The Sources of Financing Constraints *

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November 30, 2018

Abstract

In order to identify the relevant sources of firms' financing constraints, we ask what financial frictions matter for corporate policies. To that end, we build, solve, and estimate a range of dynamic models of corporate investment and financing, embedding a host of financial frictions. We focus on limited enforcement, moral hazard, and tradeoff models. All models share a common technology, but differ in the friction generating financing constraints. Using panel data on Compustat firms for the period 1980-2015 and a more recent dataset on private firms from Orbis, we determine which features of the observed data allow to distinguish among the models, and we assess which model performs best at rationalizing observed corporate investment and financing policies across various samples. Our tests, based on empirical policy function benchmarks, favor trade-off models for larger Compustat firms, limited commitment models for smaller firms, and moral hazard models for private firms. Our estimates point to significant financing constraints due to agency frictions.

Keywords: Financing constraints, financial frictions, moral hazard, limited enforcement, trade-off, dynamic contracting, agency, structural estimation, empirical policy function estimation

JEL Classification Numbers: G31, G32.

*We would like to thank Toni Whited for encouraging us to write this paper and conference participants at the American Economic Association Meetings, European Finance Association Meetings, Western Finance Association Meetings, NBER SI Capital Markets, Swiss Finance Institute Research Meetings, Lancaster University Corporate Finance Workshop, CEPR Third Annual Spring Symposium in Financial Economics, seminar participants at BI Business School, Erasmus University Rotterdam, University of Illinois, University of Toronto, University of Zurich, as well as Christopher Hennessy, Alex Karaivanov, Vincenzo Quadrini, Jean-Charles Rochet, Stephane Verani, Neng Wang, Yufeng Wu, and Josef Zechner for helpful discussions.

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1. Introduction

Corporate finance revolves around the study of financing constraints. Indeed, as pointed out forcefully by Modigliani and Miller, in the absence of frictions restricting firms' access to external financing, corporations' financing decisions are irrelevant to their valuations and real policies. But what are the sources of financing constraints? What are these frictions that limit firms' access to external financing? While there is little disagreement on their relevance, their nature is much more debated, and a host of theoretical models have been proposed to rationalize observed firm financing policies. Are firms financially constrained because tax advantages make it attractive to firms to issue loans that they might default on after adverse shocks, as in a trade-off model? Is it because firms cannot commit to honor their debt obligations, as in a limited commitment model? Or is it because firms might misreport their effective performance to lenders and divert cash flows, as moral hazard models have suggested? Moreover, are these frictions equally important across firms, if at all?

In this paper, we propose to take a step towards providing quantitative guidance regarding the sources of financing constraints by empirically evaluating a host of dynamic financing models proposed in the literature by means of structural estimation, across a variety of different data samples, and assessing their relative fit. In every estimation, we ask: Which of the proposed models provides the best description of the actual behavior of a given set of companies, if any? Do our data allow us to discriminate between the relevance of these models for a particular set of firms?

We start by laying out in a unified environment a triplet of models of dynamic firm financing that have received attention in both empirical and theoretical literature. The first is a standard trade-off model, similar to Hennessy and Whited (2007), in which tax advantages of debt encourage firms to issue defaultable debt. The relevance of tax considerations for the determination of firms' capital structures has long been highlighted in the literature. More recently, in contrast, the theoretical literature has emphasized the role of financial contracting in determining firm policies and dynamics.[1] Dynamic financial contracts arise to mitigate agency conflicts between firms' insiders and outsiders and affect corporations' financial structures, investment policies, and valuations. We

[1] See e.g. the surveys of Harris and Raviv (1991) and Zingales (2000).

account for these developments by considering, first, a model in which an optimal lending contract between lenders and shareholders determines firm financing when the latter cannot commit to honor their obligations, similar to Albuquerque and Hopenhayn (2004), or more recently in Rampini and Viswanathan (2010), and Rampini and Viswanathan (2013), and second, a model in which firms' access to external financing is curtailed by moral hazard in the presence of asymmetric information about cash flows, so that shareholders can divert cash flows from lenders, similar to Clementi and Hopenhayn (2006), Quadrini (2004), or Verani (2016). In contrast to tax-based tradeoff models, the latter dynamic agency models emphasize the state-contingent nature of financial instruments as important features of optimal dynamic contracts.

Our approach relies on recent advances in computation and estimation. On the computational side, the linear programming approach to dynamic programming, introduced by Trick and Zin (1993) and recently extended in the context of dynamic corporate finance models in Nikolov, Schmid, and Steri (2018), enables us to efficiently solve a large number of dynamic models of firm financing that are computationally challenging because of the high-dimensionality of the set of choice variables. Regarding estimation, we adopt a novel approach to structural estimation, empirical policy function estimation, introduced by Bazdresch, Kahn, and Whited (2016), that allows to trace out firms' dynamic behavior[2] by providing the empirical mapping between a corporation's actions (policies) and current characteristics (states). Since ultimately the main predictive restriction of a model is a set of policy functions dynamically linking choices and state variables, these empirical mappings constitute a natural benchmark for model evaluation and comparison. Formally, our estimator picks model parameters that minimize the distance between policy functions recovered from model simulations and the empirical benchmarks. This procedure allows to test the empirical relevance of a set of model implied policy functions and has excellent power to detect misspecification, as emphasized in Bazdresch, Kahn, and Whited (2016).

Our strategy is to evaluate all of these models by estimating their policy functions on a variety of samples and assess their relative fit with respect to the relevant empirical benchmarks. This approach exploits and emphasizes the relevance of firm heterogeneity in the data. To that end, we consider full samples and subsamples drawn from both the standard Compustat universe, as

[2]Gala and Gomes (2016) offer a related approach.

well as a more recent dataset on US private firms from Orbis. Our data thus allow us to evaluate model fit across small and large firms, public and private firms, profitable and unprofitable firms, among others. Our implementation is based on empirical mappings between investment, leverage and payout (policies), and size, profitability, and leverage (states), respectively.

We statistically compare the fit of the competing models by means of statistical tests built on Rivers and Vuong (2002). We test the null hypothesis that a pair of models is statistically indistinguishable relative to the data, versus the two alternatives that one model fits the data better than the other. As the Vuong (1989) test for model comparison for maximum likelihood estimations, our tests do not require any specific assumptions to nest the three models. Thus, they are designed to compare non-nested models as the tradeoff, limited enforcement and dynamic agency models without the need of embedding our candidate frictions in a single specification.[3]

Our estimation results favor trade-off models as best representations of the dynamic behavior of large public firms. Key to this empirical result is the observation that the empirical investment benchmark is largely unresponsive to leverage, while the leverage benchmark responds positively to profitability in this sample. This is in line with a trade-off model in which the desire to shield profits from taxes is the main driver of leverage, rather than funding investment expenditures. In this sense, our empirical approach suggests that large firms' capital structure decisions are predominantly revolving around profitability.

When it comes to smaller public firms, we find that limited commitment models provide the most adequate descriptions of corporate behavior. This result reflects that in the empirical benchmarks these firms' leverage responds negatively to profitability, while investment does so positively. This is readily rationalized in a limited commitment model, in which enforcement constraints limit a firms' leverage, and thus investment and growth, while a windfall profitability shock reduces firms' dependance on external financing and spurs growth. Accordingly, our empirical approach highlights the relevance of growth opportunities as drivers of these firms' financing policies.

Intriguingly, our estimation procedure strongly favors moral hazard specifications over alternative candidate models in the case of private firms. In our dataset, private firms tend to be smaller

[3] Paulson, Townsend, and Karaivanov (2006), and Karaivanov and Townsend (2014) follow a similar approach. They estimate non-nested models by maximum likelihood and use the Vuong (1989) test to discriminate among them.

and less profitable than public firms, they tend to invest less, and be significantly more levered. Our results suggests that cash flow diversion is highly relevant for this class of firms. Realistically, cash flow diversion is likely to be interpreted not narrowly as outright stealing of profits, but more broadly perhaps as conflicts of interest about the proper use of funds in firms that are less transparent and lacking the scrutiny of the public spotlight. One way to interpret our results then is that leverage arises as an effective device to discipline such conflicts. Key to our empirical result here is the observation that in this sample the empirical benchmark for investment responds negatively to leverage, in that highly levered firms invest less. This is consistent with a moral hazard model in which firms can invest only if insiders' interests are sufficiently aligned with those of financiers, so that insiders' incentives for cash flow diversion are minimized. This is achieved by giving them 'skin in the game', that is more equity, and thus, all else equal, less leverage. In other words, agency emerges as the main determinant of private firms' financing policies.

Remarkably, when evaluated across the entire Compustat universe, our estimations favor trade-off models. This suggests that a trade-off model provides an adequate description of the dynamic behavior of an 'average' firm in Compustat. Our results thus provide guidance to researchers looking for a 'one size fits it all' model of capital structure, for the purpose of a macroeconomic model. Such a model can be useful to evaluate the aggregate implications of debt financing, for example. Our estimations thus single out a trade-off model as a good starting point in this regard.

Our structural estimation procedure also yields point estimates of the relevant parameters across models, and thus also allow us to gauge the magnitudes of the financial frictions and agency costs necessary to rationalize observed firm behavior, both in the full sample as well as across subsamples. Reassuringly, we find that the estimates of the technological parameters are remarkably similar and consistent across model specifications, so that differences in fit can be plausibly attributed almost exclusively to their financing behavior, allowing us to identify the relevant sources of financing constraints. The parameter estimates of particular concern, therefore, relate to the unobserved agency conflicts that drive financial structure and investment, such as the degree of cash flow diversion in moral hazard models, the degree of informational asymmetries, and the amount of capital firms can abscond with in limited commitment models. Regarding cash flow diversion, our results indicate that in order to rationalize observed corporate policies, firm owners need to be able to divert around 13 cents on the dollar of profits, for private firms. Moreover, the contribution of unobservable shocks to total volatility is estimated to be substantial for these firms. On the other hand, consistent with earlier results in Nikolov, Schmid, and Steri (2018), firms can collateralize around 60 percent of their assets in limited commitment models, for smaller public firms.

Finally, we use our estimation results to infer the costs of financing constraints across models and samples. We do this by comparing firm valuations from models in which financing is constrained by one of our candidate frictions, to counterfactual specifications in which external financing is i) entirely unconstrained in that firms can costlessly access equity markets, and ii) maximally constrained in that no external financing is available and all expenditures need to be financed using internal funds, so that firms are effectively in financial autarky. The relevant costs then are embedded in the relative valuations. We find costs ranging from 20 to 30 percent across samples. Our estimates thus point to significant financing constraints due to financial frictions.

1.1. Related literature

Our paper belongs to a small, but growing literature which tries to estimate, or quantitatively evaluate, the empirical implications of the literature on dynamic agency conflicts across a variety of economic environments. In our attempt to distinguish and discriminate across different models of dynamic financing constraints, we are inspired by Karaivanov and Townsend (2014) and Karaivanov and Wright (2011), who use numerical techniques and estimation methods to distinguish among different sources of financing constraints that inhibit risk sharing among households in Thai villages, and international capital flows, respectively. Our work introduces this agenda in the context of the literature of dynamic contracting and dynamic corporate finance.

A number of recent papers have used structural estimation to evaluate models based on a single source of agency conflicts for firm dynamics and financing. For example, Li, Whited, and Wu (2016) investigate a dynamic limited commitment model to gauge the relative importance of tax advantages and agency conflicts for firm financing. Relatedly, Ai, Kiku, and Li (2016) estimate a dynamic moral hazard model in general equilibrium to assess the severity of the agency conflicts arising from effort provision for the real economy. We differ from these papers by our focus on model comparison, and estimation technique. Verani (2016) presents and estimates a model combining moral hazard and limited commitment and estimates it on macroeconomic data from Colombia, while we focus on firm-level panel data.

In our implementation of a limited commitment model, we follow the work of Albuquerque and Hopenhayn (2004), and especially Rampini and Viswanathan (2010, 2013), Zhang (2016), and Sun and Zhang (2016). Our approach is closest to Nikolov, Schmid, and Steri (2018), which emphasizes the implementation of a limited commitment model with state-contingent debt with a mixture of real world securities such as straight loans and credit lines. Bolton, Wang, and Yang (2018) present a tractable continuous-time approach in the context of non-alienability of human capital.

