large share of their regular financial assets to their own company stock and a small share to the stock of their alternative employment. In the absence of borrowing constraints, the model is more predisposed to produce positive holdings of employer stock.

The mean-variance portfolio component (TW) embodies the investment in risky financial assets derived solely from the need to finance future consumption. It decreases as the individual ages because the remaining consumption stock erodes near death. Such diminution, however, does not lessen the proportion of savings in securities i and j , which is constant over time. In fact, the ratio of consumption-driven investments readily obtains from the right-hand-side of Eq. (12) and takes the form

$$\frac{TW_{i,t}}{TW_{j,t}} \equiv \frac{\sigma_{S_i}^{-1}\theta_i}{\sigma_{S_j}^{-1}\theta_j} = \frac{\frac{\mu_{S_i}-r_f}{\sigma_{S_i}^2}}{\frac{\mu_{S_j}-r_f}{\sigma_{S_j}^2}}, \quad (13)$$

equal to 1.26 implementing the baseline calibration of Section 3. The aggregate mean-variance risk exposure – the percentage invested in industry i and the market – falls from 10.30 to 0.95 times current financial wealth as the individual nears retirement. Aggregate shares significantly above 1 at young ages reflect optimal consumption policies in excess of financial wealth.

The third column, HW, presents the Bodie-Merton-Samuelson implicit investment in securities i and j : it is an estimate of the individual exposure to risky assets of the capitalized value of wage flows. Accordingly, high risk exposures ought to relate to the industry of employment and low levels of exposure to the alternative one. Because of its negative sign in the optimal investment formula, the HW component reduces the individual's overall risky position by lowering her mean-variance share. Furthermore, the HW investment is decreasing over time, mirroring labor income substitution for risk-free asset holdings. In Table 4, the HW percentage reduces to almost one thirtieth of its initial value relative to industry i , while it never drifts from zero relative to the market because the individual elects to keep her job till the end.

The option factor, last column of Table 4 (OW), obtains from the numerical evaluation of the last component of Eq. (12). Since this component is signed negatively, a negative hedge indicates larger stock holdings while a positive one moderates the individual's positions. The idea behind this mechanism is intuitive. Because the individual's career option depreciates in the stock price underlying her current job, S_i , her behavior in face of potential losses arising from higher value of stock i , all else constant, is to take on broader long positions in stock i . A symmetric argument applies to stock j . After 10 years of work, the option's delta accounts for 32% of the individual's financial wealth contribution to stock i and 30% of what she deposits into the market index. Both OW shares diminish over time in absolute values, becoming eventually negligible as the option matures. As a matter of fact, the individual would optimally decline all job change opportunities within few years to retirement since insufficient time would remain to recover tuition expenses.

The first column of Table 4 is a composite of the three factors analyzed above. In what follows, I refer to the gradual increase in financial risk-taking resulting from the depreciation of risky human capital as the *aging effect*. On the other hand, a mature worker who has grown very entrenched in her industry tends to concentrate her holdings in the alternative one. This investment strategy connotes what I identify as the *settling effect*. What is key

to my representation, is that while the former effect depends purely on the length of the individual's working experience, irrespective of her sector of employment, the latter effect is contingent on her past career choices.

The aging effect dominates the time pattern of own-industry holdings after 20 years of employment: the share of financial wealth invested in stock i rises from 27% to 33%. The settling effect is also operative in that the percentage of equities in the market index consistently outpaces that in industry i where the individual is employed: $\left(\frac{\pi_{j,t}}{\pi_{i,t} + \pi_{j,t}}\right)$ ranges from circa 92% during the individual's early career to 56% near retirement. By difference, the individual tilts her portfolio towards her industry stock initially committing to it 8% of her total equity exposure and raising it later to as much as 44%.

Abstracting from the absolute size allocated to each risky asset, the proportion of financial wealth invested in equities decreases over time. Towards retirement, the individual prefers safer investments and contributes up to 25% of her accumulated wealth to the risk-free asset.

Table 5 summarizes the investment strategy of an individual who switches jobs within the first 10 years of work in industry i . Since the most robust fact in the empirical labor literature is the sharp decline of labor mobility with age, studying the intertemporal financial allocation of a young mover is particularly relevant and fruitful.

The individual's early exercise of her career option materializes in the third column of Table 5: both OW factors are null.

Aside from the proportion of the TW fractions, which I already showed to be constant over time, the size of each share is much reduced with respect to those of Table 4 at all investment horizons. Indeed, the exercise of the option has enriched the individual with a greater compensation, softening her positions in equities for the purpose of guaranteeing future consumption. The TW share allocated to own- (cross-)industry stock falls from 79% (100%) to 36% (45%) if the individual quits her job versus 575% (455%) and 53% (42%) if she stays.

Finally, special attention ought to be warranted to explaining the magnitude, the sign and the intertemporal variations of the optimal net policies. Recall that the settling effect designates an asset-holding position in which a worker who is much anchored to her industry invests heavily in the alternative one. The motive supporting this strategy is hedging: investors hold risky financial assets to offset their labor income risk. The first column of Table 5 validates this principle: 30 years ahead of the expected retirement date the share of equities in cross-industry stock equals 122%; 5 years away from retirement it is still as high as 71%. This, in turn, implies that investments in own-industry stock rise with age from -22% to 29%. These percentages summarize very effectively how a change of jobs when young causes large redistributions of individual financial wealth over the life-cycle. Additionally, an individual who has changed job allocates a lower fraction of equities to her current employment than she would had she not left her initial position.³⁰ The explanation lies in the loss of the extra "diversification-on-the-job" embedded in the individual's option. Consequently, to reconstruct a well-diversified portfolio after switching job, the individual needs to tilt her

³⁰The own-industry share of equities grows from -22% to 29% if the individual quits her job, versus 8% and 44% if she stays (first column of Tables 5 and 4, respectively).

assets away from her industry and towards the alternative one.

Moreover, $\pi_{i,t}$ and $\pi_{j,t}$ inflate or deflate during the individual's working years depending on the rate of decay of her human wealth, the aging effect. As in Table 4, the aging effect mirrors labor income substitution for risk-free asset holdings. Nonetheless, this does suffice to generate an increasing pattern of aggregate equity holdings over time.

In conclusion, with or without a job change, (i) an extensive fraction of financial wealth is invested in own-industry equity, even if limited relative to its cross-industry complement, and (ii) individual portfolios are significantly rebalanced over time.³¹

What differentiates permanent employment from job mobility is the fact that, in the former case, total equity holdings are positive and tangible while, in the latter case, they are positive but less sizeable due to negative initial positions in own-industry stock. De facto, as documented in Table 5, risk-free investments range between 18% and 37%, which are consistently lower numbers than those of Table 4. The prediction that, conditional on switching jobs, workers have no need for borrowing at any age is a very attractive feature of the model, particularly in light of the many borrowing constraints that investors face in reality and that prevent them from capitalizing future labor income.

4.4 Comparative Static Analysis

In this Subsection I conduct a sensitivity analysis of my findings to several changes in the benchmark parameter values. The first set of comparative statics involves the individual's degree of risk aversion. Table 6 compiles own- and cross-industry portfolio shares, along with risk-free asset investments, as functions of the expected retirement horizon and for coefficients of relative risk aversion equal to 4, 6 (baseline), 8 and 10. Each share is the algebraic sum of TW, HW and Option.

Raising risk aversion from 4 to 10 lowers the fraction of net worth allocated to equities from 4.69 (1.22) to 1.18 (0.36) when retirement is expected in 30 (5) years. Correspondingly, at all horizons to retirement, low risk averse individuals short the risk-free asset while high risk averse individuals hold large amounts of it in their portfolios. For instance, two individuals who are 10 years from retirement, identical in all respects but their degree of risk aversion, equal to 4 and 10 respectively, would invest -73% and 63% in the risk-free asset.

In addition, the reduction in equity risk exposures that accompanies higher coefficients of risk aversions is characterized by a relatively more substantial decrease in own-industry investments. These shares become negative for degrees of risk aversion equal to 8 and 10 indicating strong hedging motives. Note that, irrespective of her degree of risk aversion, the individual invests a positive and considerable fraction of her accumulated wealth to the cross-industry stock – the settling effect introduced earlier. The share of total equity held in own-industry stock by a low risk averse individual ($\gamma = 4$) raises with age from 0.36 to 0.48 times the level of current financial wealth. The comparable shares held by a high risk averse individual ($\gamma = 10$) are -5.84 and 0.30, respectively.

Table 7 summarizes optimal investment policies as I vary industry i wage risk, $\sigma_{i,i}$.

³¹Large transfers are economically important especially in view of non-negligible costs of stock market participation, as those estimated by Vissing-Jorgensen (2002) and Polkovnichenko (2004).

Increased wage volatility engenders three noteworthy effects for asset allocation: holding constant the retirement horizon, (i) the fraction of financial wealth invested in own-industry stock decreases, and (ii) the share of risk-free investment grows; intertemporally, (iii) highly volatile wages can produce increasing risk exposure to equities.

More volatile labor earnings affect the proportion of financial wealth in equities in a similar manner to higher degrees of risk aversion: the individual invests more cautiously in equities, notably reducing the relative exposure to her industry stock. 30 years ahead of retirement, she invests in industry i 1.84 or -8.34 times her accumulated wealth, depending on $\sigma_{i,i}$ being fairly low, 2%, or very steep, 10%. As she ages, she contributes less financial wealth to her own-industry stock, eventually apportioning to it 44% or 15% in the lowest and highest wage volatility scenarios, respectively. Similarly, her exposure to the cross-industry stock declines steadily over time to 43% and 38% in correspondence to the lowest and the highest $\sigma_{i,i}$ values, respectively.

Against all risk-aversion calibrations of Table 6, workers with more uncertain jobs may exhibit increasing equity patterns over time. Investors whose stream of future earnings is highly uncertain (column 4 in Table 7), and whose overall equity exposure is initially very limited, abandon their conservative positions in favor of a more balanced equity/risk-free asset mix. On the contrary, young workers receiving relatively safe labor income payments borrow massively and invest all in equities. As they grow older, they reduce their exposure to equities and shift their assets towards safer options. Levels of wage volatility as low as 2%, 4.5% or 6% produce decreasing equity patterns (first three columns of Table 7). A level of wage volatility equal to 10% is sufficient to invert the pattern (column 4 in Table 7).³²

The analyses of Tables 6 and 7 exemplify how sensitive portfolio allocation rules are to different assumptions about utility and the stochastic process for labor income. With respect to the baseline calibration, trends are generally not inverted – the qualitative implications of the model are robust to the variations in parameter values examined up to now – but the optimal equity/risk-free asset mix often rebalances substantially.

