

Generalized Disappointment Aversion, Learning, and Asset Prices [☆]

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Abstract

Contrary to leading asset pricing theories, recent empirical evidence indicates that it was costless to hedge long-term volatility in aggregate stock market returns over the last two decades, whereas investors paid large premia for insurance against the unexpected realized variance. This paper offers a generalized disappointment aversion explanation that can also account for the variance and skew risk premiums in equity returns and the implied volatility skew of index options. The model captures other puzzling features of the data including the low risk-free rate, the high equity premium, excess stock market volatility, and return predictability patterns.

Keywords: Generalized Disappointment Aversion, Learning, Price of Variance Risk, Variance and Skew Risk Premiums, Volatility Skew

JEL: D81, E32, E44, G12

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1. Introduction

The consumption-based asset pricing literature has been recently revived by generalized long-run risks frameworks ([Drechsler and Yaron, 2011](#); [Drechsler, 2013](#)) and stochastic disaster risks models ([Wachter, 2013](#); [Seo and Wachter, 2018](#)) to capture many characteristics of the equity and derivatives markets. Yet leading theories fail to explain the timing of volatility risk. In these models, investors are assumed to have a preference for early resolution of uncertainty and, therefore, they price long-term volatility in aggregate stock market returns strongly. Contrary to these well-known theoretical models, the recent empirical literature documents that financial markets compensate short-term volatility risk ([Dew-Becker, Giglio, Le and Rodriguez, 2017](#)).¹ The main aim of this paper is to explain the variance term structure, while reconciling salient properties of the risk-free rate, equity returns, index option and variance swap prices. The paper argues that generalized disappointment aversion is the key to understanding the link between investor perceptions of uncertainty shocks and the pricing of volatility risk.

In this paper, I construct an equilibrium representative agent model with generalized disappointment aversion (GDA) preferences and time-varying macroeconomic uncertainty induced by learning about an unobservable state of the consumption growth process. In the model, consumption growth follows a hidden two-state Markov process and hence the agent filters a posterior belief about the hidden regime based on past consumption growth realizations. The negative news to consumption growth implies that the posterior belief partially falls and so does the equity price. The combination of pessimistic posterior beliefs and low consumption growth rates raises the marginal utility. Crucially, the GDA preferences penalize continuation utilities below a scaled certainty equivalent and hence amplify the countercyclical dynamics of the pricing kernel, which induce the agent to particularly dislike stock market declines. I show that this mechanism can reproduce a number of stylized facts observed in the equity and derivatives markets, including (i) a low and stable risk-free rate (ii) a high equity premium and volatility of equity returns (iii) large and volatile variance and skew risk premiums (iv) predictability of excess returns by the price-dividend ratio and the variance premium (v) annualized Sharpe ratios for

¹[Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) study variance swap prices in the consumption-based models of [Drechsler and Yaron \(2011\)](#) and [Wachter \(2013\)](#) and find that these models are inconsistent with the variance term structure. Additionally, the authors find that a disaster risk model with habit formation ([Du, 2011](#)) also fails to match the data.

forward variance claims (vi) average forward variance claim prices, and (vii) high prices for at-the-money (ATM) and out-of-the-money (OTM) European put options. Thus, the asymmetric model used in this paper provides a unified resolution of the variance term structure and puzzling features observed in equity returns and derivative prices. Compared to the asymmetric asset pricing models that use [Gul's \(1991\)](#) disappointment aversion utility, my results suggest that [Routledge and Zin's \(2010\)](#) generalized disappointment aversion plays a key role in reproducing salient features of the moment risk premiums and equity index options. Furthermore, unlike leading asset pricing models with Epstein-Zin preferences, my framework with generalized disappointment aversion to downside consumption risk remains consistent with the timing of variance risk and hence provides further evidence in favor of GDA preferences.

I begin my quantitative investigation by adopting the theoretical model of [Routledge and Zin \(2010\)](#) with generalized disappointment aversion to an incomplete information setting. I assume a hidden two-state Markov switching process for consumption growth and define stock market dividends as a leverage to aggregate consumption. A rare bad state in consumption growth is calibrated to replicate the historical decline in consumption during the Great Depression, and a persistent good state reflects normal time fluctuations. [Routledge and Zin \(2010\)](#) extend [Gul's \(1991\)](#) static disappointment aversion (DA) model to a recursive setting and add an extra parameter controlling a fraction of the certainty equivalent below which the outcomes are considered disappointing. Relative to an Epstein-Zin (EZ) specification, a GDA utility introduces two additional parameters: a disappointment penalty and a disappointment threshold. I calibrate GDA preferences to be consistent with the empirical estimates ([Delikouras, 2017](#)) and compare the performance of GDA with EZ and DA. The calibration with GDA preferences matches well the traditional moments of the fundamentals, the risk-free rate, the excess equity returns, and generates excess return predictability by the price-dividend ratio, my analysis further contributes to the existing theoretical frameworks along three important dimensions.

First, the GDA model can explain the term structure of prices and returns on variance swaps. Recently, [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) document that investors are willing to pay a high premia for unexpected realized volatility in a short term, whereas risks about future volatility are unpriced in the data for longer horizons.

Further, they show that a standard calibration of an Epstein-Zin utility with a preference for early resolution of uncertainty counterfactually predicts strongly priced shocks to future volatility in models with long-run risks, rare disasters, and habit formation. In contrast, this paper’s asymmetric model is successful at reconciling the variance term structure. In my model, time-varying state beliefs generate jumps in the pricing kernel and return volatility in response to downside news to consumption growth. These fluctuations are further magnified by generalized disappointment aversion and so the model yields the large risk premium on forward variance claims at the one-month horizon. Most importantly, I calibrate the risk aversion and intertemporal elasticity of substitution parameters in GDA preferences with no preference for resolution of uncertainty. Therefore, the model does not lead to overpricing of volatility risks in the long term, consistent with the data. Overall, the GDA model can reasonably match the average Sharpe ratios and prices of variance claims for maturities from one to 12 months. Further, I show that the EZ framework leads to incorrect predictions about the price of variance risk in line with the results of [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#). Interestingly, I document that the DA preferences only exacerbate the issue by generating too large Sharpe ratios on variance claims, since volatility risks are strongly priced in the presence of high disappointment thresholds.

Second, like the variance premium, I document a large skew premium, consistent with [Kozhan, Neuberger and Schneider \(2013\)](#), and show that the calibration of the GDA model can capture the size of both moment risk premiums. In contrast, the EZ specification generates about half of the average premiums in the second and third moments of equity returns compared to benchmark results, while the DA framework performs even worse. The failure of alternative preferences and the success of the GDA model can be attributed to the differences in their impact on the risk-neutral distribution of variables. Mechanically, generalized disappointment aversion and risk aversion both stand between the physical \mathbb{P} and risk-neutral \mathbb{Q} probability measures through the Radon-Nikodym derivative defined as $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t(M_{t+1})}$. The risk aversion smoothly distorts the \mathbb{Q} -density towards the left tail through the pricing kernel, while generalized disappointment aversion puts more weight on the outcomes strictly below a scaled certainty equivalent. Since GDA preferences enable control of the disappointment threshold, it becomes instrumental in generating a fatter left tail of a \mathbb{Q} -measure compared to smooth preferences. At the

same time, DA preferences penalize too many outcomes due to a high disappointment threshold that is equal to one. As a result, these preferences cannot generate negatively skewed risk-neutral distribution of equity returns.

Third, the model with GDA preferences generates a steep volatility skew implied by one-month index options and replicates the term structure of implied volatilities. Intuitively, deep OTM options are more expensive in the GDA model because they provide insurance against disappointing events in consumption growth, which are associated with low posterior beliefs and hence declines in equity prices. Similarly, the high degree of risk aversion in the Epstein-Zin utility increases investor aversion to consumption shocks, however, I show that the implied volatilities in the EZ model remain significantly below the empirical curves. Thus, risk aversion alone cannot match the implied volatility skew in one-month option prices. The DA framework predicts a basically flat one-month volatility curve that is approximately equal to the annualized stock market volatility. When looking at one-, three-, and six-month volatilities for ATM and OTM options, the GDA calibration matches the slopes and levels of the volatility curves well. In the GDA framework, there is an upward sloping pattern in ATM volatilities and a downward sloping pattern in OTM volatilities, consistent with the data. In contrast, both EZ and DA models significantly underprice options for longer maturities similarly to the way they are priced in a one-month horizon. In a thorough comparative analysis, I show that all results associated with equity, option and swap prices are robust to different calibrations of key parameter values in three model specifications.

Related literature. This paper contributes to several strands of the literature. It is closely related to the growing literature studying the asset pricing implications of asymmetric preferences. A number of studies incorporate asymmetric preferences into the standard asset pricing frameworks ([Campbell and Cochrane, 1999](#); [Barberis and Huang, 2001](#); [Barberis et al., 2001](#); [Routledge and Zin, 2010](#)) and find that these models better explain the asset returns compared to those with symmetric preferences. In the context of generalized disappointment aversion, [Bonomo, Garcia, Meddahi and Tedongap \(2011\)](#) construct a consumption-based asset pricing model with GDA preferences and long-run volatility risks to explain the equity premium. [Bonomo, Garcia, Meddahi and Tedongap \(2015\)](#) recalibrate their model at a daily frequency to reproduce the risk-return trade-off at high- and low-frequency. [Liu and Miao \(2014\)](#) focus on production-based asset

pricing with GDA preferences. [Augustin and Tedongap \(2016\)](#) shed light on the role of GDA preferences in explaining sovereign credit spreads. [Delikouras \(2017\)](#) employ disappointment aversion to explain the cross-section of expected returns. My paper contributes to the existing literature by exploring additional implications of generalized disappointment aversion for moment risk premiums in equity returns and option prices, which are not analyzed by the aforementioned studies. Further, I document that GDA preferences can explain the term structure of variance risk. Finally, the extant literature mainly studies the impact of GDA preferences in the setting of [Bansal and Yaron \(2004\)](#), while my analysis does not rely on long-run risks in endowments. In my model, time variation in asset prices arises from endogenous fluctuations of a posterior state belief.

This paper is also related to [Schreindorfer \(2018\)](#), who documents that US consumption and dividend growth rates are more correlated in bad times than in good times. He introduces this feature into a model with GDA preferences to explain the high equity premium and some features of index options. His paper does not consider investor learning, which this paper treats as a central driver of time-varying macroeconomic uncertainty, and he does not examine the variance term structure and implied volatility curves at different horizons. In this paper, learning is a key driver of fluctuations in asset prices, conditional moments and the pricing kernel, and hence the model can reproduce excess return predictability patterns by the price-dividend ratio and the variance premium, while matching prominent features of the risk-free rate, equity returns, index options and variance swaps. In contrast, the IID model of [Schreindorfer \(2018\)](#) cannot account for time variation of asset prices, nor can it reproduce the excess return predictability observed in the data.

This paper is also related to leading asset pricing theories advocating that habit formation, rare disasters, and long-run risks in consumption are explanations for equity returns and option prices. In the context of habits, [Du \(2011\)](#) shows that an extension of the model with habit formation ([Campbell and Cochrane, 1999](#)) to include rare disasters can explain the observed implied volatility skew and further reproduce state-dependent smirk patterns in the data. Under the rare disasters umbrella, the implied volatility surface can be explained with extensions to model uncertainty about rare events ([Liu, Pan and Wang, 2005](#)), rare jumps in persistence ([Benzoni, Collin-Dufresne and Goldstein, 2011](#)), or stochastic volatility of disasters ([Seo and Wachter, 2018](#)). The long-run

risks literature generalizes the model of [Bansal and Yaron \(2004\)](#) by introducing jump risks ([Eraker and Shaliastovich, 2008](#); [Shaliastovich, 2015](#)) to explain the high premium embedded in option prices. Additionally, a few papers can explain the variance premium in equilibrium. These include long-run risks models with transient non-Gaussian shocks to fundamentals ([Bollerslev, Tauchen and Zhou, 2009](#); [Drechsler and Yaron, 2011](#)) and multiple volatility risks ([Zhou and Zhu, 2014](#)). [Drechsler \(2013\)](#) constructs an extension of the long-tun risks framework with model uncertainty to explain both the variance premium and the implied volatility skew. The mechanism of my paper is distinct from the existing literature since it points out the importance of the investor's generalized disappointment aversion for asset prices. Unlike other models, the approach in this work additionally explains the skew risk premium and the term structure of prices and returns on variance swaps.

The remainder of the paper is organized as follows. Section [2](#) reports the empirical evidence. Section [3](#) describes the economy. Section [4](#) derives asset prices in the model. Section [5](#) provides asset pricing results of the models with GDA, DA and Epstein-Zin preferences. Section [6](#) concludes. Section [A](#) of the Appendix discusses the data, Section [B](#) contains technical details of the representative agent's maximization problem, and Section [C](#) outlines the application of the projection method.

2. Variance and Skewness Risk

This section describes several methods how to quantify risk premia embedded in the variance and skewness of equity returns. First, following the discussion of [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#), I describe salient features of the term structure of variance claims in the equity index market and discuss the failure of leading asset pricing models to account for variance forward prices and returns. Second, I define and measure the one-month variance and skew risk premiums in aggregate stock returns through the lens of variance and skew swaps. Third, I construct the volatility surface by extrapolating historical volatilities implied by equity index options.

Recent empirical studies focus on the term structure of dividend and variance claims. In particular, [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) discover new stylized facts about the price of variance risk, which are at odds with well-known asset pricing theories. Their analysis is based on the pricing of volatility-linked assets, primarily variance swaps,

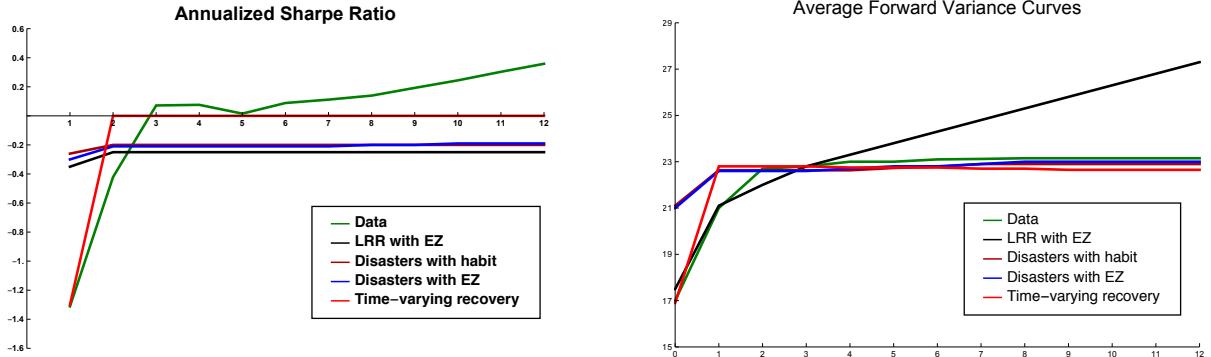


Figure 1: Average Prices and Annualized Sharpe Ratios for Forward Variance Claims. The left and right panels plot annualized Sharpe ratios and average prices for forward variance claims for the data and different models. The prices are reported in annualized volatility terms, $100 \times \sqrt{12} \times F_t^n$. The empirical lines are the curves constructed in [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) and correspond to U.S. data from 1996 to 2013.

and it yields two main results. First, they document that, over the period from 1996 to 2014, news about future volatility at horizons ranging from one quarter to 14 years is unpriced. Second, risk exposure to unexpected realized variance is significantly priced in the data. This leads to the conclusion that it was almost costless to hedge future variance in the period under consideration, whereas investors paid a lot of money for protection against extreme realized volatility in the short term.

