ISSN 2279-9362

Collegio Carlo Alberto

Two-sided reputation in certification markets

Matthieu Bouvard
Raphael Levy

No. 339
December 2013

Carlo Alberto Notebooks

www.carloalberto.org/research/working-papers

Two-sided reputation in certification markets

Matthieu Bouvard[*], Raphaël Levy[†]

December 2013

Abstract

In a market where sellers solicit certification to overcome adverse selection, we show that the profit of a monopolistic certifier is hump-shaped in his reputation for accuracy: a higher accuracy attracts high-quality sellers but sometimes repels low-quality sellers. As a consequence, reputational concerns may induce the certifier to reduce information quality, thus depressing welfare. The entry of another certifier impacts reputational incentives: when sellers can only solicit one certifier, competition plays a disciplining role and the region where reputation is bad shrinks. Conversely, this region expands when sellers can hold multiple certifications.

Keywords: reputation, certification, multihoming.

JEL codes: D82, D83

[*]McGill University, Desautels Faculty of Management, 1001 Sherbrooke West, Montreal H2J 2L2, Canada. Phone: +1 514 518 2560. matthieu.bouvard@mcgill.ca.

[†]Mannheim University, L7, 3-5, 68131 Mannheim, Germany. Phone: +49 (0)621 181 1913. raphael.levy@uni-mannheim.de.

1 Introduction

Certifiers play a critical role in markets with information asymmetries: by providing a third-party opinion, they bridge the informational gap between buyers and sellers and boost gains from trade. Many markets plagued by adverse selection would indeed be extremely thin in the absence of certification mechanisms. For instance, the 2011 final report of the US Financial Crisis Inquiry Commission (2011) emphasized that "without the active participation of the rating agencies, the market for mortgage-related securities could not have been what it became." In financial markets, the central role of certifiers is further reinforced by regulations that rely on their seal of approval.[1] However, because certifiers are themselves subject to incentive problems, they are not a perfect solution to adverse selection and might even generate inefficient trading of lemons. The unfolding of the financial crisis in 2008 suggests that credit rating agencies (henceforth CRAs) have been instrumental in misallocating capital.[2] On February 4, 2013, the U.S. Department of Justice lodged a complaint against Standard & Poors, claiming that "S&P's desire for increased revenue and market share in the RMBS and CDO markets led S&P to downplay and disregard the true extent of the credit risks posed by RMBS and CDO."[3] A report from the Security and Exchange Commission in September 2011 casts doubt on CRA's incentives to provide unbiased information, even

[1]For instance, under the Basel II regulation, banks can use credit ratings from approved agencies in the derivation of their capital requirements. The U.S. Fed has been using ratings to regulate commercial banks since the 1930s; the SEC has been using ratings for the regulation of broker-dealers or money-market funds.

[2]The same report from the FCIC states: "We conclude that the failures of credit rating agencies were essential cogs in the wheel of financial destruction. The three credit rating agencies were key enablers of the financial meltdown."

[3]RMBS stands for Residential Mortgage-Based Security, CDO stands for Collateralized Debt Obligation. Both are structured financial products that essentially consist of bundles of repackaged debt claims.

after the heavy criticism they sustained in the aftermath of the 2008-09 crisis.[4]

In this paper, we investigate a fundamental incentive mechanism for certifiers: reputation. Reputation has been a central defense of CRAs against accusations of conflict of interest and misaligned incentives. In the words of Thomas McGuire, former executive vice-president of Moody's, "what's driving us is primarily the issue of preserving our track record. That's our bread and butter."[5] However, as Mark Froeba, former senior vice-president of Moody's, suggests, rating agencies have striven in the same breath to develop a reputation for being business-friendly: "This was a systematic and aggressive strategy to replace a (...) getting-the-rating-right kind of culture with a culture that was supposed to be "business-friendly", but was consistently less likely to assign a rating that was tougher than our competitors."[6] Reputation seems to play a dual role: on the one hand, CRAs need to maintain a reputation for accuracy for their ratings to have any value; on the other hand, a reputation for leniency might help attract a larger number of issuers. The first objective of this paper is to examine how a certifier tries to find a compromise between these two conflicting aspirations. Building on this analysis, we then study the impact of competition between certifiers on their reputational incentives, in particular when simultaneous certification from different certifiers is possible, as this is typically the case for ratings. Rating agencies constitute a natural illustration of our framework, and an example we repeatedly use in the paper.[7]

[4]See for instance: "SEC critical of rating agency's controls," Financial Times, September 30, 2011. See also Cornaggia et al. (2011), who provide empirical evidence suggesting a systematic bias of rating agencies towards issuers that generate a higher turnover.

[5]Quoted by Becker and Milbourn (2011).

[6]"How Moody's sold its ratings - and sold out investors", McClatchy Newspapers, Oct. 19, 2009.

[7]The financial crisis has engendered a stream of paper on credit rating agencies, many of them involving reputational concerns. For an overview of the recent literature on CRAs, see Jeon and Lovo (2013).

However, our model is meant to capture any situation in which market frictions call for certification agencies or standard-setting organizations (Lerner and Tirole, 2006; Farhi, Lerner, and Tirole, 2005, 2013). It can be applied to any certification market where certifiers care about the size of their customer base and applicants may hold certifications from several certifiers at the same time. Examples of such market include financial audit, technical standards (e.g., ISO, CEN), school accreditations (e.g., EQUIS, AACSB) or individual proficiency tests (e.g., GMAT, GRE, TOEIC).

We first develop a static model in which a seller needs to resort to a certifier to overcome adverse selection. We show that the certifier's profit is maximum when he is perceived as neither too accurate nor too inaccurate: accuracy attracts good sellers, who have nothing to hide and prefer certification to be as credible as possible; however, a bad seller trades the cost of being proven bad against the benefit of being pooled with good sellers if undetected. It follows that too precise a certifier at some point deters bad sellers from applying for certification. In other words, accuracy is appealing to one side of the market (high-quality sellers), while the other side of the market (low-quality sellers) cares about accuracy to the extent that good sellers exert a positive externality on bad sellers who obtain certification, but intrinsically dislikes precision. The certifier wants to attract these two types of clientele with conflicting preferences over his reputation, hence needs a "balanced reputation" for accuracy. This captures the idea that reputation is essentially two-sided in certification markets. We next introduce reputation-building and show that the desire to achieve an intermediate reputation gives rise to a rebalancing effect: when perceived as inaccurate, the certifier makes extra effort to increase the precision of his signal so as to develop a reputation for accuracy; conversely, a certifier with too good a reputation decreases the precision of the information he provides in order to signal he is (bad) seller-friendly. Therefore, reputational concerns sometimes "discipline" the certifier, by inducing him to be more precise than if he was myopic; reputation is then welfare-enhancing. However, it is otherwise welfare-decreasing ("bad reputation"), as reputational concerns provide incentives to decrease information quality, which, in turn, limits gains from trade.

We then proceed to the case where several certifiers coexist in the market. Specifically, we consider a monopolist facing the entry of a second certifier, and contrast two market structures. In the first, a seller can only be certified by a single certifier ("singlehoming"). This market structure gives rise to frontal competition in which only the more reputable certifier is active. Intuitively, even though the monopoly profit of the certifier is maximized when he attracts a mix of good and bad sellers, good sellers drive the demand because they exert a positive externality on bad sellers: a certifier faces no demand unless he attracts some good sellers. Since good sellers have an unambiguous preference for accuracy, they flock to the more reputable certifier. As a result, the incumbent certifier is less likely to be ousted from the market by the entrant when his reputation is higher, which shifts his preferred reputation towards more precision. It follows that competition attenuates bad reputation effects: a monopolistic certifier who decreases precision for reputational motives provides more precise information when he is facing entry. In the second market structure, sellers may simultaneously solicit certification from more than one certifier ("multihoming"). We first show that, provided that the cost of double certification is sufficiently small, any seller who applies for certification requires certification from both certifiers. This has a dramatic impact on certifiers' incentives: instead of competing, certifiers now share the same clientele. This results in each certifier's bliss reputation being lower than in the monopoly case. Intuitively, the size of the total market for certification is maximal when the overall certification process is neither too precise nor too imprecise; in the presence of another certifier who produces an independent signal, a certifier can only compensate the additional information generated by having himself a lower reputation, all the more as the other certifier is perceived as accurate. As a result, multihoming exacerbates bad reputation effects: a monopolistic certifier who decreases precision for reputational motives under monopoly now provides even less precise information when facing the entry of a second certifier.

This paper belongs to the literature on the reputation and credibility of experts. After Sobel (1985), Benabou and Laroque (1992) and Mathis et al. (2009) have shown that reputation has a disciplining effect but is not sufficient to ensure truthful information transmission. These papers are based on a trade-off between short-term incentives to manipulate information in order to inflate the current profit and long-term incentives to build up a reputation. By contrast, we show that, even in the absence of an immediate reward from information manipulation, reputation itself can lead a certifier to decrease the quality of information. Therefore, reputation can be "bad," *i.e.*, welfare-reducing, while it is welfare-enhancing in those two papers.[8] A series of papers on "bad reputation" have investigated conditions under which reputation may have an adverse effect on welfare (Morris, 2001; Ely and Välimäki, 2003; Ely et al., 2008). However, in these papers, inefficiencies result from the attempt by a "honest" type to separate himself from types with biased preferences.

[8] Another difference with Benabou and Laroque (1992) and Mathis et al. (2009) is that our model features adverse selection in the product market, while they assume that sellers do not have any informational advantage over buyers.

On the contrary, in our model, the certifier distorts the quality of information because he wants to signal to future low-quality sellers that he is likely to produce imperfect information. Our modeling of reputation is related to models of "signal-jamming" pioneered by Holmström (1999), who studies the effort decision of a manager with career concerns. Finally, our paper is related to recent work on reputation with respect to two audiences: the emphasis on public reputation distinguishes our paper from Frenkel (2010), who studies the repeated interaction of a rating agency with the same seller (issuer), who can accumulate private information about the certifier. Bar-Isaac and Deb (2012) study a general framework where an agent tries to develop a reputation vis-à-vis several audiences.[9] We differ from their paper by focusing on the issue of information transmission in certification markets. Our contribution to the literature on reputation also lies in the fact that we analyze different market environments (monopoly, singlehoming, multihoming) and underline how reputational incentives vary across different structures.

