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BROTHERS IN ALMS? COORDINATION BETWEEN NONPROFITS ON MARKETS FOR DONATIONS

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ABSTRACT. Mission-driven nonprofit organizations compete for donations through fundraising activities. Such competition can lead to inefficient outcomes, if nonprofits impose externalities on each others' output. This paper studies the design of sustainable voluntary cooperation agreements, using a game-theoretic model of alliance formation. Two key characteristics determine the stability of cooperation: the alliance formation rule and whether the fundraising efforts of nonprofits are strategic complements or substitutes. Both affect the incentives to deviate from the cooperative agreement (by one or several nonprofits). We propose conditions on the alliance formation protocols that facilitate the stability of Pareto-optimal cooperation in fundraising.

Keywords: nonprofits, giving, coordination, endogenous coalition formation, non-distribution constraint.

JEL codes: L31, D74, L44, C72.

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> ”On 21 December 1984, unable to resist the allure of Ethiopian famine pictures, World Vision ran an Australia-wide Christmas Special television show calling on the public in that country to give it funds. In so doing it broke an explicit understanding with the Australian Council of Churches that it would not run such television spectaculars in competition with the ACC’s traditional Christmas Bowl appeal. Such ruthless treatment of ’rivals’ pays, however: the American charity is, today, the largest voluntary agency in Australia ...” (Hancock 1989: 17)

1. Introduction

Private provision of public goods in modern economies is essentially organized by nonprofit organizations. These entities constitute a sector which employs, on average, 5.6 per cent, and in some countries - Netherlands, Belgium, Canada, U.K., Israel and Ireland - over 10 per cent of the economically active population (Salamon 2010).

To a large extent, the revenues of nonprofits come from charitable donations. Given that nonprofits have to compete for donations through fundraising activities, these organizations can be considered as rational players on the philanthropic ’market’ (Andreoni 2006). Following this avenue, recent studies (Bilodeau and Slivinski 1997, Andreoni and Payne 2003, Castaneda et al. 2008, Aldashev and Verdier 2009) have modelled the nonprofit sector outcomes as equilibria of the decentralized interaction of many nonprofit organizations, each of which maximize a certain (usually impurely altruistic or "warm glow") objective function.

It is well known that one fundamental downside of this decentralized organization of the nonprofit sector is that competing for donations - when the aggregate amount of donations is relatively inelastic to fundraising - is socially wasteful (see, for instance, Rose-Ackerman 1982 and Aldashev and Verdier 2010). Occasionally, nonprofit organizations are able to overcome this problem, by designing voluntary cooperative agreements. The examples of successful coordination are the umbrella organizations that conduct joint fundraising appeals: the American United Way (Brilliant 1990), Disaster Relief Agency created by Dutch NGOs in 1993, Disasters Emergency Committee (DEC) in Britain (Smillie 1995: 116), and Belgian National Center for Development Cooperation (Similon 2009).

However, in general, constructing sustainable cooperation agreements between competing nonprofits is difficult. De Waal (1997) writes in his description of the development nonprofit sector: "[An organization that is] most determined to get the highest media profile obtains the most funds ... In doing so it prioritizes the requirements of fundraising: it follows the TV cameras, ... engages in picturesque and emotive programmes (food and medicine, best of all for children), it abandons scruples about when to go in and when to leave, and it forsakes cooperation with its peers for advertising its brand name." The opening quotation of this paper also presents an example of failed coordination between emergency relief nonprofits in Australia.

This problem naturally calls for a policy intervention. However, given that nonprofit organizations are also non-governmental, the "top-down" government intervention is unlikely to be effective, because it would be perceived as undermining the very essence of these organizations. For instance, Edwards and Hulme (1996) argue that the stronger are the links of nonprofits with the government agencies, the less effective the nonprofits are in pursuing independently their missions. There might be some scope for indirect public policies, using the basic set of tools that governments usually employ towards the nonprofit sector (subsidizing or taxing the fixed costs of setting up a nonprofit, matching grants that are given to nonprofits in proportion of private donations that they collect, tax deductibility of charitable donations). Finally, most countries have national associations of nonprofit organizations, and these platforms themselves can introduce some internal rules and codes of conduct for fundraising campaigns.

From an economist's point of view, the key question for policy design is then: what can be done to facilitate the voluntary cooperation between nonprofits and to make it more sustainable? In this paper, we provide the first analysis of this question. To do so, we build a model of endogenous nonprofit alliance formation, by exploiting game-theoretic tools used in the recent literature on endogenous coalition formation (Bloch 2003, 2009, Yi 2003, Ray 2007). In our two-stage model, at Stage 2 of the game, nonprofits engage in individual fundraising activities (with the opportunity cost being working on the project that contributes to their missions). Fundraising activity of one nonprofit can affect the donations collected by another nonprofit (either positively or negatively). Thus, nonprofits impose externalities on the each other's output. At Stage 1, nonprofits can form alliances, i.e. credibly commit to levels of fundraising that internalize the externalities among the alliance members.

The alliance formation takes place via the following process: each nonprofit announces an alliance to which it would like to belong; then, an alliance is formed on the basis of the profile of these announcements and according to a certain alliance-formation rule. We then study how outcomes differ under two alliance-formation rules: *unanimity* rule requiring that all players of the alliance unanimously agree to form *that specific* alliance, and a milder *aggregative* rule which only requires, for an alliance to form, that all its members have announced the same alliance (not necessarily the one that forms). Given these rules, we investigate under which conditions the Pareto-efficient full-coordination agreement (the grandcoalition of nonprofits) and other alliance structures are *stable,* according to standard individual or coalitional equilibrium concepts (i.e. that no single nonprofit or a group of nonprofits have better alternatives in a different alliance structure).

We find that two key characteristics that determine the stability of the voluntary coordination are the alliance formation rule and whether the fundraising efforts of nonprofits are strategic complements or substitutes. Both of these features affect the extent to which a deviation from the cooperative agreement (by one nonprofit or a group of nonprofits) leads to the fundraising effort response by non-deviating nonprofits, which, in turn, determines the relative benefits of deviation. If fundraising activities are strategic complements, full coordination is in general always individually stable. If the unanimity rule for alliance formation is imposed, full coordination is also stable to deviations by groups of nonprofits. Other alliance structures also can be stable, but they have to be *asymmetric,* i.e. formed by alliances of unequal sizes. In particular, one alliance competing against individual nonprofits can be usually sustained as a Nash equilibrium alliance structure.

When fundraising activities are strategic substitutes, full coordination is more likely to be unstable when the nonprofits' (negatively sloped) best-reply functions are steeper: breaking the coordination agreement pays off when rival nonprofits greatly reduce their fundraising efforts in response. If the less demanding aggregative rule governs the formation of coalitions, then the Pareto-efficient cooperation is more difficult to achieve. Such cooperation may be destabilized by both individual and coordinated deviations. However, when the technology of fundraising implies that there exist coalitional synergies, such that forming alliances hurts in some way the remaining nonprofits, then stability can be achieved. When this is not the case, imposing the condition that breaking an alliance requires the majority of nonprofits involved (the so-called *majority breaking protocol*) makes the efficient cooperative agreement resistant not only to individual deviations, but also to deviations by groups of nonprofits.

To the best of our knowledge, there are no papers that theoretically analyze the cooperative agreements among nonprofits.[1] The existing literature (see, in particular, Rose-Ackerman 1982, Bilodeau and Slivinski 1997, Castaneda *et al.* 2008, Aldashev and Verdier 2009, Aldashev and Verdier 2010) has mostly concentrated on fundraising competition between nonprofits. Our paper contributes to this literature by studying what features the cooperative agreements should have to allow the nonprofits to successfully coordinate the fundraising activities, so as to reduce the negative consequences of the harmful competition for donations.

The rest of the paper is organized as follows. Section 2 presents the setup for both noncooperative and cooperative behavior of nonprofits; it also introduces the process of alliance formation and the necessary game-theoretic concepts of stability of a structure of alliances. Section 3 presents the main intuitions with a numerical example. Section 4 derives the main general results, in particular, the sufficiency conditions for the stability of the Pareto-efficient coordination. Finally, Section 5 discusses how policy tools can be used to guarantee these conditions and addresses the broader implications of our analysis.

2. Setup of the model

2.1. **Donors.** Consider an economy with a continuum of donors whose total size is L. Donors consume a numeraire good and obtain warm-glow benefit from giving to charitable causes served by a finite set $N = \{1, ..., n\}$ of nonprofit organizations, each having one charitable "project". The preferences of each donor are described by a simple linear-quadratic function, similar to the one used in recent industrial-organization models of monopolistic competition (see, for example, Melitz and Ottaviano (2008)):

$$U(C, \mathbf{d}) = C + b \sum_{i=1}^{n} \omega_i d_i - \frac{\gamma}{2} \sum_{i=1}^{n} d_i^2 - \frac{1}{2} \left(\sum_{i=1}^{n} d_i \right)^2 .$$

Here, C is the consumption of a numeraire good, $\mathbf{d} = (d_i)_{i=1,..,n}$ is a vector of donations to nonprofit projects $i = 1, .., n$, ω_i is the weight attached to giving to project i by each donor and b and γ are positive parameters. The budget constraint of each donor with income I is then:

$$C + \sum_{i=1}^{n} d_i = I.$$

Using the budget constraint, the individual optimum choice for each donor satisfies the first-order condition

$$b\omega_i - 1 - \gamma d_i - \sum_{i=1}^{n} d_i = 0 \qquad \text{for all } i = 1, ..., n.$$

Summing the first-order conditions over i, we obtain

$$b \sum_{i=1}^{n} \omega_i - n = (\gamma + n) \sum_{i=1}^{n} d_i,$$

[1]Gugerty (2008) and the papers in Gugerty and Prakash (2010) study, on the basis of case studies, the performance of several forms of nonprofit self-regulation, mainly aimed at increasing accountability towards donors.

and thus,

$$\gamma d_i = b\omega_i - 1 - \frac{\left(b\sum_{i=1}^{n}\omega_i - n\right)}{\gamma + n}.$$

Then, the donation of each individual donor to nonprofit i can be written as

$$d_i = \frac{b\omega_i\left(\gamma + n - 1\right) - \gamma - b\sum_{j\neq i}\omega_j}{\gamma\left(\gamma + n\right)}. \tag{2.1}$$

The weights ω_i (and thus the amount given to each nonprofit project) are endogenous and are influenced by fundraising efforts of nonprofits, as explained below.

2.2. **Nonprofit organizations.** Nonprofit organizations are founded by a "social entrepreneur". In terms of her motivation, a social entrepreneur is impurely altruistic (à la Andreoni 1989): she receives a "warm-glow" utility which increases (linearly, for simplicity) in the output of her nonprofit. In other words, she likes to see the objectives of the sector advanced, but only if this advancement goes through *her* organization. This implies that the objective function of a nonprofit organization is maximizing the output of its project.

The production technology of the project of nonprofit i has two inputs:[2] funds F_i (rasied through fundrasing, as explained below) and time τ_i:

$$Q_i(F_i, \tau_i) = F_i \cdot \tau_i. \tag{2.2}$$

Each social entrepreneur has an endowment of 1 unit of time. She can use this time either to work on the project or to collect funds:

$$1 = \tau_i + y_i, \tag{2.3}$$

where y_i denotes the amount of time devoted to fundraising. Therefore, time is fungible and the social entrepreneur faces a well-defined trade-off: more time spent on fundraising increases the funds that can be devoted to the project; however, this comes at the cost of reducing the *time* devoted to the project.

