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Elisa Luciano
Giovanna Nicodano

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Default risk in business groups*

Elisa Luciano[†] Giovanna Nicodano[‡]

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Abstract

This paper analyzes how combining firms into either groups or conglomerates affects their credit standing, as measured by their default probabilities, recovery rates and credit spreads. Each combination offers protection against default to its affiliates, and issues debt to optimize the trade-off between tax gains and default costs. In a group, the probability of joint default turns out to be lower than that of both stand-alone firms and conglomerates. This is the bright side of credit risk in groups. The dark side is that affiliation depletes the credit worthiness of the subsidiary. Such results hold irrespective of cash-flow correlation, if affiliates are equal in size, but fade if the parent is larger.

Keywords: credit risk, structural models, groups, mergers, parent-subsidiary.

JEL classification numbers: G32, G33, G34

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[†]Università di Torino, Collegio Carlo Alberto and ICER.

[‡]Università di Torino, Collegio Carlo Alberto and ECGI. Corresponding Author. Address: Via Real Collegio 30 - 10024 Moncalieri (Torino), ITALY. giovanna.nicodano@unito.it, Ph: +39-011.670 6073, Fax 6062

1 Introduction

Empirical studies highlight changes in default risk when one company becomes affiliated to both private equity funds and other companies. To mention a few examples, distress risk increases after a private equity fund buys out a company, yet bankruptcies of buyout companies are relatively rare considering that they display over 95% leverage (Jensen, 1989, 2007; Borell and Tykvovà, 2012)). Moving away from the private equity case, the default frequency of a group affiliate responds to the credit standing of the other affiliated units (Dewaelheyns and Van Hulle, 2006). Similarly, the spread paid by SPVs is affected by the sponsors' ratings (Gorton and Souleles, 2006). A fortiori, the default probability of a stand-alone company changes when it merges with another one (Furfine and Rosen, 2011). This paper studies how credit risk varies due to these affiliations, that are quite pervasive across countries and had a relevant role during the financial crisis.

One reason for the observed change in credit standing is that affiliation brings protection against default, in the form of either implicit or contractual guarantees. These have different levels of legal enforcement and different realizations: they can take the form of cash transfers, subsidized loans, support in a restructuring, collateral provision, transfer pricing at off-market prices. The presence of such guarantees is well documented. In the US, for instance, Bodie and Merton (1992) recognize that guarantees exist even when they are not reported in balance sheets. Herring and Carmassi (2009) describe the nuances of intra-group guarantees, which range from comfort letters to legally binding agreements. In emerging countries, empirical evidence refers to Indian groups (Gopalan, Nanda and Seru (2004)), Korean chaeboels (Bae, Cheon, and Kang, (2008)) and Japanese keiretsu (Jensen, 1991).

The presence of group guarantees does not rule out selective default, i.e., the default of one affiliate firm while other group members stay solvent. This occurs because each group affiliate enjoys limited liability with respect to other group companies. This phenomenon explains why rating agencies look at the type of group support for assessing the risk of debt obligations (Emery and Cantor (2005)). On the contrary, after a merger the divisions of a conglomerate share the same credit assessment.

These three facts, namely default risk modifications when entering a group, guarantees and selective defaults, still need an explanation, up to our knowledge. Indeed, models that endogenize default risk typically consider companies as stand-alone units and disregard guarantees. This paper analyzes how combining firms affects their credit standing, as measured by their default probabilities, recovery rates and credit spreads. To this end, we propose a model for business combinations, enriched with the classical trade-off between tax gains and default costs of debt. The distinguishing feature of each business combination will be the type of protection against default among the component firms. In a parent-subsidiary structure firms are separately incorporated and maintain limited liability[1], even if they rescue each other when there are sufficient cash-flows available. In a conglomerate merger, on the contrary, the (originally stand-alone) companies become fully liable for the common debt obligations. Finally, companies that do not belong to combinations, and

[1]This is the case in major jurisdictions, including the U.S., the U.K., Germany and France (Hadden, 1996; Blumberg, 1989).

remain stand-alone, enjoy no guarantees. Since groups are an important example of parent-subsidiary organization, in the sequel we use the two terms interchangeably.

The model delivers results concerning leverage and default risk of two BBB firms, once they enter a business combination. A preview of our main results is as follows. Debt financing - and the associated tax shield - is larger for a group than for the merger. It is also larger for group-affiliated subsidiaries than for stand-alone firms. Despite this, the probability of joint default for the two companies in a group is lower than that of two comparable stand-alone units and their merger. This result, that holds for any level of cash flow correlation between firms, is due to the optimal capital structure of the group in the presence of guarantees. When the group members are identical, this entails a complete shift of the debt burden onto the subsidiary, leaving the parent unlevered. By so doing, the group optimally exploits the tax shield of debt net of default costs. What preserves the high credit standing of a group is not diversification - which is at work in mergers - but the guarantee together with limited liability. We also show that the shift of debt onto the subsidiary is not always optimal with asymmetries between the parent and the subsidiary, such as a much greater size of the parent.

The key insight of our analysis is there is a bright side in the credit quality of groups: the parent's and subsidiary's joint default probability is smaller than both the stand-alone companies' and the merger's. However, there is also a dark side: affiliation to a group considerably depletes the credit worthiness of a subsidiary, and this is mirrored in its marginal default probability, recovery rate and spread over the riskless return.

The paper is organized as follows. Section 2 presents the set up. Section 3 analyzes the stand-alone, parent-subsidiary and conglomerate cases through the BBB-numerical example. It compares their credit standing as measured by default probabilities, recovery rates and credit spreads. It does so in correspondence to the optimal leverage and for symmetric firms, i.e. units which have the same bankruptcy costs, size and volatility. Section 4 examines whether the results are robust when correlation changes. Section 5 extends the same analysis to groups made of asymmetric firms. The last Section concludes.

2 The model

We consider a no-arbitrage environment with two dates $t = \{0, T\}$ and two firms or activities. Each activity generates a random cash-flow X_i at time T, which has be to be interpreted as the profit or loss from its operational activities. For the sake of simplicity, X_i will be a Gaussian random variable. In each organizational form (stand-alone, parent-subsidiary, merger), rules for debt repayment - in particular whether another member of the groups helps in case of distress - are different[2]. For each organizational form we then determine how much of the cash-flows X_i will be paid to bondholders and shareholders. We also determine the value at time 0 of debt and equity as the expected, discounted value of the corresponding payoff. Last, we assume that debt is optimized. Each organization is

[2]The model we build upon is developed in Luciano and Nicodano (2012), starting from the stand-alone and merger case of Leland (2007). Luciano et al. (2012) also characterize the optimal guarantee.

supposed to choose the face value of debt so as to maximize the overall firm value at time 0. Let the face value of debt be F_i and let D_{0i} be its no-arbitrage value at $t = 0$. In the stand-alone case, where firms are labeled $1, 2$, we have:

$$\max_{F_i} \ \nu_{0i}(F_i) = E_{0i} + D_{0i}, i = 1, 2 \tag{1}$$

where E_{0i} is the time-0 value of each firm's equity. In the parent-subsidiary case, where the labels are p, s, we have

$$\max_{F_i} \ \nu_{0g}(F_p, F_s) = E_{0p} + D_{0p} + E_{0s} + D_{0s}, \tag{2}$$

In the merger case we have

$$\max_{F_M} \ \nu_{0M}(F_M) = E_{0M} + D_{0M}, \tag{3}$$

where the index M stands for merger.

Since this is a structural model, the optimal level of debt determines the probability of default, the recovery rate and the credit spread.