2 The model

2.1 The product market

We consider a setup with three categories of risk-neutral players: a seller, buyers and a certifier. The seller owns a product of quality $\theta \in \{\theta_g, \theta_b\}$, where $Pr(\theta = \theta_g) = \beta$. There is a continuum of competitive buyers with valuation 1 for a good-quality product ($\theta = \theta_g$) and 0 for a bad-quality product ($\theta = \theta_b$). The seller's valuation is 0 for a bad product, and λ for a good product, where λ is a continuous random variable with density f and a log-concave cumula-

[9]See also Gertner et al. (1988) and Austen-Smith and Fryer (2005) on signaling to multiple audiences.

tive distribution F on $\mathbb{R}_+$. When product quality is public information, good products are traded as long as $\lambda < 1$. We introduce frictions by assuming that product quality, as well as the reservation value λ, is private information of the seller. This informational friction calls for a certification mechanism, which enhances buyers' confidence on product quality. A natural application is the market for structured products such as Mortgage-Based Securities (MBS). In this market, the seller is a financial institution who wants to rebalance its portfolio by securitizing loans, *i.e.*, issuing bonds backed by mortgages. MBS are typically bought by institutional investors who seek exposure to the real estate market. Credit rating agencies play a fundamental role by providing information on the structured product's credit risk. Note that interpreting sellers as financial institutions is consistent with them being "sophisticated" market participants, with superior information about the quality of their products. It is also consistent with the idea that the motive for trading could be ambiguous: the seller might be hit by a liquidity shock that affects his willingness to hold on to an asset, or might simply try to take advantage of the market price being above the asset's fundamental value.

2.2 The certification process

The certifier is endowed with a technology which produces a signal $\sigma \in \{\varnothing, b\}$ on the quality of the product, with conditional distributions $\Pr(\sigma = \varnothing | \theta = \theta_g) = 1$ and $\Pr(\sigma = b | \theta = \theta_b) = \alpha + e$. The precision of the certifier's signal depends both on an enduring technological parameter $\alpha \in \{\alpha_L, \alpha_H\}$ and on some unobservable effort $e \in [-\varepsilon, \varepsilon]$ that the certifier exerts, where $0 < \varepsilon < \min\{\alpha_L, 1 - \alpha_H\}$. Effort e is allowed to be negative and involves a cost $\frac{1}{2}ce^2$: while increasing the precision of the signal takes extra effort and resources, decreasing the precision might require destroying or falsifying the information that enters the signal-generating process, or might *ex post* expose the certifier to the risk of lawsuits or regulatory sanctions.[10] Notice that a signal $\sigma = b$ provides perfect evidence that the product is of bad quality, while perfect evidence of high quality is never available.[11] We interpret σ as the certification outcome: if $\sigma = \varnothing$, the product is said to be certified; when $\sigma = b$, the seller's application is rejected.

Finally, we assume that certification involves a fixed cost $\phi < 1$ for the seller. This cost consists of a fixed and upfront fee paid to the certifier, $z\phi$, and of additional costs, $(1-z)\phi$, related to information production and product design.[12] For instance, financial claims may have to be repackaged and distributed to institutional investors, which requires the services of a range of financial intermediaries. Importantly, since the certifier is paid upfront, he has no direct incentive to make any (positive or negative) effort to change his precision, while a report-contingent payment would create incentives to lower effort so as to increase fees, even in a one-shot game.[13] Hence, effort is here purely driven by long-term reputational concerns. Note also that we take the fee, $z\phi$, to be exogenous. The question of the optimal payment structure of cer-

[10] We assume here costly negative effort, but could alternatively assume an intrinsic preference for truthtelling, which induces the certifier to exert positive effort even absent reputational concerns. We would then assess the impact of reputation by comparing the level of effort with reputational concerns to this benchmark effort level. This would give essentially similar results.

[11] This asymmetry in the distribution of the signal greatly simplifies the analysis but is not essential. We could have instead assumed that the certifier produces a signal with symmetric conditional distributions, but the analysis would then be more cumbersome.

[12] z is irrelevant in the monopoly case and can be though as being equal to 1, but will prove useful once we introduce multiple certifiers. See section 4.2.

[13] One of the sharpest criticisms following the subprime crisis was that a significant part of rating agencies' fees was indeed contingent on a favorable rating. During the Summer 2008, an agreement was found between the New York State Attorney General Andrew Cuomo and the three main credit rating agencies requiring that rating fees be charged upfront.

tification services has been extensively studied, for instance in Faure-Grimaud, Peyrache, and Quesada (2009), Bolton et al. (2012). These papers have shown that the price structure of certification, as well as the identity the party that purchases certification services, influence both the composition of the pool of applicants and certifiers' incentives to manipulate information.[14] Incorporating these effects into our model would compromise its tractability, while our objective is to insulate the role of reputation. Implicitly, we therefore assume that certification fees are rigid and cannot be adjusted to changes in the certifier's reputation. A consequence is that the certifier's profit is proportional to the total demand for certification.

2.3 Timing and information

We conclude this description of the model with the timing of the game. There are two periods; the seller and the buyers only live one period, while the certifier is long-lived with a discount factor normalized to 1. Within each period t, the game unfolds as follows:

a. The seller observes the quality $\theta_t \in \{\theta_b, \theta_g\}$ of his product and decides whether to solicit certification,

b. The certifier exerts effort e_t and produces a signal $\sigma_t \in \{b, \varnothing\}$,

c. Buyers observe σ_t and independently submit bids for the product in a second-price auction.

While effort is private information of the certifier, the signal σ is publicly observed. In other words, the certifier can determine the *ex ante* precision of the signal through costly effort, but cannot manipulate the signal *ex post.*

[14]On this issue, see also Kashyap and Kovrijnykh (2013) and Stahl and Strausz (2011).

This is reminiscent of Rayo and Segal (2010) and Kamenica and Gentzkow (2011), who model a game where a sender who wants to persuade a receiver to take a given action selects the *ex ante* distribution of a signal disclosed to the receiver. Note, however, that the structure of the signal can be adjusted at no cost and is public information in these models, while adjustments are costly and unobservable in ours. This captures the idea that (costly) manipulations of the technology are more difficult to detect than manipulations of the signal produced. In particular, this is consistent with reports on how credit rating agencies have been adjusting the information they provide to markets: rather than directly manipulating the outcome of their credit risk models (the rating itself) they adjust their models or the type of information inputted into these models (see, e.g., the 2008 SEC Report of Issues Identified in the Examination of Select Credit Rating Agencies). Notice that the seller cannot manipulate the signal produced either (unlike, for instance, in Cohn et al., 2013, who focus on sellers's reputation).

Importantly, we also assume that the certifier does not have private information on α.[15] In the beginning of period 1, all players share the common belief that $\Pr(\alpha = \alpha_H) = \rho_1$. If certification takes place in period 1, all the period 2 players observe *ex post* both the certification outcome σ_1 and the true quality of the product θ_1. We denote $\rho_2 = \Pr(\alpha = \alpha_H | \sigma_1, \theta_1)$ and will henceforth refer to ρ_t as the certifier's reputation in period t. A feature of our game is that no realization of (σ_1, θ_1) is ever out of the equilibrium path, which results in all players sharing the same beliefs all along the game in a

[15] This assumption is similar to Holmström (1999). Private information of the certifier on α would raise the issue of equilibrium multiplicity, a problem we can circumvent in a simple way when α is symmetric information.

PBE involving certification.[16]

3 Two-sided reputation: the monopoly case

3.1 Equilibrium with no certifier

Before we derive the equilibrium with certification, we describe the outcome in the product market absent certification. We assume that adverse selection precludes any trade in the absence of additional information.[17] Formally:

Assumption 1. $\beta < \min\limits_{P \in [0,1]} \frac{P}{P+(1-P)F(P)}$.[18]

Consider a candidate price P at which the seller could sell his good. A high-quality seller is willing to sell at price P if and only if $\lambda \leq P$. Bad sellers are willing to sell at any price $P \geq 0$. Buyers are willing to pay at most the expected value of the product conditional on the seller being willing to sell, that is, $\frac{\beta F(P)}{\beta F(P)+1-\beta}$. Assumption 1 ensures that this expected value is strictly smaller than P for any $P \in (0,1]$, which implies that the market has no equilibrium with trading. Intuitively, when the probability of a high-quality seller β is too small, adverse selection drives all high-quality buyers out of the market, which then collapses.

3.2 Equilibrium with certifier: period 2

We first analyze the final period ($t = 2$), in which the certifier has no reputational concerns. The certifier exerts zero effort and the precision of his signal

[16]As will be clear shortly, there is always an equilibrium with no certification, in which Bayes' rule does not apply.

[17]This is only for simplicity. What matters is that certification increases gains from trade by making it possible to trade for sellers who would not be able to trade otherwise.

[18]Note that this implies that F is differentiable at 0 (otherwise, $\frac{P}{P+(1-P)F(P)}$ cannot be bounded away from 0 for $P \in [0,1]$).

only depends on the technology α. The expected probability at $t = 2$ that a bad-type seller θ_b obtains a favorable rating ($\sigma_2 = \varnothing$) given ρ_2 is

$$q_2 \equiv \Pr(\sigma_2 = \varnothing | \theta_2 = \theta_b) = 1 - [\rho_2 \alpha_H + (1 - \rho_2)\alpha_L].$$

We characterize the period 2 equilibrium as a function of q_2, and then derive the expression of the certifier's profit as a function of ρ_2, using this simple relationship between ρ_2 and q_2.

3.2.1 Equilibrium definition

In period 2, a perfect Bayesian equilibrium consists of a probability to solicit certification for each type of good seller λ and for a bad seller, as well as Bayesian posterior beliefs on θ_2 on following (a) no certification being asked, (b) certification being asked but rejected ($\sigma_2 = b$), and (c) certification being granted ($\sigma_2 = \varnothing$). Let us first remark that, if a good seller with type λ solicits certification with positive probability, then all good sellers with reservation value below λ must solicit certification with probability one. Indeed, the benefit of certification does not depend on the reservation value λ, while the opportunity cost of selling is strictly lower for a lower λ. It follows that we can characterize a good seller's strategy by a cutoff type $\overline{\lambda}_2 \geq 0$ such that a good seller with reservation value λ solicits certification if and only if $\lambda \leq \overline{\lambda}_2$.

Let γ_2 denote the probability that a bad seller solicits certification. As long as $\gamma_2 > 0$, a bad report $\sigma_2 = b$ perfectly reveals a low-quality product, in which case there is no trade. If $\gamma_2 = 0$, observing $\sigma_2 = b$ is off path and we assume in that case that buyers consider the good to be of bad quality. Therefore, trade is never possible following $\sigma_2 = b$.[19] Following the event "no certification being

[19] We assume anyway in the next subsection that the equilibrium involves positive partic-

asked," buyers must believe that the product is of low quality. A formal proof lies in the Appendix. The intuition is as follows: since trade is impossible without certification, and since certification is relatively more attractive to good sellers, adverse selection is more severe than when no certification is available, so it is a fortiori impossible to trade with no certificate when it is available. Finally, the only transaction price we need to care about is the price that the seller obtains following certification, *i.e.*, a signal $\sigma_2 = \varnothing$. Since buyers are competitive, risk-neutral and hold the same beliefs in equilibrium, they bid up to the expected value of the good. Let $P_2 \equiv \Pr(\theta_2 = \theta_g | \sigma_2 = \varnothing)$ denote this price. Overall, a perfect Bayesian equilibrium is fully characterized by a triplet $(\overline{\lambda}_2, \gamma_2, P_2)$.