Our implementation of a discrete time moral hazard model follows the work of Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007b), DeMarzo and Fishman (2007a), and especially, Quadrini (2004), who quantitatively examines a dynamic moral hazard model when shocks can be persistent. Doepke and Townsend (2006) develop computational techniques to deal with the challenging case when persistent shocks are privately observed only. In contrast, we assume that persistent shocks are publicly observable. For the case with privately observed persistent shocks, Fu and Krishna (2016) develop an implementation of the corresponding cash flows by means of real world securities. DeMarzo, Fishman, He, and Wang (2012) and Gryglewicz, Mayer, and Morellec (2018) develop tractable continuous-time models with moral hazard and investment, while Albuquerque and Schroth (2010) and Wang and Wu (2018) estimate private benefits of control by means of a structural approach.

Regarding structural estimation and quantitative evaluation in dynamic corporate finance, our work more broadly builds on a growing and maturing literature that estimates and gauges the magnitude of financial frictions, as in Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005, 2007), Taylor (2010, 2013), Korteweg (2010), Nikolov and Whited (2014), Glover (2015), Morellec, Nikolov, and Schürhoff (2012, 2018), Wu (2017).

2. A triplet of models

In this section, we present the models that we take to the data and attempt to empirically evaluate and compare. The models themselves are fairly standard and have been widely used in the literature to address a variety of questions in corporate finance, growth and development, among others. To facilitate comparison, we present them in a unified setup that emphasizes similarities, and readily allows to identify differences. In all the models, financing constraints ultimately emerge from limited liability, which preclude straight equity financing. Critically, however, their precise nature and the amount and form of external financing differs across models and depends on the particular financial friction assumed.

The models are i) a standard trade-off model where tax advantages encourages financing with defaultable debt which is limited by rising credit spreads, ii) a limited commitment model which retains the tax advantages of debt financing, but allows for a more flexible implementation of debt structure by means of state-contingent debt repayment schedules, which need to be secured by collateralizable assets, as well as iii) a model where external financing from lenders is limited by asymmetric information, in that lenders do not observe shock realizations and financial contracts thus need to be structured such that borrowing firms are induced to revealing the truth in the presence of moral hazard.

We keep the specification of technology identical across models, so that differences in observed policies stem exclusively from different financial frictions. We outline the technology first, and then describe in detail the different sources of frictions. Perhaps slightly deviating from the standard formulations of the models familiar from the literature, we cast them as firm rather than equity value maximization problems, to facilitate comparison.

Our empirical approach thus compares how different models each based on a candidate friction fit the data. In other words, we compare a set of non-nested models. An alternative approach would entail embedding our candidate frictions in one specification, so that they would appear as nested in one single model, and then let the data inform us about the relative strength of the frictions for each sample. We prefer our approach for a variety of reasons. First, some of our models, in line with the literature, feature exogenously incomplete markets (trade-off), while others feature endogenously incomplete markets (limited enforcement and moral hazard). Similarly, some (moral hazard) feature asymmetric information, while the others do not. Conceptually, therefore, these frictions are challenging to nest. Second, an all-encompassing model with nested frictions would necessarily be computationally challenging and rather intractable. In contrast, our approach not just allows quantifying the frictions at work, but also to give guidance to researchers looking for a model to describe a particular group of firms. Third, the availability of statistical tests to assess the relative fit of the non-nested models to their empirical benchmarks allows to bypass the conceptual challenges related to the existence of an all-encompassing model.

2.1. Technology and Investment

We consider the problem of value-maximizing firms in a perfectly competitive environment. Time is discrete. We assume that all agents are risk-neutral, so that the one period interest rate r is constant.

After-tax operating profits for firm i in period t depend upon the capital stock k_{it} and shocks z_{it} and η_{it}, respectively, and are given by

$$\pi(k_{it}, z_{it}, \eta_{it}) = (1-\tau)((z_{it} + \eta_{it})k_{it}^{\alpha} - f), \tag{1}$$

where $0 < \tau < 1$ denotes the corporate tax rate, $0 < \alpha < 1$ is the capital share in production, and $f > 0$ is a fixed cost incurred in the production process. Note that a capital share less than unity captures decreasing returns to scale. The variable z_{it} reflects shocks to demand, input prices, or productivity and follows a stochastic process with bounded support $Z = [\underline{z}, \overline{z}]$, with $-\infty < \underline{z} < \overline{z} < \infty$, and described by a transition function $Q_z(z_{it}, z_{it+1})$[4]. Finally, η_{it} is an iid disturbance, which takes values η with probability κ and $-\eta$ with probability $1-\kappa$. The shock η_{it} only plays a major role in the context of dynamic moral hazard, in which it allows us to introduce asymmetric information in a tractable manner.

At the beginning of each period the firm is allowed to scale its operations by choosing its next period capital stock k_{it+1}. This is accomplished through investment i_{it}, which is defined by the standard capital accumulation rule

$$k_{it+1} = k_{it}(1-\delta) + i_{it}, \tag{2}$$

[4]In our empirical work, we parameterize z_{it} so as to provide a discrete approximation to a continuous AR(1) process with persistence ρ_z and conditional volatility σ_z.

where $0 < \delta < 1$ is the depreciation rate of capital. Given our modeling of corporate taxation, we account for a depreciation tax allowance in the form of $\tau\delta k_{it}$.

Investment is subject to capital adjustment costs. As in Bolton, Chen, and Wang (2011), for example, we follow the neoclassical literature (Hayashi, 1982) and consider convex adjustment costs for simplicity. We parameterize capital adjustment costs with the functional form

$$\Psi(k_{it+1}, k_{it}) \equiv \frac{1}{2}\psi\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}, \tag{3}$$

where the parameter ψ governs the severity of the adjustment cost.

Timing In settings that accommodate asymmetric information and moral hazard, some care must be taken with respect to the timing of decisions. While less critical in the other specifications, for the sake of comparison, we adopt the same timing across candidate models. In particular, for a set of endogenous state variables s and persistent shocks z, we will refer to $W(s_{it-1}, z_{it-1})$ as the firm values at the end of period $t-1$, that is after the realization of all the $t-1$ shocks and production, and *before* that of the time t shocks. The state variable z_{it-1} is informative about the conditional distribution of z_{it} in period t, which affects the expected returns to capital. For tractability, we assume that decisions are taken at the end of period $t-1$ about financing and investment expenditures at the beginning of period t. The latter entail adjustment costs and leaves them with a depreciated capital stock after production.

2.2. Trade-off

Our first model is a standard trade-off model in which firms aim at exploiting the tax advantage of debt financing available in the US tax code, similar to e.g. Hennessy and Whited (2007). In this setup, η_{it} is public information.

Financing At time t, firms have the option to issue one-period bonds b_{it+1}, that are due at the beginning of the next period, with interest. Limited liability implies that there are states in which firms will be unable to fully repay their debt obligations at time $t+1$. This is because internal funds after a sequence of bad shocks are so low that they are not sufficient to cover repayments. In such states, shareholders default on their commitments, creditors take over and recover a fraction of firm cash flows and assets net of bankruptcy costs. In anticipation of such states, creditors adjust the yields on debt so as to break even in expectation. In other words, they will charge a default premium Δ_{it+1} above the risk-free rate to be compensated for potential losses in default, so that the effective interest rate on bonds amounts to $r + \Delta_{it+1}$. We determine Δ_{it+1} endogenously below.

In line with the US tax code, we assume that interest payments are tax deductible, so that the effective repayment due in period $t+1$ amounts to $(1 + (1-\tau)(r + \Delta_{it+1})b_{it+1}$ only. The amount $\tau(r + \Delta_{it+1})b_{it+1}$ therefore represents a tax shield.

We assume that firms have to repay debt commitments at the beginning of the period, after realization of the shocks, and before issuing new debt and taking investment decisions. At that point, the firm is solvent if and only if

$$(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + (1-\delta)k_{it} + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1-\tau))\, b_{it-1} \geq 0. \tag{4}$$

We assume that the lenders liquidate the firm if the pair of shocks (z_{it}, η_{it}) and the policy (k_{it}, Δ_{it}) violate this solvency constraint. We can thus define the default set as $D_{it} \equiv \{(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) \in \overline{Z} \times \overline{N} \times \mathcal{R}^+ \times \mathcal{R}^+ : (4) \text{ does not hold}\}$, where $\overline{Z}$ and $\overline{N}$ denote the support for the shocks z and η respectively. To save on notation, we denote the indicator function for default as $\mathcal{I}_{D,it}$. Creditors will anticipate default states, and determine the default premium accordingly. Given risk neutrality, creditors break even in expectation if

$$\int\limits_{\overline{D}} (1 + r + \Delta_{it})\, d(z_{it}, \eta_{it}, k_t, \Delta_{it}) + \int\limits_{D} \frac{\xi(1-\delta)k_{it}}{b_{it-1}} d(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) = 1 + r$$

where ξ denotes the recovery rate in bankruptcy.

With the default premium at hand, we can determine firms' payouts. Debt and internal resources can be used to fund investment expenditures, or distributions d_{it} to shareholders. Given limited liability, seasoned equity offerings are effectively precluded. While this may initially appear restrictive, in the data equity issuances are often employee-initiated issues[5]. Employee-initiated

[5]McKoen (2015) documents the empirical relevance of employee-initiated equity issuances.

issues are not part of our model and characterize the exercise of stock options. With this caveat in mind, then, we have that

$$d_{it} \equiv (1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) - k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1-\tau))\, b_{it-1} + b_{it} \geq 0.$$

Firm problem Investment and financing policies are set to maximize firm value. Capital accumulation and financing needs reflect the persistent profitability shocks z_{it}, while debt policies additionally exploit the tax advantage and constraints. More formally, given our timing convention, firm value $W(k_{it-1}, b_{it-1}, z_{it-1})$ satisfies the following Bellman equation:

$$\begin{aligned} W(k_{it-1}, b_{it-1}, z_{it-1}) \equiv & \tfrac{1}{1+r} \max_{k_{it}, b_{it}} - k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} \\ & + \tau(r + \Delta_{it}) b_{it} \mathcal{I}_{1-D,it} - (1-\xi)((1-\delta)k_{it} + \tau\delta k_{it})\mathcal{I}_{D,it} + E_{t-1}[(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it}, b_{it}, z_{it})] \end{aligned}$$

subject to

$$(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) - k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} - (1 + (r + \Delta_{it})(1-\tau))\, b_{it-1} + b_{it} \geq 0,$$

$$\int_{\overline{D}} (1 + r + \Delta_{it})\, d(z_{it}, \eta_{it}, k_t, \Delta_{it}) + \int_D \tfrac{\xi(1-\delta)k_{it}}{b_{it-1}} d(z_{it}, \eta_{it}, k_{it}, \Delta_{it}) = 1 + r.$$

Discussion The previous paragraphs summarize a standard trade-off model of capital structure in the context of a dynamic firm investment problem, similar to, for example, Hennessy and Whited (2007). Firms' financing decisions thus reflect funding needs for investment expenditures, for example, in light of limited liability, but also the possibility of shielding profits from taxation. Financing constraints arise from a pricing mechanism in that elevated leverage increases the default set and thus raises spreads on risky debt. The type of debt contract thus entertained in this setup most likely resembles unsecured, public debt in the form of corporate bonds. Our empirical approach, detailed below, thus allows to let the data inform us about the relative relevance of these forces for various types of firms and for which type this sort of external financing is of relevance. Moreover, it will be informative about the magnitude of losses through default that lenders realistically anticipate and face.

2.3. Limited enforcement

We now relax the perhaps slightly stark assumption that firms only source of external financing is one-period debt. This assumption immediately precludes any instruments with a more state-contingent flavor such as credit lines, derivatives, or even external equity. We relax this in the context of a limited commitment model, in which, formally, we allow the payoffs of the securities available to outside investors to be contingent on the realization of the profitability shock z_{it}. While we do not take a stand on the precise implementation of this instrument by means of real world securities for the sake of this paper, we refer to Nikolov, Schmid, and Steri (2018), or the recent work by Rampini and Viswanathan (2013) or Li, Whited, and Wu (2016) for examples and estimation evidence.[6] In this context, η_{it} is public information.