The next Section is dedicated to contrast the model’s predictions with the major findings of perhaps the most frequently cited and trustworthy empirical studies on household portfolio selection. Since career options are a novelty within the life-cycle portfolio literature, it is extremely important to verify that their inclusion really permits to better represent individual portfolio choices. Hence, the importance of meticulously choosing case studies to which the results presented so far may be compared.

5 Matching Actual Portfolio Selection Criteria

The first data source that I consider is the annual report on 401(k) plan asset allocation, account balances and loan activity by VanDerhei, Holden, Copeland and Alonso (2010) for the Employee Benefit Research Institute (EBRI). EBRI and the Investment Company Institute (ICI) have developed the most comprehensive database on 401(k) plan participants

³²Predictions that originate from highly volatile labor payments are strongly supported by the data indicating that most households hold significant amounts of low-risk assets in their portfolios and increase their share of stocks with age. Farhi and Panageas (2007), among others, document this tendency.

yet assembled: their multi-source longitudinal database provides information on participant-level decisions with respect to participation, contributions, and asset allocation.

At year-end 2009, the EBRI/ICI database included statistical information about 20.7 million 401(k) plan participants in 51,852 employer-sponsored 401(k) plans holding \$1.210 trillion in assets. The EBRI/ICI database covers 40% of the universe of 401(k) plan participants, 10% of plans, and 44% of 401(k) plan assets. Its investment options are grouped into eight categories: these do not comprise the number of distinct investment options presented to a given participant, but rather the types of options presented to her. Equity funds consist of pooled accounts primarily invested in stocks such as equity mutual funds, bank collective trusts, life insurance separate accounts, and other pooled investments. Similarly, bond funds are pooled accounts primarily invested in bonds, and balanced funds are pooled accounts invested in both stocks and bonds. Company stock is equity in the plan's sponsor (the employer). Money funds consist of those funds designed to maintain a stable share price. Stable value products, such as guaranteed investment contracts (GICs) and other stable value funds, mainly insurance company products that guarantee a specific rate of return on the invested capital, are reported as one category. The last two categories are residual for other investments, such as real estate funds, and for funds that could not be identified.

Figure 2 quantifies own- and cross-industry stock ownerships, as well as risk-free asset ownership, in the model and in the EBRI/ICI database. Some clarifications are in order to explain how I have aggregated VanDerhei *et al.*'s original figures into the histograms below.

First, own-industry stock holdings correspond to the equity percentage in the plan's sponsor. This is equivalent to assuming that the worker's company stock is representative of her industry's risk-return characteristics.

Second, cross-industry stock holdings collect all equity investments – except for the fraction in company stock – as well as other risky securities. These include equity funds, balanced funds and risky bonds. GICs and money market accounts constitute risk-free investments. Therefore, in my reclassification of financial securities, risk-free investment generally indicates less risky assets.

Third, the EBRI/ICI database records asset allocation information for retirement assets only, not for all individual financial assets. Since distinct average shares for non-retirement and retirement assets are unavailable, I assume that retirement assets alone form individual portfolios.

Lastly, my simulation results are averages over the individual's working life and VanDerhei *et al.*'s calculations are averages across age groups.³³ To my knowledge no data set containing time series on household portfolio allocations is lengthy enough to allow for an exhaustive investigation of the individual's saving and investing decisions over the life-cycle. This compels me to make use of cross-sectional average shares that may differ significantly from lifetime averages if birth cohort effects are present.³⁴

Figure 2 shows that simulated portfolio shares approximate exceptionally well the per-

³³Disaggregated (simulated and actual) data across age groups are available upon request.

³⁴This is not too much of a concern in light of the simulation results in Table II: as previously uncovered by Ameriks and Zeldes (2004), decreasing equity shares with age appear in the data if age and time effects only are included in the specification, but cohort effects are not.

centages from the EBRI/ICI database.

Besides the very close match of own-industry stock holdings to the documented empirical evidence, the histograms underline that the greater part of total assets is invested in stocks and other risky securities: more than two-thirds of 401(k) participants' assets are invested in equity securities through equity funds, the equity portion of balanced funds, and company stock.

In their report, VanDerhei *et al.* stress that the share of 401(k) accounts invested in company stock has been shrinking over the past decade, falling by 1 percentage points in the sole 2009 and continuing a steady decline that started in 1999. I thus examined previous EBRI/ICI reports. The share of assets allocated to own-industry stocks equaled 19% in 1996 and 1999 and it started decreasing afterwards reaching 16% in 2002, 13% in 2005 and 10% in 2008.³⁵ This suggests changes in both plan design and participants' behaviors. VanDerhei *et al.* focus on recently hired participants to draw out information about the impact of current plan design and other factors on individual participants' decisions. They find that not only are fewer recent hires holding own-industry stock, but fewer recent hires are holding high concentrations of own-industry stock. For example, among recently hired participants, 3.9% held more than 90% of their account balance in own-industry stock in 2006 (VanDerhei *et al.* (2009), Figure 41, p. 43). Among the comparable group in 1998, 12.4% had such concentration. Irrespective of this trend in usage and concentration, observed shares invested in own-industry stock did not distance significantly from the optimal policies and remained close to the predicted average of 14%. Risk-free investments ranged from a minimum of 15% in 1999 to a maximum of 22% in 2002 and 2008.

Overall, the model's predictions are robust and the explanatory power of life-cycle options for portfolio selection mechanisms is unquestionable. Background risk exposures, transaction costs and facilitated/better information on a particular stock have been often advocated as possible explanations for the puzzling concentrated holdings in employer's stock.^{36,37} The additional diversification afforded by the individual's career option justifies her taking large financial positions in her industry stock. As a matter of fact, one could argue that, even though both channels are feasible, job mobility may provide a more easily implementable means of diversification than active trading in financial markets. Because labor markets have grown more flexible, and because career options do not entail a premium while operating in financial markets still commands a price³⁸ and demands some degree of financial knowledge, overinvestments in own-industry stocks could be anticipated.

³⁵ Although some of the decline since 2000 reflects a drop in stock prices during the 2000-2002 and 2007-2009 bear markets, the share of assets held in company stock continued to drop during the stock market's rebounds.

³⁶ Costly information acquisition helps rationalizing under-diversification in Van Nieuwerburgh and Veldkamp (2010).

³⁷ Cohen (2009) determines that workers' company loyalty, broadly defined as "an emotional tie," (p. 1214), helps explaining large proportions of employee pension wealth invested in own company stock.

³⁸ The relevance of transaction costs for individual investors is studied by Vissing-Jorgensen (2002) and Polkovnichenko (2004). Vissing-Jorgensen estimates that a per period stock market participation cost of just \$50 is sufficient to explain the choices of half of stock market nonparticipants. Similarly, Polkovnichenko concludes that participation costs of less than 1 percent of per capita labor income support equilibria with no trading in equities for 70% of the population.

As mentioned at the beginning of this manuscript, the present model lends itself to various job qualifications. Next I further exploit this flexibility by examining employment in a specific firm vis-a-vis the rest of the economy.

Key parameters for the market index are unchanged relative to those of Table I. I set firm-level volatility equal to 30%, an upper bound to the 25% average estimated by Campbell *et al.* (2001). Risk premium and dividend yield are equal to 4% and 1.64%, respectively. Figure 3 compares this alternative calibration to the empirical analysis of Ameriks and Zeldes (2004).

Ameriks and Zeldes use pooled cross-sectional data from the 1998 Survey of Consumer Finances. The survey includes information on assets both inside and outside of retirement accounts as well as demographic information. Using these data for the investigation of portfolio allocation, however, presents some disadvantages. Ameriks and Zeldes highlight that, for instance, the survey responses to questions regarding the allocation of assets held in mutual funds or in retirement accounts are categorical in nature, adding noise to the data on household portfolio shares. Another disadvantage is that the survey does not follow the same set of households over time.

Financial assets are classified into four categories: stocks, bonds, cash, and "other". The data show that the average portfolio share invested in stocks through retirement accounts is roughly 57% and that, on average, households hold in retirement accounts circa 30% of their net worth. This does not suffice to quantify individual holdings in the firm's stock. For this purpose I consider Benartzi's (2001) estimated company-stock ownership, as a percentage of the employee's *voluntary* contributions. When the plan requires the employer match to be invested in the company stock, this share is equal to 29% (Benartzi (2001), Table II, p.1753), which translates into the 5% average share reported in Figure 3 ($5 \cong 57 \cdot 0.29 \cdot 0.3$). Cross-firm holdings are pooled investments of other stocks and risky bonds while risk-free investments include cash and "other."

The histograms show that the own-firm investment predicted by the model is equal to that of the data. The aggregate share allocated to risky equities, however, differs considerably between the two. The model engenders limited positions in equities, 21% of total financial wealth, while the bulk of total assets is allocated to risk-free investments. The data exhibit equity shares of the order of 40%. I attribute this difference to the large value of firm-specific volatility: as documented in Table 7, optimal portfolio policies are very sensitive to this parameter and high levels of risk largely reduce equity holdings. The same argument explains the radical change in the relative weight allocated to risky versus risk-free securities: the optimal equity-risk-free asset mix is 85%–15% according to the baseline calibration, Figure 2, and almost reversed, 20%–80%, according to the alternative one.

These numbers illustrate how the individual's perception of her being employed in a particular firm, or belonging to a particular industry, affects the model's calibration and substantially alters the resulting optimal policies. In a similar manner, the model could be adapted to a setting featuring new employment in another firm, rather than the rest of the economy. Then, depending on the two firms being part of the same industry, the individual would need to acquire little new skills at her new job, if any, and mobility costs would be minor. All these examples underline how the present model, if properly calibrated, can

predict optimal behaviors for a great variety of labor and financial market conditions.³⁹

6 Summary

In this paper I study household portfolio decisions over the life-cycle. I extend the theoretical literature on optimal intertemporal asset allocation positing that the finite-lived individual of this economy is enabled to change job during her working years.

I find that the individual optimally bears more own-industry risk in her financial portfolio than if she did not have the extra "diversification on-the-job" embedded in her option to change career. This effect contributes to explain (i) the tendency of investors to choose "familiar" stocks, and (ii) the large exposure of young businessmen to risky investments. Another remarkable result is the large value of portfolio rebalancing over time.