Figure 1 is a reproduction of Figure 10 in [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) and quantitatively illustrates their main results. The left graph of Figure 1 compares annualized Sharpe ratios for forward variance claims in the data and in different models: a long-run risk model of [Drechsler and Yaron \(2011\)](#) with Epstein-Zin preferences, a disaster risk model of [Du \(2011\)](#) with habit formation, a time-varying disaster risk model of [Wachter \(2013\)](#) with Epstein-Zin preferences, and a rare disaster model of [Gabaix \(2012\)](#) with time-varying recovery rates. The right plot of Figure 1 compares empirical variance claim prices with those predicted by theoretical frameworks. As shown in the figure, the empirical Sharpe ratios are significantly negative for short maturities, especially for one-month variance forwards, whereas they become slightly positive for horizons from 3 to 12 months. In turn, the term structure of variance forwards is, on average, upward sloping in the data and significantly flattens with the horizon. The figure also shows that the long-run risk model ([Drechsler and Yaron, 2011](#)) and rare disaster frameworks with recursive preferences ([Wachter, 2013](#)) or habit formation ([Du, 2011](#)) fail to capture these stylized facts. Most notably, the three models generate almost flat Sharpe ratios on variance forwards, which are respectively far too small and too large for

Table 1
Summary Statistics: Variance and Skew Risk Premiums

	vp_t	sp_t
Mean	10.24	-42.12
Median	7.50	-68.11
SD	10.49	82.11
Max	83.70	447.37
Skewness	2.62	3.57
Kurtosis	14.15	16.26

This table reports monthly descriptive statistics for the conditional variance vp_t and skew sp_t premiums. The entries in the rows Mean, Median, SD, Max, Skewness, and Kurtosis report the sample average, median, standard deviation, maximum, skewness, and kurtosis, respectively. The empirical statistics of the variance premium are for the U.S. data from January 1990 to December 2016, while the data used for the construction of the skew premium are from January 1996 to December 2016.

short and long maturities compared to the data. A model with disasters and with time-varying recovery rates ([Gabaix, 2012](#)) does a better job capturing the empirical patterns, though it cannot fully explain the upward trend in Sharpe ratios for longer periods.

Closely related to the variance term structure is the risk premium in the second and third moment of returns. A large strand of literature has concentrated on the variance premium, while the skew premium has received little attention, especially from the theoretical research. This paper aims to explain both phenomena simultaneously. The variance premium can be defined as the difference between expectations of stock market return variance under the risk-neutral \mathbb{Q} and actual physical \mathbb{P} probability measures for a given horizon ([Bollerslev, Tauchen and Zhou, 2009](#); [Bollerslev, Gibson and Zhou, 2011](#); [Drechsler and Yaron, 2011](#)). Formally, a τ -month variance premium at time t is:

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}[\text{Return Variation}(t, t + \tau)] - \mathbb{E}_t^{\mathbb{P}}[\text{Return Variation}(t, t + \tau)],$$

where the total return variation is calculated over the period t to $t + \tau$. The quantity vp_t corresponds to the expected profits of a variance swap, which pays the equity's realized variance over the term of the contract. [Britten-Jones and Neuberger \(2000\)](#) and [Carr and Wu \(2009\)](#) show that this payoff can be replicated by a portfolio of European options. Like the variance swap, [Kozhan, Neuberger and Schneider \(2013\)](#) consider a skew swap with a payoff equal to the equity's realized skewness. [Bakshi, Kapadia and Madan \(2003\)](#) show that a skew contract can be replicated by a trading portfolio involving long OTM

calls and short OTM puts. I follow [Kozhan, Neuberger and Schneider \(2013\)](#) and define a τ -month skew risk premium at time t as the return on the skew swap:

$$sp_t = \frac{\mathbb{E}_t^{\mathbb{P}}[\text{Return Skewness}(t, t + \tau)]}{\mathbb{E}_t^{\mathbb{Q}}[\text{Return Skewness}(t, t + \tau)]} - 1,$$

where the total return skewness is calculated over the period t to $t + \tau$. In this paper, I focus on the one-month variance and skew risk premiums consistent with the literature. For the empirical analysis of the variance premium, I use the VIX index, S&P 500 index futures, and the S&P 500 index from the Chicago Board of Options Exchange (CBOE). The options data used to construct the skew premium is from OptionMetrics. The datasets employed in the analysis of the variance and skew measures cover the periods from January 1990 to December 2016 and from January 1996 to December 2016, respectively. I provide a description of the empirical strategy in the Appendix.

[Table 1](#) shows summary statistics for variance and skew risk premiums. A positive variance premium and a negative skew premium are consistent with the literature ([Bakshi, Kapadia and Madan, 2003](#); [Bollerslev, Tauchen and Zhou, 2009](#); [Kozhan, Neuberger and Schneider, 2013](#)). Since the prices of variance and skew swaps are, on average greater, than their corresponding payoffs, the average profits from writing these contracts are interpreted as insurance premiums associated with higher moments of equity returns. [Table 1](#) also shows that both premiums have large volatility, positive skewness, and a kurtosis coefficient much larger than three. The latter two characteristics indicate fat tails in the distributions of quantities.

The risk premiums associated with the second and third moments of equity returns equivalently correspond to the level and the slope of the implied volatility surface, a puzzling feature of equity index options that remains a challenge for equilibrium asset pricing models. I construct the empirical implied volatility surface by performing polynomial extrapolation of volatilities in the maturity time and strike prices. I use the option data from OptionMetrics for January 1996 to December 2016. I present the empirical methodology in the Appendix.

The left plot in [Figure 2](#) shows the implied volatility curve for 1-month maturity as a function of moneyness (a ratio of strike to spot price). The implied volatilities are downward sloping in moneyness and decline from around 28% to slightly above 20% for a range of moneyness from 0.9 to 1.05. This shape is known in the literature as the implied volatility skew. Note that the implied volatilities are significantly above the annualized

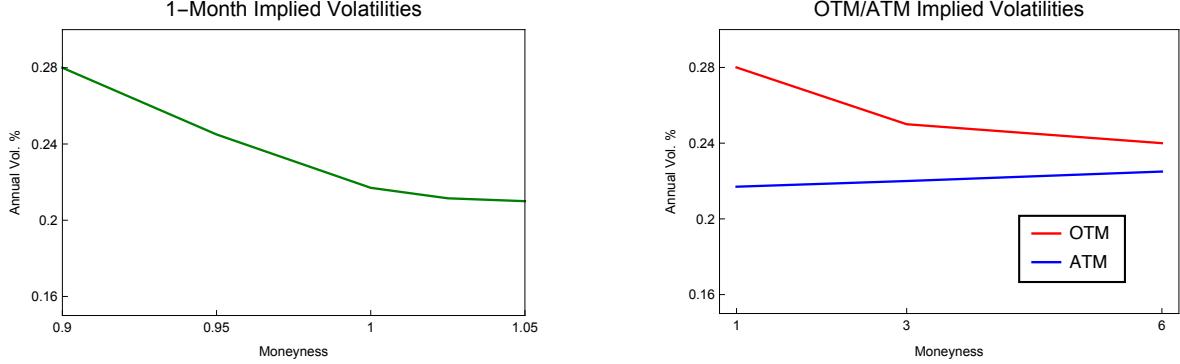


Figure 2: Empirical Implied Volatilities. The left panel plots the 1-month implied volatility curve as a function of moneyness (Strike/Spot Price) for the data. The right panel plots the empirical implied volatility curves for 0.90 OTM and ATM options as functions of the time to maturity expressed in months. The empirical statistics are for the U.S. data from January 1996 to December 2016.

stock market volatility. The right plot in Figure 2 provides the implied volatility curve for 0.90 OTM and ATM put options as functions of maturity. The graph suggests that ATM volatilities are slightly increasing in the horizon, but equal approximately 22% for 1, 3, and 6 month maturities, while OTM volatilities are slightly declining in the horizon. Furthermore, the plot confirms that OTM volatilities are strictly higher than ATM volatilities for all times to expiration. It is difficult to rationalize the level and the slope of the implied volatility surface given historical volatility of the stock market.

3. Model Setup

3.1. Generalized Disappointment Aversion Risk Preferences

The environment is an infinite-horizon, discrete-time exchange economy with a representative agent extracting utility from a consumption stream. Following the recursive utility framework of Epstein and Zin (1989, 1991), the agent's utility V_t in period t is defined as:

$$V_t = \left[(1 - \beta)C_t^\rho + \beta\mu_t^\rho \right]^{1/\rho}, \quad (1)$$

where C_t is consumption at time t , $0 < \beta < 1$ is the subjective discount factor, $\frac{1}{1-\rho} > 0$ is the intertemporal elasticity of substitution (IES), and $\mu_t = \mu_t(V_{t+1})$ is the certainty equivalent of random future utility using the t -period conditional probability distribution.

The certainty equivalent captures the generalized disappointment aversion (GDA) risk attitude as defined by Routledge and Zin (2010). These risk preferences allocate more weight on the tail events compared to the expected utility. In the Routledge and Zin (2010) model, the representative agent perceives some outcomes as "disappointing" similarly to the disappointment aversion preferences of Gul (1991). For the Gul (1991)

model, an outcome is viewed as disappointing when it is below the certainty equivalent, whereas for the [Routledge and Zin \(2010\)](#) generalized disappointment aversion specification, this outcome should be below some fraction of the implicit certainty equivalent. Formally, the certainty equivalent $\mu_t(V_{t+1})$ of GDA risk preferences is defined as:

$$\frac{[\mu_t(V_{t+1})]^\alpha}{\alpha} = \mathbb{E}_t \left[\frac{V_{t+1}^\alpha}{\alpha} \right] - \theta \mathbb{E}_t \left[\mathbb{I} \left(\frac{V_{t+1}}{\mu_t(V_{t+1})} \leq \delta \right) \left(\frac{[\delta \mu_t(V_{t+1})]^\alpha}{\alpha} - \frac{V_{t+1}^\alpha}{\alpha} \right) \right] \quad (2)$$

or equivalently:

$$\mu_t(V_{t+1}) = \left(\mathbb{E}_t \left[V_{t+1}^\alpha \cdot \frac{1 + \theta \mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))}{1 + \theta \delta^\alpha \mathbb{E}_t [\mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))]} \right] \right)^{1/\alpha},$$

where $\mathbb{I}(\cdot)$ denotes the indicator function, $1 - \alpha > 0$ is the relative risk aversion, $\delta \in (0, 1]$ and $\theta \geq 0$ represent a disappointment threshold and a disappointment penalty, respectively. The GDA risk preferences enable control for a disappointment threshold by changing δ . In this case, the outcome V_{t+1} is considered disappointing only when it is below the scaled certainty equivalent $\delta \mu_t(V_{t+1})$.

The [Routledge and Zin \(2010\)](#) preferences defined by (1) and (2) nest two preference specifications. The expected utility of [Epstein and Zin \(1989, 1991\)](#) can be obtained by setting $\theta = 0$, in which case the certainty equivalent $\mu_t(V_{t+1})$ simplifies to $(\mathbb{E}_t[V_{t+1}^\alpha])^{1/\alpha}$. Assuming $\theta \neq 0$ and $\delta = 1$, GDA preferences reduce to the [Gul \(1991\)](#) disappointment aversion utility.

3.2. Endowments and Inference Problem

A popular paradigm in the asset pricing literature is the application of a regime switching framework for modeling aggregate consumption growth.² I follow this tradition in the asset pricing literature and subject log consumption growth to hidden regime switches:

$$\Delta c_{t+1} = \mu_{s_{t+1}} + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1).$$

²Since [Hamilton \(1989\)](#) and [Mehra and Prescott \(1985\)](#), researchers have used these models to embed business cycle fluctuations in the mean growth rates and volatility of consumption growth ([Cecchetti, Lam and Mark, 1990](#); [Veronesi, 1999](#); [Ju and Miao, 2012](#); [Johannes, Lochstoer and Mou, 2016](#); [Collin-Dufresne, Johannes and Lochstoer, 2016](#)). By changing the number of states and parameters controlling the persistence and conditional distribution of regimes, these models can also embed 'peso problem' in the mean ([Rietz, 1988](#); [Barro, 2006](#); [Backus, Chernov and Martin, 2011](#); [Gabaix, 2012](#)) or persistence ([Gillman, Kejak and Pakos, 2015](#)) of consumption growth. Additionally, a proper calibration of a regime switching model can match the dynamics of long-run risks in consumption and dividend growth as studied in [Bonomo, Garcia, Meddahi and Tedongap \(2011, 2015\)](#).

The consumption volatility σ is assumed to be constant, whereas the mean growth rate $\mu_{s_{t+1}}$ is driven by a two-state Markov-switching process s_{t+1} with the state space

$$\mathcal{S} = \{1 = \text{expansion}, 2 = \text{recession}\},$$

and a transition matrix

$$\mathcal{P} = \begin{pmatrix} \pi_{11} & 1 - \pi_{11} \\ 1 - \pi_{22} & \pi_{22} \end{pmatrix},$$

where $\pi_{ii} \in (0, 1)$ are transition probabilities. I further assume $\mu_1 > \mu_2$ in order to identify states $s_{t+1} = 1$ and $s_{t+1} = 2$ as expansion and recession, respectively.

I now specify a dividend stream of the equity. There are several approaches in the literature to model dividends. A standard Lucas-type model ([Lucas Jr., 1973](#)) implies that dividends and consumption are the same in the equilibrium. However, dividends are more volatile than consumption in the data. I follow [Bansal and Yaron \(2004\)](#) and model consumption and equity dividends separately. In the endowment economy, it is commonly assumed that aggregate consumption is generated by several stochastic endowments, including dividends as one endowment, and is the sum of all these endowments in the equilibrium. Similarly to [Campbell \(1996\)](#), one can interpret other endowments as labor income.