Time devoted to fundraising brings in funds: we assume that the weights ω_i in donors' utility function depend on the fundraising effort $y_i \geq 0$ of each nonprofit i

$$\omega_i = \omega + y_i + \Delta\left(\sum_{j\neq i} y_j\right).$$

In other words, we assume that fundraising activities by a specific nonprofit i increase donors' perceived importance of that nonprofit's project, and thus the utility from giving to this project. Moreover, we assume that there may be some positive "awareness" spillovers of fundraising by other nonprofits $j \neq i$ on the willingness of each donor to give more funds to project i. For instance, being contacted by Greenpeace, a nonprofit working towards environmental issues, might raise the awareness of a donor about those issues. However, she might dislike the methods used by Greenpeace (and thus abstain from giving to this organization). Nevertheless, she now cares more about environment, and if contacted by another non-profit in the same sector (e.g. World Wildlife Fund), she is much more willing to give to WWF, as compared to a scenario in which she was not contacted by Greenpeace in the first place. Thus, the fundraising activities of Greenpeace have created positive spillovers

[2] We assume here, for simplicity, a Cobb-Douglas type of output function. We show below that most of the results extend to more general output function for nonprofits.

for WWF. Analytically, this is captured by the term $\Delta\left(\sum_{j\neq i} y_j\right)$ with $\Delta > 0$. Spillovers can also be negative and, in this case, $\Delta < 0$.

Given that nonprofits cannot, by law, distribute profits (Hansmann 1980, Weisbrod 1988), the social entrepreneur puts all the funds that she collects (net of the financial costs) into the project. Denoting, by $D_i = d_i L$, the donations collected by nonprofit i, the non-distribution constraint can be formally expressed by

$$D_i = f + cD_i + F_i,$$

where $0 \leq c < 1$ is the financial cost of collecting a unitary donation, $f > 0$ is the fixed cost of the nonprofit project, and F_i is the amount invested into the project. The non-distribution constraint pins down the amount of funds that a nonprofit invests into its project:

$$F_i = (1-c)D_i - f. \tag{2.4}$$

Using (2.1), (2.2) and (2.4)), the objective function of any nonprofit i can thus be expressed as a function of its fundraising effort and of the effort levels of other nonprofits as:

$$Q_i\left(y_i, y_{-i}\right) = (1-c)\left(\Omega + \delta y_i - \beta \sum_{j\neq i} y_j\right)(1-y_i), \tag{2.5}$$

where

$$\Omega = \frac{L\left(b\omega - 1\right)}{\gamma + n} - \frac{f}{1-c} > 0,\ \delta = \frac{Lb\left(\gamma + n - 1 + \gamma\Delta\right)}{\gamma\left(\gamma + n\right)} > 0,\ \beta = \frac{Lb(1-\gamma\Delta)}{\gamma\left(\gamma+n\right)} \lesseqgtr 0.$$

The key parameter here is β, that captures the effect on donations collected by nonprofit i of fundraising by other nonprofits. Mathematically, β is positive if $\gamma\Delta < 1$ (and is negative otherwise). Note that γ and Δ capture conceptually different characteristics: γ reflects the intrinsic preferences of donors, while Δ describes how the fundraising activities of nonprofits impact on donors' preferences.

In terms of empirically observable measures, Δ and γ can be linked to three broad dimensions. The first is the nature of the cause towards which nonprofits operate. If the issue is relatively new and unknown to donors (e.g. in case of a humanitarian emergency), it is likely that the awareness-raising spillovers (Δ) are large. Otherwise (if the issue is such that most of the potential donors already have an extensive background information about the issue), the awareness-raising spillovers are likely to be absent.

The second is the dominant technology of soliciting donations. Suppose that the fundraising technology allows for precise targeting of potential donors on the basis of certain characteristics or behavior (as in case of online solicitations, when targeting is made through professional marketing firms on the basis of consumption patterns of potential donors). In this case, the spillovers are relatively small. Contrarily, if the technology does not allow for targeting, as for instance in case of direct mailing, the spillovers are likely to be high.

The third is the degree of perceived differentiation of nonprofits' projects by donors, i.e. the extent of loyalty of donors to nonprofit 'brands'. For instance, when donors and nonprofits have been first matched to each other relatively recently, the loyalty of donors to the nonprofit 'brands' is relatively weak. This implies that donors' mobility from one nonprofit to another is relatively high. On the contrary, in a market where a donor has been giving to the same organization for years (e.g. a local religious charity), the 'brand loyalty' of donors is strong, and donors consider different nonprofits as poor substitutes.

Summing up, β is likely to be positive (i.e. more fundraising by others reduces the donations to i) in donation markets with weak donor loyalty, fundraising technologies allowing for good targeting and projects centered around issues on which donors already have good information. On the other hand, β is likely to be negative in markets with strong donor loyalty, where the fundraising technology is untargeted, and the issues in nonprofit mission require an active information-providing role for nonprofits.

Recent empirical work measures the extent of spillovers and complementarities in nonprofit fundraising. Van Diepen et al. (2009) analyze data on direct mailing solicitations for charitable giving in the Nethelands and find that solicitations by different charities are short-run complements (i.e. awareness spillovers seem to be present in the short run) and long-run substitutes (i.e. spillovers die out in the long run). This is consistent with our interpretation above: in the short run, donors do not have enough information about the issue raised by a nonprofit asking for a donation, thus the awareness-raising effect of such solicitation probably benefits other nonprofits; in the long run, donors accumulate enough information about the issue, and thus the total size of the donation market becomes constant. Two subsequent papers (Lange and Stocking 2012 and Reinstein 2012) that analyze the same question using experimental data find opposed results. In a field experiment, Lange and Stocking (2012) find that providing some donors of one charity with the opportunity to be included in the stream of solicitations of another charity does not decrease their donation to the original charity (if anything, such donations actually increase). In a lab experiment, Reinstein (2012) finds, instead, that providing more promotional information about some charity or reducing the price of giving to that charity leads to an increase of donations to this charity and to a simliar-size decrease of donations to the charities with similar goals. The framework that we develop in this paper allows to encompass both the cases of complements and substitutes.

2.3. **Nonprofit alliance formation.** To model the strategic behavior of nonprofit in terms of creating and breaking cooperative agreements, we need to introduce two further elements into our analysis: a process of alliance formation and a notion of stability of a given nonprofit alliance structure.

We adapt a very simple approach to the alliance formation process: a simultaneous game in which every nonprofit i announces a non empty alliance $A \subset N$ to which it would like to belong. For every profile of announcements $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ declared by the n nonprofits, an alliance structure (i.e. a partition) $\mathcal{S} = (A_1, A_2, .., A_m)$, with $m \leq n$, is induced.

Clearly, the rule according to which an alliance structure originates from a profile of announcements is the key issue for predicting which nonprofit alliances will emerge in equilibrium. One possibility is to assume that a particular alliance emerges if and only if all its (future) members announce exactly this particular alliance. We call this rule the *unanimity rule*. Formally, the partion

$$\mathcal{S}^U(\alpha) = \{A_1(\alpha), A_2(\alpha), ..., A_m(\alpha)\},$$

is such that every nonprofit i belongs to A_k ($k = 1, 2, .., m$) if and only if $\alpha_i = \{A_k\}$ for all $i \in A_k$ and stays as singleton otherwise.

Another possibility is to assume that a nonprofit alliance emerges if and only if all its (future) members announce the same alliance. The difference with respect to the unanimity rule is that this announced alliance may, in general, differ from the alliance that will form.

We call this rule the *aggregative rule*.[3] Formally:

$$\mathcal{S}^A(\alpha) = \{A_1(\alpha), A_2(\alpha), ..., A_m(\alpha)\},$$

such that every nonprofit i belongs to A_k $(k = 1, 2, .., m)$ if and only if $\alpha_i = \alpha_j$ for all $i, j \in A_k$ and stays as singleton otherwise.

The two rules generate different partitions after a deviation from a given alliance structure by an individual organization or an alliance of nonprofits. Under the unanimity rule, a deviation induces the remaining organizations in the alliance to split up into singletons. Contrarily, under the aggregative rule the remaining nonprofits continue to stick together. All nonprofits understand this; therefore, the strategic incentives to announce a given alliance might differ under the two rules.

A simple 3-player example illustrates the difference between the two rules clearly. Suppose there are three nonprofit organizations that compete for donations: $i = 1, 2, 3$. Let nonprofits 1 and 2 announce the grand coalition: $\alpha_1 = \alpha_2 = \{1, 2, 3\}$, whereas nonprofit 3 announces $\alpha_3 = \{3\}$. Under the unanimity rule, this profile of announcements will result in all nonprofits playing as singletons: $\mathcal{S}^U(\alpha) = \{(1), (2), (3)\}$. Instead, under the aggregative rule, the same profile of announcements results in the structure $\mathcal{S}^A(\alpha) = \{(1, 2), (3)\}$.

Of course, in reality, the alliance formation process among nonprofits is much more complex: it involves meetings, proposals and counter-proposals, back-and-forth negotiation, and so on. We can think of the rules described above as a simplified representation of this complicated process, which captures some of its basic features. Which alliance formation rule represents better the reality depends on the institutional environment in which nonprofits operate. Most coordination agreements - e.g. umbrella associations - operate in the fashion which is best described by the aggregative rule: if one nonprofit decides to exit the agreement, usually the remaining nonprofits continue to operate under the umbrella. In some cases, however, the fact that some organization exits the alliance might lead to the break-up of the remaining alliance: this occurs if the coordination is such that nonprofits put their productive assets into a common activity in a (highly) complementary fashion, e.g. NGOs during a humanitarian crisis. In our model, this would correspond to a more restrictive unanimity rule.

Next, we can define the notion of stability of a given alliance structure. An alliance structure is stable when it is induced by an announcement profile that is a *Nash equilibrium*, or, alternatively, a *coalitional equilibrium* (i.e. a Nash equilibrium profile of announcements robust to deviations by alliances)[4] of a given game of alliance formation. Let

$$Q_i(\alpha) \equiv Q_{A_k}(\overline{y}(\mathcal{S}(\alpha)))/a_k$$

(where a_k expresses the size of A_k) denote the payoff of a nonprofit belonging to alliance A_k at the fundraising equilibrium $\overline{y}$ when the alliance structure $\mathcal{S}(\alpha) = \{A_1, A_2, ., A_k, .., A_m\}$ has been induced by the announcement profile $\alpha = (\alpha_1, \alpha_2, .., \alpha_n)$. Using this shortcut in notation, we are able to define two distinct concepts of stability of alliance structures.

Definition 1. *(Nash stability) The alliance structure $\mathcal{S} = \{A_1, A_2, ..., A_m\}$ is Nash-stable if $\mathcal{S} = \mathcal{S}(\overline{\alpha})$ for some $\overline{\alpha}$, such that there exist no organization $i \in N$ with an alternative*

[3]The unanimity rule was first introduced by von Neumann and Morgenstern (1944). It is also sometimes called the *gamma-rule*, whereas the aggregative rule is also known as the *delta-rule* of coalition formation (Hart and Kurz 1983).

[4]In game-theoretic jargon, the terms "strong Nash equilibrum" and "strong stability" are used.

announcement $\alpha_i' \in \mathcal{A}$ *such that*

$$Q_i(\alpha_i', \overline{\alpha}_{N\backslash\{i\}}) > Q_i(\overline{\alpha}).$$

Definition 2. *(Coalitional stability) The alliance structure* $\mathcal{S} = \{A_1, A_2, ..., A_m\}$ *is coalitional stable if* $\mathcal{S} = \mathcal{S}(\widetilde{\alpha})$ *for some* $\widetilde{\alpha}$*, such that there exists no alliance* A *with an alternative joint announcement* α_A' *such that*

$$Q_i(\alpha_A', \widetilde{\alpha}_{N\backslash A}) \geq Q_i(\widetilde{\alpha}) \text{ for all } i \in A$$

and

$$Q_h(\alpha_A', \widetilde{\alpha}_{N\backslash A}) > Q_h(\widetilde{a}) \text{ for at least one } h \in A.$$

It is clear that $\overline{\alpha} = (\overline{\alpha}_1, \overline{\alpha}_2, .., \overline{\alpha}_n)$ corresponds to a Nash equilibrium of the announcement game and $\widetilde{\alpha} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, .., \widetilde{\alpha}_n)$ to a coalitional equilibrium to the same game. Note also that the coalitional stability is a highly demanding stability concept. By requiring the Nash stability against every alternative profile of announcements (including the one formulated by the grand coalition), it imposes the Pareto optimality on the resulting allocation.

