Uncertainty and the Politics of Employment Protection

Andrea Vindigni
Simone Scotti
Cristina Tealdi

No. 298
May 2013
Uncertainty and the Politics of Employment Protection*

Andrea Vindigni†, Simone Scotti‡ and Cristina Tealdi§

May 2013

Abstract

This paper investigates the social preferences over labor market flexibility, in a general equilibrium model of dynamic labor demand. We demonstrate that how the economy responds to productivity shocks depends on the power of labor to extract rents and on the status quo level of the firing cost. In particular, we show that when the firing cost is initially relatively low, a transition to a rigid labor market is favored by all the employed workers with idiosyncratic productivity below some threshold value. Conversely, when the status quo level of the firing cost is relatively high, the preservation of a rigid labor market is favored by the employed with intermediate productivity, whereas all other workers favor more flexibility. A more volatile environment, and a lower rate of productivity growth, i.e., “bad times,” increase the political support for more labor market rigidity only where labor appropriates of relatively large rents. The coming of better economic conditions not necessarily favors the demise of high firing costs in rigid high-rents economies, because “good times” cut down the support for flexibility among the least productive employed workers. The model described provides some new insights on the comparative dynamics of labor market institutions in the U.S. and in Europe over the last few decades, shedding some new light both on the reasons for the original build-up of “Eurosclerosis,” and for its relative persistence until the present day.

Keywords: employment protection, job creation and destruction, firing cost, idiosyncratic productivity, volatility, growth, political economy, voting, rents, status quo, path dependency, institutional divergence.

JEL Classification: D71, D72, E24, J41, J63, J65.

---

*We are especially grateful to Gilles Saint-Paul, Bjoern Bruegemann and two anonymous referees whose comments allowed us to improve significantly the quality of the paper, and the editor Christopher Taber for many valuable suggestions and insights. We also thank Daron Acemoglu, David Autor, Roland Bénabou, Olivier Blanchard, Tito Boeri, Ricardo Caballero, Robert Shimer, Iván Werning, for helpful comments. The usual disclaimer applies.
†IMT Lucca and Collegio Carlo Alberto.
‡LPMA, Université Paris 7 Diderot.
§IMT Lucca.
1 Introduction

Employment protection legislation varies significantly across OECD countries. Relatively stringent job security provisions are currently implemented in several Continental European countries such as Portugal, Spain, Greece, whereas other countries such as the U.K. and especially the U.S. are characterized by relatively flexible labor markets.\footnote{For a summary index of employment protection legislation (EPL) strictness, the reader may refer to OECD (2008).} The current institutional status quo observed across developed countries reflects the diverging paths of evolution of employment protection legislation and other labor market institutions on the two sides of the Atlantic, over the past forty years. There is evidence that in Continental Europe firing costs have gradually become higher since the early 1970’s,\footnote{See for example Caballero and Hammour (1998), Blanchard (2000), and Blanchard and Wolfers (2000).} the period traditionally associated with the build-up of “Eurosclerosis,”\footnote{“Eurosclerosis” is a term coined by Giersch (1985) to describe an economic pattern observed in Europe in the 1970’s and the early 1980’s, where countries faced simultaneously high unemployment and slow job creation, despite some evidence of recovery of economic growth. In contrast, the United States experienced in the same period an economic expansion, high job growth as well as relatively low unemployment.} and mildly reduced since the 1990’s. During the same period, the structure of the labor markets of the U.K. and particularly of the U.S. has instead changed relatively little, displaying over time a remarkably persistent, flexible regulation of labor relation.\footnote{The U.K. labor market became indeed more flexible during the 1980’s due the reforms implemented by the conservative government chaired by Mrs. Thatcher.}

Part of the institutional inertia observed seemed to be related to the fact that many European Governments have implemented simultaneously two antithetical policy measures: on one side they kept or sometimes even reinforced already strong protection policies to reduce job destruction; on the other hand they introduced new flexible, fixed-term contracts, to push job creation (Cahuc and Postel-Vinay, 2002; Boeri and Garibaldi, 2007).

The strategy to increase the flexibility of the labor market has been implemented at the margin (dual track reforms), without affecting those hired on “regular” contracts, who are still protected by high job security (Bentolila and Dolado, 1994; Saint-Paul, 1996; Cabrales and Hopenhayn, 1997; Tealdi, 2011).\footnote{Dual-track reforms have increased the flexibility of temporary contracts, leaving unchanged the EPL affecting permanent contracts; however these reforms might have changed and reshaped the size and profile of the electorate who supports EPL reforms. For instance in Spain, the intensive utilization and the extended scope of the new fixed-term contracts deregulated in the late 1980’s lead unemployed workers and temporary workers to strongly support in the late 1990’s the reduction of rigidities associated with the regular contract (Dolado et al., 2002; Bentolila et al., 2008).} Therefore, the protection of the traditional permanent contract has often not changed significantly following the introduction of temporary contracts (OECD, 2008). Furthermore, in a few major economies, such as France and Germany, there...
is hardly any evidence that in recent years firing restrictions and other forms of labor market rigidities for regular contracts have been reduced at all (OECD, 2008).

On the whole, despite the exceptions mentioned above, the big picture observed seems one of significant persistence of the high dismissal barriers legislated over past decades in several Continental European countries. This outcome is especially remarkable since the issue of the reform of rigid labor market contracts, including permanent ones, toward more flexibility has been in the past years often at the top of the political agenda of many governments of Continental Europe. Furthermore, the different macroeconomic conditions present today relative to the period of build-up of Eurosclerosis, make the relative persistence of many of the institutional rigidities typical of Continental European labor markets, including in primis EPL, an even more important and challenging open question. In this paper, we attempt to shed some new light on these puzzling facts.

More specifically, the goal of this paper, is to investigate how the interaction of institutional and economic factors affects the emergence and the potential persistence of political support for some form of employment protection regulations. In order to pursue this goal, we develop and fully characterize the solution of a general equilibrium model of dynamic labor demand, which carries three distinctive features.

First, we assume that the productivity of active firms evolves over time according to a Geometric Brownian motion and an independent Poisson process, reflecting respectively the realization of idiosyncratic productivity shocks, and exogenous quit decisions by the workers, which firms (probabilistically) anticipate. The existence of idiosyncratic uncertainty makes firms and workers, which are both ex-ante identical, ex-post heterogeneous due to their variable productivity, and leads to a non-degenerate distribution of productivity across active firms. The second feature of the model is that employed workers appropriate a rent, i.e., an economic benefit in excess of the utility of the unemployed. The extent of such rent represents the amount of money that firms need to pay to the workers in excess of their outside option, to cope with an underlying moral hazard problem, partly in the spirit of efficiency wage models (e.g., Shapiro and Stiglitz, 1984). However, the overall rent will depend not only on the monitoring technology adopted by firms, but also on politico-institutional factors potentially different across countries, but regarded in this paper as exogenous. Such politico-institutional

---

7As we will make more clear later, part of the explanation of the relatively little progress made on the road to flexibility, is due to the decisive opposition of unions as well as of some pivotal (typically leftist) political parties, which had some degree of political connection with the unions.
fundamentals will be captured in the model by one parameter reflecting the power of extraction of rents of the employed workers. In particular, we will assume it to be higher in Continental Europe than in Anglo-Saxon countries. This assumption is motivated by different bodies of literature in the broad field of comparative political economy (see for example Amable, 2009), that emphasize how labor market institutions depend on a diverse class of political institutions. In particular, in the literature on endogenous institutions and comparative politics of public finance, Ticchi and Vindigni (2010) demonstrate that the “consensual democracy,” i.e., parliamentary government coupled with proportional representation (see Lijphart, 1999, for a classical taxonomy of the different forms of democracy), typical of Continental Europe generates endogenously a political bias in favor of leftist parties, relative to the different forms of “majoritarian” democracy typical of the Anglo-Saxon countries, which tend to favor the Right. Left-wing parties are in turn naturally connected with the unusually powerful European unions, a connection which allows employed workers to extract relatively high rents from firms. In the model, the entire rent appropriated by the workers will also depend on the idiosyncratic productivity of the job, on the exogenous parameters of the stochastic processes governing the evolution of the productivity of the firm, and on the interest rate; in general equilibrium, this will generate a rich set of comparative statics results.

Third, the reservation productivity at which firms eventually decide to quit operating, and which also affects the total value of rent appropriated by employed workers, depends on a legislated tax, or firing cost, imposed on the firms upon laying-off their employees. The firing cost, which is determined through a political process based on standard majority voting, is a key endogenous variable in the model. Lower firing costs potentially harm workers since they decrease the rent appropriated by the employed. This we refer to as rent erosion effect. On the other hand, lower firing costs decrease the total cost of labor borne by the firms, and therefore increase job creation, raising the exit rate from unemployment, and increasing the welfare of

---

8 We refer to the parameter in question as the “rent extraction power” instead of the bargaining power since we do not use the standard Nash bargaining rule (or any other bargaining game such as Rubinstein, 1982, or Shaked and Sutton, 1982) to determine the equilibrium wage.

9 For instance in 2010 the trade union density as percentage of employees in the United States was 11.4% against the 38.1% in Europe and 18.1% among the OECD countries (OECD Employment Database).

10 See for instance Alesina, Glaeser and Sacerdote (2005), for a discussion of this fact. The important contribution in the literature of political economy of finance of Pagano and Volpin (2005), who show empirically that proportional representation, as opposed to a majoritarian electoral system, correlates with relatively low investors protection and relatively high employment protection, is also related. Pagano and Volpin (2005) propose in addition a theoretical model where these empirical patterns reflect causal effects of the electoral systems.

11 These parameters will include the drift and the instantaneous standard deviation of the geometric Brownian motion, which characterize respectively the average rate of growth of productivity and its volatility, and the Poisson quit rate.

12 See Section 2.2 (p. 9) for a more precise description of the sharing rule adopted and of its microfoundation.
both unemployed and employed workers. This we refer to as job creation effect.

We show that the resolution of the trade-off generated by lower firing costs, i.e., a smaller inter-temporal flow of rents, due to the rent erosion effect, versus a greater exit rate from unemployment, due to the job creation effect, depends in particular on the parameter capturing the rent extraction power of the workers. When this power is small enough, i.e., below some threshold, there is little scope to protect jobs and the associated rents with legislated firing restrictions. More precisely, in this scenario the workers are unanimously in favor of zero firing costs, regardless of the status quo employment protection level. In the opposite case where the bargaining power of the workers is above a critical threshold, we show that workers split in two opposite coalitions, favoring respectively a rigid and a flexible labor market. Specifically, when firing cost are relatively low, a transition to a rigid labor market is favored by all the employed workers with idiosyncratic productivity below some threshold value. All the unemployed and the most productive employed form a “non-connected” or extreme coalition to support low firing costs. Vice versa, when firing costs are relatively high, a rigid labor market is preferred by the employed workers with intermediate productivity, confirming the findings of Boeri and Burda (2009). A flexible labor market is instead supported by an extreme coalition involving all the unemployed, as well as the more and the less productive employed. Intuitively, regardless of the status quo, the unemployed prefer to eliminate firing costs to induce firms to open new vacancies, thus reducing the expected length of their unemployment spell.

The analysis of the politico-economic equilibrium reveals two additional insights. First, we show that a complementarity arises in the equilibrium of the model between the volatility of productivity and labor market flexibility: in a more turbulent environment, both the positive job creation effect of lower firing costs (and partially on the welfare of the employed), and the related rent erosion effect, are magnified. Second, we show that a substitutability arises in the equilibrium of the model between productivity growth and labor market flexibility, as higher productivity growth reduces both the job creation and the rent erosion effects.

The deeper significance of these results is that how unexpected productivity shocks affect the political equilibrium of the model, depends on the power of labor to extract rents, and therefore ultimately on the balance of power between leftist parties, unions, producers and their political supporters. Specifically, employment protection is more likely to emerge and

---

13 A similar type of coalition arises under some conditions in Ticchi and Vindigni (2010), where the poor may prefer to side with the rich, and support a relatively conservative fiscal policy, against the middle class party.

14 This refers to unexpected changes in the two parameters governing the evolution of the productivity of all the active firms.
persist in economies where workers have greater rent extraction power. This finding can help explain the divergent evolution of the labor market institutions in high and low rents economies, i.e., Continental Europe, where the Left was historically stronger and had gained considerable political momentum earlier, versus the U.S. and the U.K., in response to similar negative aggregate shocks.

This paper is related to a variety of different contributions including primarily previous models of political economy of labor market institutions, such as Lindbeck and Snower (1988), Bertola and Rogerson (1997), Blanchard and Giavazzi (2003), and Saint-Paul (1993), as well as the more recent contribution of Saint-Paul (2002). The main difference between our paper and Saint-Paul (2002), is that his model addresses the question of how the preferences for employment protection are affected by the rate of growth of embodied productivity within a vintage capital model. By focusing on a disembodied form of productivity growth, we obtain a number of different comparative statics results. In particular, whereas Saint-Paul finds that higher productivity growth reduces unambiguously the political support for employment protection regulation, we find that how growth affects the political equilibrium generally depends both on the bargaining power of labor, and on the status quo level of firing cost. In addition, Saint-Paul does not investigate how volatility, which plays an essential role in our model, affects the politico-economic equilibrium.

In recent papers, Boeri and Burda (2009) and Bruegemann (2012) also address the question of the persistence of rigid labor market institutions, within a version of the standard Mortensen and Pissarides (1994) framework. In particular, using a Nash bargaining wage determination mechanism, Boeri and Burda (2009) compare the outcome of flexible-wage setting in a decentralized competitive search market with a rigid-wage labour market, where the salary is independent of the individual match productivity. They show that employment protection is a necessary condition for support for collectively bargained wages to arise in equilibrium. Severance protection, in the form of a deadweight firing tax, increases the acceptance of rigid wage policies, because it further increases the utility of employed workers relative to the decentralized

---

15Examples of important episodes of leftist empowerment include the facts of Paris in May of 1968 (e.g., student and workers going on strikes, occupation of factories), and the Italian “Hot Autumn,” in Turin in 1969.
16By developing a general equilibrium model with both product and labor regulations, Blanchard and Giavazzi (2003) address the issue of deregulation. They find that currently employed workers are more in favor of the deregulation of the labor market whenever it is combined simultaneously with the deregulation of the product market.
17See also Saint-Paul (2000) for a survey of this literature.
18Even though this paper closely relates to the published paper of Saint-Paul on the political economy of firing costs (see Saint-Paul, 2002), we will make use of the wage setting rule described in the working paper version of the same work (see Saint-Paul, 1999), which we find more convincing, simple, elegant and transparent.
equilibrium. They also show that severance taxation increases the relative support for rigid wages for employed workers with intermediate productivity, confirming the predictions of our model. Using Nash bargaining for wage determination, Bruegemann (2012) finds that workers value employment protection because it increases their bargaining power in wage negotiations. By driving the distribution of match-specific productivity toward lower values, stringent protection in the past actually reduces support for future employment protection. However, when introducing involuntary separations, workers value employment protection because it delays involuntary dismissals. Moreover, workers in low productivity matches gain most since they face the highest risk of dismissal. Thus, as we argue in this paper, Bruegemann concludes that the existence of employment protection may endogenously create its own political support.

In the context of trade-off between employment protection and unemployment benefits, as emphasized by Larsen (2004), we can classify European countries according to their choice to protect more the jobs versus to support more the unemployed. The recent work by Boeri et al. (2012) addresses this topic, by exploring the political economy behind it. By characterizing employment protection and unemployment benefits as schemes redistributing between insiders and outsiders as well as across skill groups, they find that configurations characterized by less employment protection and more unemployment benefits, should emerge in countries with less compressed wage structures. In this perspective, by exploring the political economy of unemployment insurance, other papers such as Hassler et al. (2005) and Hassler and Rodríguez Mora (1999) are related to this work. Hassler et al. (2005) investigate whether generous unemployment benefits in the past raise support for unemployment benefits today. In their model unemployment benefits increase the workers’ attachment to a geographic location, increasing the support for higher unemployment benefits. While in their model wages and separations are exogenous and workers benefit from unemployment benefits, in our model they do not play any substantial role. Hassler and Rodríguez Mora (1999) show that saving and borrowing is a good substitute for unemployment insurance when turnover is high. Therefore, with high turnover, the median voter prefers low unemployment insurance. With low turnover, instead, generous unemployment insurance becomes more valuable. Even though we acknowledge the importance of potential the trade-off between employment protection legislation and unemployment benefits, as emphasized notably by Boeri et al. (2012), our model is not designed to investigate this problem.\textsuperscript{19}

\textsuperscript{19}Garibaldi and Violante (2005) study the different effect of severance payments and firing costs on unemployment and argue that, interestingly, it varies according to the degree of wage rigidity. In economies where wage rigidity is relatively high, severance payments are either neutral or have negative effects on unemployment.
This paper complements the political economy literature on inefficient redistribution (e.g., Coate and Morris, 1995; Acemoglu and Robinson, 2001), and on the persistence of various classes of policies and institutions (e.g., Fernandez and Rodrik, 1991; Coate and Morris, 1999; Bénabou, 2000; Acemoglu, and Robinson, 2000; Acemoglu, and Robinson, 2008; Acemoglu, Ticchi and Vindigni, 2011). In particular, the influential paper of Bénabou (2000) demonstrates that unequal societies, featuring potentially very different degrees of fiscal redistribution of income and of inequality, or “social contracts,” can arise and persist in a dynamic model of political economy of taxation, depending on the initial degree of income inequality. Moreover, Acemoglu, Ticchi and Vindigni (2011) demonstrate that under some conditions, such as high income inequality, states with weak fiscal capacity, i.e., with a weak ability to raise taxes and provide public goods, emerge and persists over time despite potential drastic shocks to formal political institutions.\(^{20}\)

This paper is also related to the seminal models of dynamic labor demand of Bentolila and Bertola (1990) and of Mortensen and Pissarides (1994). The paper of Bentolila and Bertola (1990) presents a partial equilibrium model with a stochastic structure identical to the one assumed here, which we extend to allow for the endogenous determination of wages, labor market flows and firing costs. The model of Mortensen and Pissarides (1994) has a dynamic general equilibrium structure, based on the assumption that firms experience productivity shocks described by a homogenous Poisson process (i.e., with constant hazard rate), rather than by a combined Geometric Brownian process and a Poisson process, as assumed here.\(^{21}\) In addition, while in Mortensen and Pissarides’ model higher volatility has a negative effect on job creation overall, in our model we obtain the opposite effect.\(^{22}\) Finally, while in the model of Mortensen and Pissarides, the heterogeneity in match quality depends on the ex-ante workers heterogeneity in productivity, in our model workers and firms are ex-ante homogeneous. However the productivity of the firms, which evolves according to a Geometric Brownian motion, generates ex-post (variable) heterogeneity in the match quality. The literature which introduces Geometric Brownian idiosyncratic uncertainty to model productivity shocks within

\(^{20}\)Therefore inefficient states, like rigid labor market institutions, have the potential to be self-stable, i.e., to generate ex-post the political constituency supporting their own future political survival.

\(^{21}\)This assumption implies that employed workers have the same marginal benefit from an increase in firing costs regardless on the level of their idiosyncratic productivity and therefore they all have potentially the same preferences over employment protection legislation. Conversely, in our model, the (almost sure) continuity of the paths of a Brownian motion allows for a potentially negative correlation between current productivity and preferences on job separation. Therefore, in our set-up more productive workers tend to demand less firing costs due to their relative insulation from the risk of job loss.

\(^{22}\)See also the comprehensive discussion of search and matching models of the labor market presented in the excellent textbook of Pissarides (2000).
a general equilibrium model of the labor market (with endogenous turnover restrictions) began with Vindigni (2002). The same literature has been further developed within a search and matching model featuring learning by doing (Nagypál, 2005; Prat, 2009), endogenous worker turnover and wage distribution (Prat, 2003 and 2006; Moscarini, 2005), and to develop a notion of rest unemployment (Alvarez and Shimer, 2011). In addition, in the general equilibrium macro-labor literature with frictions, the papers by Caballero and Engel (1993), Ramey and Watson (1997), Ljungqvist and Sargent (1998), Hopenhayn and Rogerson (1993), Bertola (1994), MacLeod, Malcomson, and Gomme (1994) and Yashiv (2000), Rogerson and Pries (2005) play an important role. See also Caballero (2007) for an excellent discussion of some of this literature.