I seek to price the equity (a levered consumption claim) with monthly log dividend growth defined as:

$$\Delta d_{t+1} = g_d + \lambda \Delta c_{t+1} + \sigma_d e_{t+1}, \quad (3)$$

where $e_{t+1} \sim N(0, 1)$ is the idiosyncratic shock of dividend growth, and $\lambda > 0$ is the leverage ratio on expected consumption growth. I use the growth rate of dividends g_d to match the long-run consumption growth, and the volatility of dividends σ_d to match the annual 11.04% dividend growth volatility observed in the data. In addition, the chosen value of the leverage parameter enables me to match the observed correlation between annual consumption and dividend growth.

The investor knows the true parameters and distribution of shocks in the model but does not observe the state s_{t+1} of the economy. Consequently, he forms a posterior belief about the hidden state s_{t+1} , conditional on the observable history of consumption and dividend growth rates at time t :

$$\mathcal{F}_t = \left\{ (\Delta c_\tau, \Delta d_\tau) : 0 \leq \tau \leq t \right\}.$$

The inference problem is to derive the evolution of $\pi_t = \mathbb{P}(s_{t+1} = 1 | \mathcal{F}_t)$ given the initial belief π_0 (the stationary prior). In this paper, I relax the assumption of perfect rationality of Bayesian investors, assuming instead an alternative learning rule motivated by the behavioral literature. Evidence from psychology and finance suggests that people are influenced by extrapolative bias when making their decisions (Hirshleifer, 2001; Barberis and Thaler, 2003). Thus, I assume that the agent has an additional behavioral bias in the following form: the belief is updated by taking a weighted average of Bayes rule and the last period belief. Denoting the weight on the old belief by ω , the agent updates π_{t+1} through the following rule:

$$\pi_{t+1} = (1 - \omega)\pi_{t+1}^B + \omega\pi_t, \quad \omega \in (0, 1) \quad (4)$$

where π_{t+1}^B denotes the rational belief obtained through Bayes rule

$$\pi_{t+1}^B = \frac{\pi_{11}f(\Delta c_{t+1}|s_{t+1} = 1)\pi_t + (1 - \pi_{22})f(\Delta c_{t+1}|s_{t+1} = 2)(1 - \pi_t)}{f(\Delta c_{t+1}|s_{t+1} = 1)\pi_t + f(\Delta c_{t+1}|s_{t+1} = 2)(1 - \pi_t)},$$

where

$$f(\Delta c_{t+1}|s_{t+1} = i) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(\Delta c_{t+1} - \mu_{s_{t+1}})^2}{2\sigma^2}}.$$

In the quantitative analysis, extrapolative bias puts 50% weight on the belief. This number helps with matching the negative skewness in the simulated excess returns and reproducing return predictability by the price-dividend ratio.

4. Asset Prices

The representative agent consumes C_t in period t and invests the remaining wealth in multiple assets. The agent maximizes his utility subject to a budget constraint:

$$W_{t+1} = (W_t - C_t)R_{t+1}^\omega,$$

where R_{t+1}^ω is the return on the total (unobservable) wealth W_t . Additionally, the return R_{t+1}^ω satisfies:

$$R_{t+1}^\omega = \sum_{i=1}^N \omega_{i,t} R_{i,t+1} \quad \wedge \quad \sum_{i=1}^N \omega_{i,t} = 1,$$

where $\omega_{i,t}$ is the fraction of the t -period wealth invested in the i -th asset with gross real return $R_{i,t+1}$. In equilibrium, the representative investor makes his consumption and portfolio decisions subject to the endogenous determination of asset prices and markets clearing conditions.

Following Routledge and Zin (2010), it can be shown (see Appendix B) that the gross return $R_{i,t+1}$ on the i -th traded asset satisfies:

$$\mathbb{E}_t [M_{t+1} R_{i,t+1}] = 1, \quad (5)$$

where the pricing kernel of the economy is defined as:

$$M_{t+1} = \underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1}}_{M_{t+1}^{CRRRA}} \cdot \underbrace{\left(\frac{V_{t+1}}{\mu_t(V_{t+1})} \right)^{\alpha-\rho}}_{M_{t+1}^{EZ}} \cdot \underbrace{\left(\frac{1 + \theta \mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))}{1 + \theta \delta^\alpha \mathbb{E}_t [\mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))]} \right)}_{M_{t+1}^{GDA}}. \quad (6)$$

There are different components of the pricing kernel. The first part M_{t+1}^{CRRRA} is the pricing kernel of the time-separable power utility. The second multiplier M_{t+1}^{EZ} is the adjustment of Epstein-Zin preferences, which allow a separation between the coefficient of risk aversion and elasticity of intertemporal substitution. The third component M_{t+1}^{GDA} represents the generalized disappointment aversion adjustment. When the agent's utility is below a predefined fraction of the certainty equivalent, more weight is attached to the pricing kernel. The generalized disappointment aversion thus magnifies the countercyclical dynamics of the pricing kernel. For better understanding of the key role of GDA, I consider the calibration of preference parameters $\alpha = \rho$. In this case, the agent has no preference for resolution of uncertainty, and the pricing kernel simplifies to:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1} \cdot \left(\frac{1 + \theta \mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))}{1 + \theta \delta^\alpha \mathbb{E}_t [\mathbb{I}(V_{t+1} \leq \delta \mu_t(V_{t+1}))]} \right).$$

4.1. Risk-free Rate and Equity Returns

I solve the model numerically due to the lack of an analytical solution for equilibrium returns. I first need to solve for the return on the wealth portfolio R_{t+1}^ω (the return on the aggregate consumption claim) and then the equity return $R_{e,t+1}$ (the return on the aggregate dividend claim), which are implicitly defined by the equation (5). Denoting the equity price by P_t^e , the returns on the wealth portfolio and equity can be rewritten as:

$$R_{t+1}^\omega = \frac{W_{t+1}}{W_t - C_t} = \frac{\frac{W_{t+1}}{C_{t+1}}}{\frac{W_t}{C_t} - 1} \cdot e^{\Delta c_{t+1}} \quad \wedge \quad R_{t+1}^e = \frac{P_{t+1}^e + D_{t+1}}{P_t^e} = \frac{\frac{P_{t+1}^e}{D_{t+1}} + 1}{\frac{P_t^e}{D_t}} \cdot e^{\Delta d_{t+1}}.$$

I conjecture that the wealth-consumption ratio $\frac{W_t}{C_t} = G(\pi_t)$ and the price-dividend ratio $\frac{P_t^e}{D_t} = H(\pi_t)$ are functions of the state belief π_t . Substituting R_{t+1}^ω and R_{t+1}^e into (5), I

apply the projection method (Judd, 1992) to approximate $G(\pi_t)$ and $H(\pi_t)$ by a basis of complete Chebyshev polynomials. The details of the numerical solution algorithm are provided in Appendix C. Having solved for wealth-consumption and price-dividend ratios, I can simulate asset pricing moments associated with the one-period risk-free rate, equity returns and price-dividend ratio. Further, I can numerically calculate the pricing kernel and all asset prices, including implied volatilities and quantities in the skew and variance risk premiums.

4.2. Prices and Returns of Variance Swaps

I further analyze the term structure of variance claims predicted by different models. I consider an n -month variance swap that is a claim to realized variance over months $t+1$ to $t+n$. Given the discrete nature of the model, total variance of the return is simply equal to the sum of conditional variances RV_{t+i} in each subperiod i . Following Dew-Becker, Giglio, Le and Rodriguez (2017), the price of an n -month variance swap is

$$VS_t^n = \mathbb{E}_t^Q \left[\sum_{i=1}^n RV_{t+i} \right].$$

In turn, the price of the zero coupon forward claim on realized variance is

$$F_t^n = \mathbb{E}_t^Q [RV_{t+n}].$$

In other words, F_t^n is equal to the risk-neutral expectation of return variance realized during the n -th month from the current period, whereas F_t^0 is naturally defined as the realized variance in the current period. Next, I define the return on the n -month variance forward as a return on the trading strategy where investors buy the n -month forward at time t and sell it in the next period as a forward claim with maturity $n-1$. The proceeds from selling the forward are then used to purchase a new n -month variance at price F_{t+1}^n . Formally, the excess return of an n -period variance forward is

$$R_{t+1}^n = \frac{F_{t+1}^{n-1} - F_t^n}{F_t^n}.$$

4.3. The Variance and Skew Risk Premiums

The focus of this paper is on the monthly variance and skew risk premiums associated with equity returns. The t -time monthly variance premium vp_t is defined as the difference between risk-neutral and physical expectations of the total return variance between time

t and $t + 1$. As in Drechsler and Yaron (2011), the variance premium equals:

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}(var_{t+1}^{\mathbb{Q}}(r_{e,t+2})) - \mathbb{E}_t^{\mathbb{P}}(var_{t+1}^{\mathbb{P}}(r_{e,t+2})), \quad (7)$$

where $var_{t+1}^{\mathbb{Q}}(r_{e,t+2})$ and $var_{t+1}^{\mathbb{P}}(r_{e,t+2})$ are $(t + 1)$ -period conditional variances of the log return $r_{e,t+2} = \ln(R_{e,t+2})$ under the risk-neutral \mathbb{Q} and physical \mathbb{P} probability measures, respectively. As noted in Drechsler and Yaron (2011), the variance premium is decomposed into two components denoted as the level difference and drift difference. The level difference, defined as

$$var_t^{\mathbb{Q}}(r_{e,t+1}) - var_t^{\mathbb{P}}(r_{e,t+1}),$$

reflects the difference in the conditional return variance under the risk-neutral and physical measures. The drift difference, defined as

$$[\mathbb{E}_t^{\mathbb{Q}}(var_{t+1}^{\mathbb{Q}}(r_{e,t+2})) - var_t^{\mathbb{Q}}(r_{e,t+1})] - [(\mathbb{E}_t^{\mathbb{P}}(var_{t+1}^{\mathbb{P}}(r_{e,t+2})) - var_t^{\mathbb{P}}(r_{e,t+1}))],$$

incorporates the difference in the expected change of $var_{t+1}(r_{e,t+2})$ under the measures \mathbb{Q} and \mathbb{P} .

The t -time monthly skew premium is defined as the expected payoff of the skew swap, a contract paying the difference between the implied skew and the realized skew of the index return between time t and $t + 1$ (Kozhan, Neuberger and Schneider, 2013). The monthly implied and realized skews simply equal the risk-neutral and physical expectations of the index return skewness denoted by $\mathbb{E}_t^{\mathbb{Q}}(skew_{t+1}^{\mathbb{Q}}(r_{e,t+2}))$ and $\mathbb{E}_t^{\mathbb{P}}(skew_{t+1}^{\mathbb{P}}(r_{e,t+2}))$, respectively. As in Kozhan, Neuberger and Schneider (2013), I further express the skew risk premium as a percentage of the implied skew that results in the definition of the skew risk premium as stated below:

$$sk_t = \frac{\mathbb{E}_t^{\mathbb{P}}(skew_{t+1}^{\mathbb{P}}(r_{e,t+2}))}{\mathbb{E}_t^{\mathbb{Q}}(skew_{t+1}^{\mathbb{Q}}(r_{e,t+2}))} - 1.$$

The quantity sk_t reflects the dollar amount of profit per \$1 investment in the implied skew.

4.4. Implied Volatilities

I now describe how I compute model-based option prices and solve for their Black-Scholes implied volatilities. Consider a European put option written on the price of the equity that is traded in the economy. Note that the equity price should not include dividend payments; that is, options are written on the ex-dividend stock price index.

Using the Euler condition (5), the relative price $\mathcal{O}_t(\pi_t, \tau, K) = \frac{P_t^o(\pi_t, \tau, K)}{P_t^e(\pi_t)}$ of the τ -period European put option with the strike price K , expressed as a ratio to the initial price of the equity P_t^e , should satisfy:

$$\mathcal{O}_t(\pi_t, \tau, K) = \mathbb{E}_t \left[\prod_{k=1}^{\tau} M_{t+k} \cdot \max \left(K - \frac{P_{t+\tau}^e}{P_t^e}, 0 \right) \right]. \quad (8)$$

It is worth noting that a put price P_t^o depends on the equity price P_t^e , whereas the normalized price \mathcal{O}_t does not. One can express the ratio $\frac{P_{t+\tau}^e}{P_t^e}$ in terms of the dividend growth rates and price-dividend ratios on the equity and hence the state belief π_t provides sufficient information for the calculation of the option prices. Specifically, I compute model-based European put prices $\mathcal{O}_t = \mathcal{O}_t(\pi_t, \tau, K)$ via Monte Carlo simulations. I convert them into Black-Scholes implied volatilities with properly annualized continuous interest rate $r_t = r_t(\pi_t)$ and dividend yield $q_t = q_t(\pi_t)$. Thus, given the time to maturity τ , the strike price K , the risk-free rate r_t , and dividend yield q_t , the implied volatility $\sigma_t^{\text{imp}} = \sigma_t^{\text{imp}}(\pi_t, \tau, K)$ solves the equation:

$$\begin{aligned} \mathcal{O}_t &= e^{-r_t \tau} \cdot K \cdot N(-d_2) - e^{-q_t \tau} \cdot N(-d_1), \\ d_{1,2} &= \frac{\ln \left(\frac{1}{K} \right) + \tau \left(r_t - q_t \pm \frac{(\sigma_t^{\text{imp}})^2}{2} \right)}{\sigma_t^{\text{imp}} \sqrt{\tau}}. \end{aligned} \quad (9)$$

5. Calibration and Quantitative Results

In this section, I first calibrate the cash-flow processes for consumption and dividend growth. The chosen parameters are consistent with the historical US data from January 1930 to December 2016. To better understand the role of generalized disappointment aversion in the consumption-based asset pricing economy of this paper, I consider three specifications of preference parameters: the benchmark model (GDA) with generalized disappointment aversion preferences, a pure disappointment aversion economy (DA) with linear preferences and infinite elasticity of intertemporal substitution, and an Epstein-Zin framework (EZ). The comparison of GDA and DA isolates the contribution of disappointment aversion, while the comparison of GDA and EZ illustrates the impact of the representative agent's preference for early resolution of uncertainty. Since the model does not allow an analytical solution, I solve for equilibrium pricing ratios using the projection method (Judd, 1992) with Chebyshev interpolation. Having solved the

model, I generate 10,000 simulations of the economy and report statistics of cash-flows and asset prices corresponding to their empirical counterparts.