3.2.2 No-certification equilibrium

Before we characterize the equilibrium with certification, notice that there always exists a no-certification equilibrium, $(\overline{\lambda}_2 = \gamma_2 = 0, P_2 \leq \phi)$. This equilibrium is sustained by any buyers' out-of-equilibrium beliefs which result in a price smaller than ϕ following certification. All along the paper, we will refine the equilibrium by imposing out-of-equilibrium beliefs which attribute a deviation toward "more certification" to the type most likely to benefit from it, that is, the high-quality seller, who is more likely to obtain certification.[20] The refinement imposes to consider $P_2 = 1$ in the out-of-equilibrium event where a product is certified. Therefore, the no-certification equilibrium does not survive the refinement and we simply disregard it.

ipation of bad types, which makes this assumption unnecessary.

[20] This is essentially equivalent to D1, although D1 does not apply strictly speaking, because we have a continuum of types. However, the relevant type is only θ here. λ is indeed irrelevant: since buyers do not care about λ, the sale price only reflects buyers' information on θ.

3.2.3 Equilibrium with certification

Consider a candidate equilibrium $(\overline{\lambda}_2, \gamma_2, P_2)$. We have:

$$P_2 = \Pr(\theta = \theta_g | \sigma_2 = \varnothing) = \frac{\beta F(\overline{\lambda}_2)}{\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2 q_2} \in [0,1].$$

A good seller with reservation value λ solicits certification iff $P_2 - \phi \geq \lambda$. If $P_2 \leq \phi$, no seller ever solicits certification (no-certification equilibrium). We assume that $P_2 > \phi$ and will check that it is true *ex post*.

Consider some $q_2 \in [1-\alpha_H, 1-\alpha_L]$. If $P_2 > \phi$, $(\overline{\lambda}_2, \gamma_2, P_2)$ must satisfy

$$P_2 - \phi = \overline{\lambda}_2 \tag{1}$$

$$\gamma_2 \in \underset{\tilde{\gamma} \in [0,1]}{\text{argmax}}\ \tilde{\gamma}(q_2 P_2 - \phi) \tag{2}$$

$$P_2 = \frac{\beta F(\overline{\lambda}_2)}{\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2 q_2} \tag{3}$$

We immediately see that we indeed have $P_2 > \phi$: if $\gamma_2 = 0$, then $P_2 = 1 > \phi$. If $\gamma_2 \geq 0$, we must have $P_2 \geq \frac{\phi}{q_2} > \phi$. Notice also that, since P_2 is a decreasing function of γ_2, the solution to (2) must be unique.

We restrict attention to the case where $\gamma_2 \in (0,1)$. This is not essential for our results but simplifies the analysis, as it ensures that the certifier's profit function is differentiable everywhere. Necessary and sufficient conditions for interiority are

Assumption 2. $\phi < 1 - \alpha_H$ *and* $\beta < \frac{1}{1 + \max_{q_2 \in [1-\alpha_H, 1-\alpha_L]} \frac{1}{\phi}(1 - \frac{\phi}{q_2}) F(\frac{1-q_2}{q_2}\phi)}$

The first inequality states that the certification cost is smaller than the lowest possible probability for a bad seller to obtain certification. Therefore, the bad seller is willing to solicit certification with positive probability for all

q_2. This also implies that $P_2 < 1$. The second inequality ensures that there are *ex ante* too many bad types to sustain an equilibrium in which a bad seller always solicits certification (the proof of Lemma 1 formally establishes this). Note that this condition imposes the same type of constraint as Assumption 1, namely that the adverse selection problem is severe. Under assumption 2, solving for the system ((1),(2),(3)) yields:

Lemma 1. *For all $q_2 \in [1-\alpha_H, 1-\alpha_L]$, the period-2 equilibrium is such that:*

$$\begin{cases} \overline{\lambda}_2 &= \frac{1-q_2}{q_2}\phi \\ \gamma_2 &= \frac{\beta}{1-\beta}(1-\frac{\phi}{q_2})\frac{1}{\phi}F(\frac{1-q_2}{q_2}\phi) \\ P_2 &= \frac{\phi}{q_2} \end{cases}$$

Proof. Suppose first that γ_2 is interior: (2) implies $P_2 = \frac{\phi}{q_2}$, which allows to derive $\overline{\lambda}_2 = \frac{1-q_2}{q_2}\phi$ and $\gamma_2 = \frac{\beta}{1-\beta}(1-\frac{\phi}{q_2})\frac{1}{\phi}F(\frac{1-q_2}{q_2}\phi)$. Assumption 2 ensures that $0 < \gamma_2 < 1$, so γ_2 is indeed interior. Since we have established uniqueness, this is the only possible solution of the system. □

Since the certifier charges a fixed fee $z\phi$, his expected profit in period 2 is given by

$$[\beta F(\overline{\lambda}_2) + (1-\beta)\gamma_2]z\phi = \beta z(1 - \frac{1-q_2}{q_2}\phi)F(\frac{1-q_2}{q_2}\phi). \tag{4}$$

3.2.4 The costs and benefits of reputation

The certifier's profit depends on the odds ratio $\frac{1-q_2}{q_2}$, a measure of the probability for a bad seller to be certified, and hence of the certifier's (perceived) precision ρ_2. From the expression in (4), the impact of a change in precision on the certifier's profit is a priori ambiguous. Intuitively, a higher expected precision has two effects: (a) the probability of obtaining certification decreases for bad sellers, while it is unchanged for good sellers (q_2 decreases); (b) conditional on certification, buyers are willing to pay a higher price (P_2 increases). Combining these two effects, a higher reputation for accuracy unambiguously raises the participation of good sellers ($\overline{\lambda}_2$ increases).[21] As for bad sellers, the impact of reputation is unclear. On the one hand, the price conditional on certification is higher, both because the signal is more precise and because more good sellers participate. This tends to increase the demand of bad types. On the other hand, the probability of certification is lower, which decreases their incentive to ask for certification. Which of these effects dominate depends on the parameters. We make the following assumption:

Assumption 3.
$F(\frac{\alpha_L}{1-\alpha_L}\phi) \leq (1 - \frac{\alpha_L}{1-\alpha_L}\phi)f(\frac{\alpha_L}{1-\alpha_L}\phi)$ *and* $F(\frac{\alpha_H}{1-\alpha_H}\phi) \geq (1 - \frac{\alpha_H}{1-\alpha_H}\phi)f(\frac{\alpha_H}{1-\alpha_H}\phi)$

In particular, Assumption 3 is always satisfied when $(1-x)F(x)$ is concave. Replacing q_2, we rewrite the profit as a function of ρ_2 :

$$\pi_2(\rho_2) \equiv \beta z \left(1 - \frac{\rho_2\alpha_H + (1-\rho_2)\alpha_L}{1 - [\rho_2\alpha_H + (1-\rho_2)\alpha_L]}\phi\right) F\left(\frac{\rho_2\alpha_H + (1-\rho_2)\alpha_L}{1 - [\rho_2\alpha_H + (1-\rho_2)\alpha_L]}\phi\right),$$

and derive from Assumption 3 the following proposition:

Proposition 1. $\exists \rho_2^* \in (0,1), \pi_2'(\rho_2) \geq 0$ *on* $[0, \rho_2^*]$ *and* $\pi_2'(\rho_2) \leq 0$ *on* $[\rho_2^*, 1]$

Proof. In the Appendix. □

Proposition 1 points that the certifier's profit is a hump-shaped function of his reputation for accuracy. For low levels of precision (low ρ_2, *i.e.* high q_2), a higher reputation for accuracy increases profit: more high-quality sellers solicit

[21] Note that, if a higher precision was also increasing the probability that good sellers are certified, this effect would be magnified.

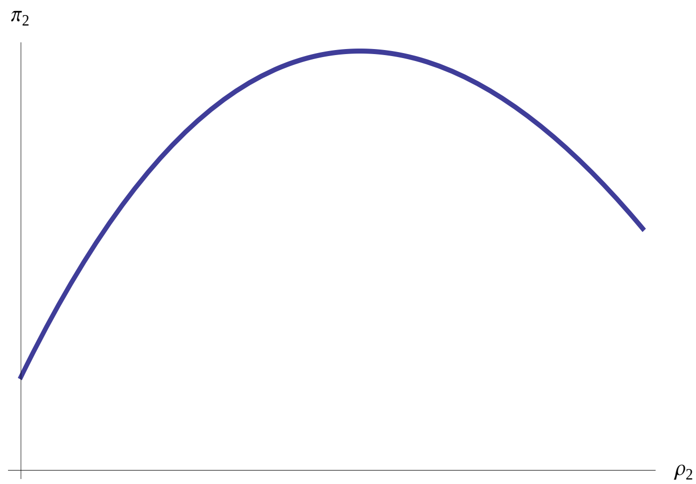

Figure 1: The certifier's profit in period 2

certification and, even if the demand from low-quality sellers may decrease, this decline is slower than the increase in the demand of high-quality sellers. For higher reputations, low-quality sellers' demand γ_2 drops as their probability to obtain certification dwindles. Under Assumption 3, this drop is too large to be outweighed by the increase in good seller's participation. Consequently, the overall profit of the certifier decreases beyond a certain level of expected precision. Overall, Proposition 1 suggests that the reputation for accuracy is essentially two-sided: while a good seller always prefers a more accurate certifier, a bad seller would like the certifier to be neither too accurate nor too imprecise. This results in total demand being maximized for a level of expected accuracy which is not maximal: a certifier can be perceived as "too accurate". The certifier then wishes to develop a reputation for being more accurate when his perceived precision ρ_2 is low and, conversely, to develop a reputation for being less accurate when ρ_2 is large. Therefore, the direction of reputational incentives is ambiguous and depends on the certifier's current reputation, as we will see in the next subsection.

Notice that, if Assumption 3 was not satisfied, the certifier's profit would be monotonic in the reputation. The analysis of reputational incentives would then be essentially similar to previous contributions (e.g., Benabou and Laroque, 1992 or Mathis et al., 2009). Assumption 3 allows to focus on the novel case where the certifier's profit is a non-monotonic function of the reputation and to highlight that reputational concerns naturally create ambiguous incentives.

3.2.5 Welfare

Since only good-quality products generate gains from trade, total welfare in period 2 is an increasing function of the certifier's reputation ρ_2 ($\overline{\lambda}_2$ increases in ρ_2). The certifier does not internalize total welfare because he cares about attracting bad sellers, although they do not generate any surplus. In the model, this is driven by the assumption of a fixed price, which prevents the certifier from screening out bad types, for instance by offering menus of contracts and contingent payments. However, any mechanism by which the certifier could extract rents from bad sellers without jeopardizing too much his attractiveness to good sellers would qualitatively generate the same effects.