Increasing the individual's aversion to risk or the degree of uncertainty of her wage function does not generally invert observed trends – the qualitative implications of the model are robust to variations in parameter values – but the optimal equity/risk-free asset mix often varies substantially.

The model matches the empirical evidence on own-industry stock holdings very closely and, if properly calibrated, it can predict optimal policies for a great variety of labor and financial market conditions.

This framework enables a wide range of future research. It can be adapted, for example, to analyze optimal portfolio policies within an economy in which the individual can repeatedly alternate between her initial job and the later one. No-borrowing constraints, no-short-sale constraints, participation and trading costs could also be included.

³⁹ Although not reported in this version of the paper, I examine how the model's predictions can be applied to other puzzles in the financial literature. These are the private equity premium puzzle documented by Moskowitz and Vissing-Jorgensen (2002) and Hintermaier and Steinberger (2005) and the motion of wealth-to-income ratios with age studied by Gentry and Hubbard (2000). All results are available upon request.

7 Appendix A

7.1 The option to switch careers

7.1.1 Specification over the state space

Since the career option of Eq. (6) is completely characterized by the individual's human capital wealth, the present value of her future labor earnings, some algebraic manipulations are necessary to specify $V_0(w_i, w_j, \tau_1)$ over the state space created by the two risky assets. Letting $\mathfrak{S}_{(\cdot),0,t} \equiv \frac{S_{(\cdot),t}}{S_{(\cdot),0}}$, I re-express wage earnings in terms of current stock values. Equations (2) and (4) yield

$$\frac{w_{i,t}}{w_{i,0}} = e^{\{\gamma_i t\}} (\mathfrak{S}_{i,0,t})^{\frac{\sigma_{i,i}}{\sigma_{S_i}}} (\mathfrak{S}_{j,0,t})^{\frac{\sigma_{i,j}}{\sigma_{S_j}}}, \quad (14)$$

$$\frac{w_{j,t}}{w_{j,0}} = e^{\{\gamma_j \tau_1\}} (\mathfrak{S}_{i,0,t})^{\frac{\sigma_{j,i}}{\sigma_{S_i}}} (\mathfrak{S}_{j,0,t})^{\frac{\sigma_{j,j}}{\sigma_{S_j}}} \quad (15)$$

where γ_i and γ_j are defined by

$$\gamma_i \equiv \alpha_i - \frac{\sigma_{i,i}^2}{2} - \frac{\sigma_{i,j}^2}{2} - \frac{\sigma_{i,i}}{\sigma_{S_i}} \left(\mu_{S_i} - \delta_{S_i} - \frac{\sigma_{S_i}^2}{2} \right) - \frac{\sigma_{i,j}}{\sigma_{S_j}} \left(\mu_{S_j} - \delta_{S_j} - \frac{\sigma_{S_j}^2}{2} \right), \quad (16)$$

$$\gamma_j \equiv \alpha_j - \frac{\sigma_{j,j}^2}{2} - \frac{\sigma_{j,i}^2}{2} - \frac{\sigma_{j,j}}{\sigma_{S_j}} \left(\mu_{S_j} - \delta_{S_j} - \frac{\sigma_{S_j}^2}{2} \right) - \frac{\sigma_{j,i}}{\sigma_{S_i}} \left(\mu_{S_i} - \delta_{S_i} - \frac{\sigma_{S_i}^2}{2} \right). \quad (17)$$

Equations (14) and (15) illustrate that, in the presence of strictly positive cross-industry volatilities, a rise in any security price generates a positive wealth effect. However, because my baseline calibration assumes $\sigma_{i,i} > \sigma_{i,j}$ ($\sigma_{j,j} > \sigma_{j,i}$), wages in industry i (j) are driven primarily by the evolution of stock i (j), conducing to disproportionate increases in each job's labor income and to a potential substitution effect of the current career for the alternative one. Employing Eqs. (14) and (15), the value of the job switching option assumes the more convenient form

$$\begin{aligned} V_0(w_i, w_j, \tau_1) \equiv & \sup_{\tau_1 \in S_0, \tau_2 - \Delta} E \left[\xi_{\tau_1} \left(w_{j,0} e^{\{\gamma_j \tau_1\}} (\mathfrak{S}_{i,0,t})^{\frac{\sigma_{j,i}}{\sigma_{S_i}}} (\mathfrak{S}_{j,0,t})^{\frac{\sigma_{j,j}}{\sigma_{S_j}}} \frac{e^{\{\eta_j(\tau_2 - \tau_1)\}} - e^{\{\eta_j \Delta\}}}{\eta_j} \right) \right. \\ & \left. - \xi_{\tau_1} \left(w_{i,0} e^{\{\gamma_i \tau_1\}} (\mathfrak{S}_{i,0,t})^{\frac{\sigma_{i,i}}{\sigma_{S_i}}} (\mathfrak{S}_{j,0,t})^{\frac{\sigma_{i,j}}{\sigma_{S_j}}} \frac{e^{\{\eta_i(\tau_2 - \tau_1)\}} - 1}{\eta_i} + f \right) \right], \quad (18) \end{aligned}$$

or, under the proper risk-neutral measure,

$$V_0(w_i, w_j, \tau_1) \equiv \sup_{\tau_1 \in S_{0, \tau_2 - \Delta}} \tilde{E} \left[e^{\{-r\tau_1\}} \left(w_{j,0} e^{\{\gamma_j \tau_1\}} \left(\tilde{\mathfrak{S}}_{i,0,t} \right)^{\frac{\sigma_{j,i}}{\sigma_{S_i}}} \left(\tilde{\mathfrak{S}}_{j,0,t} \right)^{\frac{\sigma_{j,j}}{\sigma_{S_j}}} \frac{e^{\{\eta_j(\tau_2 - \tau_1)\}} - e^{\{\eta_j \Delta\}}}{\eta_j} \right) \right. \\ \left. - e^{\{-r\tau_1\}} \left(w_{i,0} e^{\{\gamma_i \tau_1\}} \left(\tilde{\mathfrak{S}}_{i,0,t} \right)^{\frac{\sigma_{i,i}}{\sigma_{S_i}}} \left(\tilde{\mathfrak{S}}_{j,0,t} \right)^{\frac{\sigma_{i,j}}{\sigma_{S_j}}} \frac{e^{\{\eta_i(\tau_2 - \tau_1)\}} - 1}{\eta_i} + f \right) \right]. \quad (19)$$

7.1.2 Numerical methods

To solve the individual's optimization problem I rely on a two-dimensional binomial lattice for the underlying risky securities. Broadie and Detemple (1996, Appendix B) suggest a computationally efficient binomial routine for the pricing of American options on a single underlying asset. I extend their routine to the present two-dimensional environment in a way that allows me to produce a time series of optimal exercise boundaries in addition to the pricing of the career switching spread option. I begin by succinctly describing the peculiarities of the lattice method. The efficient routine of Broadie and Detemple (1996) does not require to store the entire tree in memory: only the information related to the current time step is required. I determine the step amplitude via Hull and White's (1988) equations adjusted for dividends as in Broadie and Detemple (1996). The range of an "up" movement in the binomial tree is expressed by

$$Up_i = \frac{tmp_i + \sqrt{tmp_i^2 - 4a_i^2}}{2a_i}, \quad (20)$$

where $tmp_i = a_i^2 + b_i + 1$, $a_i = e^{\{(r - \delta_{S_i})dt\}}$, and $b_i = a_i^2 \left(e^{\{\sigma_{S_i}^2 dt\}} - 1 \right)$. The down movement $Down_i$ is set so that $Up_i Down_i = 1$. The risk-neutral probabilities for security i , $P_i^{Up} = \frac{(a_i - Down_i)}{(Up_i - Down_i)}$ and $P_i^{Down} = 1 - P_i^{Up}$, determine the state prices over the four potential outcomes at each node. These equal $AD^{uu} = e^{\{-rdt\}} P_i^{Up} P_j^{Up}$, $AD^{ud} = e^{\{-rdt\}} P_i^{Up} P_j^{Down}$, $AD^{du} = e^{\{-rdt\}} P_i^{Down} P_j^{Up}$ and $AD^{dd} = e^{\{-rdt\}} P_i^{Down} P_j^{Down}$. Discounting risk-neutral probabilities initially to obtain Arrow-Debreu prices reduces the computational burden by saving a multiplication at each node. Finally, the stock price ladders are computed recursively via the formula $S_{i,0} Up^l Down^{n-l}$, where l represents the position in the ladder relative to the smallest possible realization of security i at step n . This sidesteps the need of relatively time-consuming power functions. The routine requires to input the tuition cost, f , the wages parameters, α_i , α_j , $\sigma_{i,i}$, $\sigma_{M,M}$, $\sigma_{i,M}$, and $\sigma_{M,i}$, and the stocks parameters μ_{S_i} , μ_{S_M} , δ_{S_i} , δ_{S_M} , σ_{S_i} , σ_{S_M} . Initial values for the stock prices, $S_{i,0}$ and $S_{M,0}$, as well as for the wages in both industries, $w_{i,0}$ and $w_{M,0}$, also need to be specified. Finally, the user must enter the risk-free rate of interest, r , and the individual's time to retirement, τ_2 , along with the spell during which the individual reinvests in human capital, Δ . The algorithm returns the value of the spread option, along with the optimal exercise boundary at each point in time in terms of positions in the binomial ladder, stock prices, and wages.

