Collegio Carlo Alberto

A Model of Educational Investments, Social Concerns and Inequality

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No. 405
April 2015 (Revised, February 2018)

Carlo Alberto Notebooks
www.carloalberto.org/research/working-papers

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A Model of Educational Investment, Social Concerns and Inequality\textsuperscript{1}

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February 2018

Abstract

We consider a model in which educational investments entail productivity gains, signaling power, and social status. The latter depends on the agent’s relative achievement in one of three dimensions: (i) innate skills, (ii) level of schooling, and (iii) income. We study the three scenarios separately and characterize the conditions under which social concerns increase or decrease educational and income inequality. Inequality increases (decreases) if education lowers the stigma suffered by low skilled workers less (more) than it boosts the prestige enjoyed by high skilled workers. We discuss the expected results of some policies in light of these findings.

\textit{JEL Classification:} D03, D10, I20, I21.

\textit{Keywords:} education, signaling, social status, inequality.

\textsuperscript{1}We are grateful to participants in the seminars at IMT Lucca and NUS, and in the conferences SAET 2014 (Tokyo), EEA-ESEM 2014 (Toulouse), EUI Alumni Workshop and ASSET 2014 (Aix-en-Marseille). Edoardo Grillo gratefully acknowledges Unicredit&Universities Foscolo Foundation for its generous financial support through the Foscolo Europe fellowship.

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I. Introduction

The choice regarding which level of education to acquire is paramount in an individual’s life. This decision not only shapes an agent’s job market perspectives - and thus his expected career path and economic well-being - but it also plays a fundamental role in determining an individual’s social position. Sociological studies have long recognized education as one of the main determinants of an agent’s social prestige and status attainment (Haller and Portes, 1973; Hollingshead, 2011). In particular, it has been noted that in many societies, low educational achievements can lead individuals to suffer from social stigma (Solga, 2002), while high educational levels convey social prestige (Archer et al., 2005).

In principle, the correlation between an individual’s level of schooling and his social standing can arise both directly, as education entails social recognition per se, and indirectly because it is associated with other key determinants of social status such as intelligence or income. Using data from the World Values Survey, Table 1 shows that, despite substantial heterogeneity in the extent and the relative strength of the two relationships (probably reflecting socioeconomic and cultural differences across countries), an individual’s self-reported social position is positively associated with both his educational attainment and his income, even after controlling for sociodemographic and economic characteristics.¹

Starting with Veblen (1899), an extensive literature in economics testifies to the importance of status-seeking behavior in many economically relevant settings (see Weiss and Fershtman, 1998, for a survey and our discussion of the literature below for more details). Particularly related to our paper, Solnick and Hemenway (1998) and Nikolaev (2016) show through survey data that individuals care about their relative level of schooling. Interestingly, Solnick and Hemenway (1998) shows that, among all dimensions, individuals are particularly interested in their relative position in terms of income, education, and perceived ability as

¹Table 1 reports the data of a small sample of countries from all continents. In the Data Appendix, we show that both relationships hold true (at the 95% confidence interval) for 50 out of the 57 countries covered in the World Values Survey and for which observations on the variables used in the empirical analysis are available.
Table 1: Regression of Self-Reported Social Status on Income and Education.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\beta_{Inc}$</th>
<th>$\beta_{Edu}$</th>
<th>Country</th>
<th>$\beta_{Inc}$</th>
<th>$\beta_{Edu}$</th>
<th>Country</th>
<th>$\beta_{Inc}$</th>
<th>$\beta_{Edu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.261 (.024)</td>
<td>0.122 (.020)</td>
<td>Japan</td>
<td>0.160 (.014)</td>
<td>0.053 (.020)</td>
<td>Nigeria</td>
<td>0.373 (.022)</td>
<td>0.128 (.019)</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.194 (.018)</td>
<td>0.055 (.015)</td>
<td>Mexico</td>
<td>0.100 (.012)</td>
<td>0.049 (.0145)</td>
<td>Russia</td>
<td>0.247 (.019)</td>
<td>0.077 (.017)</td>
</tr>
<tr>
<td>China</td>
<td>0.443 (.025)</td>
<td>0.061 (.020)</td>
<td>Morocco</td>
<td>0.607 (.031)</td>
<td>0.167 (.023)</td>
<td>South Africa</td>
<td>0.232 (.014)</td>
<td>0.164 (.016)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.394 (.016)</td>
<td>0.137 (.021)</td>
<td>Netherlands</td>
<td>0.266 (.031)</td>
<td>0.270 (.023)</td>
<td>Sweden</td>
<td>0.286 (.0264)</td>
<td>0.103 (.0223)</td>
</tr>
<tr>
<td>India</td>
<td>0.084 (.008)</td>
<td>0.075 (.007)</td>
<td>New Zeland</td>
<td>0.110 (.025)</td>
<td>0.138 (.044)</td>
<td>USA</td>
<td>0.380 (.018)</td>
<td>0.115 (.022)</td>
</tr>
</tbody>
</table>

Notes: Coefficients obtained from ordered probit regressions for each country of self-reported social status (measured on 1-5 scale with 1 being the lowest class and 5 the highest class) against income level, educational attainment and a set of controls. Controls are: gender, age, marital status, employment status, dummy variable for religiosity, dummy variable for whether the household saved money during last year, degree of financial satisfaction, and a variable measuring whether the job of the respondent is manual or intellectual (see Data Appendix for details). Standard deviations in parenthesis. Data source: World Values Survey, wave 6, 2010-2014.

Building upon these premises, we investigate how concerns for social status combine with standard economic incentives and affect educational and income inequality within a society. We consider a model of signaling in which education is a costly investment that affects an agent’s well-being in two ways. It determines future wages (and thus income), and it influences the agent’s social status. In particular, education can trigger additional positive or negative feelings that stem from comparing one’s own position with the average position that endogenously emerges in the population. Underachievements with respect to this endogenous benchmark lead to social stigma and thus negatively affect the agent’s overall utility. At the opposite, overachievements with respect to the average yield social prestige and have a positive effect on utility. Importantly, we allow the impact of underachievements on the utility to be higher, lower or equal than the one of overachievements.

Focusing on the nature of social comparisons, we model three channels through which an individual’s educational level can affect his social status. First, if social ranking is determined
on the basis of an individual’s intrinsic characteristics (e.g., her intelligence or personality), then the level of schooling may act as a signal of these skills. In this case, we will say that people care about their *perceived type*. Second, social concerns can be defined in terms of *educational level*, i.e., people may care about educational achievements per se, and higher educational levels may directly yield higher social prestige. Finally, social status may be determined on the basis of *income*, in which case individuals may decide to invest in education because this fosters their earnings and thus allows them to climb the social ladder.