This article also complements the extensive literature that ties firing costs to labor market performance. Specifically, our work relates to the large empirical literature on the impact of firing costs using macrodata and microdata. Studies using aggregate data include the earlier works of Bertola (1990), Lazear (1990), and the more recent contribution of DiTella and MacCulloch (2005), among others. There are also a handful of studies examining the impact of firing costs using microdata, including Kugler (1999), Oyer and Schaefer (2000, 2002), Acemoglu and Angrist (2001), Kugler, Jimeno, and Hernanz (2003), Kugler and Pica (2004), Autor, Donohue III and Schwab (2006) and Autor, Kerr and Kugler (2007). Overall, the empirical analysis using data for several OECD countries finds a quite clear relationship between measures of EPL and labour market flows. Countries with more stringent regulations are found to have, everything else being equal, more employment stability, but higher unemployment duration. However, the results are more mixed when analyzing empirically

---

23 These authors define as “rest unemployment” the activity whereby a worker, who intends to reallocate, waits for its current industry’s condition to improve. This concept differs from the notion of “search unemployment,” which involves the actual attempt of a worker to move to a better industry and it is therefore more costly.


25 See for example the interesting contribution of Bertola (1992), who studies the effect of labor turnover costs on the average employment level in a partial equilibrium model of labor demand.

26 Although international comparisons may be difficult if data are not comparable. See Blanchard and Portugal (2001) and OECD (2004).
the relationship between measures of firing costs and unemployment rates (e.g., Guell, 2010). Interestingly, numerical simulations show that our model generates a smooth, hump-shaped relation between equilibrium employment and the stringency of EPL, broadly coherent with these empirical patterns.

The paper is organized as follows. Section 2 lays down the foundations of the model, whose economic equilibrium is obtained and characterized in Section 3. The political equilibrium, its properties and some important applications of the model are characterized in Sections 4, 5 and 6, respectively. Section 7 concludes with a brief discussion of the accomplishments of the paper and presents some directions for potential future work. All the proofs are either in the main text or are reported in the appendix.

2 The Economy

2.1 Basic Environment

The economy is a small and open one, populated by a continuum of measure one of risk neutral workers who always consume all of their disposable income. Workers can be employed or unemployed, and discount future welfare at rate \( r \) equal to the real interest rate. Hence, letting \( \{y_t\}_{t=0}^{\infty} \) denote the uncertain future income stream of a worker, his preferences can be represented as

\[
E_t \left\{ \int_t^{\infty} e^{-r(\tau-t)} y_\tau \, d\tau \right\},
\]

where \( E_t \) denotes the expected value operator, conditional on the information available at time \( t \). Firms are created by a small set of risk neutral entrepreneurs, by paying a fixed cost \( C \).

The available production technology is Leontief, allowing a firm to produce some amount of output per unit of time by hiring one worker only. There are no search frictions, and therefore firms fill their vacancy instantaneously. The productivity \( x \) of each firm is normalized to one at the moment when the firm is created, but it varies over time due to the realization of two independent types of random idiosyncratic shocks. Specifically, \( x \) follows a Geometric Brownian process, whose stochastic differential is represented by

\[
dx = \mu x \, dt + \sigma x \, dW,
\]

\( ^{27} \)Henceforth, we will omit to specify the dependence of endogenous variables on time whenever that does not cause any confusion.

\( ^{28} \)In this respect, the model is different from a matching model with search frictions à la Mortensen and Pissarides, where both sides of the market have to wait for a trading partner. See Coles and Petrongolo (2008) for a comparative discussion between different classes of models of job creation.
where $W$ stands for a Wiener process. The parameters $\mu \in \mathbb{R}_+$ and $\sigma \in \mathbb{R}_{++}$ indicate respectively the drift and the instantaneous standard deviation of $x$.\footnote{It is well known that the stochastic differential equation (2) has explicit solution $x_t$ described by}

\[ x_t = x_0 \exp \left\{ \left[ \mu - \frac{\sigma^2}{2} \right] t + \sigma W_t \right\}. \tag{3} \]

The expression $x_t$ is clearly always positive, a result which matches the natural economic fact that firms cannot produce an output with negative value.

In addition, each production unit is also subject to a Poisson shock with arrival rate $\lambda$, which reflects a potential exogenous voluntary quit of the worker, driving permanently the productivity of the firm to zero.\footnote{In other words, the stochastic process governing the evolution of the productivity of a firm is a compound (Geometric) Brownian and (homogenous) Poisson process.} Importantly, in case of quits firms do not pay any legislated layoff cost, which will apply only to firings (the difference between quits and firings in the model will be clarified below).

To ensure the existence of an equilibrium where some workers are not employed, we restrict the parameters according to the following assumption:

**Assumption 1** $\mu < \sigma^2/2$.

Assumption 1 will be used only in the special case of $\lambda = 0$.\footnote{If $\mu \geq \sigma^2/2$, the steady state rate of job destruction is equal to zero, and therefore the model features a long run equilibrium with full employment for any level of the firing cost (see Subsection 3.2).} Because productivity is variable, a firm may eventually decide to stop producing and to lay-off the worker. When this event happens, the firm pays a mandatory firing cost $F$ for dismissing the worker, which represents a pure deadweight loss, i.e., the corresponding income is entirely wasted. The firing cost $F$ is chosen by the society through a standard political process based on majority voting, described in greater detail in Section 4.

The value of a firm $J(x \mid R, \theta)$ active at time $t \in \mathbb{R}_+$, i.e., the expected present discounted value of the stream of profits gross of the layoff cost as a function of its productivity $x$, conditionally on the endogenous reservation productivity $R$ and on the endogenous job creation rate $\theta$, can be written as

\[
J(x \mid R, \theta) = \sup_{T \in \mathcal{T}_t} E_t \left\{ \int_t^{T \wedge T_\lambda} e^{-r(T - t)} \left[ x - w(x \mid R, \theta) \right] d\tau - Fe^{-r(T - t)} 1_{T \leq T_\lambda} \mid \mathcal{F}_t \right\}, \tag{4}
\]

where $\mathcal{F}_t = \sigma \{ W_s : s \leq t \}$ denotes the filtration generated by the Wiener process $W$. Notice that the supremum is taken over the set $\mathcal{T}_t$ of possible stopping times within $[t, +\infty)$. However, the actual random separation time is equal to the minimum between $\bar{T}$, i.e., the random time at which the firm decides to stop producing, and the arrival time $T_\lambda$ of the exogenous Poisson
quitting shock of the worker. By standard arguments, the value function $\mathcal{J}(\cdot | R, \theta)$ satisfies the following Bellman-Wald functional equation

$$r \mathcal{J}(x | R, \theta) = \max \left\{ x - w(x | R, \theta) + \frac{1}{dt} \mathbb{E}(d\mathcal{J}) - \lambda \mathcal{J}, -rF \right\},$$

(5)

which characterizes the optimal stopping problem of the firm. The right-hand-side of (5) is the maximum between the continuation value of the asset corresponding to the value of the firm, and the flow-equivalent (or annuity value) of the endogenous firing cost $F$. The continuation value is equal to the flow payoff generated by the match, plus the expected capital gain, which is decomposed in the change of the value of the asset due to the Brownian shock, $\mathbb{E}(d\mathcal{J})/dt$, and to the Poisson quitting shock, $-\lambda \mathcal{J}$, respectively.

The solution of the optimization problem represented by equation (5) involves the implementation of a barrier-control policy. The firm closes down, lays-off the worker and pays the mandatory firing cost $F$ as soon as its productivity reaches a reservation level $R$, corresponding to an optimally set threshold. The optimal stopping rule of the firm is characterized in the appendix of the paper. There, we solve the free-boundary problem represented by the differential equation associated with the functional equation (5), or Hamilton-Jacobi-Bellman equation, and the related optimality conditions (i.e., the value matching and the smooth pasting conditions, which generate a set of mixed boundary conditions). For future reference, we define the random calendar time $\tilde{T}_x(R)$ at which the stochastic process describing the productivity of a firm active at time $t \in \mathbb{R}_+$ with $x_t = x$, reaches the absorbing barrier $R$ (ignoring the Poisson quitting shock) as

$$\tilde{T}_x(R) = \inf \{ \tau \in [t, +\infty) : x_\tau = R \mid x_t = x \}.$$  

(6)

The value of a firm $\mathcal{J}(\cdot | R, \theta)$ also satisfies the initial value condition following from the standard assumption of free entry, which implies that firms earn no pure profits in equilibrium, since the ex-ante value of job creation, corresponding to the initial level of productivity $x = 1$ is equalized to the cost hiring set-up $C$. Formally, free entry of vacancies implies that

$$\mathcal{J}(1 | R, \theta) = C.$$  

(7)


33 We use this notation to indicate the Dynkin operator associated with stochastic differential equation (2).

34 A general result shows that the stopping region defined by $S = \{ x \in \mathbb{R} : \mathcal{J}(x | R, \theta) = -F \}$ is of the form $(0, \hat{x})$, where $\hat{x}$ is given by a smooth pasting condition (e.g. Pham, 1009, ch. 5).

35 Note that $\tilde{T}_x(R)$ does not depend explicitly on $t$ because of the time-homogeneity property of transition density of Brownian processes.
2.2 Wage Setting Mechanism

We now describe the wage setting mechanism. It is useful to begin by dividing expression (1) into a pair of recursive equations satisfied by the values of employment and unemployment. By standard arguments, the value $W(x | R, \theta)$ of working in a firm with idiosyncratic productivity $x \in (R, +\infty)$, and the value $U(R, \theta)$ of unemployment satisfy the following system of functional equations

$$rW(x | R, \theta) = w(x | R, \theta) + \frac{1}{dt} \mathbb{E}(dW) + \lambda [U(R, \theta) - W(x | R, \theta)],$$

(8)

and,

$$rU(R, \theta) = b + \theta [W(1 | R, \theta) - U(R, \theta)],$$

(9)

where $w(x | R, \theta)$ is the wage rate paid by the firm to the worker, $b$ is the exogenous level of unemployment compensation (or value of leisure). Both equation (8) and (9) decompose as usual the asset value into the payoff and the related capital-gain. In particular, the capital gain in equation (8) includes the expected variation of the value of the asset due to realization of the Brownian shock, and the Poisson shock, which turns an employed worker into an unemployed. The capital gain in recursion (9) reflects that an unemployed worker becomes employed with flow probability $\theta$, and initial productivity $x = 1$.

The wage rate is determined according to the same wage setting mechanism proposed in Saint-Paul (1999), where a worker employed in a firm with productivity $x$ earns a salary such that the corresponding value of employment is equal to the value of unemployment plus a fraction $\beta$ of the expected present discounted value of the output stream produced by the firm. The sharing rule in question, represented by the equation (10) reported below, can be given a micro-foundation which reflects the rents that firms need to pay to the workers to cope with an underlying moral hazard problem, partially in the spirit of efficiency wage models (e.g., Shapiro and Stiglitz, 1984), consisting in appropriating of part of the current output. The main difference between equation (10) and the sharing rule featured in the model of Shapiro and Stiglitz (1984) is in the rent appropriated by an employed worker which represents a fixed markup over the value of unemployment in their model, and a variable markup in the model presented in this paper,\(^{36}\) reflecting the idiosyncratic productivity of the firm. Also, impor-

---

\(^{36}\)This is because, unlike in the model of Shapiro and Stiglitz (1984), the reward from misconduct (moral hazard) in Saint-Paul’s model is, as already mentioned, getting access to a technology which allows to steal (moral hazard problem) an exogenous fraction of the firm’s (variable) output. Even though the stealing activity is not verifiable by a court, the worker may be caught (with constant probability) in the process of stealing, in which case it is fired. The efficiency wage is therefore the price which deters workers from trying to steal.
tantly, the rent appropriated by the employed depend not only on the monitoring technology available to the firm, but also on the fraction of output that they can appropriate, which can be interpreted as reflecting the political power of organized labor. This is relatively high in Continental Europe, and relatively low in the Anglo-Saxon countries (see the discussion of this point in the Introduction).\textsuperscript{37}

\[
W(x \mid R, \theta) = \mathcal{U}(R, \theta) + \beta V(x \mid R).
\]

In this expression, \(W(x \mid R, \theta)\) and \(\mathcal{U}(R, \theta)\) are defined recursively by (8) and (9), and

\[
V(x \mid R) = \mathbb{E}_t \left\{ \int_t^{T_x(R) \wedge T_\lambda} e^{-r(t-\tau)} x_\tau d\tau \mid x_t = x \right\},
\]

represents the expected present discounted value of the future output stream generated by a firm having at time \(t\) a productivity level \(x_t = x\), up to the minimum between the endogenous absorption time \(T_x(R)\) defined by (6), and the arrival time \(T_\lambda\) of the exogenous Poisson quitting shock. The expected value in (10) is computed with respect to the probability distribution of the random time \(T_x(R) \wedge T_\lambda\). Finally, the parameter \(\beta \in (0, 1)\) in (10) represents the rent extraction power of employed workers.

The sharing rule (10) has several important implications for the model. First, the firing cost affects wages only indirectly, i.e., by reducing the reservation productivity, rather than also directly, i.e., by affecting the relative bargaining power of workers and firms.\textsuperscript{38} Second, the extraction by the workers of a (variable) rent over and above the value of unemployment leads to involuntary unemployment, since the unemployed are willing to work for a wage lower than the wage paid to the employed, but firms are nonetheless unwilling to hire them due to the underlying moral hazard problem existing at the microeconomic level. While the effects of employment protection legislation over the bargaining power of the firms are also potentially interesting, we intend to focus the attention only on the role of the firing cost in extending the duration of jobs, which the sharing rule (10) allows us to do. Third, the sharing rule (10) will

\textsuperscript{37}Moreover, Acemoglu and Newman (2001) document the interesting fact that even though Continental European economies have potentially access to the same monitoring technology as the U.S. and the U.K, clearly they have significantly less investment in monitoring (which also raises the rent of the employed along with pro-Left political institutions). An explanation is provided by Gordon (1996), who argues that the differences in corporate structures (including the level of monitoring) observed across countries reflect the level of control over the natural tendency of corporate bureaucracies to expand their sizes. In the U.S, corporate bureaucracies have been allowed to do so more than anywhere else in Continental Europe.

\textsuperscript{38}This is the case, for example, if wages are set with Nash bargaining, in which case higher firing costs (which firms are supposed to pay), make workers stronger at the bargaining table by increasing the cost of a negotiation breakdown for the firms (see for example Pissarides, 2000, p. 42).
imply that separations are decided unilaterally by firms, rather than “consensually” as they do under Nash bargaining, and therefore the break-up of a “match” reflects, in a proper sense, the “firing” of the worker by the firm.\footnote{Under a Nash bargaining mechanism, the decision to dissolve an employment relationship is always consensual. This is because whenever the surplus of the match is negative, both $\mathcal{J}(x) < 0$ and $W(x) < \mathcal{U}$, due to the transferability of utility. Therefore, the firm prefers to dissolve the match and the worker prefers to quit rather than continuing working. See Pissarides (2000, p. 42) for further discussion.} See also Footnote 40 for further discussion of this issue.

Notice that because the worker is fired and the match broken at the moment the absorbing barrier $R$ is reached, it is the case that $V(R) = 0$. This fact and the sharing rule (10) imply that the following terminal condition

$$W(R \mid \theta) = \mathcal{U}(R \mid \theta),$$

according to which the value of employment at the reservation productivity $R$ is equal to the value of unemployment, is also (mechanically) true. As demonstrated in the appendix, the wage schedule implied by the sharing rule (10) reads\footnote{Note that the sharing rule (10) implies that separations are decided unilaterally by firms, i.e., at the reservation productivity, employed workers would prefer to go on with the match rather than splitting, as the (potential) wage rate is greater than the flow value of unemployment since from equation (10) and equation (13) we have that $w(R) = b + \theta \beta V(1 \mid R) > \mathcal{U}$. Conversely, if wages are set by bargaining à la Nash, workers earn a zero rent at the margin of job destruction, and $w(R) = \mathcal{U}$, since the net surplus of a match is equal to zero at that point. This implies that separations are always mutually optimal for firms and workers, therefore there is no difference between layoffs and quits and no “firing” of workers ever takes place. See Pissarides (2000, p. 42) for further details.}

$$w(x \mid R, \theta) = b + \theta \beta V(1 \mid R) + \beta x.$$  

Clearly, the wage is greater than $b$ for any value of $x$; moreover, it increases both with idiosyncratic productivity and with the exit rate from unemployment in partial equilibrium. Closed-form expressions (for given $R$ and $\theta$) are also computed in the appendix both for $V(\cdot \mid R)$ and for $\mathcal{J}(\cdot \mid R, \theta)$ and read, respectively

$$V(x \mid R) = \frac{x}{r + \lambda - \mu} - \frac{R^{1-\alpha} x^\alpha}{r + \lambda - \mu},$$

and

$$\mathcal{J}(x \mid R, \theta) = \frac{(1 - \beta) x}{r + \lambda - \mu} - \frac{b \beta}{r + \lambda} - \frac{\theta \beta}{r + \lambda} \left( \frac{1 - R^{1-\alpha}}{r + \lambda - \mu} \right) - \frac{(1 - \beta) R^{1-\alpha} x^\alpha}{\alpha (r + \lambda - \mu)},$$

where $\alpha$ corresponds to the negative root of the characteristic polynomial associated with the differential equation satisfied by $\mathcal{J}(\cdot \mid R, \theta)$, whose expression is reported in the appendix.
Note finally that in partial equilibrium wages decreases with the reservation productivity, since workers expect to earn a higher flow of rent as $R$ decreases, an effect which raises the wage rate earned per unit of time.

3 Economic Equilibrium

3.1 Aggregation

In this subsection, we begin the description of the economic equilibrium of the model, assuming that a steady state featuring positive job creation and job destruction exists. In our model each firm is created at some point in time, and experiences thereafter the realization of idiosyncratic shocks to its productivity, until the time when the absorbing barrier $R$ is reached. While the duration of the life-span of each firm is random, the evolution over time of the cohort of firms created at the same point in time is deterministic, since every cohort of new firms is formed by a continuum of units. Therefore, by the law of large numbers, the deterministic fraction of firms of each cohort that are still active at any point in time following their creation, corresponds to the survival probability of a firm from the same cohort up to that time.

Because the transition density function of the stochastic process (2) describing the dynamics of productivity is time-homogenous, the random time $\tilde{T}(R) = \tilde{T}_1(R) - t$ elapsed since the time $t$ of creation of a firm (with productivity is $x_t = 1$), at the moment when absorption takes place, does not depend on the calendar time of creation of the firm. Therefore, we can write the probability distribution of $\tilde{T}(R)$ as follows

$$P\{\tilde{T}(R) \wedge T_\lambda > \tau\} = \int_{R^+} \tilde{p}_\lambda(1, \xi; \tau) d\xi,$$

where $\tilde{p}_\lambda(1, \cdot; \tau)$ denotes the time-homogenous transition density function of $x$, conditional on the absence of absorption or exogenous quit at rate $\lambda$ since the moment of creation of the firm $t$ up to time $t + \tau$.