Financing Formally, in every period firms can sell a portfolio of securities whose payoffs to investors are contingent on the realization of next periods profitability shocks z_{it+1} and η_{it+1}. Selling such a portfolio at time t thus raises an amount $b_{it} \equiv \frac{1}{1+r} E_t[p_{it+1}(z_{it+1}, \eta_{it+1}) + b_{it+1}(z_{it+1}, \eta_{it+1})]$, where $p_{it+1}(z_{it+1}, \eta_{it+1})$ is the cash flow transferred to the investors contingent on the realization of the two shocks, and $b_{it+1}(z_{it+1}, \eta_{it+1})$ is the residual present value of future promised repayments. For the sake of our analysis here, we think of these state contingent payments as repayments to a lender, which need to be fully collateralized. That is, we require that

$$p_{it+1}(z_{it+1}, \eta_{it+1}) + b_{it+1}(z_{it+1}, \eta_{it+1}) \leq \theta(1-\delta)k_{it+1}, \qquad \forall z_{it+1}, \eta_{it+1}.$$

where θ denotes the fraction of capital that can be pledged as collateral. Similar to the case of straight debt considered in the trade-off model above, these state-contingent debt instruments can be used to fund investment expenditures and distributions to shareholders, d_{it}, jointly with internal resources. We retain the assumption of limited liability on the shareholders' side, which requires that

$$d_{it} \equiv (1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) - k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} + \tau r b_{it-1} - p_{it+1}(z_{it+1}, \eta_{it+1}) \geq 0$$

[6] Our estimation procedure does not require to fully specify the implementation of the optimal contracts as their state-contingent features are embedded in observed corporate policies.

Firm problem The firm's problem is then to choose investment and state-contingent financing plans so as to maximize firm value, subject to constraints. Formally, appealing to our timing assumption, firm value $W(k_{it-1}, b_{it-1}, z_{it-1})$ satisfies the following Bellman equation:

$$\begin{aligned} W(k_{it-1}, b_{it-1}, z_{it-1}) \quad = \quad & \frac{1}{1+r} \max_{k_{it}, b_{it}(z_{it}, \eta_{it}), p_{it}(z_{it}, \eta_{it})} -k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} + \tau r b_{it-1} \\ & + E_{t-1}[(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it}, b_{it}(z_{it}, \eta_{it}), z_{it})] \end{aligned}$$

subject to

$$b_{it} \equiv \frac{1}{1+r} E_t[p_{it+1}(z_{it+1}, \eta_{it+1}) + b_{it+1}(z_{it+1}, \eta_{it+1})] \tag{5}$$

$$\begin{aligned} & (1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) - k_{it} + (1-\delta)k_{it} - \Psi(k_{it}, k_{it-1}) + \tau\delta k_{it} \\ & + \tau r b_{it-1} - p_{it+1}(z_{it+1}, \eta_{it+1}) \geq 0, \forall z_{it+1}, \eta_{it+1} \end{aligned} \tag{6}$$

$$p_{it+1}(z_{it+1}, \eta_{it+1}) + b_{it+1}(z_{it+1}, \eta_{it+1}) \leq \theta(1-\delta)k_{it+1}, \forall z_{it+1}, \eta_{it+1}. \tag{7}$$

Discussion Models of firm financing based on limited enforcement have been studied extensively theoretically in Albuquerque and Hopenhayn (2004) and Rampini and Viswanathan (2010, 2013), for example. Financing constraints here arise from the enforcement or collateral constraints in expressions (7) that tie firms' debt capacity to tangible assets. In this sense, all debt is secured and firms' financing and investment decisions are tied together through the collateral resulting from firms' investment. Such models thus emphasize the relevance of firm investment and funding needs for capital structure. Moreover, the type of debt contract arising in this setup thus most likely resembles secured bank debt, perhaps, given its state-contingent nature, likely in the form of credit lines, as in Nikolov, Schmid, and Steri (2018). Our empirical approach thus gives us a means to let the data speak to for which type of firms this sort of external financing is most relevant. Additionally, our estimations can give guidance on the extent to which firms' assets can effectively serve as collateral in financing deals.

2.4. *Moral hazard*

We now embed a dynamic moral hazard problem into our setup by assuming that lenders cannot observe the realization of all shocks and therefore have to rely on shareholders' report. Naturally, all else equal, shareholders have an incentive to underreport realized shocks as it allows them to pocket a larger share of realized cash flows by repaying less debt. An incentive compatible contract therefore designs a repayment schedule such that shareholders are always better off truthfully reporting. This can be achieved by implementing state-contingent repayments satisfying a number of constraints as detailed below.

Given that shocks are unobservable to lenders, some care must be taken with the specification of information sets. Further, as previously, we realistically want to allow for serially correlated shocks z_{it}. To keep the analysis transparent, we assume, as before, that z_{it} follows a Markov Chain, which is publicly observable[7], especially for the lender. However, conditional on a particular realization of z_t, realized cash flows are also impacted by the iid disturbance η_{it}. To give rise to a meaningful dynamic moral hazard problem, critically, we assume that η_{it} is observable by shareholders, but *unobservable* by lenders.

In this context, a lending contract amounts to a sharing rule that splits a firm's resources between payments to the lender, p_{it}, and payments to the shareholder, that is, dividends, d_{it} in a fully state-contingent manner. An optimal contract between shareholders and lenders maximizes the firm value W_{it}, subject to incentive constraints, promise keeping, as well as limited liability constraints. In this context, the incentive constraints amount to requiring that under the contract shareholders are always better off sticking to the contract and revealing the true realization of η_{it}, rather then underreporting realized cash flows and diverting cash flows. At this stage, we capture the cash flows that can be diverted by misreporting $\hat{\eta}_{it}$ rather than the true η_{it} so as to reap benefits according to the general 'diversion function' $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$. We will discuss economically motivated functional forms below.

In this setting with dynamic moral hazard, it is convenient to use the equity value of the firm, V_{it}, as a state variable. Clearly, then, the value of debt can be recovered, from $b_{it} = W_{it} - V_{it}$.

[7] See Doepke and Townsend (2016) for analyses of privately observable persistent shocks

Similarly, the contract picks state-contingent dividend payments d_{it} as controls, so that the payments to the lender p_{it} are recovered from the resource constraint, as detailed below. Accordingly, we accommodate tax deductability of interest on debt, $\tau r b_{it} = \tau r(W_{it} - V_{it})$, which translates into an adjusted discount rate for the firm ($1/(1+(1-\tau)r)$ rather than $1/(1+r)$) and a penalty for foregone tax deductions on debt for the amount of $\tau r V_{it}$.

More formally, the firm value function satisfies

$$\begin{aligned} W(k_{it-1}, V_{it-1}, z_{it-1}) &= \max_{k_{it}, V_{z_{it},\eta_{it}}, d_{z_{it},\eta_{it}}} \frac{1}{1+(1-\tau)r}[-k_{it} - \Psi(k_{it}, k_{it-1}) + (1-\delta)k_{it} + \tau\delta k_{it} - r\tau V_{it-1} \\ &\quad + E_{t-1}[(1-\tau)\pi(k_{it}, z_{it}, \eta_{it}) + W(k_{it}, V_{z_{it},\eta_{it}}, z_{it})]] \end{aligned}$$

subject to

$$V_{it-1} = \frac{1}{1+r}E_{t-1}[d_{z_{it},\eta_{it}} + V_{z_{it},\eta_{it}}], \tag{8}$$

$$d_{z_{it},\eta_{it}} + V_{z_{it},\eta_{it}} \geq d_{z_{it},\hat{\eta}_{it}} + V_{z_{it},\hat{\eta}_{it}} + \mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it}), \quad \forall z_t, \forall \hat{\eta}_{it}, \tag{9}$$

$$d_{z_{it},\eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}, \tag{10}$$

$$V_{z_{it},\eta_{it}} \geq 0, \quad \forall z_{it}, \forall \eta_{it}, . \tag{11}$$

Here, V_{it-1} is the equity value at the end of period $t-1$, and the promise keeping constraint states that the (state-contingent) dividend payments to shareholders and equity value at the end of period t have to add up to V_{t-1} in expectation. The incentive constraints state that shareholders are always better reporting the true realization of the iid shock η_{it} and receiving a dividend $d_{z_{it},\eta_{it}}$ and continuation value $V_{z_{it},\eta_{it}}$, rather than misreporting $\hat{\eta}_{it}$ and pocketing the diverted cash flow, captured by the diversion function $\mathcal{D}(k_{it}, z_{it}, \eta_{it}, \hat{\eta}_{it})$, as well as the dividends and continuation values under the misreported cash flows, $d_{z_{it},\hat{\eta}_{it}} + V_{z_{it},\hat{\eta}_{it}}$. The diversion function, in its most straightforward specification, is just $\lambda(\pi(k_{it}, z_{it}, \eta_{it}) - \pi(k_{it}, z_{it}, \hat{\eta}_{it}))$, where $1-\lambda$ captures potential losses in cash flow diversion. Finally, in every state, dividend payments and equity values have to be non-negative, reflecting shareholders' limited liability.

Given our timing assumption, all the cash flows accrue intra-period, so that payments p_{it} to the lender are simply the mirror image of contractually designed dividend payments to shareholders.

In other words, we must have that these respective payments exhaust available resources, so that, state by state,

$$p_{it} = -k_{it} - \Psi(k_{it}, k_{it-1}) + (1-\delta)k_{it} + \tau\delta k_{it} + \tau r(W_{it} - V_{it}) + \pi(k_{it}, z_{it}, \eta_{it}) - d_{it}.$$

This formulation emphasizes the trade-off between payments to shareholders and lenders, which must respect promise keeping and incentive constraints laid out above.

Discussion In dynamic moral hazard models, financing constraints arise from asymmetric information between financiers and insiders. Strictly speaking, such asymmetry of information gives rise to the possibility that insiders 'divert' or blatantly steal cash flows. Realistically, cash flow diversion is likely to be interpreted not narrowly as outright stealing of profits, but more broadly perhaps as conflicts of interest about the proper use of funds in firms that are less transparent and lack the scrutiny of the public spotlight. In other words, financing decisions in a moral hazard setting prominently reflect agency conflicts and informational asymmetries. These agency conflicts are mitigated by the optimal contract by providing insiders with equity, that is, skin-in-the-game, that prevents them from stealing, as the incentive constraints, expressions (9) in our recursive formulation indicate. The contracts also dictates how equity, and thus leverage, dynamically evolves. Our empirical approach thus not only gives us a way to assess for which firms such conflicts are of primary importance, but also enables us to provide guidance regarding the magnitudes of potential cash flow diversion λ and the amount of asymmetric information η.

3. Model computation and estimation

In this section, we describe the two key steps that enable us to bring the models we laid out in Section 2 to the data. Section 3.1 describes the numerical solution method we use for model computation. Section 3.2 presents the structural estimation procedure and the statistical tests we use for model comparison.

3.1. Solution method: Linear programming

The triplet of models we laid out in Section 2 has no closed-form solution. Thus, we solve the dynamic programs all of them numerically. In addition, the numerical solution of the limited commitment and moral hazard models is computationally challenging. The presence of state-contingent policies introduces a large number of control variables that makes the curse of dimensionality excessively severe for standard iterative computational methods.[8]

We overcome this difficulty by adopting the linear programming (LP) representation of dynamic programming problems with infinite horizon (Ross (1983)), building on Trick and Zin (1993), and Trick and Zin (1997). We exploit and extend linear programming methods to efficiently solve for the value and policy functions. Linear programming methods, while common in operations research, have been introduced into economics and finance in Trick and Zin (1993, 1997). We follow Nikolov, Schmid, and Steri (2018) to extend the LP approach to setups common in dynamic corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

To start with, any finite dynamic programming problem with infinite horizon can be equivalently formulated as a linear programming problem (LP). The LP representation associates every feasible decision at each grid point on the state space with a constraint. Specifically, the three models can be formulated as LP problems as follows:

$$\min_{W_{k,u,z}} \sum_{k=1}^{nk} \sum_{u=1}^{nu} \sum_{z=1}^{nz} W_{k,u,z} \tag{12}$$

$$s.t.$$

$$W_{k,u,z} \geq R_{k,u,z,a} + \sum_{z'=1}^{nz} \beta Q_z(z,z') W_{k'(a),u'(a),z'} \qquad \forall k,u,z,a, \tag{13}$$

[8] Because of the presence of several occasionally non-binding collateral constraints, the models cannot be solved numerically by interior point methods. In principle, all models can be solved on a discrete grid by standard iterative methods as value and policy function iteration. However, as discussed above, the application of these methods to the contracting models in Section 2 is computationally problematic.

where u denotes the promised utility variable, namely b_{it} for the tradeoff and the limited commitment model and v_{it} for the moral hazard model; nk, nu, and nz are the number of grid points on the grids for $k_{i,t}$, $u_{i,t}$, and $z_{i,t}$ respectively; $W_{k,u,z}$ is the value function on the grid point indexed by k, u, and z; a is an index for a *feasible* action on the grid for both capital, promised utility, and payouts, and $R_{k,u,z,a}$ denotes the return function corresponding to the action a starting from the state indexed by k, u, and z; β is the appropriate discount rate; $Q_z(z, z')$ is the transition matrix of the Markov chain driving profitability shocks; $k'(a)$ and $u'(a)$ denote the future values for the state variables given the current firm's decisions. For a formal proof, we refer to Ross (1983).