We characterize the least-cost separating equilibrium of the model when social concerns are defined separately in each of the three dimensions described above. We show that the existence of social pressures triggers a rat race among individuals (Akerlof, 1976) in all three scenarios and thus lead to an increase in educational investments.

Moreover, we also show that the dimension of social comparisons (perceived type, education, or income) and the relative incidence of social prestige versus social stigma impact differently on the two forces that shape educational returns: (i) the direct benefit of education (i.e., the marginal gain in utility that schooling entails due to enhancements in human capital and to the social benefits it conveys), and (ii) the signaling benefit (i.e., the gain in utility that more productive individuals enjoy by signaling their type to firms and peers). Importantly, in a separating equilibrium, both low skilled workers and high skilled workers enjoy the direct benefit, whereas only high skilled workers enjoy the signaling benefit.

Building on this, our main contribution is to define the conditions under which social concerns lead to a widening or to a reduction of educational and income inequality. Intuitively, whenever social concerns leave the direct benefit constant but increase the signaling benefit, then only high types increase their educational investment. As a result, educational and income inequality increase. This is exactly what happens when social comparisons are based on agents’ perceived type. If instead social comparisons are made in terms of educational levels, it is the direct benefit that goes up, while the signaling benefit stays constant. Then, educational and income inequality decrease (increase) if social concerns raise the direct benefit
for low skilled workers more (less) than what they do for high skilled workers. This happens when underachievements have a larger (smaller) incidence than overachievements. Finally, if the relevant dimension of social comparisons is income, then both the direct benefit of the two types and the signaling benefit enjoyed by high types get inflated. As such, educational and income inequality decrease if and only if the increase in the direct benefit of low skilled workers overcomes the increase that high skilled workers experience in both the direct and the signaling benefit. This happens when underachievements loom larger than overachievements and the signaling benefit is not too large.

We then discuss the expected impact of various types of policy interventions and socioeconomic trends in light of our results. The model suggests that the government may lower inequality by launching campaigns that are aimed at improving the “social image” of educational achievements. Indeed, campaigns of this kind (often labeled under slogans such as “school is cool” or “learning is cool”) are a common tool to contrast school drop-outs especially among minorities and, consequently, to reduce differences in educational and job-related outcomes. On the contrary, messages that boost the social prestige enjoyed by high-income earners (e.g., TV shows, or commercials of high-end products) may widen inequality. As discussed in Corneo and Jeanne (2001), this kind of social pressures may have contributed to the increasing level of inequality and low economic growth that many western countries recently experienced.

We also study the effect of policies and socioeconomic trends that may affect the educational costs of the low skilled (e.g., policies that provide scholarships and dedicated help) or boost the monetary return of education (e.g., the increase in the skill premium in the last decades). Results in these cases are in line with standard predictions across all our model specifications.

The paper is organized as follows. In the remainder of the Introduction, we discuss the related literature. Section II introduces the model. Section III contains the equilibrium analysis. Section IV studies how social concerns impact educational and income inequality
and investigates some policy implications of the model. Section V discusses some extensions of the baseline model. Section VI concludes. The Appendix contains some of the proofs. Longer proofs and a more detailed analysis of the above-mentioned extensions are included in the Online Appendix.

**Literature Review**

By studying how standard economic incentives and concerns for social status combine to affect agents’ educational choices, we connect two strands of the economic literature.

On the one hand, the analysis of the economic incentives that underlie an agent’s decision to invest in education has always been a cornerstone of labor economics. In this respect, both human capital models (see Becker, 1962, 1975, for the pioneering contributions to the field, and Blundell et al., 2009, for a review) and signaling models (see Spence, 1973, and Riley, 2001, for a review) reach the same conclusion despite fundamental differences in their assumptions as well as in their welfare and policy implications. That conclusion is that individuals invest in education because higher levels of schooling lead to higher income. From an empirical point of view, the causal relationship between schooling and earnings has been tested extensively and generally confirmed through the estimation of so-called Mincerian equations (see Lemieux, 2006, and the references therein).

On the other hand, the importance of social prestige, peer pressures, conformism and status-seeking behavior in shaping individual choices has a long tradition in economics (Veblen, 1899; Duesenberry, 1949; Frank, 1985), and it has been acknowledged in a variety of situations (see Weiss and Fershtman, 1998, for a survey). These include consumption patterns (Bernheim, 1994; Hopkins and Kornienko, 2004), financial behavior (Barberis and Thaler, 2003), and engagement in pro-social activities (Bénabou and Tirole, 2006). As discussed in Postlewaite (1998), concerns about social status can be direct (i.e., agents care about status per se) or instrumental (i.e., agents care about status because it affects future consumption). Furthermore, they can be rationalized by evolutionary arguments or positive
assortative matching (Kalmijn, 1994; Rege, 2008; Chiappori et al., 2009; Eika et al., 2014).

Linking educational choices with social status considerations, Austen-Smith and Fryer (2005) consider a setting in which different social groups perceive educational achievements in conflicting ways (the so-called acting white phenomenon), while Zimmermann (2003) and Cipollone and Rosolia (2007) exploit some natural experiments to empirically identify the effects of peer pressures in influencing educational choices. On a related vein, Bursztyn and Jensen (2014) shows experimentally how adherence to prevailing social norms affects pupils’ educational investments. From a theoretical point of view, Gallice (2009) studies how conformism and status-seeking behavior may shape agents’ schooling decisions. Finally, as already discussed above, Solnick and Hemenway (1998) and Nikolaev (2016) use survey data to show that individuals care about how their level of schooling compares with the ones of their peers. We contribute to this literature by explicitly modeling the various channels through which educational achievements can impact an individual’s social status and thus affect inequality in the society.

From a methodological point of view, we capture the fact that agents care about the social status implications of their educational choices through the notion of interdependent preferences. In particular, we postulate that agents compare their own position in a particular dimension with the average position in the population determined through rational expectations. Such an approach (commonly referred to as “Keeping up with the Joneses”) has a long tradition in the literature (Duesenberry, 1949; Pollack, 1976). More recently, it has been used to study the relation between wealth and growth (Harbaugh, 1996; Futagamia and Shibata, 1998; Corneo and Jeanne, 2001) and inequality and happiness (Hopkins, 2008), or to model social concerns about income and social class (Clark and Oswald, 1996; Gallice and Grillo, 2016).