At time $s$, the flow of workers from unemployment into employment, equivalent to the mass of newly created production units, has measure $\theta_s(1 - L_s)$, where $L_s$ denotes the total mass of employed workers at $s$. Therefore, assuming that the economy begins operating at time zero, the total employment $L_t$ at time $t$ can be decomposed as the integral sum of the firms created over the period $[0, t]$, weighting the mass of firms of each cohort$^{41}$ by the survival probability up to time $t$ of their “representative” unit, so that

$^{41}$We remind that each active firm hires one worker only.
\[ L_t = \int_0^t \theta_{t-s} (1 - L_{t-s}) \mathbb{P} \{ \bar{T}(R) \land T_{\lambda} > s \} \, ds. \] (16)

In the steady state, all aggregate labor market outcomes are stationary, and therefore \( L_t = L, \theta_t = \theta, \) and \( \delta_t = \delta, \) where \( \delta_t \) indicates the aggregate job destruction rate. Moreover, the labor market flows-balance condition

\[ \delta L = \theta (1 - L), \] (17)

equating the number of jobs destroyed per unit of time, \( \delta L, \) to the number of jobs created, \( \theta (1 - L), \) also applies in the steady state, and therefore \( L = \theta/(\delta + \theta).42 \) Combining the steady state form of expression (16), obtained by imposing the condition of stationarity and letting \( t \to +\infty, \) and equation (17), the aggregate steady state job destruction rate \( \delta \) can be written as

\[ \delta = \frac{1}{\int_0^{+\infty} \mathbb{P} \{ \bar{T}(R) \land T_{\lambda} > t \} \, dt}. \] (18)

This result completes the description of the partial economic equilibrium of the model.

### 3.2 Characterization of the Economic Equilibrium

For any given level of firing cost implemented, the economic equilibrium of the model is defined by a pair of equations in two endogenous variables, the reservation productivity \( R \) and the exit rate from unemployment \( \theta. \) The first of these equations is the free entry condition, which can be computed from (7) and (15) and reads

\[ \frac{(1 - \beta)}{r + \lambda - \mu} - \frac{b}{r + \lambda} - \frac{\theta \beta}{(r + \lambda)} \left( \frac{1 - R^{1-\alpha}}{r + \lambda - \mu} \right) - \frac{(1 - \beta) R^{1-\alpha}}{\alpha (r + \lambda - \mu)} = C. \] (19)

The second equation corresponds to the value matching condition, which arises from the solution of the optimal stopping problem of the firm (see the appendix), and establishes the continuity of the firm’s value function upon closing down. This equation reads

\[ \frac{(1 - \beta) R}{r + \lambda - \mu} - \frac{b}{r + \lambda} - \frac{\theta \beta}{r + \lambda} \left( \frac{1 - R^{1-\alpha}}{r + \lambda - \mu} \right) - \frac{(1 - \beta) R}{\alpha (r + \lambda - \mu)} = -F. \] (20)

Equation (19) can be used to obtain the expression of the rate of job creation, as a function of the reservation productivity, or

\[ \theta = \frac{r + \lambda}{\pi \beta (1 - R^{1-\alpha})} \left[ (1 - \beta) R^{1-\alpha} + \pi (r + \lambda - \mu) \left( \frac{1 - \beta}{r + \lambda - \mu} - \frac{b}{r + \lambda} - C \right) \right]. \] (21)

\[ ^{42} \text{Out of the steady state, instead, employment evolves according to the first order differential equation} \]
\[ dL_t/dt = (1 - L_t) \theta_t - \delta_t L_t. \]
Subtracting member-by-member equation (19) and equation (20) one obtains a single equation defining implicitly the equilibrium value of \( R \), or

\[
\frac{1 - \beta}{r + \lambda - \mu} \left(1 - R + \frac{R - R^{\alpha}}{\alpha}\right) = C + F. \tag{22}
\]

Equation (22) states that the expected present discounted value of the flow of gross profits of a firm is equal to the sum of the fixed set-up cost and the firing cost. The economic equilibrium of the model has a recursive structure. Equation (22) defines a downward-sloping relation between \( R \) and \( F \), which determines the unique equilibrium value of the reservation productivity, as a function of a set of exogenous parameters and of the firing cost (which are endogenous in the political equilibrium of the model, but are still treated as given at this stage). Finally, the equilibrium value of \( \theta \) can be computed using the equilibrium value of \( R \) and equation (20). Since equation (20) defines a strictly upward sloping locus in the \((R, \theta)\) plane, the economic equilibrium of the model is unique, and determined at the stage up to the endogenous firing cost \( F \).

Note that equation (20) implies that around the separation point the firm is making losses (i.e., its value is negative), but the existence of firing costs prevent what Caballero (2007) defines as “zoombie” firms to shed unproductive workers.

**Remark 1** According to equation (21), the job creation rate is always positive if the following sufficient condition holds

\[
C \leq C_{MAX} = \frac{1 - \beta}{r + \lambda - \mu} - \frac{b}{r + \lambda}. \tag{23}
\]

**Remark 2** Since the productivity of a firm is always non-negative due to the assumption that its dynamics (up to voluntary quits) is described by a Geometric Brownian process, see equation (2), the reservation productivity \( R \) has a lower bound at zero (potentially reached in an infinite time). Equation (22) therefore implies that the level of the firing cost obtaining as

\[
P \{\hat{T}_{\alpha_0} (R) < T_\lambda\} - P \{\tilde{T}_{\alpha_0} (\hat{R}) < T_\lambda\} = P \left\{ \inf_{0 < s < T_\lambda} z_0 + \sigma W_s + \eta s > \hat{R} \right\} = e^{-\frac{\lambda - \beta}{\sqrt{2\lambda + \frac{\sigma^2}{2}}} z_0 - \frac{\beta - \beta}{\sigma}}.
\]

The RHS of this expression decreases with \( F \) since the reservation productivity falls, and so does the probability that \( \hat{T} (R) < T_\lambda \).
If $F = \hat{F}$, firms never close down and therefore the model has a steady state where the rate of job destruction is $\lambda$ (i.e., separations are only due to exogenous quits). It is straightforward to verify, using equation (19) that $\theta$ tends to a positive limit, $\theta \to \theta_0$, as $R \to 0$;\footnote{We remind the reader that the given the existence of a monotonic, one-to-one relation between firing costs and reservation productivity, it is equivalent, for the analysis of the equilibrium, to let $R \to 0$, and to let $F \to \hat{F}$.} moreover, standard arguments and expression (24) imply that steady state employed is equal to

$$L(R = 0) = \frac{(r + \lambda - \mu) \left( \frac{1-\beta}{r + \lambda - \mu} - \frac{b}{r + \lambda} - C \right)}{(r + \lambda - \mu) \left( \frac{1-\beta}{r + \lambda - \mu} - \frac{b}{r + \lambda} - C \right) + \frac{\lambda \beta}{r + \lambda}}.$$ \hspace{1cm} (25)

To ensure the existence of a steady state featuring both endogenous and exogenous job destruction, we introduce the ad hoc restriction that the firing cost cannot be higher than some given threshold $F_{\text{MAX}} \in (0, \hat{F})$. Summarizing, the firing and the set-up cost are subject the set of restrictions reported in the following assumption.

**Assumption 2** $F < F_{\text{MAX}} < \hat{F}$, where $\hat{F}$ is defined in equation (24) and $C \leq C_{\text{MAX}}$, where $C_{\text{MAX}}$ is defined by (23).

We conclude this subsection by reporting the expression of $\delta$, which is computed in the appendix, the expression of the ergodic probability density function of productivity across active firms, which is also computed in the appendix of the paper, in the case where $F > 0$, and of the expected value of the duration of a job conditional on its current productivity.

**Proposition 1** If $F \in (0, F_{\text{MAX}}]$, the steady state aggregate job destruction rate, $\delta$, reads,

$$\delta = \frac{\lambda}{(1 - R^{-\phi_2})},$$ \hspace{1cm} (26)

and where

$$\delta_0 = \frac{(\sigma^2/2) - \mu}{R^+} = \frac{\eta}{\ln(R)},$$ \hspace{1cm} (27)

is the aggregate job destruction rate if there is no Poisson quitting shock (i.e., $\lambda = 0$), with $R^+ = |\ln R|$, and $\eta = [\mu - (\sigma^2/2)]$.\footnote{A proof is available upon request from the authors that $\delta \to \delta_0$ as $\lambda \to 0$.} In addition, the ergodic cross-sectional distribution of productivity across firms, $\Psi_\lambda(\cdot)$, has probability density function $\psi_\lambda(x)$ represented by\footnote{We are especially grateful to Bjoern Bruegemann for his help in the computation of the ergodic distribution of productivity across active firms.}

$$\psi_\lambda(x) = \frac{\phi_2 \phi_1}{(\phi_2 - \phi_1)} \left\{ \left( \right\{ (\mathbb{1}_{x \geq 1} + \mathbb{1}_{R < x < 1^2}^{1/2}_x^{1/2} - R^{\phi_1 - \phi_2}) R^{\phi_1 - \phi_2} \right) x^{-\phi_1 - 1} \left( 1 - R^{-\phi_2} \right)^{-1} \right\},$$ \hspace{1cm} (28)
where \( \mathbb{1} \) denotes the indicator function defined in the standard way, and \( \phi_1 \) and \( \phi_2 \) are constant defined in the equation (72) reported in the appendix, with \( \phi_1 > 0, \phi_2 < 0 \) and \( \phi_2 - \phi_1 < 0 \).

**Proof.** See appendix. \( \square \)

**Remark 3** Equation (22) implies that \( R = 1 \) obtains in the limit case where \( F = 0 \) and \( C = 0 \), i.e., the reservation productivity is equal to the standardized initial productivity level. As a result, both the rate of job creation (21) and the rate of job destruction (27) are infinite, that is people find and lose jobs instantaneously (and the economy is “fully flexible”).\(^{48}\) Therefore the expected duration of any spell of employment and of unemployment is infinitesimal. Workers are constantly matched and existing matches are constantly destroyed. Workers alternate infinitesimal spells of employment with infinitesimal spells of unemployment in such a way that the fraction of time spent in unemployment is strictly positive. Moreover, the cross-sectional distribution of productivity across employment is a Dirac distribution at \( x = 1 \).

Finally, for the limit case in which \( \lambda = 0 \) and \( R = 0 \) we show (see appendix) that employment is equal to 1.

Under an additional condition on the fundamental parameters of the model, the cross-sectional distribution of productivity has finite mean value.

**Assumption 3** \( \mu < \lambda - (\sigma^2/2) \).

**Corollary 1** The mean value \( \mathbb{E}_\Phi[x] \) of the ergodic distribution (28) is equal to

\[
\mathbb{E}_\Phi[x] = \frac{\lambda}{\lambda - \mu - (\sigma^2/2)} \frac{1 - R^{1-\phi_2}}{1 - \phi_2}.
\]

**Proof.** Straightforward integration of equation with respect to the density defined by equation (28). \( \square \)

An important corollary of Proposition 1 concerns how EPL affects the mean level of productivity across establishments \( \mathbb{E}_\Phi[x] \), which is equivalent to the mean value of the productivity of labor (as one firms hires one worker only).

**Corollary 2** Higher firing costs reduce the equilibrium average productivity of labor.

\(^{48}\)As already anticipated, if we introduced search frictions by assuming a matching function, both sides of the market, firms and workers, would be rationed, i.e., find a match in a positive expected time. Conversely, in the current model, vacancies are filled immediately and only workers experience “wait unemployment.” However, we conjecture that the structure of the preferences would still depend on employment status and idiosyncratic productivity similarly to our model, and therefore the structure of the equilibrium would not essentially change, depending on properties of the particular search technology assumed.
Proof. Straightforward differentiation of equation (29).

In our economy, this result is not surprising since firing costs correspond to a pure dead-weight loss, and have no potential role in improving the competitive allocation of resources.\(^{49}\) The result, however, is not necessarily true in economies featuring some market failure. See for example Ramey and Watson (1997) or Chari, Restuccia and Urrutia (2005), for a rationale of why dismissal costs may improve the allocation of resources and TFP relative to the Walrasian benchmark.\(^{50}\)

Furthermore, we can compute the expected value of lifetime of a firm conditionally on its productivity.

**Proposition 2** The expected duration of a job with current productivity \(x\) reads

\[
E_t [\bar{T}_x (R) \wedge T_x] = \frac{1}{\lambda} \left[ 1 - \left( \frac{x}{R} \right)^{\phi^2} \right].
\] (30)

Proof. See appendix. \(\blacksquare\)

**Corollary 3** If \(\lambda = 0\) (i.e., \(T_\lambda = +\infty\)), the expected duration of a firm with productivity \(x\) reads

\[
E_t [\bar{T}_x (R)] = \frac{\ln(x) - \ln(R)}{|\eta|}.
\] (31)

Proof. Straightforward application of de l’Hospital’s rule to equation (30). \(\blacksquare\)

Differentiation of equation (31) shows the intuitive result that higher embodied productivity growth and current productivity increase, for given \(R\), the life expectancy of a firm by driving productivity away from \(R\).\(^{51}\) Also, for a given \(R\), higher instantaneous variance reduces the expected duration of a job by making a critical productivity downfall more likely to happen. This effect is instead dampened in general economic equilibrium, where higher \(\sigma\) reduces the reservation productivity (see below), and therefore tends to increase the expected lifetime of a firm. Overall \(E_t [\bar{T}_x (R)]\) is therefore a non-monotonic function of \(\sigma\).

---

\(^{49}\)Bertola (1994) demonstrates an analogous result in an endogenous growth model where firms experience idiosyncratic productivity shocks, and shows that EPL has a monotonic negative effect on the rate of growth. Similarly, in Saint-Paul (2002), firing costs artificially extend the life-cycle of relatively obsolete firms and slow down the pace of technological renewal of the economy.

\(^{50}\)Interestingly, Chari, Restuccia and Urrutia (2005) report evidence (see figure 1, p. 5) of a positive correlation between EPL and TFP (relative to US level) across European countries. Of course, where the correlation in question uncovers any causal effect or not remains an open question.

\(^{51}\)It is also the case that in general economic equilibrium (see below) productivity growth increases the expected duration of a production unit by reducing the reservation productivity. Therefore both the partial and the general economic equilibrium effects of \(\mu\) on the expected duration of a production unit go in the same direction.
3.3 Comparative Statics

The economic equilibrium of the model has a number of comparative statics properties, some of which are non-standard, which are discussed next, with the qualification that the firing cost will still be treated as an exogenous parameter at this stage of the analysis of the model (general economic equilibrium).\(^5\)

1. A higher value of the firing cost \(F\) reduces the reservation productivity \(R\) since firms prefer to hold on longer when layoffs are more costly; as a result, both the aggregate rate of job destruction \(\delta\) and the exit rate from unemployment \(\theta\) (which are increasing in \(R\)) fall. It follows from equation (17) that an higher firing cost has overall ambiguous effects on the level of equilibrium employment.\(^3\) Nevertheless, our numerical simulations (see figure 1) show that the relation between firing cost and equilibrium employment is humped-shaped: employment first decreases and then increases with the stringency of EPL.

2. A higher value of the rent extraction power \(\beta\) reduces the reservation productivity, the aggregate job destruction rate and the exit rate from unemployment.\(^4\) The reservation productivity falls since in partial equilibrium job creation is lower when workers appropriate more rents; as a result, firms prefer to hold on for a longer time, i.e., \(R\) decreases with \(\beta\). The intuition is, as in Bertola (1990), that the firing cost also acts as hiring cost. Therefore, in equilibrium cumulated profits should be large enough to cover that cost; when \(\beta\) increases the firm’s flow profit is lower, so that the firm needs to stay longer in business to cover the firing cost. The aggregate job destruction rate falls as a result of the fact that the reservation productivity is lower (see equation (26)). The exit rate from unemployment decreases with \(\beta\), because of its direct effect and of its general equilibrium effect (i.e., through the reservation productivity) on \(\theta\), which are both negative.

3. Higher volatility \(\sigma\) decreases the equilibrium reservation productivity, because in a more turbulent environment the option value of a job is higher for the firm. The impact of \(\sigma\) on the steady state aggregate rate of job destruction \(\delta\) is instead ambiguous. This is because \(\delta\) decreases with \(\sigma\) through \(R\), due to the negative effect that volatility has on

\(^{5}\)In the remaining, unless stated, we keep the assumption that the exogenous job destruction rate \(\lambda\) is strictly positive.

\(^{3}\)This is a very well known and general result, first pointed out by Bentolila and Bertola (1990) and more recently, among others, by Blanchard and Portugal (2001).

\(^{4}\)See also Saint-Paul (1999), Proposition 2 and 3.
the reservation productivity, and to the fact that $\delta$ increases with $R$. However, $\sigma$ has an ambiguous effect on $\delta$ in partial equilibrium, i.e., holding $R$ constant.$^{55}$

4. Higher volatility *stimulates* job creation, i.e., it raises $\theta$. This is a consequence of the *convexity effect* of volatility, which increases the value of a firm and therefore drives up job creation.$^{56}$ Notice that this is a general equilibrium effect, which dominates over the negative partial equilibrium effect that $\sigma$ has on $\theta$ due to the fact that $\theta$ increases with $R$, and that $R$ decreases in equilibrium with $\sigma$, as we already know.

5. A higher value of the drift coefficient $\mu$ *increases* the equilibrium reservation productivity. This is because higher productivity growth raises both the expected output produced by a match, and the cost of labor by increasing the value of the rent appropriated by the employed. The second effect dominates, inducing firms to dismiss workers sooner; as a result, $R$ raises with $\mu$. The impact of $\mu$ on $\delta$ is instead ambiguous, since $\mu$ has a positive indirect effect due to the increment of $R$, which leads to more job destruction, but an ambiguous direct effect on it.$^{57}$ Our numerical simulations show that the effect of $\mu$ on $\delta$ is hump-shaped: first it decreases and then it increases as the drift of the productivity increases (figure 5). The impact of $\mu$ on $\theta$ is also ambiguous, since both the value of output and the cost of labor are increasing in $\mu$.

6. A higher value of $\lambda$ has ambiguous effects on the reservation productivity. This is because as the exogenous separation rate increases, firms may prefer to rely more on quits (which are not subject to firing cost) than on dismissal, and therefore keep the worker longer. In addition, a higher quit rate has ambiguous effects on the aggregate job destruction rate, as the direct ambiguous effect of $\lambda$ on $\delta$ comes along with the ambiguous indirect effect through the reservation productivity. Our numerical simulations show that the effect of $\lambda$ on $\delta$ is hump-shaped: first it decreases and then it increases as the exogenous job destruction rate increases (figure 6). Finally, job creation also depends ambiguously on $\lambda$ since a higher quit frequency has a standard negative effect on profitability, by reducing the *ex-ante* value of creating a production unit. However, a higher quit rate also reduces

$^{55}$Note that whenever $\lambda = 0$, $\delta_0$ monotonically increases with $\sigma$ in partial equilibrium, i.e., holding $R$ constant; however, it decreases with $\sigma$ through $R$. Therefore, the overall effect on $\delta_0$ is also ambiguous.

$^{56}$Conversely, in a matching model with endogenous separations, and where idiosyncratic uncertainty is described by a homogenous Poisson process, a higher arrival rate of productivity shocks reduces job creation (see for example Pissarides, 2000, p. 10).

$^{57}$Note that whenever $\lambda = 0$, $\mu$ has a negative direct effect on $\delta_0$ and a positive indirect effect through $R$. Therefore, the overall effect on $\delta_0$ is also ambiguous.
the expected duration of the time during which the firm makes negative profits, but it is prevented from closing-down by the firing cost, i.e., it relaxes the actual degree of labor market rigidity. On the whole, more frequent quits have therefore an interesting ambiguous effect on equilibrium employment.

7. A higher a value of $C$ has a direct negative effect on profitability, for given wages, by raising the cost of creating a firm, and therefore reduces the extent of job creation. However, a higher set-up cost $C$ also reduces the reservation productivity (for a given $F$), and it reduces job destruction. Both of these effects are due to the fact that higher set-up costs reduce wages (by reducing the incentives of creating jobs and therefore $\theta$), and they both induce firms to go on longer with a worker. The overall effect of $C$ on equilibrium employment is therefore ambiguous.

Proof. See appendix.

3.4 Numerical Simulations

To complete our understanding of the model, here we present a simple calibration. We choose realistic parameters following Saint-Paul (1999), Bentolila and Bertola (1990) and Nagypál (2005). This calibration is meant to be a useful instrument to better understand the model dynamics; however, it is not expected to match the real data. We consider the following set of parameter values: $\mu = 0.0008$, $\sigma = 0.18$ and $\lambda = 0.1$. The unemployment utility $b$ is set equal to 0 since unemployment benefits do not play a particular role in this model.