5.1. Calibrated Parameters

I begin with the parameters of a regime-switching process for aggregate consumption growth $(\pi_{11}, \pi_{22}, \mu_1, \mu_2, \sigma)$. As in [Bansal and Yaron \(2004\)](#), I make the model's time-averaged consumption statistics consistent with observed annual log consumption growth from 1930 to 2016. I calibrate a two-state regime-switching model of monthly consumption growth with the recession state mimicking large declines like the Great Depression, and the expansion state reflecting normal business cycle fluctuations. Therefore, I set $\pi_{11} = 1151/1152$ and $\pi_{22} = 47/48$. These numbers imply an average duration of the high-growth state of about $(1 - \pi_{11})^{-1} = 96$ years and the low-growth state of about $(1 - \pi_{22})^{-1} = 4$ years. Furthermore, the unconditional probability of being in expansion $\bar{\pi}_{11} = (1 - \pi_{22})/(2 - \pi_{11} - \pi_{22})$ results in $\bar{\pi}_{11} = 0.96$ and hence the agent experiences one 4-year depression per century, consistent with the historical data. For the mean growth rate, consumption tends to grow, on average, at the annual rates of about $\mu_1 \times 12 = 2.08\%$ and $\mu_2 \times 12 = -4.6\%$ in the expansion and recession states, respectively. The depression state is consistent with an average annual decline in the real, per capita log consumption growth during the Great Depression and is less severe than rare disasters, defined as a drop in annual consumption growth larger than 10 percent ([Rietz, 1988](#); [Barro, 2006](#)). I calibrate the consumption volatility σ to match the observed standard deviation.³

I now turn to calibrating parameters in the dividend process. I regress the annual dividends on the annual consumption covering the period 1930-2016 and find the leverage ratio is around 2.5, a conservative number within an interval of plausible values from 1.5 to 4.5. The leverage ratio is an important parameter for two reasons. First, it controls the volatility of dividends in normal times. Second, it determines the decline of dividends in

³The reason for calibrating the model with only two regimes is twofold. First, I want to maintain parsimony for the sake of convenient interpretation of results. Second, I do not introduce additional risks in consumption growth as considered in other papers (more states, non-Gaussian shocks in consumption growth, alternative information settings, etc.) in order to isolate and emphasize the impact of learning and GDA risk preferences. Of course, a model with more regimes would lead to richer consumption dynamics. For example, it could introduce long-run risks in consumption as studied in [Bonomo, Garcia, Meddahi and Tedongap \(2011, 2015\)](#). Alternatively, a framework with a multidimensional learning problem ([Collin-Dufresne, Johannes and Lochstoer, 2016](#); [Johannes, Lochstoer and Mou, 2016](#)) could additionally contribute to the dynamics of the model. Although taking into account all these risks channels would certainly improve the model's performance, I show that a combination of learning about an unobservable state and the agent's GDA risk attitude alone can reproduce a wide array of dynamic asset pricing phenomena observed in the equity and derivatives markets.

Table 2
Parameter Values

Parameter	Description	Value
β	Discount factor	0.9989
$1 - \alpha$	Risk aversion	1/1.5
$1/(1 - \rho)$	EIS	1.5
θ	Disappointment aversion	4.66
δ	Disappointment threshold	0.928
π_{11}	Transition probability from expansion to expansion	0.9991
π_{22}	Transition probability from recession to recession	0.9787
μ_1	Consumption growth in expansion	0.17(3)
μ_2	Consumption growth in recession	-0.38(3)
σ	Consumption volatility	0.6928
g_d	Mean adjustment of dividend growth	-0.2519
σ_d	Std. deviation of dividend growth shock	3.11
λ	Leverage ratio	2.6

This table reports parameter values in the benchmark model with GDA preferences. All parameters are calibrated at a monthly frequency.

the recession state. As a result, increasing the leverage would increase the payoffs of put options, conditional on the realization of recession. Compare the model's performance with models in existing literature, particularly the literature on rare disasters, I set the leverage ratio $\lambda = 2.6$, corresponding to the value used in [Seo and Wachter \(2018\)](#). I further follow the literature and set g_d to equalize the long-run dividend and consumption growth. The standard deviation of the dividend process σ_d is used to generate large annual dividend volatility observed in the data.

In the GDA model, I choose preference parameters in line with existing studies. I set the subjective discount factor $\beta = 0.9989$ and the EIS $1/(1 - \rho) = 1.5$ to match the first and second moments of the risk-free rate. The coefficient of relative risk aversion is $1 - \alpha = 1/1.5$. In this case, the representative investor has no preference for resolution of uncertainty and, thus, asset prices are driven by the generalized disappointment aversion channel in the economy. I jointly choose the disappointment aversion parameter $\theta = 4.66$ and the disappointment threshold $\delta = 0.928$ to match the high equity premium observed in the data. In the economy, the representative investor penalizes disappointing utilities 5.66 times more than normal time outcomes. The degree of disappointment aversion is consistent with the empirical literature that reports a range of values from 3.29 to 8.41 ([Delikouras, 2017](#)). Note that prominent statistics of the variance and skew premiums, the variance term structure and the implied volatility surface are not directly targeted in the model calibration. Surprisingly, these complex features of the data compare well with

the model-generated statistics, which endogenously arise in the model due to fluctuations in the posterior state belief. Table 2 summarizes the calibrated values of the GDA model.

For the DA model, I shut off the generalized disappointment aversion channel by setting $\delta = 1$. Furthermore, the DA specification does not exhibit any curvature in the pricing kernel associated with the relative risk aversion, which is set at $1 - \alpha = 0$, or elasticity of intertemporal substitution, which is equal to $(1 - \rho)^{-1} = \infty$. I only adjust the disappointment aversion to match the observed equity premium. The remaining parameters are fixed at the benchmark values. For the EZ model, I turn off all (generalized) disappointment aversion by setting $\theta = 0$. In this case, the representative agent only exhibits a preference for early resolution of uncertainty, a popular workhorse in the asset pricing literature. Further, I increase the risk aversion to $1 - \alpha = 5$, so the model matches the equity premium observed in the data. Other parameters correspond to those in the GDA specification.

5.2. Asset Pricing Implications

Before discussing the asset pricing implications of GDA, DA, and EZ models, I look at the cash-flow dynamics predicted by a two-state regime switching model. Panel A in Table 3 compares the annualized consumption and dividends moments of the data with those implied by the calibration in this paper. The model-based averages of the mean and volatility of consumption and dividend growth come out close to their empirical counterparts, although mean dividend growth is slightly higher in the simulations. The autocorrelation of cash-flows is also in line with the empirical estimates. The leverage parameter chosen allows me to capture the observed correlation between consumption and dividends. Overall, one can see that a two-state regime-switching model of consumption and dividend growth matches the key empirical statistics well.

5.2.1. Risk-free Rate and Equity Returns

Panel B in Table 3 reports the key annualized moments of the risk-free rate, equity returns, and the price-dividend ratio for three model specifications: GDA, DA, and EZ. All three models do a good job of accounting for the salient features of equity returns, as all predict the low risk-free rate, the large equity premium, and volatility of excess returns. In addition, the volatility of the risk-free rate and the level of the log price-dividend ratio correspond well to the empirical estimates under all specifications.

Table 3
Cash Flows, Stock Market Returns and Predictability

	Data	GDA	DA	EZ
β		0.9989	0.9989	0.9989
$1 - \alpha$		1/1.5	0	5
$1/(1 - \rho)$		1.5	∞	1.5
θ		4.66	0.5	0
δ		0.928	1	
Panel A: Cash Flows				
$E(\Delta c)$	1.83	1.88	1.88	1.88
$\sigma(\Delta c)$	2.22	2.25	2.25	2.25
$ac1(\Delta c)$	0.50	0.34	0.34	0.34
$E(\Delta d)$	1.44	1.91	1.91	1.91
$\sigma(\Delta d)$	11.04	10.50	10.50	10.50
$ac1(\Delta d)$	0.19	0.27	0.27	0.27
$corr(\Delta c, \Delta d)$	0.55	0.55	0.55	0.55
Panel B: Returns				
$E(r_f)$	0.81	1.13	1.32	1.25
$\sigma(r_f)$	1.87	2.20	0.00	1.44
$E(r_e - r_f)$	5.22	5.82	5.49	5.70
$\sigma(r_e - r_f)$	19.77	20.58	18.15	19.51
$E(pd)$	3.11	3.05	3.05	3.04
$\sigma(pd)$	0.33	0.15	0.14	0.15
$skew(r_e - r_f)[M]$	-0.47	-0.64	-0.44	-0.57
$kurt(r_e - r_f)[M]$	10.06	5.13	5.55	5.12
Panel C: Predictability				
$\beta(1y)$	3.12	8.78	7.71	8.24
$R^2(1y)$	7.00	8.79	4.75	7.40
$\beta(3y)$	7.05	16.74	14.33	15.70
$R^2(3y)$	14.67	15.50	8.99	13.57
$\beta(5y)$	12.34	19.98	17.00	18.70
$R^2(5y)$	27.26	17.26	10.50	15.52

Panel A reports moments of consumption and dividend growth rates denoted by Δc and Δd .

Panel B reports moments of the log risk-free rate r_f , the excess log equity returns $r_e - r_f$, and the log price-dividend ratio pd . Panel C reports results of the predictive regression of

h -year future excess log equity returns constructed as $r_{t+1 \rightarrow t+12h}^{ex} = \sum_{i=1}^{12h} (r_{e,t+i} - r_{f,t-1+i})$

on the lagged annualized dividend yield $dp_{t-11 \rightarrow t}$. Specifically, the slope estimates $\beta(h)$ and $R^2(h)$ are based on the linear projection:

$$r_{t+1 \rightarrow t+12h}^{ex} = \text{Intercept} + \beta(h) \times dp_{t-11 \rightarrow t} + \varepsilon_{t+12h},$$

where $h = 1, 3$ and 5 years. The moments and regression outputs are for the data and three models: GDA, DA, and EZ. The table entries for moments are annualized statistics except for the [M] rows, which provide skewness and kurtosis of monthly excess log returns. The empirical moments are for the U.S. data from January 1930 to December 2016. For each model, I simulate 10,000 economies sampled at a monthly frequency with a sample size equal to its empirical counterpart. I obtain moments, the slope coefficients $\beta(h)$ and $R^2(h)$ for each simulation and report averages of sample statistics over all 10,000 artificial series. I use the common notations for the sample mean E , standard deviation σ , autocorrelation $ac1$, skewness $skew$, and kurtosis $kurt$ of the series.

The main shortcoming of the three models is the low volatility of the log price-dividend ratio. Despite this low volatility, the GDA model can generate the patterns of return predictability by dividend yields observed in the data. Indeed, Panel C in Table 3 runs regressions of future excess returns on the annualized dividend-price ratio over various forecasting horizons. The extrapolative bias in learning helps generate increasing patterns of the slopes and R^2 s in the three models. In terms of magnitude, the GDA specification closely matches the R^2 statistics for a 1-year and 3-year period, while it slightly underestimates forecasting power for a 5-year horizon relative to the data. The remaining models with DA and EZ preferences have qualitatively similar predictions, however, they produce smaller regression coefficients and R^2 .

The bottom of Panel B also presents the higher moments of excess returns at the monthly frequency (labeled [M] in the table). These statistics are instructive about the return distribution that would have a direct impact on the higher moment risk premiums and option prices.⁴ All three specifications qualitatively respect the negative skewness and excess kurtosis in monthly returns. The negative skewness in the distribution of beliefs helps capture the third moment of returns in the data. Since the cash-flow model of consumption and dividend growth rates does not introduce stochastic volatility or jumps of a different size in the dynamics of fundamentals, all three frameworks slightly underestimate the excess kurtosis in returns.

5.2.2. The Price of Variance Risk

Figure 3 compares the empirical and model-based term structure of Sharpe ratios and prices for forward variance claims. These graphs assess how well different preferences can explain the patterns in the data. The left plot in Figure 3 shows that the GDA model does a good job matching the overall shape of annualized Sharpe ratios. In particular, it generates a curve that is very steep for the one-month returns and then has a positive but small slope for the longer horizons. The figure also documents that both EZ and DA specifications fail to reconcile the concave and upward shape of the term structure. Consistent with the findings of [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#), the calibration with Epstein-Zin preferences generates almost constant Sharpe ratios, thus, underpricing volatility risk in the short term and overpricing future realized variance in the long term.

⁴In a power-utility economy setting, [Bakshi, Kapadia and Madan \(2003\)](#) theoretically show how changing the skewness and kurtosis of the physical density of returns alters the (first) three moments of the risk-neutral distribution.

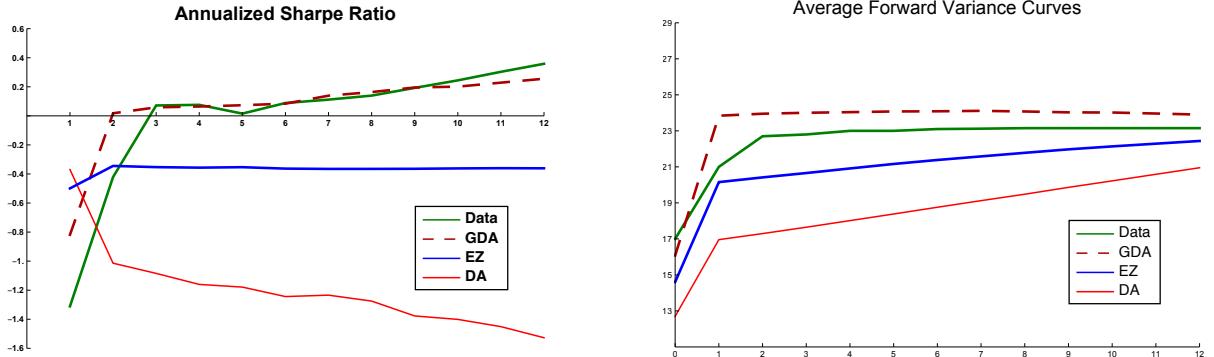


Figure 3: Sharpe Ratios and Average Term Structure in Different Models. The left and right panels plot annualized Sharpe ratios and average prices for forward variance claims for the data and three models: GDA, DA, and EZ. The prices are reported in annualized volatility terms, $100 \times \sqrt{12} \times F_t^n$. The empirical lines are the curves constructed in [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) and correspond to the U.S. data from 1996 to 2013. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The simulation results are averages of sample moments based on these 10,000 artificial series.

The results for the DA model show that disappointment aversion generates even higher Sharpe ratios for the longer maturities, while the one-month forwards are underpriced compared to the data as well as to models with GDA and EZ preferences.

The interpretation of our results in the EZ economy is similar to the intuition provided by [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#). The risk-averse investors consider the states with high expected future volatility as periods of low lifetime utility. With the Epstein-Zin preferences, low utility increases the pricing kernel and, thus, investors with a strong preference for early resolution of uncertainty require large compensation for future consumption volatility. The economic intuition for the results with DA preferences is slightly different. In this case, even though the coefficient of relative risk aversion is very low, investors are extremely averse to expected future volatility due to disappointment aversion. Because increased volatility leads to more disappointing outcomes in consumption growth, investors are willing to pay high prices for forward variance swaps that can hedge their concerns about future disappointment events. Note though, that a too high disappointment threshold actually dampens volatility of the pricing kernel in the short term, so Sharpe ratios tend to be higher for longer maturities than for shorter horizons.