Finally, in terms of modeling techniques, we borrow extensively from the literature on expectation-based reference dependence and the related notion of personal equilibrium due to Köszegi and Rabin, 2006 and 2007 (see Abeler et al., 2011 and Ericson and Fuster, 2011).
for experimental evidence on this approach).

II. The Model

A unit mass of individuals is made of two types of agents who differ in terms of their innate productivity: low types have a productivity equal to \( w_l \), and high types have a productivity equal to \( w_h = w_l + \Delta w \), where \( \Delta w > 0 \) measures the baseline productivity gap. The proportion of high types in the population is given by \( \alpha \in (0, 1) \) and this is common knowledge.

Each agent can make an educational investment denoted by \( e \in \mathbb{R}_+ \) (see Section V and the Online Appendix for a discussion of what would happen if we differentiate among different types of educational investments). Achieving educational level \( e \) entails a cost \( c(e \mid \psi) = \theta_\psi e^2 \), where \( \psi \in \{l, h\} \) indicates the agent’s type, and the condition \( \theta_l > \theta_h \) holds. Thus, low types face a higher cost of studying than high types. The specific functional form for \( c(e \mid \psi) \) is assumed for analytical tractability. Our analysis and results remain valid for other cost functions that are strictly increasing and strictly convex in educational investments and that satisfy the single-crossing property.

Educational investments lead to productivity gains. If an individual of type \( \psi \) achieves educational level \( e \), then his actual productivity becomes \( w_{\psi} + \beta \cdot e \). We refer to \( \beta \) as to the marginal monetary benefit of education. In line with the empirical literature (see Lemieux, 2006, and the references therein), we assume that this benefit is weakly positive, namely \( \beta \geq 0 \). We also assume that \( \beta \) is independent of the agent’s type. This is done to simplify the exposition. We discuss the case in which the marginal monetary benefit is positively related to the agent’s type in Section V of the paper and in Section III of the Online Appendix (for empirical evidence in support of such an hypothesis see Blackburn and Neumark, 1993; Weiss, 1995; Card, 1999; Belzil and Hansen, 2002). All of our qualitative results remain valid in this richer framework.

In addition to investing in education, individuals supply labor and receive a wage equal
to their expected productivity. Thus, an agent’s income depends on two components that are both associated with his observable level of education. The first component captures the returns to signaling and depends on the beliefs that firms have about the agent’s type. The second component measures the productivity enhancements that stem from the educational investment, i.e. the increase in human capital conveyed by education. Formally, the income of an individual who attains educational level $e$ is given by

$$I(e) = E[w \mid \pi(e, \tilde{e}_l, \tilde{e}_h)] + \beta \cdot e,$$

where $E[w \mid \pi(e, \tilde{e}_l, \tilde{e}_h)]$ denotes the firms’ expectation of the agent’s innate productivity. In particular, $\pi(e, \tilde{e}_l, \tilde{e}_h)$ denotes the firms’ beliefs about the agent’s type. These beliefs depend on the agent’s level of education $e$ and on the firms’ conjecture about the distribution of educational achievements in the population, $(\tilde{e}_l, \tilde{e}_h)$. Formally, $\tilde{e}_\psi$ is a cdf on $\mathbb{R}_+$ describing the distribution of educational achievements among types $\psi \in \{l, h\}$. When no confusion arises, we will omit to specify the dependency on $(\tilde{e}_l, \tilde{e}_h)$ and simply denote firms’ beliefs as $\pi(e)$ and their assessment about the agent’s innate productivity as $E[w \mid e]$.

Notice that we rule out the possibility that employers could learn the productivity of employees (and adjust salaries accordingly) by observing them on the workplace. Thus, we assume that salaries are fully determined by educational choices. However, employers’ learning about employees’ productivity can be an important feature of labor markets (see Lange, 2007; Arcidiacono et al., 2010 for empirical evidence on this topic). In Section V of the paper and in Section III of the Online Appendix, we discuss an extension of our model, which allows for the possibility of employers’ learning and we show that our results always carry over to this setting and that additional predictions may arise when learning is sufficiently strong (see Alós-Ferrer and Prat, 2012, for a general discussion of employers’ learning in a model of job market signaling).
The consumption utility of an agent of type $\psi$ can thus be defined as:

$$U(e \mid \psi) = E[w \mid e] + \beta \cdot e - \frac{\theta_\psi}{2}e^2. \quad (2)$$

Our main departure from the literature is the assumption that an agent's well-being is determined not only by his consumption utility given by (2), but also by an additional social component that stems from comparing one's own achievements with a benchmark level. Formally, let $\mu(\cdot) : \mathbb{R} \to \mathbb{R}$ be a function such that for every $x, y \in \mathbb{R}_+$:

$$\mu(x - y) = \begin{cases} 
\eta \cdot (x - y) & \text{if } x - y > 0, \\
\eta \lambda \cdot (x - y) & \text{if } x - y \leq 0.
\end{cases} \quad (3)$$

In (3), $x$ is the agent's achievement and $y$ is the reference level that we will shortly define. The parameter $\eta > 0$ thus measures the relevance of social concerns, i.e., their relative importance with respect to consumption utility. The parameter $\lambda$ instead captures the incidence of an underachievement (i.e., a situation in which $x < y$) with respect to an overachievement (i.e., a situation in which $x > y$). Thus, if $\lambda > 1$ (or $\lambda < 1$), underachievements have a higher (or lower) impact than overachievements.\(^3\) There is ample evidence that low relative levels of education and income lead to social stigma, whereas high relative levels convey social prestige (see Solga, 2002; Hopkins and Kornienko, 2004; Archer et al., 2005 and the references therein). However, to the best of our knowledge, there are no studies that establish a systematic pattern for the relative strength of these two forces. In our analysis, we thus allow for both possibilities, $\lambda \gtrless 1$.

As discussed in the Introduction, we postulate that social concerns can arise in different dimensions, and that individuals use the average level that emerges in the population as the

\(^3\)Since a change in $\lambda$ may modify the concavity or convexity of the function $\mu(x - y)$, our approach extends the analysis by Clark and Oswald (1998) to an environment with incomplete information. More in general, notice that in our setting social status is a cardinal concept. See Bilancini and Boncinelli (2012) for a comparison of cardinal and ordinal social comparisons in a context of redistribution.
benchmark to evaluate their relative achievement. Agents’ reference level is thus given by the “average achievement” $y$ computed through rational expectations. In this respect, our approach is in line with models of endogenous reference point formation (see Kőszegi and Rabin, 2006 and 2007, and, in particular, their notion of personal equilibrium).