In table 1, we study the effect of an increase of the firing cost $F$ on the reservation productivity, the aggregate job destruction rate, the exit rate from unemployment and on employment, keeping the rent extraction power constant. Both reservation productivity and aggregate destruction decrease with $F$, as well as the exit rate from unemployment. The relationship between firing cost and employment is hump-shaped, as shown in figure 1 and in line with the corresponding result of Bentolila and Bertola (1990). For small values of the firing cost the job creation effect prevails and employment is lower; however, for larger values of the firing cost, the job destruction effect is dominant and employment increases.

Table 2 shows the effect of an increase of the rent extraction power on the reservation productivity, the exit rate from unemployment and on employment. Both the reservation pro-

---

Note that this effect is absent in a framework where firms do not make losses at the separation margin, as they do in our model.
ductivity and the exit rate from unemployment are decreasing in \( \beta \). In addition, employment, which is a function of \( \theta \) and \( \delta \),\(^{59}\) is also decreasing in \( \beta \), as shown in figure 2.\(^{60}\)

Table 3 shows the effect of an increase of the discounting parameter \( r \) on employment, keeping the firing cost constant. An increase of the discounting rate implies that firms discount more the future and therefore the discounted value of the firing cost (which is paid in the future, from the viewpoint of the job-creation time) is lower. This effect increases job creation, and as a consequence employment is higher (figure 3).\(^{61}\)

In table 4, we study the effect of an increase of the exogenous job destruction rate \( \lambda \) on the equilibrium value of a firm \( J \). Our numerical simulations indicate that in some cases the relationship is hump-shaped: first, the value of the firm increases and then it decreases as a function of \( \lambda \) (figure 4). This shows that a higher value of the quit rate can increase the value of the firm, by reducing the expected duration of the period where the firm operates but makes negative profits, and therefore by relaxing the effective stringency of EPL.\(^{62}\)

Table 5 shows the effect of an increase of the drift coefficient of productivity \( \mu \) on the aggregate job destruction rate \( \delta \). While \( \mu \) has a positive indirect effect through the reservation productivity, due to the fact that as the exogenous separation rate increases firms may prefer to keep the worker longer (by relying more on quits than on layoffs), it has an ambiguous direct effect on it. Therefore, the overall effect of \( \mu \) on \( \delta \) is ambiguous. Our numerical simulations show that the effect of \( \mu \) on \( \delta \) is hump-shaped: first it decreases and then it increases as the drift coefficient of productivity increases (figure 5).

Table 6 shows the effect of an increase of the exogenous job destruction rate \( \lambda \) on the aggregate job destruction rate \( \delta \). Since \( \lambda \) has both an ambiguous indirect effect through the reservation productivity and an ambiguous direct effect, the overall effect of \( \lambda \) on \( \delta \) is ambiguous. Our numerical simulations show that the effect of \( \lambda \) on \( \delta \) is hump-shaped: first it decreases and then it increases as the exogenous job destruction rate increases (figure 6).

These results complete the characterization of the general economic equilibrium of the model. The next section characterizes the general political economy equilibrium, i.e., the general economic equilibrium with endogenous firing costs.

---

59 As shown in Section 3.1, the steady-state expression for employment is a function of \( \theta \) and \( \delta \), that is \( L = \theta/(\delta + \theta) \).

60 Note that for low values of the firing cost \( F \), employment is decreasing in \( \beta \) since the effect on the reservation productivity dominates the effect on job creation.

61 See also Bentolila and Bertola (1990), who first pointed out how the consequences of layoffs restrictions depend on slenderest rate through the same discounting effect operating here.

62 This result, again, is in contrast with the standard search and matching model, where the relationship between the exogenous job destruction rate and the value of the firm is monotonically decreasing since separations are always jointly optimal for both partners (see Pissarides, 2000, p. 13).
4 Politics

4.1 The Political Mechanism

We assume that a given level of the firing cost $F = F_0$ is initially implemented, representing the status quo level of employment protection, and that the economy is in the corresponding stationary equilibrium. The status quo value of $F$ may be changed as a result of a majority voting process. We assume that voting on the firing cost takes place only once, immediately after an unexpected shock to the exogenous variables of the model occurs, potentially affecting the rent extraction power of the workers, and the drift and standard deviation of the Brownian process describing the evolution of productivity, when the economy is in the politico-economic equilibrium corresponding to $F = F_0$.\(^{63}\) The new legislated firing cost corresponds to any point of the policy space, i.e., the interval $[0, F_{MAX}]$.\(^{64}\)

For analytical reasons, it is convenient to assume that workers vote for the effective level of employment protection, i.e., the reservation productivity $R$, rather than for legal employment protection, i.e., the level of the firing cost $F$. Since the bijective relation between $F$ and $R$ exists according to equation (22), voting on $F$ is equivalent to voting on the corresponding level of $R$. The relevant policy space is thus the interval $[R_{MIN}, R_{MAX}]$, where we remind that $R_{MIN}$ is defined as the reservation productivity corresponding to the maximum feasible level of the firing cost $F_{MAX}$, and $R_{MAX}$ is defined as the reservation productivity corresponding to the minimum feasible level of the firing cost $F_{MIN} = 0$.

It must be emphasized at this point that if the legislated firing cost differ from $F_0$, the transition to the new politico-economic equilibrium is instantaneous\(^ {65}\) for $\langle \theta, J(x \mid R, \theta), w(x \mid R, \theta), W(x \mid R), U(R) \rangle$, which are all functions of jump-variables only.\(^ {66}\) Since the

\(^{63}\)The assumption that voting takes place only once rules out the interesting but potentially complicated effects that the anticipation of the future political equilibria has on the current voting decision of the workers (see Hassler, Storesletten, Rodríguez Mora and Zilibotti, 2000, and Acemoglu, Ticchi and Vindigni, 2011, for examples of dynamic political games based on repeated majority voting).

\(^{64}\)The assumption that voting takes place immediately after a shock hits the economy reflects the fact that it is optimal for the majority of workers to vote immediately rather then to wait. This is because, if a majority of workers is in favor of changing the status quo, it is strictly better-off by doing it as soon as possible; vice versa, if the majority is in favor of preserving the status quo after the shock, it gains nothing by postponing voting.

\(^{65}\)Bruegemann (2007) investigates the case in which a change in employment protection is implemented with time delays. He shows that in countries with flexible labour markets, workers are less in favor of moving towards an equilibrium with more stringent protection, due to the increased risk of dismissal before the actual implementation. The difficulties in introducing protection could therefore represent an alternative explanation for differences in employment protection across countries.

\(^{66}\)Equation (22) implies that $R$ depends only on $F$ and therefore it immediately adjusts to the steady state value corresponding to the new value of $F$. Equation (21) implies that $\theta$ only depends on $R$ and therefore it also adjusts instantaneously. Finally, the expressions of the value functions of all the workers show that the only endogenous variables on which they depend are $R$ and $\theta$, which as stated above, are jump-variables. Therefore,
welfare of all the workers jumps instantaneously to the new steady state level, in deciding how to vote workers simply compare their value across different steady states. Instead, the state-variables (i.e., the cross-sectional distribution of productivity, and the level of employment) do not change instantaneously (and in particular at the instant of voting), but gradually converge to their new steady state.

It is not possible to characterize the political equilibrium of the model using the median voter theorem since, as we already know, the preferences of a set of positive measure of agents do not satisfy the single-peakness (and neither the single-crossing) property. Nonetheless, we are able to demonstrate that the social preferences over employment protection regulation induced by majority voting do not indeed cycle, i.e., that a political equilibrium always exists. In particular, it is possible to demonstrate that a unique Condorcet winner supported at unanimity exists regardless on what the status quo level of the firing cost is, provided the rent extraction power of the workers is below some threshold value. When the rent extraction power of the workers exceeds the threshold value in question, a unique political equilibrium still exists, but the corresponding policy is not voted at unanimity and, more importantly, the political equilibrium can depend on the status quo level of employment protection. While admittedly somewhat extreme, this feature of the model allows us to complete its solution relatively simply. To make progress in the characterization of the political equilibrium, we compute next the expressions of the value functions of all workers.

4.2 The Structure of the Preferences over Labor Market Regulation

In this subsection, we describe the preferences over employment protection regulations of all workers. We begin by computing the values of the unemployed and of the employed workers. Combining equations (8), (9), (10), (13) and eliminating \( \theta \) in the resulting expression by using (21), the value of unemployment \( U(R) \) can be written as

\[
U(R) = \frac{1 - \beta}{r + \lambda - \mu} \frac{(1 - \beta) R^{1-\alpha}}{(r + \lambda - \mu) \alpha}.
\]

Straightforward differentiation of (32) shows that the value of unemployment is strictly increasing in \( R \). Intuitively, unemployed workers would be strictly better-off in a fully flexible labor market where layoffs are not constrained in any way and where therefore the exit rate from unemployment is as high as it can be.

the value functions also adjust instantaneously.
The value of employment $W(\cdot \mid R)$ is instead computed by combining equations (14) and (32), and reads

$$W(x \mid R) = \frac{1 - \beta}{r + \lambda - \mu} - \frac{(1 - \beta) R^{1-\alpha}}{(r + \lambda - \mu) \alpha} + \beta x - \frac{R^{1-\alpha} x^\alpha}{r + \lambda - \mu}. \quad (33)$$

Equation (33) allows us to determine how the welfare of employed workers depends on a marginal increment in $R$.

**Lemma 1** Let $R = R_0$ denote the status quo reservation productivity. All workers employed in firms with productivity $x \in (x^*, +\infty)$, where

$$x^* = \left[ \beta \pi (1 - \beta)^{-1} \right]^{\frac{1}{\alpha}}, \quad (34)$$

benefit strictly from a marginal increment in labor market flexibility (i.e. an infinitesimally higher value of $R$), all workers in firms with productivity $x \in (R_0, x^*)$ are made strictly worse-off, and all workers in firms with productivity $x = x^*$ are indifferent.\(^68\)

**Proof.** A straightforward differentiation of equation (33) shows that $\partial W(x \mid R) / \partial R \geq 0$ for any $R$ if $x \geq x^*$, and that $\partial W(x \mid R) / \partial R = 0$ if $x = x^*$. \(\blacksquare\)

Lemma 1 tells us that the workers employed by relatively productive firms (i.e., with $x > x^*$) are made better-off if the labor market becomes marginally more flexible, while the workers employed by relatively unproductive firms are made worse-off. Intuitively, employment protection involves some benefits, due to the extension of the duration of the rent appropriated by the employed, but also costs, due to its adverse general equilibrium effect on job creation.

By differentiating equation (33), the total effect of a marginal increment of $R$ on the welfare of the workers with productivity $x$ can be decomposed in two parts, corresponding respectively to the marginal gain, $\partial U(R) / \partial R$, and to the marginal loss, proportional to $\partial V(x \mid R) / \partial R$. The gain of increasing marginally $R$ is the same for all workers, independently of their individual productivity, since it is due entirely to the corresponding variation of the value of unemployment (expressed by the sum of the first two terms in (33)), which as we know is positive. Conversely, the loss caused by more flexibility, due to the reduction of the value of the rent appropriated by the employed (i.e., the fourth term in (33)), can be shown to be decreasing in $x$.\(^69\) This result implies that relatively more productive workers lose relatively less by a relaxation of

\(^{67}\)For convenience, we remind the reader that $\pi \equiv |\alpha|$.

\(^{68}\)If $R_0 \geq x^*$, the interval $(R_0, x^*)$ is of course empty, in which case all employed workers are made strictly better-off if the firing cost is relaxed infinitesimally.

\(^{69}\)This follows from a straightforward differentiation of $\partial V(x \mid R) / \partial R$ with respect to $x$.  

27
the firing discipline, and explains why the workers with productivity above \( x^* \) are better-off in a more flexible labor market, and vice versa.\(^{70}\) We remark that the threshold \( x^* \) does not depend on the initial reservation productivity \( R_0 \) since, as equation (33) shows, the value of each employed worker is either strictly increasing or strictly decreasing in \( R \) if \( x \neq x^* \), and it does not depend on \( R \) if \( x = x^* \).

The analysis of the equilibrium conducted so far, may suggest that all workers with productivity below \( x^* \) are always harmed by more flexibility. However, this intuition is not correct because Lemma 1 only determines how a marginal increase in labor market flexibility affects the welfare of employed workers, as opposed to a discrete increase in \( R \). If \( R \) increases from \( R = R_0 \) to a higher value \( R = R' \), i.e., if labor market flexibility increases by a non-infinitesimal amount, then a set of jobs of non-zero measure, corresponding to the firms with productivity in the interval \([R_0, R']\), that were initially prevented from closing down by the tighter firing restrictions, is instantaneously destroyed. It can be shown that a set of positive measure of least productive workers exists, who are better-off if \( R = R' \) than they are in the status quo, despite the fact that they are fired if the reform is implemented. Intuitively, this is because the function \( W = W(x|R_0) \) is continuous in \( x \) over the range \([R_0, +\infty)\).\(^{71}\) This means that the welfare of the employed workers with productivity in a small right-neighborhood of \( R_0 \) (i.e., with \( x \approx R_0 \)) is approximately equal to \( U(R_0) \). Moreover, voting involves the choice between two alternatives which, for the workers with productivity \( x \approx R_0 \) is approximately equivalent to the choice between being unemployed in a relatively rigid (i.e., with \( R = R_0 \)) and in a relatively flexible economy (i.e., with \( R = R' \)). Given that the value of the unemployed is everywhere increasing in \( R \), it is clear that the workers employed by firms whose idiosyncratic productivity is sufficiently close to the status quo reservation productivity \( R_0 \) will vote for a more flexible labor market. This argument can be stated more formally as follows.

**Lemma 2** In voting between the two alternatives \( R_0 \) and \( R' \), where \( R' > R_0 \), the status quo is preferred by the employed workers with a level of idiosyncratic productivity \( x \in (x'^*_0, x^*) \) where \( x^* \) is defined by (34), and \( x'^*_0 \) is defined by equation (35). All the unemployed, and the employed with productivity \( x \in [R_0, x'^*_0) \cup (x^*, +\infty) \), form a coalition voting for the alternative \( R' \).

**Proof.** Let \( x'^*_0 \) be the expression of the productivity level at which workers are indifferent

---

\(^{70}\)This result depends on the nature of the stochastic process governing the dynamics of productivity, and in particular on the persistence property of the geometric Brownian motion. If the realizations of \( x \) were governed by a homogeneous Poisson process, an infinitesimal increment of \( R \) would have the same effect on the lifetime utility of all of the employed.

\(^{71}\)In particular, we remind that the terminal condition (12) implies the continuity of \( W - W(\cdot|R) \) at \( x = R \), for any value of \( R \).
between the two policy alternatives considered above. Letting \( W(\cdot | R_0) \) denote the value of employment in the status quo labor market, as a function of idiosyncratic productivity, and letting \( U(R') \) denote the value of unemployment in the reformed labor market, the threshold \( x_0' \) is defined implicitly by the equation \( W(x_0'| R_0) = U(R') \), which can be written (using the expression \( x^* \) reported in equation (34)) as

\[
x_0' - (R_0)^{1-\alpha} (x_0')^\alpha = (x^*)^\alpha \left[ (R')^{1-\alpha} - (R_0)^{1-\alpha} \right]. \tag{35}
\]

Since the left-hand-side of this equation is strictly increasing in \( x_0' \), and equal to zero if \( x_0' = R_0 \), whereas its left-hand-side is strictly positive, equation (35) has always a unique solution over the range \((R_0, +\infty)\). Hence, it exists a semi-closed set of positive measure \([R_0, x_0']\), such that the workers employed in firms with productivity in this interval are strictly better-off as unemployed in the more flexible labor market with \( R = R' \), than as employed in the status quo equilibrium with \( R = R_0 \). We summarize the last set of results in the following lemma, which will be used later on in the characterization of the political equilibrium level of firing cost.

A particularly important implication of the analysis leading to Lemma 2 is the existence of a set of positive measure of workers, i.e., the employed in firms with productivity smaller than \( x^* \), whose preferences over \( R \) are not single-peaked. The value of these workers is equal to \( W(x| R) \) for any \( R \) such that \( R_{MIN} \leq R \leq x \) and, by Lemma 1, it is strictly decreasing in \( R \) over the same range. However, the value of the same workers is equal to \( U(R'| R) \) (since they are fired) for any \( R \) such that \( x \leq R \leq 1 \), which as we know is strictly increasing in \( R \), i.e., their preferences have two peaks. The presence of a set of positive measure of agents with non single-peaked preferences in the policy variable implies the violation of one of the assumptions of the median voter theorem, which therefore can not be applied to solve for the political equilibrium of the model.

5 Political Equilibria

With no loss of generality, we will henceforth assume for analytical convenience that set-up costs, which are exogenous, are normalized to zero; therefore \( R_{MAX} (F_{MIN} = 0) = 1 \). Also, before proceeding to characterize the political equilibria of the model, we introduce the following pair of definitions.

**Definition 1** \( R = R^* \) is a political equilibrium conditional on the status quo, if it defeats any other alternative in pairwise comparisons conditionally on \( R = R_0 \).
Definition 2 $R = R^*$ is an unconditional political equilibrium, if it defeats any other alternative in pairwise comparisons regardless on the status quo value of $R$.

We also define the threshold $\hat{\beta}$ as the unique value of $\beta$ such that $x^*(\beta)$ is equal to the initial productivity $x = 1$,\(^{72}\) or
\[
\hat{\beta} = \frac{1}{1 + \pi}.
\] (36)

Proposition 3 If $\beta \leq \hat{\beta}$, where $\hat{\beta}$ is defined by (36), then the unique unconditional political equilibrium of the model involves setting $R = 1$, and this choice is preferred at unanimity to any alternative.

Proof. See appendix. □

Proposition 3 tells us that as long as the rent extraction power of the employed is relatively low, a fully flexible labor market is politically stable, in the sense that workers prefer it at unanimity to any possible alternative, whatever the status quo is. The intuition for this result is that when the rents appropriated by the employed are small enough, workers have little reason to protect them by demanding any job security provisions, since the costs of employment protection are larger than the corresponding gains for any positive value of $F$.

To complete the characterization of the political equilibrium, let us define $\bar{x}$ as the productivity level such that a worker is indifferent between being employed in the most rigid economy, i.e., where $R = R_{MIN} = R(F_{MAX})$, in a firm with idiosyncratic productivity equal to $\bar{x}$, and unemployed in the most flexible economy possible, i.e., where $R = R_{MAX} = R(F_{MIN} = 0) = 1$. Formally, $\bar{x}$ is defined implicitly by equation (35), setting $R_0 = R_{MIN}$ and $R_{MAX} = 1$ (see above). Finally, let $\lambda_\psi \{\cdot\}$ indicate the Lebesgue-Stieltjes measure induced by the stationary distribution of productivity across active firms $\Psi_\lambda(\cdot)$, characterized by Proposition 1, for any given initial level of job security provisions.\(^{73}\)

Proposition 4 Suppose that $\beta > \hat{\beta}$, where $\hat{\beta}$ is defined by expression (36). We have that:

1. If $R_0 < \bar{x}$, then $R = R_{MIN}$ is the unique conditional political equilibrium if
\[
\lambda_\psi \{(\bar{x}, x^*)\} L \geq \frac{1}{2},
\] (37)

\(^{72}\)Since $x^*(\cdot)$ is strictly increasing in $\beta$ and onto $(0, 1)$, the equation in question has always a unique solution.

\(^{73}\)There is no confusion between “$\lambda$” defined as the arrival rate of the exogenous quit shock, and “$\lambda_\psi \{\cdot\}$” indicating the measure of any Borel set $\mathcal{A} \in \mathcal{B}(\{R_0, +\infty\})$. Moreover, the density of the stationary distribution (28) is absolutely continuous w.r.t. Lebesgue measure. As a direct consequence, single points are null set for $\lambda_\psi$ and, in particular the measure does not distinguish open and closed sets.
where \( L \) denotes the level of employment corresponding to the initial level of labor market rigidity. Vice versa, \( R = R_{\text{MAX}} = 1 \) is the unique conditional political equilibrium if the reverse of condition (37) holds.