Default probability

The concept of default probability has to be tailored to each organizational form. Consider stand-alone firms first. Each stand-alone firm either defaults or not. The marginal default probability π_i is the probability that cash-flows net of taxes, X_i^n, be smaller than the face value of debt F_i at T :

$$\pi_i = P\left[X_i^n < F_i\right] \qquad\qquad i = 1, 2$$

Since each stand-alone is either in default or not, four mutually-exclusive, joint occurrences are possible. These are joint default, which occurs with probability π_{12}; joint survival; selective default of either 1or 2. Joint default occurs when both cash flows are below the face values of debt:

$$\pi_{12} = P\left[X_1^n < F_1, X_2^n < F_2\right]$$

Selective default occurs when one firm survives while the other is in default. Its probability - which we denote as π_i^- - is the difference between the marginal and joint default probabilities. Consider firm 1:

$$\pi_1^- = P\left[X_1^n < F_1, X_2^n > F_2\right] = \pi_1 - \pi_{12}$$

Joint survival probability $\bar{\pi}_{12}$ is computed considering that joint survivorship excludes both joint default and selective default of both firms:

$$\begin{aligned} \bar{\pi}_{12} &= P\left[X_1^n > F_1, X_2^n > F_2\right] \\ &= 1 - \pi_1^- - \pi_2^- - \pi_{12} \\ &= 1 - \pi_1 - \pi_2 + \pi_{12} \end{aligned}$$

Consider now a group. In the case of a parent, the event triggering default is equal to the one of a stand-alone company:

$$\pi_p = P\left[X_p^n < F_p\right]$$

The default of a subsidiary instead occurs only if the parent cannot provide funds for avoiding it. The marginal default probability is equal to:

$$\pi_s = P\left[X_s^n < F_s, X_p^n < h(X_s^n)\right]$$

where $h(X_s^n)$ is the level of the parent cash flow below which support is not provided, else the parent would be drawn into default by its subsidiary. This no-rescue level will be determined below.

Selective default of the parent occurs as in the stand-alone case, while selective default of the guaranteed firm occurs if the parent is solvent but has insufficient funds for support:

$$\pi_p^- = P\left[X_p^n < F_p, X_s^n > F_s\right] = \pi_p - \pi_{ps}$$

$$\pi_s^- = P\left[F_p < X_p^n < h(X_s^n), X_s^n < F_s\right]$$

Joint default still occurs when each cash flow is lower than the respective face value of debt:

$$\pi_{ps} = P\left[X_p^n < F_p, X_s^n < F_s\right]$$

Joint survival occurs when each firm reimburses its debt holders, either with or without support:

$$\bar{\pi}_{ps} = P\left[X_p^n > h(X_s^n), X_s^n < F_s\right] + P\left[X_p^n > F_p, X_s^n > F_s\right]$$

We label the first event as "joint survival with rescue", the second as "joint survival without rescue" and the corresponding probabilities as $\bar{\pi}_{ps}^r, \bar{\pi}_{ps}^{\bar{r}}$, so that by definition:

$$\bar{\pi}_{ps} = \bar{\pi}_{ps}^r + \bar{\pi}_{ps}^{\bar{r}}.$$

Figure 1 presents the corresponding events in terms of the two firms' cash flows. Event A is joint survival with rescue, event D joint survival without rescue.

Insert here figure 1

So the relevant probabilities take complex forms for parent-subsidiary groups, when guarantees mitigate the default triggering condition.

Finally, in the conglomerate merger case we either have joint defaut, occurring with probability:

$$\pi_M = P\left[X_M^n < F_M\right]$$

or no default.

Recovery rate

For all of the previous organizations, the expected loss is the difference between the full repayment, F_i, and the recovery, $D_{0i}(1+r_T)$, where r_T is the riskless rate over the period $(0,T)$. The percentage expected loss is then equal to:

$$\frac{F_i - D_{0i}(1+r_T)}{F_i}$$

for $i = 1, 2, p, s, M$. By taking the ratio of each company's expected loss to the corresponding default probability, we get the loss given default, and therefore the recovery rate

$$R_i = 1 - \frac{F_i - D_{0i}(1+r_T)}{F_i \times \pi_i} \tag{4}$$

Credit spread

Last but not least, the spread y is the difference between the rate of return on zero-coupon debt, which is the ratio between the face and present value of debt, and the riskless rate:

$$y_i = (F_i/D_{0i})^{1/T} - 1 - r_T \tag{5}$$

The spread is an overall measure of credit quality, since it incorporates information on both recovery and default probability - via the ratio of the face to the fair value of debt, F_i/D_{0i}.

As in all structural models, the previous quantities (loss, recovery, spread) are endogenous. By comparing such quantities across different organizations we will be able to discuss their comparative credit standing. In order to facilitate the comparison - but without loss of generality - we assume that, when two firms form a group, firm 1 becomes the parent, while firm 2 is the subsidiary.[3] This means that $X_1 = X_p, X_2 = X_s$, where equality is in distribution. The merger cash flow is simply the sum of the merged firms' profits, since - consistent with the other cases - we assume the absence of operational synergies: $X_M = X_1 + X_2$.

2.1 Stand-alone companies

This Section describes how net cash flow of a stand-alone X_i^n is determined, provides a restatement of the default-triggering condition $X_i^n < F_i$ and establishes how debt of a stand-alone is valued.

Let the two activities 1 and 2 be separately incorporated and independently managed. Each one can issue a zero-coupon bond with face value F_i at time 0. They have an incentive to do so as interest on debt is a deductible expense. However, debt increases the probability of default, which is assumed to cost a fraction α_i of cash-flows X_i, when they are positive. So, debt generates the standard trade-off between tax gains and default costs. In our setting, both are random when the leverage decision is taken and credit standing is evaluated, since both depend on the random realization of cash-flows at the terminal date.

[3]Luciano and Nicodano (2012) endogenously characterize which firm should be the supporting one. They also show that mutual support - the parent supports the subsidiary and viceversa - is suboptimal. This is why it is never considered in this paper.

Debt is due at T and debt holders have absolute priority with respect to shareholders. Debt is paid after taxes, though. Since the promised interest payment or tax shield is X_i^Z

$$X_i^Z = F_i - D_{0i} \tag{6}$$

taxable income is the cash-flow X_i net of interest payment:

$$X_i - X_i^Z \tag{7}$$

There is a flat tax rate τ_i on cash-flows net of the tax shield. It follows[4] that cash-flows, net of tax payments, are

$$X_i^n = \begin{cases} 0 & X_i < 0 \\ X_i & 0 < X_i < X_i^Z \\ X_i(1-\tau_i) + \tau X_i^Z & X_i > X_i^Z \end{cases} \tag{8}$$

We know that default occurs when net cash-flow at T is smaller than the face value of the debt:

$$X_i^n < F_i \tag{9}$$

Having defined the default threshold X_i^d as

$$X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_{0i} = \frac{F_i - \tau_i X_i^Z}{1-\tau_i} \tag{10}$$

the default triggering condition (9) can be written in terms of the pre tax cash-flows as $X_i < X_i^d$. The payoff D_i to lenders at time T will equal F_i when $X_i > X_i^d$ and the firm is solvent. In the event of default, bondholders will receive a fraction $(1-\alpha_i)$ of the cash-flow X_i when this is positive. This happens after the firm has paid taxes, if cash-flows are above the tax shield. The payoff to lenders is therefore

$$\begin{cases} (1-\alpha_i)X_i & 0 < X_i < X_i^Z \\ (1-\alpha_i)X_i - \tau_i(X_i - X_i^Z) & X_i^Z < X_i < X_i^d \\ F_i & X_i > X_i^d \end{cases}$$

The value of debt at time 0 can be written as

$$D_{0i}(F_i) = (1+r_T)^{-1}\mathbb{E}\left[\begin{array}{c} (1-\alpha_i)X_i \ \mathbf{1}_{\{0<X_i<X_i^Z\}} + \\ \left[(1-\alpha_i)X_i - \tau_i(X_i - X_i^Z)\right] \mathbf{1}_{\{X_i^Z<X_i<X_i^d\}} + \\ +F_i \ \mathbf{1}_{\{X_i>X_i^d\}} \end{array}\right] \tag{11}$$

where $\mathbf{1}_{\{\bullet\}}$ is the indicator function.

The payoff to shareholders at time T is cash-flow less taxes and the repayment of principal, when the difference is positive: $\max(0, X_i^n - F_i)$.

By no arbitrage the value of equity is simply[5]

$$E_{0i}(F_i) = (1+r_T)^{-1}\mathbb{E}[\max(0, X_i^n - F_i)] \tag{12}$$

[4]Following Leland (2007), we assume that no tax refunds are paid to the firm when $X_i < X_i^Z$.