2.4. **Non-cooperative interaction between nonprofits.** Consider first the situation in which every nonprofit acts individually and non-cooperatively, with the objective of maximizing its output (by choosing the amount of time it devotes to fundraising, y_i), taking as given other nonprofits' fundraising efforts, y_{-i}. In other words, for every $i = 1, ..., n$, the problem is

$$\max_{y_i} Q_i\left(y_i, y_{-i}\right) = \max_{y_i} \; (1-c)\left(\Omega + \delta y_i - \beta \sum_{j \neq i} y_j\right)(1-y_i). \tag{2.6}$$

First-order conditions for an interior equilibrium of the game played among nonprofits, denoted $\overline{y} = (\overline{y}_1, \overline{y}_2, .., \overline{y}_n)$, imply that for every $i = 1, ..., n$:

$$\frac{\partial Q_i\left(y_i, y_{-i}\right)}{\partial y_i} = \underbrace{\delta(1-y_i)(1-c)}_{\text{marginal fundraising benefit}} - \underbrace{(1-c)(\Omega + \delta y_i - \beta y_{-i})}_{\text{marginal fundraising cost}}$$

Intuitively, at the equilibrium each nonprofit equates the marginal benefit of additional fundraising (in terms of project output) to its marginal (opportunity) cost.[5] Using symmetry, it is easy to obtain the following Nash equilibrium fundraising levels:[6]

$$\overline{y}_i = \frac{\delta - \Omega}{2\delta - \beta\left(n-1\right)}, \tag{2.7}$$

and Nash equilibrium payoffs of nonprofits as

$$Q_i\left(\overline{y}\right) = \frac{(1-c)\,\delta\left(\delta + \Omega - \beta\left(n-1\right)\right)^2}{\left(2\delta - \beta\left(n-1\right)\right)^2}.$$

Note also that by (2.6) $Q_i\left(y_i, y_{-i}\right)$ is strictly concave in y_i, given that $c < 1$ and $\delta > 0$:

$$\frac{\partial^2 Q_i\left(y_i, y_{-i}\right)}{\partial y_i^2} = -2\delta(1-c) < 0.$$

How the output of nonprofit i is affected by fundraising activities of some other nonprofit j crucially depend on the sign of β:

[5]See the Appendix for the more general case.

[6]We assume here that $\delta > \Omega$ and $\beta < 2\delta/(n-1)$.

$$\frac{\partial Q_i\left(y_i, y_{-i}\right)}{\partial y_j} = -\beta\left(1-c\right)\left(1-y_i\right) \lesseqgtr 0 \text{ for } \beta \gtreqless 0, \text{ for all } j \neq i.$$

Moreover,

$$\frac{\partial^2 Q_i\left(y_i, y_{-i}\right)}{\partial y_i \partial y_j} = \beta(1-c) \gtreqless 0 \text{ for } \beta \gtreqless 0.$$

Thus, when $\beta > 0$ (more fundraising by others reduces the donations to i), nonprofits impose negative externalities on each other's output, and fundraising efforts are strategic complements (in other words, any increase in rivals' fundraising raises the incentives for a nonprofit to devote more time to fundraising). Contrarily, when $\beta < 0$ (more fundraising by others increases donations given to i), nonprofits impose positive externalities on each other's output, whereas fundraising efforts are strategic substitutes: higher effort by rival nonprofits increase the donations given to i, thus rising its marginal opportunity cost of diverting one unit of time from the project to the fundraising activity.

2.5. **Interaction between nonprofit alliances.** Next, suppose that nonprofits can organize their common actions in alliances $A \subset N$. The *grand coalition* N is the largest possible alliance and corresponds to the full coordination among nonprofits in the sector. Alternatively, nonprofits can coordinate their actions to some intermediate levels. These are expressed by alliance structures (partitions) denoted with $\mathcal{S} = \{A_1, A_2, ..., A_m\}$, i.e. representing any collection of nonprofits in alliances $A_k \subset N$ having null intersection and summing up to N.

To obtain a well-defined interaction for nonprofits forming alliances A_k in all feasible coalition structures $\mathcal{S} = \{A_1, A_2, ..., A_m\}$, we assume that members inside A_k fully coordinate their individual actions, and the alliance payoff is simply expressed as $Q_{A_k} = \sum_{i \in A_k} Q_i(y_i, y_{-i})$, i.e. the sum of every nonprofit's project output. Note that in our setting the main benefit for nonprofits to create alliances is to coordinate their fundraising effort levels, thus internalizing the externalities (negative or positive) that the nonprofits can impose on each other's output. In this case, the outcome obtained by the grandcoalition is always Pareto-efficient (from the point of view of nonprofits' objective functions).

Moreover, we assume for simplicity that within every alliance there is an equal-sharing allocation rule.[7] In addition, every nonprofit alliance behave *à la* Nash against rival alliances of nonprofits and therefore act to maximize the sum of the joint output of all its members, taking as given the actions of nonprofits that do not belong to this alliance. Formally, for every $A_k \in \mathcal{S}$, the objective function is

$$\max_{(y_i)_{i \in A_k}} \sum_{i \in A_k} (1-c)\,(\Omega + \delta y_i - \beta \sum_{j \neq i} y_j)(1-y_i).$$

The first-order condition of this problem implies, for every member of the alliance, $i \in A_k$ in a generic partition of m alliances $\mathcal{S} = (A_1, ., A_k, .., A_m)$,

$$\underbrace{\delta(1-y_i)(1-c) + \textstyle\sum_{h \in A_{k \setminus i}} \beta(1-y_h)(1-c)}_{\text{social marginal fundraising benefit}} - \underbrace{(1-c)(\Omega + \delta y_i - \beta y_{-i})}_{\text{marginal fundraising cost}} = 0. \tag{2.8}$$

[7]Note that when the participants to an alliance are symmetric (as is the case of our model), the equal-sharing allocation rule can also be obtained endogenously by assigning to every participant, as in Hart and Kurz (1983), the Owen value (Owen 1977) for transferrable-utility games (i.e. the Shapley value with *a priori* coalition structures).

This expression indicates that every nonprofit participating in alliance A_k sets its fundraising level to equate the marginal cost of fundraising to the marginal social (coalitional) benefit. Comparing the conditions (2.4) and (2.8), we see that when nonprofits' fundraising activities impose positive (negative) externalities on each other's output, the level of fundraising chosen by an alliance member is higher (lower) than that of a nonprofit playing non-cooperatively. Two extreme cases are particularly important. The first is the grand coalition alliance (with size $|A_k| = a_k = n$). In this case the equilibrium fundraising effort for each i is

$$\widetilde{y}_i = \frac{1}{2}\frac{\delta - \Omega - \beta\,(n-1)}{\delta - \beta\,(n-1)}.$$

We can compare it to the Nash equilibrium with all singletons (2.7). It is easy to see that equilibrium fundraising effort under grand coalition is higher than that in the non-cooperative all-singleton Nash equilibrium (i.e. $\widetilde{y}_i > \overline{y}_i$) when nonprofits impose positive externalities on each others' output, i.e. when $\beta < 0$. Analogously, $\widetilde{y}_i > \overline{y}_i$ when $\beta > 0$.[8] The payoff of a nonprofit inside the grand coalition is

$$Q_i(\widetilde{y}) = \frac{1}{4}\frac{\left(\Omega + \delta - \beta\,(n-1)\right)^2(1-c)}{\delta - \beta\,(n-1)}.$$

The second case is when an alliance A_k compete against all other nonprofits $j \in N\backslash A_k$. In this case the payoffs of each nonprofit i inside alliance A_k and that of a nonprofit outside the alliance (playing as singleton) are given, respectively, by

$$\begin{aligned}
Q_i(\overline{y}) &= \frac{\left(\delta - \beta\,(a_k-1)\right)\left(\beta\,(n-1) - \delta - \Omega\right)^2(\beta + 2\delta)^2(1-c)}{\left(2\beta\delta(3-n-a_k) + 4\delta^2 + \beta^2(2-2n+a_k\,(n-a_k))\right)^2},\\
Q_j(\overline{y}) &= \frac{\delta\left(\beta\,(n-1) - \delta - \Omega\right)^2\left(2\delta - \beta\,(a_k-2)\right)^2(1-c)}{\left(2\beta\delta(3-n-a_k) + 4\delta^2 + \beta^2(2-2n+a_k\,(n-a_k))\right)^2}.
\end{aligned}$$

Using the expressions above it is easy to check that $Q_i(\overline{y}) < Q_j(\overline{y})$: due to the fundraising externalities, being in the fringe of nonprofits guarantees an advantage if compared to being a member of an alliance that coordinates the fundraising effort of its members.

3. An example

In this section, we explore some of the main intuitions of the model with a numerical example, before deriving the general results in the next section. Consider a donations market with nonprofits, fix the financial cost of collecting a unit of donations c to 0.1, and the parameters of donor preferences Ω and δ at 0.1 and 1, respectively. On the basis of these values, we have constructed two figures. Figure 1 maps the payoffs of three categories of players: those inside the grand coalition, denoted with $Q_i\left(\widetilde{y}\right)$, those inside an alliance of a smaller size A_k, denoted with $Q_i\left(\overline{y}\right)$, and those outside coalition A_k playing as singletons, denoted with $Q_j\left(\overline{y}\right)$. Let, in Figure 1, $\beta = 0.1$, i.e. nonprofits impose negative fundraising externalities on each other and fundraising efforts be strategic complements. Figure 2 presents a similar mapping, but for the value of $\beta = -0.2$ (positive externalities and strategic substitutes).

[8] It is easy to check the boundaries required on β for $\overline{y}_i \in (0,1)$ under any alliance structure. In particular, for $\beta > 0$, $\beta < (\delta - \Omega)/(n-1)$. This condition also ensures that individual best-responses are contraction, requiring, in turn, $|\beta| < 2\delta/(n-1)$.

First of all, note that regardless of the sign of externalities, for every feasible nonprofit alliance, $Q_i(\widetilde{y})$ is larger than $Q_i(\overline{y})$. In other words, the payoff of nonprofits inside the grand coalition is always higher than that of nonprofits in an alliance of smaller size. This implies that the grand coalition is both Nash and coalitionally stable under the unanimity rule. If the unanimity rule is used, once the grand coalition is formed, none of the nonprofits have an incentive to deviate: clearly, such a deviation would imply the break-up of the grand coalition into singletons, and the payoff of any nonprofit would fall from point A to point J on Figure 1 (and similarly on Figure 2).[9] This is also true for any sub-coalition of nonprofits (this can be seen from the fact that the dotted blue line is everywhere below the dashed red line): no coalition $A_k \subset N$ can improve upon $\{N\}$ by deviating jointly, because the grandcoalition fully internalizes the externalities that nonprofits impose on each other.[10]

Second, when nonprofits impose negative externalities on each other and fundraising efforts are strategic complements ($\beta > 0$, see Figure 1), and alliances are formed using the unanimity rule, the above intution on Nash and coalitional stability also applies for any alliance smaller than the grand coalition. In fact, note that the payoff curve $Q_i(\overline{y})$ is always positively sloped, and thus any point on it is higher than point J. Intuitively, given the strategic complementarity, deviation from the alliance (individually or jointy) implies that all nonprofits increase their fundraising efforts, and since they impose negative externalities on each other, such deviation reduces the payoffs of all of them, including the deviating one(s). This is sufficient to discipline them and thus guarantees the stability of the initial alliance.