The generalized disappointment averse investors also have an asymmetric risk attitude towards downside risk in consumption growth. Unlike disappointment averse agents, they place more weight on outcomes that are sufficiently deep in the left tail of the consumption distribution. One can generate higher volatility of the pricing kernel by controlling the disappointment threshold and, consequently, this leads to higher Sharpe

ratios for the one-month forward variance swaps relative to the DA and EZ economies. Furthermore, absent a preference for resolution of uncertainty in the calibration of GDA preferences, volatility risk is not overpriced on the longer end of the term structure. Therefore, generalized disappointment averse agents are afraid of unexpected realized volatility but are willing to pay less for insurance against future volatility, consistent with the empirical evidence.

The right plot in Figure 3 displays the average prices of forward variance claims in the data and three models. The graph shows that neither model can fully capture a concave shape of the empirical term structure. The DA economy generates the lowest prices among the three specifications. The term structure in the model with Epstein-Zin preferences increases slightly, but remains strictly below the empirical curve. The GDA economy matches the average prices of forward variance claims with a one-month maturity and captures the flatness on the upper end well, though it cannot fully account for the curvature and level of the term structure.

Table 4 provides statistics on the monthly returns on forward variance claims for the one, three, six, and 12 month maturities in the data and three models. In the data, the average returns for the one-month horizon are strongly negative, whereas all others remain positive. Also, return volatilities are much higher in the short term relative to the longer maturities. Finally, there is high skewness and kurtosis in the returns for the one-month maturity, however, both decrease substantially with the horizon. In terms of the model-based results, the GDA framework is the only one that closely matches the negative average returns for the one-month horizon and generates positive average returns for the longer maturities. All three models overstate the standard deviation and generate smaller skewness and kurtosis for the one-month returns. Note that the models reasonably replicate a drop in the volatility, skewness, and kurtosis of returns after the first month, though GDA preferences seem to perform slightly better.

5.2.3. The Variance and Skew Risk Premiums

Benchmark Model: GDA. In Table 5, I collect moments of the variance premium and conditional variances of the market return under the actual and risk-neutral probability measures. Panel A in Table 5 shows that the GDA model is able to generate a large and volatile variance premium. Although the mean and volatility of the variance premium predicted by the benchmark are slightly larger than in the data, the empirical

Table 4
Higher Moments of Variance Forward Returns

	Maturity (months)	Data	GDA	DA	EZ
β			0.9989	0.9989	0.9989
$1 - \alpha$			1/1.5	0	5
$1/(1 - \rho)$			1.5	∞	1.5
θ			4.66	0.5	0
δ			0.928	1	
Mean	1	-308.98	-281.13	-141.72	-189.38
	3	8.58	0.81	-46.92	-22.98
	6	5.58	1.72	-48.25	-20.67
	12	21.29	6.28	-43.17	-14.46
SD	1	235.34	412.71	436.16	433.73
	3	117.51	89.14	48.08	69.46
	6	67.99	61.34	43.57	57.13
	12	60.33	30.04	32.34	37.42
Skewness	1	6.17	3.38	2.76	3.06
	3	2.68	2.09	2.47	2.18
	6	1.27	1.60	2.34	1.93
	12	0.99	1.01	2.01	1.58
Kurtosis	1	59.49	21.06	14.27	17.49
	3	17.13	11.02	15.18	11.44
	6	6.33	7.70	13.65	9.56
	12	4.37	4.87	10.35	7.38

This table reports moments of the returns for variance claims at horizons of one, three, six, and 12 months. The mean and standard deviation statistics are annualized, while the skewness and kurtosis statistics are calculated for the monthly returns. The moments are for the data and three models: GDA, DA, and EZ. The empirical statistics are the estimates from [Dew-Becker, Giglio, Le and Rodriguez \(2017\)](#) and correspond to the U.S. data from 1996 to 2013. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The simulation results are averages of sample moments based on these 10,000 artificial series.

estimates easily fall into the 90% model-based confidence interval (not reported in the table) of the corresponding model-based statistics. It is well-known that the variance premium distribution is fat-tailed with positive skewness and kurtosis. The GDA model qualitatively respects the non-normality of the distribution, although the sample skewness and kurtosis statistics are smaller relative to the data. Panel A in Table 5 also demonstrates that the GDA framework is able to account for the first and second moments of the variance premium with empirically consistent conditional return variances under both probability measures. Specifically, the total return variance is more volatile under the risk-neutral distribution relative to the physical distribution. Additionally, both volatilities are persistent, as they are in the data.

Empirical literature further documents return predictability by the variance pre-

Table 5
Variance Premium and Predictability

	Data	GDA	DA	EZ
β		0.9989	0.9989	0.9989
$1 - \alpha$		1/1.5	0	5
$1/(1 - \rho)$		1.5	∞	1.5
θ		4.66	0.5	0
δ		0.928	1	
<hr/>				
Panel A: Variance Premium				
$E(vp)$	10.24	13.39	1.88	5.28
$\sigma(vp)$	10.49	13.57	1.77	5.51
$skew(vp)$	2.62	1.03	2.04	1.50
$kurt(vp)$	14.15	3.11	8.33	4.90
$\sigma(var_t^{\mathbb{P}}(r_e))$	26.14	20.89	13.23	17.88
$ac1(var_t^{\mathbb{P}}(r_e))$	0.81	0.89	0.89	0.91
$\sigma(var_t^{\mathbb{Q}}(r_e))$	34.34	37.11	14.35	23.20
$ac1(var_t^{\mathbb{Q}}(r_e))$	0.82	0.88	0.89	0.91
$skew(var_t^{\mathbb{Q}}(r_e))$	3.45	1.11	2.67	1.60
$kurt(var_t^{\mathbb{Q}}(r_e))$	20.72	3.41	12.19	5.42
<hr/>				
Panel B: Predictability				
$\beta(1m)$	0.81	0.68	3.37	1.46
$R^2(1m)$	2.63	1.80	1.06	1.59
$\beta(3m)$	0.89	0.63	3.08	1.34
$R^2(3m)$	8.32	4.75	2.83	4.25
$\beta(6m)$	0.61	0.56	2.72	1.20
$R^2(6m)$	7.26	8.09	4.87	7.29

Panel A reports moments of the conditional variance premium vp , market return variances $var_t^{\mathbb{P}}(r_e)$ and $var_t^{\mathbb{Q}}(r_e)$ under the physical \mathbb{P} and risk-neutral \mathbb{Q} probability measures, respectively. The Panel A entries are monthly statistics. Panel B reports results of the predictive regression of h-month future excess log equity returns constructed as $r_{t+1 \rightarrow t+h}^{ex} = \sum_{i=1}^h (r_{e,t+i} - r_{f,t-1+i})$ on the lagged variance premium vp_t . Specifically, the slope estimates $\beta(h)$ and $R^2(h)$ are based on the linear projection:

$$100 \times r_{t+1 \rightarrow t+h}^{ex} = \text{Intercept} + \beta(h) \times vp_t + \varepsilon_{t+h},$$

where $h = 1, 3$ and 6 months. The moments and regression outputs are for the data and three models: the benchmark model with generalized disappointment aversion preferences GDA, a pure disappointment aversion specification with linear preferences and infinite elasticity of intertemporal substitution DA, and an Epstein-Zin economy EZ. The empirical statistics are for the U.S. data from January 1990 to December 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. I obtain moments, the slope coefficients $\beta(h)$ and $R^2(h)$ for each simulation and report averages of sample statistics over all 10,000 artificial series. I use the common notations for the sample mean E , standard deviation σ , autocorrelation $ac1$, skewness $skew$, and kurtosis $kurt$ of the series.

mium. To study this predictive relationship, I regress the one-, three-, and six-month cumulative excess log returns, which are expressed in percentages, on the lagged monthly variance premium. Consistent with the existing literature, the "Data" column of Panel B in Table 5 indicates a positive impact as measured by positive and slightly decreasing

Table 6
Skew Premium

	Data	GDA	DA	EZ
β		0.9989	0.9989	0.9989
$1 - \alpha$		1/1.5	0	5
$1/(1 - \rho)$		1.5	∞	1.5
θ		4.66	0.5	0
δ		0.928	1	
$E(sp)$	-42.12	-39.50	3.56	-20.68
$\sigma(sp)$	82.11	54.27	16.31	37.43
$skew(sp)$	3.57	1.05	1.46	1.09
$kurt(sp)$	16.26	33.63	13.77	26.31
$E(skew_t^{\mathbb{P}}(r_e))$	-87.52	-53.88	-32.43	-49.93
$\sigma(skew_t^{\mathbb{P}}(r_e))$	173.59	14.57	19.03	16.34
$E(skew_t^{\mathbb{Q}}(r_e))$	-177.73	-98.80	-31.95	-66.08
$\sigma(skew_t^{\mathbb{Q}}(r_e))$	92.33	38.86	19.39	25.07

This table reports moments of the conditional skew premium sp , market return skewness $skew_t^{\mathbb{P}}(r_e)$ and $skew_t^{\mathbb{Q}}(r_e)$ under the physical \mathbb{P} and risk-neutral \mathbb{Q} probability measures, respectively. The entries are monthly statistics. The moments are for the data and three models: GDA, DA, and EZ. The empirical statistics are for the U.S. data from January 1996 to January 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The simulation results are averages of sample moments based on these 10,000 artificial series. I use the common notations for the sample mean E , standard deviation σ , skewness $skew$, and kurtosis $kurt$ of the series.

regression coefficients. In addition, there is an increasing predictive power as measured by increasing R^2 's over longer horizons. The GDA model replicates these empirical findings by matching the magnitude of coefficients and R^2 statistics.

Table 6 compares descriptive statistics of the skew premium and related variables in the data and the models. Consistent with the results in [Kozhan, Neuberger and Schneider \(2013\)](#), I quantify its size by looking at the profit of a skew swap, a financial contract paying the difference between the implied skew and the realized skew of the equity return. As shown in the table, the GDA specification produces sizable mean and volatility of the skew premium that correspond well to the historical values. The GDA model generates a positive skewness and an excess kurtosis in the conditional distribution of the skew premium. Table 6 also demonstrates the first and second moments of the return skewness under risk-neutral and physical distributions. The conditional mean of the return skewness under both measures is significantly negative, although the model cannot fully capture the size observed in the data. The main drawback of the GDA model is lower volatility of the realized and implied skew.

Pure Disappointment Aversion and Epstein-Zin Specifications. GDA, DA and EZ

models provide a good fit with equity returns. However, the key difference between the three specifications can be observed in light of the variance and skew risk premium statistics. Table 5 shows that disappointment aversion alone produces mean and volatility of the variance premium approximately seven times smaller than the benchmark values. Turning off the GDA channel also leads to a significant reduction in the volatility of return variance in the DA model. As the variance premium decreases, its predictive power for the excess log returns also suffers. This is manifested in the lower R^2 and empirically inconsistent regression coefficients. The asset pricing implications of the DA model are further augmented by Table 6. The table shows that a disappointment aversion specification alone cannot reproduce the skew premium statistics. In particular, the skew premium proves to be positive. Furthermore, the bottom part of Table 6 shows that the DA model predicts the smallest first and second moments of return skewness of the three models.

Next, I turn off any source of (generalized) disappointment aversion and consider a representative agent with Epstein-Zin preferences. According to Table 5, the EZ model increases the mean and volatility of the variance premium by a factor of three relative to the DA model, however, sample statistics are less than half of the benchmark numbers. The small variance premium is a result of the reduced volatility in the conditional variances, especially under the risk-neutral measure. For instance, the volatility of the squared VIX time series remains significantly lower relative to the data and the GDA model. Although persistent belief revisions still generate high R^2 's, a reduced variance premium results in the too-high regression coefficients. The last column in Table 6 displays the impact of relative risk aversion on the skew premium. The results for the EZ model show that the risk-neutral return density becomes more distorted towards the left tail and hence the model can generate roughly a half of mean and volatility of the skew premium observed in the data. Although the EZ framework predicts the correct size, it cannot fully capture the magnitudes. Overall, these results indicate the important role of generalized disappointment aversion in generating a correct risk neutral distribution of equity returns.

5.2.4. The Term Structure of Implied Volatilities

I further examine the asset pricing implications of all models for equity index options. The top graph of Figure 4 compares the 1-month volatility curves for the data and

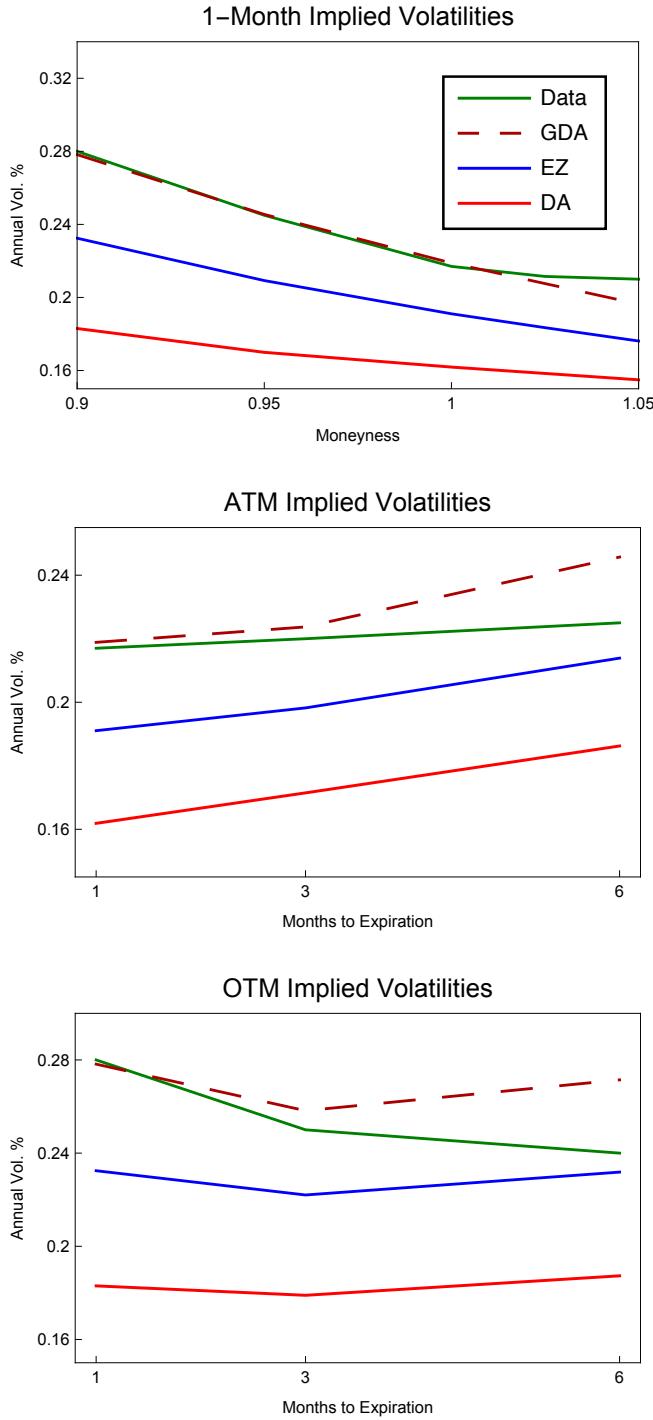


Figure 4: Benchmark Calibration and Sensitivity Analysis: Volatility Term Structure. The top panel plots the 1-month implied volatility curve as a function of moneyness (Strike/Spot Price) for the data and three models: GDA, DA, and EZ. The middle and bottom panels plot the empirical and model-based implied volatility curves for ATM and OTM options as functions of the time to maturity expressed in months. The empirical statistics are for the U.S. data from January 1996 to December 2016. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The model-based curves are calculated for option prices using the annualized model-implied interest rate $r_t(\pi_t)$ and dividend-yield $q_t(\pi_t)$ in each period. The simulation results are averages of implied volatilities based on these 10,000 artificial series.

three models. The implied volatilities are expressed as a function of moneyness ranging from 0.9 to 1.05. The plot shows that the empirical implied volatilities are declining in moneyness, a pattern also known in the literature as the volatility skew. The top panel of Figure 4 shows that the DA implied volatilities for the 1-month maturity are flat and approximately equal to the realized stock market volatility. One apparent candidate to generate a steep volatility skew is high risk aversion. Although increasing risk aversion in the EZ model improves the model performance, it cannot fully account for the level in implied volatilities. In contrast, the GDA framework can fit the option prices very closely. The middle and bottom plots of Figure 4 additionally present the term structure of implied volatilities for ATM and 0.90 OTM options. In the data, ATM volatilities slightly increase over the horizon, while a downward trend can be observed for OTM volatilities. The model-based results clearly indicate that neither DA nor EZ models can match the level of the empirical curves. In contrast, the benchmark with generalized disappointment aversion can explain overall patterns and magnitudes of the empirical ATM and 0.90 OTM volatilities for one, three, and six month maturities well.