The relative standing of an individual - and thus his social position - is defined over three different dimensions: perceived type, educational achievements, and income. We analyze these three scenarios separately to best highlight the forces that social comparisons in a specific dimension activate and the impact that these forces have on inequality measures. In Section V and in Section III of the Online Appendix we discuss a setting in which all three dimensions are relevant at the same time and we show that in this case the equilibrium would be determined by a combination of the very same forces that we identify in the three separate scenarios. Thus, our insights generalize to the case of multidimensional social concerns.

For comparative purposes, we first define what happens when one sets $\eta = 0$ so that social concerns have no role. We refer to such a situation as the benchmark model, and we denote it by the superscript $B$. The utility of an individual who chooses educational level $e$ is then given by:

$$V^B (e | \psi) = U (e | \psi).$$  \hfill (4)

Now, consider the case in which social concerns exist (i.e., $\eta > 0$) and are defined over the perceived type of an individual (we denote such a scenario by the superscript $T$). The perceived type of an individual who achieves an educational level equal to $e$ is given by $E[w | e]$. The agent’s total utility can thus be expressed as:

$$V^T (e | \psi) = U (e | \psi) + \mu (E[w | e] - \bar{w}),$$  \hfill (5)

where $\bar{w} = \alpha \int_0^{+\infty} E[w | x] d\tilde{e}_{h}^{T} (x) + (1 - \alpha) \int_0^{+\infty} E[w | x] d\tilde{e}_{l}^{T} (x) = \alpha w_h + (1 - \alpha) w_l$ is the reference level and $(\tilde{e}_{l}^{T}, \tilde{e}_{h}^{T})$ denote agents’ behavior.

As a second scenario, suppose that social concerns are defined over education levels (su-
perscript $E$). In this case, the total utility of an individual is given by:

$$V^E(e \mid \psi) = U(e \mid \psi) + \mu (e - \bar{e})$$

and the appropriate reference level is $\bar{e} = \alpha \int_0^{+\infty} x \tilde{e}^E_h(x) \, dx + (1 - \alpha) \int_0^{+\infty} x \tilde{e}^E_l(x) \, dx$, where $(\tilde{e}^E_l, \tilde{e}^E_h)$ describe the behavior of different types.

Finally, social concerns can be defined over income (superscript $I$). Agent’s total utility then becomes:

$$V^I(e \mid \psi) = U(e \mid \psi) + \mu (I(e) - \bar{I})$$

and the reference level is $\bar{I} = \alpha \int_0^{+\infty} I(x) \, d\tilde{e}^I_h(x) + (1 - \alpha) \int_0^{+\infty} I(x) \, d\tilde{e}^I_l(x)$.

### III. Equilibrium Characterization

We now characterize the equilibria of the game. Because we are dealing with a dynamic game with incomplete information (firms do not know the agents’ types), we adapt the notion of symmetric Perfect Bayesian Equilibrium (PBE). Then, both types choose educational investments to maximize their utility given that the reference levels $\bar{w}, \bar{e},$ and $\bar{I}$ are determined as discussed in Section II, and the expected productivity of individuals is computed by observing their educational choices and by applying Bayes rule whenever possible. We provide a formal definition of the equilibrium concept in the Online Appendix.

As it is standard in signaling games, several PBE exist. To refine the set of equilibria, we use the Intuitive Criterion (Cho and Kreps, 1987). In particular, if education is sufficiently costly, then this refinement selects a unique equilibrium - the so-called least-cost separating equilibrium - regardless of social concerns. However, the specific features of this equilibrium depend on the nature and the relevance of social concerns. As a technical point, notice that since the function $c(e \mid \psi)$ satisfies the single-crossing property and $\beta \geq 0$, if $\theta_h$ is sufficiently lower than $\theta_l$, separation can be achieved by having individuals simply choosing their first-best level of education. As we are interested in analyzing the interactions between signaling,
human capital accumulation and social concerns, we assume that $\theta_h$ is instead sufficiently high. Moreover, we also assume that achieving high levels of education is sufficiently costly for low types, i.e., $\theta_l$ is also relatively high.\footnote{If $\lambda > 1$, this last assumption is irrelevant. Instead, when $\lambda \leq 1$, the assumption guarantees that in equilibrium low types do not mix between different educational levels. We refer the reader to Section II in the Online Appendix for further details.} These two assumptions, which we impose throughout the paper, simplify the equilibrium analysis without affecting any of our insights.

**The Benchmark Case without Social Concerns**

To highlight how social concerns influence agents’ choices, it is useful to first discuss what happens in their absence, namely when $\eta = 0$. Proposition 1 shows that in equilibrium, the low types choose their first-best level of education equating the marginal monetary benefit of education ($\beta$) and its marginal cost ($\theta_le$). In contrast, high types choose the minimum level of education that achieves separation from low types (Riley, 1979).

**Proposition 1.** Suppose that the utility of individuals is given by (4). Then, the unique equilibrium outcome satisfying the Intuitive Criterion is given by

$$\left(e_l^B, e_h^B\right) = \left(\frac{\beta}{\theta_l}, \frac{\beta + \sqrt{2\theta_l \Delta_w}}{\theta_l}\right).$$

(8)

**Equilibria with Social Concerns**

Now, let social concerns affect individuals’ utility as discussed in Section II. Proposition 2 describes agents’ optimal educational choices when social concerns are defined over each of the three possible dimensions: perceived types (scenario $s = T$), educational achievements (scenario $s = E$), or income (scenario $s = I$). Its proof extends standard techniques to accommodate for the fact that in scenario $s = E$ and in scenario $s = I$ the reference level for social comparison is determined endogenously (see Section II of the Online Appendix).

**Proposition 2.** For each scenario $s \in \{T, E, I\}$, there is a unique equilibrium outcome satisfying the Intuitive Criterion. In this equilibrium, agents choose the level of education
The educational investment of both types is weakly increasing in the relevance of social concerns ($\eta$). This relationship is strict for all educational choices, but $e^T_i$.