7.2 Optimal portfolio policies: formal derivations

Let me now consider the optimal life-cycle portfolio. The wealth process is given by

$$\begin{aligned} dW_t &= \pi_{i,t} [\mu_{S_i} dt + \sigma_{S_i} dz_{i,t}] + \pi_{j,t} [\mu_{S_j} dt + \sigma_{S_j} dz_{j,t}] + (W_t - \pi_{i,t} - \pi_{j,t}) r dt + w_{(\cdot),t} dt - c_t dt \\ &= \pi_{i,t} \sigma_{S_i} (\theta_i dt + dz_{i,t}) + \pi_{j,t} \sigma_{S_j} (\theta_j dt + dz_{j,t}) + W_t r dt + w_{(\cdot),t} dt - c_t dt \end{aligned}$$

and

$$d\xi_t = -\xi_t (r dt + \theta_i dz_{i,t} + \theta_j dz_{j,t})$$

so that

$$\begin{aligned} d(W_t \xi_t) &= d(W_t) \xi_t + d(\xi_t) W_t + d\langle W, \xi \rangle_t \\ &= \phi_{i,t} dz_{i,t} + \phi_{j,t} dz_{j,t} + \xi_t (w_{(\cdot),t} - c_t) dt \end{aligned}$$

I denote $\phi_{i,t} \equiv \xi_t \pi_{i,t} \sigma_{S_i} - \xi_t W_t \theta_i$. In addition, the dynamic budget constraint is

$$\underbrace{\xi_t W_t}_{\text{financial risk}} = E_t \left[\underbrace{\int_t^T \xi_v c_v^* dv}_{\text{desired risk}} - \underbrace{\int_t^{\tau_2} \xi_v w_{i,v} dv}_{\text{endowed risk}} \right] - \underbrace{\xi_t V_t(w_i, w_j, \tau_1)}_{\text{endowed risk}},$$

in which I solve the conditional expectations in closed forms to obtain

$$\begin{aligned} \xi_t W_t &= \xi_t c_t^* \frac{1 - e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}} - \xi_t w_{i,t} \frac{e^{\{\eta_i(\tau_2-t)\}} - 1}{\eta_i} \\ &\quad - \xi_t V_t(w_i, w_j, \tau_1). \end{aligned} \tag{21}$$

The dynamic budget constraint serves not only as a balance sheet equality in value, but also as a risk balance sheet equality. Its right-hand side expresses the individual's desired risk exposures implied in the optimal consumption flow (i.e., the individual's liability) net of endowed risk exposures from initial wage income and job switching opportunities (i.e., the individual's non-financial assets). The optimal portfolio policies are those that match the financial risk exposures of the individual's portfolio (the left-hand side of the budget constraint) with her net desired risk exposures. Thus, the optimal investment policy may be interpreted as resulting from liability driven investing procedures at the level of the individual. Based on the Clark-Ocone formula,

$$\phi_{i,t} = \mathfrak{D}_{i,t} [\xi_t W_t].$$

I proceed by deriving the Malliavin derivative of the first two terms in Eq. (21). The Malliavin derivative for the first term is

$$\begin{aligned}
& \mathfrak{D}_{(\cdot),t} \left[E_t \left[\int_t^T \xi_v c_v^* dv \right] \right] \\
&= -\theta_i \frac{R-1}{R} \left[\xi_t c_t^* \frac{1 - e^{\{-\mathcal{X}(T-t)\}}}{\mathcal{X}} \right] \\
&= -\theta_i \frac{R-1}{R} E_t \left[\int_t^T \xi_v c_v^* dv \right]
\end{aligned} \tag{22}$$

and that for the second term is

$$\mathfrak{D}_{(\cdot),t} E_t \left[\int_t^{\tau_2} \xi_v w_{i,v} dv \right] = \left((\sigma_{i,(\cdot)} - \theta_{(\cdot)}) \frac{e^{\{\eta_i(\tau_2-t)\}} - 1}{\eta_i} \right) \xi_t w_{i,t} = (\sigma_{i,(\cdot)} - \theta_{(\cdot)}) E_t \left[\int_t^{\tau_2} \xi_v w_{i,v} dv \right]. \tag{23}$$

As regards the third term in Eq.(21), no closed form for the Malliavin derivative can be obtained due to the presence of a stochastic stopping time. I will approximate the Malliavin derivative for the spread option via numerical methods.

Substituting Eqs. (22) and (23) into the Clark-Ocone formula, I find

$$\begin{aligned}
\phi_{(\cdot),t} &= -\theta_{(\cdot)} \frac{R-1}{R} E_t \left[\int_t^T \xi_v c_v^* dv \right] - (\sigma_{i,(\cdot)} - \theta_{(\cdot)}) E_t \left[\int_t^{\tau_2} \xi_v w_{i,v} dv \right] \\
&\quad - \mathfrak{D}_{(\cdot),t} [\xi_t V_t(w_i, w_j, \tau_1)].
\end{aligned}$$

From the definition of $\phi_{i,t}$ above, I determine the dollar portfolio policy invested in stock i to support the optimal consumption stream in Eq. (8).

Using the expression for W_t in Eq. (11), I find

$$\begin{aligned}
\pi_{(\cdot),t} &= W_t \sigma_{S(\cdot)}^{-1} \theta_{(\cdot)} + \xi_t^{-1} \sigma_{S(\cdot)}^{-1} \phi_{(\cdot),t} \\
&= \left(\xi_t^{-1} E_t \left[\int_t^T \xi_v c_v^* dv - \int_t^{\tau_2} \xi_v w_{i,v} dv \right] - \xi_t^{-1} \xi_t V_t(w_i, w_j, \tau_1) \right) \sigma_{S(\cdot)}^{-1} \theta_{(\cdot)} \\
&\quad - \xi_t^{-1} \sigma_{S(\cdot)}^{-1} \theta_{(\cdot)} \frac{R-1}{R} E_t \left[\int_t^T \xi_v c_v^* dv \right] - \xi_t^{-1} \sigma_{S(\cdot)}^{-1} (\sigma_{i,(\cdot)} - \theta_{(\cdot)}) E_t \left[\int_t^{\tau_2} \xi_v w_{i,v} dv \right] \\
&\quad - \xi_t^{-1} \sigma_{S(\cdot)}^{-1} \mathfrak{D}_{(\cdot),t} [\xi_t V_t(w_i, w_j, \tau_1)] \\
&= \frac{\sigma_{S(\cdot)}^{-1} \theta_{(\cdot)}}{R} E_t \left[\int_t^T \xi_{t,v} c_v^* dv \right] - \sigma_{S(\cdot)}^{-1} \sigma_{i,(\cdot)} E_t \left[\int_t^{\tau_2} \xi_{t,v} w_{i,v} dv \right] - \sigma_{S(\cdot)}^{-1} \mathfrak{D}_{(\cdot),t} [V_t(w_i, w_j, \tau_1)]
\end{aligned} \tag{24}$$

or, using the closed-form expressions from Eq. (21), $\pi_{(\cdot),t} = \frac{\sigma_{S(\cdot)}^{-1} \theta_{(\cdot)} (1 - e^{\{-\mathcal{X}(T-t)\}})}{R\mathcal{X}} c_t^* - \frac{\sigma_{S(\cdot)}^{-1} \sigma_{i,(\cdot)} (e^{\{\eta_i(\tau_2-t)\}} - 1)}{\eta_i} w_{i,t} - \sigma_{S(\cdot)}^{-1} \mathfrak{D}_{(\cdot),t} [V_t(w_i, w_j, \tau_1)]$.

8 Appendix B

This appendix extends my study on the impact of labor flexibility on optimal life-cycle portfolio decisions. The model built in the text of the paper assumes that a worker's wage is perfectly correlated with her employer's stock – her human capital constitutes the holdings of her employer's stock and of put options on that stock – and that correlation between industry wages is null. Because the return on these implicit options is higher both when one's own industry does poorly and when one's alternative industry does well, studying the instance of zero correlation between wages is particularly insightful. This case shows the value of introducing the option mechanism and it provides the economic intuition behind it.⁴⁰ To the contrary, in the instance of two highly correlated companies, the benefit from holding the job-switching option is modulated and, in the limit case of perfect positive correlation, the option is worthless.⁴¹

This appendix demonstrates how an economy with less than perfectly correlated labor and financial markets (or, correspondingly, correlated industry wages) can be transformed to restore the canonical model specification in the text of the paper. I propose a three-step procedure to convert a given set of parameter values for stocks and wages into corresponding figures that can be directly inputted in the solution algorithm illustrated in Appendix A. Specifically, this technology creates an artificial economy that is formally isomorphic to the actual one and that reproduces the dynamics of the original model.

The Section to follow summarizes labor and financial markets accessible to the individual.

8.1 Original model specification

In the baseline model I posit a financial market comprised of a risk-less asset and two risky dividend-paying assets. The two risky security prices, S_i , and S_j , follow Itô processes

$$\begin{aligned} \frac{dS_{i,t}}{S_{i,t}} &= (\mu_{S_i} - \delta_{S_i}) dt + \sigma_{S_i} dz_{i,t} & t \in [0, T]; S_{i,0} > 0 \text{ given} \\ \frac{dS_{j,t}}{S_{j,t}} &= (\mu_{S_j} - \delta_{S_j}) dt + \sigma_{S_j} dz_{j,t} & t \in [0, T]; S_{j,0} > 0 \text{ given} \end{aligned},^{42} dz_{i,t} dz_{j,t} = 0 \quad (25)$$

where δ_{S_i} and δ_{S_j} are dividend yields, μ_{S_i} and μ_{S_j} are instantaneous expected rates of return and σ_{S_i} and σ_{S_j} are instantaneous volatility coefficients. $z_{i,t}$ and $z_{j,t}$ are Brownian motion processes and T is the individual's finite fixed planning horizon. The implied market prices

⁴⁰A befitting example is that of an individual with a Master of Business Administration (MBA). The commitment of MBA programs is to help individuals to acquire new skills that can be exercised throughout the course of their career. Education and training in a wide range of disciplines enhance the individual's ability to pursue many careers and her flexibility to move among rather dissimilar industries or companies. To this individual the job-switching option is most consequential.

⁴¹Intuitively, positing perfect positive correlation in wages is equivalent to assuming a unique source of risk impacting both companies. Under this condition, the individual is indifferent between being employed in either industry and her job-switching option is valueless.

⁴²Equation (25) corresponds to Eq. (2) in the text of the paper.

of risk are denoted by $\theta_i \equiv \sigma_{S_i}^{-1} (\mu_{S_i} - r_f)$ and $\theta_j \equiv \sigma_{S_j}^{-1} (\mu_{S_j} - r_f)$, where $r_f (> 0)$ is the instantaneous market rate of interest, and it is constant over time.