[5]Note that (11) is an implicit equation, since X_i^Z and X_i^d are themselves a function of D_{0i} through (6)

2.2 Parent-subsidiary

We now examine the parent-subsidiary case. This Section uses the net cash flow of single firms, X_p^n, X_s^n, in order to determine the rescue threshold $h(X_s)$ and establishes how debt of the subsidiary - who benefits from rescue - and equity of the parent - who provides it - are affected.[6]

The parent company enjoys limited liability. If the subsidiary defaults, i.e. $X_s < X_s^d$, the parent transfers funds in order to rescue it only if three circumstances occur. First, if debt to its own bondholders has been paid, $X_p > X_p^d$. Second, if the cash-flows of the subsidiary are non-negative, $X_s > 0$. The parent would otherwise bear an operational loss that it could have avoided. Third, if rescue does not endanger her survival. Put together, these restrictions mean that transfer occurs if and only if the subsidiary is in default and the parent's cash-flows exceed a threshold $h(X_s)$:

$$\begin{cases} X_s < X_s^d, \\ X_p > h(X_s) \end{cases} \tag{13}$$

The function $h(X_s)$ is represented as a kinked line[7] in Figure 1. In the figure the occurrence of these conditions is event A, i.e. joint survival with rescue. When rescue occurs, the parent transfers exactly what the subsidiary is short of in order to repay debt, namely $F_s - X_s^n$. The amount of the transfer then is

$$(F_s - X_s^n)\mathbf{1}_{\{A\}}.$$

The group shareholders are assumed to choose the face value of debt in the parent and in the subsidiary so as to maximize levered group value. The group value ν_{0g} is:

$$\nu_{0g} = \nu_0(F_p, F_s) = E_{0p} + D_{0p} + E_{0s} + D_{0s} \tag{14}$$

For the sake of simplicity, we write down the model assuming that the tax rate and default costs do not differ across the two units. Therefore, $\alpha_i = \alpha$ and $\tau_i = \tau$. This assumption is removed later.

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and (10). Numerical methods are required for its solution. Since D_{0i} determines the thresholds and the latter enter the equity value, the solution approach for finding firm value ν_{0i} consists in finding a fixed point for D_{0i} and then determine X_i^Z, X_i^d and E_{0i}.

[6] The default thresholds X_i^d, $i = p, s$ and the tax shield X_i^Z are related to debt principal and present value by (6) and (10), as in the stand-alone case.

[7] Its expression is :

$$h(X_s) = \begin{cases} 0 & X_s < 0 \\ k - \dfrac{X_s}{1-\tau_h} & 0 \le X_s < X_s^Z \\ K - X_s \dfrac{1-\tau_s}{1-\tau_h} & X_s^Z \le X_s < X_s^d \end{cases}$$

where, with equal tax rates for the two firms ($\tau_s = \tau_h = \tau$),

$$\begin{aligned} k &= \frac{P_s}{1-\tau} + X_h^d \\ K &= X_h^d + X_s^d \end{aligned}$$

The only difference in payoff-relevant events with respect to the stand-alone case is the existence of the event A. The cash-flow accruing to *shareholders of the parent* company is equal to the one of the stand-alone company, less the transfer. As a consequence, it depends on both principals F_p and F_s:

$$\max(0, X_p^n - F_p) - (F_s - X_s^n)\mathbf{1}_{\{A\}} \tag{15}$$

The equity value is

$$E_{0p}(F_p,F_s) = (1 + r_T)^{-1}\mathbb{E}\left[\max(0, X_p^n - F_p) - (F_s - X_s^n)\mathbf{1}_{\{A\}}\right] \tag{16}$$

The payoff to *subsidiary lenders* is the same as in the stand-alone case, in the states where the subsidiary defaults since no transfer takes place.[8] These states are formally characterized as

$$B = \begin{cases} X_s < X_s^Z \\ X_p > X_p^d \\ X_p^n - F_p < F_s - X_s^n \end{cases}$$

$$C = \begin{cases} X_s^Z < X_s < X_s^d \\ X_p > X_p^d \\ X_p^n - F_p < F_s - X_s^n \end{cases}$$

and are drawn in Figure 1. The main difference between events B or C is in whether the subsidiary does not pay taxes ($X_s < X_s^Z$) or pays them ($X_s^Z < X_s < X_s^d$). The payoff to lenders must instead be augmented by the transfer if event A occurs. There, it would have been X_s^n. Including the transfer, it becomes $X_s^n + (F_s - X_s^n) = F_s$.

Since the subsidiary debt is the present expected value of these final payoffs, it becomes:

$$D_{0s}(F_s, F_p) =$$

$$(1 + r_T)^{-1}\,\mathbb{E}\left[\begin{array}{c} X_s(1-\alpha)\mathbf{1}_{\{B\}}+ \\ +\left[X_s(1-\alpha) - \tau(X_s - X_s^Z)\right]\mathbf{1}_{\{C\}}+ \\ +F_s\left[\mathbf{1}_{\{A\}} + \mathbf{1}_{\{X_s > X_s^d\}}\right] \end{array}\right] \tag{17}$$

The payoffs to *lenders of the parent* do not change with respect to the stand-alone case, as the transfer to the subsidiary occurs only after the service of the parent debt. Similarly, *equity holders of the subsidiary* are unaffected, as the transfer occurs for the sake of servicing debt. As a consequence equations (11) and (16) still hold for $i = p$ and $i = s$ respectively.

[8] We are assuming that there is no consolidation of assets in the event of default of the parent - which seems consistent with what happens in most real-world cases (Samson, 2001).

2.3 Conglomerates

The conglomerate merger case - introduced in Leland (2007) - obtains when the two activities $X_i,\ i = 1, 2$, are incorporated as divisions into one company. The merger cash-flow X_m is the sum of the cash-flows of the original activities:

$$X_m = X_1 + X_2 \tag{18}$$

The merger issues a unique debt. Its face value F_m maximizes the merger value ν_{0m}

$$\nu_{0m} = \nu_0(F_m) = E_{0m}(F_m) + D_{0m}(F_m) \tag{19}$$

where $E_{0m}(F_m)$ and $D_{0m}(F_m)$ are computed as in the stand-alone case, since the two cash flows remunerate together debt and equity, exactly as the cash-flow of a stand-alone firm.[9] Equations (11) and (16) still hold for $i = M$. From the point of view of computing debt and equity, tax shields, default thresholds, survival and default probabilities, there is no difference between the stand-alone and merger case. Only the definition of the underlying cash flow is.

3 Credit risk: a base case

We numerically study the credit risk implications of the organizational structures outlined above. The values of the parameters in a base case come from Leland (2007), who calibrates them to firms that issue BBB-rated unsecured debt when stand-alone. Companies are symmetric, in the sense that they have identically distributed cash-flows, the same tax and default cost rate. The parameters in the base case are the following: the debt maturity T is equal to five years, consistent with investment grade evidence. The annual riskless interest rate is 5%, and the expected cash-flow for each activity, $\mathbb{E}(X_i)$, is chosen so that its present value is 100. The standard deviation of these cash flows is 49.2. If annual cash-flows are independent in time, this is consistent with an annual standard deviation of 22. This implies an annualized volatility, σ, in percent of the present value of cash flow, of 22%. The correlation coefficient between the cash flows is equal to 0.2. Finally, the tax rate and the default cost parameter are respectively set to 20% and to 23%, so as to generate optimal leverage and recovery rates consistent with the ones observed in a BBB stand-alone firm.

We obtain the optimal capital structure and credit risk indicators of table 1. The first column reports values for a stand-alone. The second and the third refer to a parent and a subsidiary respectively, while the fourth one depicts the overall parent-subsidiary group. The last column refers to a conglomerate.

Insert here Table 1

We start by comparing the credit features of a stand-alone company versus a parent-subsidiary group first (column one versus two-to-four). The stand-alone company versus conglomerate (column one versus five) will follow. In the end we will be able to compare the group and the conglomerate (columns four and five).

[9] X_m^Z and X_m^d are defined as in (6) and (10).