3.3 Reputation building: equilibrium in period 1

In period 2, given that the certifier has no reputational concerns, he picks the cost-minimizing level of effort $e_2^* = 0$. However, the certifier has an incentive to try and build a reputation in period 1 because the accuracy of his signal conveys information about his type α : when the certifier provides effort e_1, the posterior probability that the certifier is the accurate type α_H following a bad signal is given by:[22]

$$\begin{aligned}\rho^{+}(e_1) &\equiv \Pr(\alpha = \alpha_H | \theta_1 = \theta_b, \sigma_1 = b) \\ &= \frac{\rho_1(\alpha_H + e_1)}{\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1} = \rho_1 + \frac{\rho_1(1-\rho_1)(\alpha_H - \alpha_L)}{\rho_1\alpha_H + (1-\rho_1)\alpha_L + e_1}.\end{aligned}$$

Conversely, certification of a bad quality product triggers an updating from ρ_1 down to

$$\begin{aligned}\rho^{-}(e_1) &\equiv \Pr(\alpha = \alpha_H | \theta_1 = \theta_b, \sigma_1 = \varnothing) \\ &= \frac{\rho_1(1 - \alpha_H - e_1)}{1 - \rho_1\alpha_H - (1-\rho_1)\alpha_L - e_1} = \rho_1 - \frac{\rho_1(1-\rho_1)(\alpha_H - \alpha_L)}{1 - \rho_1\alpha_H - (1-\rho_1)\alpha_L - e_1}.\end{aligned}$$

Finally, since good-quality products are certified with probability 1 regardless of the certifier's accuracy, observing a good product being certified in period 1 is uninformative; the posterior belief then equals the prior:

$$\rho^{=}(e_1) \equiv \Pr(\alpha = \alpha_H | \theta_1 = \theta_g, \sigma_1 = \varnothing) = \rho_1.$$

As in period 2, the equilibrium in period 1 features a cutoff type $\overline{\lambda}_1(e_1)$, the probability for a bad seller to solicit certification $\gamma_1(e_1)$, and a transaction price $P_1(e_1)$ following $\sigma_1 = \varnothing$, which can all be expressed as a function of the probability for a bad seller to obtain certification $q_1(e_1) \equiv 1 - [\rho_1\alpha_H + (1 - \rho_1)\alpha_L] - e_1$:

$$\overline{\lambda}_1(e_1) = \frac{1 - q_1(e_1)}{q_1(e_1)}\phi,$$

$$\gamma_1(e_1) \in \operatorname*{argmax}_{\tilde{\gamma}\in[0,1]} \tilde{\gamma}\left[q_1(e_1)P_1(e_1) - \phi\right],$$

[22]In order to avoid heavy notation, we do not explicitly write the dependence of the posterior ρ_2 on the prior ρ_1, except in the Appendix.

$$P_1(e_1) = \frac{\beta F[\overline{\lambda}_1(e_1)]}{\beta F[\overline{\lambda}_1(e_1)] + (1-\beta)\gamma_1(e_1) q_1(e_1)}.$$

We have expressed the equilibrium variables as a function of the effort of the certifier e_1, which should also be chosen optimally. In equilibrium, the certifier maximizes his expected profit in period 2 net of effort costs, taking as given the other players' expectation on effort, and this expectation has to be correct. Formally,

$$e_1^* \in \underset{e_1 \in [-\epsilon, \epsilon]}{\operatorname{argmax}} \, \mu(e_1^*) \left\{ [1 - q(e_1)] \pi_2[\rho^+(e_1^*)] + q(e_1) \pi_2[\rho^-(e_1^*)] \right\} - c \frac{e_1^2}{2} \tag{5}$$

where

$$\mu(e_1) = \frac{(1-\beta)\gamma_1(e_1)}{\beta F\left[\overline{\lambda}_1(e_1)\right] + (1-\beta)\gamma_1(e_1)}$$

The term within brackets is the expected future profit given effort e_1 when other players expect effort e_1^*, conditional on the product being of low quality (if the product is of high quality, the future profit is independent of effort). $\mu(e_1^*)$ is the probability that the seller is of low quality when effort e_1^* is expected: the effort decision of the certifier affects the composition of the pool of applicants, hence the probability that effort impacts reputation.

Under the assumption that c is sufficiently large (see the Appendix for a formal condition), we derive the following proposition on the equilibrium level of effort in period 1:

Proposition 2. *In period 1, there is a unique equilibrium level of effort given by the function* $e_1^*(\rho_1)$, *with* $e_1^*(\rho_1)$ *continuous on* $[0, 1]$. *In addition, there is a threshold* $\overline{\rho} \in (0, 1)$ *such that:*

- $e_1^*(\overline{\rho}) = e_1^*(0) = e_1^*(1) = 0,$
- $e_1^*(\rho_1) > 0$ *for* $\rho_1 \in (0, \overline{\rho}),$

- $e_1^*(\rho_1) < 0$ *for* $\rho_1 \in (\overline{\rho}, 1)$.

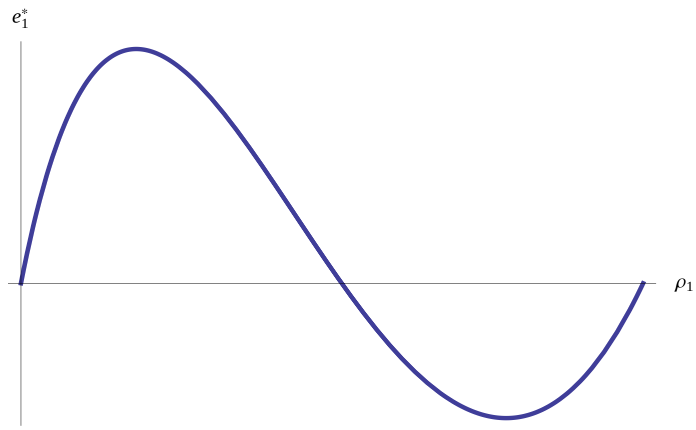

Figure 2: Equilibrium effort in period 1

Proof. In the Appendix. □

Proposition 2 establishes that the overall effect of reputation-building on welfare is ambiguous. Reputation has a disciplining effect when ρ_1 is low: as compared to the no-reputation case, the certifier increases the precision of his signal so as to build up credibility vis-à-vis good sellers; however, there is "bad reputation" when ρ_1 is high: a certifier decreases precision in order to achieve a more balanced reputation and attract more future bad sellers. With respect to the standard results in the literature on reputation, our model exhibits two (related) distinctive features: first, the qualitative impact of reputation, *i.e.* whether effort is higher or lower than in the no-reputation case, critically depends on the prior reputation; second, reputational concerns have no impact on effort for some intermediate value of the prior reputation (at $\rho_1 = \overline{\rho}$). In one-sided reputation models, the distortion from the static preferred action is highest in the intermediate region of beliefs where the prior uncertainty is maximum because attempts to influence beliefs are more effective. With two-sided reputation, there is a point at which the certifier is equally concerned about catering to good and bad sellers, so that incentives are only driven by the effort cost. Notice finally that the certifier's incentives to manipulate the precision of his signal are purely driven by reputation, that is, he has no short-term incentive to distort information production. This contrasts with existing models of reputation of experts, such as Benabou and Laroque (1992) or Mathis et al. (2009), in which the expert trades off long-term reputational incentives against short-term incentives to distort information in order to reap immediate profits. In these models, the expert always prefers being perceived as more accurate, but is at some point willing to milk his reputation to enjoy higher current benefits. Hence, while reputation might not be enough to perfectly discipline the expert, there is more information transmission when he cares about his reputation than when he does not. On the contrary, in our model, for $\rho_1 > \overline{\rho}$, there is less information provided when the certifier cares about his reputation.

4 Multiple certifiers: single- and multihoming.

In this section, we study how the presence of a second certifier affects reputational incentives. Specifically, we assume that certifier A is a monopoly in the first period but faces the entry of a second certifier, B, in period 2. We contrast two market structures, which we show have opposite implications regarding incentives to produce accurate information: in the first one ("singlehoming"), the seller is constrained to solicit certification from one certifier only; in the second ("multihoming"), a seller may solicit both certifiers. A seller who applies for certification still bears a fixed cost $(1-z)\phi$, plus a cost $z\phi$ paid to each certifier solicited. A seller who singlehomes therefore pays a total cost ϕ, as in the monopoly case, while a seller who multihomes pays a total certification cost of $(1+z)\phi$. Finally, we assume that the period 2 reputation of the potential entrant ρ_2^B is *ex ante* unknown and is drawn from the uniform distribution on $[0,1]$.[23]

4.1 Singlehoming

We start with the case where the seller is constrained to singlehome. We slightly modify the timing to allow for entry:

In period 1,

1a. The seller observes the quality θ_1 of his product and decides to solicit certification from A or not,

1b. If solicited, certifier A makes effort e_1 and publishes the signal $\sigma_1 \in \{\varnothing, b\}$,

1c. Buyers observe the signal and independently submit bids for the product.

In period 2,

2a. ρ_2^B is realized and observed by all parties,

2b. B decides to enter or not,

2c. The seller observes the quality θ_2 and of his product and decides which certifier to solicit, if any,

[23] Our results would hold for any distribution. Randomness allows to smooth profit functions, it is introduced only for technical reasons.

2d. The certifier j who has been solicited chooses effort e_2^j and publishes the signal $\sigma_2^j \in \{\varnothing, b\}$,

2e. Buyers observe the signal and independently submit bids for the product.

We assume that certifier B does not observe the true precision of certifier A, but only A's reputation (as all the other players). Letting ρ_t^j denote the reputation of certifier $j \in \{A, B\}$ in period t, we derive the following lemma.

Lemma 2. *In period 2, the only active certifier is the one with the higher reputation for accuracy. The profit of the active certifier is equal to the monopoly profit.*

Proof. In the Appendix. □

Under singlehoming, the certifier with the higher reputation captures all the market in period 2. As already discussed, good sellers exert a positive externality on bad sellers, who enjoy being pooled with the better types. The certifier is therefore unable to attract bad sellers unless he attracts good sellers. Because good sellers have a clear preference for accuracy, they pick the certifier with the higher reputation, which leaves the certifier with the lower reputation inactive. This dynamic is reminiscent of the literature on two-sided markets in which platforms have a similar incentive to cater to the side of the market which exerts the strongest positive externality on the other side (see, for instance, Caillaud and Jullien, 2003) Notice that it is also possible that only the certifier with the lower reputation is active, but our refinement allows to rule out this unnatural equilibrium.