Sadly, this discipline is much weaker if the alliance formation is made using the aggegative rule. Consider the grand coalition (point A). If one nonprofit deviates, given the aggregative rule, the remaining nine stay together, which means that the payoff of the deviator increases up to point B. This is because the smaller alliance increases the fundraising efforts of its members only slightly, whereas the deviator can enjoy the benefits of tapping into the market share, which is greatest when the initial alliance is the largest possible (this reflects the case of World Vision discussed in the opening quote of this paper). Thus, the grand coalition is not generally stable under the aggregative rule, even when $\beta > 0$. In our example, it is also true for several smaller alliances: in fact, point D is located higher than point C, point F higher than point E, etc. The slope, however, is decreasing, and at some point, some alliance becomes Nash stable: given an alliance of size 4, the incentive of each member to deviate is absent (G is higher than H).

What about the situation with positive externalities and when fundraising efforts are strategic substitutes ($\beta < 0$, see Figure 2)? Free-riding is now even more convenient than before, because when the alliance members increase their fundraising, the outsider nonprofit gains donors with no additional effort (and - because of strategic substitutes - has actually an incentive to reduce her fundraising effort). Notice that for a wider range of a_k, the payoff of an outsider $Q_j(\overline{y})$ dominates both that of the alliance members $Q_i(\overline{y})$ and that of the grand coalition members $Q_i(\widetilde{y})$. One implication is that even small alliances might now be unstable, under the institutional setting which is most favorable to the formation of alliances

[9]In the next section we show that, under the unanimity rule, this result continues to hold in a more general setup (see Corollary 2).

[10]Under the unanimity rule, we prove that this result extends to a more general setup in which fundraising efforts are either strategic complements (Proposition 2) or strategic substitutes and best-replies are contraction (Proposition 3).

(i.e. the unanimity rule). In fact, the point I (the payoff of members of alliance of size 3) is located lower than point J (the payoff under all-singleton competition). This is an effect similar to the "merger paradox" in industrial organization literature (Salant et al. 1983, Deneckere and Davidson 1985): when a small alliance forms and increases its fundraising effort, the fringe of nonprofits reduce its own (since here $\beta < 0$) to the extent to damage the alliance itself.

We can also slightly generalize this example by varying the total number of nonprofits and verifying which of all of the possible coalitions structures are Nash and coalitionally stable under aggregative and unanimity rules. Table 1 lists all the stable coalition structures, for both the strategic complements and strategic substitutes cases. The patterns that we find confirm the above intuitions. First, in general, there are many Nash stable structures under the unanimity rule, whereas stability under the aggregative rule is much more difficult to obtain. As we said above, the unanimity rule makes the deviation much more costly, because a deviation leads to a much more aggressive competition. Second, there are fewer Nash stable structures, even under the unanimity rule when the externalities are positive ($\beta < 0$). This occurs essentially because of the "merger paradox" effect.

[**Table 1 about here**]

4. General results

We are now ready to provide some general results on the stability of given structures of nonprofit alliances. We will present our main results on the stability of nonprofits alliances adopting a general output function

$$Q_i\left(y_i, y_{-i}\right) = Q(F_i(y_i, y_{-i}), \tau_i(y_i)), \tag{4.1}$$

assumed twice continuously differentiable and concave in y_i for every $i = 1, 2, .., n$.[11] At this point, it is also useful to define formally the notion of positive (negative) externalities. We say that nonprofit $j (\neq i)$ imposes, through its fundraising effort y_j, *a negative (positive) externality* on the output of nonprofit i if $\partial Q_i\left(y_i, y_{-i}\right) / \partial y_j$ is negative (positive).

Since the grandcoalition outcome - i.e. full coordination - is Pareto-efficient, we concentrate our analysis on the question of stability of the grandcoalition of nonprofits. Besides the grandcoalition, other alliance structures may also be coalitionally stable. However, since by definition a coalitionally stable alliance structure must be Pareto-optimal, no coalitionally stable alliance structures can be made only of symmetric alliances. This result is expressed in the next proposition. From this simple result, we derive two further important corollaries.

Proposition 1. *Regardless of the rule of alliance formation, no partition of nonprofits* $\mathcal{S}^E = \{A_1, A_2, ..., A_m\}$, *such that every alliance possesses the same size (cardinality)* $a_1 = a_2 = ... = a_k = ... = a_m$, *can be coalitionally stable.*

Proof. See Appendix. □

Corollary 1. *The alliance structure with all nonprofits as singletons is always Nash stable and never coalitionally stable.*

Proof. Follows directly from Proposition 1. □

[11] The Appendix describes the conditions on the output function required for fundraising activities of nonprofits to be strategic complements or substututes.

Corollary 2. *Under the unanimity rule of alliance formation, full coordination is always Nash stable.*

Proposition 1 helps to see that if stable confederations of nonprofits different from the grandcoalition exist, they have to be made of asymmetric alliances, thus exploiting some sort of free-riding advantages in fundraising activities. The example in table 1 shows that for high number of nonprofits competing in the donors' market, a stable alliance structure under unanimity rule is represented by one coalition of nonprofits that coordinate their fundrasing efforts competing with a fringe of rivals. Corollary 1 is an obvious consequence of Proposition 1. Corollary 2 highlights the fact that since, under the unanimity rule, an individual nonprofit's decision to leave the grandcoalition breaks the alliance of the remaining nonprofits completely, given that the grandcoalition allocation is efficient, it must also be Nash stable. What remains to be analyzed is under which circumstances the grandcoalition is *coalitionally* stable against asymmetric alliance structures. The sections that follow are devoted to finding the conditions that guarantee the coalitional stability of the grandcoalition of nonprofits under both the unanimity and aggregative rules of alliance formation.

4.1. **Coalitional stability under the unanimity rule and strategic complements.** As was indicated by the numerical example in the previous section, the first key feature for the coalitional stability of the grandcoalition of nonprofits is the strategic complementarity of their fundraising activities. If the grandcoalition of nonprofits works as a device to discipline the fundraising activity of every nonprofit, everyone can benefit from such a discipline. If a subcoalition of nonprofits deviates from a joint agreement by increasing its fundraising activity and remaining nonprofits increase their fundraising as well (strategic complementarity), then we can prove that all feasible asymmetric deviations are not convenient. We start by presenting a lemma that characterizes, under complementarity, the level of fundraising effort of every nonprofit in a Nash equilibrium of the game played between an alliance of nonprofits that coordinate their activities and all remaining nonprofits that act as singletons.[12] [13]

Lemma 1. *Let the fundraising activities of nonprofits be strategic complements. Then, at the fundraising equilibrium associated with any alliance structure made of one alliance and all remaining nonprofits as singletons, the following condition holds: if fundraising activity of every nonprofit imposes negative (positive) externalities on other nonprofits, then an organization that belongs to the alliance exerts a lower (higher) fundraising effort than every singleton nonprofit.*

Proof. See Appendix. □

Intuitively, nonprofits inside the alliance choose their fundraising efforts so as to internalize the externality that they impose on each other's output. However, no such internalization of externalities occurs outside the alliance. Under negative externalities, this implies that the outsiders disregard the negative impact that their actions impose on other nonprofits and thus engage in more fundraising than the alliance members. Under positive externalities, given that the actions of alliance members create positive effects also on the *outsiders'* output,

[12]The Lemmata that follows exploit the logic analogous to the one in Currarini and Marini (2006).

[13]Note that in the specific model presented in Section 2, strategic complementarity in fundraising efforts always coincides with negative externalities that nonprofits impose on each other's output (both are driven by $\beta > 0$), whereas strtategic substitutability with positive externalities (when $\beta < 0$). In a more general setup, such coincidence needs not to hold.

the outsiders free-ride to some extent on the actions of alliance members, thus putting lower fundraising effort than the nonprofits inside the alliance.

Lemma 1 also allows us to compare the payoffs of nonprofits inside and outside the alliance at the fundraising equilibrium. We characterize this in the following

Lemma 2. *Let the fundraising activities of nonprofits be strategic complements. Then at any alliance structure made of one alliance and all remaining nonprofits as singletons, every nonprofit inside the alliance obtains a lower payoff than any nonprofit acting alone.*

Proof. See Appendix. □

Lemma 2 establishes that at the fundraising equilibrium of the second-stage game, any organization belonging to the alliance produces a lower project output as compared to a nonprofit that does not belong to the alliance. The intuition comes from the fact that the discipline necessary to internalize the externalities is individually costly (in terms of output) and the nonprofits outside the alliance avoid bearing this cost.

We can now show that under the unanimity rule of alliance formation, strategic complementarity is sufficient for the stability (against a deviation by any group of nonprofits) of the full coordination.

Proposition 2. *Let the fundraising activities of nonprofits be strategic complements and alliance formation occur by unanimity rule. Then, the full coordination by nonprofits is coalitionally stable.*

Proof. See Appendix. □

Intuitively, the argument runs as follows. A group of nonprofits (or an individual nonprofit) have an interest in deviating from the full coordination agreement only if by doing so it obtains a higher payoff. However, under the unanimity rule, this deviation implies that the remnant of the grandcoalition breaks down into singleton nonprofits. In this case, we find ourselves in the second-stage game with the alliance structure described by Lemmata 1 and 2. Lemma 2 has shown that the non-deviant nonprofits (which now find themselves as singletons) are better off than those in the deviating group. However, this would mean that the sum of payoffs of organizations in deviating and non-deviant groups must be higher than the sum of payoffs in the grandcoalition, which is impossible because the grandcoalition structure is Pareto-efficient.

Proposition 2 underlines the role that the unanimity rule and the strategic complementarity of fundraising efforts of nonprofits play for the (coalitional) stability of the nonprofit coordination. The role of the unanimity rule lies in making sure that the deviation by a group from the full coordination agreement automatically implies that the deviating group finds itself playing against a set of singleton organizations in the second-stage game. By itself, this is not sufficient to discourage a deviation by a *group* of nonprofits. For this, we also need the strategic complementarity. It guarantees that given that the deviating group changes its behavior, under strategic complementarities the non-deviant organizations change their behavior in the same direction, and this greatly hurts the deviating group, making sure that the incentive to deviate is absent.

In terms of real-life behavior of nonprofits, this result tells us the following. The nonprofit confederations are often organized in such a way that a break-up of a confederation would occur by several nonprofits separating away and taking with them some key assets of the confederation (e.g. the brand name, the key contacts in the government or large foundations).

In this case, what can prevent such break-ups is the way in which fundraising functions in the nonprofit sector. If the break-up would induce the remaining nonprofits to increase their fundraising efforts, then the separating nonprofits would be seriously damaged by such intensification of fundraising competition. This is sufficient to prevent the dissolution of the large-scale confederations of nonprofits.

Proposition 2 is particularly important for the real-life applications in the nonprofit world, because the crucial threat for nonprofit confederations is not that single members might exit the confederation; it is that the break-away might be organized jointly by several member organizations. Often, this threat intensifies when there is a generational change in the leadership of one or several organizations, and the founding leaders of the confederation are no longer key decision-makers. Proposition 2 tells us that as far as there is enough strategic complementarity in fundraising efforts, the risk of such break-away is small.

This strategic complementarity in fundraising is likely to be related to the elasticity of donations pool to the overall fundraising effort, i.e. how likely the fundraising efforts by nonprofits to attract new donors (as compared to competing for the existing ones). Aldashev and Verdier (2010) show that when the pool of donations is fixed, NGO fundraising efforts are strategic complements, while when fundraising attracts also new donors (e.g. by raising awareness), fundraising efforts become strategic substitutes. In the next sub-section, we study the stability of the grandcoalition under strategic substitutes.

4.2. **Coalitional stability under the unanimity rule and strategic substitutes.** Under strategic substitutes, the threat of the break-up is likely to be more serious, as the break-away group is unlikely to be damaged as seriously as in the case of strategic complementarities. Nevertheless, the full coordination can still be coalitionally stable. We prove in this sub-section that under the unanimity rule of alliance formation, even after removing strategic complementarity of fundraising efforts, the grandcoalition remains coalitionally stable, under the additional requirement that the individual best-reply function of a nonprofit to any deviation in the fundraising efforts of its' rivals satisfies the contraction property.