3.1 Groups versus stand-alone firms

Leverage

In order to understand the credit risk of different organizations, we must first look at their leverage. Table 1 highlights two important features: the group is highly levered with respect to two stand-alone firms; and all the group debt is in the subsidiary, instead of being shared with the parent. This extreme capital structure obtains in the absence of moral hazard. Indeed, the best solution to the tax- bankruptcy trade off is to raise capital via the subsidiary, given the possibility of supporting it when insolvent. Support often saves the subsidiary from bankruptcy, so as to avoid the loss of value inherent in default. At the same time, the subsidiary does default when its rescue increases bankruptcy costs so much that it would drag the parent into default. The resulting face value of debt in the subsidiary is higher than in the two stand-alone firms (219 versus 114.4). This characteristic of the optimal solution is consistent with the empirical evidence in Dewaelheyns and Van Hulle (2006), who notice that the "decreased potential costs of financial distress allow group members to ex ante take on more debt, thus realizing more tax gains"[10]. The Table indeed shows that the group pays lower taxes than two stand-alone firms (25.37 versus 17.62 times 2).

Default probabilities

Leverage impacts on the marginal default probabilities. These change from $\pi_1 = \pi_2 = 11.2\%$ for stand-alone companies to $\pi_s = 46.54\%$ for the subsidiary. The latter is much more likely to default than a stand-alone, in spite of receiving support, because of its extreme leverage. This is consistent with the increase in the number of default occurences observed after companies' buyouts (Borell and Tykvovà, 2012). As for the unlevered parent, its marginal default probability is just the probability that its cash flows turn negative, $\pi_p = 0.34\%$.

Rescue of the subsidiary takes place more or less in one half of the cases, since the corresponding probability is $\bar{\pi}_s^r = 52.35\%$. However, in the absence of rescue but keeping the same leverage, the default probability would be even larger - actually close to one ($\pi_s = 99.3\%$). This is a possible explanation for the relatively low number of defaults of buyout targets in highly leveraged transactions (Jensen, 1989, Jensen, 2007, Andrade and Kaplan, 1998). It also justifies concerns on the contribution to systemic risk, i.e. the destabilizing effects of parent-subsidiary structures, since the default of a subsidiary, which occurs in 46.54% of the cases, endangers its lenders. Another figure that reveals such dark side of group credit risk and related concerns is that joint survival occurs in more than one half of the cases ($\bar{\pi}_{ps}$= 52.92%) while the two stand-alone firms survive with 79.73% probability.

Concerns should be mitigated by the observation that the joint default probability, $\pi_{ps} = 0.34\%$, is much lower than it would be without within-group support. For the comparable

[10] Such leverage produces a considerable increase in the group value ν_{0g}^* (165.91) with respect to that of a stand alone (162.46).Thus parent-subsidiary links, that preserve limited liability while allowing for state-contingent support, create value for financiers. Luciano and Nicodano (2012) show this holds in general. However, the value of the parent falls with respect to the stand alone situation (from 81.23 to 49.2), as it discounts the povision of support to its subsidiary

stand-alone firms joint default occurs 2% of the times (π_{12} =2.08%). So, the risk that the whole group goes bankrupt diminishes in spite of its high leverage, thanks to group support. This may explain the low occurrence of group defaults in highly leveraged transactions (see Altman et al., 2008). This is also good news for the contribution of parent-subsidiary structures to systemic risk, and represents the bright side of group credit quality.

Recovery rate

The recovery rate falls from 48.1% for the stand-alone company to 31.2% for the subsidiary. Indeed, the parent is more likely to be unable to support its subsidiary when the latter losses are larger - leaving these low recovery cases to lenders. The decline in recovery when the default probability increases - or, equivalently, the increase in loss given default - is an important feature to capture, since empirical evidence supports it (see for instance Altman et al., 2008).

Spread

As a consequence of high default probability and low recovery, the credit spread of the subsidiary dramatically increases with respect to the stand-alone company. Over five years, it becomes 8.4% versus 1.26%.

3.2 Conglomerate versus stand-alone

We now compare the conglomerate merger and its divisions to the stand-alone firms.

Leverage

In order to understand credit risk we look first at the optimal leverage which determines it. The conglomerate raises more debt than the two stand-alone firms. The overall debt of the conglomerate (117.4) is greater than the sum of two stand-alone debts (114.4). It derives from the fact that divisions of a conglomerate diversify away some risk when the correlation between their cash flows is lower than one (Lewellen, 1971). The merger can increase its leverage because the tax advantages induced by the level of debt are not completely offset by the increase in expected default costs, thanks to risk diversification.[11]

Default probability

Risk sharing is at work in a merger, because debt is issued against a diversified portfolio of assets. This does not prevent the conglomerate to be more likely to default than two stand-alone companies: in spite of being backed by a diversified portfolio, debt increases so much with respect to the stand-alone case that the merger default probability is higher than the one of a stand-alone firm. It reaches 6.5% instead of 2%, the joint default probability of two stand-alone units. This happens since there is no limited liability. If one activity performs poorly and the other is profitable, the former can drag the latter into default. This is the well-known Sarig effect (Sarig, 1985).

Recovery rate

[11] The merger value is greater than twice the value of two stand-alone firms, thanks to this optimal leverage. Leland (2007) shows that this is not always the case, becasue of the loss in limited liability.

For the same risk sharing and mutualization - or lack of limited liability - reason, merger is accompanied by higher recovery than a stand-alone. We have 56.5% instead of 48.1%, since two cash flows back up the common debt and losses of one division can be mitigated by profits of the other. So, when going from a stand-alone company to a merger, the effect on default probability and recovery go into opposite directions.

Spread

A priori, the opposite effects just described could lead to either an increse or a decrease in spread. It turns out that, for BBB firms with equal size, risk, tax and default rates, the increase in recovery outweighs that in default probability. The spread is reduced with respect to a stand-alone. It reaches 0.6% instead of 1.26% for each of its divisions. This simply says that, for BBB firms, the overall credit quality improves because of mutualization, in spite of higher leverage.

3.3 Group versus conglomerate

We now turn to the comparison between a parent-subsidiary group and a conglomerate.

Leverage

Our model shows that group debt is greater than for conglomerates (219 versus 117.4 in terms of face value). We explain this with the possibility of setting two different levels of debt in the group affiliates, using the support option if and only if the parent survives. In a merger, on the contrary, the divisions have to support each other even when this leads to higher bankruptcy costs. The parent-subsidiary structure has higher optimal leverage because it exploits the benefits of risk sharing without incurring into its costs.[12]

Default probability

Higher leverage usually leads to higher default probability. Looking again at Table 1, we discover instead that the probability of joint default is 0.34% for the parent-subsidiary group and 6.5% for the merger. This result obtains even if the optimal leverage ratio is greater for groups than for conglomerates, 70% for the former and 54.8% for the latter, i.e. total debt is 219 instead of 117.4. We explain it as follows. The specificities of the group - namely limited liability and separate incorporation - allow it to choose not only the amount of debt, but also its location (at the subsidiary versus the parent level) so as to perfectly eploit the benefits of risk sharing while sheltering the parent's cash flows from being unusefully wasted in bankrutcy costs.

To sum up, parent-subsidiary structures considerably deplete the credit worthiness of subsidiaries - measured in terms of marginal default probability, recovery rate and spread. This is the dark side of group credit risk. There is a bright side too: the parent's and subsidiary's joint default probability is smaller than both the stand-alone companies' and

[12] As a consequence,the value of a group, $\nu_{0g}^{*} = 165.91$, exceeds that of a conglomerate, $\nu_{0m}^{*} = 163.15.$, in spite of its higher spread, debt and default probability.

the merger's. The contribution to systemic risk of parent-subsidiary structures will depend on the relative importance in the economy of these dark and bright sides. An important question is whether an increase in cash flow correlation brings group joint default probability, thus shrinking the bright side of groups. In the next sections we show that previous results on joint default risk extend not only to very large values of the cash-flow correlation, but also to the case of asymmetric firms, because they are a straighforward implication of the option to provide support, that in turn is a consequence of corporate limited liability.

4 Default risk as correlation changes

Cash-flow correlation tends to be high when the combined firms belong to the same industry. Correlation may also increase during a crisis. This Section analyzes changes in the credit standing of BBB symmetric firms as their cash flow correlation varies.