Lemma 2 implies that the entry decision of B is somewhat irrelevant: even if B always enters (enter is a weakly dominant strategy, as there is no entry cost), the incumbent only loses the market whenever $\rho_2^B > \rho_2^A$, which happens with prior probability $1 - \rho_2^A$. Therefore, the period 2 expected profit of the incumbent reads $\pi_2^{sh}(\rho_2^A) = \rho_2^A \pi_2(\rho_2^A)$. As for the period-1 choice of effort, we derive the following result, where $e_1^{sh}(\rho_1)$ denotes the (unique) equilibrium level of effort under singlehoming:

Proposition 3. *Under singlehoming, competition mitigates bad reputation effects:*

$$e_1^*(\rho_1) < 0 \Rightarrow e_1^*(\rho_1) < e_1^{sh}(\rho_1)$$

Proof. In the Appendix □

When the seller singlehomes, competition lowers incentives for the certifier to pander to bad types. Intuitively, the inefficiency in the monopoly case stems from the excessive weight the certifier puts on bad sellers relative to good ones. Competition in part corrects this bias by increasing the value of attracting good sellers. One shows easily that the function $\pi_2^{sh}(\rho) = \rho\pi_2(\rho)$ reaches its maximum at a point strictly larger than ρ_2^*, the certifier's bliss reputation under monopoly. As a result, the region in which certifier A's reputation is "too accurate" shrinks, or even disappears (if the profit function under singlehoming is nondecreasing). Note, however, that the overall impact of competition is unclear: the threat of being displaced also scales down the expected profit in period 2; this, in turn, lowers the expected benefit of effort for lower values of ρ_1^A, thereby undermining the disciplining effect of reputation.

4.2 Multihoming

In this subsection, we relax the constraint that the seller has to choose one certifier and allow him to "multihome," *i.e.* solicit certification from both A and B. Except for this change (at stage 2.c, the seller can now apply for certification from certifier A, B, or both), the timing is identical to the one in subsection 4.1. We assume that the signals produced by certifiers A and B are independently distributed conditional on product quality. This assumption is inessential (we only need that A's signal is no sufficient statistics for B's signal and vice versa) but simplifies the analysis. Given that both certifiers make 0 effort in period 2, the probability that a bad-type seller obtains certification if he multihomes reads

$$q_2^{mh} \equiv [1 - (\rho_2^A \alpha_H + (1 - \rho_2^A)\alpha_L)][1 - (\rho_2^B \alpha_H + (1 - \rho_2^B)\alpha_L)].$$

We also impose that the seller simultaneously applies for ratings if he applies for more than one. Our results would hold if we allowed for sequential applications, as long as they are public. However, we abstract from the issue of shopping, whereby the seller secretly asks for a rating and discloses it only when it is good enough.[24] Finally, to ensure an interior solution, we make the following assumption, which is the counterpart of Assumption 2 in the monopoly case:

Assumption 4.
$\phi(1+z) < (1-\alpha_H)^2$ *and* $\beta < \frac{1}{1+\max_{q_2^{mh} \in [(1-\alpha_H)^2, (1-\alpha_L)^2]} \frac{1}{\phi}(\frac{1}{1+z} - \frac{\phi}{q_2^{mh}})F(\frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi)}$.

The first inequality ensures that the lowest possible probability of being certified twice is high enough, so that the demand of low quality sellers is always positive. The second one allows to make sure that there is never full participation of low-quality sellers.

Under Assumption 4, we derive the following lemma:

[24] Rating shopping is analyzed in Skreta and Veldkamp (2009), and Bolton et al. (2012).

Lemma 3. *There is no singlehoming in equilibrium: the seller either multihomes, i.e. solicits certification from both certifiers, or does not solicit certification at all.*

Intuitively, an equilibrium in which only one certifier is active is possible only if the cost of an incremental certificate is too high, and/or if buyers believe that a seller who multihomes has a sufficiently high probability of being a bad seller. Our refinement imposes to attribute a deviation towards multihoming ("more certification") to the high-quality seller. In addition, Assumption 4 ensures that the cost of extra certification is small enough, hence the result. Note that this is consistent with the empirical observation that multi-rated issuances are pervasive in the market for corporate credit ratings: Chen et al. (2009) reports that the overwhelming majority of large corporate bond issues have at least two ratings.[25] Again, a consequence of Lemma 3 is that the entry decision of B is trivial: since he always gets positive demand, he enters regardless of his and the incumbent's reputation.[26] Therefore, rather than generating competition, a market structure where multihoming is possible results in both certifiers sharing the same clientele. Indeed, the equilibrium is such that both certifiers enjoy the same profit, no matter their respective reputations:

Lemma 4. *For any ρ_2^B, both certifiers make the same period-2 profit:*

$$\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B) = \beta \frac{z}{1+z}[1-(1+z)\frac{1-q_2^{mh}(\rho_2^A, \rho_2^B)}{q_2^{mh}(\rho_2^A, \rho_2^B)}\phi]F[(1+z)\frac{1-q_2^{mh}(\rho_2^A, \rho_2^B)}{q_2^{mh}(\rho_2^A, \rho_2^B)}\phi]$$

[25] In their sample of 8,767 bonds taken from the Barclays Capital Bond Index, 99.5% of bond issues are rated by S&P and Moody's and 70% are rated by Fitch. On this, see also Bongaerts et al. (2012). Doherty et al. (2012) introduce the possibility for a previously rated issuer to apply for a second rating when another rating agency enters the market. However, they do not study the decision to apply for multiple ratings simultaneously.

[26] Besides, the fact that the entrant's reputation is *ex ante* unknown is irrelevant in the multihoming case.

Viewed *ex ante*, certifier A's continuation profit in period 2 reads

$$\pi_2^{mh}(\rho_2^A) = \int_0^1 \tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B)\, \mathrm{d}\rho_2^B$$

Let $\rho_2^{*mh}(\rho_2^B)$ the bliss reputation of certifier A given ρ_2^B.

Proposition 4. *For all* $\rho_2^B, \rho_2^{*mh}(\rho_2^B) < \rho_2^*$ *and* $\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} \leq 0$.

Proof. In the Appendix □

As a corollary, the bliss period-2 reputation under multihoming is always lower than under monopoly. Notice that this does not rely on the monopoly profit being hump-shaped, i.e. on Assumption 3. Assumption 3 implies that $\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B)$ is either nonincreasing or hump-shaped in ρ_2^A, but the profit under multihoming could as well be hump-shaped if the monopoly profit was increasing in reputation. In other words, the range of parameters for which the two-sidedness of reputation becomes relevant expands as one considers different market structures.

The intuition for Proposition 4 is as follows: because he faces a tradeoff between the credibility of certification and the need to attract bad sellers who fear they might not be able to trade, a monopolistic certifier obtains a profit which is maximized for some interior value of the odds ratio $\frac{1-q_2}{q_2}$, which corresponds to the profit-maximizing "informativeness" of the overall certification process. When a second certifier is active in the market, the other certifier can only compensate the additional information which the latter generates by having himself a lower reputation. And the more reputable the other certifier, the more so.[27]

[27] Notice that $z\phi$ has the effect of lowering the bliss reputation of the certifier even further. Indeed, the certification fee plays a screening role, making certification less attractive for bad sellers, who accordingly need to be attracted through a lower reputation.

As lower reputations become relatively more desirable to a certifier under multihoming than under a monopoly, reputational concerns could, in turn, adversely impact information production in period 1. Before we establish this formally, let us make the assumption that certifiers submit their ratings simultaneously, meaning that a certifier cannot learn about the type of the other by observing his signal before choosing his own effort. Let us also assume that the function $(1-x)F(x)$ is concave. We derive the following result, where $e_1^{mh}(\rho_1)$ denotes the (unique) equilibrium level of effort under multihoming:

Proposition 5. *Multihoming exacerbates bad reputation effects:*

$$e_1^*(\rho_1) < 0 \Rightarrow e_1^{mh}(\rho_1) < e_1^*(\rho_1).$$

Proof. In the Appendix. □

When the seller can multihome, the entry of a second certifier has the opposite effect of exclusive competition. Because certifiers do not compete to attract sellers but instead share the same customer base, a reputation for accuracy becomes less valuable than in the monopoly case. This exacerbates bad reputation effects: when reputation for accuracy is high enough to generate a negative effort for a monopolist, then entry provides further incentives to decrease effort. Note that we have assumed, for simplicity, that competition only takes place in period 2, which makes the analysis of period 1 more tractable. If there were two competitors in both periods, the welfare impact of multihoming in period 1 would be ambiguous: (a) on the one hand, multihoming sometimes lowers incentives to exert effort, which adversely impacts welfare; (b) on the other hand, extra information is conveyed because the competitor produces an independent signal.

5 Conclusion

Recent years have evidenced a compelling need for efficient certification: technologies become more complex, market participants are more sophisticated, which increases the scope for information asymmetries; there has been an increasing demand for green or fair trade products. All these evolutions tie in with a more influential role for certifiers. In addition, externalities call for regulation, as in financial markets. A few certification intermediaries may accordingly exert a considerable influence on the allocation of resources in the whole economy. The question of the ability and incentives of certifiers to generate and transmit accurate information is therefore absolutely critical. In this paper, we argue that reputation in certification markets is essentially two-sided, in that certifiers use their reputation to attract different types of sellers with conflicting preferences for precision. In particular, a certifier can be perceived as too accurate, which creates ambiguous incentives for information production: the certifier may then voluntarily lower the precision of the information he provides to the market. In addition, the presence of multiple certifiers, while providing more diverse information, sometimes exacerbates incentives to reduce information precision.

Our paper offers several possibilities of extension in different directions. First, we abstract from the issue of optimal pricing by certifiers in order to insulate the impact of reputation on the certifier's profit. While assuming some rigidity in prices (*i.e.*, prices cannot instantly adjust to changes in reputation) seems a reasonable assumption in the short run and in a heavily regulated environment, it would be interesting to study how pricing interacts with reputation in a context where a certifier tries to attract different types of applicants. In particular, the interplay between reputation and pricing under competition seems particularly promising. Second, the idea that reputation is multi-sided, in that it reflects the desire to attract different clienteles, could generate new interesting insights in other markets. In particular, two-sided markets where a platform connects two types of agents (e.g., media, operating systems) would constitute a natural application. Finally, in the spirit of the literature on multi-sided communication, an interesting question our paper raises is whether a sender willing to build a two-sided reputation should talk privately or publicly to each of his audiences.

References

Austen-Smith, D., and R. Fryer, 2005, "An economic analysis of acting white," *The Quarterly Journal of Economics*, 120(2), 551–583.

Bar-Isaac, H., and J. Deb, 2012, "Reputation for a servant of two masters," working paper, New York University.

Becker, B., and T. Milbourn, 2011, "How did increased competition affect credit ratings?" *Journal of Financial Economics*, 101(3), 493–514.

Benabou, R., and G. Laroque, 1992, "Using privileged information to manipulate markets: Insiders, gurus, and credibility," *The Quarterly Journal of Economics*, 107(3), 921–958.

Bolton, P., X. Freixas, and J. Shapiro, 2012, "The credit ratings game," *The Journal of Finance*, 67(1), 85–11.

Bongaerts, D., K. Cremers, and W. N. Goetzmann, 2012, "Tiebreaker: Certification and multiple credit ratings," *The Journal of Finance*, 67(1), 113–152.

Caillaud, B., and B. Jullien, 2003, "Chicken & egg: Competition among intermediation service providers," *RAND journal of Economics*, 34(2), 309–328.

Chen, Z., A. Lookman, N. Schürhoff, and D. Seppi, 2009, "Why Ratings Matter: Evidence from the Lehman Brothers Index Rating Redefinition," working paper.