Lemma 3. *At the fundraising equilibrium associated to any alliance structure made of one alliance and all remaining nonprofits as singletons, the following condition holds: for every nonprofit i belonging to alliance A, we have that: (i) $r_i\left(\overline{y}_{A\setminus\{i\}}, \overline{y}_{N\setminus A}\right) \leq \overline{y}_i$ under positive externalities, and (ii) $r_i\left(\overline{y}_{A\setminus\{i\}}, \overline{y}_{N\setminus A}\right) \geq \overline{y}_i$ under negative externalities, where $r_i(.)$ denotes the individual best-reply of every nonprofit $i \in A$.*

Proof. See Appendix. □

Lemma 3 makes an out-of-equilibrium statement. Take any alliance structure made of one alliance plus singletons, and consider an individual nonprofit inside the alliance. Given this particular alliance structure, the Lemma states that if every single nonprofit belonging to alliance A were to play according to its individual best-reply, then, under negative externalities, the fundraising level that it would choose in the second-stage game would be higher than the level that it chooses being inside the alliance and coordinating its behavior with the alliance members. Under positive externalities, the opposite is true: this nonprofit's hypothetical fundraising would be lower than the level it chooses cooperatively inside the alliance.

The key point behind this lemma is that the nonprofit playing "as if" it were alone would not internalize the externality that it imposes on the nonprofits remaining inside the alliance.

Thus, under negative (positive) externalities, this free-riding nonprofit would choose a higher (lower) fundraising level as compared to its level inside the alliance. Note that, contrarily to Lemma 1, this result does not require that nonprofits' fundraising levels are strategic complements.

We can now prove the following

Proposition 3. *Let every nonprofit's best-reply function be a contraction. Then, under the unanimity rule of alliance formation, the full coordination is coalitionally stable.*

Proof. See Appendix. □

Let a group of nonprofits decide to break out of the full coordination agreement. Under the unanimity rule, they find themselves playing (in the second stage) against singleton nonprofits. Under strategic substitutes (and the best-reply function being a contraction), the reaction of any singleton nonprofit to the change in fundraising by nonprofits inside the alliance (as compared to their fundraising under full coordination) is to change its fundraising effort in the opposite direction, but by a smaller amount. However, under such a change and under positive or negative externalities, the nonprofits outside the alliance still choose their fundraising levels without internalizing their effect on the nonprofits inside the alliance. Then, similarly to Lemma 2, the output of a nonprofit outside the alliance is still higher than that of an alliance member. This means that it is impossible for any group of nonprofits to jointly increase their output as compared to that under full coordination.

This finding implies that strategic complementarity is not crucial for reducing the incentive of a group of nonprofits to break the full coordination agreement. The interests of the break-away group is damaged sufficiently by the fact that under the best-reply being a contraction, the singleton nonprofits do not change their fundraising levels sufficiently to internalize the externality that they impose on the break-away alliance members. However, it remains crucial that the groups of nonprofits are formed by the unanimity rule: the break-away group must be facing the set of singleton nonprofits for the damage described above to serve as a threat.

In real-life applications, sometimes an increase in one nonprofit's fundraising might induce other nonprofits to reduce their fundraising. This occurs, for example, when an aggressive fundraising campaign by a nonprofit forces other nonprofits to retreat or to switch to searching for other donors. In this case, the nonprofits' actions are strategic substitutes. This might put the stability of full coordination in danger. Proposition 3 says that as far as the size of such a reaction by other nonprofits is not bigger than the increase in fundraising by the deviating nonprofit, the stability of the grandcoalition is not at risk.

4.3. **Stability under the aggregative rule.** Clearly, the strategic incentives to announce a given alliance differs under the aggregative rule. In this sub-section, we extend some of the results obtained above for the unanimity rule of alliance formation to the aggregative rule. This can be done by noting that if the stability of full coordination holds under unanimity, it must hold *a fortiori* when forming alliances hurt for some reasons the remaining players (i.e. under *negative synergies*, defined formally below). When, instead, synergies are positive, the aggregative rule of alliance formation makes the full coordination more difficult than under the unanimity rule. Let us first define formally the concept of synergies.

Definition 3. *Negative (positive) synergies among nonprofits are present if for every feasible alliance structure* $\mathcal{S}$ *and alliance* $A \in \mathcal{S}$, $Q_i(y(\mathcal{S}')) < Q_i(\overline{y}(\mathcal{S}))$ $(Q_i(\overline{y}(\mathcal{S}')) > Q_i(\overline{y}(\mathcal{S})))$ *for every* $i \in A$, *where* $\mathcal{S}'$ *is obtained from* $\mathcal{S}$ *by simply merging alliances of nonprofits in* $\mathcal{S}\setminus\{A\}$.

Note that the concept of synergies is different from that of externalities: synergies refer to the formation of alliances, whereas externalities refer to the actions of individual nonprofits. It is necessary to introduce the notion of synergies for the analysis of coalition formation under the aggregative rule, whereas the notion of externalities suffices for the analysis under the unanimity rule. Intuitively, this is because under the aggregative rule, the payoffs of deviating nonprofits off the equilibrium path is calculated taking into account that the remaining nonprofits continue to stick together in an alliance, whereas under the unanimity rule, off-the-equilibrium path payoffs are calculated taking into account that the deviating group of nonprofits faces only singletons as rivals.

Negative synergies occur, for instance, if there exist economies of scale in fundraising or in other common activities that, e.g. by reducing the alliance members' costs, hurt in some way all remaining nonprofits outside the alliance. From the above definition it follows that under *negative synergies*, for every nonprofit $i \in A$,

$$Q_i\left(\overline{y}\left(\mathcal{S}^U\right)\right) > Q_i\left(\overline{y}\left(\mathcal{S}^A\right)\right),$$

where $\mathcal{S}^U = \left(\{A\}, \{j\}_{j\in N\setminus A}\right)$ and $\mathcal{S}^A = (\{A\}, \{N\setminus A\})$. This fact explains the natural extension of some of the results obtained above to the aggregative rule.

Corollary 3. *If negative synergies hold for all nonprofits, full coordination is always Nash stable under the aggregative rule of alliance formation.*

Proof. From Corollary 2 we know that the grand coalition is always Nash stable under the unanimity rule of alliance formation. With negative synergies this must, *a fortiori,* holds under the aggregative rule. □

The coalitional stability of the grandcoalition under the aggregative rule is in general not so straightforward as the Nash stability. This is because we cannot be sure that Lemmata 1 - 3 still hold in presence of positive synergies in alliances of nonprofits. However, in one case the coalitional stability under the unanimity rule fully extends to the aggregative rule. This occurs when, in an alliance only a majority of nonprofits can deviate from the full-coordination agreement. If the decision to dissolve the grand coalition can be taken only by *a majority* of its members, then every deviating alliance of nonprofits by definition consists of a number of members greater or equal than $|N|/2$.[14]

Definition 4. *A majority breaking protocol holds in any arbitrary alliance of nonprofits* $A \subset N$ *if and only if the decision to deviate from* A *can be taken only by a majority of its members, i.e.,by every* $A' \subset A$ *with* $|A'| \geq |A|/2$.

We can show that Lemmata 1 and 3 easily apply to every alliance structure $\mathcal{S}^A = (\{A\}, \{N\setminus A\})$, in which $|A| \geq |N|/2$. In this case, in every alliance structure $\mathcal{S}^A$ the following results hold: (i) $\overline{y}_i \leq \overline{y}_j$ under negative externalities in fundraising, and (ii) $\overline{y}_i \geq \overline{y}_j$

[14]Any game of coalition formation based on a unanimity or aggregative rule can be constrained to a majority protocol by simply restricting the coalitional payoff of every coalition $A \subset N$ to be equal to zero for $|A| < |N|/2$ as happens in majority games. See Ray 2007: 289, for a detailed discussion.

under positive externalities in fundraising, for every nonprofit $i \in A$ and $j \in N \backslash A$.[15] As a result, Lemma 2 can also be applied and, therefore, $Q_i\left(\overline{y}\left(\mathcal{S}^A\right)\right) \leq Q_j\left(\overline{y}\left(\mathcal{S}^A\right)\right)$, for every nonprofit $i \in A$ and $j \in N \backslash A$. This implies, in turn, the following:

Proposition 4. *Let the fundraising activities of nonprofits be strategic complements and a majority breaking protocol hold in N. Then, under the aggregative rule of alliance formation, full coordination is coalitionally stable.*

Proof. Follows directly from Proposition 2 and Definition 4. □

Proposition 5. *Let every nonprofit's best-reply be a contraction and a majority breaking protocol hold in N. Then, under the aggregative rule of alliance formation, full coordination is coalitionally stable.*

Proof. Follows directly from Proposition 3 and Definition 4. □

The intuition for the above results is as follows. Since a majority breaking protocol is imposed by assumption to the grand coalition of nonprofits, the only feasible deviations that can concretely take place are those made by groups of nonprofits with a size greater or equal than half of all the nonprofits operating in the market. In this case, under the aggregative rule of alliance formation the deviating group, say A, will compete against a smaller alliance $N \backslash A$ and, therefore, any of its member $i \in A$ will exert a fundraising effort greater than any $j \in N \backslash A$ under positive externalities and a smaller one under negative externalities. This can be easily proved (by extending Lemma 1) for fundraising efforts that are either strategic complements or strategic substitutes with individual best-replies being contractions. The fact that small groups are relatively more advantaged than big groups in the fundraising market depends on the absence of synergies, on the symmetry of players and on the presence of externalities. As a result, we have (by simple extension of Lemma 2) that at the fundraising equilibrium every nonprofit in $N \backslash A$ is better off than any nonprofit belonging to A. Therefore, by the efficiency of the grand coalition, it is impossible that a deviating majority of nonprofits improves upon its allocation of output obtained in the grand coalition.

Table 2 resumes the taxonomy of the general results that we have obtained.

[**Table 2 about here**]

5. Discussion

As some nonprofit practitioners write, "Umbrella campaigns, such as the United Way, have traditionally been one means of reducing wasteful competition among nonprofits. But [...] pressures to allow greater donor choice have had the effect of reviving such competition. Although it may be desirable in principle to reduce incentives for organizations to engage in socially wasteful competition, it is more challenging in practice to develop either new social institutions, or policies that limit such spending" (Cordes and Rooney 2003). Our model provides indications on the type of policies that might increase the stability of nonprofit cooperation, by taking into account the strategic effects and incentives that we have shown are crucial for nonprofits in decicing whether to stick to the proposed agreements.

[15]To economize on space, we omit these proofs. They are available upon request.

First, some public policies can directly affect the strategic complementarity of fundraising efforts of nonprofits, via β. For instance, if the awareness campaigns about the issues towards which most nonprofits operate is done by the public sector entities (e.g. the ministry of health), then the awareness spillovers are likely to be small (Δ is low). Alternatively, if government subsidizes the cost of fundraising campaigns via a technology that allows for precise targeting of donors, e.g. consumer-analytics based solicitations via Internet (as compared to non-targeting technologies, such as direct mailing), this would reduce the spillovers (and thus the value of Δ). Both policies would increase the strategic complementarity of fundraising and, according to our theory, stabilize the Pareto-efficient cooperation between nonprofits.

Second, public policies can affect the returns to fundraising actions of nonprofits. For instance, tax deductibility of donations or matching grants clearly increase the "prize" that a deviating nonprofit obtains under the aggregative rule (in terms of Figures 1 and 2, this would make the curve Q_j steeper). Such policies make sustainable nonprofit cooperation more difficult. On the contrary, a large direct grant would make the funds each nonprofit has a relatively more abundant input in its production function, and thus would increase the opportunity cost of time spent for fundraising. This, in turn, would reduce the incentives to deviate from cooperative agreements, and make cooperation easier.