2. If \( R_0 \geq \bar{x} \), then \( R = R_{\text{MIN}} \) is the unique conditional political equilibrium if

\[
\lambda_{\Psi}\{(R_0, x^*)\} L > \frac{1}{2}.
\]

Vice versa, \( R = R_{\text{MAX}} = 1 \) is the unique political equilibrium if the reverse of condition (38) holds.

**Proof.** See appendix. ■

Proposition 4 describes the basic structure of the political equilibrium, which has the following characteristics. First, according to Proposition 4 the political equilibrium always exists, and it involves either the choice of an unregulated labor market (i.e., \( R = 1 \)) or of the most rigid labor market possible (i.e., \( R = R_{\text{MIN}} \)). Second, according to Proposition 4, labor market rigidity is supported by the workers who are employed in firms which have an intermediate level of idiosyncratic productivity when voting occurs. Vice versa, flexibility is supported by an extreme coalition made up by the workers employed by the more and by the less productive firms, and also by all the unemployed.74

**Remark 4** If \( \beta = \hat{\beta} \), equation (35) implies that \( x^* = \bar{x} = 1 \). Since \( \bar{x} \) is decreasing in \( \beta \), and \( x^* \) is increasing \( \beta \) and equal to 1 when \( \beta = \hat{\beta} \), we have that \( \bar{x} < x^* \) and \( R_0 < x^* \) for any \( \beta \) greater than \( \hat{\beta} \). It follows that the sets of workers in favor of a rigid labor market defined in two cases of Proposition 4 are both non-empty.

### 5.1 Some Comparative Statics Properties of the Political Equilibrium

In this subsection, we characterize how some endogenous elements of the equilibrium of the model, i.e., the thresholds \( x^* \), \( \bar{x} \) and \( \hat{\beta} \), are affected by the key exogenous parameters.

We begin by observing that the threshold value \( x^* \) depends directly on the rent extraction power of the employed \( \beta \) and indirectly (i.e., through \( \pi \)) on the drift \( \mu \) and on the instantaneous standard deviation \( \sigma \). In particular, it can be shown with a straightforward differentiation of equation (34) that \( x^* \) is strictly increasing in \( \beta \). This result is not surprising since employment

---

74 As the coalition of low and high productive individuals becomes larger, economies having initially a very high firing cost, which allows the survival of low-productivity jobs and generates a highly dispersed productivity distribution, will tend to support reforms increasing the flexibility of the labor market.
protection is attractive for the employed only if the rents that they capture, which are propor-
tional to $\beta$, are large enough to compensate for the general equilibrium distortions caused
by the firing cost, i.e., for the reverse of the job creation effect on their utility. Perhaps more
surprisingly, we also find that how $x^*$ is affected by the other two parameters of interest, i.e.,
$\sigma$ and $\mu$, depends on the rent extraction power of the employed in the sense explained by the
following lemma.

**Lemma 3** Higher volatility and lower growth both increase the threshold $x^*$ defined in (34)
if $\beta \in (\beta^*, 1)$ where $\beta^* = e/(e + \pi)$, where “$e$” denotes Euler’s number. Higher volatility and
lower growth both decrease $x^*$ if $\beta \in (0, \beta^*)$, and do not affect $x^*$ if $\beta = \beta^*$.

**Proof.** It follows from the differentiation of expression (34). ■

In words, when the power of rent extraction of employed workers is relatively high, i.e.,
if $\beta > \beta^*$, more volatility increases the productivity $x^*$ of the marginal worker, as defined in
Lemma 1, and vice versa; intuitively, this result is due to the following reason. In a more
volatile economy, both the positive impact of higher labor market flexibility on the utility
of the employed through the job creation effect, and its negative impact through the rent
erosion effect are magnified, reflecting the existence of a complementarity between flexibility
and volatility.

Which of these two opposite forces dominates over the other depends on $\beta$. If $\beta > \beta^*$,
the rent is a relatively important component of the welfare of the employed, and therefore the
magnification of the utility loss due to the rent erosion effect dominates over the magnification
of the utility gain due to the job creation effect. As a result, the threshold $x^*$ expressing the
productivity of the marginal worker has to increase in order to ensure a greater insulation
from the risk of job destruction of the pivotal worker.\textsuperscript{75} The opposite happens if the power of
rent extraction of the employed is relatively small, i.e., if $\beta < \beta^*$, and as a result the critical
productivity level $x^*$ falls.\textsuperscript{76}

\textsuperscript{75}We remind that, because the paths of a Brownian process are (almost surely) continuous, the current
productivity level of a firm exhibits some degree of persistence in the future. Therefore, the matches that are
relatively productive in the present are exposed to a lower risk of destruction in the future, all else equal. This
is reflected in fact that, as already remarked, the rent of the employed decreases less with $R$ the greater is $x$,
i.e., the cross-partial derivative of $V$ with respect to $x$ and $R$ is strictly positive.

\textsuperscript{76}Our numerical simulations (available from the authors upon request) show that, consistently with the theory,
the welfare of the unemployed is positively affected by the interaction of higher volatility of output growth and
lower firing cost. This effect is stronger whenever the firing cost is low because the magnification effect of job
creation due to higher uncertainty, is stronger in a relatively flexible economy. In addition, for low values of $\beta$,
i.e., $\beta < \beta^*$, the job creation effect prevails on the rent erosion effect and therefore the welfare of the employed
increases as a consequence of higher volatility of the rate of output growth and lower firing cost. However, for
Similarly, it is possible to verify that lower productivity growth increases both the marginal gain, through the job creation effect, and the marginal loss, through the rent erosion effect, caused by more flexibility. When $\beta$ is relatively high, the magnification of the marginal loss dominates, and therefore $x^*$ must increase relative to its initial value in order to make the marginal worker more insulated from the risk of job destruction, reducing its marginal loss from more flexibility. The converse is true if $\beta$ is relatively low, in which case the threshold $x^*$ must decrease.

The next lemma clarifies how the second productivity threshold $\bar{x}$ mentioned in Proposition 4 is affected by the parameters of interest.

**Lemma 4** The productivity level $\bar{x}$ defined in (35) with $R_0 = R_{MIN}$ and $R' = 1$ increases with $\sigma$ and decreases with $\beta$ and $\mu$.

**Proof.** See appendix. □

It is not surprising that $\bar{x}$ decreases with $\beta$ since being an insider becomes more valuable when rents are higher, and this creates more political support for job security provisions among the employed. Higher volatility $\sigma$ has qualitatively the opposite effect of $\beta$ on $\bar{x}$, because in a more volatile environment the risk of a critical fall of productivity down to the absorbing barrier $R$ is higher. As a result, the least productive workers expect to earn less rents, for any given level of the firing cost; this effects erodes the political support for rigidity at the bottom of the distribution of productivity, i.e., $\bar{x}$ increases. Finally, a higher value of the drift coefficient $\mu$ is found to reduce $\bar{x}$. Intuitively, a higher value of $\mu$ means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in the future. In particular, a higher value of $\mu$ induces the workers employed at the moment of voting in low productivity firms to become relatively more “optimistic” about their future productivity, and therefore about the future amount of rents that they can appropriate of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of low job security provisions.

Finally, Lemma 5 clarifies how the threshold value $\hat{\beta}$ depends on the parameters governing the dynamics of productivity.

**Lemma 5** The threshold $\hat{\beta}$ defined by equation (36) increases with $\sigma$ and it decreases with $\mu$.

---

It is not surprising that $\bar{x}$ decreases with $\beta$ since being an insider becomes more valuable when rents are higher, and this creates more political support for job security provisions among the employed. Higher volatility $\sigma$ has qualitatively the opposite effect of $\beta$ on $\bar{x}$, because in a more volatile environment the risk of a critical fall of productivity down to the absorbing barrier $R$ is higher. As a result, the least productive workers expect to earn less rents, for any given level of the firing cost; this effects erodes the political support for rigidity at the bottom of the distribution of productivity, i.e., $\bar{x}$ increases. Finally, a higher value of the drift coefficient $\mu$ is found to reduce $\bar{x}$. Intuitively, a higher value of $\mu$ means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in the future. In particular, a higher value of $\mu$ induces the workers employed at the moment of voting in low productivity firms to become relatively more “optimistic” about their future productivity, and therefore about the future amount of rents that they can appropriate of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of low job security provisions.

Finally, Lemma 5 clarifies how the threshold value $\hat{\beta}$ depends on the parameters governing the dynamics of productivity.

**Lemma 5** The threshold $\hat{\beta}$ defined by equation (36) increases with $\sigma$ and it decreases with $\mu$.

---

It is not surprising that $\bar{x}$ decreases with $\beta$ since being an insider becomes more valuable when rents are higher, and this creates more political support for job security provisions among the employed. Higher volatility $\sigma$ has qualitatively the opposite effect of $\beta$ on $\bar{x}$, because in a more volatile environment the risk of a critical fall of productivity down to the absorbing barrier $R$ is higher. As a result, the least productive workers expect to earn less rents, for any given level of the firing cost; this effects erodes the political support for rigidity at the bottom of the distribution of productivity, i.e., $\bar{x}$ increases. Finally, a higher value of the drift coefficient $\mu$ is found to reduce $\bar{x}$. Intuitively, a higher value of $\mu$ means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in the future. In particular, a higher value of $\mu$ induces the workers employed at the moment of voting in low productivity firms to become relatively more “optimistic” about their future productivity, and therefore about the future amount of rents that they can appropriate of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of low job security provisions.

Finally, Lemma 5 clarifies how the threshold value $\hat{\beta}$ depends on the parameters governing the dynamics of productivity.

**Lemma 5** The threshold $\hat{\beta}$ defined by equation (36) increases with $\sigma$ and it decreases with $\mu$. 

---

It is not surprising that $\bar{x}$ decreases with $\beta$ since being an insider becomes more valuable when rents are higher, and this creates more political support for job security provisions among the employed. Higher volatility $\sigma$ has qualitatively the opposite effect of $\beta$ on $\bar{x}$, because in a more volatile environment the risk of a critical fall of productivity down to the absorbing barrier $R$ is higher. As a result, the least productive workers expect to earn less rents, for any given level of the firing cost; this effects erodes the political support for rigidity at the bottom of the distribution of productivity, i.e., $\bar{x}$ increases. Finally, a higher value of the drift coefficient $\mu$ is found to reduce $\bar{x}$. Intuitively, a higher value of $\mu$ means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in the future. In particular, a higher value of $\mu$ induces the workers employed at the moment of voting in low productivity firms to become relatively more “optimistic” about their future productivity, and therefore about the future amount of rents that they can appropriate of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of low job security provisions.

Finally, Lemma 5 clarifies how the threshold value $\hat{\beta}$ depends on the parameters governing the dynamics of productivity.

**Lemma 5** The threshold $\hat{\beta}$ defined by equation (36) increases with $\sigma$ and it decreases with $\mu$. 

---

It is not surprising that $\bar{x}$ decreases with $\beta$ since being an insider becomes more valuable when rents are higher, and this creates more political support for job security provisions among the employed. Higher volatility $\sigma$ has qualitatively the opposite effect of $\beta$ on $\bar{x}$, because in a more volatile environment the risk of a critical fall of productivity down to the absorbing barrier $R$ is higher. As a result, the least productive workers expect to earn less rents, for any given level of the firing cost; this effects erodes the political support for rigidity at the bottom of the distribution of productivity, i.e., $\bar{x}$ increases. Finally, a higher value of the drift coefficient $\mu$ is found to reduce $\bar{x}$. Intuitively, a higher value of $\mu$ means that employed workers, for any level of current idiosyncratic productivity, expect to become relatively more productive in the future. In particular, a higher value of $\mu$ induces the workers employed at the moment of voting in low productivity firms to become relatively more “optimistic” about their future productivity, and therefore about the future amount of rents that they can appropriate of, conditionally on remaining employed. The greater optimism makes these workers more reluctant to give up their position of insiders by voting in favor of low job security provisions.

Finally, Lemma 5 clarifies how the threshold value $\hat{\beta}$ depends on the parameters governing the dynamics of productivity.

**Lemma 5** The threshold $\hat{\beta}$ defined by equation (36) increases with $\sigma$ and it decreases with $\mu$. 

---

high values of $\beta$, i.e., $\beta > \beta^*$, the opposite is true: the rent erosion effect dominates and therefore the welfare of the employed decreases as a consequence of higher volatility of output growth and lower EPL.
Proof. Straightforward differentiation of equation (36) using the fact that \( |\alpha| > 0 \) and the expression of \( \alpha \) reported in equation (52) in the appendix.

Lemma 5 has the implication that it is more likely to obtain an equilibrium with unanimous political support for full flexibility (i.e., \( R = 1 \)) in a more volatile economic environment and in presence of lower productivity growth. Intuitively this is because, as we already know (Lemma 3), if workers appropriate of relatively low rents, i.e., if \( \beta \) is below \( \beta^* \), the magnification of the job creation effect due to higher flexibility dominates over the magnification of the rent erosion effect, i.e., the marginal utility gain from higher flexibility for employed workers increases relative to the marginal utility loss. As a result, the political support for flexibility increases; since \( \hat{\beta} = (1 + \pi)^{-1} < \beta^* = e(e + \pi)^{-1} \), this is the case around \( \hat{\beta} \) and therefore \( \hat{\beta} \) increases. Following the same logic, if productivity growth is higher, the existence of unanimous support for full flexibility is less likely to emerge in the low rents zone.

6 On The Rise and Persistence of Eurosclerosis

In this section, we use the set of results demonstrated in Section 5 to investigate how unexpected shocks to the main parameters of the model, i.e., \( \beta, \sigma \) and \( \mu \), affect the political equilibrium of the model characterized in Proposition 4. In particular, the goal of this section is to shed some new light on the major stylized facts on the comparative dynamics of labor market institutions in the U.S. and in Europe over the last few decades, briefly reviewed in the introduction of the paper. Before continuing, we remind that a useful property of the model (previously remarked) is that the value functions of all workers only depend on jump-variables, i.e., workers make their voting decisions “comparing steady states.” Moreover, because the state-variables of the model, i.e., the level of employment and the cross-sectional distribution of productivity, are not affected by an exogenous shock on impact, and because voting takes place immediately after the realization of the shock occurs, this affects voting decisions and the political equilibrium of the model through the jump-variables only.\footnote{The interaction between shocks and institutions has been first highlighted by Blanchard and Summers (1987) and Blanchard and Wolfers (2000).}

6.1 The Breakdown of a Flexible Economy

It has been widely remarked that the divergence of the labor market institutions of Continental Europe and of the U.S. has begun in the aftermath of the major negative macroeconomic
shocks, increasing volatility and reducing productivity growth, occurred during the 1970’s.\footnote{See for example Gottschalk and Moffitt (1994) and Comin and Philippon (2005) for evidence of increased earnings and output volatility since the 1970’s.} The model presented in this paper can shed some light on this fact since it implies that institutional divergence can occur if the same negative shock hits economies which are relatively flexible to begin with, but which differ in terms of the ability of labor to appropriate rents. According to Proposition 4, in a relatively flexible economy (i.e., with $R_0 \geq \bar{x}$) a transition to a rigid labor market is favored by the employed workers with productivity in the interval $(R_0, x^*)$, which has measure induced by $\Psi_{\lambda}(x)$ equal to

$$\lambda_{\Psi}\{(R_0, x^*)\} L \equiv [\Psi(\lambda(x^*)) - \Psi(R_0)] L, \tag{39}$$

where we remind that $L$ is the level of employment in the equilibrium obtaining with the initial level of the firing cost. Expression (39) defines the size of the coalition for rigidity, which depends on $\beta$, $\sigma$ and $\mu$, and also on the status quo level of employment protection regulation $R_0$. Since we assume that workers vote directly on the reservation productivity, neither the level of employment nor the distribution of productivity are immediately affected by the shock, which changes instantaneously only the productivity cutoff $x^*$. In particular, as we already know, $x^*$ increases in $\beta$, which means that if labor becomes stronger, as it has been the case in many Continental European countries since the late 1960’s (e.g., Caballero and Hammour, 1998), more workers find themselves employed in firms with productivity in the range $(R_0, x^*)$. As a result, the size of the coalition for rigidity increases with the bargaining power of the employed, reflecting the resulting greater scope for rent appropriation available to the insiders. Moreover, according to Lemma 3, how the size of the coalition for rigidity is affected by a shock to $\sigma$ and $\mu$ also depends on the value of $\beta$. In particular, in an economy where labor appropriates of relatively high rents, i.e., where $\beta > \beta^*$, the threshold productivity level $x^*$ increases as volatility increases and as productivity growth slows down, i.e., as “bad times” come; the opposite is true in a relatively low rents economy, i.e., where $\beta < \beta^*$. Since the threshold $x^*$ is the only element of (39) affected by the economic shock in question, we conclude that bad business conditions, such as those experienced by most industrialized economies during the 1970’s, increase the political support for the transition to a more rigid labor market in high rents economies, such as those of Continental Europe. However, the same type of shock does not make labor market rigidity more appealing politically in low rents economies, such as the U.S., where the size of the coalition for rigidity actually shrinks. As we already know, this result depends on the complementarity existing between volatility and
flexibility and on the substitutability existing between growth and flexibility, which imply that
greater volatility and lower productivity growth boost at the same time the job creation effect
and the rent erosion effect of flexibility. It follows that employed workers demand an higher
firing cost only if the composition of these two opposite effects is such that the marginal gain
in utility from flexibility decreases relative to the corresponding marginal loss, which is the
case if the rent is a relatively important component of their welfare.

Finally, how the status quo level of employment protection regulation affects the size of the
coalition of employed workers in favor of the transition to a more rigid labor market can be
determined by differentiating expression (39) with respect to $R_0$. Using the expression of the
ergodic cross-sectional distribution of productivity across employment, reported in equation
(77) in the appendix of the paper, it is possible to show that the measure of the set $(R_0, \pi^*)$
decreases with $R_0$, i.e., the size of a coalition in favor of adopting more stringent job security
provisions is larger in an economy that is initially relatively more rigid. This result also concurs
in explaining the institutional divergence of European and American labor markets during the
1970’s, since it has been documented (e.g., Blanchard, 2000) that Continental Europe was
already somewhat more rigid than the U.S. at the beginning of that period.

6.2 Good Times and Labor Market Reforms

At the present moment, the political debate over labor market institutions in Continental
Europe is centered around the elimination of part of their rigidity. According to Bean (1998),
the macroeconomic environment strongly affects the incentives for, and the feasibility of, labour
market reforms. In particular, a boom, i.e., “good times”, is the ideal time to undertake such
reforms for two main reasons. First of all, the opposition to the reforms will be less strong,
whilst budgetary constraints will be less likely to constrain expenditures. Furthermore, if such
reforms are successful, they will allow the boom to continue longer.

Our model, which focuses on the political support for labor market deregulation from work-
ers with different levels of productivity, implies that a simple relation between macroeconomic
well-being and the political feasibility of labor market reforms increasing flexibility need not
exist. Intuitively, this is because if economic conditions improve for all workers, e.g., if pro-
ductivity grows at a higher rate, the value of the rent appropriated by the employed increases,
making the least productive workers more willing to support high job security provisions shel-
tering them from the job destruction process.

To determine how a change in the rent extraction power of the employed, and a positive
productivity shock hitting a relatively rigid economy, i.e., with $R_0 < \bar{x}$, affect the political equilibrium, we begin by observing that according to Proposition 4 a reform consisting in increasing labor market flexibility is supported by all the unemployed. In addition, the coalition for flexibility includes the employed with productivity in the upper and lower tail of the distribution, i.e., with $x \in [R_0, \bar{x})$ and with $x \in (x^*, +\infty)$. A higher value of $\beta$ decreases without ambiguity the size of the coalition for reforms since $x^*$ is strictly increasing, and $\bar{x}$ is strictly decreasing in $\beta$, reflecting that preserving a rigid status quo is more appealing for employed workers when they can extract more rents from firms. To determine what effect $\mu$ has on the two threshold levels of productivity $x^*$ and $\bar{x}$, we need to distinguish between the two different cases contemplated by Lemma 3, corresponding to values of $\beta$ smaller or greater than the threshold $\beta^*$. The case of $\beta > \beta^*$ is more relevant here since it reflects the high-rents economies of Continental Europe. If $\beta > \beta^*$ Lemmas 3 and 4 imply that the effect of $\mu$ on $x^*$ is such that $\partial x^*/\partial \mu < 0$, and its effect on $\bar{x}$ is such that $\partial \bar{x}/\partial \mu < 0$.