Cohn, J., U. Rajan, and G. Strobl, 2013, "Credit ratings: strategic issuer disclosure and optimal screening," working paper.

U.S. Commission FCIC, 2011, "The Financial Crisis Inquiry Report."

Cornaggia, J., K. Cornaggia, and J. Hund, 2011, "Credit ratings across asset classes: A=A?," working paper.

Doherty, N. A., A. V. Kartasheva, and R. D. Phillips, 2012, "Information effect of entry into credit ratings market: the case of insurers' ratings," *Journal of Financial Economics*, 106, 308–330.

Ely, J., D. Fudenberg, and D. Levine, 2008, "When is reputation bad?," *Games and Economic Behavior*, 63(2), 498–526.

Ely, J., and J. Välimäki, 2003, "Bad reputation," *The Quarterly Journal of Economics*, 118(3), 785–814.

Farhi, E., J. Lerner, and J. Tirole, 2005, "Certifying new technologies," *Journal of the European Economic Association*, 3(2-3), 734–744.

Farhi, E., J. Lerner, and J. Tirole, 2013, "Fear of rejection? Tiered certification and transparency," *The RAND Journal of Economics*, forthcoming.

Faure-Grimaud, A., E. Peyrache, and L. Quesada, 2009, "The ownership of ratings," *The RAND Journal of economics*, 40(2), 234–257.

Frenkel, S., 2010, "Repeated Interaction and Rating Inflation: A Model of Double Reputation," working paper.

Gertner, R., R. Gibbons, and D. Scharfstein, 1988, "Simultaneous signalling to the capital and product markets," *The RAND Journal of Economics*, 173–190.

Hall, K., 2009, "How Moody's Sold Its Ratings - and Sold Out Investors," *McClatchy Newspapers, October*, 18.

Holmström, B., 1999, "Managerial incentive problems: A dynamic perspective," *The Review of Economic Studies*, 66(1), 169–182.

Jeon, D.-S., and S. Lovo, 2013, "Credit rating industry: A helicopter tour of stylized facts and recent theories," *International Journal of Industrial Organization*, forthcoming.

Kamenica, E., and M. Gentzkow, 2011, "Bayesian persuasion," *American Economic Review*, 101(6), 2590–2615.

Kashyap, A., and N. Kovrijnykh, 2013, "Who should pay for ratings and how?," working paper.

Lerner, J., and J. Tirole, 2006, "A model of forum shopping," *The American economic review*, 96(4), 1091–1113.

Mathis, J., J. McAndrews, and J. Rochet, 2009, "Rating the raters: are reputation concerns powerful enough to discipline rating agencies?," *Journal of Monetary Economics*, 56(5), 657–674.

Morris, S., 2001, "Political correctness," *Journal of Political Economy*, 109(2), 231–265.

Rayo, L., and I. Segal, 2010, "Optimal information disclosure," *Journal of Political Economy*, 118(5), 949–987.

Skreta, V., and L. Veldkamp, 2009, "Ratings shopping and asset complexity: A theory of ratings inflation," *Journal of Monetary Economics*, 56(5), 678–695.

Sobel, J., 1985, "A theory of credibility," *The Review of Economic Studies*, 52(4), 557–573.

Stahl, K., and R. Strausz, 2011, “Who should pay for certification?,” working paper.

6 Appendix (For Online Publication)

6.1 Monopoly

Let us first show that the seller cannot trade without asking for certification. Let P_{nc} be the price at which a seller can sell if he does not solicit certification and suppose that $P_{nc} > 0$. P_2 denotes the price the seller obtains when he is granted certification.

There are three cases:

- $P_{nc} > P_2 - \phi$: no good seller ever asks for certification, a fortiori no bad seller. Under Assumption 1, it is impossible to find a positive trading price with no certification.

- $P_{nc} < P_2 - \phi$: a good seller never tries to sell with no certification, hence $P_{nc} > 0$ is impossible.

- $P_{nc} = P_2 - \phi$: a good seller is indifferent between asking for certification or not, therefore a bad seller has a strict preference for not asking for certification. This yields: $P_2 = 1$ and $P_{nc} = 1 - \phi$. The total mass of good sellers who sell their goods is $\beta F(1-\phi)$. Let x be the fraction of them who do not solicit certification. The maximal willingness to pay of buyers upon no certification being asked reads $\frac{\beta F(1-\phi)x}{\beta F(1-\phi)x+1-\beta} < \frac{\beta F(1-\phi)}{\beta F(1-\phi)+1-\beta} < 1-\phi$ thanks to Assumption 1. This contradicts $P_{nc} = 1 - \phi$.

In any case, $P_{nc} > 0$ leads to a contradiction: it is impossible to trade at a positive price with no certification. $\square$

Proof of Proposition 1 Let us define $g(x) \equiv \beta z(1-x)F(x)$ and $k(\rho) \equiv \frac{\rho\alpha_H+(1-\rho)\alpha_L}{1-(\rho\alpha_H+(1-\rho)\alpha_L)}\phi$.

Both functions $F(x)$ and $1-x$ log-concave, so $g(x)$ is also log-concave in x. Since g is positive on $(0,1)$, it is also quasi-concave on $(0,1)$. $k(\rho_2) \in (0,1)$ when $\rho_2 \in [0,1]$ and $k'(\rho_2) \geq 0$. Therefore, $\pi_2(\rho_2) = g[k(\rho_2)]$ is quasiconcave in ρ_2 on $[0,1]$.

$\pi_2'(0)$ has the same sign as $g'(k(0)) = \beta z[(1-\frac{\alpha_L}{1-\alpha_L}\phi)f(\frac{\alpha_L}{1-\alpha_L}\phi)-F(\frac{\alpha_L}{1-\alpha_L}\phi)] \geq 0$.

$\pi_2'(1)$ has the sign of $g'(k(1)) = \beta z[(1-\frac{\alpha_H}{1-\alpha_H}\phi)f(\frac{\alpha_H}{1-\alpha_H}\phi)-F(\frac{\alpha_H}{1-\alpha_H}\phi)] \leq 0$. Because π_2 is quasi-concave, it cannot change monotonicities more than once. Therefore, π_2 is a unimodal function: there is a unique $\rho_2^* \in (0,1)$ such $\pi_2'(\rho_2^*) = 0$. □

Proof of Proposition 2 Note first that the conditions we impose in Assumption 2 to ensure that the participation of low-quality types γ is interior have to be adjusted to account for the impact of effort on the probability of detection. Specifically, the following assumption is sufficient to ensure that that $\gamma_1(e_1) \in (0,1)$ for all ρ_1 and $e_1 \in [-\epsilon, \epsilon]$.

Assumption 5. *$\phi < 1-\alpha_H-\epsilon$ and $\beta < \frac{1}{1+\max\limits_{q_1\in[1-\alpha_H-\epsilon,1-\alpha_L+\epsilon]}\frac{1}{\phi}(1-\frac{\phi}{q_1})F(\frac{1-q_1}{q_1}\phi)}$*

γ_1 is interior, hence it equals $\frac{\beta}{1-\beta}(1-\frac{\phi}{q_1})\frac{1}{\phi}F(\frac{1-q_1}{q_1}\phi)$ (see the Proof of Lemma 1).

Therefore, we have $\frac{(1-\beta)\gamma_1}{\beta F(\bar{\lambda}_1)+(1-\beta)\gamma_1} = \frac{q_1-\phi}{q_1-\phi+\phi q_1}$.

Let us define the following function (where the dependence of q_1, ρ^+ and ρ^- on both ρ_1 and e_1 is made explicit):

$$L(\rho_1,e_1) \equiv \frac{q_1(\rho_1,e_1)-\phi}{q_1(\rho_1,e_1)-\phi+\phi q_1(\rho_1,e_1)}[\pi_2(\rho^+(\rho_1,e_1))-\pi_2(\rho^-(\rho_1,e_1))]-ce_1$$

A solution to (5) is either $e_1^* = -\epsilon$ if $L(\rho_1, -\epsilon) < 0$, $e_1^* = \epsilon$ if $L(\rho_1, \epsilon) > 0$, or e_1^* such that $L(\rho_1, e_1^*) = 0$.

Let us assume that c is large enough that $\sup_{\rho_1, e_1} L_2(\rho_1, e_1) < 0$.[28] This ensures that there is a unique solution e_1^* to (5) for all ρ_1.[29]

Furthermore, L is continuously differentiable in each argument, so $e_1^*(\rho_1)$ is continuous in ρ_1.

Consider $\rho_1 \in \{0, 1\}$. $\forall e_1, \rho^+(\rho_1, e_1) = \rho^-(\rho_1, e_1) = \rho_1$, so $L(\rho_1, e_1) = -ce_1$.

Therefore, we have $e_1^*(0) = e_1^*(1) = 0$.

By the implicit function theorem, when the solution to (5) is interior,

$$\frac{\partial e_1^*}{\partial \rho_1}(\rho_1) = -\frac{L_1(\rho_1, e_1^*(\rho_1))}{L_2(\rho_1, e_1^*(\rho_1))}$$

$$= \frac{\frac{(\alpha_H - \alpha_L)\phi^2}{[q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)]^2}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))]}{L_2(\rho_1, e_1^*)}$$

$$- \frac{\frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}\left\{\pi_2'[\rho^+(\rho_1, e^*(\rho_1))]\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e^*(\rho_1)) - \pi_2'[\rho^-(\rho_1, e^*(\rho_1))]\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e^*(\rho_1))\right\}}{L_2(\rho_1, e_1^*)}$$

We have:

$$\frac{\partial \rho^+}{\partial \rho_1}(\rho_1, e_1) = \frac{(\alpha_H + e_1)(\alpha_L + e_1)}{[\rho_1 \alpha_H + (1 - \rho_1)\alpha_L + e_1]^2} \geq 0$$

and

$$\frac{\partial \rho^-}{\partial \rho_1}(\rho_1, e_1) = \frac{(1 - \alpha_H - e_1)(1 - \alpha_L - e_1)}{[\rho_1(1 - \alpha_H) + (1 - \rho_1)(1 - \alpha_L) - e_1]^2} \geq 0.$$

This implies, recalling that $e_1^*(0) = e_1^*(1) = 0$ and that $L_2(\rho_1, e_1^*(\rho_1))) < 0$:

- $\frac{\partial e_1^*}{\partial \rho_1}(0)$ has the sign of $\pi_2'(0)[\frac{\partial \rho^+}{\partial \rho_1}(0,0) - \frac{\partial \rho^-}{\partial \rho_1}(0,0)] = \pi_2'(0)[\frac{\alpha_H}{\alpha_L} - \frac{1-\alpha_H}{1-\alpha_L}] \geq 0$.

[28] $L_i(.,.)$ refers to the partial derivative of L with respect to the i-th variable.

[29] This assumption is sufficient to get uniqueness, but not necessary. It is indeed enough that $L(\rho_1, e_1) = 0 \Rightarrow L_2(\rho_1, e_1) < 0$; but since a solution to $L(\rho_1, e_1) = 0$ can only be defined implicitly, this is much more cumbersome to write.