5.3. Sensitivity Analysis

In this section, I conduct an extensive sensitivity analysis to examine the impact of alternative parameter choices in the three preference specifications on asset prices. I show that my main findings remain robust to many different calibrations of preferences. In particular, I confirm the key importance of generalized disappointment aversion in reconciling salient features of the skew and variance risk.

Figure 5 provides sensitivity results for the risk-free rate, the equity premium and equity volatility, the price-dividend ratio, and the moment risk premiums for a broad range of parameter choices in the three models. In this sensitivity exercise, I consider three preference specifications and change a key parameter in each one, while holding the remaining values the same as in the original. In the GDA model, I vary the disappointment threshold between 0.91 and 0.94. In the DA model, I change the disappointment aversion between 0.3 and 0.65. In the EZ model, the results are provided for relative risk aversion ranging from 3 to 6.5. The panels in Figure 5 present the model-based average statistics implied by the GDA (a dashed line), DA (a red line), and EZ (a blue line) frameworks. The asset pricing moments are expressed as a function of a varying parameter, which is indicated on the corresponding axis.

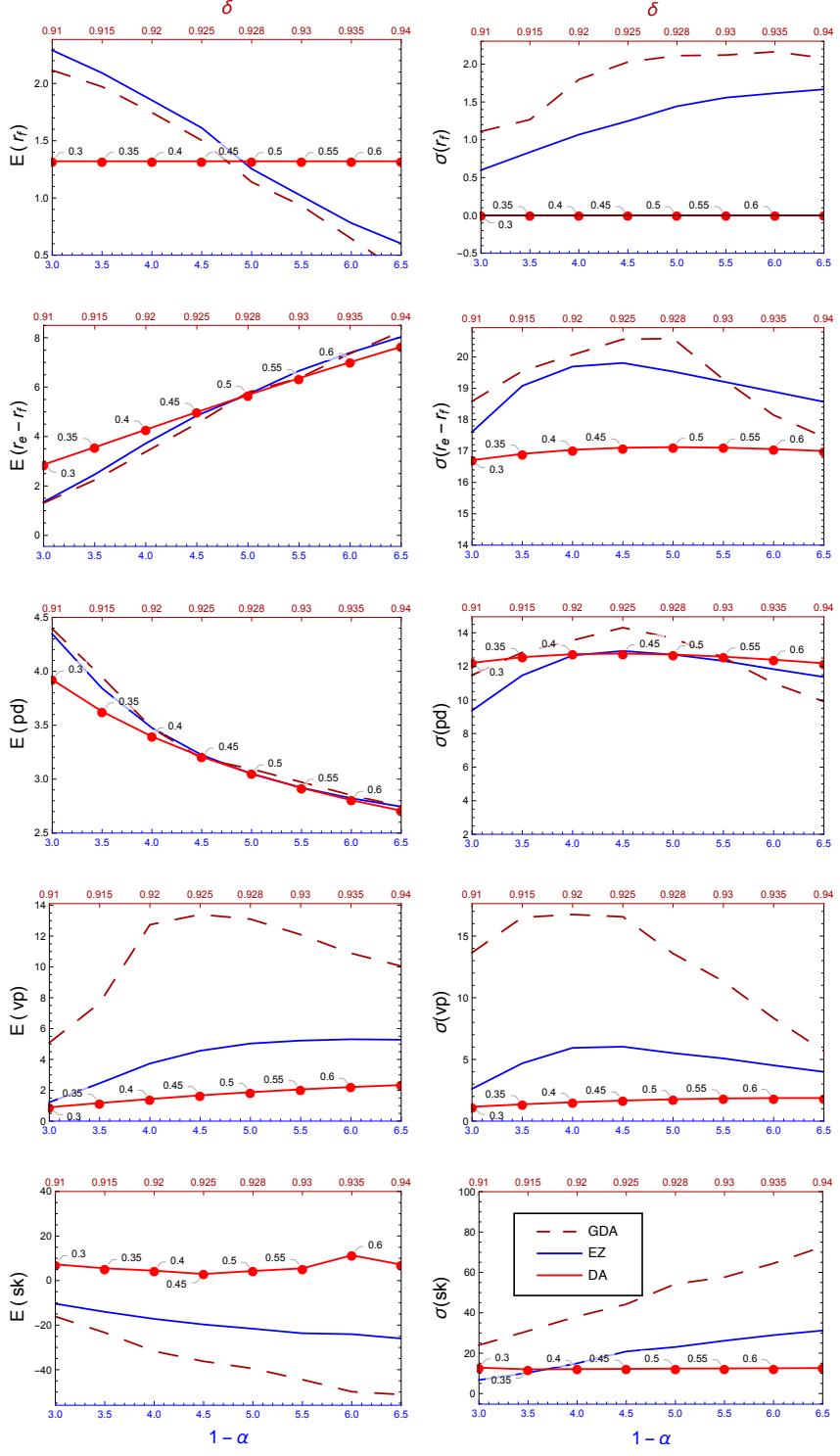


Figure 5: Sensitivity of Asset Prices to Preference Specifications and Parameter Values. The figure plots asset pricing moments in three models (GDA, DA, and EZ) with different preference calibrations. In each model, a single parameter is changed while others are fixed at original values. Specifically, I change the disappointment threshold, the disappointment aversion and the relative risk aversion over a range of values in the GDA, DA and EZ models, respectively. For each model specification, I simulate 10,000 economies at a monthly frequency. The entries of the figure are sample average statistics (annualized for the risk-free rate, the equity premium and the price-dividend ratio; monthly for variance and skew risk premiums) based on these 10,000 artificial series. I use the common notations for the average E and standard deviation σ of the series.

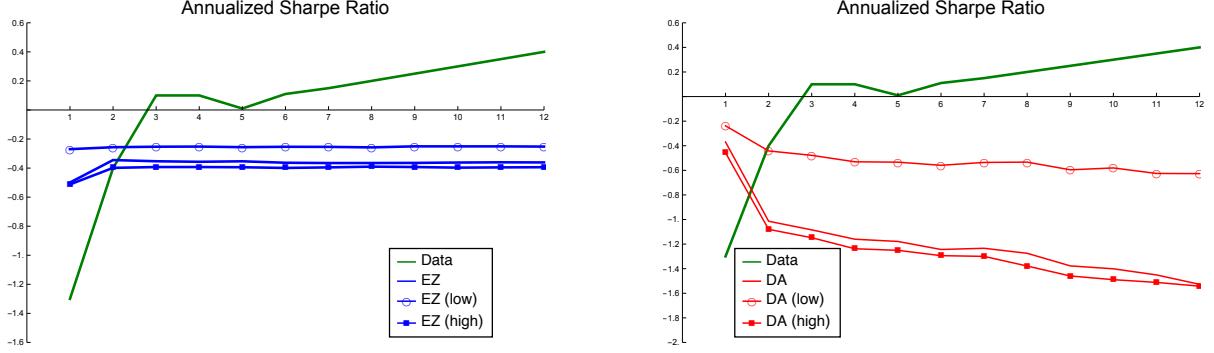


Figure 6: Sensitivity of Sharpe Ratios to Preference Specifications and Parameter Values. The left and right panels plot annualized Sharpe ratios for forward variance claims for different calibrations of the EZ and DA models, respectively. For the EZ specification, the left graph displays the results of the benchmark calibration of Epstein-Zin preferences with relative risk aversion equal to 5, and two EZ economies with lower (3) and higher (6.5) risk aversion. For the DA specification, the right graph displays the results of the benchmark calibration of DA preferences with disappointment aversion equal to 0.5, and two DA economies with lower (0.30) and higher (0.65) disappointment aversion. For each model specification, I simulate 10,000 economies at a monthly frequency. The simulation results are averages of sample moments based on these 10,000 artificial series.

Figure 5 shows that the risk-free rate decreases with the disappointment threshold and a risk aversion in the GDA and EZ models, while it is equal to a constant $-12 \ln \beta$ regardless of disappointment aversion in the DA framework. Further, the equity premium increases and equity prices decline in δ , θ , and $1 - \alpha$. Intuitively, when the agent faces more disappointing outcomes or becomes more averse to low consumption growth rates, he demands larger premiums in expected returns for bearing the additional risk in consumption growth. The impact of δ and $1 - \alpha$ on the volatility of asset prices is similar in the GDA and EZ models: a higher disappointment threshold or a higher risk aversion leads to a more volatile risk-free rate, while the volatility of equity returns and the price-dividend ratio exhibits a hump-shaped pattern with a maximum approximately in the middle of the parameter intervals considered. In the DA model, increasing the disappointment aversion slightly increases the equity volatility, while the risk-free rate remains unaffected due to linear preferences and the infinite elasticity of intertemporal substitution. Overall, the magnitude of changes in the risk-free rate, equity returns, and the price-dividend ratio are quite comparable across the three preference specifications, especially in terms of the performance of the GDA and EZ frameworks. These findings suggest that all three preference specifications can reasonably explain first and second moments of equity returns by adjusting a key preference parameter. In contrast, the four bottom panels in Figure 5 indicate the crucial importance of generalized disappointment aversion for generating significant risk premiums in higher moments of equity returns.

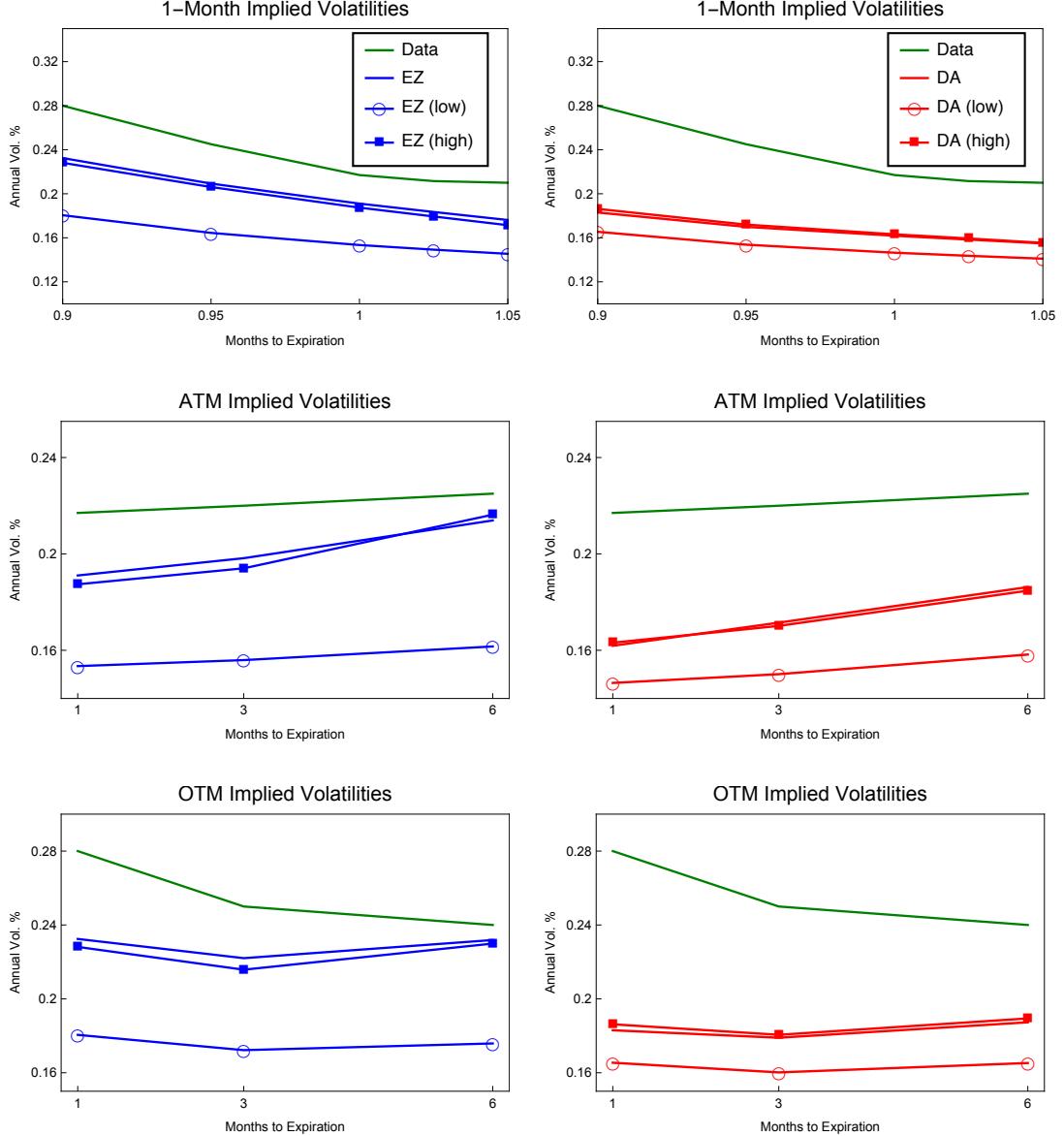


Figure 7: Sensitivity of Implied Volatilities to Preference Specifications and Parameter Values. The left and right panels in each row plot the 1-month implied volatility curve as a function of moneyness (Strike/Spot Price), implied volatility curves for ATM and OTM options as functions of the time to maturity expressed in months for different calibrations of the EZ and DA models, respectively. For the EZ specification, the left graphs display the results of the benchmark calibration of Epstein-Zin preferences with relative risk aversion equal to 5, and two EZ economies with the lower (3) and higher (6.5) risk aversion. For the DA specification, the right graphs display the results of the benchmark calibration of DA preferences with disappointment aversion equal to 0.5, and two DA economies with the lower (0.30) and higher (0.65) disappointment aversion. For each model specification, I simulate 10,000 economies at a monthly frequency. The simulation results are averages of implied volatilities based on these 10,000 artificial series.