What can the model tell us about how more favorable macroeconomic conditions affect the political viability of reforms of European labor markets? Unfortunately, not a clear-cut message. This is because, as remarked above, the two thresholds $x^*$ and $\bar{x}$ defining the coalition for rigidity tend to move in the same direction, i.e., they both decrease if $\mu$ increases. In particular, the measure of the set $(x^*, +\infty)$ of most productive employed workers who stand for flexibility, or

$$\lambda_\Psi \{(x^*, +\infty)\} L = [1 - \Psi_\lambda (x^*)] L,$$

increases as $x^*$ decreases. However, the same type of shock also tends to cut down the political support for flexibility among the workers located at the lower tail of the distribution of productivity, i.e., the set $(R_0, \bar{x})$ with measure

$$\lambda_\Psi \{(R_0, \bar{x})\} L = [\Psi_\lambda (\bar{x}) - \Psi_\lambda (R_0)] L,$$

by moving out of it some workers with productivity below the lower bound $\bar{x}$. Intuitively, this is because good economic conditions boost the rents that employed workers can potentially obtain, and therefore make some of them more reluctant to give up their position of insiders. As a result, whether the extent of the political support for rigidity among the workers with intermediate productivity, i.e., the measure of the set $(\bar{x}, x^*)$, increases or decreases cannot be established a priori. This result is particularly important since it implies that the way labor market institutions evolve in response to a worsening and to an improvement of aggregate business conditions respectively may be strikingly asymmetric. In particular, whereas a bad
economic shock may cause the breakdown of a relatively flexible economy, a good shock hitting a rigid high-rents economy need not have the effect of triggering the opposite transition to a more flexible labor market. This result is broadly consistent with, and provides a novel explanation for the dynamics of labor market institutions observed in Continental Europe in the recent years, which have shown little tendency to revert to flexibility, long after the original negative shocks favoring the build-up of Eurosclerosis have vanished.\footnote{It must be emphasized that the persistence of high levels of rent extraction power on the part of employed workers, is important according to our model to explain the lack of reversibility displayed by Continental European labor market institutions. This is documented empirically by Saint-Paul (2004), who finds no evidence of a decline in the rents of employed workers in Europe during the 1990’s, with the exception of Ireland. See also Möller and Aldashev (2005), who document that employed workers have been able to appropriate of persistently higher rents in Germany than in the U.S., since the early 1980’s.}

We conclude this section by remarking that our results question to some degree the validity of the argument that good times are necessarily good also for reforms. Cutting down the rents appropriated by the employed, i.e., reducing the value of $\beta$, is an important pre-requisite of a potentially successful reform of a rigid labor market. However, favorable economic shocks do not have clear-cut consequences for the political feasibility of a reform aimed at making a rigid labor market more flexible.

7 Conclusions

The aim of this paper is to explain the diverging comparative dynamics of one labor market institution, employment protection legislation, in Europe and in the U.S. over the last few decades. At the methodological level, the paper represents an innovation to the existing literature, since the model presented relies on the novel assumption that the dynamics of productivity is described by a Geometric Brownian process (whose dynamics features some degree of persistence) and an homogenous Poisson process rather than, as usually assumed, solely by an homogenous Poisson process (which has instead no memory, as new productivity levels are drawn from an exogenous distribution and are independent on current productivity levels). This assumption is important since it implies that the preferences on employment protection legislation of the workers are potential affected by their own idiosyncratic productivity at the moment of voting. This is because relatively more productive workers gain relatively little from a more stringent regulation of dismissals, due to the persistence of their current productivity implied by the continuity of the paths of the Brownian motion. Within this novel framework, a key substantive result demonstrated is the broad importance of the rents that employed workers are able to extract from firms. The capacity of labor to appropriate rents and labor
turnover regulations have appeared to be closely linked as part of rigid politico-economic equilibria. This result is intuitive since if rents are low, there is clearly little scope to demand their protection with stringent job security provisions. Moreover, and perhaps more surprisingly, how labor market institutions are affected by economic shocks, has also been found to depend on the extent of the rents appropriated by labor, as well as on the status quo level of the firing cost.

The results provided by the analysis of the political equilibrium of the model, have then been used to demonstrate that the different ability of labor to extract rents, can explain the diverging pattern of institutional evolution experienced by Continental Europe and by the U.S., in response to the similar major negative shocks experienced during the 1970’s on both sides of the Atlantic. In addition, a novel potential explanation has been provided of why the institutional rigidity typical of Continental European labor markets, which emerged quite a long time ago, has largely persisted (with some qualifications such as short-term employment contracts) up to the present day, long after the major shocks originally favoring its creation have vanished. More generally, an important implication of our model is that once stringent job security provisions are put in place, they have the potential to be persistent across different economic conditions, i.e., there exists a potential scope for hysteresis and path dependency in labor market institutions.

8 Appendix

In the following, we shall refer to two regions defined implicitly by the variational inequality (5): the continuation region \( \mathcal{C} \) where \( rV(x | R, \theta) = x - w(x | R, \theta) + \mathbb{E}(dJ) / dt - \lambda J \), and the stopping region \( \mathcal{S} = \mathcal{C} \) where \( J(x | R, \theta) = -F \). Moreover, we remark that the set-up cost \( C \) is an additive term in the value function \( J \), therefore we can perform all computations first assuming \( C = 0 \), and then adding the real hiring cost.

8.1 Derivation of the Expected PDV of Output

The integral in (10) can be broken down recursively to obtain the following recursion, satisfied by the functional \( V(\cdot | R) \) over the region \( \mathcal{C} \) of productivity levels such that firms continue operating,

\[
rV(x | R) = x + \frac{1}{dt} \mathbb{E}(dV) - \lambda V(x | R).
\] (40)
Using the Feynman-Kac representation, we find that the second order differential equation satisfied by $V(\cdot \mid R)$ over the continuation region $C$ reads

$$\frac{1}{2} \sigma^2 x^2 V''(x \mid R) + \mu x V'(x \mid R) - (r + \lambda) V(x \mid R) + x = 0. \quad (41)$$

The general integral of this equation is represented by

$$V(x \mid R) = \frac{x}{r + \lambda - \mu} + D_1 x^\nu + D_2 x^\alpha, \quad (42)$$

where $\nu$ and $\alpha$ denote respectively the positive and negative root of the relevant characteristic polynomial associated with (41), identical to the corresponding characteristic polynomial $[(\sigma^2/2) + \mu \epsilon - r - \lambda]$ associated with (49). The expression of equation (14) follows from (42), excluding the positive root $\nu$ by setting $D_1 = 0$ in (42) for the standard reason (e.g., Dixit, 1993, p. 25), i.e., to prevent the fundamental of the asset to become negligible relatively to its option value as $x \uparrow +\infty$, and taking into account the boundary condition $V(R \mid R)$. Refer to equation (51) below for the explicit expression of $D_2$.

8.2 Derivation of the Wage Schedule

Combining equations (9), (10), and setting $x = 1$ (the initial productivity of a new firm), the flow-value of unemployment can be expressed as

$$r U(R, \theta) = b + \theta \beta V(1 \mid R). \quad (43)$$

Substituting for $W(x \mid R, \theta)$ using terminal relation defined by (12), and using the fact, implied by (10), that $E_p(dW) / dt = \beta E_p(dV) / dt$, the recursion (8) can be written as

$$(r + \lambda) [U(R, \theta) + \beta V(x \mid R)] = w(x \mid R, \theta) + \beta \frac{1}{dt} E_p(dV) + \lambda U(R, \theta). \quad (44)$$

Also, by combining equations (43) and (44), we obtain that

$$w(x \mid R, \theta) = b + \theta \beta V(1 \mid R) + (r + \lambda) \beta V(x \mid R) - \beta \frac{1}{dt} E_p(dV). \quad (45)$$

Finally, substituting in this equation the expression of $V(x \mid R)$ provided by (40), (45) can be written as the expression reported in equation (13).

8.3 Solution of the Optimal Stopping Problem of the Firms

Firms face a standard problem of optimal stopping in continuous time, which is formalized by the Bellman-Wald equation (5). It is well known that the solution of this class of problems (e.g.,
Dixit, 1993; Dixit and Pindyck, 1994; Peskir and Shiryaev, 2006; Stokey, 2008) is characterized in terms of a productivity threshold $R$, such that the continuation value of the asset exceeds the value of the asset-upon stopping as long as $x > R$ and is exceeded by it if $x < R$, with the two values matching at $x = R$. The optimal stopping rule of the firm is to continue producing as long as $x$ remains above $R$, and to close down, firing the workers and paying the associated layoff cost $F$, as soon as the absorbing barrier $R$ is first reached. On the continuation region $\{x \in \mathbb{R}_+ : x > R\}$, therefore, the functional equation (5) corresponds to the equation

$$
(r + \lambda)J(x | R, \theta) = x - w(x | R, \theta) + \frac{1}{dt} \mathbb{E}(dJ),
$$

while at the absorbing barrier $R$, the following value matching (or continuous fit) condition

$$
J(R | R, \theta) = -F
$$

must hold, establishing the continuity of the value function $J(\cdot | R, \theta)$ upon stopping. A second functional relation, the smooth pasting (or smooth fit) condition, must also hold for the stopping rule to be optimal. This condition states that the value function is differentiable with continuity along the curve separating the continuation region from the stopping region. Here, the continuation value of the firm upon stopping is equal to $-F$, and therefore the smooth pasting condition implies that

$$
J'(R | R, \theta) = 0.
$$

Equation (46) is interpreted as a second order ordinary differential equation in the unknown function $J(\cdot | R, \theta)$, i.e., an Hamilton-Jacobi-Bellman equation. This allows us to transform the optimal stopping problem of the firm into a free-boundary problem (or Stefan’s problem), with mixed boundary conditions.\(^{80}\) The Hamilton-Jacobi-Bellman equation satisfied by $J(\cdot | R, \theta)$ over the continuation region reads\(^{81}\)

$$
\frac{1}{2} \sigma^2 x^2 J''(x | R, \theta) + \mu x J'(x | R, \theta) - (r + \lambda) J(x | R, \theta) + x - w(x | R, \theta) = 0.
$$

\(^{80}\)Free-boundary problems are a special class of boundary value problems arising in the theory of partial differential equations (e.g., Brown and Churchill, 2012).

\(^{81}\)The first two terms of the RHS of equation (49) correspond to the Dynkin operator $\mathcal{D}$ associated with the diffusion process described by the stochastic differential equation (2) of the optimization problem of the firm, whose variational formulation reads

$$
\min [-\mathcal{D}J + \lambda J + rJ - (x - w), J + F] = 0.
$$

For a complete characterization, we can show that the previous variational inequality has a unique viscosity solution, (e.g. Crandall, Ishii, and Lions, 2009). Moreover, this viscosity solution is regular, and therefore it is a classical solution. In particular, we can show that this solution is of class $C^1$ on the domain and $C^2$ inside the continuation region $C$.  

41
Using the expression of the wage rate $w(x)$ reported in (13), the general integral of this equation is found to be of the type
\[
J(x \mid R, \theta) = \frac{(1 - \beta) x}{r + \lambda - \mu} - \frac{b + \theta \beta V(1 \mid R)}{r + \lambda} + D_1 x^\alpha + D_2 x^\alpha,
\]
with $D_1$ and $D_2$ standing for constants to be determined, and with $\rho$ and $\alpha$ standing for the positive and for the negative root of the characteristic polynomial
\[
\frac{\sigma^2}{2} \epsilon (\epsilon - 1) + \mu \epsilon - (r + \lambda)
\]
associated with (49). By a standard argument (e.g., Dixit, 1993, p. 25), the root $\rho$ must be eliminated by setting the constant $D_1$ equal to zero. This is because otherwise the fundamental of the asset would become negligible, relatively to its option value, as $x \uparrow +\infty$.\footnote{From a mathematical point of view, we can show that the viscosity solution is controlled by a strict supersolution of affine type (see Pham, 2009, ch. 4) thanks to a verification theorem. As a consequence $D_1 = 0$.} The value of $D_2$ is instead determined through the smooth pasting condition, which implies that
\[
D_2(R) = -\frac{(1 - \beta) R^{1-\alpha}}{(r + \lambda - \mu) \alpha}.
\]
The expression of equation (15) then follows. Moreover, since the negative root $\alpha$ of (50) reads
\[
\alpha = \frac{1}{\sigma^2} \left\{ \frac{\sigma^2}{2} - \mu - \frac{1}{2} \sqrt{(\sigma^2 - 2\mu)^2 + 8\sigma^2(r + \lambda)} \right\},
\]
it can be verified with straightforward algebra that
\[
\frac{\partial \alpha}{\partial \sigma} > 0 \text{ and } \frac{\partial \alpha}{\partial \mu} < 0 \text{ and } \frac{\partial \alpha}{\partial \lambda} < 0.
\]
Finally, the endogenous absorbing barrier $R$ is computed using equation (50), setting as already remarked $D_1 = 0$, together with the boundary condition $J(x \mid R, \theta) = F$.

### 8.4 Proof of Proposition 1

The strategy of the proof consists in first looking at the special case where $\lambda = 0$, and then generalizing the results obtained to the case of $\lambda > 0$.

#### 8.4.1 Preliminary Results for the Special Case $\lambda = 0$: Job Destruction Rate and Ergodic Distribution

In order to describe the evolution of the productivity of a firm it is convenient to consider, rather than the original process $x$, the transformed process $z \equiv \ln(x)$. It is known (e.g., Dixit, 1993) that, since $x$ represents a Geometric Brownian process with drift $\mu$ and instantaneous standard
deviation $\sigma$, $z$ is a linear Brownian process with mean $\eta = [\mu - (\sigma^2/2)]$, and instantaneous standard deviation $\sigma$. Notice that, because the initial value of $x$ is normalized to one, the initial value of $z$ is equal to zero. Moreover, the drift $\eta$ of the transformed process is negative, since $\mu < \sigma^2/2$ by assumption.

Next, focusing the attention with no loss of generality on a firm created at time $s = 0$, define $p(z_0, z; t)$ as the probability density function of $z$, conditional on the fact that the process $z$ has never reached the barrier $\hat{R} = \ln(R)$ within the time interval $(0, t)$, starting at $z(0) = z_0$. We can write the conditional distribution function corresponding to the density $p(z_0, z; t)$ as

$$P \left\{ z(t) > z \mid z(\tau) > \hat{R}, \quad \forall \tau \in (0, t) \right\} = \int_{z}^{+\infty} p(z_0, \zeta; t) \, d\zeta = P(z_0, z; t). \quad (54)$$

Using this expression, we can write the probability that the process $z$ has not yet been absorbed at $\hat{R}$ up to time $t$, as $P(z_0, \hat{R}; t)$. Moreover, defining $\hat{T}(\hat{R})$ as the random time elapsed since the creation of the firm in question, at which the process describing the evolution of the (log of) its productivity first reaches the barrier $\hat{R}$, we obviously have that

$$P \left\{ \hat{T}(\hat{R}) > t \right\} = P(z_0, \hat{R}; t). \quad (55)$$

Since our objective is to compute the probability distribution of a first passage time of a Brownian motion, it is natural to look at the Kolmogorov backward partial differential equation satisfied by its transition density. It is known (e.g., Cox and Miller, 1965, ch. 5) that the function $P(z_0, \hat{R}; t)$ satisfies the Kolmogorov backward partial differential equation, and therefore we can write that

$$\frac{1}{2} \sigma^2 \frac{\partial^2 P}{\partial z^2} (z_0, \hat{R}; t) + \eta \frac{\partial P}{\partial z_0} (z_0, \hat{R}; t) = \frac{\partial P}{\partial t} (z_0, \hat{R}; t), \quad (56)$$

given the pair of boundary conditions

$$P(\hat{R}, \hat{R}; t) = 0 \quad \text{and} \quad \lim_{z_0 \to +\infty} P(z_0, \hat{R}; t) = 1. \quad (57)$$

The first boundary condition reflects the fact that absorption immediately occurs if $z_0 = \hat{R}$, and the second boundary condition reflects the fact that absorption occurs with probability zero in a finite time, if the process $z$ starts at an initial position infinitely distant from the barrier. For our purpose, it is convenient to solve the boundary value problem represented by (56) and (57) with the Laplace transform method. Let $\mathcal{L}(\cdot; z_0, \hat{R})$ indicate the Laplace transform of $P(z_0, \hat{R}; t)$, defined as

$$\mathcal{L}(\rho; z_0, \hat{R}) = \int_{0}^{+\infty} e^{-\rho t} P(z_0, \hat{R}; t) \, dt. \quad (58)$$
By transforming both sides of equation (56) using the theorem of differentiation of the original, and the fact that at the moment of creation, the probability that the productivity of a firm is equal to $R$ is zero, i.e.,

$$P(z_0, \hat{R}; 0) = 1,$$

it can be shown that $\mathcal{L}(\rho; \cdot, \hat{R})$ satisfies the following second order ordinary differential equation

$$\frac{1}{2} \sigma^2 \frac{d^2 \mathcal{L}}{dz_0^2} (\rho; z_0, \hat{R}) + \eta \frac{d \mathcal{L}}{dz_0} (\rho; z_0, \hat{R}) = \rho \mathcal{L} (\rho; z_0, \hat{R}) - 1, \quad (59)$$

subject to the pair of transformed boundary conditions

$$\mathcal{L} (\rho; \hat{R}, \hat{R}) = 0 \text{ and } \lim_{z_0 \to +\infty} \mathcal{L} (\rho; z_0, \hat{R}) = \frac{1}{\rho}. \quad (60)$$

Letting, $\vartheta (\rho) = \left[ -\eta - \sqrt{\eta^2 + 2 \sigma^2 \rho} \right] / \sigma^2$, denote the negative root of the characteristic polynomial $[(\sigma^2 e^2 / 2) + \eta e - \rho]$ associated with (59), as a function of $\rho$, the solution of equation (59) subject to (60), is found to be

$$\mathcal{L} (\rho; z_0, \hat{R}) = \frac{1}{\rho} \left[ 1 - e^{\vartheta (\rho)(z_0 - \hat{R})} \right]. \quad (61)$$