Figure 1 illustrates the results of Proposition 2 by plotting agents’ optimal choices as a function of the relevance of social concerns ($\eta$) in the three different scenarios $s \in \{T, E, I\}$, and by comparing them with the benchmark case.6

The educational investments characterized in Proposition 2 highlight the channels through which social concerns affect agents’ equilibrium choices. In particular, an educational investment entails two kinds of benefits. First, a *direct benefit* associated with the monetary and social gains that education generates. By its very nature, this benefit is enjoyed by both types of individuals and it thus influences the educational choices of all individuals. Second,
a signaling benefit given by the fact that high skilled workers can use education to signal their superior skills. In a separating equilibrium, signaling only benefits the high types and thus the signaling benefit only affects their educational choices.

In the benchmark model, the direct benefit is purely monetary and equals $\beta$ for both types, whereas the signaling benefit equals $\Delta w$ (we discuss the case in which the direct benefit of education varies with the worker’s type in Section V of the paper and in Section III of the Online Appendix). Now, consider how the existence and the nature of social concerns modify these two benefits. First, social pressures may inflate the direct benefit because, on top of the marginal monetary return $\beta$, educational investments can now reduce the social stigma suffered by low types and increase the social prestige enjoyed by high types. Second, social concerns can also amplify the signaling benefit because being recognized as a high type may yield social prestige. Depending on the dimension of social comparison one or both of these channels may be active.

When social concerns are defined over perceived type, they only boost the signaling benefit. As a result, the educational investment of high types increases with respect to the benchmark case, while the educational investment of low types stays constant (left chart in Figure 1). If instead social concerns are defined over the level of schooling, then they inflate only the direct benefit. Both types thus increase their investment in education. Importantly, the effect of an increase in the importance of social concerns will be stronger for low types or for high types depending on whether underachievements loom larger than overachievements ($\lambda > 1$, middle chart in Figure 1) or vice versa ($\lambda < 1$). Finally, if social concerns are defined over income, then they affect both the direct and the signaling benefit. On the one hand, a higher level of education entails a productivity gain, which translates into a higher wage and thus enhances the individual’s social status. On the other hand, signaling oneself as a high skilled worker yields a discrete wage increase of size $\Delta w$, which also generates additional social prestige for the high types. As a result, the educational investment of both types increases with respect to the benchmark case (right chart of Figure 1).
Table 2 summarizes the previous discussion by showing whether social concerns (measured by $\eta$) increase with respect to the benchmark model the direct benefit and the signaling benefit of the two types of individual in each of the three scenario $s \in \{T, E, I\}$. The table thus also enables pairwise comparisons across scenarios. For instance, it highlights that the differences between the perceived type and the education scenarios hinge on the fact that in the first case social concerns only inflate the signaling benefit (enjoyed by high types), while in the second case, they only increase the direct benefits of the two types. Similarly, it shows that in the income scenario social pressures increase both the direct benefits of the two types (as it happens in the education scenario) and the signaling benefit of the high type (as it happens in the perceived type scenario).

**Table 2:** Increase in the Direct Benefit and the Signaling Benefit due to Social Concerns.

<table>
<thead>
<tr>
<th>Scenario (s)</th>
<th>Direct Benefit</th>
<th>Signaling Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low types</td>
<td>High types</td>
</tr>
<tr>
<td><em>Perceived Type (T)</em></td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><em>Education (E)</em></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><em>Income (I)</em></td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**IV. Social Concerns and Inequality**

Proposition 2 characterizes the channels through which social concerns modify the educational choices at the individual level. Given that social concerns tend to inflate agents’ educational investments, it follows that the average level of education in the population always gets distorted upward with respect to the benchmark case (see Figure 1). Now, we show that social concerns’ effects on inequality are subtler and vary across scenarios depending on how social motivations affect different types of agents.
To investigate this issue, define the education gap between low types and high types as:

$$\Delta_e^s = e^s_h - e^s_l$$  \hspace{1cm} \text{for } s \in \{B,T,E,I\}. \hspace{1cm} (9)$$

Similarly, define the income gap as:

$$\Delta_i^s = I(e^s_h) - I(e^s_l)$$  \hspace{1cm} \text{for } s \in \{B,T,E,I\}. \hspace{1cm} (10)$$

By construction, $$\Delta_i^s = \beta \Delta_e^s + \Delta_w$$. The income gap is thus the sum of two terms: the monetary returns associated with the education gap, $$\beta \Delta_e^s$$, and the baseline productivity gap, $$\Delta_w$$. Since both $$\beta$$ and $$\Delta_w$$ are positive and constant in $$\eta$$ and $$\lambda$$, the education gap and the income gap always move in the same direction following a change in the parameters capturing social concerns.

Below, we investigate how the education and the income gaps vary with the parameters of our model. We then derive some policy implications. In particular, we consider two sets of comparative statics: those with respect to the parameters capturing social concerns and those with respect to the parameters capturing the standard costs and benefits of educational investments.

**Inequality and the parameters capturing social concerns.**

The parameters that capture the relevance and the incidence of social concerns are $$\eta$$ and $$\lambda$$. Proposition 3 focuses on $$\eta$$, the relevance of social concerns. It shows that inequality is increasing in $$\eta$$ whenever the incidence of social stigma is lower than the one of social prestige (i.e., $$\lambda < 1$$).

**Proposition 3.** If $$\lambda < 1$$, both the education gap and the income gap are increasing in the relevance of social concerns, $$\eta$$, in all scenarios $$s \in \{T,E,I\}$$. If $$\lambda = 1$$, then the gaps are strictly increasing if $$s \in \{T,I\}$$ and constant if $$s = E$$.

To understand Proposition 3, recall that individuals’ optimal choices are shaped by two
forces: the direct benefit and the signaling benefit. As discussed in the previous section, an increase in the relevance of social concerns leads to (i) an increase in the direct benefit enjoyed by both types, and (ii) an increase in the signaling benefit experienced by high types. Moreover, when overachievements loom larger than underachievements (i.e., $\lambda < 1$), the direct benefit of high types increases more than the one of low types. As a result, independently of the dimension of social comparison, high types always have a stronger incentive to invest in education. Thus, the education and income gaps widen.

Figure 2 summarizes the discussion by showing how the education gap in each of three scenarios compares with the one obtained in the benchmark case. Note that inequality is more responsive to $\eta$ when social concerns are defined over income. Indeed, in this case social comparisons affect both the difference in the direct benefits of the two types and the signaling benefit experienced by the high type. Instead, when social comparisons are based on perceived types (education), the only active force is the one that operates through the signaling benefit (the difference in direct benefits).