It is evident from Figure 5 that a pure disappointment aversion model can produce a high mean and volatility of equity returns, however, the model-generated variance premium is too small and less volatile compared to the data. Moreover, disappointment aversion alone cannot reproduce salient moments of the skew premium at all. In the DA

setting, increasing the disappointment aversion parameter θ does not improve the model performance, as the variance and skew risk premium moments are not very sensitive to changes in θ . The EZ economy provides a better fit with the data. In particular, when the risk aversion increases from 3 to 5, the mean variance premium increases from less than 2 to around 5, while the skew premium declines from around -10% to -20%. However, note that the mean and volatility of the variance premium actually start to decline at some point and, thus, the higher risk aversion will move the model away from the data. The comparative analysis with respect to the disappointment threshold in GDA preferences provides overall patterns in variance and skew risk premiums similar to those generated by different risk aversion parameters in Epstein-Zin preferences. However, with generalized disappointment aversion, the magnitude of variance and skew risk premiums is significantly amplified. The sensitivity analysis in Figure 5 confirms that the distribution of the stochastic discount factor, necessary to reconcile the empirical asset pricing moments, is attributable to the agent's generalized disappointment aversion and cannot be supported by any parameter values in alternative preference specifications.

Figure 6 examines the role of the risk aversion coefficient and the disappointment aversion parameter in explaining the observed annualized Sharpe ratios for forward variance claims. The two panels in Figure 6 compare the results generated by the original EZ and DA calibrations with those predicted by the lower and higher degrees of risk aversion and disappointment aversion in the EZ and DA models. The graphs show that the magnitude of the average Sharpe ratios does not change much in response to an increase in $1 - \gamma$ or θ . The reason for this result is that further increasing $1 - \gamma$ or θ makes the pricing kernel slightly more volatile and so Sharpe ratios for the variance claims are only slightly larger. In contrast, lower $1 - \gamma$ or θ lead to a substantial reduction in the pricing kernel volatility. As a result, the Sharpe ratios become flatter and smaller in absolute values. Similarly, Figure 7 shows that the volatility surface implied by equity index options is not very sensitive to the increase in risk aversion or disappointment aversion parameters. The plots in Figure 7 show that increasing relative risk aversion to 6.5 in the EZ model or disappointment aversion to 0.65 in the DA model generates the 1-month implied volatilities and the implied volatility curves for ATM and OTM options that are very close to the results of the original preference calibrations. For smaller values of both aversion coefficient, the implied volatility curves respond more and become flatter in both cases.

6. Conclusion

This paper builds an equilibrium model in which the agent has GDA preferences and consumption growth follows a hidden Markov-switching process. I show that the combination of the investor's aversion to tail events and fluctuating economic uncertainty due to learning about a hidden state of the economy can explain a wide variety of asset pricing phenomena. The GDA model reproduces the term structure of variance risk, captures the large variance and skew risk premiums in equity returns, and generates a realistic volatility surface implied by index options, while simultaneously matching the prominent features of data. In particular, I show that the GDA calibration matches the salient moments of the fundamentals, the unconditional mean and volatility of the risk-free rate and excess equity returns, and the level of the log price-dividend ratio, while investor learning and tail aversion further generate the empirical predictability of future excess returns by the price-dividend ratio and the variance premium.

The success of the model with GDA preferences is attributable to endogenously varying uncertainty and agent tail aversion, which has a large impact on asset prices, particularly on the variance term structure, moment risk premiums, and implied volatilities. Further, in a calibrated specification of this model, the representative agent has no preference for resolution of uncertainty and exhibits generalized disappointment aversion. Therefore, he is willing to pay a high premium for unexpected realized volatility, while news about future volatility is unpriced, consistent with the data. To emphasize the importance of GDA preferences, I consider alternative models with disappointment aversion and preferences for early resolution of uncertainty. Although all three specifications can reasonably match moments of equity returns, the GDA model outperforms the other two calibrations by additionally capturing salient features of variance and skew risk premiums, variance swap and option prices. These results suggest the important role of generalized disappointment aversion and learning in asset pricing models.

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Appendix

A. Data

A1. Consumption, Dividends, and Market Returns

I follow [Bansal and Yaron \(2004\)](#) and construct the real per capita consumption growth series (annual due to the frequency restriction) for the longest sample available 1930-2016. In the literature, consumption is defined as the sum of personal consumption expenditures on nondurable goods and services. I download the data from the U.S. National Income and Product Accounts (NIPA) as provided by the Bureau of Economic Analysis. I first apply the seasonally adjusted annual quantity indexes from Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product, Quantity Indexes, A:1929-2016) to the corresponding series from Table 2.3.6. (Real Personal Consumption Expenditures by Major Type of Product, Chained Dollars, A:1995-2016) to obtain real personal consumption expenditures on nondurable goods and services for the sample period 1929-2016. I further retrieve mid-month population data from NIPA Table 7.1. to convert both real consumption series to per capita terms.

I measure the total market return as the value-weighted return including dividends, and the dividends as the sum of total dividends, on all stocks traded on the NYSE, AMEX, and NASDAQ. The dividends and value-weighted market return data are monthly and are retrieved from the Center for Research in Security Prices (CRSP). To construct the monthly nominal dividend series, I use the CRSP value-weighted returns including and excluding dividends of CRSP common stock market indexes (NYSE/AMEX/NASDAQ/ARCA), denoted by RI_t and RE_t , respectively. Following [Hodrick \(1992\)](#), I construct the price series P_t by initializing $P_0 = 1$ and iterating recursively $P_t = (1 + RI_t)P_{t-1}$. Next, I compute normalized nominal monthly dividends $D_t = (RI_t - RE_t)P_t$. The proxy of the risk-free return $R_{f,t+1}$ is the 1-month nominal Treasury bill. The nominal annualized dividends are constructed by summing the corresponding monthly dividends within the year. Finally, I retrieve the inflation index from CRSP to deflate all quantities to real values.

A2. The Variance Premium Data

I define risk premiums associated with higher moments of equity returns consistent with the existing literature. For the variance risk premium, I closely follow [Bollerslev, Tauchen and Zhou \(2009\)](#), [Bollerslev, Gibson and Zhou \(2011\)](#), [Drechsler and Yaron](#)

(2011) and Drechsler (2013), while the empirical strategy and key definitions of the skew risk premium are in line with Bakshi, Kapadia and Madan (2003) and Kozhan, Neuberger and Schneider (2013).

Under the no-arbitrage assumption, the risk-neutral conditional expectation of the return variance is equal to the price of a variance swap, a forward contract on the realized variance of the asset. Since the Chicago Board of Options Exchange (CBOE) calculates the VIX index as a measure of the 30-days ahead risk-neutral expectation of the variance of the S&P 500 index, I use the VIX index as a proxy for the risk-neutral expectation of the market's return variation. The VIX is quoted in annualized standard deviation. Hence, I first take it to a second power to transform to variance units and then divide by 12 to obtain monthly frequency. Thus, I obtain a new series defined as $[VIX]_t^2 = \frac{VIX_t^2}{12}$. I further use the last available observation of $[VIX]_t^2$ in a particular month as a measure of the risk-neutral expectation of return variance in that month.

For the objective expectation of return variance, a second component in the variance premium, I calculate a one-step-ahead forecast from a simple regression similar to Drechsler (2013). I first calculate the measure of the realized variance by summing the squared daily log returns on the S&P 500 futures and S&P 500 index obtained from the CBOE. The constructed series are denoted by FUT_t^2 and IND_t^2 , respectively. Subsequently, I estimate the following regression:

$$FUT_{t+1}^2 = \beta_0 + \beta_1 \cdot IND_t^2 + \beta_2 \cdot [VIX]_t^2 + \varepsilon_{t+1}. \quad (\text{A1})$$

The actual statistical expectation is measured by the one-period ahead forecast given by (A1). I refer to the resulting series as the realized variance and denote it by RV_t . Theoretically, the variance premium should be non-negative in each period. Thus, I truncate the difference between the implied series of $[VIX]_t^2$ and RV_t from below by 0.

For the empirical strategy above, I obtain the data series of the VIX index, S&P 500 index futures, and S&P 500 index from the CBOE. The main restriction on the length of the constructed monthly variance premium is the VIX index, reported by the CBOE only from January 1990. Using high-frequency data would provide a finer estimation precision of the realized variance, but the empirical statistics of the variance premium remain largely consistent with the existing estimates in the literature (Drechsler and Yaron, 2011; Drechsler, 2013).

A3. Option Prices

For the empirical analysis of the skew risk premium and implied volatility surface, I use European options written on the S&P 500 index and traded on the CBOE. The option data set covers the period from January 1996 to December 2016 and is from OptionMetrics. Option data elements include the type of options (call/put) along with the contract's variables (strike price, time to expiration, Greeks, Black-Scholes implied volatilities, closing spot prices of the underlying) and trading statistics (volume, open interest, closing bid and ask quotes), among other details.

The empirical estimates of the conditional skew risk premium are computed in line with the methodology of [Kozhan, Neuberger and Schneider \(2013\)](#). The empirical strategy consists of calculating fix and floating legs for the skew swap, which correspond to the risk-neutral and physical expectations of the return skewness. Please refer to [Kozhan, Neuberger and Schneider \(2013\)](#) for a detailed description. To construct the empirical implied volatility curves, I first compute the moneyness for each observed option using the daily S&P 500 index on a particular trading day. I filter out all data entries with non-standard settlements. I use the remaining observations to construct the implied volatility surface for a range of moneyness and maturities. In particular, I follow [Christoffersen and Jacobs \(2004\)](#) and perform polynomial extrapolation of volatilities in the maturity time and strike prices. This strategy makes use of all available options and not only those with a specific maturity time. The fitted values are further used to construct the implied volatility curves for 1-, 3-, and 6-month maturities.

B. Representative Agent's Maximization Problem

A representative agent starts with an initial wealth denoted by W_0 . Each period t , the agent consumes C_t consumption goods and invests in N assets traded on the competitive market. Denote the fraction of the total t -period wealth W_t invested in the i -th asset with gross real return $R_{i,t+1}$ by $\omega_{i,t}$. Then, the agent's budget constraint in period t takes the form:

$$W_{t+1} = (W_t - C_t)R_{t+1}^\omega \quad (\text{B2})$$

where

$$\sum_{i=1}^N \omega_{i,t} = 1 \quad \text{and} \quad R_{t+1}^\omega = \sum_{i=1}^N \omega_{i,t} R_{i,t+1}. \quad (\text{B3})$$

The agent chooses the allocation $\{C_t, \omega_{1,t}, \dots, \omega_{N,t}\}$ in period t in order to maximize (1) subject to (B2) and (B3).

The Bellman equation becomes:

$$J_t = \max_{C_t, \omega_{1,t}, \dots, \omega_{N,t}} \left\{ (1 - \beta) C_t^\rho + \beta [\mu_t(J_{t+1})]^\rho \right\}^{1/\rho}$$

with the constraints (B2) and (B3). I guess optimal value function of the form $J_t = \phi_t W_t$. Using this conjecture of J_t and the form of μ_t from (2), I rewrite the Bellman equation as:

$$\phi_t W_t = \max_{C_t, \omega_{1,t}, \dots, \omega_{N,t}} \left\{ (1 - \beta) C_t^\rho + \beta \left[\mathbb{E}_t \left[(\phi_{t+1} W_{t+1})^\alpha \mathcal{K}(\phi_{t+1} W_{t+1}) \right]^{\rho/\alpha} \right]^{1/\rho} \right\},$$

where

$$\mathcal{K}(x) = \frac{1 + \theta \mathbb{I}\{x \leq \delta \mu_t(x)\}}{1 + \theta \delta^\alpha \mathbb{E}_t [\mathbb{I}\{x \leq \delta \mu_t(x)\}]}. \quad (B3)$$

Note that the function \mathcal{K} defined above is homogeneous of degree zero.

