Intermediate Mathematics

Instructor: Bertrand Lods, Elena Vigna

Course description

This is an introductory course on ordinary differential equations with some applications. Fundamental prerequisites are elementary calculus and linear algebra. The main objective of the course is to become familiar with a wide range of ordinary differential equations, which typically arise in economic and financial modelling, and to be able to approach them and solve the easiest ones. Although elementary integration should be part of basic calculus, I will nevertheless give an adequate introduction to Riemann integrals, before starting with the core topics of the course. Complex numbers will also be introduced as preliminary material for the course. Thus, the course will consist of two main parts: preliminaries (integrals and complex numbers) and ordinary differential equations.

Problem Sets

A number of problem sets will be assigned after the completion of each main part. The problem sets are intended to help students to familiarize with the introduced techniques, in preparation for the final exam: assignments will not be collected and marked.

Exam

There will be a final exam at the end of the course, consisting in both written and oral examination.

Course outline

1. Preliminaries
(a) Riemann integrals
1. Introduction: the integral as an area
2. Analytic definition of the integral
3. Characterization of integrable functions
4. Class of integrable functions and properties of integrals
5. The mean value theorem for integrals
6. The fundamental theorem of calculus
7. Elementary integrals
8. Integration techniques: integration by substitution and by parts, integration of rational functions
9. Some results on elementary functions that can be integrated explicitly
10. Integral dependent on one parameter: continuity and differentiability
(b) Complex numbers
1. Algebraic form and polar form of complex numbers
2. De Moivre's formula and the roots of a complex number
3. Exponential form of complex numbers and Euler formulae
4. The fundamental theorem of algebra
2. Ordinary Differential Equations (ODEs)
(a) Preliminary examples: population models
(b) Definitions and terminology: n-th order differential equations and systems of ODEs
(c) The initial-value problem (IVP) or Cauchy problem
(d) The existence and uniqueness theorem for the IVP
(e) Solution of some common first-order ODEs: linear, exact differential, separable, homogeneous equations, Bernoulli, Riccati
(f) Linear differential equations: general solution of homogeneous and complete linear ODE
(g) Solution of some common higher order ODEs: linear differential equations with constant coefficients, Euler equations

Textbooks

For the preliminaries:

- Courant R., Fritz, J. "Introduction to Calculus and Analysis" vol. I, reprint of the 1989 ed., Springer, Berlin, 1999.
- Rudin W., "Principles of Mathematical Analysis", McGraw-Hill, New York, 1976.

For ordinary differential equations:

- Boyce W.E., Di Prima R.C., "Elementary Differential Equations and Boundary Value Problems", Wiley, New York, 1977.
- Braun M., "Differential Equations and Their Applications", Springer Verlag, New York, 1978.
- Coddington E.A., "An Introduction to Ordinary Differential Equations", Prentice-Hall, Englewood Cliffs, 1981.
- Rainville E.D., Bedient P.E., "Elementary Differential Equations", Mac Millan, New York, 1974.
- Reiss E.L., Callegari A.J., Ahluwalia D.S., "Ordinary Differential Equations with Applications", Holt Rinehart and Winston, New York, 1976.
- Ross S.L., "Introduction to Ordinary Differential Equations", Wiley, New York, 1989.