Now, consider the case in which social stigma is stronger than social prestige ($\lambda > 1$).

**Proposition 4.** If $\lambda > 1$, then the education gap and the income gap are:

(i) increasing in $\eta$ if social concerns are defined over perceived types;

(ii) decreasing in $\eta$ if social concerns are defined over educational achievements;

(iii) increasing (resp., decreasing) in $\eta$ if social concerns are defined over income and the baseline productivity gap, $\Delta_w$, is high enough (resp., low enough).
Contrary to the case with $\lambda < 1$, Proposition 4 highlights that there are now situations in which inequality decreases as social concerns become more relevant (see Figure 3 for a graphical illustration). The intuition hinges on the fact that social concerns impact the incentives of different types differently.

More precisely, because underachievements weigh more than overachievements, the increase in the direct benefit of a low type induced by social comparisons is larger than the one of a high type. This pushes toward a decrease in educational and income inequality. However, social concerns also boost the signaling benefit enjoyed by high types and this pushes toward an increase in both types of inequality.

Inequality thus increases if social concerns are defined over perceived types (top left panel in Figure 3) because, in this case, the effect of $\eta$ on the direct benefit is null. Inequality instead falls when social concerns are defined over educational achievements (top right panel) because in this case it is the signaling benefit that remains constant. Finally, if the relevant dimension of social comparisons is income, then the two channels identified above push in opposite directions. Inequality thus increases when the signaling benefit channel dominates.
- a condition that is verified when the productivity gap $\Delta_w$ is sufficiently large (bottom left panel). Rather, it decreases when the direct benefit channel dominates, i.e., when $\Delta_w$ is sufficiently small (bottom right panel).

The comparative statics of the education gap and income gap with respect to the incidence of underachievements and overachievements (as captured by $\lambda$) display similar features. An increase in $\lambda$ boosts low types’ direct benefit with respect to high types’ one. Thus, it pushes toward a decrease in the education and income gaps. However, an increase in $\lambda$ also increases the signaling benefit, and this tends to amplify the two gaps. As a result, inequality may increase or decrease depending both on the dimension of social comparisons and on the relative force of these two channels.

**Proposition 5.** The education gap and the income gap are:

(i) increasing in $\lambda$ if social concerns are defined over perceived types;

(ii) decreasing in $\lambda$ if social concerns are defined over educational achievements;

(iii) increasing (resp., decreasing) in $\lambda$ if social concerns are defined over income and the baseline productivity gap, $\Delta_w$, is high enough (resp., low enough).

Thus far, our analysis sheds light on the possible channels through which social concerns may affect individual and aggregate educational choices. In particular, we highlighted the effects of social motivations on inequality. The results suggest that the government can try to reduce inequality by influencing the nature and the relevance of social concerns. In this respect, our model suggests that campaigns aimed at promoting the social image of education may reduce inequality, whereas messages that push the role of innate skills or boost the social prestige enjoyed by high-income earners may amplify it.

Several policies commonly implemented in the context of educational choices are in line with these findings. Examples include campaigns that aim at raising pupils’ awareness about the “social value” of education. These contrast with more “income-oriented” messages that agents may get from other media or their peers. Campaigns of this type are often marketed with names such as “school is cool”, “learning is cool”, or the alike. They are common in
many countries, especially at the local level and they are often implemented in segregated areas to contrast negative peer pressures and fight school dropouts. Their ultimate goal is to reduce inequality in educational achievements as well as in job-related outcomes. References to the social value of education also often appear in presidential speeches and governmental declarations.\textsuperscript{7} Alternative policies that use different instruments but share the same philosophy are those that aim at providing a social environment that is more favorable to the achievement of high educational levels (say, the “moving to opportunity” program that was launched in the 1990s in some US cities).

\textit{Inequality and the parameters capturing standard costs and benefits of education.}

We now investigate how inequality reacts to changes in the educational costs of the low types (these are captured by the parameter $\theta_l$) and in the benefits of education (these are captured by the parameters $\beta$ and $\Delta_w$).\textsuperscript{8} For instance, a reduction in $\theta_l$ can be read as a policy aimed at providing weaker students with dedicated help, while a change in $\beta$ or $\Delta_w$ can be linked to socioeconomic trends that boost the importance of human capital and the education and skill premia in the “Information Age” (see, for instance, Acemoglu and Autor, 2011, and the references therein).

\textbf{Proposition 6.} The education gap and the income gap are always decreasing in $\theta_l$ and increasing in $\Delta_w$. The income gap is always increasing in $\beta$. The education gap is constant in $\beta$ if social concerns are defined over perceived types or educational achievements, and increasing, decreasing or constant in $\beta$ if social concerns are defined over income and, respectively, $\lambda < 1$, $\lambda > 1$ or $\lambda = 1$.

\textsuperscript{7}From President Obama’s speech in Alington, VA on September 8th 2009: “And no matter what you want to do with your life, I guarantee that you will need an education to do it [...] And this is not just important for your own life and your own future. What you make of your education will decide nothing less than the future of this country. What you are learning in school today will determine whether we as a nation can meet our greatest challenges in the future. [...] We need every single one of you to develop your talents, skills and intellect so you can help solve our most difficult problems. If you do not do that, if you quit on school, you are not just quitting on yourself, you are quitting on your country.”

\textsuperscript{8}Notice that the educational costs of the high types (i.e., the parameter $\theta_h$) never influence individual optimal choices (see Section III). As such, they do not affect inequality either.
Proposition 6 states that the income gap reacts to changes in $\theta_l$, $\beta$ and $\Delta w$ in the same way independently of the dimension of social comparisons. The same holds for the education gap if we consider changes in $\theta_l$ and $\Delta w$. In this respect, the comparative statics with respect to these parameters are robust to the presence and to the type of social concerns.

Instead, the way in which the education gap changes in response to changes in $\beta$ is not univocal. To understand this, observe that if social concerns are defined over perceived types or educational attainments, then an increase in $\beta$ simply increases the marginal direct monetary benefit of education by the same amount for both types.\(^9\) As such, educational choices get shifted upwards but the education gap remains constant. However, if social comparisons are defined over income, then an increase in $\beta$ also raises the marginal direct social benefit of schooling. A larger $\beta$ pushes individuals to work more, translating into higher incomes for both types. This in turn modifies average income and consequently the social prestige of high types and the social stigma of low types. Therefore, and in line with the discussion of Proposition 4, low types increase their educational investment by more than high types (and the education gap thus shrinks) if stigma looms larger than prestige (i.e., $\lambda > 1$). The opposite occurs if prestige is more salient than stigma (i.e., $\lambda < 1$).