Third, given that the effectiveness of these policies crucially depends on the alliance-formation rule, policies affecting these institutional characteristics might also coalitionally influence nonprofit cooperation. We have seen that cooperation is more difficult to reach under the aggregative rule. Consider now a policy mandating that nonprofits willing to conduct joint fundraising appeals have to register a trademark for the joint appeal with a government agency, and that any future request of a member of the joint appeal to exit automatically implies the dissolution of the trademark (essentially, this implies imposing the unanimity rule). By raising the cost of alliance members' sticking together when a member of the alliance deviates, such a policy makes the dissolution of the cooperative agreement upon deviation more likely, which strategically increases its stability, as shown by our model. Similarly, a somewhat less drastic policy - requiring that the formation of a new joint fundraising appeal (while one such appeal is alreary in place) can only be done by a majority of the members of the current appeal implies imposing the majority breaking protocol, and - as our results indicate - this would stabilize the Pareto-optimal full coordination.

Two caveats are worth mentioning. First, the discussion of efficiency throughout the model assumes that we look at the efficiency taking only the payoffs of nonprofits into account. The beneficiaries of the nonprofit projects are not portrayed explicitly. It is quite possible that in a more complete model (i.e. the one that includes the beneficiaries as active players), the analysis of efficiency substantially differs from the one developed here. For instance, in a setting with the Samaritan's dilemma (Buchanan 1975), the efficient output level by the nonprofits would be lower than the one derived here. Second, while our paper concentrates on the nonprofit interaction along the fundraising dimension, there are several other key dimensions along which nonprofit compete and (possibly) coordinate (e.g. location of operations (Koch 2007) or emphasis on urgent versus long-run projects (Brown and Minty 2006)). Both avenues call for future work.

6. Appendix

Preliminaries

We assumed that every nonprofit alliance behave *à la* Nash against rival alliances of nonprofits and therefore act to maximize the sum of the joint output of all its members, taking as given the actions of nonprofits that do not belong to this alliance. Formally, for every $A_k \in \mathcal{S}$, the objective function is

$$\max_{y_{A_k} \in Y_{A_k}} Q_{A_k} = \max_{y_{A_k} \in Y_{A_k}} \sum_{i \in A_k} Q_i \left[F_i(y_{A_k}, y_{N \setminus A_k}), \tau_i(y_i) \right].$$

The first-order condition of this problem implies, for every member of the alliance, $i \in A_k$,

$$\frac{dQ_{A_k}}{dy_i} = \frac{\partial Q}{\partial F_i} \frac{dF_i}{dy_i} + \frac{\partial Q}{\partial \tau_i} \frac{d\tau_i}{dy_i} + \sum_{h \in A_k \setminus \{i\}} \frac{\partial Q}{\partial F_h} \frac{dF_h}{dy_i} = 0, \tag{6.1}$$

which, using the non-distribution and time constraints, becomes

$$\frac{\partial Q}{\partial F_i}(1-c)\frac{\partial D_i}{\partial y_i} + \sum_{h \in A_k \setminus \{i\}} \frac{\partial Q}{\partial F_h}(1-c)\frac{\partial D_h}{\partial y_i} = \frac{\partial Q}{\partial \tau_i}, \text{ for } \forall i \in A_k. \tag{6.2}$$

This expression indicates that every nonprofit participating in alliance A_k sets its fundraising level to equate the marginal cost of fundraising to the marginal social (coalitional) benefit. Note that when all nonprofits act noncooperatively, the interior Nash equilibrium $\overline{y} = (\overline{y}_1, \overline{y}_2, .., \overline{y}_n)$, implies, for every $i = 1, ..., n$:

$$\frac{dQ_i}{dy_i} = \frac{\partial Q}{\partial F_i} \frac{dF_i}{dy_i} + \frac{\partial Q}{\partial \tau_i} \frac{d\tau_i}{dy_i} = 0, \tag{6.3}$$

which, using the non-distribution and time constraints (6.3) becomes:

$$\frac{\partial Q}{\partial F_i}(1-c)\frac{\partial D_i}{\partial y_i} = \frac{\partial Q}{\partial \tau_i}. \tag{6.4}$$

Suppose now that the best reply of every i-th nonprofit with respect to other organizations' fundraising activity is a single-valued function; let's denote it with

$$r_i(y_{-i}) = \arg\max_{y_i} Q_i\left(y_i, y_{-i}\right)$$

for every $i \in N$. Implicitly differentiating the first-order condition at the equilibrium $\overline{y}$ as

$$\frac{dQ_i(r_i(\overline{y}_{-i}), \overline{y}_{-i})}{dy_i} \equiv 0,$$

we obtain the generic expression for the slope of the best-reply function of every nonprofit as:

$$\frac{dr_i(y_j)}{dy_j} = -\frac{\partial^2 Q_i / \partial y_i \partial y_j}{\partial^2 Q_i / \partial y_i^2} \text{ for } \forall j \neq i. \tag{6.5}$$

From (6.5), given the concavity of output in fundraising effort (i.e. $\partial Q_i^2 / \partial y_i^2 < 0$), we obtain

$$\text{sign } \frac{dr_i(y_j)}{dy_j} = \text{sign } \frac{\partial^2 Q_i}{\partial y_i \partial y_j} = \text{sign } \frac{\partial Q}{\partial F_i}(1-c)\frac{\partial^2 D_i}{\partial y_i \partial y_j} - \frac{\partial^2 Q}{\partial \tau_i \partial y_j} \text{ for } \forall j \neq i. \tag{6.6}$$

Therefore, when the marginal returns on fundraising increase sufficiently with the fundraising effort exerted by other nonprofits, i.e. the donation function $D_i\left(y_i.y_{-i}\right)$ exhibits sufficiently high increasing differences in $(y_i.y_{-i}) \in Y_i \times Y_{-i}$, every nonprofit's best-reply is non-decreasing in fundraising effort of its rivals.[16] In other words, in this case, fundraising efforts of nonprofits are strategic complements. Conversely, when marginal returns on fundraising are negative, by (6.6) nonprofits' fundraising efforts are strategic substitutes and best-reply functions are non-increasing. These properties will be extensively used in the proofs of Lemmata and Propositions presented below.

The question of existence of a fundraising equilibrium $\overline{y} = (\overline{y}_1, \overline{y}_2, .., \overline{y}_n)$, either played between singletons or among alliances of nonprofits (see next section), does not pose, in our setup, particular problems. Since by (2.3) nonprofits' choice sets are nonempty, compact and convex, and their objective functions (4.1) are continuous and strictly quasi-concave (by the assumption of strict concavity), a coalitional fundraising profile (i.e. a non-improvable equilibrium among individual or alliances of nonprofits) exists. The proof is rather standard and we do not report it here, to economize on space.[17]

Concerning the uniqueness of equilibrium, note that most results of our paper are obtained with either nonprofits fundraising efforts being strategic complements (Lemmata 1 and 2, Proposition 1) or strategic substitutes with best-replies that satisfy the contraction property (Lemma 4, Proposition 2). The contraction property is sufficient to guarantee the existence of a unique (Nash) fundraising equilibrium $\overline{y}$ played among singletons or alliances of nonprofits. The property of increasing differences of nonprofits' payoffs is sufficient to guarantee the existence, in our setup, of a greatest and a least elements within the set of fundraising equilibria (see Topkis 1998). Since in our framework one of these two elements Pareto dominates (for nonprofits) all others elements, this specific unique selection will be considered in the following analysis.

Proof of Proposition 1

We have assumed that, in every alliance A_k, each member receives the equal split payoff $Q_i = Q_{A_k}(\overline{y})/a_k$. Since in the symmetric alliance structure $\mathcal{S}^E$ the unique fundraising equilibrium profile $\overline{y} = (\overline{y}_1, \overline{y}_2, ..., \overline{y}_m)$ must be symmetric, $Q_{A_h}(\overline{y}) = Q_{A_r}(\overline{y})$ for every A_h, $A_r \in \mathcal{S}^E$ and $Q_i(\overline{y}) = Q_j(\overline{y})$ for every $i \in A_h$ and $j \in A_r$, i.e, every nonprofit obtains the same payoff. The efficiency of the profile y^e associated to the grandcoalition N implies, for every nonprofit $i \in N$

$$Q_i(y^e) \geq Q_i(\overline{y})$$

and, for at least one $j \in N$,

$$Q_j(y^e) > Q_j(\overline{y}).$$

Note that by the continuity and strict concavity of every Q_i on y_i, compactness of every nonprofit's strategy set Y_i and then of $Y = (Y_1 \times Y_2 \times .. \times Y_n)$, the (efficient) cooperative profile y^e played by the grandcoalition, i.e.

$$y^e = \arg\max_{y \in Y_N} \sum_{i \in N} Q_i(F_i(y_i, y_{-i}), \tau_i(y_i)),$$

[16] A real-valued function $f(x, y)$ has increasing (decreasing) differences in $(x, y) \in (X \times Y)$ whenever $f(x, y') - f(x, y'')$ is increasing (decreasing) for every $y'' > y'$. When $f(x, y)$ is continuously differentiable in $\mathbb{R}^2$, it exhibits increasing (decreasing) differences if and only if $\frac{\partial^2 f}{\partial x \partial y} \geq (\leq) 0$ (see Topkis 1998).

[17] For a proof of the existence of a coalitional equilibrium see, for instance, Ray and Vohra (1997). See also Haeringer (2002).

exists and is unique. Hence, $y^e \neq \overline{y}$, and since at $\overline{y}$ every nonprofit receives the same payoff, it must be that

$$\sum_{i\in N} Q_i\left(y^e\right) > \sum_{A_k\in\mathcal{S}^E}\sum_{i\in A_k} Q_i\left(\overline{y}\right),$$

Therefore, every nonprofit in $\mathcal{S}^E$ would gain by announcing $\alpha_i = \{N\}$ and forming the grandcoaliton, thus every symmetric alliance structure $\mathcal{S}^E$ different from N could be improved upon and can never be coalitionally stable. ■

Proof of Lemma 1

We need to show that when $Q_i\left(y_i, y_{-i}\right)$ exhibits increasing differences in $\left(y_i, y_{-i}\right) \in Y_i \times Y_{-i}$, at the fundraising equilibrium $\overline{y}$ played by every alliance structure of the form

$$\mathcal{S} = \left(\{A\}, \{j\}_{j\in N\backslash A}\right)$$

the following holds: (i) negative externalities in fundraising $\left(\frac{\partial Q_j}{\partial y_i} \leq 0\right)$ imply that the fundraising level of every nonprofit in alliance $i \in A$ and every nonprofit acting as singleton $j \in N\backslash A$ are such that $\overline{y}_i \leq \overline{y}_j$; (ii) positive externalities in fundraising $\left(\frac{\partial Q_j}{\partial y_i} \geq 0\right)$ imply that $\overline{y}_i \geq \overline{y}_j$ for every $i \in A$ and $j \in N\backslash A$.