The solution of the LP above would require to store an extremely large matrix, because state-contingent decisions render the number of constraints in the problem enormous. Precisely, the set of feasible actions a is a highly dimensional object for both the limited commitment model (due to state-contingent debt repayments) and the moral hazard model (due to state-contingent dividends and promised equity values). Computational requirements would therefore be excessive. Thus, we implement constraint generation, a standard operation research technique to attack problems with a large number of constraints. First, we solve a relaxed problem with the same objective function. Second, we use the current solution to identify the constraints it violates. Third, we add one of the violated constraints, namely the most violated one, to the relaxed problem. We iterate the procedure until all constraints are satisfied.

To practically implement the constraint generation procedure, we need to deal with another computational issue. The selection of the most violated constraint involves searching over an extremely large vector of grid points for all the state-contingent control variables. The computational burden would still be excessive for the two contracting models we solve. To do so, we implement a separation oracle, an auxiliary mixed-integer programming problem to identifies the most violated constraint[9]. Appendix A outlines the constraint-generation algorithm and the separation oracle for the three models.

We implement the codes with Matlab® and CPLEX® as a solver for the linear and mixed-integer programming problems. Our workstation has a CPU with 24 cores and 124GB of RAM.

[9]Separation oracles are standard tools in operation research, as described in Schrijver (1998) and Vielma and Nemhauser (2011).

The models are solved with five grid points for the idiosyncratic shock, 21 grid points for capital, 17 grid points for current promised utility. All control variables are chosen on a continuous grid up to CPLEX numerical precision, which is 1e-6.

3.2. Estimation method

The dynamic corporate finance literature relies typically on two estimation methods: the Simulated Method of Moments or SMM, (Hennessy and Whited, 2005, 2007, Taylor, 2010) and the Simulated Maximum Likelihood or SML (Morellec, Nikolov, and Schürhoff, 2012, 2016). We depart from the literature and rely on an alternative method that is most readily identified as Indirect Inference or II in the terminology of Gourieroux, Monfort, and Renault (1993).

Unlike, SMM or SML, II relies on an auxiliary model for estimating the structural parameters. The auxiliary model is an approximation of the true data generating process. Typically, this approach is suitable in cases where the likelihood is not available in closed form and is computationally infeasible. In our case, we choose II as it constitutes a natural framework for comparing models. In particular, for the auxiliary model, we choose the model policy functions. The competing models that we consider do not share the same parameters, but their policy functions are common. We can thus compare in this framework both nested and non-nested models. Below we further motivate our choice of estimation method and describe its implementation.

We estimate the key structural parameters of interest using II. However, we estimate some of the model parameters separately. For example, we set the risk-free interest rate, r, equal to the average one-year Treasury rate over the sample period. We set the corporate tax rate equal to 20%. This rate is an approximation of the corporate tax rate relative to personal taxes. Finally, we set the probability κ of observing a high realization of the iid cash flow shock to 50%. Because κ and η always appear as a product in the tradeoff and limited enforcement models, they are not separately identified. Setting $\kappa = 0.5$ guarantees that the iid shock η has zero mean and does not directly affect average profitability.

We then estimate ten parameters: the curvature of the profit function, α; the fixed production cost, f; the serial correlation of $ln(z)$, ρ_z; the standard deviation of the innovation of $ln(z)$, σ_z; the depreciation of capital, δ; the capital adjustment cost, ψ; the magnitude of the iid cash flow shock, η; the recovery rate, ξ; the fraction of capital that can be pledged as collateral, θ; and the diversion parameter, λ.

3.2.1. Empirical policy functions

A policy function[10] is an association between an optimal choice of the firm, for example investment or financing, and its currently observable state. In accordance with this definition, we write the policy function as

$$\mathbf{w} = P(\mathbf{x}) \tag{14}$$

where $\mathbf{x}$ is a vector of (possibly transformed) state variables and $\mathbf{w}$ is a vector of policy variables of the model. For all models, we choose k_{it}, b_{it}, and z_{it} as states, and k_{it+1}, b_{it+1}, and d_{it+1} as policy variables. Thus, $\mathbf{x}_{it} = \{k_{it}, b_{it}, z_{it}\}$ and $\mathbf{w}_{it} = \{k_{it+1}, b_{it+1}, d_{it+1}\}$.

One challenge when working with policy functions is that some state or control variables are unobservable. We tackle this challenge by working with observable transformations of these variables. For example, the state variable z is unobservable. In this case, we use zk^{α}/k, firm profitability, that is observable.

We now characterize the empirical counterpart of the policy function $\mathbf{w} = P(\mathbf{x})$. One way to do so is linear approximation. This approach however will fail to capture the non linearities embedded in our models. We select then a semiparametric approach.

We consider the following specification for each control variable

$$\mathbf{w}_{it}^{n} = P^{n}(\mathbf{x}_{it}) + \mathbf{u}_{it}^{n} \tag{15}$$

where n is the n^{th} element of the policy vector $\mathbf{w}_{it}^{n}$, u_{it}^{n} is the specification error with $E[\mathbf{u}_{it}^{n} \mid \mathbf{x}_{it}] = 0$. We estimate the function $P(\mathbf{x}_{it})$

[10]The exposition follows Bazdresch, Kahn, and Whited (2016).

We use a series approximation functions $p_j(\mathbf{x}_{it})$ where $j = 1, ...J$ to estimate the policy function $P(\mathbf{x}_{it})$. In particular, as $J \to \infty$, the expected mean square difference between the $P(\mathbf{x}_{it})$ and a linear combination of $p_j(\mathbf{x}_{it})$ approaches zero, that is

$$\lim_{J\to\infty} E\left(\sum_{j=1}^{J} h_j p_j(\mathbf{x}_{it}) - P(\mathbf{x}_{it})\right)^2 . \tag{16}$$

We find that a power series with linear, quadratic, and all cross-products performs well. We have experimented with several alternative series functions. We observe that our results are immune to the particular choice of the series functions.

3.2.2. *Structural estimation: Indirect inference*

We use Indirect Inference to structurally estimate our set of models. The Indirect Inference method relies on an auxiliary model. While the auxiliary model is an approximation of the true data generating process, it captures the most important features of the data. Empirical policy functions constitute a natural candidate for an auxiliary model as they characterize the solution of the model. Below we detail the estimation procedure.

We define the vector of observed data $\mathbf{v}_{it} \equiv (\mathbf{w}_{it}, \mathbf{x}_{it})$, where $i = 1, ..., n$ indexes firms and $t = 1, ..., T$ indexes time. Similarly, we define the vector of simulated data $\mathbf{v}_{it}^s$, where $s = 1, ...S$ is the number of times we simulate the model. The simulated data vector, $\mathbf{v}_{it}^s(\beta)$, depends on the vector of structural parameters β. We define the estimating equation as

$$g(\mathbf{v}_{it}, \beta) = \frac{1}{nT} \sum_{i=1}^{n} \sum_{t=1}^{T} \left[\mathbf{h}(\mathbf{v}_{it}) - \frac{1}{S} \sum_{s=1}^{S} \mathbf{h}(\mathbf{v}_{it}^s(\beta))\right] \tag{17}$$

where $\mathbf{h}(.)$ is the parameter vector from (16) defining the empirical policy functions. The dimension of $\mathbf{h}$ is larger than the dimension of the vector of structural parameters β. The Indirect Inference estimator for β is given by

$$\widehat{\beta} = \arg\min_{\beta} g(\mathbf{v}_{it}, \beta)' \widehat{W}_{nT} g(\mathbf{v}_{it}, \beta)$$

where $\widehat{W}_{nT}$ is a positive definite weighting matrix that converges in probability to a deterministic positive definite matrix W.

3.2.3. Testing and model selection

We now build a set of tests that help us evaluate the relative performance of the competing models. To do so, we use the test in Rivers and Vuong (2002), that allows to test whether each pair of models is statistically distinguishable relative to the data. The test in Rivers and Vuong is suited to statistically compare the fit of a pair of non-nested models estimated using the method of moments and can be applied to our indirect inference estimation.[11] This test is the counterpart of the Vuong (1989) model comparison test for maximum likelihood estimation, used for example in Paulson, Townsend, and Karaivanov (2006) and Karaivanov and Townsend (2014). As the Vuong (1989) test, it does not require any specific functional form or other assumptions to nest the three models.

We consider two separate estimations for which we use the same set of data moments, $\mathbf{h}\left(\mathbf{v}_{it}\right)$, but that use different economic models, so that the simulated data differ across estimations. Let the parameter vector from the first estimation be β_1, and let the parameter vector from the second estimation be β_2. We want to test the null hypothesis that the two models are asymptotically equivalent, that is $H_0 : Q(\beta_1) = Q(\beta_2)$, where $Q(\beta_k)$, $k = 1, 2$, are the estimation minimands.

Rivers and Vuong show that under the null the test statistics

$$\nu_{nT} = \frac{1}{\sqrt{nT}} \frac{\widehat{Q}(\widehat{\beta}_1) - \widehat{Q}(\widehat{\beta}_2)}{\widehat{\sigma}_{nT}},$$

has a standard normal distribution, where $\widehat{Q}(\widehat{\beta}_k)$, $k = 1, 2$ are the minimands under the estimated parameter vectors, and $\widehat{\sigma}_{nT}$ is a consistent estimator of the limiting variance of $\widehat{Q}(\widehat{\beta}_1) - \widehat{Q}(\widehat{\beta}_2)$. We detail the computation of $\widehat{\sigma}_{nT}$ in Appendix B.

In one tailed tests, we contrast the null hypothesis with the alternatives that model $k = 1$ is asymptotically better than model $k = 2$, that is $H_1 : Q(\beta_1) < Q(\beta_2)$, and that model $k = 2$ is asymptotically better than model $k = 1$, that is $H_2 : Q(\beta_1) > Q(\beta_2)$.

[11] See also Hall and Pelletier (2011).

4. Empirical results

Our strategy is to first empirically establish policy function benchmarks for a variety of samples. With these benchmarks at hand, our estimation procedure then identifies the best fitting model from our selection of candidate models by comparing empirical benchmarks with policy functions recovered from simulations. We consider a variety of samples drawn from both the standard Compustat universe, as well as a more recent dataset on US private firms from Orbis, and describe our data below. Our ensuing discussion of the empirical benchmarks and those implied from model simulations, and the point estimates, is complemented by various counterfactuals.

4.1. Data and sample splits

Our main sample is drawn from the Compustat database for the 1965 to 2015 period. We exclude firms that do not have a stock exchange code (EXCHG) equal to 10 or 11. We remove firms that operate in the financial sector (four-digit SIC code between 4900 and 4999) or in regulated sectors (four-digit SIC code between 6000 and 6999 or between 9000 and 9999). We also drop firms with less than two consecutive year of data to be able to compute growth rates when required. Our resulting sample includes 39,433 firm-year observations.

We then compute the following variables. Investment is CAPX/AT; net book leverage is (DLTT+DLC-CHE)/AT; profitability is OIBDP/AT; dividends are (DVT+PSSTKC-PSTKRV)/AT; log size is the natural logarithm of PPENT; market-to-book is (DLTT+DLC+PRCCF x CHSO)/AT. Following, for example, Hennessy and Whited (2005) and Hennessy and Whited (2007), for the estimation procedure we remove firm fixed effect of each state and control variable. To reconcile the average levels of the control variables in the data and in the model, we add the sample mean into each variable after removing fixed effects.

To quantify the importance of the different types of frictions behind each model, we estimate the three models on different subsamples of firms. These subsamples are formed by splitting the sample in two on each on the (rescaled) state variables, namely (log) size, profitability, and leverage. Firms in the top and bottom 30th percentiles are respectively assigned to the subsamples with high and low values for the sorting variable.