V. Extensions of the Baseline Model

The model that we propose (henceforth, the baseline model) is flexible enough to accommodate a number of extensions that capture relevant features of the labor market and current educational systems. In what follows, we briefly report the main insights that stem from the analysis of four different extensions. We refer the interested reader to the Online Appendix for further details.

\(^9\)In Section V of the paper and Section III of the Online Appendix we discuss how the education gap reacts to changes in the marginal direct monetary benefit of education when this is not equal across types.
Type-Dependent Marginal Returns of Education

An extensive empirical literature shows that more skilled individuals enjoy higher benefits from education than less skilled ones (Blackburn and Neumark, 1993; Weiss, 1995; Card, 1999; Belzil and Hansen, 2002). In the context of our model, we can capture this feature by assuming that the marginal effect of education on the productivity of an individual of type \( \psi \in \{l, h\} \) is given by \( \beta_\psi \), with \( \beta_l < \beta_h \). Importantly, \( \beta_\psi \) is employee’s private information and it is not observed by the firm. All the insights of the baseline model generalize to such a setting. A least-cost separating equilibrium still exists in every scenario and the patterns of educational and income inequality remain qualitatively similar to the ones identified in the baseline model, with the only difference being that the cutoff level of \( \lambda \) above which the education and income gaps decreases in \( \eta \) in scenarios \( s = E \) and \( s = I \) goes up. Intuitively, to reduce inequality social concerns must now increase the direct benefit of low types against the one of high types by a greater extent because they must offset the gap in marginal productivity. This requires underachievements to have a larger impact than overachievements with respect to the baseline model.

Employers’ Learning about Individuals’ Types

The baseline model assumes that firms can neither observe nor learn an individual’s type. In reality, although firms may initially be uncertain about an agent’s productivity, they can observe him at work and update their initial beliefs (for empirical studies on the importance of employers’ learning, see, for instance, Lange, 2007; Arcidiacono et al., 2010). In line with Alós-Ferrer and Prat (2012), we can allow for learning by postulating that, on top of what they infer from the agent’s level of schooling, employers can further update their beliefs about the worker’s type based on a second signal \( \sigma \) that they receive after they hire the worker. In this setting, as long as \( \sigma \) is not too informative (i.e., learning is not too effective), the least-cost equilibria identified in the baseline model remain the unique equilibria compatible with the Intuitive Criterion. Additional pooling equilibria can instead emerge if learning is
very precise.

Heterogeneity between Different Educational Paths

The baseline model assumes that “education” is a homogeneous signaling tool. However, in reality, there is ample variation in the educational paths that an individual can pursue and not all the paths entail the same monetary and social benefits. To incorporate this into our model, assume that there exist two different educational paths. The first one, which we label as “practical”, entails a higher marginal monetary benefit. The second one, which we label as “theoretical”, is instead more effective in separating low types and high types and entails a higher social prestige in the education scenario. If social concerns are absent, then in equilibrium separation occurs with the two types choosing the practical path and replicating the outcome characterized in Proposition 1. Low types choose the practical track because it maximizes their monetary return, while high types, although they could separate themselves also by investing in the theoretical track, pursue the practical track because (in equilibrium) it allows them to better leverage the advantage they have in terms of educational costs. A similar pattern (namely, both types choose the practical track and the equilibrium levels of education replicate the ones obtained in the baseline model) arises also in the perceived type scenario (because social concerns do not affect the direct benefit) and in the income scenario (because social concerns amplify the advantage that the practical major has in terms of monetary returns). Instead, in the educational scenario, social pressures can push individuals to pursue the theoretical track if this track is sufficiently more prestigious than the practical one.

Multidimensional Social Comparisons

The main analysis investigates the effects of the three possible dimensions of social concerns (perceived type, education, and income) in isolation. If we allow all three dimensions to be simultaneously at work with different weights, then a unique least-cost separating
equilibrium still exists and it is shaped by the very same forces we identified analyzing the three “pure” scenarios. Importantly, such scenarios can be regarded as the limit cases of this more general model when the weights of two out of three dimensions approach 0. All the qualitative results about educational and income inequality smoothly carry over to the multidimensional setting.

VI. Conclusion

An agent’s level of education is an important determinant of his social status. The effect can be direct because high educational attainments convey prestige per se or indirect because high levels of schooling are correlated with other variables that determine social standing such as income or intelligence. Starting from these considerations, we investigated how social status considerations interact with standard economic incentives and influence individuals’ and aggregate educational choices. In particular, we discussed the implications that stem from defining an agent’s social status over three different dimensions: (i) his perceived ability, (ii) his educational attainment, and (iii) his income.

Social concerns inflate the average educational level in the society and, more importantly, they influence inequality both in terms of educational achievements and income. In this respect, our analysis showed that the impact of social status considerations on inequality varies depending on the predominant dimension over which social concerns are defined and on the relative importance of underachievements with respect to overachievements.

Clearly, and in line with the empirical evidence that we provided in the Introduction, country-specific cultural traits (or differences across ethnic or sociodemographic groups) are likely to have an impact on the relevance that perceived ability, education, and income have in determining an individual’s social status. Our model may provide a starting point to investigate both theoretically and empirically the origin and the evolution of these traits as well as their role in shaping the patterns of inequality within and across countries. We leave such analysis to future research.
VI. Appendix

Throughout the appendix, we define $H(\alpha, \eta, \lambda) = 1 + \alpha \eta \lambda + (1 - \alpha) \eta$ and $G(\alpha, \eta, \lambda) = (1 - \alpha) \eta (\lambda - 1)$. When no confusion arises, we simply write $H$ and $G$. Notice that $H$ is increasing in $\eta$ for all $\alpha$ and $\lambda$, while $G$ is increasing (respectively, decreasing) in $\eta$ if $\lambda > 1$ (respectively, $\lambda < 1$) for all $\alpha$.

It is immediate to verify that Proposition 2 implies that the education gaps in the various scenarios are given by $\Delta^T_e = \sqrt{2\Delta_w H} \Delta^E_e = \frac{\sqrt{G^2 + 2\theta I \Delta_w - G}}{\theta I}$ and $\Delta^I_e = \sqrt{\beta G^2 + 2\theta I \Delta_w H} - \beta G$. Furthermore, for each scenario $s \in \{T, E, I\}$, the income gap is given by $\Delta^s_I = \beta \Delta^s_e + \Delta_w$.