The finite-lived individual of this economy finances consumption and investments by earning a per period salary, along with earnings on accumulated financial assets. The wage rates offered in industry i and in the rest of the economy are given by

$$\begin{aligned} \frac{dw_{i,t}}{w_{i,t}} &= \alpha_i dt + \sigma_{i,i} dz_{i,t} & t \in [0, \tau_2], \quad w_{i,0} > 0 \text{ given} \\ \frac{dw_{j,t}}{w_{j,t}} &= \alpha_j dt + \sigma_{j,j} dz_{j,t} & t \in [0, \tau_2], \quad w_{j,0} > 0 \text{ given} \end{aligned}, \quad {}^{43} \quad dz_{i,t} dz_{j,t} = 0. \quad (26)$$

The instantaneous expected wage growth rates are indicated by $\alpha_i (> 0)$ and $\alpha_j (> 0)$, while $\sigma_{i,i}$ ($\sigma_{j,j}$) is the volatility of wage i (j) with respect to the i^{th} (j^{th}) Brownian motion. Retirement is an irreversible labor income state beginning at time τ_2 , $\tau_2 \in [0, T]$.

Lastly, given the dynamics in Eq. (25), $dz_{i,t} dz_{j,t} = 0$ is a necessary and sufficient condition for the two stocks to be uncorrelated. The same argument applies to Eq. (26).

8.2 Construction of the artificial economy

Consider now a given economy whose stocks and wages' dynamics depart from those of Eqs. (25) and (26). This Section exposes a procedure to "translate" the set of equations characterizing the actual economy into new dynamic equations consistent with the specification of Section 8.1. This technology is implementable via a three-step process.

1. Determine stock and wage parameters of the actual economy.
2. Re-establish orthogonality between wages. This is achieved by decomposing the wage rate received under the alternative job in two independent components: a measure of the current wage rate and a residual wage increment.
3. Re-establish perfect correlation between stocks and wages. This requires to construct two portfolios of the original assets whose optimal shares are chosen to guarantee perfect correlation with the wage rates defined in Step 2.

8.2.1 Defining the actual economy

The first step consists of identifying the parameter values that fully describe the given actual economy. These include dividend yields, instantaneous expected growth rates and volatility coefficients of both stock returns as well as instantaneous expected wage growth rates and volatilities. While the investment opportunity set of the actual economy needs be identical to that of the original economy – the individual can freely trade in the same two risky securities and risk-free asset – the dynamics of the labor income rates are assumed to satisfy

$$\begin{aligned} \frac{dw'_{i,t}}{w'_{i,t}} &= \alpha_i dt + \sigma_{w'_i} dz'_{i,t}, & \sigma_{w'_i} dz'_{i,t} &\equiv \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t} \\ \frac{dw'_{j,t}}{w'_{j,t}} &= \alpha_j dt + \sigma_{w'_j} dz'_{j,t}, & \sigma_{w'_j} dz'_{j,t} &\equiv \sigma_{j,i} dz_{i,t} + \sigma_{j,j} dz_{j,t} \end{aligned}, \quad dz_{i,t} dz_{j,t} = 0. \quad (27)$$

⁴³Equation (26) corresponds to Eq. (4) in the text of the paper.

Equation (27) states that each industry's wage rate is impacted by two sources of risk: the former originates from the individual's industry of employment (own-industry risk, $\sigma_{i,i}$ and $\sigma_{j,j}$); the latter originates from the alternative industry of employment (cross-industry risk, $\sigma_{i,j}$ and $\sigma_{j,i}$).

The effect of adding a cross-industry component of risk is twofold. First, in spite of the orthogonality between $z_{i,t}$ and $z_{j,t}$, $w'_{i,t}$ is correlated with $w'_{j,t}$. Specifically, indicating $dz'_{i,t}dz'_{j,t} = \rho_{i',j'}dt$, the correlation coefficient $\rho_{i',j'}$ derives from

$$\frac{dw'_{i,t}}{w'_{i,t}} \frac{dw'_{j,t}}{w'_{j,t}} = \sigma_{w'_i} \sigma_{w'_j} \rho_{i',j'} dt = (\sigma_{i,i} \sigma_{j,i} + \sigma_{i,j} \sigma_{j,j}) dt$$

and equals $\rho_{i',j'} = \frac{\sigma_{i,i} \sigma_{j,i} + \sigma_{i,j} \sigma_{j,j}}{\sigma_{w'_i} \sigma_{w'_j}} = \frac{\sigma_{i,i} \sigma_{j,i} + \sigma_{i,j} \sigma_{j,j}}{\sqrt{(\sigma_{i,i}^2 + \sigma_{i,j}^2)(\sigma_{j,i}^2 + \sigma_{j,j}^2)}} \neq 0$.⁴⁴

Second, each industry's stock and wage processes are no longer perfectly correlated. Defining $\rho_{i,i'}$ and $\rho_{j,j'}$ according to $dz_{i,t}dz'_{j,t} = \rho_{i,i'}dt$ and $dz_{j,t}dz'_{j,t} = \rho_{j,j'}dt$, Eqs. (25) and (27) permit to calculate the correlation between stock and wage functions, namely

$$\begin{aligned} \frac{dS_{i,t}}{S_{i,t}} \frac{dw'_{i,t}}{w'_{i,t}} &= \sigma_{S_i} \sigma_{w'_i} \rho_{i,i'} dt = \sigma_{S_i} \sigma_{i,i} dt, \quad \rho_{i,i'} = \frac{\sigma_{S_i} \sigma_{i,i}}{\sigma_{S_i} \sigma_{w'_i}} = \frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \neq 1 \\ \frac{dS_{j,t}}{S_{j,t}} \frac{dw'_{j,t}}{w'_{j,t}} &= \sigma_{S_j} \sigma_{w'_j} \rho_{j,j'} dt = \sigma_{S_j} \sigma_{j,j} dt, \quad \rho_{j,j'} = \frac{\sigma_{S_j} \sigma_{j,j}}{\sigma_{S_j} \sigma_{w'_j}} = \frac{\sigma_{j,j}}{\sqrt{\sigma_{j,i}^2 + \sigma_{j,j}^2}} \neq 1. \end{aligned}$$

Only a financial asset with returns generated by stochastic fluctuations in both $z_{i,t}$ and $z_{j,t}$ would permit to re-establish perfect correlation.⁴⁵

8.2.2 Restoring orthogonality between wages

The second step consists of retrieving orthogonality between wages. Re-expressing the wage process in the market as a function of that in industry i yields

⁴⁴ The last equality in the formula of $\rho_{i',j'}$ follows from the definition of $\sigma_{w'_i} dz'_{i,t}$ and $\sigma_{w'_j} dz'_{j,t}$ in Eq. (27).

⁴⁵ Assume a fictitious risky asset, S , whose rate of return follows the Itô process

$$\frac{dS_t}{S_t} = (\mu_S - \delta_S) dt + \sigma_{S_i} dz_{i,t} + \sigma_{S_j} dz_{j,t} \quad t \in [0, T]; \quad S_0 > 0 \text{ given.}$$

In this case, perfect correlation between $\frac{dS_t}{S_t}$ and $\frac{dw'_{i,t}}{w'_{i,t}}$ could be re-introduced by simply scaling the dynamics of S to satisfy

$$\frac{dS_t^*}{S_t^*} = (\mu_{S^*} - \delta_{S^*}) dt + v_{S^*} (\sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t}) \quad t \in [0, T]; \quad S_0^* > 0 \text{ given,}$$

in which v_{S^*} is the proper scale factor.

While this example is purely illustrative, it provides the intuition behind the mechanism of restoring perfect correlations between stocks and wages. Applying the same line of reasoning, the third step of the procedure presented here demonstrates how perfect correlations can be re-obtained by constructing fictitious securities deriving their returns from optimal combinations of the existing risky and risk-free assets.

$$\frac{dw'_{j,t}}{w'_{j,t}} \equiv \psi_j dt + \beta_j \frac{dw'_{i,t}}{w'_{i,t}} + \sigma_{\varepsilon_j} d\varepsilon_{j,t}, \quad \sigma_{\varepsilon_j} d\varepsilon_{j,t} \equiv \lambda_i dz_{i,t} + \lambda_j dz_{j,t}, \quad (28)$$

where $\varepsilon_{j,t}$ is a Brownian motion process with λ_i and λ_j chosen to satisfy

$$d\varepsilon_{j,t} \frac{dw'_{i,t}}{w'_{i,t}} = \frac{1}{\sigma_{\varepsilon_j}} [\lambda_i dz_{i,t} + \lambda_j dz_{j,t}] [\alpha_i dt + \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t}] = 0. \quad (29)$$

Equation (28) defines the new parameters of the model, which will serve to restore orthogonality between stocks and wages in Step 3. Substituting the definitions both of industry i 's wage rate and of the newly defined Brownian motion $\varepsilon_{j,t}$ in Eq. (28) yields

$$\begin{aligned} \frac{dw'_{j,t}}{w'_{j,t}} &= \psi_j dt + \beta_j [\alpha_i dt + \sigma_{i,i} dz_{i,t} + \sigma_{i,j} dz_{j,t}] + \lambda_i dz_{i,t} + \lambda_j dz_{j,t} \\ &= (\psi_j + \beta_j \alpha_i) dt + (\beta_j \sigma_{i,i} + \lambda_i) dz_{i,t} + (\beta_j \sigma_{i,j} + \lambda_j) dz_{j,t}. \end{aligned} \quad (30)$$

The unknown parameters in Eq. (30) derive from solving the following system of equations

$$\begin{cases} \psi_j + \beta_j \alpha_i = \alpha_j \\ \beta_j \sigma_{i,i} + \lambda_i = \sigma_{j,i} \\ \beta_j \sigma_{i,j} + \lambda_j = \sigma_{j,j} \\ \lambda_i \sigma_{i,i} + \lambda_j \sigma_{i,j} = 0 \end{cases}. \quad (31)$$

Few algebraic manipulations provide the values

$$\begin{aligned} \psi_j &= \alpha_j - \frac{\sigma_{i,i}\sigma_{j,i} + \sigma_{i,j}\sigma_{j,j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \alpha_i \\ \beta_j &= \frac{\sigma_{i,i}\sigma_{j,i} + \sigma_{i,j}\sigma_{j,j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \\ \lambda_i &= \frac{\sigma_{i,j}(\sigma_{i,i}\sigma_{j,i} - \sigma_{i,j}\sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2} \\ \lambda_j &= \frac{\sigma_{i,i}(\sigma_{i,i}\sigma_{j,j} - \sigma_{i,j}\sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}. \end{aligned} \quad (32)$$

I indicate the portion of wage j that is independent of wage i by $\frac{dw''_{j,t}}{w''_{j,t}}$, such that

$$\frac{dw''_{j,t}}{w''_{j,t}} = \frac{dw'_{j,t}}{w'_{j,t}} - \beta_j \frac{dw'_{i,t}}{w'_{i,t}} = \psi_j dt + \sigma_{\varepsilon_j} d\varepsilon_{j,t}, \quad (33)$$

⁴⁶These parameters permit to compute the volatility of $\varepsilon_{j,t}$, $\sigma_{\varepsilon_j} = \sqrt{\lambda_i^2 + \lambda_j^2} = \frac{|\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}$.

in which $\frac{dw'_{i,t}}{w'_{i,t}} \frac{dw''_{j,t}}{w''_{j,t}} = 0$,⁴⁷ according to Eq. (29). The size of $w''_{j,t}$ – the wage increment earned in the market economy – determines whether the individual should exercise her career option and leave industry i .⁴⁸

This second step has transformed the wage processes of a given general model to fit the equations of the canonical specification. One last step is necessary to reinstate perfect correlations between stocks and wages so as to reaffirm the original economy.