The Return on the Aggregate Consumption Claim Asset. I further conjecture that the consumption C_t is homogeneous of degree one in wealth at the optimum, that is $C_t = b_t W_t$. Then, I obtain the Bellman equation:

$$\phi_t^\rho = \left\{ (1 - \beta) \left(\frac{C_t}{W_t} \right)^\rho + \beta \left(1 - \frac{C_t}{W_t} \right)^\rho \left[\mathbb{E}_t \left[(\phi_{t+1} R_{t+1}^\omega)^\alpha \mathcal{K}(\phi_{t+1} R_{t+1}^\omega) \right]^{\rho/\alpha} \right] \right\} \quad (B4)$$

or equivalently

$$\phi_t^\rho = \{(1 - \beta) b_t^\rho + \beta (1 - b_t)^\rho y_t^*\} \quad (B5)$$

where

$$y_t^* = \left[\mathbb{E}_t \left[(\phi_{t+1} R_{t+1}^\omega)^\alpha \mathcal{K}(\phi_{t+1} R_{t+1}^\omega) \right]^{\rho/\alpha} \right].$$

Taking the FOC of the right side of a simplified Bellman equation (B4) with respect to C_t , I find:

$$(1 - \beta) \left(\frac{C_t}{W_t} \right)^{\rho-1} = \beta \left(1 - \frac{C_t}{W_t} \right)^{\rho-1} y_t^*.$$

or using the notations:

$$(1 - \beta) b_t^{\rho-1} = \beta (1 - b_t)^{\rho-1} y_t^*. \quad (B6)$$

Solving for y_t^* from the last equation and substituting it into (B5), I deduce:

$$\phi_t = (1 - \beta)^{\frac{1}{\rho}} b_t^{\frac{\rho-1}{\rho}} = (1 - \beta)^{\frac{1}{\rho}} \left(\frac{C_t}{W_t} \right)^{\frac{\rho-1}{\rho}}$$

Shifting one period ahead the formula for ϕ_t and substituting the resulting form of ϕ_{t+1} into (B6), I obtain:

$$(1 - \beta)C_t^{\rho-1} = \beta(W_t - C_t)^{\rho-1} \left[\mathbb{E}_t \left[(1 - \beta)^{\alpha/\rho} \left(\frac{C_{t+1}}{W_{t+1}} \right)^{\alpha \frac{\rho-1}{\rho}} (R_{t+1}^\omega)^\alpha \mathcal{K}(\phi_{t+1} R_{t+1}^\omega) \right] \right]^{\rho/\alpha}.$$

Then, I rewrite the equation above as:

$$C_t^{\rho-1} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{\frac{W_{t+1}}{(W_t - C_t)}} \right)^{\alpha \frac{\rho-1}{\rho}} (R_{t+1}^\omega)^\alpha \mathcal{K} \left(\left(\frac{C_{t+1}}{\frac{W_{t+1}}{W_t - C_t}} \right)^{\frac{\rho-1}{\rho}} R_{t+1}^\omega \right) \right]^{\rho/\alpha}.$$

and derive the asset pricing restriction for the return on the total wealth R_{t+1}^ω :

$$\mathbb{E}_t \left[\left\{ \underbrace{\left(\beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1} R_{t+1}^\omega \right)^{1/\rho}}_{z_{t+1}} \right\}^\alpha \mathcal{K} \left(\underbrace{\left(\beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1} R_{t+1}^\omega \right)^{1/\rho}}_{z_{t+1}} \right) \right]^{1/\alpha} = 1.$$

Define R_{t+1}^c the return on the consumption endowment. In equilibrium, $R_{t+1}^c = R_{t+1}^\omega$ and, as in Routledge and Zin (2010), using the definition of the certainty equivalent (2) and the function \mathcal{K} , the return R_{t+1}^c should satisfy the equation:

$$\mu_t(z_{t+1}) = 1 \quad (\text{B7})$$

where

$$z_{t+1} = \left(\beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1} R_{t+1}^c \right)^{1/\rho}.$$

Rewriting R_{t+1}^c in the form:

$$R_{t+1}^c = \frac{W_{t+1}}{W_t - C_t} = \frac{\frac{W_{t+1}}{C_{t+1}}}{\frac{W_t}{C_t} - 1} \cdot \frac{C_{t+1}}{C_t} = \frac{\xi_{t+1}}{\xi_t - 1} \cdot \frac{C_{t+1}}{C_t},$$

the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ can be found from the functional equation:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^\alpha \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}} \cdot \mathcal{K}(z_{t+1}) \right] = 1.$$

The Return on the Aggregate Dividend Asset. Following Routledge and Zin (2010), the portfolio problem for the obtained values ϕ_{t+1} reads as follows:

$$\max_{\omega_{1,t}, \dots, \omega_{N,t}} \mu_t(\phi_{t+1} R_{t+1}^\omega),$$

subject to the constraints $\sum_{i=1}^N \omega_{i,t} = 1$ and $R_{t+1}^\omega = \sum_{i=1}^N \omega_{i,t} R_{i,t+1}$. Taking the FOC with respect to the weight $\omega_{i,t}$, I derive:

$$\mathbb{E}_t [\phi_{t+1}^\alpha (R_{t+1}^\omega)^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)] R_{i,t+1}] = 0.$$

Taking the difference between the i -th and j -th FOCs, I thus obtain:

$$\mathbb{E}_t [\phi_{t+1}^\alpha (R_{t+1}^\omega)^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)] (R_{i,t+1} - R_{j,t+1})] = 0.$$

Multiplying the last equation by $\omega_{j,t}$ and summing over all possible values of j , I further obtain:

$$\begin{aligned} & \mathbb{E}_t \left[\phi_{t+1}^\alpha (R_{t+1}^\omega)^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)] R_{i,t+1} \underbrace{\sum_{j=1}^N \omega_{j,t}}_{=1} \right] = \\ &= \mathbb{E}_t \left[\phi_{t+1}^\alpha (R_{t+1}^\omega)^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)] \underbrace{\sum_{j=1}^N R_{j,t+1} \omega_{j,t}}_{=R_{t+1}^\omega} \right] \end{aligned}$$

or

$$\begin{aligned} & \mathbb{E}_t [\phi_{t+1}^\alpha (R_{t+1}^\omega)^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)] R_{i,t+1}] = \\ &= \mathbb{E}_t [\phi_{t+1}^\alpha (R_{t+1}^\omega)^\alpha [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)]]. \end{aligned} \quad (\text{B8})$$

Following Epstein and Zin (1989, 1991), it is straightforward to show that $\phi_{t+1} = \frac{z_{t+1}}{R_{t+1}^\omega}$ holds in equilibrium. Using these equilibrium conditions and the definition of μ_t , I have:

$$\begin{aligned} & \mathbb{E}_t [\phi_{t+1}^\alpha (R_{t+1}^\omega)^\alpha [1 + \theta \mathbb{I}(\phi_{t+1} R_{t+1}^\omega < \delta \mu_t)]] = \mathbb{E}_t [z_{t+1}^\alpha [1 + \theta \mathbb{I}(z_{t+1} < \delta \mu_t)]] = \\ & \mathbb{E}_t \left[1 + \theta \delta^\alpha \mathbb{I}(z_{t+1} < \delta \underbrace{\mu_t(z_{t+1})}_{=1}) \right] \underbrace{\mu_t(z_{t+1})^\alpha}_{=1} = \mathbb{E}_t [1 + \theta \delta^\alpha \mathbb{I}(z_{t+1} < \delta)]. \end{aligned} \quad (\text{B9})$$

Combining (B8)-(B9) and using the equilibrium condition $R_{t+1}^c = R_{t+1}^\omega$, I finally obtain the asset pricing restriction for the gross return $R_{i,t+1}$:

$$\mathbb{E}_t \left[\frac{z_{t+1}^\alpha (R_{t+1}^c)^{-1} (1 + \theta \mathbb{I}(z_{t+1} < \delta)) R_{i,t+1}}{1 + \theta \delta^\alpha \mathbb{E}_t [\mathbb{I}(z_{t+1} < \delta)]} \right] = 1, \quad (\text{B10})$$

Moreover, the pricing kernel M_{t+1} is:

$$M_{t+1} = \frac{z_{t+1}^\alpha (R_{t+1}^c)^{-1} (1 + \theta \mathbb{I}(z_{t+1} < \delta))}{1 + \theta \delta^\alpha \mathbb{E} [\mathbb{I}(z_{t+1} < \delta)]}.$$

Rewriting $R_{i,t+1}$ in the form:

$$R_{i,t+1} = \frac{P_{i,t+1} + D_{i,t+1}}{P_{i,t}} = \frac{\frac{P_{i,t+1}}{D_{i,t+1}} + 1}{\frac{P_{i,t}}{D_{i,t}}} \cdot \frac{D_{i,t+1}}{D_{i,t}} = \frac{\lambda_{t+1} + 1}{\lambda_t} \cdot \frac{D_{i,t+1}}{D_{i,t}},$$

the price-dividend ratio of the i -th asset $\lambda_t = \frac{P_{i,t}}{D_{i,t}}$ can be found from the functional equation:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\alpha-1} \frac{D_{i,t+1}}{D_{i,t}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{K}(z_{t+1}) \cdot (\lambda_{t+1} + 1) \right] = \lambda_t.$$

C. Numerical Algorithm

This technical appendix provides the description of the numerical method used to solve the model. Following the notation from the paper, aggregate consumption growth

$$\Delta c_{t+1} = \ln \left(\frac{C_{t+1}}{C_t} \right)$$

is given by:

$$\Delta c_{t+1} = \mu_{s_{t+1}} + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1).$$

Whereas the consumption volatility σ is assumed to be constant, the mean growth rate $\mu_{s_{t+1}}$ is driven by a two-state Markov-switching process s_{t+1} with the state space:

$$\mathcal{S} = \{1 = \text{expansion}, 2 = \text{recession}\},$$

a transition matrix

$$\mathcal{P} = \begin{pmatrix} \pi_{11} & 1 - \pi_{11} \\ 1 - \pi_{22} & \pi_{22} \end{pmatrix}$$

and transition probabilities $\pi_{ii} \in (0, 1)$. Let

$$\mathcal{X}(y_1, y_2, y_3) = \frac{1 + \theta \mathbb{I} \left\{ \beta e^{\rho y_1} \left(\frac{y_2}{y_3 - 1} \right) \leq \delta^\rho \right\}}{1 + \theta \delta^\alpha \mathbb{E}_t \left[\mathbb{I} \left\{ \beta e^{\rho y_1} \left(\frac{y_2}{y_3 - 1} \right) \leq \delta^\rho \right\} \right]},$$

then, the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ satisfies the equation:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}(\Delta c_{t+1}, \xi_{t+1}, \xi_t) \right] = 1, \quad (\text{C1})$$

and the price-dividend ratio $\lambda_t = \frac{P_t}{D_t}$ of the asset with gross return R_{t+1} (I skip the subscript i for convenience) is given by:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1} + \Delta d_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}(\Delta c_{t+1}, \xi_{t+1}, \xi_t) \cdot \frac{\lambda_{t+1} + 1}{\lambda_t} \right] = 1. \quad (\text{C2})$$

I apply the projection method of Judd (1992) to solve for the equilibrium pricing functions when the state of the economy is unobservable.

The Return on the Aggregate Consumption Claim Asset. I conjecture the wealth-consumption ratio of the form $\xi_t = G(\pi_t)$, where π_t is the posterior belief defined by (4). I seek to approximate the functional form of $G(\pi_t)$, which solves (C1). I approximate $G(\pi_t)$ by a basis of complete Chebyshev polynomials $\Psi = \{\Psi_k(\pi_t)\}_{k=0}^n$ of order n with coefficients $\psi = \{\psi_k\}_{k=0}^n$:

$$G(\pi_t) = \sum_{k=0}^n \psi_k \Psi_k(\pi_t) \quad \forall \pi_t \in [0, 1]^5. \quad (\text{C3})$$

I further define the function:

$$\begin{aligned} \Gamma(\pi_t; j) &= \mathbb{E}_{t,j} \left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}(\Delta c_{t+1}, \xi_{t+1}, \xi_t) \right] = \\ &= \beta^{\frac{\alpha}{\rho}} \int e^{\alpha y} \left(\frac{G(B(y, \pi_t))}{G(\pi_t) - 1} \right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}(y, G(B(y, \pi_t)), G(\pi_t)) f(y, j) dy \end{aligned} \quad (\text{C4})$$

where $B(y, \pi_t)$ is given by:

$$B(y, \pi_t) = \frac{(1-q)f(y, 1)(1-\pi_t) + pf(y, 2)\pi_t}{f(y, 1)(1-\pi_t) + f(y, 2)\pi_t}$$

and $f(y, j)$ is the probability density function of a normal distribution $N(\mu_{S_t}, \sigma^2)$ conditional on $S_t = j$. Substituting $G(\pi_t)$ from (C3) and $\Gamma(\pi_t; j)$ from (C4) into (C1), I obtain the residual function:

$$R^c(\pi_t; \psi) = (1 - \pi_t)\Gamma(\pi_t, 1) + \pi_t\Gamma(\pi_t, 2) - 1.$$

The objective is to choose the unknown coefficients ψ in order to make the residual function $R^c(\pi_t; \psi)$ close to zero $\forall \pi_t \in [0, 1]$. I apply the orthogonal collocation method. Formally, I evaluate the residual function in the collocation points $\{r_k\}_{k=1}^{n+1}$ given by the roots of the $n + 1$ order Chebyshev polynomial ⁶ and then solve the system of $n + 1$ equations:

$$R^c(r_k; \psi) = 0 \quad \forall k = 1, \dots, n + 1$$

for $n + 1$ unknowns $\psi = \{\psi_k\}_{k=0}^n$. Let $\tilde{\xi}_t = \tilde{G}(\pi_t) = \sum_{k=0}^n \tilde{\psi}_k \Psi_k(\pi_t)$ denote an approximation of the wealth-consumption ratio.

⁵I adjust the domain of the Chebyshev polynomials from $[-1, 1]$ to the domain of the state variable π_t which is $[0, 1]$.

⁶Again, I adjust the domain of the Chebyshev polynomials from $[-1, 1]$ to the domain of the state variable π_t which is $[0, 1]$.

The Return on the Aggregate Dividend Asset. I conjecture the price-dividend ratio of the form $\lambda_t = H(\pi_t)$, where π_t is the posterior belief defined by (4). Now, I seek to approximate the functional form of $H(\pi_t)$, which solves the equation (C2). I approximate $H(\pi_t)$ by a basis of complete Chebyshev polynomials $\Upsilon = \{\Upsilon_k(\pi_t)\}_{k=0}^n$ of order n with coefficients $v = \{v_k\}_{k=0}^n$:

$$H(\pi_t) = \sum_{k=0}^n v_k \Upsilon_k(\pi_t) \quad \forall \pi_t \in [0, 1]. \quad (\text{C5})$$

Define the function:

$$\begin{aligned} \Lambda(\pi_t; j) &= \mathbb{E}_{t,j} \left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1} + \Delta d_{t+1}} \left(\frac{\tilde{\xi}_{t+1}}{\tilde{\xi}_t - 1} \right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\left(\Delta c_{t+1}, \tilde{\xi}_{t+1}, \tilde{\xi}_t\right) \cdot \frac{\lambda_{t+1} + 1}{\lambda_t} \right] = \\ &= \beta^{\frac{\alpha}{\rho}} \iint e^{(\alpha+\lambda-1)y+g_d+z} \left(\frac{\tilde{G}(B(y, \pi_t))}{\tilde{G}(\pi_t) - 1} \right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\left(y, \tilde{G}(B(y, \pi_t), \tilde{G}(\pi_t)\right) \cdot \\ &\quad \cdot \frac{H(B(y, \pi_t))}{H(\pi_t) - 1} f(y, j) g(z, j) dy dz, \end{aligned} \quad (\text{C6})$$

where $f(y, j)$ and $g(z, j)$ are probability density functions of normal distributions $N(\mu_{S_{t+1}}, \sigma)$ and $N(g_d, \sigma_d)$, respectively, conditional on $S_{t+1} = j$. Substituting $H(\pi_t)$ from (C5) and $\Lambda(\pi_t; j)$ from (C6) into (C2), I obtain the residual function:

$$R^d(\pi_t; v) = (1 - \pi_t)\Lambda(\pi_t, 1) + \pi_t\Lambda(\pi_t, 2) - 1.$$

Next, I evaluate the residual function $R^d(\pi_t; \psi)$ in the collocation points $\{s_k\}_{k=1}^{n+1}$ given by the roots of the $n+1$ order Chebyshev polynomial and solve the system of $n+1$ equations

$$R^d(s_k; v) = 0 \quad \forall k = 1, \dots, n+1$$

for $n+1$ unknowns $v = \{v_k\}_{k=0}^n$.