Proof of (i): Suppose by contradiction that, for every $i \in A$ and $j \in N\backslash A$, $\overline{y}_i > \overline{y}_j$ under negative externalities. By strict concavity of nonprofit's payoff $Q_i\left(\overline{y}\right)$, if one nonprofit in A were to switch to an effort equal to $\overline{y}_j < \overline{y}_i$, the derivative of Q_i with respect to y_i at the equilibrium would be such that

$$\frac{\partial Q_i\left(\overline{y}\right)}{\partial y_i} < \frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right)}{\partial y_i}, \tag{6.7}$$

where $\overline{y} = \left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right)$ denotes the fundraising equilibrium level with one nonprofit in A exerting the fundraising level $\overline{y}_j$ of a singleton nonprofit $j \in N\backslash A$ instead of $\overline{y}_i$, while all remaining nonprofits in A (denoted now as the alliance $A\backslash\{i\}$) playing $\overline{y}_i$ as before. Next, by the property of increasing differences assumed for $Q_i\left(y_i, y_{-i}\right)$, we have that

$$\frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right)}{\partial y_i} \leq \frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_i\right)}{\partial y_i}, \tag{6.8}$$

where $\overline{y} = \left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_i\right)$ denotes the equilibrium fundraising level with nonprofit $i \in A$ exerting the fundraising level $\overline{y}_j$ instead of $\overline{y}_i$ and one nonprofit $j \in N\backslash A$ now switching to the fundraising level $\overline{y}_i > \overline{y}_j$. Using the fact that nonprofits are all identical *ex ante* and, therefore, their payoffs are symmetric,[18] we also can write

$$\frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_i\right)}{\partial y_i} = \frac{\partial Q_j\left(\overline{y}_i, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_j\right)}{\partial y_j}, \tag{6.9}$$

which, by definition, corresponds to the equilibrium first-order condition of every nonprofit $j \in N\backslash A$ playing as singleton,

$$\frac{\partial Q_j\left(\overline{y}_i, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_j\right)}{\partial y_j} = \frac{\partial Q_j\left(\overline{y}\right)}{\partial y_j} = 0. \tag{6.10}$$

[18]Symmetric payoffs imply, for every $i, j \in N$, that $Q_i\left(y_i, y_j, y_{N\backslash\{i\cup j\}}\right) = Q_j\left(y_j, y_i, y_{N\backslash\{i\cup j\}}\right)$.

Together, the expressions (6.7)-(6.10) imply that

$$\frac{\partial Q_i(\overline{y})}{\partial y_i} < 0,$$

i.e., that

$$\frac{dQ_i(\overline{y})}{dy_i} = \frac{\partial Q}{\partial F}\frac{dF_i}{dy_i} + \frac{\partial Q}{\partial \tau_i}\frac{d\tau_i}{dy_i} < 0 \tag{6.11}$$

for every $i \in A$. Therefore, since negative externalities in fundraising imply in (6.11) that

$$\sum_{j \in A_k \setminus \{i\}} \frac{\partial Q}{\partial F_j}\frac{dF_j}{dy_i} \leq 0,$$

the condition (6.11) is contradicted.

Proof of (ii): This proof follows the same logic as in (i). Again, by contradiction, assume that, for every $i \in A$ and $j \in N \setminus A$, $\overline{y}_i < \overline{y}_j$ under positive externalities. By strict concavity of nonprofit's payoff (first inequality), increasing differences (second inequality) and symmetric payoffs (first equality) and the equilibrium first-order conditions of every $j \in N \setminus A$ playing as singleton (second equality), we obtain

$$\begin{aligned}\frac{\partial Q_i(\overline{y})}{\partial y_i} &> \frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\setminus\{i\}}, \overline{y}_{N\setminus A}\right)}{\partial y_i} \geq \frac{\partial Q_i\left(\overline{y}_j, \overline{y}_{A\setminus\{i\}}, \overline{y}_{(N\setminus A)\setminus\{j\}}, \overline{y}_i\right)}{\partial y_i} = \\ &= \frac{\partial Q_j\left(\overline{y}_i, \overline{y}_{A\setminus\{i\}}, \overline{y}_{(N\setminus A)\setminus\{j\}}, \overline{y}_j\right)}{\partial y_j} \equiv \frac{\partial Q_j(\overline{y})}{\partial y_j} = 0\end{aligned}$$

for every $i \in A$. Therefore, since positive externalities imply in (6.11) that

$$\sum_{j \in A_k \setminus \{i\}} \frac{\partial Q}{\partial F_j}\frac{dF_j}{dy_i} \geq 0,$$

the condition (6.11) is contradicted. ■

Proof of Lemma 2

We have to prove that at a fundraising equilibrium $\overline{y} = \left(\overline{y}_A, \{\overline{y}_j\}_{j \in N\setminus A}\right)$, with some nonprofits grouped in one alliance and all remaining nonprofits playing alone, nonprofits' payoffs respect the inequality $Q_j(\overline{y}) \geq Q_i(\overline{y})$, $i \in A$ and $j \in N \setminus A$.

Using the definition of equilibrium, we can write

$$Q_j(\overline{y}) \geq Q_j\left(\overline{y}_A, \overline{y}_{(N\setminus A)\setminus\{j\}}, \overline{y}_i\right), \tag{6.12}$$

expressing the simple fact that if we let any nonprofit $j \in N \setminus A$ playing alone switch its fundraising level with that of any nonprofit playing in alliance $i \in A$, then its payoff, by definition, will not improve. By Lemma 1 we know that under negative externalities $\overline{y}_i \leq \overline{y}_j$ for every $i \in A$ and $j \in N \setminus A$ and, contrarily, under positive externalities, $\overline{y}_i \geq \overline{y}_j$ for every $i \in A$ and $j \in N \setminus A$. Therefore, regardless of the sign of fundraising externalities, if we let an nonprofit in A to play $\overline{y}_j$ instead of $\overline{y}_i$, for every nonprofit playing as singleton we obtain

$$Q_j\left(\overline{y}_A, \overline{y}_{(N\setminus A)\setminus\{j\}}, \overline{y}_i\right) \geq Q_j\left(\overline{y}_j, \overline{y}_{A\setminus\{i\}}, \overline{y}_{(N\setminus A)\setminus\{j\}}, \overline{y}_i\right). \tag{6.13}$$

Next, by the symmetry of all nonprofits, switching strategies implies switching payoffs, and we can write

$$Q_j\left(\overline{y}_j, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_i\right) = Q_i\left(\overline{y}_i, \overline{y}_{A\backslash\{i\}}, \overline{y}_{(N\backslash A)\backslash\{j\}}, \overline{y}_j\right) = Q_i\left(\overline{y}\right). \tag{6.14}$$

Therefore, by (6.12)-(6.14) we finally obtain that, for all $i \in A$ and $j \in N\backslash A$,

$$Q_j\left(\overline{y}\right) \geq Q_i\left(\overline{y}\right).$$

■

Proof of Proposition 2

Suppose that a group of nonprofits $A \subset N$ could deviate *profitably* from $\{N\}$ with an alternative announcement $\alpha_i' = \{A\}$ of all its members, inducing, by the unanimity rule, the alliance structure $\mathcal{S}^U\left(\alpha_A', \widetilde{\alpha}_{N\backslash A}\right) = \left(\{A\}, \{j\}_{j\in N\backslash A}\right)$. As a result, it must be that, for every $i \in A$

$$Q_i\left(\overline{y}\right) > Q_i\left(y^e\right), \tag{6.15}$$

where $Q_i\left(y^e\right)$ indicates the payoff obtained by every nonprofit at the (efficient) cooperative profile y^e played by the grandcoalition, By Lemmata 1 and 2 at the fundraising equilibrium $\overline{y}$ associated to the alliance structure $\mathcal{S} = \left(\{A\}, \{j\}_{j\in N\backslash A}\right)$ we have that

$$Q_j\left(\overline{y}\right) \geq Q_i\left(\overline{y}\right),$$

and, by expression (6.15), for every $j \in N\backslash A$ playing as singleton

$$Q_j\left(\overline{y}\right) > Q_i\left(y^e\right). \tag{6.16}$$

Therefore

$$\sum_{i\in A} Q_i\left(\overline{y}\right) + \sum_{j\in N\backslash A} Q_j\left(\overline{y}\right) > \sum_{i\in N} Q_i\left(y^e\right),$$

which is a contradiction of the efficiency of the grandcoalition strategy profile y^e. ■

Proof of Lemma 3

We need to show that if every nonprofit in an alliance $A \subset N$ were to play according to its individual best-reply, defined as

$$r_i\left(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right) = \arg\max_{y_i}\ Q_i(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}),$$

against all other nonprofits playing their equilibrium fundraising levels, we would have that $r_i\left(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right) \leq \overline{y}_i$ under positive externalities and $r_i\left(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right) \geq \overline{y}_i$ under negative externalities.

Let us define

$$\overline{y}_A = \arg\max_{y_A}\ Q_A(y_A, \overline{y}_{N\backslash A}) = \arg\max_{y_A} \sum_{i\in A} Q_i((y_A, \overline{y}_{N\backslash A}).$$

By the definition of profile $\overline{y}_A$ for alliance A, we have

$$Q_A\left(\overline{y}_A, \overline{y}_{N\backslash A}\right) \geq Q_A\left(y_i', \overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right), \tag{6.17}$$

for any arbitrary $y' \in Y_i$. Next, suppose, by contradiction, that under positive externalities, the best-reply $r_i\left(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right) > \overline{y}_i$.

By the definition of best-reply of an nonprofit $i \in A$, we have

$$Q_i\left(r_i\left(\overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right), \overline{y}_{A\backslash\{i\}}, \overline{y}_{N\backslash A}\right) \geq Q_i\left(\overline{y}_A, \overline{y}_{N\backslash A}\right). \tag{6.18}$$

Moreover, for every nonprofit $k \in A\backslash\{i\}$, positive externalities imply

$$\sum_{k\in A\backslash\{i\}} Q_k\left(r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{N\backslash A}\right),\overline{y}_{A\backslash\{i\}},\overline{y}_{N\backslash A}\right) > \sum_{k\in A\backslash\{i\}} Q_k\left(\overline{y}_A,\overline{y}_{N\backslash A}\right). \tag{6.19}$$

Therefore, jointly (6.18) and (6.19) contradict the expression (6.17). The statement for the case of negative externalities is proven analogously. ■

Proof of Proposition 3

We have to show that, for every $y,y\prime \in \mathbb{R}^{n-1}$ if

$$\left\|r_i\left(y_{N\backslash\{i\}}\right)-r_i\left(y'_{N\backslash\{i\}}\right)\right\| \leq \phi\left\|y_{N\backslash\{i\}}-y'_{N\backslash\{i\}}\right\|$$

with $\phi<1$ and $\|.\|$ defining the Euclidean norm on space $\mathbb{R}^{n-1}$, then, under the unanimity rule of alliance formation, the grandcoalition is coalitionally stable. Under the unanimity rule, when an alliance A of nonprofits decides to deviate from the grandcoalition N, the alliance structure $\mathcal{S}^U=(\{A\},\{j\}_{j\in N\backslash A})$ forms as a result.