In addition, we also consider a subsample of private firms. We obtain data on US private firms from the Orbis database in the period from 2003 to 2012, as described in Kalemli-Ozcan, Sorensen, Villegas-Sanchez, Volosovych, and Yesiltas (2015). We remove observations with missing, zero, or negative values for total assets. We drop firms with total assets less than 100,000 USD or less than 10 employees. Due to data restrictions, data on dividends are not available. In addition, market-to-book ratios cannot be computed for private firms in that there are no market share prices for them. We proxy investment as the growth rate of total assets, leverage as the ratio of the difference between total assets and shareholder funds and total assets, log size as the natural logarithm of total assets, and profitability as the ratio between profit and losses before taxes and total assets. We drop firm-year observations with missing values for investment, leverage, and profitability. We are left with an unbalanced panel with 83,823 firm-year observations.[12]

Table 1 reports the summary statistics of the aforementioned variables in the model. We winsorize all variables at the one percent level. Our Compustat sample is representative in that it exhibits sample statistics in line with the extant literature. Aa fraction of total assets, the average profitability of our Compustat sample is around 15%, asset growth around 12%, net book leverage around 13%, and payouts around 2.2%. The average log size is around 6.5, with market-to-book somewhere around 1.4. When we split firms according to the 30th and 70th percentile, we find that larger firms invest less than smaller ones, have higher leverage, and are more profitable. Regarding payout, smaller and larger firms are similar.

In comparison to public firms, private firms are less profitable, significantly smaller, invest less, and are far more levered, with a remarkable debt-to-asset ratio around 60%. While this figure appears rather high, it is in line with Huynh, Paligorova, and Petrunia (2012), who discuss that it is mainly driven by higher short-term leverage than public firms. In addition, private firms exhibit a significant heterogeneity across firms, as a standard deviation around thirty percent suggests.

[12]Not surprisingly, private firms are numerous in comparison to public firms in a given time period.

4.2. Identification and model comparison

4.2.1. Identification

For each model, global identification of the parameter vector requires a one-to-one mapping between the vector of model deep parameters β and a subset of the parameters of the auxiliary model with the same dimension. Local identification requires the gradient $\frac{\partial h(v^s_{it}(\beta))}{\partial \beta}$ of the auxiliary model with respect to the deep parameters to have full rank. This condition has an intuitive interpretation. Identification requires that every estimated deep parameter in β has a differential impact on the firms' investment, financing, and payout policies as characterized by the set of auxiliary parameters $h(v^s_{it}(\beta))$.

Because the choice of β, and in particular of the financing parameters ξ, θ, and λ affects the optimal policy functions, the aptness of the three models to rationalize observed corporate policies depends on how parameters can be chosen to minimize the distance between the policies implied by the model and their real-data counterparts, described by $h(v_{it})$. For each model, the estimation procedure selects parameters to minimize the distance function in Equation 17.

Importantly, different financial frictions have a different importance for different types of firms, that in turn make different corporate decisions. Intuitively, the financial constraints that restrict the most the access to external financing for large mature firms are likely to bet different than those that restrict small young firms. As a consequence, the observed investment, financing, and payout policies of different subsets of firms can significantly differ, and the model that better describes such policies is likely not to be the same for all firms in the economy.

4.2.2. Model comparison: Empirical policy functions

We now turn to the basis of our model evaluation and comparison procedure, namely empirical policy function benchmarks. These benchmarks provide an empirical description of the mapping between corporate actions (policies) and current firm characteristics (state variables). When firm characteristics change over time with the economic environment, this mapping allows to trace out firm dynamics in a natural way. By recovering policy functions in a analogous manner from simulations of each of our candidate models and comparing them with the empirical benchmarks, we obtain a natural procedure to evaluate the adequacy of a model's dynamic implications. At the same time, it provides us with parameter estimates and a distribution theory that allows us to compare the dynamic implications across models not just qualitatively, but also formally statistically. Our implementation is based on empirical mappings between the main choices in our model, namely investment, leverage and payout, and the main state variables, namely size, profitability, and leverage, respectively.

Figure 1 provides a first illustration of our approach, and helps validate it by visually examining a subset of our estimated empirical policy benchmarks. It plots the mappings between investment and size, and future leverage and current leverage, respectively, across samples and models. Qualitatively, the emerging patterns are well known and have been widely documented in the literature, in that the link between investment and size is negative, so that relatively larger firms invest less, and the mapping between current leverage and future leverage is positive, so that leverage exhibits hysteresis. The policy benchmarks estimated from the data further show that these particular patterns obtain across all samples, be it in larger or smaller, or private, firms. Moreover, in simulated data, at the estimated parameter values that we will discuss in detail below, the mappings recovered from model simulations replicate these patterns quite well. Intuitively, this reflects that the production function in all models exhibits decreasing returns to scale, leading to a negative relation between investment size, and observable profitability is persistent, leading to leverage hysteresis. Providing validation to our approach, these results show that indeed our models have the potential to rationalize relevant aspects of the dynamic behavior of firms. On the other hand, the policies underlying the mappings considered here are driven by technological properties that are common across our models, and illustrate that these particular benchmarks are not informative about the nature of financial frictions at work in the data. However, as we now document in our main results, careful inspection of informative policy benchmarks allows us to tease out the relevant financial frictions driving financing constraints and discriminate across models and across samples, both economically and statistically. We note that all our estimates are based on the entire set of empirical benchmarks, whether informative regarding discrimination across models or not. Specifically, we consider investment, future leverage, and payout versus size; investment, future leverage, and payout versus leverage; and investment, future leverage, and payout versus profitability.

4.3. Estimation results

We now describe the empirical patterns, as represented by empirical policy benchmarks, that are informative about the sources of financing constraints, across samples. Going beyond model validation, for each of our main samples, large and small public firms, and private firms, we identify a number of policies that are qualitatively distinct across these samples. Critically, we show that the implications of our candidate models for these policies are qualitatively rather distinct as well, suggesting a particular model as a most relevant economic representation of a typical firm in these samples. Importantly, we document that these qualitative differences are borne out statistically as well by our estimator, thereby identifying a particular model, and thus a friction, as the main source of financing constraints for any of our samples. Our estimator naturally also delivers parameter estimates that not only allow us to evaluate the economic relevance of particular financial frictions, but also to quantify their magnitudes.

4.3.1. Large Public Firms

Figure 2 presents a subset of empirical benchmarks for large public firms, and the corresponding estimated policy functions from our candidate models. The leftmost panel shows that the empirical investment benchmark is flat and thus essentially unrelated to leverage, suggesting that funding needs are not major drivers of leverage here. Relatedly, profits are often paid out rather than reinvested, as the middle panel shows. Critically, leverage is strongly positively related to profitability, as documented in the rightmost panel.

Visually, the figure suggests qualitatively that policy functions estimated from a trade-off model can by and large account for the dynamic behavior of large, public firms most adequately. Intuitively, this is in line with the notion that in such a model profits and the desire to shield profits from taxes are major drivers of payout and leverage decisions, rather than funding needs. As such they describe well the behavior of profitable, large firms, which have rather direct access to external financing from equity markets. The policies estimated from a limited enforcement model on this sample suggest a strong connection between funding needs, investment, and leverage, as profits are more likely reinvested than paid out, and higher profitability reduces leverage. Similarly, a moral hazard prescribes a counterfactually steep positive link between leverage and profitability, as the optimal contract requires that insiders receive more skin in the game after persistent rises in profitability in order not to divert resources. At these low levels of leverage, this raises debt capacity.

Table 2 formally confirms these qualitative results statistically, based on estimation on the entire set of benchmarks. Regarding model comparison, the relevant results are in the last row, reporting the values of the objective function, namely the distance between empirical benchmarks and the model implied policies, to be minimized. The lowest value, 2.805, computed from the trade-off model thus provides the best relative fit across the candidate models. Clearly, a moral hazard model struggles to account for the dynamic behavior for large, public firms, while a limited enforcement model does better, but not as well as the trade-off specification.

The table also provides guidance regarding the magnitudes of the implied financial frictions. The relatively high estimated recovery rates on debt, ξ, of about 60 percent reflect the elevated leverage ratios of large firms. While less reliable given their inferior fit, the same observation is reflected in high pledgeability of assets, θ, and a low diversion parameter, λ, suggesting a lower need for incentive provision via skin-in-the-game, and thus a higher debt capacity. Critically, we emphasize that the estimates for the technological parameters are in a very similar range across candidate models. Therefore, the differences in dynamic behavior we identify are unlikely driven by technology, but plausibly by differences in financial frictions.

4.3.2. Small Public Firms

Figure 3 reports qualitative results for small, public firms by means of a number of relevant and informative empirical benchmarks and corresponding policies recovered from candidate models. On this sample, the rightmost panel shows that the empirical link between profitability and leverage is negative, so that more profitable firms choose lower leverage, in contrast to the case of large firms. Intuitively, this is in line with a setting in which tax shields effects are not a main driver of leverage policies, but perhaps funding needs are. Consistent with that notion, as the middle panel shows, investment responds positively to profitability. Payout is also positively related to profitability, but empirically that link is weaker than with large firms.

Visually, the policies estimated from a limited enforcement model provide the closest approximation of the empirical benchmarks. This is consistent with the notion that in such models enforcement constraints limit firms' investment and growth, so that higher profitability reduces firms' dependence on external financing and thereby leverage, and boosts investment. The relevance of funding needs and limits in this case is consistent with firms with growth options that are operating at a smaller scale, so that profitability shocks accelerate their growth to an optimal scale. On the other hand, a trade-off model predicts an excessively high level of leverage, and a very steep link between payouts and profitability. Similarly, a moral hazard model predicts a positive link between profitability and leverage on that sample, as the optimal contract prescribes raising insiders stake to give proper incentives not to divert resources, which raises debt capacity at these levels of leverage.

Table 3 provides the formal statistical evaluation of our model comparison. The objective function is minimized at 4.552 in case of the limited enforcement specification, given econometric content to the assertion that this model describes the dynamic behavior of small public firms most adequately. A moral hazard model struggles to account for the dynamic behavior of these firms as well, while a trade-off specification provides a better fit, but worse than limited enforcement.

In terms of magnitudes, the parameter estimates suggest that firms can collateralize around 60 percent of their assets in this sample. This estimate is consistent with earlier estimation results e.g. in Nikolov, Schmid, and Steri (2018), obtained with alternative estimation methods, such as SMM. This observation validates the relevance of the parameter estimates based on our procedure as well. Finally, we emphasize again that the estimates for the technological parameters are fairly uniform across candidate specifications, suggesting that the distinct dynamic behavior across them can mainly be attributed to differences in financial frictions.

4.3.3. Private Firms

Figure 4 displays a subset of empirical benchmarks that are informative about the dynamic behavior of private firms. In this sample, investment responds negatively to leverage, as documented in the leftmost panel. This is inconsistent with both trade-off and limited enforcement specifications, whose policies exhibit a strong positive link between investment and leverage. Given persistent shocks and adjustment costs, investment tends to come with a history of past funding needs, leading levered firms to invest more, all else equal. Intriguingly, in the moral hazard specification, highly levered firms tend to invest less, as private firms do. In that setting, firms can invest and grow when the threat of diversion is low. This is the case when insiders have sufficient skin in the game, or in other words, a sufficient equity stake so that, all else equal, leverage falls is at the observed levels of leverage. The relevant empirical benchmark is thus consistent with an environment in which agency conflicts are a key determinant of investment policies.

The moral hazard model is also consistent with a setting in which investment responds positively to profitability, while leverage is largely unaffected by size, as the middle and rightmost graphs document. In both trade-off and limited enforcement models leverage responds quite steeply to size, positively with rising profits in the trade-off model and negatively with declining funding needs in the limited enforcement case. In the moral hazard setup two agency-driven effects are at play that work in opposite directions. To begin with, weaker incentives need to be given to insiders in larger firms in the first place, as growth opportunities decline with size, raising debt capacity. On the other hand, given fixed costs, persistent profits tend to rise with size as the optimal prescribes increasing equity in order to provide insiders with incentives not to steal, reducing leverage at the observed high levels of leverage, all else equal. The result is a policy which is largely unrelated to size. The latter effect also explains why investment is increasing with profitability, as in the data.

We report the results of the formal model comparison in Table 4. In line with the preceding qualitative discussion, the objective function is minimized at 5.129 in case of the moral hazard specification, with both trade-off and limited enforcement models doing roughly equally worse with values above 7. This establishes not only qualitatively, but also statistically the moral hazard model as providing the most adequate representation of the dynamic behavior of the private firms in our sample.