8.2.3 Restoring perfect correlation between stocks and wages

The third step consists of constructing two portfolios of the original assets that are perfectly correlated with $w'_{i,t}$ and $w''_{j,t}$, respectively. These portfolios, which we indicate as $S'_{i,t}$, $S'_{j,t}$, are defined and constructed by the dynamic strategies $x_{i,i}$, $x_{i,j}$, $x_{j,i}$, $x_{j,j}$ such that

$$\begin{aligned} \frac{dS'_{i,t}}{S'_{i,t}} &= x_{i,i} \left[\frac{dS_{i,t}}{S_{i,t}} - r_f dt \right] + x_{i,j} \left[\frac{dS_{j,t}}{S_{j,t}} - r_f dt \right] + r_f dt \\ &= \left[x_{i,i} (\mu_{S_i} - \delta_{S_i}) + x_{i,j} (\mu_{S_j} - \delta_{S_j}) + r_f (1 - x_{i,i} - x_{i,j}) \right] dt + \sigma_{S'_i} dz'_{i,t}, \end{aligned} \quad (34)$$

where $\sigma_{S'_i} dz'_{i,t} \equiv x_{i,i} \sigma_{S_i} dz_{i,t} + x_{i,j} \sigma_{S_j} dz_{j,t}$, and

$$\begin{aligned} \frac{dS'_{j,t}}{S'_{j,t}} &= x_{j,i} \left[\frac{dS_{i,t}}{S_{i,t}} - r_f dt \right] + x_{j,j} \left[\frac{dS_{j,t}}{S_{j,t}} - r_f dt \right] + r_f dt \\ &= \left[x_{j,i} (\mu_{S_i} - \delta_{S_i}) + x_{j,j} (\mu_{S_j} - \delta_{S_j}) + r_f (1 - x_{j,i} - x_{j,j}) \right] dt + \sigma_{S'_j} dz'_{j,t} \end{aligned} \quad (35)$$

⁴⁷It is straightforward to verify that $\frac{dw'_{i,t}}{w'_{i,t}} \frac{dw''_{j,t}}{w''_{j,t}} = 0$. Defining $\rho_{i',j''}$ to satisfy $dz'_{i,t} dz_{j,t} = \rho_{i',j''} dt$,

$$\frac{dw'_{i,t}}{w'_{i,t}} \frac{dw''_{j,t}}{w''_{j,t}} = \sigma_{w'_i} \sigma_{\varepsilon_j} \rho_{i',j''} dt = (\sigma_{i,i} \lambda_i + \sigma_{i,j} \lambda_j) dt$$

which is equivalent to obtaining

$$\begin{aligned} \rho_{i',j''} &= \frac{\sigma_{i,i} \lambda_i + \sigma_{i,j} \lambda_j}{\sigma_{w'_i} \sigma_{\varepsilon_j}} \\ &= \frac{\sigma_{i,i} \frac{\sigma_{i,j} (\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2} + \sigma_{i,j} \frac{\sigma_{i,i} (\sigma_{i,i} \sigma_{j,j} - \sigma_{i,j} \sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2} \frac{|\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\ &= \frac{\frac{\sigma_{i,i} \sigma_{i,j} (\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}) - \sigma_{i,j} \sigma_{i,i} (\sigma_{i,i} \sigma_{j,j} - \sigma_{i,j} \sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2} \frac{|\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}|}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\ &= 0. \end{aligned}$$

⁴⁸Notice that the wage increment $w''_{j,t}$ can be either positive or negative.

in which $\sigma_{S'_j} dz'_{j,t} \equiv x_{j,i} \sigma_{S_i} dz_{i,t} + x_{j,j} \sigma_{S_j} dz_{j,t}$.

The newly created $S'_{i,t}$ and $S'_{j,t}$ can be interpreted as artificial Exchange-Traded Funds (ETFs). Indeed, these securities would allow the individual to trade index portfolios of the original risky assets just as they do shares of stocks. The optimal combination of stock i and stock j in each ETF derives from solving the following system of equations

$$\begin{cases} x_{i,i} \sigma_{S_i} = \chi \sigma_{i,i} \\ x_{i,j} \sigma_{S_j} = \chi \sigma_{i,j} \\ x_{j,i} \sigma_{S_i} = \kappa \lambda_i \\ x_{j,j} \sigma_{S_j} = \kappa \lambda_j \\ \sigma_{S_i} = \sigma_{S'_i} \\ \sigma_{S_j} = \sigma_{S'_j} \end{cases} \quad (36)$$

The first two equations ensure that $S'_{i,t}$ and $w'_{i,t}$ are perfectly correlated – they change jointly up to a scale factor χ . Similarly, the third and fourth equations guarantee a perfect correlation between $S'_{j,t}$ and $w'_{j,t}$, with corresponding scale factor κ . The last two equations, which are functional to determine the unknown scale factors, set the volatilities of the fictitious risky assets equal to those of the original assets.⁴⁹ Solving for the unknown parameters yields

$$\begin{aligned} \chi &= \frac{\sigma_{S_i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \\ x_{i,i} &= \frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \\ x_{i,j} &= \frac{\sigma_{i,j}}{\sigma_{S_j} \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \\ \kappa &= \frac{\sigma_{S_j} \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{|\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}|} \\ x_{j,i} &= \left(1_{\{(\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}) > 0\}} - 1_{\{(\sigma_{i,j} \sigma_{j,i} - \sigma_{i,i} \sigma_{j,j}) < 0\}} \right) \frac{\sigma_{S_j} \sigma_{i,j}}{\sigma_{S_i} \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \\ x_{j,j} &= \left(1_{\{(\sigma_{i,i} \sigma_{j,j} - \sigma_{i,j} \sigma_{j,i}) > 0\}} - 1_{\{(\sigma_{i,i} \sigma_{j,j} - \sigma_{i,j} \sigma_{j,i}) < 0\}} \right) \frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}},^{50} \end{aligned}$$

in which κ , $x_{j,i}$ and $x_{j,j}$ utilize the optimal solution for λ_i and λ_j given in Eq. (32). The optimal strategies $x_{i,i}$, $x_{i,j}$, $x_{j,i}$, $x_{j,j}$ permit to verify that $\rho_{i',i'}$ and $\rho_{j',j''}$, defined according to $dz'_{i,t} dz'_{i,t} = \rho_{i',i'} dt$ and $dz'_{j,t} dz'_{j,t} = \rho_{j',j''} dt$, equal 1. In particular,

$$\frac{dS'_{i,t}}{S'_{i,t}} \frac{dw'_{i,t}}{w'_{i,t}} = \sigma_{S'_i} \sigma_{w'_i} \rho_{i',i'} dt = (x_{i,i} \sigma_{S_i} \sigma_{i,i} + x_{i,j} \sigma_{S_j} \sigma_{i,j}) dt,$$

⁴⁹Notice that, since markets are complete, both portfolios can be freely rescaled given certain wage dynamics. In this sense, neither of the last two conditions is restrictive.

⁵⁰ $1_{\{A\}}$ denotes the indicator function assuming value 1 if event A occurs and 0 otherwise.

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$$\begin{aligned}
\rho_{i',i'} &= \frac{x_{i,i}\sigma_{S_i}\sigma_{i,i} + x_{i,j}\sigma_{S_j}\sigma_{i,j}}{\sigma_{S'_i}\sigma_{w'_i}} \\
&= \frac{\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\sigma_{S_i}\sigma_{i,i} + \frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\sigma_{S_j}\sigma_{i,j}}{\sqrt{\left(\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2 + \left(\frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\
&= \frac{\frac{\sigma_{S_i}^2(\sigma_{i,i}^2 + \sigma_{i,j}^2)}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}}{\sqrt{\frac{\sigma_{S_i}^2(\sigma_{i,i}^2 + \sigma_{i,j}^2)}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \\
&= 1.
\end{aligned}$$

Similarly,

$$\frac{dS'_{j,t}}{S'_{j,t}} \frac{dw''_{j,t}}{w'_{j,t}} = \sigma_{S'_j}\sigma_{\varepsilon_j}\rho_{j',j''}dt = (x_{j,i}\sigma_{S_i}\lambda_i + x_{j,j}\sigma_{S_j}\lambda_j) dt,$$

leading to

$$\begin{aligned}
\rho_{j',j''} &= \frac{x_{j,i}\sigma_{S_i}\lambda_i + x_{j,j}\sigma_{S_j}\lambda_j}{\sigma_{S'_j}\sigma_{\varepsilon_j}} \\
&= \frac{\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\sigma_{S_i}\frac{\sigma_{i,j}(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j})}{\sigma_{i,i}^2 + \sigma_{i,j}^2} - \frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\sigma_{S_j}\frac{\sigma_{i,i}(\sigma_{i,i}\sigma_{j,j} - \sigma_{i,j}\sigma_{j,i})}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\left(\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2\sigma_{S_i}^2 + \left(\frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2\sigma_{S_j}^2\frac{\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}}} \\
&= \frac{\frac{\sigma_{S_j}(\sigma_{i,i}^2 + \sigma_{i,j}^2)(\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j})}{(\sigma_{i,i}^2 + \sigma_{i,j}^2)\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}}{\sqrt{\frac{\sigma_{S_j}^2(\sigma_{i,i}^2 + \sigma_{i,j}^2)}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\frac{\sigma_{i,j}\sigma_{j,i} - \sigma_{i,i}\sigma_{j,j}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}}} \\
&= 1
\end{aligned}$$