Leverage

Leland (2007) shows that a purely financial merger leaves unaffected leverage and default risk of the two component firms if their cash-flow correlation is equal to 1. This is because the distinctive characteristic of conglomerates is diversification. We might expect a similar result to hold also for parent-subsidiaries, if we focus on diversification only. We might forecast that the optimal face value of debt in groups converges to the stand-alone level, as correlation among cash-flow increases, since the transfer from the parent to the subsidiary becomes less likely. This intuition is incorrect, because it does not take into consideration the possibility of sheltering the group from bankrupcty by reducing the leverage of the firm that provides support. Taking this into consideration too, we infer that the parent-subsidiary is still able to lever more than two stand-alone firms, even when correlation is equal to one. Indeed, with perfect correlation, both cash flows can be used to service debt, when they are positive; on the other hand, only the subsidiary incurs into costly default when both are negative, since the parent preserves limited liability. This reasoning works provided that the parent stays unlevered, otherwise also the parent defaults. Our numerical results confirm this intuition: debt in the parent continues to be zero, leverage in the subsidiary is still high and the parent-subsidiary is subject only to a minor decrease in total leverage[13]. Figure 2, top , reports the optimal leverage ratio for the three types of organizations.

Insert here Figure 2

As correlation increases and the effect of diversification vanishes, the optimal leverage of a conglomerate indeed converges to that of a stand-alone company. On the contrary, the one of groups falls from over 80% to less than 70%, a figure which remains 30% higher than the leverage of conglomerates and stand-alone firms.

[13]As correlation increases, so does the face value of debt. But the value of equity increases by more given its option-like payoff. This brings about a reduction in leverage, which remains larger than the one in the stand-alone.

Default probabilities

In the top panel of Figure 3 we plot the merger's , two-stand-alone firms' and group's joint probabilities of default.

Insert here Figure 3

The group default probability is always low, below 1%, and almost insensitive to cash-flow correlation. It is always smaller than the merger's, thanks to limited liability. It is also smaller than the two stand-alone companies', unless correlation is very low (in Figure 3, -0.5 or -0.8). The good survival properties of groups are thus robust to industry composition and economic downturns, in sharp contrast to those of the merger. Its default probability is as high as 10% for a 0.8 cash-flow correlation, but is already higher than 5% with uncorrelated cash-flows.

As the bottom part of the figure shows, these effects occur at the expense of the subsidiary, unless cash flow correlation is very low. Its default probability is dramatically higher with respect to a stand-alone, unless correlation approaches -1. In this last case diversification makes the guarantee so effective that the default probabilities of a stand-alone and a subsidiary are almost indistinguishable.

For all levels of correlation, the subsidiary default probability is smaller than it would be in the absence of guarantees. This becomes evident when we compare the subsidiary (in green) with a stand-alone with the same leverage (in blue), as in the bottom part of Figure 3. The default probability of a subsidiary ranges from 10% to 55%, compared with 95% for the highly levered stand-alone. This shows that the paradox of relatively infrequent defaults, which has been observed in highly leveraged transactions (Jensen, 1989, Jensen, 2007, Andrade and Kaplan, 1998), may obtain even with high cash-flow correlation.

Recovery rate

The mid plot of Figure 2 represents the recovery rate: the inequalities across organizational forms hold throughout the correlation range. The spread sensitivity to correlation is higher in the merger case. Cross subsidization makes the merger recovery both higher and more sensitive to correlation than under no support (stand-alone) and under conditional rescue (group), since pure risk sharing is at work.

Spread

The bottom plot of Figure 2 represents the credit spread. Again, apart from the fact that the inequalities across organizational forms hold throughout the correlation range, we notice that - as correlation goes down - the merger spread benefits from cross-subsidization. The group spread is much higher, because of high leverage and conditional rescue only, and does not vanish as correlation tends to -1.

We can summarize the results from Figures 2 and 3 as follows: the leverage of the group exceeds the one of both the conglomerate and the corresponding stand-alone. The marginal default probability and the spread of the subsidiary are higher than the merger's and stand-alone's. Its recovery rate is smaller than for mergers and stand-alone firms. However, joint default probability stays much lower.

5 Asymmetric companies

In this Section we consider the credit risk of firm combinations that are not identical. In particular, we analyze the cases of lower default costs (Table 2), higher volatility (Table 3) and smaller size (Table 4) for the subsidiary.[14] And we restore the $\rho = 0.2$ assumption, so as to be able to make comparisons with our base case.

Different default costs

Insert here Table 2

Higher default costs in the parent ($\alpha = 75\%$) do not change the optimal capital structure in a group, as the whole debt burden is still borne by the subsidiary (with $\alpha = 23\%$ as in the base case) only. A zero leverage for the parent's leverage was indeed optimal even with lower default costs. Given the absence of debt, the parent will never incur into bankruptcy costs, which do not therefore affect credit quality in the subsidiary either. Thus the overall credit risk assessment of groups is insensitive to default costs in the parent. Comparison across Table 2 and Table 1 reveals that the effect of higher α is to reduce optimal leverage, both in the merger (44% instead of 55%) and in the stand-alone (30.9% instead of 52%) cases. This cannot prevent the recovery rate from falling, but leads to a lower joint default probability in both the merger (2.9% instead of 6.5%) and stand-alone companies (0.77% instead of 2.08%), relative to the base case. The group, with an unchanged optimal leverage of the base case (70.33%), still enjoys lower joint default probability (0.34%) but has higher spread (8.4%). This indicates that the parent-subsidiary structure is able to shield firm activities from asymmetric bankruptcy costs, preserving both the bright and dark sides of its credit quality.

Different cash-flow volatility

Insert here Table 3

Consider the case in which the annualized volatility of the subsidiary cash flow ($\sigma = 44\%$) is twice as large as in the parent ($\sigma = 22\%$). A riskier subsidiary turns out to have slightly larger face value of debt (223 up from 219), because the tax shield now delivers higher tax savings. Such an increase in debt and larger cash-flow volatility implies that the subsidary's credit quality worsens. Its default probability increases (48.7% instead of 46.5%); the corresponding recovery rate falls (20.2% instead of 31.2%) and the spread reflects this (10.9% up from 8.4%)[15].This increase in the spread is moderate compared to that in a stand-alone firm, that jumps to 6.2% from 1.26%, since the face value of debt increases to 83 up from 57.2. Conglomerates only marginally increase debt (up to 118 from

[14] We also examined the opposite cases, in which the subsidiary is costlier, less volatile and larger than its parent company. We neglect them in our tables since we find that the corresponding value of the group is lower.

[15] Also in the group case, we find that as default probability grows for higher volatility the recovery rate falls. This is a stylized fact in the literature on credit risk (Altman et al., 2008).

117.4) and diversification opportunities help contain the increase in the cost of debt (from 0.6% to 2%).

Thus spreads in firm combinations, both groups and mergers, are less sensitive to changes in cash flow volatility than in the stand-alone firms. This is also true for joint default probabilities: the one of mergers falls slightly to 6.35% (down from 6.5% in the base case), the one of groups is unchanged at 0.34%, the one of the stand-alone firms reaches 5.64 (up from 2.08 in the base case). Thus, the credit quality of firm combinations appear to be less sensitive to a highly volatile cash-flows in one of their units. However, the marginal default probability for the subsidiary is still higher than the stand-alone firm's (48.71% instead of 37.09%). Both the bright and the dark side of credit risk in business groups carry over to a situation of asymmetric risk.

Different size

Insert here Table 4

Let us explore the case in which the parent is five times as large as its subsidiary, in the sense that the mean of the final cash-flow is such a multiple ($\mathbb{E}(X_p) = 5\mathbb{E}(X_s)$). All other parameters, including volatility, are equal to the base case in both units. Suchs size asymmetry is irrelevant to the credit assessment of stand-alone companies, because all optimal choices are simply scaled up or down, with ratios and probabilities unchanged.