The estimates for the moral hazard specification in this case are of independent interest. They not only allow to infer the magnitude of potential profit diversion implied by the data, but also that of insiders' private information about shocks. Regarding profit diversion, the point estimate for λ is

0.13, indicating that observed policies imply that insiders could divert 13 cents on a dollar profits, unless given incentives to do otherwise by the optimal contract. To put that number in some context, we observe that the corresponding estimates for λ in case of large and small public firms are an order of magnitude lower, indeed just barely a few percentage points. This comparison corroborates the notion that agency conflicts and informational asymmetries, perhaps stemming from reduced disclosure requirements, are key determinants of private firms' policies. Indeed, our point estimates for η indicate that these are substantially larger for private firms than the corresponding estimates for either type of public firms considered, so that unobservable shocks account for a substantial fraction of private firms' total volatility.

4.3.4. Subsample estimation: Leverage and Profitability

While it is economically natural to examine model fit across the firm size distribution, methodologically our procedure more broadly suggests testing and comparing models by means of sorting firms along the relevant state variables, that is, not only size (assets), but also leverage and profitability. We describe the results of the corresponding estimates in this subsection. In particular, we implement our estimation procedure on subsamples identified as he top and bottom terciles of firms sorted, first, on leverage, and second, on profitability.

Table 5 reports estimates based on the leverage sorts, and shows that differences in estimated financing parameters can account rather well for variation in capital structures across firms. From the perspective of a trade-off model, the debt of high leverage firms is supported by substantially higher estimated recovery rates, ξ, than that of low leverage firms. Similarly, estimates of pleadgeability, θ in the context of a limited enforcement model are higher for the former type of firms. Intriguingly, higher leverage ratios are rationalized in a moral hazard model by a lower diversion parameter, λ. A lower ability to divert or steal resources from the company implies that insiders need to be given less incentives, so less skin in the game and equity, and thus supports more leverage. Our estimates are consistent with that notion.

Our estimates also reveal further notable differences between high and low leverage firms. For example, low leverage firms tend to be exposed to substantially more volatile shocks, as indicated by the estimates for σ_z. As such volatility is partially offset by lower fixed costs, an intuitive picture of a low leverage firm emerges in our setting as a perhaps younger firm facing more uncertainty, but lacking the commitments in the form of wages that established, mature firms face, in line with the empirical evidence that smaller firms tend to be less levered.

In contrast to the leverage splits discussed, Table 6 presenting sorts of firms along profitability, shows that differences in profitability across firms are likely attributable to technological differences across firms rather than financing conditions. While this is economically plausible, it is statistically borne out in our candidate models. Indeed, estimated financing parameters are quite similar across sample splits and models. On the other hand, there are stark differences in estimated technological parameters across profitability splits, which are also economically plausible. Indeed, less profitable firms face fixed costs an order of magnitude higher than those of profitable firms, and returns to scale, α uniformly lower than those of profitable firms. The notion that financing conditions are not the main drivers of differences in profitability is also in line with the observation in Table 1 that the variation in leverage across profitability sorts are limited.

4.3.5. Full Sample Estimation

So far, we have presented and discussed results based on economically and methodologically motivated splits of our data, to identify frictions relevant especially for a particular subset of firms. We now take a step back and ask, which of our candidate models provides the most adequate description of the 'average' firm, especially the typical firm from the entire Compustat universe? Rather than identifying a particular economic source of financing constraints, this investigation provides guidance to researchers looking to find a 'one size fits it all' model of capital structure. Such a model can be useful in the context of a macroeconomic model, such as an analysis of the aggregate effects of debt financing, perhaps, or the impact of monetary policy on firms' credit conditions, to name a few examples.

Table 7 reports the results of our estimation across the entire Compustat universe. As indicated by the minimized objective functions across candidate models in the last row, the trade-off model statistically provides the best representation of the dynamic behavior of a typical firm in Compustat. This results partially reflects an estimated recovery rate, ξ, of around 0.57, which, reassuringly, falls between the corresponding estimate in the cases of smaller and larger public firms. Consistently, the estimate for the pledgeability parameter, θ, in case of the limited enforcement model falls between the ones obtained in the corresponding size splits, giving credence to our procedure. However, the limited enforcement model produces a somewhat larger distance between empirical policy benchmarks and simulated ones than does the trade-off specification. Clearly, the moral hazard model does not provide an adequate representation of the dynamic behavior of Compustat firms, as indicated by the large approximation error. This is in line with our result that this model specification appears to be most relevant to describe private firms.

Overall, our results prioritize trade-off specifications over alternative candidate models as an adequate description of the typical Compustat firm. For researchers that are primarily looking for a suitable description of an aggregate or representative firm, our estimations thus single out a trade-off setup as a good starting point in this regard.

4.4. Model selection: statistical tests

Table 8 reports the results of our model selection based on the statistical tests described in section 3.2.3. The table compares the tradeoff, limited commitment, and moral hazard model on the samples of firms we consider in the estimations in section 4.3.

The columns of the table refer, from left to right, to the comparisons between the tradeoff (TO) and the limited enforcement (LE) models, between the tradeoff (TO) and the moral hazard (MH) models, and between the limited enforcement (LE) and the moral hazard (MH) models. The rightmost column summarizes the model, if any, that provides a better description of the observed empirical policy functions on the corresponding subsample. The results in the table suggest that the data features that characterize our empirical policy function benchmarks suffice to statistically distinguish across models in all subsamples. For all pairs of models and all subsamples, we reject the null hypothesis of asymptotically equivalent models at the ten percent significance level.

Consistent with our point estimation results, our tests strongly indicate the tradeoff model as the best representation of the dynamic behavior of large public firms. On this subsample, we accept the alternative hypotheses that the tradeoff model has a better fit than both the limited enforcement and the moral hazard models at the one percent level. The limited enforcement model provides a better description of corporate behavior than the moral hazard model. Corporate policies of small public firms appear to be better described by the limited enforcement model, which is asymptotically better than the others at the one percent level. In addition, our tests strongly favor the tradeoff model over the moral hazard model on the same subsample. The third row of the table instead shows that the policies estimated from a moral hazard model provide the closest approximation of the empirical benchmarks. The moral hazard model provides a better fit that both the tradeoff and the limited enforcement models at the one percent level. We also reject the hypothesis that the tradeoff model is asymptotically equivalent to the limited enforcement model against the alternative hypothesis that it better fits the policies of private firms at the ten percent significance level.

Finally, the bottom five rows of the table confirm the statistical significance of our estimation results on the remaining samples of firms. The tradeoff model provides the closest approximation of corporate policies for firms with high leverage and high profitability, and for the full sample of Compustat firms. The limited enforcement model better describes the policies of low leverage firms, and the moral hazard model offers the most accurate characterization of the dynamic behavior of low profitability firms.

4.5. Counterfactuals

We can use our estimation results to infer the costs of financing constraints across models and samples. We do this by comparing firm valuations from models in which financing is constrained by one of our candidate frictions, to counterfactual specifications in which external financing is i) entirely unconstrained in that firms can costlessly access equity markets, and ii) maximally constrained in that no external financing is available and all expenditures need to be financed using internal funds, so that firms are effectively in financial autarky. The relevant costs then are embedded in the relative valuations. Overall, our estimates point to significant financing constraints due to financial frictions.

Table 9 documents a first set of such counterfactuals. In the middle row, it reports firm valuations by means of Tobin's Q for all of our estimated models, across our main samples. Keeping the relevant estimated parameters fixed in each case, it compares those to the financial autarky case in each specification (first row), and to the financially unconstrained version (last row). A number of observations are in order. First, going from the first to the second row, we see that access to external financing even subject to financial frictions in any of the forms considered, creates substantial value as Tobin's Q raises significantly relative to the autarky case across the board. However, there is also considerable heterogeneity in value gains. For example, for large firms, being able to exploit tax advantages by issuing defaultable bonds more than doubles firm valuations (1.88/0.84), while value gains would be around 15 percent if their external financing were subject to asymmetric information and diversion in a moral hazard context (1.02/0.88), for example. Second, passing from the second to the third row, we find that financing constraints give rise to substantial costs in terms of valuations, with a fair amount of heterogeneity again. For example, for small firms being subject to collateral constraints due to enforcement frictions lowers their valuations by about 20 percent relative to the unconstrained benchmark, as they inhibit growth. However, the costs of financing constraints would increase to about 80 percent if these firms would exhibit asymmetric information and the threat of diversion in an environment with moral hazard. We note that for each sample, the costs of financing constraints are lowest in case of the best fitting among our candidate models. This pattern is consistent with an environment in which, from the set of debt contracts offered across models, picking the one from the best fitting model is value-maximizing.

In table 10, we investigate the costs of alternative financial frictions for each sample more closely. While the above paragraph already hinted at this, we now fix, for each sample, the relevant estimated parameters for the best fitting model, and only change the financial frictions parameters, ξ, θ, and λ to the point estimates obtained from the corresponding specification in each sample. This approach allows us to isolate the differential effects of our candidate financial frictions from changes in other estimated parameters. The results are quite revealing. For example, if a private firm's access to external finance would not be constrained by asymmetric information, but tax-related trade-offs or limited enforcement, the costs of financing constraints would be substantial, in the order of magnitude of about 50 to 55 percent. This reflects the notion that these firms are neither very profitable, nor investing much for seeking growth. On the other hand, if a large firms' external financing would be constrained by limited enforcement, or informational asymmetries and moral hazard, these costs of alternative constraints would be in the order of 36 to 46 percent. This reflects the observation that these firms are profitable, investing less, and likely not subject to significant asymmetric information.

5. Conclusion

We propose to empirically evaluate the sources of financing constraints across firms by drawing on recent advances in modeling, computing, and estimation. To that end, we develop, solve, and estimate a range of dynamic models of corporate investment and financing, with the objective of empirically identifying the quantitatively most prevalent financial frictions underlying firms' financing constraints. Our approach encompasses tax and default based models of firms' financial structure, as well as dynamic contracting models featuring limited commitment and dynamic moral hazard in the presence of asymmetric information. Critically, our estimation procedure based on empirical policy function benchmarks readily lends itself to developing tests that allow to empirically compare and discriminate between candidate models across various samples. Specifically, we evaluate and compare the fit of our candidate models both on the standard Compustat universe and a dataset on private firms coming from Orbis, as well as various subsample splits. Our tests, based on empirical policy function benchmarks, favor trade-off models for larger Compustat firms, limited commitment models for smaller Compustat firms, and moral hazard models for private firms.

Our estimation procedure also allows to gauge the magnitude of various financial frictions proposed in the dynamic contracting literature. The parameter estimates of particular concern relate to the unobserved agency conflicts that drive financial structure and investment, such as the degree of cash flow diversion in moral hazard models, and the amount of capital firms can abscond with in limited commitment models. Regarding cash flow diversion, our results indicate that in order to rationalize observed corporate policies for private firms, firm owners need to be able to divert about 13 cents on the dollar of profits. On the other hand, consistent with earlier results in Nikolov, Schmid, and Steri (2018), firms can collateralize about 60 percent of their assets in limited commitment models.

Our work aims at providing some first guidance regarding the quantitative significance across firms of various financial frictions proposed in the recent literature. Clearly, by focusing on limited enforcement, moral hazard, and tradeoff models, many prominent and promising mechanisms and models have been left out. We view subjecting these approaches to tests of the kind we propose in this paper as an important agenda going forward.

Appendix A: Solution by mixed-integer programming

The following constraint generation algorithm converges to the unique fixed point of our Bellman problems. To save notation, the firm index i is omitted and primes denote variables at time $t+1$.

1. solve the problem in (12) with an initial random subset of constraints for each state (k,u,z);
2. if all constraints $a \in \Gamma^n(k,u,z)$, for all (k,u,z), are satisfied, terminate the algorithm (where $\Gamma^n(k,u,z)$ is the set of feasible actions at iteration n);
3. for each state (k,u,z), add the constraint $a \in \Gamma^n(k,u,z)$ that generates the highest violation in (13) with respect to the current solution $W^n(k,u,z)$;
4. solve the problem with the current set of constraints;
5. go back to step 2.

The separation oracles for the three problems are specified as follows.

