

MEASURE THEORY

Instructor: Bertrand Lods

The course introduces to the theory of Lebesgue integration. After defining appropriate structures of sets such as algebras, semi- and sigma-algebras, we provide the notion of Lebesgue-Stieltjes measures, and their construction from semi-algebras to sigma-algebras. We then constructively define integrals with respect to a Lebesgue-Stieltjes measure and state their most relevant properties. The relationship with classical Riemann integration and the connection with probability theory are also examined.

Exam

Written exam at the end of the course.

Course Outline

- classes of subsets: algebras, semi-algebras, sigma-algebras, monotone classes
- measures: definition and properties, finite- and sigma-additivity; construction of measures on sigma-algebras; completions of measures; Lebesgue-Stieltjes measures
- measurable functions
- Lebesgue-Stieltjes integrals: construction and properties
- convergence theorems
- null-measure sets and properties holding almost everywhere
- comparison with Riemann integration
- Radon-Nikodym's theorem

Textbooks

Detailed lecture notes will be provided. Reference books are:

- Folland, G.B. (1999). Real analysis. Wiley.