Finally, consider the stochastic dynamic equations (27), (33), (34), (35). Since $\rho_{i',i'}=1$, $\rho_{j',j''}=1$ and $\rho_{i',j''}=0$, then $\text{corr}\left(\frac{dS'_{i,t}}{S'_{i,t}}, \frac{dS'_{j,t}}{S'_{j,t}}\right)=\rho_{i',j''}=0$. Indeed, indicating $dz'_{i,t}dz'_{j,t} = \rho_{i',j''}dt$

$$\frac{dS'_{i,t}}{S'_{i,t}} \frac{dS'_{j,t}}{S'_{j,t}} = \sigma_{S'_i}\sigma_{S'_j}\rho_{i',j''}dt = (x_{i,i}x_{j,i}\sigma_{S_i}^2 + x_{i,j}x_{j,j}\sigma_{S_j}^2) dt,$$

and

$$\begin{aligned}
\rho_{i',j'} &= \frac{x_{i,i}x_{j,i}\sigma_{S_i}^2 + x_{i,j}x_{j,j}\sigma_{S_j}^2}{\sigma_{S_i'}\sigma_{S_j'}} \\
&= \frac{\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \sigma_{S_i}^2 + \frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \sigma_{S_j}^2}{\sqrt{\left(\frac{\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2 \sigma_{S_i}^2 + \left(\frac{\sigma_{S_i}\sigma_{i,j}}{\sigma_{S_j}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2 \sigma_{S_j}^2} \sqrt{\left(\frac{\sigma_{S_j}\sigma_{i,j}}{\sigma_{S_i}\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2 \sigma_{S_i}^2 + \left(\frac{-\sigma_{i,i}}{\sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}}\right)^2 \sigma_{S_j}^2}} \\
&= \frac{\frac{\sigma_{i,i}\sigma_{i,j}\sigma_{S_i}\sigma_{S_j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2} - \frac{\sigma_{i,i}\sigma_{i,j}\sigma_{S_i}\sigma_{S_j}}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}{\sqrt{\frac{\sigma_{S_i}^2(\sigma_{i,i}^2 + \sigma_{i,j}^2)}{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \sqrt{\frac{\sigma_{S_j}^2(\sigma_{i,i}^2 + \sigma_{i,j}^2)}{\sigma_{i,i}^2 + \sigma_{i,j}^2}}} \\
&= 0.
\end{aligned}$$

8.3 An application to positive correlation in wage innovations

This Subsection implements the three-step methodology presented above to industry 22 in French's 49 industry portfolios, "Electrical Equipment."⁵¹ To construct 49 industry portfolios, each NYSE, AMEX, and NASDAQ stock is assigned to an industry portfolio at the end of June of year t based on its four-digit Standard Industrial Classification code at that time. Returns from July of year t to June of year $t + 1$ are then computed. For industry 22, the standard deviation of annual returns over the period 1968-1994 equals 21%. In Rosenberg and Guy's (1976) analysis, "Electrical Equipment" has a beta of 1.27. Assuming a 6% market return, this implies an industry stock return, net of the dividend yield, equal to 3.46%. Accordingly, the dynamics of the two risky securities are given by

$$\begin{aligned}
\frac{dS_{i,t}}{S_{i,t}} &= (0.051 - 0.0164) dt + 0.210dz_{i,t} \quad t \in [0, T]; \quad S_{i,0} = 100 \text{ given} \\
\frac{dS_{j,t}}{S_{j,t}} &= (0.06 - 0.0164) dt + 0.157dz_{j,t} \quad t \in [0, T]; \quad S_{j,0} = 100 \text{ given} \quad , \quad dz_{i,t}dz_{j,t} = 0. \quad (37)
\end{aligned}$$

The wage equations are characterized in conformity with Davis and Willen's (2000) study of occupation-level components of individual income innovations. In their paper, Davis and Willen analyze several properties of these innovations, including their covariance with aggregate equity returns, long-term bond returns and returns on selected industry-level equity returns. Using repeated cross sections of the Current Population Survey, they find that selected industry-level equity portfolios are significantly correlated with income innovations for several occupations. For each occupation they identify one or more industries that account for a large fraction of the occupation's employment: industry 2, "Electrical Equipment" is matched to "Electrical Engineers." For "Electrical Engineers," the correlation between income innovations and industry-level returns is 0.45⁵² and the standard deviation of inno-

⁵¹Source: K. French Data Library, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

⁵²Source: Davis and Willen (2000), Table 7, p. 54.

vations to the occupation-level component of earnings equals 3.78%.⁵³ The stochastic wage process associated with "Electrical Engineers" is

$$\frac{dw'_{i,t}}{w'_{i,t}} = 0.02dt + 0.085dz'_{i,t}, \quad 0.085dz'_{i,t} \equiv 0.038dz_{i,t} + 0.076dz_{j,t}, \quad dz_{i,t}dz_{j,t} = 0, \quad (38)$$

where $\sigma_{i,i} = \sqrt{\sigma_{i,i}^2 \left(\frac{1}{\rho_{i,i'}^2} - 1 \right)} = \sqrt{0.038^2 \left(\frac{1}{0.45^2} - 1 \right)} = 0.076$. Similarly,

$$\frac{dw'_{j,t}}{w'_{j,t}} = 0.02dt + 0.09dz'_{j,t}, \quad \sigma_{w'_j} dz'_{j,t} \equiv 0.078dz_{i,t} + 0.045dz_{j,t}, \quad dz_{i,t}dz_{j,t} = 0, \quad (39)$$

where $\sigma_{j,i} = \sqrt{\sigma_{j,j}^2 \left(\frac{1}{\rho_{j,j'}^2} - 1 \right)} = \sqrt{0.045^2 \left(\frac{1}{0.5^2} - 1 \right)} = 0.078$.⁵⁴

As illustrated in Subsubsection 8.2.2, the portion of wage j that is independent of wage i is equal to

$$\frac{dw''_{j,t}}{w''_{j,t}} = \psi_j dt + \sigma_{\varepsilon_j} d\varepsilon_{j,t}, \quad (40)$$

in which $\psi_j = 0.02 - \frac{0.038 \cdot 0.078 + 0.076 \cdot 0.045}{0.038^2 + 0.076^2} \cdot 0.02 = 0.002$ and $\sigma_{\varepsilon_j} = \frac{|0.076 \cdot 0.078 - 0.038 \cdot 0.045|}{\sqrt{0.038^2 + 0.076^2}} = 0.05$. This definition transforms the wage processes of the actual economy so to fit Eq. (26).

Finally, as shown in Subsubsection 8.2.3, the dynamics of the artificially constructed ETFs, $S'_{i,t}$ and $S'_{j,t}$, follow

$$\frac{dS'_{i,t}}{S'_{i,t}} = \left[x_{i,i} (\mu_{S_i} - \delta_{S_i}) + x_{i,j} (\mu_{S_j} - \delta_{S_j}) + r_f (1 - x_{i,i} - x_{i,j}) \right] dt + \sigma_{S'_i} dz'_{i,t}, \quad (41)$$

where $\sigma_{S'_i} dz'_{i,t} \equiv x_{i,i} \sigma_{S_i} dz_{i,t} + x_{i,j} \sigma_{S_j} dz_{j,t}$, and

$$\frac{dS'_{j,t}}{S'_{j,t}} = \left[x_{j,i} (\mu_{S_i} - \delta_{S_i}) + x_{j,j} (\mu_{S_j} - \delta_{S_j}) + r_f (1 - x_{j,i} - x_{j,j}) \right] dt + \sigma_{S'_j} dz'_{j,t}$$

in which $\sigma_{S'_j} dz'_{j,t} \equiv x_{j,i} \sigma_{S_i} dz_{i,t} + x_{j,j} \sigma_{S_j} dz_{j,t}$.

The optimal dynamic strategies defining the ETFs portfolios are $x_{i,i} = \frac{0.038}{\sqrt{0.038^2 + 0.076^2}} = 0.45$, $x_{i,j} = \frac{0.210 \cdot 0.076}{0.157 \cdot \sqrt{0.038^2 + 0.076^2}} = 1.20$, $x_{j,i} = \frac{0.157 \cdot 0.076}{0.210 \cdot \sqrt{0.038^2 + 0.076^2}} = 0.67$ and $x_{j,j} = \frac{-0.038}{\sqrt{0.038^2 + 0.076^2}} = -0.45$. Perfect correlation between stocks and wages is thus re-established and the newly created set of parameters can be used in the numerical algorithm for the derivation of op-

⁵³Source: Davis and Willen (2000), Tables 2 and 4, pp. 48 and 50.

⁵⁴ $\rho_{j,j'}$ is the correlation between income innovations and market index returns. It is approximated by the arithmetic average of all correlation coefficients between income innovations and industry-level returns.

timal consumption and portfolio policies. Table 3 in the text reports simulation results for the case of industry 22.

8.4 Comparative statics: an algebraic exploration

The analysis of Subsection 8.2 permits to measure algebraically the directional changes in optimal investment policies when transforming the actual economy into the artificial equivalent economy.

By definition of $\sigma_{w'_i}$, regardless of the sign of $\sigma_{i,j}$, $\sigma_{w'_i} > \sigma_{i,i}$. As shown in Section 4, higher wage volatilities reduce the fraction of financial wealth invested in own-industry stock and, if large enough, they may produce increasing equity patterns over time. In this sense, the effect of non-zero correlations between wage innovations is to generate more conservative portfolios. Additionally, all else constant, cross-industry risk can either inflate or deflate industry j 's wage volatility. In particular,

$$\begin{aligned} &\text{if } \sigma_{i,j}\sigma_{j,i} < 0, \text{ then } \sigma_{\varepsilon_j} > \sigma_{j,j} \text{ iff } \left\{ \sigma_{j,j} > \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i} - \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \right\}; \\ &\text{if } \sigma_{i,j}\sigma_{j,i} > 0, \text{ then } \sigma_{\varepsilon_j} > \sigma_{j,j} \text{ iff } \left\{ 0 < \sigma_{j,j} < \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i} + \sqrt{\sigma_{i,i}^2 + \sigma_{i,j}^2}} \text{ or } \sigma_{j,j} > \frac{\sigma_{i,j}\sigma_{j,i}}{\sigma_{i,i}} \right\}. \end{aligned} \quad (42)$$

Under these conditions, the individual would invest more cautiously in equities, both own- and cross-industry stock (Table 7). On the contrary, for values of $\sigma_{j,j}$ outside the sets defined in Eq. (42), the artificially constructed economy features low uncertainty in the wage of the alternative industry leading to increased exposure to risky assets.