Proof of Propositions 3 and 4. We only consider changes in the education gap as these fully determine changes in the income gap as well (see the main text). Since $\frac{\partial H}{\partial \eta} > 0$, $\Delta^T_e = \sqrt{2\Delta_w H} \theta_I$ is strictly increasing in $\eta$. If concerns are defined over education ($s = E$), $\frac{\partial \Delta^E_e}{\partial \eta} = \frac{(1-\alpha)(\lambda-1)}{\theta I} \left( \frac{G}{\sqrt{G^2 + 2\theta I \Delta_w}} - 1 \right)$. Since $G < \sqrt{G^2 + 2\theta I \Delta_w}$, the previous expression is positive if $\lambda < 1$, equal to 0 if $\lambda = 1$, and negative if $\lambda > 1$. Finally, if social concerns are defined over income levels ($s = I$), the derivative of the education gap with respect to $\eta$ is given by $\frac{\partial \Delta^I_e}{\partial \eta} = \frac{\Delta_w (1-\alpha + \alpha \lambda)}{\sqrt{\beta G^2 + 2\theta I \Delta_w H}} - \frac{\beta (1-\alpha) (\lambda - 1)}{\theta I} \left( 1 - \frac{\beta G}{\sqrt{\beta G^2 + 2\theta I \Delta_w H}} \right)$. Since $\beta G < \sqrt{\beta G^2 + 2\theta I \Delta_w H}$, the previous expression is positive for any $\lambda \leq 1$. If instead $\lambda > 1$, the expression is positive if and only if $\Delta_w \geq \frac{2}{\theta I} \left( \frac{\beta (1-\alpha) (\lambda - 1)}{(1-\alpha + \alpha \lambda)} \right)^2$.

Proof of Proposition 5. We again only focus on the education gap. The result is immediate in scenario $s = T$ as $e^T_h$ is increasing in $\lambda$, while $e^T_I$ is constant in it. In scenario $s = E$, $\frac{\partial \Delta^E_e}{\partial \lambda} = \eta (1-\alpha) \frac{G - \sqrt{G^2 + 2\theta I \Delta_w}}{\theta I \sqrt{G^2 + 2\theta I \Delta_w}}$. Because $\sqrt{G^2 + 2\theta I \Delta_w} > G$, such derivative is negative for all values of parameters. Finally, consider scenario $s = I$, then $\frac{\partial \Delta^I_e}{\partial \lambda} = \eta \frac{\beta (1-\alpha) G + a \theta I \Delta_w - (1-\alpha) \beta \sqrt{\beta G^2 + 2\theta I \Delta_w H}}{\theta I \sqrt{\beta G^2 + 2\theta I \Delta_w H}}$. The derivative is positive for high values of $\Delta_w$ and negative for low values of $\Delta_w$. This concludes the proof.

Proof of Proposition 6. If $s \in \{T, E\}$, the education gap is constant with respect to $\beta$. Thus, if $s \in \{T, E\}$, $\Delta^s_e$ is constant in $\beta$ and $\Delta^s_I$ is increasing in $\beta$ (recall that $\Delta^s_I = \beta \Delta^s_e + \Delta_w$).
Instead, if \( s = I \), \( \frac{\partial \Delta^I_t}{\partial \beta} = \frac{G}{\theta_1} \left( \frac{\beta G}{\sqrt{\beta^2 G^2 + 2 \theta_1 \Delta_w H}} - 1 \right) \). If \( \lambda \leq 1 \), the previous expression is positive since \( G < 0 \) and the term inside the parenthesis is negative. Instead, if \( \lambda > 1 \) the derivative is negative as \( G > 0 \) and \( \frac{\beta G}{\sqrt{\beta^2 G^2 + 2 \theta_1 \Delta_w H}} < 1 \). As for the income gap, by definition we have

\[
\frac{\partial \Delta^I_t}{\partial \beta} = \Delta^I + \beta \frac{\partial \Delta^I}{\partial \beta} = \frac{1}{\theta_1} \cdot \left( \frac{\sqrt{G^2 + 2 \theta_1 \Delta_w H - G \beta}}{G^2 + 2 \theta_1 \Delta_w H} \right)^2 > 0.
\]

Now consider changes in \( \theta_1 \). Obviously, for each scenario \( s \in \{ T, E, I \} \), \( \frac{\partial \Delta^T_t}{\partial \theta_1} = \beta \frac{\partial \Delta^T}{\partial \theta_1} \). Thus, we only focus on the education gap. Observe that, \( \frac{\partial \Delta^T_t}{\partial \theta_1} = -G \frac{\Delta_w}{\theta_1^2 \sqrt{2 \theta_1 \Delta_w H}} < 0 \). Hence, the education gap in scenario \( T \) is decreasing in \( \theta_1 \).

Instead, \( \frac{\partial \Delta^E_t}{\partial \theta_1} = \frac{1}{\theta_1^2} \left( \frac{G \sqrt{G^2 + 2 \theta_1 \Delta_w H - G^2 - \theta_1 \Delta_w H}}{G^2 + 2 \theta_1 \Delta_w H} \right) \). If \( \lambda < 1 \), all terms in the parenthesis are negative. If \( \lambda > 1 \), then \( G^2 + \theta_1 \Delta_w > G \sqrt{G^2 + 2 \theta_1 \Delta_w} \). Thus, \( \Delta^E_t \) is decreasing in \( \theta_1 \). Finally, consider scenario \( s = I \). Then, \( \frac{\partial \Delta^I_t}{\partial \theta_1} = \frac{1}{\theta_1^2} \left( \frac{G \beta \sqrt{G^2 + 2 \theta_1 \Delta_w H - G^2 \beta^2 - \theta_1 \Delta_w H}}{G^2 \beta^2 + 2 \theta_1 \Delta_w H} \right) \). Once more, if \( \lambda < 1 \), all terms in the parenthesis are negative and if \( \lambda > 1 \), then \( G^2 \beta^2 + \theta_1 \Delta_w > G \beta \sqrt{G^2 \beta^2 + 2 \theta_1 \Delta_w} \).

Thus, \( \Delta^I_t \) is decreasing in \( \theta_1 \). Finally, the comparative statics with respect to \( \Delta_w \) follow immediately from the fact that in each scenario \( e^s_h \) is increasing in \( \Delta_w \), while \( e^s_i \) is constant in it (see Proposition 2).

\[\square\]

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