Since $\eta < 0$, and since $z_0 = 0$, the following pair of equalities holds

$$\mathcal{L} (0; 0, \hat{R}) = \lim_{\rho \to 0} \frac{1}{\rho} \left[ 1 - e^{-\vartheta (\rho) \hat{R}} \right] = \frac{\hat{R}}{\eta}, \quad (62)$$

where the second equality follows by applying de l'Hospital's theorem to compute the limit. Finally, using (62), and the fact that (55) and (58) imply that

$$\mathcal{L} (0; 0, \hat{R}) = \int_{0}^{+\infty} \mathbb{P} \{ T(R) > t \} \, dt, \quad (63)$$

the steady state rate of aggregate job destruction $\delta_0$ when $\lambda = 0$, characterized by (27), can be expressed as

$$\delta_0 = \frac{1}{\mathcal{L} (0; 0, \hat{R})} = \frac{1}{\hat{R}^+} \left( \frac{\sigma^2}{2} - \mu \right),$$

which corresponds to the expression reported in (27). To complete the proof of Proposition 1, we need to characterize the ergodic cross-sectional distribution of productivity across active firms. Using again the transformation $z \equiv \ln (x)$, we can write the steady state cross-sectional distribution of $z$ across employment as

$$\dot{\Psi} (z) = \mathbb{P} \{ Z \leq z \} = 1 - \mathbb{P} \{ Z > z \}. \quad (64)$$
Also, using the expression of the transition density of \( z \) conditional on non-absorption defined in (54), we can write that, in the ergodic steady state of the model
\[
P \{ Z > z \} = \frac{\theta (1 - L)}{L} \int_{-\infty}^{t} \left[ \int_{z}^{+\infty} p(z_0, \zeta; t - s) \, d\zeta \right] ds.
\]
This expression corresponds to the integral sum of the number of firms created since the infinitely remote past, which have survived up to time \( t \) and have productivity at \( t \) greater than \( z \), weighted by the steady state level of employment. Next, using the expression of \( \delta \) derived above and reported in (27), the fact that equation (17) implies that \( \theta (1 - L)/L = \delta \), and changing variables, we can also write \( P \{ Z > z \} \) as
\[
P \{ Z > z \} = \frac{\eta}{R} \int_{0}^{+\infty} P(z_0, z; t) \, dt,
\]
where \( P(z_0, z; t) \) is defined as in (54). Using (65) we can then write (64) as
\[
\bar{\Psi}(z) = 1 - \frac{\eta}{R} \int_{0}^{\infty} P(z_0, z; t) \, dt.
\]
To make progress in characterizing \( \bar{\Psi}(\cdot) \), we consider the Kolmogorov backward differential equation satisfied by \( P(z_0, z; t) \), which is equivalent to equation (56), together with the pair of boundary conditions, which have the usual interpretation,
\[
P \left( \hat{R}, z; t \right) = 0 \text{ and } \lim_{z_0 \to +\infty} P(z_0, z; t) = 1.
\]
It is again convenient to solve the backward equation (56) subject to (67), with the Laplace transform method. Defining the Laplace transform of the function \( P(z_0, z; t) \) as
\[
\mathcal{L}(\rho; z_0, z) = \int_{0}^{+\infty} e^{-\rho t} P(z_0, z; t) \, dt,
\]
the expression of \( \bar{\Psi}(\cdot) \) can be directly obtained by computing the limit of \( \mathcal{L}(\rho; z_0, z) \) as \( \rho \downarrow 0 \).
To compute the expression of the Laplace transform (68), we begin by transforming equation (56) in the following pair of second order ordinary differential equations,
\[
\begin{align*}
\frac{1}{2} \sigma^2 \frac{d^2}{dz_0^2} \mathcal{L}(\rho; z_0, z) + \eta \frac{d}{dz_0} \mathcal{L}(\rho; z_0, z) &= \rho \mathcal{L}(\rho; z_0, z) - 1, \text{ if } z_0 > z; \\
\frac{1}{2} \sigma^2 \frac{d^2}{dz_0^2} \mathcal{L}(\rho; z_0, z) + \eta \frac{d}{dz_0} \mathcal{L}(\rho; z_0, z) &= \rho \mathcal{L}(\rho; z_0, z), \text{ if } z_0 \leq z,
\end{align*}
\]
where \( \mathcal{L} = \mathcal{L}(\rho; z_0, z) \), along with the pair of transformed boundary conditions obtained from (67)
\[
\mathcal{L}(\rho; \hat{R}, z) = 0, \text{ and } \lim_{z_0 \to +\infty} \mathcal{L}(\rho; z_0, z) = \frac{1}{\rho}.
\]
Some tedious but simple algebra implies that the solution to (8.4.1) and (8.4.1) subject to (69) is

$$L(\rho; z_0, z) = \frac{\phi_2}{\rho} e^{\phi_1(\hat{z} - z)} \frac{e^{\phi_1(z_0 - \hat{z})} - e^{\phi_2(z_0 - \hat{z})}}{\phi_2 - \phi_1}, \text{ if } z_0 \leq z,$$

(70)

$$L(\rho; z_0, z) = \frac{\phi_2}{\rho} e^{\phi_1(\hat{z} - z)} \frac{e^{\phi_1(z_0 - \hat{z})} - e^{\phi_2(z_0 - \hat{z})}}{\phi_2 - \phi_1} + \frac{1}{\rho} \left[ 1 - \frac{\phi_2 e^{\phi_1(z_0 - \hat{z})} - \phi_1 e^{\phi_2(z_0 - \hat{z})}}{\phi_2 - \phi_1} \right], \text{ if } z_0 > z,$$

(71)

where we have used the definitions

$$\phi_1 = -\frac{\eta}{\sigma^2} + \sqrt{\frac{\eta^2}{\sigma^4} + \frac{2\lambda}{\sigma^2}} \text{ and } \phi_2 = -\frac{\eta}{\sigma^2} - \sqrt{\frac{\eta^2}{\sigma^4} + \frac{2\lambda}{\sigma^2}}.$$

(72)

### 8.4.2 The General Case $\lambda > 0$: Job Destruction Rate and Ergodic Distribution

Let $p_\lambda(z_0, z; t)$ be the density of the process $z = \ln(x)$ conditional on the fact that the exogenous time $T_\lambda$ is after $t$ and on the fact that $z$ has never hit the barrier $\hat{R}$ over the interval $[0, t]$. We then define

$$P_\lambda(z_0, z; t) = \mathbb{P}\left\{ Z > z \text{ and } T_\lambda > t \mid \inf_{s \in [0, t]} Z_s > \hat{R} \right\}.$$

$$= \mathbb{P}\left\{ Z > z \mid \inf_{s \in [0, t]} Z_s > \hat{R} \right\} \times \mathbb{P}\{ T_\lambda > t \}$$

$$= \int_z^{+\infty} p_\lambda(z_0, \zeta; t) d\zeta = \int_z^{+\infty} p_0(z_0, \zeta; t) e^{-\lambda t} d\zeta = P_0(z_0, z; t) e^{-\lambda t},$$

(73)

where the fourth equality comes from the independence of $T_\lambda$ with respect to the filtration $\mathcal{F}_t = \sigma\{W_s : s \leq t\}$ generated by $W$ and using the properties of the exponential law of $T_\lambda$.

We then have the following factorization for $P_\lambda(z_0, z; t)$ into the two marginal distribution functions forming it, or

$$P_\lambda(z_0, z; t) = P_0(z_0, z; t) e^{-\lambda t}.$$

(73)

The inverse cumulative distribution of productivity can be computed using the theorem of Fubini, and reads

$$\Psi_\lambda(z) = \mathbb{P}\{ Z_t > z \} = \delta \int_{-\infty}^{t} \int_z^{+\infty} p_\lambda(z_0, \zeta; t - s) d\zeta ds = \delta \int_{0}^{+\infty} P_\lambda(z_0, z; t) dt,$$

where the last equality come from a change of variable. We remark at this stage that $\Psi_\lambda(z)$ represents the Laplace transform of $P_0(z_0, z; t)$ at frequency $\lambda$, that is

$$\Psi_\lambda(z) = \int_{0}^{+\infty} P_0(z_0, z; t) e^{-\lambda t} dt = L(\lambda; z_0, z),$$

(74)

The details of the following algebraic derivations are available upon request from the authors.
where $L(\lambda; z_0, z)$ is defined by (70) if $z_0 \leq z$, and by (71), if $z_0 > z$, with $\rho = \lambda$ in both cases.

We now continue the proof by introducing the Kolmogorov backward differential equation (recall that the time variable is $-t$) for $P_0(z_0, z; t)$,

$$
\frac{1}{2}\sigma^2 \frac{\partial^2}{\partial z^2} P_0(z_0, z; t) + \eta \frac{\partial}{\partial z_0} P_0(z_0, z; t) = \frac{\partial P_0}{\partial t}(z_0, z; t),
$$

and the related Laplace transform, $L(\rho; z_0, z) = \int_0^{+\infty} P_0(z_0, z; t)e^{-\rho t} dt$.

We can now compute the aggregate job destruction rate $\delta$ using relation (18). One can easily show that $\delta = 1/L(\lambda; 0, \hat{R})$. Another way to find the same result is based on the autocoerence of the distribution, requiring that $\int_{R}^{+\infty} \psi_\lambda(x) dx = 1$, where $\psi_\lambda(\cdot)$ is defined by (40). This condition is equivalent to

$$
\int_{R}^{+\infty} \frac{\delta^2 \phi_1}{\lambda(\phi_2 - \phi_1)} \left\{ \left[ \mathbb{I}_{x \geq 1} + \mathbb{I}_{R < x < 1} x^{\phi_1 - \phi_2} \right] - R^{\phi_1 - \phi_2} \right\} x^{-\phi_1 - 1} dx = 1,
$$

and it is satisfied if the aggregate rate of job destruction is equivalent to the expression reported in equation (26), reading

$$
\delta = \frac{\lambda}{(1 - R^{\phi_2})}.
$$

Finally, using the previously established fact that $\Psi_\lambda(z) = L(\rho = \lambda; z_0, z)$ with $z = \ln(x)$, $z_0 = 0$ and $\hat{R} = \ln(R)$, we have that

$$
\Psi_\lambda(x) = \begin{cases} 
\frac{\phi_2}{\lambda} \frac{1 - R^{\phi_1 - \phi_2} x^{-\phi_1}}{\phi_2 - \phi_1} & \text{if } x \geq 1 \\
\frac{\phi_2}{\lambda} \frac{1 - R^{\phi_1 - \phi_2} x^{-\phi_1}}{\phi_2 - \phi_1} + \frac{1}{\lambda} \left[ 1 - \frac{\phi_2 x^{-\phi_1} - \phi_1 x^{-\phi_2}}{\phi_2 - \phi_1} \right] & \text{if } R < x < 1
\end{cases}
$$

Then, after we compute the ergodic distribution of $x$ using the fact that $\psi_\lambda(x) = \delta(1 - \delta\Psi_\lambda)(x)/\partial x$, we have that

$$
\psi_\lambda(x) = \begin{cases} 
\frac{\delta \phi_2 \phi_1}{\lambda} \frac{1 - R^{\phi_1 - \phi_2} x^{-\phi_1 - 1}}{\phi_2 - \phi_1} & \text{if } x \geq 1 \\
\frac{\delta \phi_2 \phi_1}{\lambda} \left[ \frac{x^{-\phi_2 - 1} - R^{\phi_1 - \phi_2} x^{-\phi_1 - 1}}{\phi_2 - \phi_1} \right] & \text{if } R < x < 1
\end{cases}
$$

This expression can be rewritten more compactly as in the equation (28) reported in the main text.

### 8.5 Proof of Proposition 2

We begin by remarking that the law of the hitting time $\bar{T}_z(\hat{R})$ of the threshold $\hat{R}$ by the transformed process $z$ is the same as the law of the hitting time $\bar{T}_x(R)$ of the threshold $R$ by the original process $x$, due to the bijectivity of the logarithmic function.
Next, we compute the Laplace transform of the random time $\bar{T}_z (\hat{R}) \wedge T_\lambda$, namely $\phi(z, \rho) = \mathbb{E}[e^{-\rho(T_z (\hat{R}) \wedge T_\lambda)}]$, where $\rho \geq 0$ is the parameter capturing the frequency of the transform. A direct application of Feynman-Kac formula (see for instance Borodin and Salminen, 1996, pp. 90-94), shows that $\phi(z, \rho)$ satisfies the following differential equation

$$\frac{1}{2}\sigma^2 \frac{\partial^2 \phi}{\partial z^2}(z, \rho) + \eta \frac{\partial \phi}{\partial z}(z, \rho) - (\lambda + \alpha)\phi(z, \rho) = -\lambda.$$ 

The associated boundary conditions are $\phi(\hat{R}, \rho) = 1$ and $\lim_{z \to +\infty} \phi(z, \rho) = \lambda/ (\lambda + \rho)$, which follows since $\mathbb{P}\left\{T_\lambda < \bar{T}_z (\hat{R})\right\} \to 1$ as $z \to +\infty$. It can be verified that this equation has the unique solution, which can be expressed in terms of $x$ and $R$ as $z = \ln(x)$, and $\hat{R} = \ln(R)$, given by

$$\phi(x, \rho) = \frac{\lambda}{\lambda + \rho} + \left(1 - \frac{\lambda}{\lambda + \rho}\right) \exp \left[\frac{\ln(R) - \ln(x)}{\sigma^2} \left(\eta + \sqrt{2(\lambda + \rho)\sigma^2 + \eta^2}\right)\right].$$

Using the theorem of differentiation under integral sign, we then have that

$$\mathbb{E}[\bar{T}_x (R) \wedge T_\lambda] = -\frac{\partial \phi}{\partial \rho}(x, 0).$$

A simple computation gives us the expression reported in equation (30).

### 8.6 Proof of Remark 3

By using equation (17), we get the following expression for employment

$$L = \frac{\theta}{\theta + \delta_0}. \quad (79)$$

Using the expression for $\delta_0$ and $\theta$ as in equations (27) and (21), with $C = 0$, and multiplying numerator and denominator by $\hat{R}^+$, we get, using expression (79), the following equation

$$L = \frac{\hat{R}^+ \theta}{\hat{R}^+ \theta + (\frac{\sigma^2}{2} - \mu)}. \quad (80)$$

By applying l’Hôpital’s rule, taking the first derivative of the numerator and denominator of equation (80), we can compute the closed form solution for the employment in the limit case in which $R = 1$ ($\Leftrightarrow F = 0$):

$$\lim_{R \to 1} \frac{\hat{R}^+ \theta}{\hat{R}^+ \theta + (\frac{\sigma^2}{2} - \mu)} = \lim_{R \to 1} \frac{\frac{1}{R} \theta + \hat{R}^+ \frac{d \theta}{dR}}{\frac{1}{R} \theta + \hat{R}^+ \frac{d \theta}{dR}} = 1.$$
8.7 Comparative Statics Properties of the Economic Equilibrium

Properties (1) and (2) follow from a straightforward implicit differentiation of equations (19) and (22).

To prove property (3), we begin by noticing that equation (22) implies that the equilibrium reservation productivity $R$ depends on $\sigma$ only through $\alpha$. Differentiating implicitly the equilibrium reservation productivity $R$ with respect to $\alpha$ in equation (22) we obtain that

$$\frac{dR}{d\alpha} = R \frac{1 - R^\alpha + \ln(R^\alpha)}{\alpha (1 - \alpha) (1 - R^\alpha)},$$

and this expression is negative since the denominator of (81) is positive, while the numerator is negative. The denominator of (81) is positive since $\alpha > 0$ and since equation (22) implies that $R < 1$ for any positive value of $F$, so that $R^\alpha > 1$; the fact that $1 - a + \ln a < 0$ for every $a \neq 1$ also implies that the numerator of (81) is negative. The result that in equilibrium $R$ decreases with $\sigma$ follows since $\partial \alpha / \partial \sigma > 0$ by (53).

To prove property (4) it is useful to establish first the following preliminary result. Letting as before $R$ denote the equilibrium reservation productivity defined by equation (22), using the expression of $dR/d\alpha$ provided by (81) and rearranging terms, we have that

$$\frac{d}{d\alpha} \left( R^{1-\alpha} \frac{1 - R^\alpha + R^\alpha \ln(R^\alpha)}{\alpha (1 - \alpha) (1 - R^\alpha)} \right) > 0,$$

since, as already remarked, $R^\alpha > 1$ and moreover $1 - a + a \ln(a) > 0$ for any $a > 1$.

Observe next that the equilibrium job creation rate $\theta$ defined by (21) depends on $\sigma$ only through $\alpha$, and that $\alpha$ affects $\theta$ both directly and indirectly through $R$ (which as we already know by (22) depends on $\sigma$ only through $\alpha$). It follows that how $\sigma$ affects $\theta$ depends on the total derivative of $\theta$ with respect to $\alpha$. By equation (21), the total derivative with respect to $\alpha$ of the schedule $\theta = \theta(R(\alpha), \alpha)$ representing the equilibrium job creation rate as a function of $\alpha$ can be written as

$$\frac{d\theta}{d\alpha} = \frac{(r + \lambda) (1 - \beta) \pi \beta d \left( R^{1-\alpha} \right)}{\pi \beta (1 - R^{1-\alpha})} \frac{dR}{d\alpha} + \frac{(r + \lambda) \pi \beta (1 - R^{1-\alpha})}{\pi \beta (1 - R^{1-\alpha})}$$

since equation (21) can also be written as

$$\theta R^{1-\alpha} = \frac{(r + \lambda) (1 - \beta) R^{1-\alpha}}{\pi} - r \left[ (r + \lambda - \mu) b - (1 - \beta) + (r + \lambda - \mu) C \right],$$

and this expression is negative since the denominator of (81) is positive, while the numerator is negative. The denominator of (81) is positive since $\alpha > 0$ and since equation (22) implies that $R < 1$ for any positive value of $F$, so that $R^\alpha > 1$; the fact that $1 - a + \ln a < 0$ for every $a \neq 1$ also implies that the numerator of (81) is negative. The result that in equilibrium $R$ decreases with $\sigma$ follows since $\partial \alpha / \partial \sigma > 0$ by (53).

To prove property (4) it is useful to establish first the following preliminary result. Letting as before $R$ denote the equilibrium reservation productivity defined by equation (22), using the expression of $dR/d\alpha$ provided by (81) and rearranging terms, we have that

$$\frac{d}{d\alpha} \left( R^{1-\alpha} \frac{1 - R^\alpha + R^\alpha \ln(R^\alpha)}{\alpha (1 - \alpha) (1 - R^\alpha)} \right) > 0,$$

since, as already remarked, $R^\alpha > 1$ and moreover $1 - a + a \ln(a) > 0$ for any $a > 1$.
combining (83) and (84) we also have that
\[
\frac{d\theta}{d\alpha} = \frac{(r + \lambda)(1 - \beta) + \beta \theta}{\pi \beta (1 - R^{1-\alpha})} \frac{d\theta (R^{1-\alpha})}{d\alpha} + \frac{(r + \lambda)(1 - \beta) R^{1-\alpha}}{\alpha^2 \beta (1 - R^{1-\alpha})}, \tag{85}
\]
which is positive since as we already know by (82) \(d\left(R^{1-\alpha}\right)/d\alpha > 0\) and obviously \(R^{1-\alpha} < 1\). The result that in equilibrium \(\theta\) increases strictly with \(\sigma\) follows since \(\delta\alpha/\delta\sigma > 0\) by (53).

Finally, property (5) can be demonstrated by observing that the equilibrium reservation productivity \(R\) determined by equation (22) depends on \(\mu\) both directly and through \(\alpha\), i.e., we have that
\[
R \equiv R(\alpha(\mu), \mu). \tag{86}
\]
The total derivative of this expression with respect to \(\mu\) can be represented as
\[
\frac{dR}{d\mu} = \frac{\partial R}{\partial \alpha} \frac{\partial \alpha}{\partial \mu} + \frac{\partial R}{\partial \mu}.
\]
A straightforward implicit differentiation of (22) implies that \(\delta R/\delta \mu > 0\), i.e., holding \(\alpha\) constant, \(R\) increases with \(\mu\). Moreover, we know from (81) that \(R\) is strictly decreasing in \(\alpha\) and we know from (53) that \(\delta\alpha/\delta \mu < 0\). It follows that \(dR/d\mu > 0\).

We can write the job destruction rate as a function of \(\mu\) using (27) and (86) as
\[
\delta \equiv \delta (R(\alpha(\mu), \mu), \mu).
\]
The total derivative of this expression with respect to \(\mu\) can be expressed as
\[
\frac{d\delta}{d\mu} = \frac{\partial \delta}{\partial R} \frac{dR}{d\mu} + \frac{\partial \delta}{\partial \mu}.
\]
A straightforward differentiation of equation (27) shows that the direct effect of \(\mu\) on \(\delta\) is negative, i.e., \(\delta\delta/\partial \mu < 0\), and also that \(\delta\) increases with \(R\), i.e., \(\delta\delta/\partial R > 0\). Since as just demonstrated \(R\) is overall an increasing function of \(\mu\), i.e., \(dR/d\mu > 0\), we conclude that the sign of \(d\delta/d\mu\) is ambiguous.