Definition 1 (Separation Oracle - Tradeoff)

$$\max_{a=\{k',b'\}} R_{k,k',b,b',z} + \sum_{z'=1}^{nz} Q_z(z,z')\frac{1}{1+r} W_{k'(a),b'(a),z'} - W_{k,b,z} \tag{18}$$

$s.t.$

$$p(i_k) \in \{0,1\} \qquad \forall i_k = 1,...,n_k \tag{19}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{20}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k) k^G(i_k) \tag{21}$$

$$R_{k,k',b,b',z} = \frac{1}{1+r}(-k' + (1-\delta)k' - \Psi(k',k) + \tau\delta k' + \tau(r+\Delta')b'\mathcal{I}_{1-D'}$$
$$-(1-\xi)((1-\delta)k' + \tau\delta k')\mathcal{I}_{D'} + E[(1-\tau)\pi(k',z',\eta')] \tag{22}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k) k^G(i_k) \tag{23}$$

$$d_{k,k',b,b',z} \geq 0 \tag{24}$$

Definition 2 (Separation Oracle - Limited Enforcement)

$$\max_{a=\{k',b(z',\eta'),p(z',\eta')\}} R_{k,k',b,b(z',\eta'),p(z',\eta'),z} + \sum_{z'=1}^{nz} Q_z(z,z')\frac{1}{1+r}W_{k'(a),b'(a),z'} - W_{k,b,z} \tag{25}$$

$s.t.$

$$b = E[p(z',\eta') + b(z',\eta')] \tag{26}$$

$$p(z',\eta') + b(z',\eta') \leq \theta k'(1-\delta) \quad \forall z',\eta' \tag{27}$$

$$p(i_k) \in \{0,1\} \qquad \forall i_k = 1,...,n_k \tag{28}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{29}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{30}$$

$$R_{k,k',b,b(z',\eta'),p(z',\eta'),z} = \frac{1}{1+r}(-k' + (1-\delta)k' - \Psi(k',k) + \tau\delta k' + \tau r b$$

$$+E[(1-\tau)\pi(k',z',\eta')] \tag{31}$$

$$d_{k,k',b,b(z',\eta'),p(z',\eta'),z} \geq 0 \tag{32}$$

$$b(z',\eta') \geq 0 \tag{33}$$

Definition 3 (Separation Oracle - Moral Hazard)

$$\max_{a=\{k',V(z',\eta'),d(z',\eta')\}} R_{k,k',V,V(z',\eta'),d(z',\eta'),z} + \sum_{z'=1}^{nz} Q_z(z,z')\frac{1}{1+r(1-\tau)}W_{k'(a),V'(a),z'} - W_{k,V,z} \tag{34}$$

$s.t.$

$$V = \frac{1}{1+r}E[d(z',\eta') + V(z',\eta')] \tag{35}$$

$$d(z',\eta') + V(z',\eta') \geq d(z',\hat{\eta}') + V(z',\hat{\eta}') + \mathcal{D}(k,z,\eta,\hat{\eta}), \quad \forall z, \forall\hat{\eta} \tag{36}$$

$$p(i_k) \in \{0,1\} \qquad \forall i_k = 1,...,n_k \tag{37}$$

$$\sum_{i_k=1}^{n_k} p(i_k) = 1 \tag{38}$$

$$k' = \sum_{i_k=1}^{n_k} p(i_k)k^G(i_k) \tag{39}$$

$$R_{k,k',V,V(z',\eta'),d(z',\eta'),z} = \frac{1}{1+r(1-\tau)}(-k' + (1-\delta)k' - \Psi(k',k) + \tau\delta k' - \tau r V$$

$$+E[(1-\tau)\pi(k',z',\eta')] \tag{40}$$

$$d_{z',\eta'} \geq 0 \tag{41}$$

$$V_{z',\eta'} \geq 0 \tag{42}$$

Equations (19), (20), (28), (29), (37), and (38) define the variables $p(i_k)$ that have the role to select a grid point for capital on the grid $k^G(i_k)$ and linearize the term k'^{α} in the production function and the adjustment cost function. The default conditions for the tradeoff model and all products of variables are incorporated in the separation oracles using standard mixed-integer formulations. The computation of the law of motion for future debt is obtained by interpolation with the logarithmic formulation of Vielma and Nemhauser (2011).

Appendix B: Variance estimation in model selection tests

Following Rivers and Vuong (2002) and Hall and Pelletier (2011), to which we refer the reader for an exhaustive treatment, $\widehat{\sigma}_{nT}$ is constructed as

$$\widehat{\sigma}_{nT} = \widehat{R'_{nT}}\ \widehat{V_{nT}}\ \widehat{R_{nT}},$$

where $\widehat{V_{nT}}$ is a consistent estimator of $\lim_{nT\to\infty} Var\left[\frac{1}{(nT)^{0.5}}\sum_{i=1}^{n}\sum_{t=1}^{T}\xi_{it}\right]$, based on the sample counterpart $\widehat{\xi}_{it}$ of ξ_{it}. The vector $\widehat{\xi}_{it}$ is constructed as $\widehat{\xi}_{it} = [\widehat{a}'_{1it}, \widehat{b}'_{1it}, \widehat{a}'_{2it}, \widehat{b}'_{2it}]$, where, for $k = 1, 2$,

$$\widehat{a}'_{kit} = \mathbf{h}(\mathbf{v}_{it}) - \frac{1}{S}\sum_{s=1}^{S}\mathbf{h}(\mathbf{v}_{it}^{s}(\widehat{\beta}_k)) - g(\mathbf{v}_{it}, \widehat{\beta}_k)$$

and

$$\widehat{b}'_{kit} = vech\{\psi_{\mathbf{h}(\mathbf{v}_{it})}\psi'_{\mathbf{h}(\mathbf{v}_{it})} - \widehat{\Omega}_k\}.$$

$\psi_{\mathbf{h}(\mathbf{v}_{it})}$ is the influence function of the vector of the functions of moments, and $\widehat{\Omega}_k$ is the covariance of such vector of functions, computed by covarying the influence function with itself as in Nikolov and Whited (2014).

The matrix $\widehat{R_{nT}}$ is constructed as

$$\widehat{R_{nT}} = \begin{bmatrix} \widehat{R_{nT}}^{(1)} \\ -\widehat{R_{nT}}^{(2)} \end{bmatrix}$$

where, for models $k = 1, 2$:

$$\widehat{R_T}^{(k)} = \begin{bmatrix} 2\{\widehat{\Omega}_k\}^{-1}g(\mathbf{v}_{it}, \widehat{\beta}_k) \\ -\widehat{\Delta}_{nT}^{(k)}B'_i\{g(\mathbf{v}_{it}, \widehat{\beta}_k) \otimes g(\mathbf{v}_{it}, \widehat{\beta}_k)\} \end{bmatrix},$$

where

$$\widehat{\Delta}_{nT}^{(k)} = L_k[\{\widehat{\Omega}_k\}^{-1} \otimes \{\widehat{\Omega}_k\}^{-1}]B_k.$$

The matrices L_i and B_i are selection matrices defined such that

$$vech[\{\widehat{\Omega}_k\}^{-1}] = L_i vec[\{\widehat{\Omega}_k\}^{-1}]$$

and

$$vec[\{\widehat{\Omega}_k\}^{-1}] = B_i vech[\{\widehat{\Omega}_k\}^{-1}].$$

References

Ai, Hengjie, Dana Kiku, and Rui Li, 2016, Quantifying the impact of moral hazard: Evidence from a structural estimation, Working Paper, University of Minnesota.

Albuquerque, Rui, and Hugo Hopenhayn, 2004, Optimal lending contracts and firm dynamics, *The Review of Economic Studies* 71, 285–315.

Albuquerque, Rui, and Enrique Schroth, 2010, Quantifying private benefits of control from a structural model of block trades, *Journal of Financial Economics* 96, 33–55.

Bazdresch, Santiago, Jay Kahn, and Toni Whited, 2016, Estimating and testing dynamic corporate finance models, Working Paper, University of Rochester.

Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of Tobin's q, corporate investment, financing, and risk management, *The Journal of Finance* 66, 1545–1578.

Bolton, Patrick, Neng Wang, and Jinqiang Yang, 2018, Liquidity and risk management, executive compensation, and investment with inalienable human capital: optimal contracting and corporate valuation, *Journal of Finance* forthcoming.

Clementi, Gian Luca, and Hugo Hopenhayn, 2006, A theory of financing constraints and firm dynamics, *Quarterly Journal of Economics* 121, 229–265.

Cooley, Thomas, and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American Economic Review* 91, 1286–1310.

DeMarzo, Peter, and Michael Fishman, 2007a, Agency and optimal investment dynamics, *Review of Financial Studies* 20, 151–188.

———, 2007b, Optimal long-term financial contracting, *Review of Financial Studies* 20, 2079–2128.

———, Zhiguo He, and Neng Wang, 2012, Dynamic agency and the q theory of investment, *Journal of Finance* 67, 2295–2340.

Doepke, Matthias, and Robert Townsend, 2006, Dynamic mechanism design with hidden inco me and hidden actions, *Journal of Economic Theory* 126, 235–285.

Fu, Shiming, and Vijay Krishna, 2016, Dynamic financial contracting with persistent private information, Working Paper, University of Rochester.

Gala, Vito, and Joao F. Gomes, 2016, Investment without q, Working Paper, University of Pennsylvania.

Glover, Brent, 2015, The expected cost of default, *Journal of Financial Economics* 119, 284–299.

Gomes, Joao F., 2001, Financing investment, *The American Economic Review* 91, 1263–1285.

Gryglewicz, Sebastian, Simon Mayer, and Erwan Morellec, 2018, Agency conflicts and short- vs long-termism in corporate policies, Working Paper, Swiss Finance Institute.

Hall, Alastair R, and Denis Pelletier, 2011, Nonnested testing in models estimated via generalized method of moments, *Econometric Theory* 27, 443–456.

Hennessy, Christopher A., and Toni M. Whited, 2005, Debt dynamics, *The Journal of Finance* 60, 1129–1165.

———, 2007, How costly is external financing? evidence from a structural estimation, *The Journal of Finance* 62, 1705–1745.

Huynh, Kim P, Teodora Paligorova, and Robert J Petrunia, 2012, Capital structure decisions: Insights from private firms, in *Workshop on Productivity Determinants Within Firms and the Role*.

Kalemli-Ozcan, Sebnem, Bent Sorensen, Carolina Villegas-Sanchez, Vadym Volosovych, and Sevcan Yesiltas, 2015, How to construct nationally representative firm level data from the orbis global database, Discussion paper, National Bureau of Economic Research.

Karaivanov, Alexanderr, and Robert Townsend, 2014, Dynamic financial constraints: Distinguishing mechanism design from exogenously incomplete regimes, *Econometrica* 82, 887–959.

Karaivanov, Alexander, and Mark Wright, 2011, Distinguishing across models of international capital flows, Working Paper, Simon Fraser University.

Korteweg, Arthur, 2010, The net benefits to leverage, *The Journal of Finance* 65, 2137–2170.

Li, Shaojin, Toni Whited, and Yufeng Wu, 2016, Collateral, taxes, and leverage, *Review of Financial Studies* 29, 1453–1500.

McKoen, Stephen B., 2015, Employee option exercise and equity issuance motives, Working Paper.

Morellec, Erwan, Boris Nikolov, and Norman Schürhoff, 2012, Corporate governance and capital structure dynamics, *Journal of Finance* 67, 803–848.

———, 2018, Agency conflicts around the world, *Review of Financial Studies* 11, 4232–4287.

Nikolov, Boris, Lukas Schmid, and Roberto Steri, 2018, Dynamic corporate liquidity, *Journal of Financial Economics*, forthcoming.

Nikolov, Boris, and Toni M. Whited, 2014, Agency conflicts and cash: Estimates from a structural model, *Journal of Finance* 69, 1883–1921.

Paulson, Anna L, Robert M Townsend, and Alexander Karaivanov, 2006, Distinguishing limited liability from moral hazard in a model of entrepreneurship, *Journal of political Economy* 114, 100–144.

Quadrini, Vincenzo, 2004, Investment and liquidation in renegotiation-proof contracts with moral hazard, *Journal of Monetary Economics* 51, 861–885.

Rampini, Adriano A., and S. Viswanathan, 2010, Collateral, risk management, and the distribution of debt capacity, *The Journal of Finance* 65, 2293–2322.

———, 2013, Collateral and capital structure, *Journal of Financial Economics* 109, 466–492.

Rivers, Douglas, and Quang Vuong, 2002, Model selection tests for nonlinear dynamic models, *The Econometrics Journal* 5, 1–39.

Ross, Sheldon M., 1983, *Recursive Methods in Economic Dynamics* (Academic Press: New York, NY).