The asymmetry makes it profitable for the group to raise debt from the parent, because it allows to exploit the tax shield without compromising the parent's provision of support to the small subsidiary. The leverage ratio in the parent grows from zero to 51%, while that of the subsidiary is unchanged relative to the base case (100%). The parent cash-flow is large enough to be able to rescue its subsidiary despite its positive debt commitment: the rescue probability is 66.56%, up from 52.35% in the base case. The credit quality of the parent drops, as its default probability increases to 4.5% from 0.34% in the symmetric case. The subsidiary has lower default probability than in the base case (33.04% instead of 46.54%). However, the joint default probability of the group is now higher than the one of two stand-alone companies (4.55% relative to 2.05%). Thus, differences in size smooth both the dark and the bright sides of credit risk in business groups.

6 Summary and concluding remarks

Our analysis provides an assessment of credit risk when firms become interdependent. It contributes to the literature by showing how state-contingent support in business groups, together with a tax-bankruptcy trade-off, modifies default probabilities, recovery rates and credit spreads in affiliated companies. We explain why the prediction of default frequency improves when the credit standing of the other affiliated units is taken into account (Dewaelheyns and Van Hulle, 2006). More than that, it rationalizes selective defaults and the discriminating assignments of ratings by most agencies when a parent-subsidiary link is in place (Emery and Cantor, 2005). Finally, our groups resemble private equity arrangements, in that they display relatively infrequent defaults which have been observed in LBOs and MBOs.

The key insights of our analysis are as follows. Groups appear to unevenly distribute leverage and default risk among the affiliate firms, even when they are identical as stand-alone firms. The default probability of subsidiaries increases, due to an optimal debt as high as four times the one of alternative firm organizations, while the parent has low, even zero, leverage. Interestingly, such risk shifting obtains without any moral hazard: raising debt through a guaranteed company provides the same tax relief with lower expected bankruptcy costs. As a consequence of this uneven distribution of debt, the default probability of individual subsidiaries increases, but the joint probability that all firms simultaneously default decreases, when compared to alternative organization. In this respect, groups appear to have good own survival properties, much better than conglomerate mergers that exploit diversification but cannot protect their divisions by using limited liability. This observation is reinforced when cash-flow correlation is high, i.e. when affiliates belong to the same industry or a systemic crisis is occurring. Under these circumstances the merger suffers from scarce diversification opportunities. On the contrary, these persist in groups because of the unlevered parent, that makes its cash flow available for the subsidiary when this is unable to service its debt. Both the positive and negative credit risk properties of parent-subsidiary structures carry over to the case of affiliates with heterogenous bankruptcy costs and cash flow volatility. They smooth out with differences in size.

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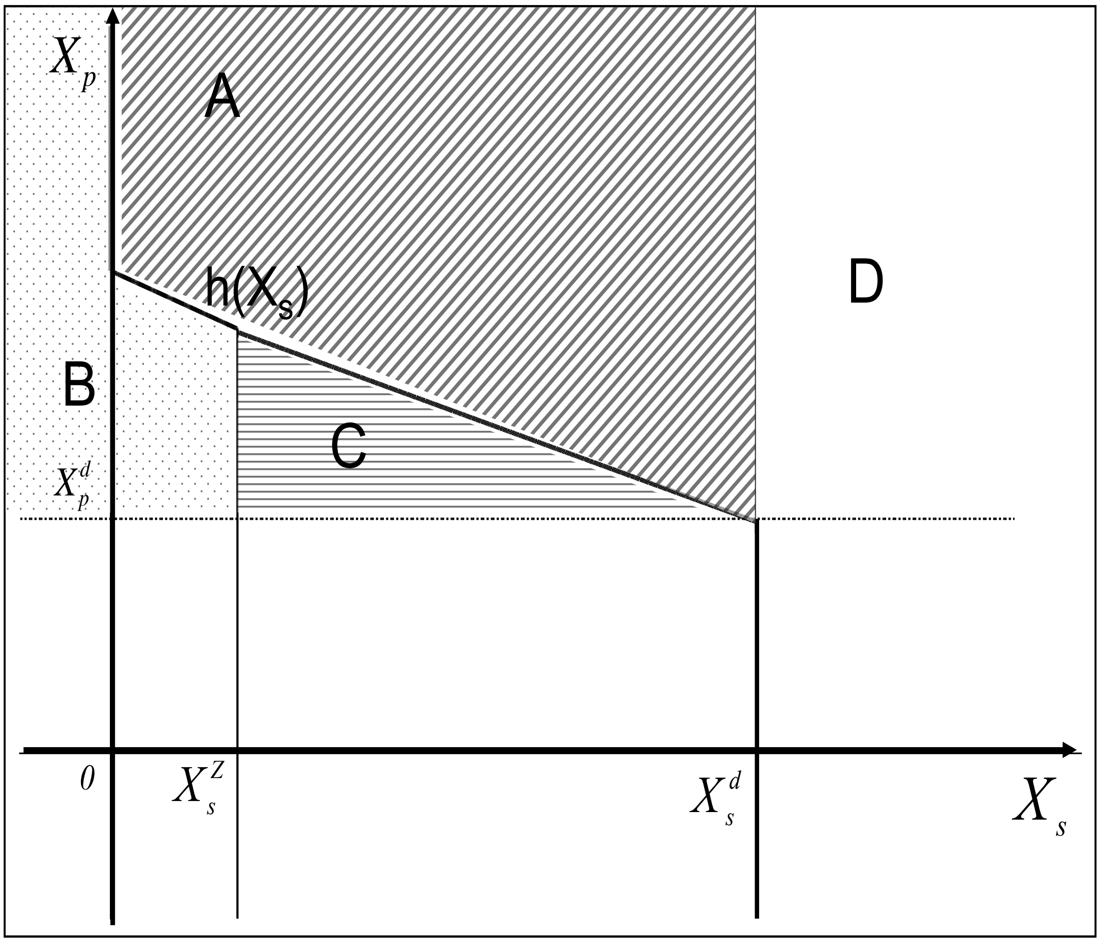


Figure 1: This figure shows the cash flows combinations ensuring the rescue of an insolvent subsidiary, as well as the areas of joint or selective defaults with infinitesimal ownership. It represents the cash flow of the subsidiary on the horizontal axis and of the parent on the vertical axis. Event A is the area which presents diagonal lines. Event B is the dotted area. Event C is the area with horizontal lines. Event D is the white area between area A and the horizontal line at level X_p^d.