To understand why productivity growth has an ambiguous effect on job creation, notice that \(\theta\) depends on \(\mu\) in a variety of ways. In particular, \(\mu\) affects \(\theta\) directly, but also indirectly through \(\alpha\) and through the equilibrium reservation productivity (itself a function of \(\alpha\) and \(\mu\)). Using (21) and (86), we can write the equilibrium job creation rate as a function of \(\mu\) as
\[
\theta \equiv \theta (R(\alpha(\mu), \mu), \alpha(\mu), \mu).
\]
The total derivative of this expression with respect to \(\mu\) can be expressed as
\[
\frac{d\theta}{d\mu} = \frac{\partial \theta}{\partial \alpha} \frac{\partial \alpha}{\partial \mu} + \frac{\partial \theta}{\partial R} \frac{dR}{d\mu} + \frac{\partial \theta}{\partial \mu} \tag{87}
\]
A straightforward differentiation of equation (21), shows that the direct effect of \( \mu \) on \( \theta \) is positive, i.e., \( \partial \theta / \partial \mu > 0 \). Moreover, we know that by (21) in equilibrium \( \theta \) increases with \( R \), i.e., \( \partial \theta / \partial R > 0 \), and that \( R \) increases with \( \mu \), i.e., \( dR/d\mu > 0 \). This implies that both the second and the third component of the total derivative of \( \theta \) with respect to \( \mu \) (87) are positive. However, by (85), we know that \( \alpha \) has a positive direct effect on \( \theta \), i.e., \( B_\alpha > 0 \), which means that the first term of \( d\theta/d\mu \) is negative. Which of the opposite effects of \( \mu \) on \( \theta \) dominates over the other cannot be established a priori.

### 8.8 Proof of Proposition 3

To determine the voting decision of the employed workers under the condition stated in Proposition 3, i.e., that \( \beta \leq \hat{\beta} \), we begin by noticing that since the threshold \( x^* \) defined in (34) is strictly increasing in \( \beta \), and that at \( x^* = 1 \) at \( \beta = \hat{\beta} \). Therefore the condition \( \beta \leq \hat{\beta} \) is equivalent to

\[
x^* \leq 1.
\] (88)

Some simple algebra shows that expression (88) implies that

\[
x^* - (R)^{1-\alpha} (x^*)^\alpha \leq (x^*)^\alpha \left[ 1 - (R)^{1-\alpha} \right],
\]

for any possible value of \( R \), i.e., in the interval \([R_{MIN}, 1]\) (including the status quo \( R_0 \)), where we remind that \( R_{MAX} = 1 \) as we are now assuming \( C = 0 \). This result in turn implies that, for any \( x \in [R_0, x^*] \), and for any value of \( R \),

\[
\mathcal{W}(x \mid R) \leq \mathcal{W}(x^* \mid R) \leq \mathcal{U}(1 \mid R),
\] (89)

since \( \mathcal{W}(\cdot \mid R) \) is strictly increasing in \( x \). Condition (89) implies therefore that all the workers who are employed in the status quo in firms with productivity \( x \in [R, x^*] \) are better-off as unemployed in the most flexible economy possible, for any level of \( R \). Moreover since, as we already know, the value of the unemployed is everywhere strictly increasing in \( R \), their voting is trivial as \( \mathcal{U}(1 \mid R) \) is maximized at \( R = 1 \). We conclude that if \( \beta \leq \hat{\beta} \), all the unemployed and all the employed workers with productivity in the interval \([R_0, x^*] \), for any \( x^* \leq 1 \), are in favor of implementing the reform generating maximum labor market flexibility, i.e., \( R = 1 \).

Lastly, we consider the case of the employed workers with productivity greater than 1. We know from Lemma 1 that their value is strictly decreasing in \( R \) on the interval \([x, R_{MAX}] \), for any \( x > 1 \). This is because these workers gain nothing from any level of job protection (they
expect not to be fired even when \( R \) approaches 1), and therefore they potentially bear only the general equilibrium costs of EPL. Therefore their value is also maximized at \( R = 1 \), i.e. they are all in favor of zero firing costs.

We conclude that when \( \beta \leq \hat{\beta} \), all workers, employed (regardless on productivity) and unemployed are in favor of a transition to \( R = 1 \) (i.e., driving to zero any positive existing level of the firing cost, whatever it is).

### 8.9 Proof of Lemma 4

We remind that \( \bar{x} \) is defined implicitly by equation (35), setting \( R_0 = R_{MIN} \) and \( R' = 1 \). The fact that \( \partial \bar{x} / \partial \beta < 0 \) follows immediately from the fact that the left-hand-side of equation (35) is increasing in \( \bar{x} \) while the right-hand-side of the same equation is decreasing in \( \beta \). Also, equation (35) implies that \( \bar{x} \) depends on \( \sigma \) and \( \mu \) only through \( \alpha \). By differentiating implicitly \( \bar{x} \) in equation (35) with respect to \( \alpha \), we obtain that

\[
\frac{\partial \bar{x}}{\partial \alpha} = \bar{x} \frac{\left[ 1 - (R_{MIN})^{1-\alpha} \right] (1 - \beta) \beta^{-1} + \alpha^2 (R_{MIN})^{1-\alpha} \bar{x}^\alpha [\ln(\bar{x}) - \ln(R_{MIN})]}{\alpha^2 \left[ \bar{x} - \alpha (R_{MIN})^{1-\alpha} \bar{x}^\alpha \right]}. 
\]

The sign of this expression is positive since both the numerator and the denominator are obviously positive. The proof of Lemma 4 follows from how \( \alpha \) depends on \( \sigma \) and \( \mu \) (see (53)).

### 8.10 Proof of Proposition 4

The proof is articulated in three parts. We first show that the possible outcome of a majority voting process are only three, i.e., the status quo \( R_0 \), \( R_{MIN} \) and 1. Then, we show that the social preference relation induced by majority voting, denoted as \( > \), is transitive (i.e., there are no Condorcet cycles in voting between the alternatives in question). Finally, we use these preliminary results to demonstrate that the political equilibrium always exists, and it is either \( R = 1 \) or \( R = R_{MIN} \).

#### Claim 1

Suppose that \( \exists R' < R_0 \) such that \( R' > R_0 \), then \( R_{MIN} \) (i.e., maximum rigidity) defeats any \( R \) such that \( R \leq R' \) in pairwise comparisons.

**Proof.** Lemma 1 implies that \( R' \) is preferred to \( R_0 \) by all and only the employed workers with productivity above \( x^* \). Moreover, the value of these workers \( W(\cdot | R) \) is strictly increasing in \( R \), for any \( R > R_0 \). In addition, all the unemployed prefer \( R' \) to \( R_0 \) since their value \( U(R) \)
is strictly increasing in $R$ for any $R$. It follows that if a majority of workers prefer $R'$ to $R_0$, then the same majority of workers prefers $R_{MIN}$ to any $R$ such that $R \leq R'$. ■

**Claim 2** Suppose that $\exists R = R'' > R_0$ such that $R'' > R_0$, then $R = 1$ (i.e., maximum flexibility) defeats any $R$ such that $R \geq R''$ in pairwise comparisons.

**Proof.** Lemma 1 and Lemma 2 imply that $R''$ is preferred to $R_0$ by all the employed workers with productivity above $x^*$, whose value, $W(x \mid R'')$, is strictly increasing in $R''$. In addition, $R''$ is preferred to $R_0$ by all the employed workers with productivity $x \in [R_0, x''_0)$, where $x''_0$ is defined as the level of productivity such that $W(x''_0 \mid R_0) = U(R'')$. Moreover, the value of the unemployed, $U(R'')$, is strictly increasing in $R''$. It follows that both these sets of workers, and the workers who are unemployed in the status quo, prefer $R = 1$ to any $R$ such that $R \geq R''$. ■

Claim 1 and Claim 2 imply that, in order to characterize the political equilibrium of the model, we can restrict the attention to the choice between three possible levels of the reservation productivity, the status quo $R_0$, $R_{MIN}$ and $R = 1$, since any other value of $R$ is defeated in pairwise comparisons by at least one of these alternatives, i.e., it is not a Condorcet winner.

The rest of the proof leads to the demonstration of the existence of a political equilibrium and to its characterization. We consider separately the outcomes of the voting over $R$ in the two possible cases of $R_0 < \bar{x}$ and of $R_0 \geq \bar{x}$, where $\bar{x}$ is defined as the unique productivity level such that $W(\bar{x} \mid R_{MIN}) = U(1)$.

**Case 1** $R_0 < \bar{x}$.

We rely on the following preliminary result.

**Claim 3** Let $\bar{x}_0$ be defined as the unique productivity level such that $W(\bar{x}_0 \mid R_0) = U(1)$; we have that $\bar{x} < \bar{x}_0$ if $R_0 > R_{MIN}$, and $\bar{x} = \bar{x}_0$ if $R_0 = R_{MIN}$.

**Proof.** Consider the equation $W(x \mid R) = U(1)$. Differentiating implicitly this equation with respect to $R$, we obtain that

$$\frac{\partial x}{\partial R} = -\frac{(1 - \beta) (1 - \alpha)}{\beta \pi} \frac{1 - x^\alpha \beta \pi (1 - \beta)^{-1}}{R^\alpha + \pi R x^{1-\alpha}}.$$  

The denominator of this expression is positive, and therefore the sign of $\partial x/\partial R$ is positive if $1 - x^\alpha \beta \pi (1 - \beta)^{-1} < 0$. A simple manipulation of this expression shows that $\partial x/\partial R > 0$ if $x < x^*$. We already know (see Remark 4) that if $\beta > \hat{\beta}$, then $\bar{x} < x^*$; using the same argument
as in Remark 4, we conclude that $\bar{x}_0 < x^*$ if $\beta > \hat{\beta}$. Since $\max\{\bar{x}, \bar{x}_0\} < x^*$, we have that $\partial x/\partial R > 0$ for $x \in \{\bar{x}, \bar{x}_0\}$ and, because $R_0 > R_{MIN}$, this implies that $\bar{x}_0 > \bar{x}$. The second statement of the claim is obvious.

We know from Lemma 1 that $R_{MIN}$ is preferred strictly to $R_0$ by the employed with productivity $x \in (R_0, x^*)$. This set has Lebesgue-Stieltjes (henceforth LS) measure\(^{84}\) $\lambda_0 = \lambda_\Psi\{(R_0, x^*)\} L$. Moreover, according to Lemma 2, $R_0$ is preferred to $R = 1$ by the workers with productivity $x \in (\bar{x}_0, x^*)$, which has LS measure $\lambda_1 = \lambda_\Psi\{(\bar{x}_0, x^*)\} L$. Finally, $R_{MIN}$ is preferred to $R = 1$ by the workers with productivity $x \in (\bar{x}, x^*)$, which has LS measure $\lambda_2 = \lambda_\Psi\{(\bar{x}, x^*)\} L$. From Claim 3, if $R_0 > R_{MIN}$, then $\bar{x}_0 < \bar{x}$ implies that $(\bar{x}_0, x^*) \subset (\bar{x}, x^*)$ and therefore the following chain of inequalities holds: $\lambda_1 < \lambda_2 < \lambda_0$. We distinguish two sub-cases, 1A and 1B, depending on the value of $\lambda_0$.

Sub-case 1A: $\lambda_0 \leq 1/2$. In this case, $\lambda_1$ and $\lambda_2$ and also both lower than 1/2 since $\lambda_1 < \lambda_2 < \lambda_0$. This means that $R_0 > R_{MIN}$, $1 > R_0$ and $1 > R_{MIN} \Rightarrow R = 1$ defeats any alternative.

Sub-case 1B: $\lambda_0 > 1/2$. We have to consider three possibilities. (1) $\lambda_1$ and $\lambda_2$ are also both greater than 1/2. In this case, we have that $R_{MIN} > R_0$, $R_0 > 1$ and $R_{MIN} > 1 \Rightarrow R_{MIN}$ defeats any alternative. (2) $\lambda_1 < 1/2$ and $\lambda_2 \geq 1/2$. In this case we have that $R_{MIN} > R_0$, $1 > R_0$, and $R_{MIN} > 1 \Rightarrow R_{MIN}$ defeats any alternative. (3) $\lambda_1$ and $\lambda_2$ are both lower than 1/2. In this case, we have that $R_{MIN} > R_0$, $1 > R_0$, and $1 > R_{MIN} \Rightarrow R = 1$ defeats any alternative.

We conclude that the social preference relation $>$ induced by majority voting is transitive, i.e., a conditional political equilibrium exists. Moreover, $R = 1$ defeats any alternative in pairwise comparisons if $\lambda_2 \geq 1/2$ and, vice versa, $R = 1$ defeats any alternative in pairwise comparisons if $\lambda_2 < 1/2$.

If $R_0 = R_{MIN}$, Claim 3 implies that $\bar{x}_0 = \bar{x}$, i.e., $\lambda_1 = \lambda_2$. It is straightforward to deduce that the political equilibrium exists also in this special case, and it is the same as described above.

**Case 2** $R_0 \geq \bar{x}$.

In this case, we have that $\lambda_2 = \lambda_0$. The proof is almost identical to the one relative to that of Case 1 and it is therefore omitted.

\(^{84}\)We remind that $\Psi_\lambda(\cdot)$ is defined as a distribution function across employment, and thus we need to multiply $\lambda_\Psi$ by $L$ to compute the size of the coalition in question.
References


Table 1: Effect of the firing cost $F$ on the aggregate job destruction rate $\delta$, the reservation productivity $R$, the exit rate from unemployment $\theta$ and employment $L$.

<table>
<thead>
<tr>
<th>$F$</th>
<th>$\delta$</th>
<th>$R$</th>
<th>$\theta$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.9397</td>
<td>0.9467</td>
<td>1.6485</td>
<td>0.6369</td>
</tr>
<tr>
<td>0.03</td>
<td>0.6686</td>
<td>0.9241</td>
<td>1.1697</td>
<td>0.6363</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5487</td>
<td>0.9067</td>
<td>0.9586</td>
<td>0.6360</td>
</tr>
<tr>
<td>0.07</td>
<td>0.4771</td>
<td>0.8918</td>
<td>0.8327</td>
<td>0.6358</td>
</tr>
<tr>
<td>0.09</td>
<td>0.4283</td>
<td>0.8786</td>
<td>0.7443</td>
<td>0.6357</td>
</tr>
<tr>
<td>0.10</td>
<td>0.4091</td>
<td>0.8724</td>
<td>0.7134</td>
<td>0.6356</td>
</tr>
<tr>
<td>0.12</td>
<td>0.3776</td>
<td>0.8609</td>
<td>0.6587</td>
<td>0.6356</td>
</tr>
<tr>
<td>0.14</td>
<td>0.3526</td>
<td>0.8501</td>
<td>0.6152</td>
<td>0.6357</td>
</tr>
<tr>
<td>0.18</td>
<td>0.3152</td>
<td>0.8304</td>
<td>0.5504</td>
<td>0.6359</td>
</tr>
<tr>
<td>0.20</td>
<td>0.3006</td>
<td>0.8212</td>
<td>0.5252</td>
<td>0.6360</td>
</tr>
<tr>
<td>0.24</td>
<td>0.2769</td>
<td>0.8040</td>
<td>0.4845</td>
<td>0.6363</td>
</tr>
<tr>
<td>0.28</td>
<td>0.2584</td>
<td>0.7880</td>
<td>0.4529</td>
<td>0.6368</td>
</tr>
<tr>
<td>0.30</td>
<td>0.1497</td>
<td>0.7803</td>
<td>0.4396</td>
<td>0.6370</td>
</tr>
</tbody>
</table>

$\beta=0.4; \ C=0.01; \ r=0.05.$
Table 2: Effect of the worker’s *rent extraction power* $\beta$ on the reservation productivity $R$, the exit rate from unemployment $\theta$ and employment $L$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$R$</th>
<th>$\theta$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.8822</td>
<td>1.1963</td>
<td>0.7308</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8724</td>
<td>0.7134</td>
<td>0.6356</td>
</tr>
<tr>
<td>0.5</td>
<td>0.8598</td>
<td>0.4358</td>
<td>0.5376</td>
</tr>
<tr>
<td>0.6</td>
<td>0.8425</td>
<td>0.2611</td>
<td>0.4366</td>
</tr>
<tr>
<td>0.7</td>
<td>0.8168</td>
<td>0.1465</td>
<td>0.3326</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7729</td>
<td>0.0709</td>
<td>0.2257</td>
</tr>
<tr>
<td>0.9</td>
<td>0.6692</td>
<td>0.0234</td>
<td>0.1163</td>
</tr>
</tbody>
</table>

$F=0.1; C=0.01; r=0.05.$
Table 3: Effect of the discounting rate $r$ on employment $L$.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.5989</td>
</tr>
<tr>
<td>0.02</td>
<td>0.6090</td>
</tr>
<tr>
<td>0.03</td>
<td>0.6184</td>
</tr>
<tr>
<td>0.04</td>
<td>0.6275</td>
</tr>
<tr>
<td>0.05</td>
<td>0.6356</td>
</tr>
<tr>
<td>0.06</td>
<td>0.6435</td>
</tr>
<tr>
<td>0.07</td>
<td>0.6510</td>
</tr>
<tr>
<td>0.08</td>
<td>0.6582</td>
</tr>
<tr>
<td>0.09</td>
<td>0.6650</td>
</tr>
</tbody>
</table>

$F=0.1; \beta=0.4; C=0.01.$
Table 4: Effect of the exogenous job destruction rate $\lambda$ on the value of a firm $J$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>17.9735</td>
</tr>
<tr>
<td>0.3</td>
<td>22.6915</td>
</tr>
<tr>
<td>0.5</td>
<td>25.9522</td>
</tr>
<tr>
<td>0.7</td>
<td>35.7953</td>
</tr>
<tr>
<td>0.9</td>
<td>40.3404</td>
</tr>
<tr>
<td>1.0</td>
<td>41.5931</td>
</tr>
<tr>
<td>1.2</td>
<td>41.4432</td>
</tr>
<tr>
<td>1.4</td>
<td>37.6643</td>
</tr>
<tr>
<td>1.6</td>
<td>31.3408</td>
</tr>
<tr>
<td>1.8</td>
<td>23.5653</td>
</tr>
<tr>
<td>2.0</td>
<td>15.9251</td>
</tr>
</tbody>
</table>

$\beta=0.4; C=0.01; r=0.05; x=0.3$. 
Table 5: Effect of the drift of the productivity $\mu$ on the aggregate job destruction rate $\delta$. 

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1.7535</td>
</tr>
<tr>
<td>0.03</td>
<td>1.7521</td>
</tr>
<tr>
<td>0.06</td>
<td>1.7512</td>
</tr>
<tr>
<td>0.09</td>
<td>1.7503</td>
</tr>
<tr>
<td>0.12</td>
<td>1.7506</td>
</tr>
<tr>
<td>0.15</td>
<td>1.7520</td>
</tr>
<tr>
<td>0.18</td>
<td>1.7544</td>
</tr>
<tr>
<td>0.20</td>
<td>1.7563</td>
</tr>
</tbody>
</table>

$\beta=0.4; \ C=0.01; \ r=0.05; \ \sigma=0.7.$
Table 6: Effect of the exogenous job destruction rate $\lambda$ on the aggregate job destruction rate $\delta$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.8706</td>
</tr>
<tr>
<td>0.14</td>
<td>1.8143</td>
</tr>
<tr>
<td>0.18</td>
<td>1.7814</td>
</tr>
<tr>
<td>0.20</td>
<td>1.7703</td>
</tr>
<tr>
<td>0.24</td>
<td>1.7506</td>
</tr>
<tr>
<td>0.28</td>
<td>1.7493</td>
</tr>
<tr>
<td>0.30</td>
<td>1.7478</td>
</tr>
<tr>
<td>0.34</td>
<td>1.7481</td>
</tr>
<tr>
<td>0.38</td>
<td>1.7514</td>
</tr>
<tr>
<td>0.40</td>
<td>1.7544</td>
</tr>
<tr>
<td>0.44</td>
<td>1.7616</td>
</tr>
<tr>
<td>0.48</td>
<td>1.7695</td>
</tr>
<tr>
<td>0.50</td>
<td>1.7746</td>
</tr>
</tbody>
</table>

$\beta=0.4$; $C=0.01$; $r=0.05$; $\sigma=0.8$. 
Figure 1: Employment $L$ for different values of the firing cost $F$. 
Figure 2: Employment $L$ for different values of the rent extraction power $\beta$. 
Figure 3: Employment $L$ for different values of the discounting rate $r$. 

71
Figure 4: Value of the firm $J$ for different values of the exogenous job destruction rate $\lambda$. 
Figure 5: Aggregate job destruction rate $\delta$ for different values of the drift of the productivity $\mu$. 
Figure 6: Aggregate job destruction rate $\delta$ for different values of the exogenous job destruction rate $\lambda$. 