- $\frac{\partial e_1^*}{\partial \rho_1}(1)$ has the sign of $\pi_2'(1)[\frac{\partial \rho^+}{\partial \rho_1}(1,0) - \frac{\partial \rho^-}{\partial \rho_1}(1,0)] = \pi_2'(1)[\frac{\alpha_L}{\alpha_H} - \frac{1-\alpha_L}{1-\alpha_H}] \geq 0.$

By continuity of $e_1^*(\rho_1)$ and from $e_1^*(0) = e_1^*(1) = 0$, there exists at least a $\overline{\rho}$ such that $e_1^*(\overline{\rho}) = 0$.

$\overline{\rho}$ is such that $L(\overline{\rho}, 0) = 0$, which is equivalent to

$$\pi_2[\rho^+(\overline{\rho}, 0)] = \pi_2[\rho^-(\overline{\rho}, 0)]$$

$\rho^+(\overline{\rho}, 0) \neq \rho^-(\overline{\rho}, 0)$ because $\overline{\rho} \notin \{0, 1\}$, so the single-peakedness of π_2 implies that

$$\pi_2'[\rho^+(\overline{\rho}, 0)] < 0 < \pi_2'[\rho^-(\overline{\rho}, 0)]$$

We derive that

$$\frac{\partial e_1^*}{\partial \rho_1}(\overline{\rho}) = -\frac{\frac{q_1(\overline{\rho},0)-\phi}{q_1(\overline{\rho},0)-\phi+\phi q_1(\overline{\rho},0)} \left\{ \pi_2'[\rho^+(\overline{\rho}, 0)] \frac{\partial \rho^+}{\partial \rho_1}(\overline{\rho}, 0) - \pi_2'[\rho^-(\overline{\rho}, 0)] \frac{\partial \rho^-}{\partial \rho_1}(\overline{\rho}, 0) \right\}}{L_2(\overline{\rho}, 0)} < 0.$$

Therefore, by continuity of $e_1^*(\rho_1)$, $\overline{\rho}$ must be unique. From uniqueness of $\overline{\rho}$ and from $\frac{\partial e^*}{\partial \rho_1}(\overline{\rho}) < 0$, we derive that $e_1^*(\rho_1) > 0$ for $\rho_1 \in (0, \overline{\rho})$ and $e_1^*(\rho_1) < 0$ for $\rho_1 \in (\overline{\rho}, 1)$.

□

6.2 Singlehoming

Proof of Lemma 2 Let denote by $P_j (j = A, B)$ the price that a seller obtains following certification from certifier j. Our refinement imposes that a deviation from the more reputable to the less reputable certifier is attributed to a bad seller, while a deviation in the opposite direction comes from a good seller. This allows to rule out the unnatural equilibrium where good sellers solicit the less reputable certifier.

If $\rho_2^A = \rho_2^B$, let us assume that all seller types go to A. This assumption is innocuous, as $\rho_2^A = \rho_2^B$ is a zero probability event. If $\rho_2^A \neq \rho_2^B$, suppose w.l.o.g. that $\rho_2^A > \rho_2^B$.

- Suppose $P_A < P_B$: this implies that no good seller ever goes to A. Therefore, P_A cannot be pinned down by Bayes' rule. Since $\rho_2^A > \rho_2^B$, our refinement imposes that $P_A = 1$, which violates $P_A < P_B$.

- Suppose $P_A = P_B = 1$: this is impossible, as a low-quality seller would then have, from Assumption 2, an incentive to solicit, say, A, which is inconsistent with $P_A = 1$.

- Suppose $P_A = P_B < 1$: then good sellers are indifferent between A and B. But, since $\rho_2^A > \rho_2^B$, bad sellers must prefer strictly B to A, so P_A should be equal to 1.

- Therefore, we must have $P_A > P_B$. This implies that no good seller ever goes to B, hence no bad seller either. P_B cannot be pinned down by Bayes' rule, and our refinement imposes $P_B = 0$. All the sellers who solicit certification then go to A and the price P_A is then determined as in the monopoly case. $\square$

Proof of Proposition 3 Remember that we have $\pi_2^{sh}(\rho) = \rho\pi_2(\rho)$. Let us define

$$L^{sh}(\rho_1, e_1) \equiv \frac{q_1(\rho_1, e_1) - \phi}{q_1(\rho_1, e_1) - \phi + \phi q_1(\rho_1, e_1)}[\rho^+(\rho_1, e_1)\pi_2(\rho^+(\rho_1, e_1)) - \rho^-(\rho_1, e_1)\pi_2(\rho^-(\rho_1, e_1))] - ce_1.$$

A solution to the incumbent's problem is either $e_1^{sh} = -\epsilon$ if $L^{sh}(\rho_1, -\epsilon) < 0$, $e_1^{sh} = \epsilon$ if $L^{sh}(\rho_1, \epsilon) > 0$, or e_1^{sh} such that $L^{sh}(\rho_1, e_1^{sh}) = 0$. As in the monopoly case, we impose that c is large enough, so that $\frac{\partial L^{sh}}{\partial e_1}(\rho_1, e_1) < 0$ for all (ρ_1, e_1). This ensures the uniqueness of e_1^{sh}.

If $e_1^* = -\epsilon$, the result is obvious.

If $-\epsilon < e_1^* \leq 0$, e_1^* is defined by

$$L(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^* = 0.$$

In order to compare e_1^* and e_1^{sh}, let us derive $L^{sh}(\rho_1, e_1^*)$:

$L^{sh}(\rho_1, e_1^*) = \frac{q_1(\rho_1,e_1^*)-\phi}{q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)}[\rho^+(\rho_1, e_1^*)\pi_2(\rho^+(\rho_1, e_1^*)) - \rho^-(\rho_1, e_1^*)\pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^*$

$= \frac{q_1(\rho_1,e_1^*)-\phi}{q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)}[(1-\rho^-(\rho_1, e_1^*))\pi_2(\rho^-(\rho_1, e_1^*)) - (1-\rho^+(\rho_1, e_1^*))\pi_2(\rho^+(\rho_1, e_1^*))]$

Furthermore, $e_1^* \leq 0 \Leftrightarrow \pi_2(\rho^+(\rho_1, e_1^*)) \leq \pi_2(\rho^-(\rho_1, e_1^*))$.

Since $\frac{q_1(\rho_1,e_1^*)-\phi}{q_1(\rho_1,e_1^*)-\phi+\phi q_1(\rho_1,e_1^*)} > 0$, we have:

$$-\epsilon < e_1^* \leq 0 \Rightarrow L^{sh}(\rho_1, e_1^*) \geq [\rho^+(\rho_1, e_1^*) - \rho^-(\rho_1, e_1^*)]\pi_2(\rho^+(\rho_1, e_1^*)) \geq 0.$$

Finally, from $\frac{\partial L^{sh}}{\partial e_1}(\rho_1, e_1) < 0$, we derive $e_1^* \leq e_1^{sh}$. $\square$

6.3 Multihoming

Proof of Lemma 3 Let P_{AB}, P_A and P_B the prices that buyers are willing to pay following certification by both A and B, A only, and B only. Suppose furthermore that $\rho_A \geq \rho_B$.

- Consider an equilibrium in which no seller multihomes. Assume first that no seller ever goes to B. From Assumption 2, we know that there is neither zero nor full participation of bad sellers. Therefore, P_A is given by the indifference condition of bad sellers: $P_A = \frac{\phi}{1-\rho_A\alpha_H-(1-\rho_A)\alpha_L}$. P_{AB}

and P_B cannot be derived from Bayes' rule. Our refinement imposes that P_{AB} be set to 1, as good sellers always have stronger incentives to deviate to multihoming than bad sellers.

In order for such an equilibrium to exist, we must therefore have $1-z\phi < P_A$.

Since $P_A + z\phi \leq \frac{\phi}{1-\alpha_H} + z\phi$, and $\phi \leq \frac{(1-\alpha_H)^2}{1+z}$ (from Assumption 4), we conclude:

$P_A + z\phi \leq 1 - \alpha_H - \frac{z\alpha_H(1-\alpha_H)}{1+z} < 1.$

This establishes that a good seller who solicits A only should deviate and solicit an extra rating.

If the seller never goes to B (*i.e.* P_A cannot be pinned down by Bayes' rule), the result is a fortiori true because $\rho_A \geq \rho_B$, so incentives to deviate are even larger.

If some good to A, some go to B, but none multihomes, then we know from the singlehoming case that those who go to B should deviate to A.

This proves that there is no equilibrium with no multihoming.

- Suppose now that there is both singlehoming and multihoming in equilibrium. A good seller must then be indifferent between multihoming and singlehoming, say with A only. Then, the bad seller strictly prefers to singlehome, so we must have $P_{AB} = 1$. The indifference condition for the good seller thus reads $P_A = 1 - z\phi$, which is impossible under Assumption 4, as we have just seen.

Therefore, singlehoming does not occur in equilibrium. □

Proof of Lemma 4 As in the monopoly case, an equilibrium features a triple $(\overline{\lambda}_2^{mh}, \gamma_2^{mh}, P_2^{mh})$. Consider some $q_2^{mh} \in [(1-\alpha_H)^2, (1-\alpha_L)^2]$. If $P_2^{mh} > (1+z)\phi$, we must have:[30]

$$P_2^{mh} - \overline{\lambda}_2^{mh} = (1+z)\phi \tag{6}$$

$$\gamma_2^{mh} \in \underset{\tilde{\gamma}\in[0,1]}{\operatorname{argmax}}\, \tilde{\gamma}[q_2^{mh} P_2^{mh} - (1+z)\phi] \tag{7}$$

$$P_2^{mh} = Pr(\theta_2 = \theta_g | \sigma_2^A = \sigma_2^B = \varnothing) = \frac{\beta F(\overline{\lambda}_2^{mh})}{\beta F(\overline{\lambda}_2^{mh}) + (1-\beta)\gamma_2^{mh} q_2^{mh}}. \tag{8}$$

The solution of this system must be unique, as P_2^{mh} is a decreasing function of γ_2^{mh}. Furthermore, at any solution involving certification with positive probability, we have $P_2^{mh} > (1+z)\phi$, so $\overline{\lambda}_2^{mh} > 0$. Finally, under Assumption 4, the solution of the system is interior.

For all $q_2^{mh} \in [(1-\alpha_H)^2, (1-\alpha_L)^2]$, the interior solution is:

$$\begin{cases} \overline{\lambda}_2^{mh} &= \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi \\ \gamma_2^{mh} &= \frac{\beta}{1-\beta}(\frac{1}{1+z} - \frac{\phi}{q_2^{mh}})\frac{1}{\phi}F(\frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi) \\ P_2^{mh} &= \frac{\phi}{q_2^{mh}} \end{cases}$$

The profit as a function of q_2^{mh} equals:

$$[\beta F(\overline{\lambda}_2^{mh}) + (1-\beta)\gamma_2^{mh}]z\phi = \beta\frac{z}{1+z}[1 - \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi]F[\frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi].$$

One derives the desired result by replacing q_2^{mh} by its value. □

Proof of Proposition 4

[30] Notice that we rule out again the equilibrium in which $\overline{\lambda}_2^{mh} = \gamma_2^{mh} = 0$, which does not survive our refinement anyway. Thus, we can always apply Bayes' rule to pin down P_2^{mh}.