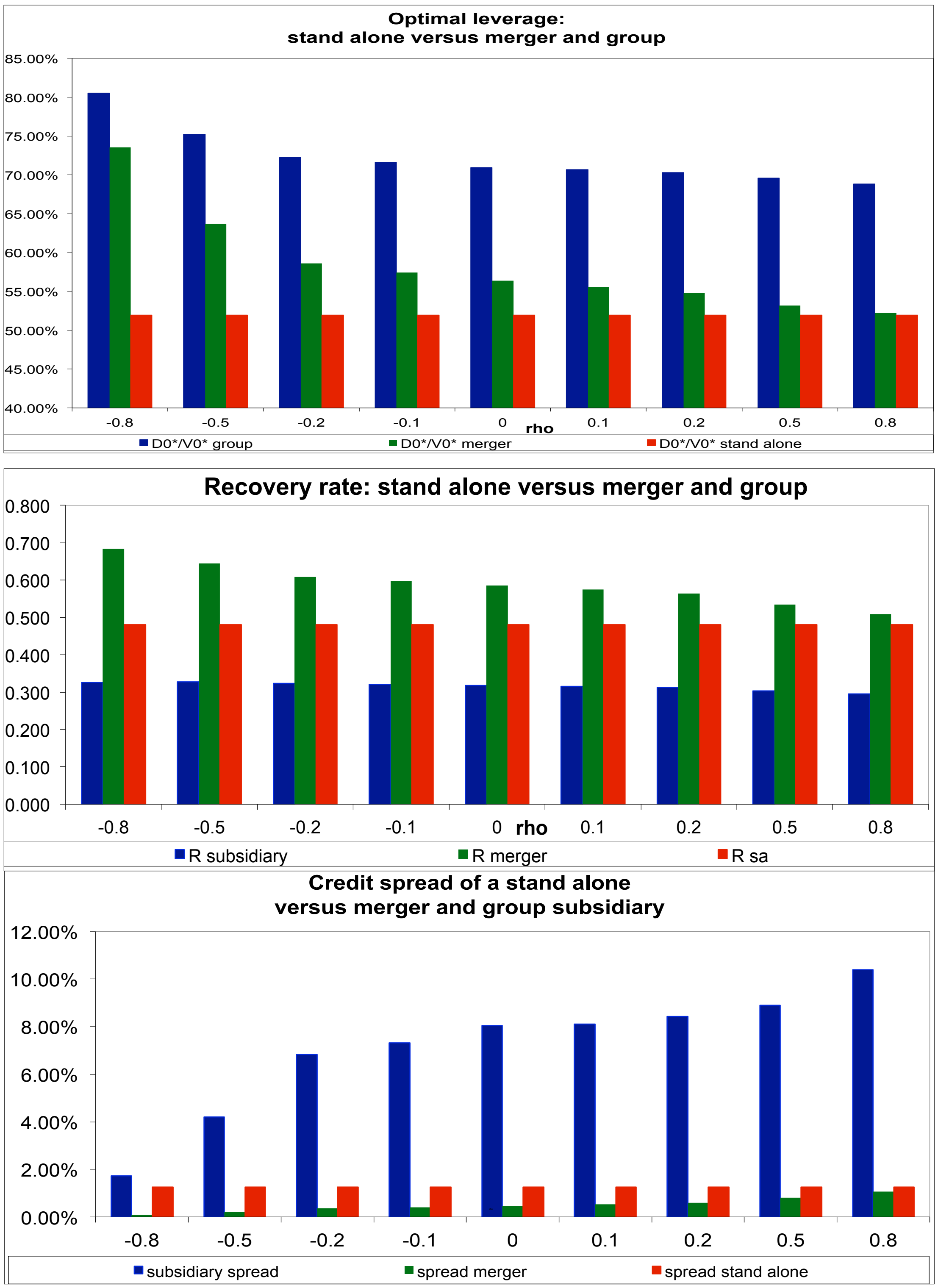
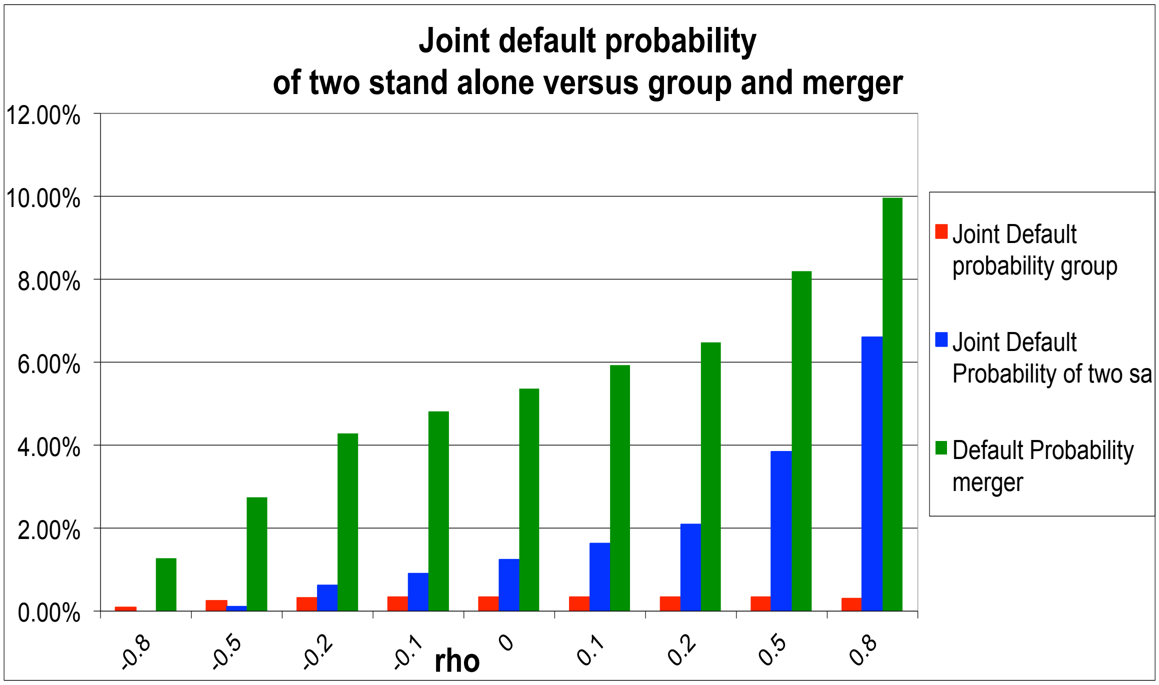


Figure 2: This figure depicts optimal leverage, recovery rate and credit spreads for the different organizational forms, as correlation between cash flows increases.



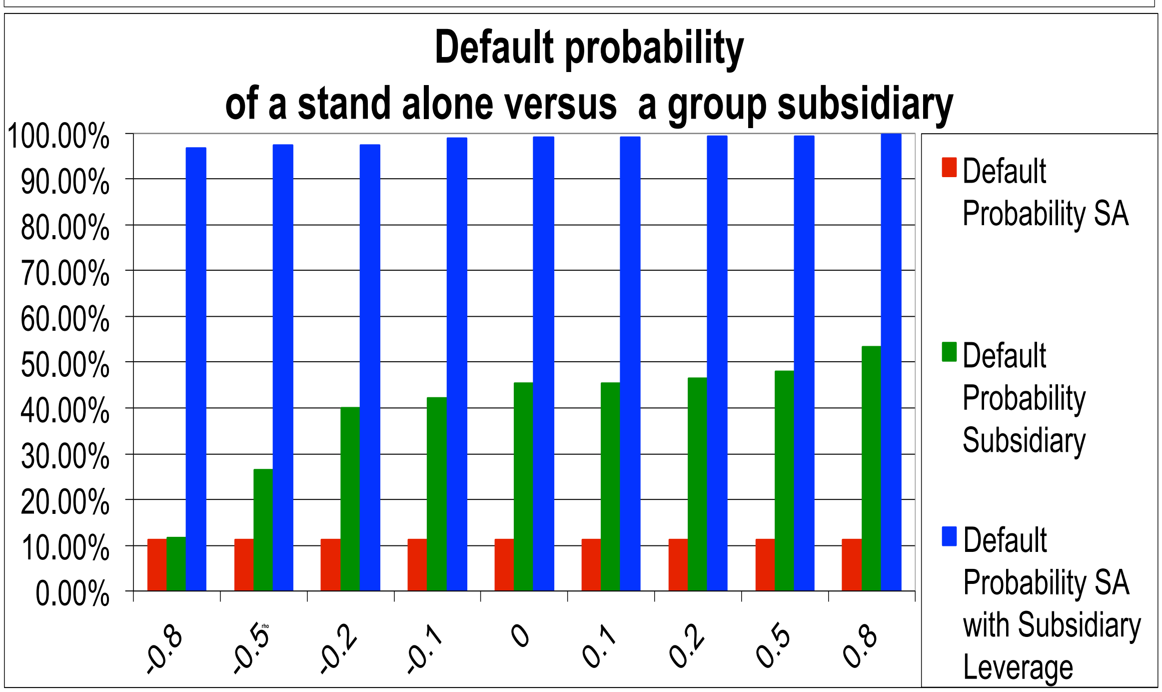


Figure 3: This figure depicts the joint default probability of the parent-subsidiary, merger and two stand alone firms (top) and the default probability of a stand alone versus a group subsidiary (bottom). The probabilities are depicted as functions of the correlation between cash flows.