Therefore, at the resulting fundraising equilibrium profile $\overline{y}=\left(\overline{y}_A,\left\{\overline{y}_j\right\}_{j\in N\backslash A}\right)$, all nonprofits i in alliance A coordinate their fundraising, such that

$$\overline{y}_A = \underset{y_A\in Y_A}{\arg\max}\sum_{i\in A} Q\left[F_i(y_A,y_{N\backslash A}),\tau_i(y_A)\right].$$

Take the choice $\overline{y}_i$ of an arbitrary nonprofit $i\in A$ and the choice $\overline{y}_j$ of an arbitrary nonprofit $j\in N\backslash A$ playing as singleton. We need to show the same result of Lemma 1, i.e. that (i) under negative externalities in fundraising, $\overline{y}_i\leq\overline{y}_j$ and (ii) under positive externalities, $\overline{y}_i\geq\overline{y}_j$. Suppose not and, in particular, suppose that $\overline{y}_i<\overline{y}_j$ under positive externalities. Applying symmetry, we have that the equilibrium fundraising level of every singleton $j\in N\backslash A$ is exactly similar to the one that a singleton nonprofit $i\in A$ facing a fundraising profile

$$\overline{y}'_{N\backslash\{i\}}=\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{(N\backslash A)\backslash\{j\}},\overline{y}_i\right)$$

would optimally play (using its best-reply). In other words, we can write

$$\overline{y}_j=r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{N\backslash A\backslash\{j\}},\overline{y}_i\right),$$

and, therefore,

$$\overline{y}_j-\overline{y}_i=r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{(N\backslash A)\backslash\{j\}},\overline{y}_i\right)-\overline{y}_i.$$

Next, given that by Lemma 3, positive externalities imply

$$r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{N\backslash A}\right)\leq\overline{y}_i,$$

we have

$$\overline{y}_j-\overline{y}_i\leq r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{(N\backslash A)\backslash\{j\}},\overline{y}_i\right)-r_i\left(\overline{y}_{A\backslash\{i\}},\overline{y}_{N\backslash A}\right),$$

in which both sides are positive. The latter expression contradicts the fact that the best-reply $r_i(.)$ is a contraction. The case of negative externalities, i.e. (ii), can be proven analogously. ■

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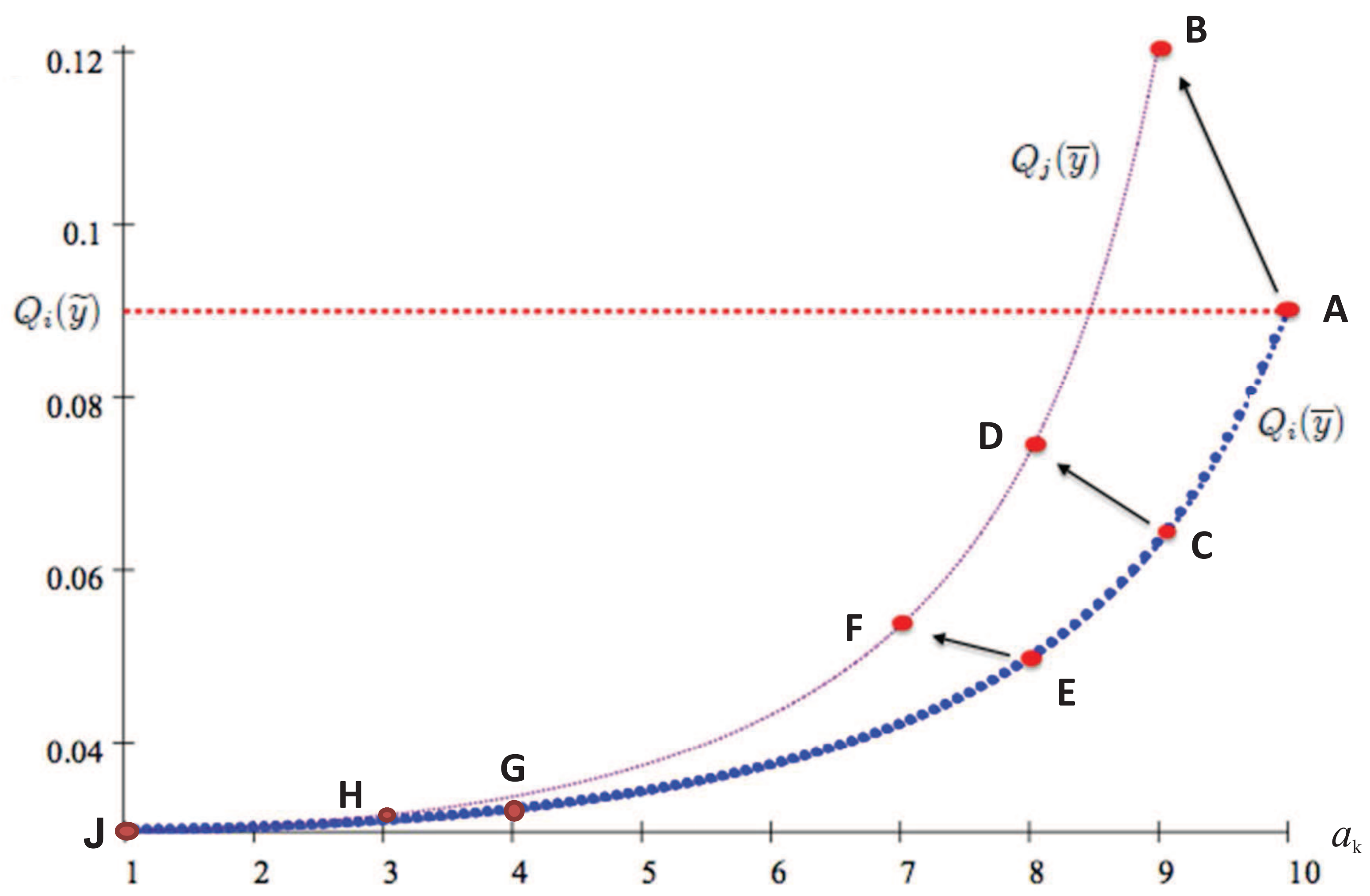

Figure 1. Payoffs when fundraising efforts are strategic complements (*β>0*)

Notes: Dashed line = payoff of the grand coalition member. Dotted line = payoff of a member in a coalition of size a_k. Continuous line = payoff of a nonprofit playing as singleton in the presence of a coalition of size a_k. Values of parameters are fixed at $\Omega = 0.1, \delta = 1$, c = 0.1, n =10, β= 0.1.

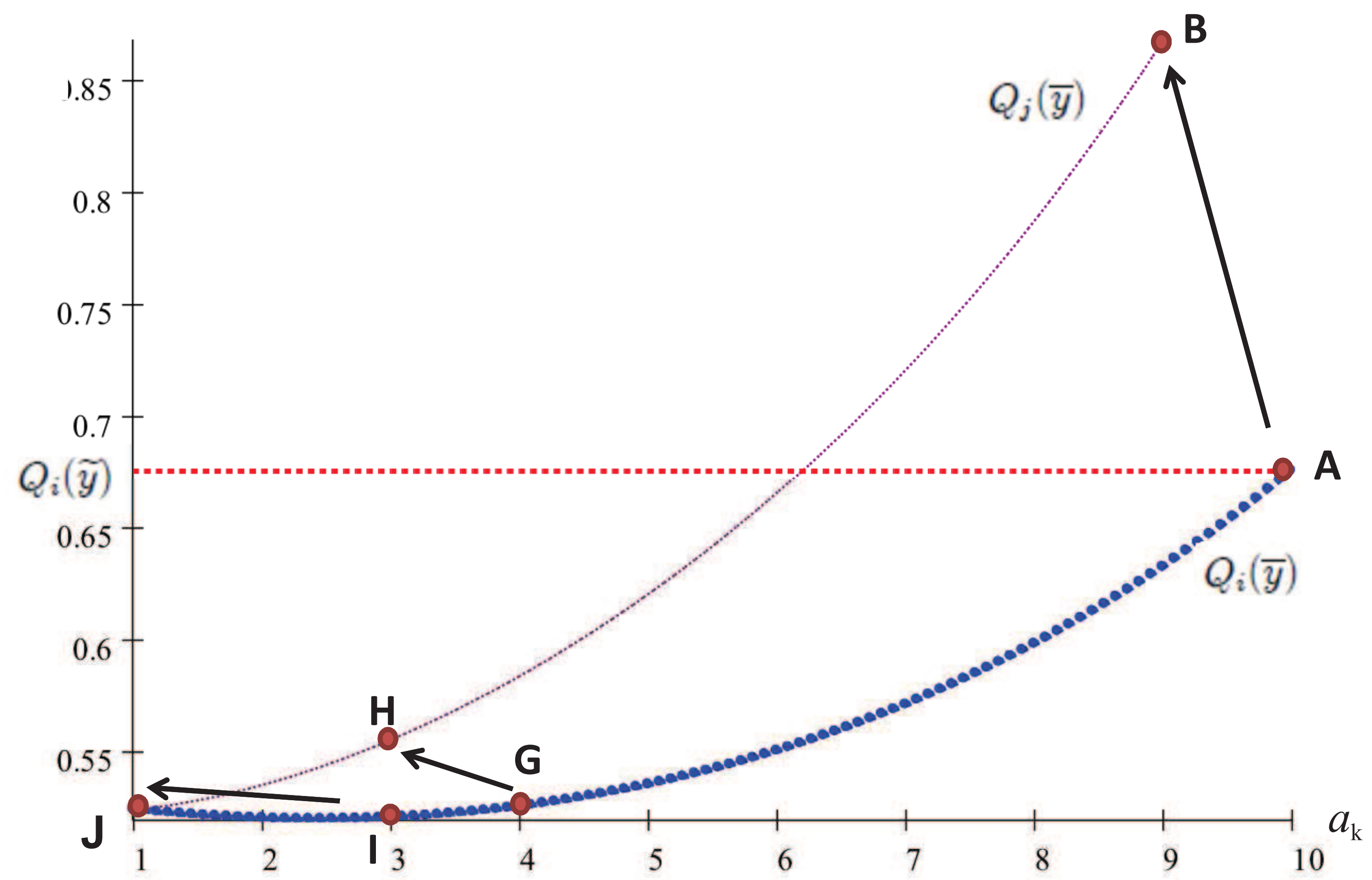

Figure 2. Payoffs when fundraising efforts are strategic substitutes (*β<0*)

Notes: Dashed line = payoff of the grand coalition member. Dotted line = payoff of a member in a coalition of size a_k. Continuous line = payoff of a nonprofit playing as singleton in the presence of a coalition of size a_k. Values of parameters are fixed at $\Omega = 0.1, \delta = 1$, c = 0.1, n =10, $\beta = -0.2$.

Table 1. Nash stable alliance structures, for different total number of nonprofits

A. Unanimity Rule

	$\beta = 0.1$	$\beta = -0.2$
N = 2	{1,1}; {2}*	{1,1}; {2}*
N = 3	{1,1,1}; {2,1}; {3}*	{1,1,1}; {2,1}; {3}*
N = 4	{1,1,1,1}; {2,1,1}; {2,2}; {3,1}; {4}*	{1,1,1,1}; {2,1,1}; {3,1}; {4}*
N = 5	{1,1,1,1,1}; {2,1,1,1}; {3,1,1}; {4,1}; {5}*	{1,1,1,1,1}; {3,1,1}; {4,1}; {5}*
N = 6	{1,1,1,1,1,1}; {2,1,1,1,1}; {3,1,1,1}; {2,2,2}; {3,3}; {3,2,1}; {4,1,1}; {4,2}; {5,1}; {6}*	{1,1,1,1,1,1}; {3,1,1,1}; {3,3}; {4,1,1}; {5,1}; {6}*
….	….	….
N > 6	At least all alliance structures $(A_k, \{j\}_{j \in N \setminus A_k})$ with $a_k \leq n$; $\{N\}^*$	At least alliance structures $(A_k, \{j\}_{j \in N \setminus A_k})$ with $a' \leq a_k \leq n$ and a' sufficiently large to overcome the 'merger paradox'; $\{N\}^*$

Note: Values of parameters are fixed at *Ω = 0.1, δ = 1, c = 0.1*. Asterisk (*) denotes structures that are also coalitionally stable.

B. Aggregative Rule

	$\beta = 0.1$	$\beta = -0.2$
N = 2	{2}	{2}
N = 3	{3}	none
N = 4	None	none
N = 5	None	none
N = 6	None	none
…	None	none

Note: Values of parameters are fixed at *Ω = 0.1, δ = 1, c = 0.1*. Asterisk (*) denotes structures that are also coalitionally stable.

Table 2. Summary of general results

Properties of interaction between nonprofits ⇨ Alliance formation rule ⇩	FUNDRAISING STRATEGIC COMPLEMENTS	FUNDRAISING STRATEGIC SUBSTITUTES
UNANIMITY RULE	Grandcoalition is Nash stable and coalitionally stable	Grandcoalition is Nash stable and coalitionally stable, if best response functions are contraction
AGGREGATIVE RULE	Grandcoalition is Nash stable and coalitionally stable, under majority breaking protocol or negative synergies	Grandcoalition is Nash stable and coalitionally stable, if best response functions are contraction and under majority breaking protocol or negative synergies