Some notation and preliminary computations In order to compare the profits under monopoly and multihoming, let us first define $t(x, \rho_2^B) \equiv (1+z)\frac{x+(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)\phi}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}$. Recalling that

$$g(x) = \beta z(1-x)F(x),$$

$$k(\rho) = \frac{\rho\alpha_H + (1-\rho)\alpha_L}{1-(\rho\alpha_H + (1-\rho)\alpha_L)}\phi,$$

$$q_2^{mh}(\rho_2^A, \rho_2^B) = [1-(\rho_2^A\alpha_H + (1-\rho_2^A)\alpha_L)][1-(\rho_2^B\alpha_H + (1-\rho_2 B)\alpha_L)],$$

one rewrites

$$\pi_2(\rho_2) = g[k(\rho_2)]$$

$$\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B) = \frac{1}{1+z}g[t(k(\rho_2^A), \rho_2^B)].$$

Proof We have established that $g(.)$ is quasi-concave on $[0,1]$. Furthermore, $t(k(\rho_2^A), \rho_2^B) = \frac{1-q_2^{mh}}{q_2^{mh}}(1+z)\phi \in (0,1)$ from Assumption 4. Finally, from $k'(\rho) > 0$ and $t_1(.,.) > 0$, we derive that $\tilde{\pi}_2^{mh}(\rho_2^A, \rho_2^B)$ is quasi-concave in ρ_2^A.[31]

By definition of ρ_2^*, we have $g'(k(\rho_2^*)) = 0$.

$$\frac{\partial \pi_2^{mh}}{\partial \rho_2^A}(\rho_2^*, \rho_2^B) = \frac{1}{1+z}g'[t(k(\rho_2^*), \rho_2^B)]t_1(k(\rho_2^*), \rho_2^B)k'(\rho_2^*).$$

This expression is negative since $t(x, \rho_2^B) > x$ for all (x, ρ_2^B) and $g'(x) < 0$ for $x \geq k(\rho_2^*)$.

This implies that $\rho_2^{*mh}(\rho_2^B) < \rho_2^*$ for all ρ_2^B.

If $\rho_2^{*mh} > 0$, we have

$$\frac{\partial \pi_2^{mh}}{\partial \rho_2^A}(\rho_2^{*mh}, \rho_2^B) = \frac{1}{1+z}g'[t(k(\rho_2^{*mh}), \rho_2^B)]t_1(k(\rho_2^{*mh}), \rho_2^B)k'(\rho_2^{*mh}) = 0.$$

[31] $t_i(x, \rho_2^B)$ refers to the partial derivative of t with respect to the i-th variable.

Since t_1 and k' are positive and g is unimodal, we derive that $t(k(\rho_2^{*mh}), \rho_2^B)$ is equal to some constant (actually $k(\rho_2^*)$). It follows that

$$\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} = -\frac{t_2(k(\rho_2^{*mh}), \rho_2^B)}{t_1(k(\rho_2^{*mh}), \rho_2^B)k'(\rho_2^{*mh})}.$$

Finally, $t_2(.,.) > 0$ implies that $\frac{\partial \rho_2^{*mh}(\rho_2^B)}{\partial \rho_2^B} \leq 0$.

Furthermore, $\frac{\partial \pi_2^{mh}}{\partial \rho_2^A}(0, \rho_2^B) < 0$ holds only when ρ_2^B is large enough. In this case, the profit function is decreasing on $[0, 1]$ and $\rho_2^{*mh} = 0$. □

Proof of Proposition 5 Let us first prove the following lemma:

Lemma 5. *The following implication holds:*

$$\begin{aligned} & \pi_2(\rho^+(\rho_1, e_1)) \leq \pi_2(\rho^-(\rho_1, e_1)) \\ \Rightarrow \quad & \pi_2^{mh}(\rho^+(\rho_1, e_1)) - \pi_2^{mh}(\rho^-(\rho_1, e_1)) \leq \pi_2(\rho^+(\rho_1, e_1)) - \pi_2(\rho^-(\rho_1, e_1)). \end{aligned}$$

Proof. Using the notation introduced earlier, we have

$$\pi_2^{mh}(\rho) = \frac{1}{1+z}\int_0^1 g[t(k(\rho), \rho_2^B)] \, \mathrm{d}\rho_2^B \text{ and } \pi_2(\rho) = g[k(\rho)].$$

Dropping arguments of ρ^+ and ρ^-, one writes

$$\pi_2^{mh}(\rho^+) - \pi_2^{mh}(\rho^-) - [\pi_2(\rho^+) - \pi_2(\rho^-)]$$

$$= \frac{1}{1+z}[\int_0^1 g[t(k(\rho^+), \rho_2^B)] - g[t(k(\rho^-), \rho_2^B)] \, \mathrm{d}\rho_2^B - [g[k(\rho^+)] - g[k(\rho^-)]].$$

One notices first that

$$\frac{1}{1+z}[\int_0^1 g[t(k(\rho^+), \rho_2^B)] - g[t(k(\rho^-), \rho_2^B)] \, \mathrm{d}\rho_2^B$$

$$= \frac{1}{1+z}\int_0^1 \int_{k(\rho^-)}^{k(\rho^+)} g'(t(s,\rho_2^B))t_1(s,\rho_2^B)\,\mathrm{d}s\,\mathrm{d}\rho_2^B$$

$$= \int_0^1 \frac{1}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}\int_{k(\rho^-)}^{k(\rho^+)} g'(t(s,\rho_2^B))\,\mathrm{d}s\,\mathrm{d}\rho_2^B.$$

and that $g(k(\rho^+) - g(k(\rho^-)) = \int_{k(\rho^-)}^{k(\rho^+)} g'(s)\,\mathrm{d}s$.

We derive

$$\pi_2^{mh}(\rho^+) - \pi_2^{mh}(\rho^-) - [\pi_2(\rho^+) - \pi_2(\rho^-)]$$

$$= \int_0^1 \frac{1}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)}\left\{\int_{k(\rho^-)}^{k(\rho^+)}[g'(t(s,\rho_2^B)) - g'(s)]\,\mathrm{d}s + (\rho_2^B\alpha_H + (1-\rho_2^B)\alpha_L)[g(k(\rho^+) - g(k(\rho^-))]\right\}\mathrm{d}\rho_2^B$$

Given that $\rho^+ \geq \rho^-$, we have $k(\rho^+) \geq k(\rho^-)$.

Furthermore, $t(s,\rho_2^B) = (1+z)\frac{s+(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)\phi}{1-(\rho_2^B\alpha_H+(1-\rho_2^B)\alpha_L)} > s$ for all ρ_2^B.

Since g is concave, we derive that $\int_{k(\rho^-)}^{k(\rho^+)}[g'(t(s,\rho_2^B)) - g'(s)]\,\mathrm{d}s \leq 0$.

It follows that

$$\pi_2(\rho^+) - \pi_2(\rho^-) \leq 0 \Rightarrow \pi_2^{mh}(\rho^+) - \pi_2^{mh}(\rho^-) - [\pi_2(\rho^+) - \pi_2(\rho^-)] \leq 0.$$

□

We now turn to the Proof of Proposition 5. Let us define

$$L^{mh}(\rho_1,e_1) \equiv \frac{q_1(\rho_1,e_1)-\phi}{q_1(\rho_1,e_1)-\phi+\phi q_1(\rho_1,e_1)}[\pi_2^{mh}(\rho^+(\rho_1,e_1)) - \pi_2^{mh}(\rho^-(\rho_1,e_1))] - ce_1$$

A solution to the incumbent's problem is either $e_1^{mh} = -\epsilon$ if $L^{mh}(\rho_1,-\epsilon) < 0$, $e_1^{mh} = \epsilon$ if $L^{mh}(\rho_1,\epsilon) > 0$, or e_1^{mh} such that $L^{mh}(\rho_1,e_1^{mh}) = 0$. As usual, we impose that c is large enough, so that $\frac{\partial L^{mh}}{\partial e_1}(\rho_1,e_1) < 0$ for all (ρ_1,e_1). This ensures the uniqueness of e_1^{mh}.

If $-\epsilon < e_1^* \leq 0$, e_1^* is defined by

$$L(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*))] - ce_1^* = 0$$

In order to compare e_1^* and e_1^{mh}, let us derive $L^{mh}(\rho_1, e_1^*)$:

$$L^{mh}(\rho_1, e_1^*) = \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2^{mh}(\rho^+(\rho_1, e_1^*)) - \pi_2^{mh}(\rho^-(\rho_1, e_1^*))] - ce_1^*$$

$$= \frac{q_1(\rho_1, e_1^*) - \phi}{q_1(\rho_1, e_1^*) - \phi + \phi q_1(\rho_1, e_1^*)}[\pi_2^{mh}(\rho^+(\rho_1, e_1^*)) - \pi_2^{mh}(\rho^-(\rho_1, e_1^*)) - (\pi_2(\rho^+(\rho_1, e_1^*)) - \pi_2(\rho^-(\rho_1, e_1^*)))].$$

Using Lemma 5, we derive from $e_1^* \leq 0$ that $L^{mh}(\rho_1, e_1^*) \leq 0$. We conclude from $\frac{\partial L^{mh}}{\partial e_1}(\rho_1, e_1) < 0$ that

$$-\epsilon < e_1^* \leq 0 \Rightarrow e_1^{mh} \leq e_1^*.$$

Suppose that $e_1^* = -\epsilon$. We must therefore have

$$\frac{q_1(\rho_1, -\epsilon) - \phi}{q_1(\rho_1, -\epsilon) - \phi + \phi q_1(\rho_1, -\epsilon)}[\pi_2(\rho^+(\rho_1, -\epsilon)) - \pi_2(\rho^-(\rho_1, -\epsilon))] + c\epsilon < 0.$$

From Lemma 5, we have

$$\pi_2^{mh}(\rho^+(\rho_1, -\epsilon)) - \pi_2^{mh}(\rho^-(\rho_1, -\epsilon)) \leq \pi_2(\rho^+(\rho_1, -\epsilon)) - \pi_2(\rho^-(\rho_1, -\epsilon))$$

We conclude that

$$\frac{q_1(\rho_1, -\epsilon) - \phi}{q_1(\rho_1, -\epsilon) - \phi + \phi q_1(\rho_1, -\epsilon)}[\pi_2^{mh}(\rho^+(\rho_1, -\epsilon)) - \pi_2^{mh}(\rho^-(\rho_1, -\epsilon))] < -c\epsilon,$$

which implies that $e_1^{mh} = -\epsilon$.

Finally,

$$e_1^* \leq 0 \Rightarrow e_1^{mh} \leq e_1^*.$$

$\square$