Table 1: Credit risk in the base case, different organizational forms, $\rho = 0.2$

Variables	Symbols	Values				
		Stand Alone	Parent	Subsidiary	Group	Merger
Optimal Face Value of Debt	F^*	57.20	0	219	219	117. 4
Default Threshold	X^{d*}	67.75	0	248.169	NA	69.87
Tax Shield	X^{Z*}	14.98	0	102.32	NA	14.00
Value of Optimal Debt	D_0^*	42.22	0	116.68	116.68	44.70
Value of Optimal Equity	E_0^*	39.01	49.2	0.037	49.237	36.88
Optimal Leverage Ratio	D_0^*/ν_0^*	52%	0	99,9%	70.33%	55%
Expected Bankruptcy Costs	DC_0^*	0.90	0	7.98	7.98	1.24
Expected Tax Payments	T_0^*	17.62	19.95	5.42	25.37	25.62
Credit Spread	y	1.26%	NA	8.4%	NA	0.6%
Recovery Rate	R	48.1%	NA	31.2%	NA	56.5%
Marginal default Probability	π_i	11.18%	0.34%	46.54%	NA	NA
Selective Default Probability	pi_i^-	9.09%	NA	46.2%	NA	NA
Joint Default Probability	$\pi_{12}, \pi_{ps}, \pi_m$	2.08%	NA	NA	0.34%	6.5%
Joint Survival Probability with rescue	$\bar{\pi}_{ps}^r$	NA	NA	NA	52.35%	NA
Joint Survival Probability	$\bar{\pi}_{12}, \bar{\pi}_{ps}, \bar{\pi}_m$	79.73%	NA	NA	52.92%	93.5%

This table reports the credit risk assessment of different organizational forms under the base case parameters. The first column refers to one Stand Alone firm, the second and the third to one Parent company and its Subsidiary. Figures in fourth column, devoted to Groups, report the sum of the figures of Parent and Subsidiary, unless they are percentages. The last column depicts the Merger. The Group and Merger figures must be compared with two Stand Alone firms, unless they are percentages. NA stands for "not applicable".

Table 2: Credit risk with different αs, different organizational form $\alpha_s = 0.23,\ \alpha_p = 0.75, \rho = 0.2$

Variables	Symbols	Values				
		Stand Alone (α=0.75)	Parent	Subsidiary	Group	Merger
Optimal Face Value of Debt	F^*	33	0	219	219	93
Default Threshold	X^{d*}	39.247	0	248.17	NA	110.86
Tax Shield	X^{Z*}	8.01	0	102.32	NA	21.58
Value of Optimal Debt	D_0^*	24.99	0	116.68	116.68	71.42
Value of Optimal Equity	E_0^*	55.84	49.2	0.037	49.237	91.05
Optimal Leverage Ratio	D_0^*/ν_0^*	30.9 2%	0	99,9%	70.33%	44%
Credit Spread	y	0.7%	NA	8.4%	NA	0.4%
Recovery Rate	R	6.2%	NA	5.3%	NA	31.48%
Default Probability	π_i	3.62%	0.34%	46.54%	NA	NA
Selective Default Probability	π_i^-	2.84%	0	46.2%	NA	NA
Joint Default Probability	$\pi_{12}, \pi_{ps}, \pi_m$	0.77%	NA	NA	0.34%	2.9%
Joint Survival Probability with rescue	$\bar{\pi}_{ps}^r$	NA	NA	NA	52.35%	NA
Joint Survival Probability	$\bar{\pi}_{12}, \bar{\pi}_{ps}, \bar{\pi}_m$	85.98%	NA	NA	52.93%	97.1%

This table reports the credit risk assessment of different organizational forms when one firm has proportional bankruptcy costs equal to $\alpha = 0.75$, while the other is equal to the base case ($\alpha = 0.23$). The first column refers to one Stand Alone firm with $\alpha = 0.75$, the second and the third to one Parent company, with $\alpha = 0.75$, and its Subsidiary, with $\alpha = 0.23$). Figures in fourth column, devoted to Groups, report the sum of the figures of Parent and Subsidiary, unless they are percentages. The last column depicts a Merger, with one division as in the base case and the other with $\alpha = 0.75$. The Group and Merger figures must be compared with two Stand Alone firms with different alfas, unless they are percentages. NA stands for "not applicable"

Table 3: Credit risk with different volatilities, different organizational forms, $\rho = 0.2, \sigma_s = 44\%, \sigma_p = 22\%$

Variables	Symbols	Values				
		Stand Alone(σ =44%)	Parent	Subsidiary	Group	Merger
Optimal Face Value of Debt	F^*	83	0	223	223	118
Default Threshold	X^{d*}	95.19	0	248.169	NA	69.50
Tax Shield	X^{Z*}	34.25	0	102.32	NA	17.01
Value of Optimal Debt	D_0^*	48.75	0	106.83	106.83	83.97
Value of Optimal Equity	E_0^*	36.10	60.29	3.01	63.30	79.28
Optimal Leverage Ratio	D_0^*/ν_0^*	57.46%	0	97.3%	62.8%	51.4%
Credit Spread	y	6.2%	NA	10.9%	NA	2%
Recovery Rate	R	1.64%	NA	20.2%	NA	negative
Marginal Default Probability	π_i	37.09%	0.34%	48.71%	NA	NA
Selective Default Probability	π_i^-	31.44%	0.01%	48.39%	NA	NA
Joint Default Probability	$\pi_{12}, \pi_{ps}, \pi_m$	5.64%	NA	NA	0.33%	6.35%
Joint Survival Probability with rescue	$\bar{\pi}_{ps}^r$	NA	NA	NA	40.19%	NA
Joint Survival Probability	$\bar{\pi}_{12}, \bar{\pi}_{ps}, \bar{\pi}_m$	57.38%	NA	NA	50.74%	93.65%

This table reports the credit risk assessment of different organizational forms when one firm has annualized cash flow volatility equal to $\sigma = 44\%$, while the other is equal to the base case ($\sigma = 22\%$). The first column refers to one Stand Alone firm with $\sigma = 44\%$; the second and the third to one Parent company, with $\sigma = 22\%$, and its Subsidiary, with $\sigma = 44\%$. Figures in fourth column, devoted to Groups, report the sum of the figures of Parent and Subsidiary, unless they are percentages. The last column depicts a Merger, with one division as in the base case and the other with $\sigma = 44\%$. The Group and Merger figures must be compared with two Stand Alone firms with different volatilities, unless they are percentages. NA stands for "not applicable"

Table 4: Credit risk with different size, different organizational forms, $\rho = 0.2$

Variables	Symbols	Values					
		Stand Alone(1/3)	Stand Alone(5/3)	Parent	Subsidiary	Group	Merger
Optimal Face Value of Debt	F^*	19	95	63.33	121.33	184.66	115
Default Threshold	X^{d*}	22.512	112.56	75. 39	138. 90	NA	136. 62
Tax shield	X^{Z*}	4.951	24.77	15. 08	51. 07	NA	28. 53
Value of Optimal Debt	D_0^*	14.05	70.24	48.25	70.26	118.51	86.47
Value of Optimal Equity	E_0^*	13.11	65.54	47.15	0	47.15	76.51
Optimal Leverage Ratio	D_0^*/ν_0^*	51.73%	51.73%	50.58%	100%	71.54%	53.06%
Credit spread	y	1.2%	1.2%	0.6%	6.5%	NA	0.9%
Recovery Rate	R	85.2%	85.2%	38.4%	20.96%	NA	32.1%
Marginal Default Probability	π_i	11.09%	11.10%	4.54%	33.04%	NA	NA
Selective Default Probability	π_i^-	9.04%	9.03%	0%	28.49%	NA	NA
Joint Default Probability	$\pi_{12}, \pi_{ps}, \pi_m$	2.05%	2.05%	NA	NA	4.55%	5.97%
Joint Survival Probability with rescue	$\bar{\pi}_{ps}^r$	NA	NA	NA	NA	66.56%	NA
Joint Survival Probability	$\bar{\pi}_{12}, \bar{\pi}_{ps}, \bar{\pi}_m$	79.87%	79.87%	NA	NA	66.56%	94%

This table reports the credit risk assessment of different organizational forms when one firm is five times as large as the other ($E(X_i) = 5E(X_j)$). The first and second columns refer to the smaller and the larger Stand Alone firm respectively; the third and the fourth to the larger Parent and its Subsidiary. Figures in fifth column, devoted to Groups, report the sum of the figures of Parent and Subsidiary, unless they are percentages. The last column depicts a Merger, with one smaller and one larger division. The Group and Merger figures must be compared with the sum of the two Stand Alone firms, unless they are percentages. NA stands for "not applicable"