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Table I
Benchmark Parameters

Description	Parameter Value
Expected Stock Growth Rates (μ_S)	0.052, ^(L) 0.071, ^(H) .060 ^(M)
Stock Volatilities (σ_S)	0.125, ^(L) 0.210, ^(H) 0.157 ^(M)
Stock Dividend Rates (δ_S)	0.0164
Risk-Free Rate (r_f)	0.02
Discount Factor (β)	0.03
Risk Aversion (γ)	6
Time to Build (Δ)	1
Tuition (f)	\$9,000
Expected Wage Growth Rates (α)	0.02
Wage Volatilities ($\sigma_{i,i}, \sigma_{j,j}$)	0.045, ^(L) 0.085, ^(H) 0.045 ^(M)
Initial Wages (w_0)	\$24,199
Initial Financial Wealth (W_0)	\$20,000
Retirement Age (τ_2)	65
Death Age (T)	90

Notes: ^(L) Low-volatility industry parameter value; ^(H) High-volatility industry parameter value; ^(M) Market parameter value.

Table II
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon
(Option to Switch Jobs from a Low-volatility Industry to the Market)

Fractions of Current Financial Wealth	FW=TW	FW=TW-HW	FW=TW-HW-OW
30 Years to Retirement			
Own-Industry Risky Investment	0.17	-3.05	0.38
Cross-Industry Risky Investment	0.16	49.47	4.26
Risk-Free Investment	0.67	-45.42	-3.64
20 Years to Retirement			
Own-Industry Risky Investment	0.17	0.15	0.27
Cross-Industry Risky Investment	0.16	3.02	1.79
Risk-Free Investment	0.67	-2.17	-1.06
10 Years to Retirement			
Own-Industry Risky Investment	0.17	0.30	0.31
Cross-Industry Risky Investment	0.16	0.78	0.68
Risk-Free Investment	0.67	-0.09	0.01
5 Years to Retirement			
Own-Industry Risky Investment	0.17	0.32	0.33
Cross-Industry Risky Investment	0.16	0.45	0.42
Risk-Free Investment	0.67	0.22	0.25

Source: Author's calculations.

Notes:

- ⁽¹⁾ FW stands for financial wealth.
- ⁽²⁾ TW stands for total wealth, the mean-variance efficient portfolio.
- ⁽³⁾ HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
- ⁽⁴⁾ OW stands for option wealth, the job-switching option's delta.

Table III
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon
(OPTION TO SWITCH JOBS FROM A HIGH-VOLATILITY INDUSTRY TO THE MARKET)

Fractions of Current Financial Wealth	FW=TW	FW=TW-HW	FW=TW-HW-OW
30 Years to Retirement			
Own-Industry Risky Investment	0.13	-4.41	-2.46
Cross-Industry Risky Investment	0.16	3.35	1.18
Risk-Free Investment	0.71	2.06	2.28
20 Years to Retirement			
Own-Industry Risky Investment	0.13	-0.78	-0.43
Cross-Industry Risky Investment	0.16	0.78	0.36
Risk-Free Investment	0.71	1.00	1.06
10 Years to Retirement			
Own-Industry Risky Investment	0.13	-0.16	-0.10
Cross-Industry Risky Investment	0.16	0.34	0.26
Risk-Free Investment	0.71	0.82	0.84
5 Years to Retirement			
Own-Industry Risky Investment	0.13	-0.02	-0.01
Cross-Industry Risky Investment	0.16	0.24	0.22
Risk-Free Investment	0.71	0.78	0.79

Source: Author's calculations.

Notes:

- ⁽¹⁾ FW stands for financial wealth.
- ⁽²⁾ TW stands for total wealth, the mean-variance efficient portfolio.
- ⁽³⁾ HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
- ⁽⁴⁾ OW stands for option wealth, the job-switching option's delta.

Table IV
Optimal Portfolio Shares as Functions of the Expected Retirement Horizon⁵⁵
(Permanent Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	FW =	TW –	HW –	OW
30 Years to Retirement				
Own-Industry Risky Investment	0.38	5.75	5.69	-0.32
Cross-Industry Risky Investment	4.26	4.55	0.00	0.30
Risk-Free Investment	-3.64	7.18	10.66	0.15
20 Years to Retirement				
Own-Industry Risky Investment	0.27	2.32	2.11	-0.05
Cross-Industry Risky Investment	1.79	1.84	0.00	0.05
Risk-Free Investment	-1.06	2.90	3.94	0.02
10 Years to Retirement				
Own-Industry Risky Investment	0.30	0.85	0.56	0.00
Cross-Industry Risky Investment	0.68	0.68	0.00	0.00
Risk-Free Investment	0.03	1.07	1.04	0.00
5 Years to Retirement				
Own-Industry Risky Investment	0.33	0.53	0.21	0.00
Cross-Industry Risky Investment	0.42	0.42	0.00	0.00
Risk-Free Investment	0.25	0.66	0.40	0.00

Source: Author's calculations.

Notes:

- (1) FW stands for financial wealth.
- (2) TW stands for total wealth, the mean-variance efficient portfolio.
- (3) HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.
- (4) OW stands for option wealth, the job-switching option's delta.

⁵⁵Row 1, column 1, 0.38 = financial wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 2, 5.75 = total wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 3, 5.69 = human wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 4, -0.32 = option's delta invested in own-industry stock as a fraction of accumulated total financial wealth.

Table V

Optimal Portfolio Shares as Functions of the Expected Retirement Horizon⁵⁶

(Switch to the Market within the First 10 Years of Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	FW =	TW –	HW –	OW
30 Years to Retirement				
Own-Industry Risky Investment	-0.18	0.79	0.97	0.00
Cross-Industry Risky Investment	1.00	1.00	0.00	0.00
Risk-Free Investment	0.18	1.25	1.07	0.00
20 Years to Retirement				
Own-Industry Risky Investment	-0.09	0.67	0.76	0.00
Cross-Industry Risky Investment	0.85	0.85	0.00	0.0
Risk-Free Investment	0.24	1.07	0.83	0.00
10 Years to Retirement				
Own-Industry Risky Investment	0.08	0.46	0.38	0.00
Cross-Industry Risky Investment	0.59	0.59	0.00	0.00
Risk-Free Investment	0.33	0.74	0.41	0.00
5 Years to Retirement				
Own-Industry Risky Investment	0.18	0.36	0.18	0.00
Cross-Industry Risky Investment	0.45	0.45	0.00	0.00
Risk-Free Investment	0.37	0.56	0.19	0.00

Source: Author's calculations.**Notes:**⁽¹⁾ FW stands for financial wealth.⁽²⁾ TW stands for total wealth, the mean-variance efficient portfolio.⁽³⁾ HW stands for human wealth, the Bodie-Merton-Samuelson exposure to the risky asset.⁽⁴⁾ OW stands for option wealth, the job-switching option's delta.

⁵⁶ Row 1, column 1, -0.18 = financial wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 2, 0.79 = total wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 3, 0.97 = human wealth invested in own-industry stock as a fraction of accumulated total financial wealth; row 1, column 4, 0.00 = option's delta invested in own-industry stock as a fraction of accumulated total financial wealth.

Table VI
Optimal Portfolio Shares as Functions of the Degree of Relative Risk Aversion
(Permanent Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	Coefficient of Risk Aversion			
	4	6	8	10
30 Years to Retirement				
Own-Industry Risky Investment	1.68	0.38	-2.31	-6.89
Cross-Industry Risky Investment	3.01	4.26	6.39	8.07
Risk-Free Investment	-3.69	-3.64	-3.08	-0.18
20 Years to Retirement				
Own-Industry Risky Investment	1.23	0.27	-0.33	-0.70
Cross-Industry Risky Investment	2.19	1.79	1.44	1.13
Risk-Free Investment	-2.42	-1.06	-0.11	0.58
10 Years to Retirement				
Own-Industry Risky Investment	0.72	0.30	0.09	-0.04
Cross-Industry Risky Investment	1.00	0.68	0.51	0.40
Risk-Free Investment	-0.73	0.03	0.40	0.63
5 Year to Retirement				
Own-Industry Risky Investment	0.59	0.32	0.19	0.11
Cross-Industry Risky Investment	0.63	0.42	0.31	0.25
Risk-Free Investment	-0.22	0.26	0.50	0.64

Source: Author's calculations.

Notes: Benchmark coefficient of relative risk aversion equal to 6.

Table VII
Optimal Portfolio Shares as Functions of Industry i Wage Volatility
(Permanent Employment in a Low-volatility Industry)

Fractions of Current Financial Wealth	Industry i Wage Volatility			
	2%	4.5%	6%	10%
30 Years to Retirement				
Own-Industry Risky Investment	1.84	0.38	-1.46	-8.34
Cross-Industry Risky Investment	2.45	4.26	5.26	6.67
Risk-Free Investment	-3.29	-3.64	-2.80	2.67
20 Years to Retirement				
Own-Industry Risky Investment	1.30	0.27	-0.22	-0.59
Cross-Industry Risky Investment	1.70	1.79	1.59	0.99
Risk-Free Investment	-2.00	-1.06	-0.37	0.60
10 Years to Retirement				
Own-Industry Risky Investment	0.63	0.30	0.14	0.07
Cross-Industry Risky Investment	0.70	0.68	0.63	0.46
Risk-Free Investment	-0.33	0.03	0.22	0.47
5 Year to Retirement				
Own-Industry Risky Investment	0.44	0.32	0.26	0.15
Cross-Industry Risky Investment	0.43	0.42	0.41	0.38
Risk-Free Investment	0.13	0.26	0.34	0.47

Source: Author's calculations.

Notes: Benchmark coefficient of industry wage volatility equal to 4.5%.

Figure 1
Value of the Career Option as a Function of the Expected Retirement Horizon

Source: Author's calculations: option to switch jobs from a low-volatility industry to the market.

Figure 2
Own- and Cross-Industry Stock Ownerships and Risk-free Asset Ownership:
Simulation Results (Baseline Calibration) vs. Actual Portfolio Allocation

Source: Author's calculations, VanDerhei, Holden, Copeland and Alonso (2009), Figure 20, p. 23.

Figure 3
Own- and Cross-Firm Stock Ownership and Risk-free Asset Ownership:
Simulation Results (Alternative Calibration) vs. Actual Portfolio Allocation

Source: Author's calculations; Ameriks and Zeldes (2004), Table 2, p. 59.