Schrijver, Alexander, 1998, *Theory of linear and integer programming* (John Wiley & Sons).

Sun, Qi, and Mindy X. Zhang, 2016, Financing intangible capital, Working Paper.

Taylor, Lucian A, 2010, Why are ceos rarely fired? evidence from structural estimation, *The Journal of Finance* 65, 2051–2087.

———, 2013, Ceo wage dynamics: Estimates from a learning model, *Journal of Financial Economics* 108, 79–98.

Trick, Michael A., and Stanley E. Zin, 1993, A linear programming approach to solving stochastic dynamic programs, Working Paper.

——— , 1997, Spline approximations to value functions: A linear programming approach, *Macroeconomic Dynamics* 1, 255–277.

Verani, Stephane, 2016, Aggregate consequences of dynamic credit relationships, Working Paper, Board of Governors of the Federal Reserve.

Vielma, Juan Pablo, and George L Nemhauser, 2011, Modeling disjunctive constraints with a logarithmic number of binary variables and constraints, *Mathematical Programming* 128, 49–72.

Vuong, Quang H, 1989, Likelihood ratio tests for model selection and non-nested hypotheses, *Econometrica: Journal of the Econometric Society* pp. 307–333.

Wang, Wenyu, and Yufeng Wu, 2018, Managerial control benefits and takeover market efficiency, .

Wu, Yufeng, 2017, What's behind smooth dividends? evidence from structural estimation, *The Review of Financial Studies* 31, 3979–4016.

Zhang, Mindy X., 2016, Who bears firm-level risk? implications for cash flow volatility, Working Paper.

Table 1
Descriptive Statistics

The table reports summary statistics for firm characteristics across the subsamples of firms in this paper. Data of public firms are from Compustat and cover an unbalanced panel of 39,433 firm-year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 83,823 firm-year observations from 2003 to 2012. Subsamples of small vs large, profitable vs unprofitable, and high vs low leverage firms are obtained from Compustat. Firms are split according to firm characteristics in correspondence to the 30th and 70th percentile respectively. Size is measured as total assets in million dollars, and profitability, dividends, and leverage are scaled by total assets. μ denotes means, and σ denotes standard deviations. All variables are winsorized at the 1 percent level.

	Profitability		Investment		Leverage		Dividends		Log(Size)		Market/Book	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Compustat	0.151	0.060	0.124	0.252	0.132	0.142	0.022	0.045	6.480	0.927	1.367	0.677
Large	0.155	0.045	0.090	0.178	0.187	0.124	0.021	0.040	8.281	0.726	1.215	0.494
Small	0.140	0.074	0.150	0.313	0.045	0.149	0.020	0.048	4.874	0.783	1.581	0.854
High Leverage	0.130	0.049	0.084	0.231	0.324	0.234	0.010	0.046	6.620	0.728	1.164	0.367
Low Leverage	0.173	0.065	0.112	0.219	0.042	0.038	0.036	0.042	5.945	0.789	1.796	0.967
Profitable	0.249	0.046	0.111	0.156	0.133	0.120	0.052	0.056	6.438	0.819	2.044	0.914
Unprofitable	0.058	0.048	0.076	0.295	0.178	0.163	0.000	0.034	6.304	0.826	1.009	0.445
Private	0.066	0.034	0.073	0.109	0.644	0.050	NA	NA	3.863	0.196	NA	NA

Table 2
Estimation: Large Public Firms

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on top terciles of Compustat firms sorted by size. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

Parameter Estimates: Large Public Firms			
	TO	LE	MH
α	0.778	0.808	0.741
	(0.001)	(0.001)	(0.001)
f	0.704	0.776	0.670
	(0.943)	(0.112)	(0.028)
ρ_z	0.834	0.779	0.834
	(0.001)	(0.001)	(0.001)
σ_z	0.292	0.305	0.364
	(0.002)	(0.001)	(0.002)
δ	0.169	0.126	0.188
	(0.001)	(0.001)	(0.001)
ψ	0.130	0.132	0.162
	(0.005)	(0.001)	(0.001)
η	0.312	0.329	0.260
	(0.005)	(0.006)	(0.003)
ξ	0.600		
	(0.007)		
θ		0.727	
		(0.005)	
λ			0.039
			(0.000)
Obj. Fun.	2.805	5.233	26.536

Table 3
Estimation: Small Public Firms

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on bottom terciles of Compustat firms sorted by size. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

Parameter Estimates: Small Public Firms			
	TO	LE	MH
α	0.765	0.764	0.772
	(0.004)	(0.002)	(0.002)
f	0.897	0.837	0.880
	(0.641)	(0.130)	(0.011)
ρ_z	0.813	0.774	0.775
	(0.004)	(0.002)	(0.003)
σ_z	0.313	0.312	0.286
	(0.006)	(0.004)	(0.002)
δ	0.182	0.182	0.187
	(0.002)	(0.003)	(0.001)
ψ	0.155	0.151	0.139
	(0.004)	(0.003)	(0.001)
η	0.263	0.250	0.230
	(0.007)	(0.012)	(0.001)
ξ	0.547		
	(0.013)		
θ		0.647	
		(0.011)	
λ			0.021
			(0.000)
Obj. Fun.	5.108	4.552	11.453

Table 4
Estimation: Private Firms

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on a sample of private firms from Orbis. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

Parameter Estimates: Private Firms			
	TO	LE	MH
α	0.569	0.605	0.630
	(0.001)	(0.002)	(0.002)
f	5.012	5.287	5.176
	(0.064)	(0.052)	(0.736)
ρ_z	0.796	0.745	0.745
	(0.001)	(0.001)	(0.001)
σ_z	0.240	0.309	0.309
	(0.002)	(0.001)	(0.005)
δ	0.089	0.058	0.058
	(0.000)	(0.001)	(0.000)
ψ	0.174	0.202	0.203
	(0.005)	(0.003)	(0.003)
η	0.486	0.423	0.391
	(0.003)	(0.004)	(0.007)
ξ	0.449		
	(0.001)		
θ		0.541	
		(0.014)	
λ			0.130
			(0.002)
Obj. Fun.	7.140	7.587	5.129

Table 5
Estimation: High versus Low Leverage Firms

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the top and bottom terciles of firms sorted by leverage. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

	Parameter Estimates					
	High Leverage Firms			Low Leverage Firms		
	TO	LE	MH	TO	LE	MH
α	0.807	0.796	0.728	0.752	0.712	0.642
	(0.000)	(0.001)	(0.001)	(0.004)	(0.003)	(0.024)
f	1.252	1.194	1.247	0.661	0.796	0.706
	(0.669)	(0.014)	(0.008)	(0.396)	(0.029)	(0.024)
ρ_z	0.878	0.735	0.874	0.824	0.747	0.672
	(0.001)	(0.001)	(0.001)	(0.002)	(0.003)	(0.004)
σ_z	0.222	0.248	0.199	0.322	0.324	0.335
	(0.000)	(0.001)	(0.000)	(0.003)	(0.001)	(0.014)
δ	0.182	0.166	0.181	0.196	0.183	0.202
	(0.000)	(0.002)	(0.001)	(0.001)	(0.003)	(0.015)
ψ	0.105	0.122	0.116	0.156	0.132	0.109
	(0.019)	(0.001)	(0.001)	(0.006)	(0.001)	(0.024)
η	0.381	0.363	0.378	0.262	0.317	0.235
	(0.003)	(0.003)	(0.003)	(0.009)	(0.005)	(0.012)
ξ	0.858			0.412		
	(0.001)			(0.004)		
θ		0.866			0.252	
		(0.007)			(0.003)	
λ			0.017			0.077
			(0.000)			(0.011)
Obj. Fun.	2.982	4.735	4.817	3.836	3.667	3.797

Table 6
Estimation: High versus Low Profitability Firms

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the top and bottom terciles of firms sorted by profitability. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

	Parameter Estimates					
	High Profitability Firms			Low Profitability Firms		
	TO	LE	MH	TO	LE	MH
α	0.789	0.788	0.712	0.575	0.570	0.575
	(0.003)	(0.001)	(0.001)	(0.001)	(0.000)	(0.002)
f	0.528	0.617	0.607	6.092	6.563	6.388
	(0.148)	(0.014)	(0.030)	(0.043)	(0.060)	(0.005)
ρ_z	0.779	0.728	0.845	0.600	0.542	0.513
	(0.002)	(0.003)	(0.001)	(0.005)	(0.002)	(0.008)
σ_z	0.308	0.266	0.237	0.177	0.272	0.236
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.004)
δ	0.162	0.176	0.154	0.078	0.063	0.072
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
ψ	0.134	0.117	0.139	0.058	0.089	0.071
	(0.001)	(0.003)	(0.001)	(0.004)	(0.001)	(0.001)
η	0.216	0.176	0.206	0.348	0.340	0.340
	(0.004)	(0.004)	(0.001)	(0.004)	(0.003)	(0.008)
ξ	0.532			0.651		
	(0.003)			(0.010)		
θ		0.668			0.699	
		(0.010)			(0.012)	
λ			0.035			0.148
			(0.000)			(0.004)
Obj. Fun.	2.593	3.218	10.251	3.538	6.248	3.130

Table 7
Estimation: Full Compustat Sample

The table reports parameter estimates for the tradeoff (TO), limited enforcement (LE), and moral hazard (MH) models using empirical policy functions. All models are estimated on the full sample of public firms from Compustat. α denotes the curvature of the production function, f the fixed production cost, ρ_z the persistence of the profitability shock, σ_z the volatility of the profitability shock, δ the depreciation rate, ψ the capital adjustment cost parameter, η the size of the iid shock to profits, ξ the recovery rate parameter in the tradeoff model, θ the tangibility parameter in the limited enforcement model, and λ the diversion parameter in the moral hazard model. Obj. Fun. denotes the goodness of fit measures as the minimized criteria for the empirical policy function estimation, multiplied by one hundred. Standard errors are in parentheses.

	Parameter Estimates: Public Firms		
	TO	LE	MH
α	0.765	0.797	0.814
	(0.001)	(0.000)	(0.000)
f	0.666	0.823	0.849
	(0.047)	(0.030)	(0.008)
ρ_z	0.853	0.649	0.732
	(0.001)	(0.002)	(0.001)
σ_z	0.334	0.349	0.277
	(0.000)	(0.002)	(0.000)
δ	0.160	0.141	0.180
	(0.000)	(0.001)	(0.000)
ψ	0.135	0.195	0.165
	(0.004)	(0.002)	(0.000)
η	0.319	0.216	0.243
	(0.002)	(0.002)	(0.001)
ξ	0.577		
	(0.000)		
θ		0.706	
		(0.007)	
λ			0.028
			(0.000)
Obj. Fun.	2.879	3.779	18.388

Table 8
Model Comparison

The table reports the results of the Rivers and Vuong (2002) statistical tests to compare the tradeoff, limited enforcement, and moral hazard model on different subsamples of firms. TO denotes the tradeoff model, LE denotes the limited enforcement model, and MH the moral hazard model. Data of public firms are from Compustat and cover an unbalanced panel of 39,433 firm-year observations from 1965 to 2015. Data on private firms are from Orbis and cover an unbalanced panel of 83,823 firm-year observations from 2003 to 2012. Subsamples of small vs large, profitable vs unprofitable, and high vs low leverage firms are obtained from Compustat. Firms are split according to firms characteristic in correspondence to the top and bottom tercile respectively. (***), (**) and (*) denote statistical significance at the one, five, and ten percent levels respectively.

	Comparison			
	TO vs LE	TO vs MH	LE vs MH	Best Fit
Large	TO***	TO***	LE***	TO
Small	LE***	TO***	LE***	LE
Private	TO*	MH***	MH***	MH
High Leverage	TO***	TO***	LE*	TO
Low Leverage	LE*	MH*	LE*	LE
High Profitability	TO***	TO***	LE***	TO
Low Profitability	TO***	MH**	MH***	MH
All Public Firms	TO***	TO***	LE***	TO

Table 9

Counterfactuals: Autarky versus No Financial Constraints

The table reports values for Tobin's Q from the following counterfactual exercise. For the subsamples of large, small, and private firms, data are simulated from all models (tradeoff, TO; limited enforcement, LE; moral hazard, MH) under the estimated parameters under three specifications, denoted as "Autarky", "Baseline", and "Unconstrained". "Autarky" refers to a model in which the firm has no access to external financing, "Baseline" to the estimated specification, and "Unconstrained" to a model in which the firm faces no financing frictions (i.e. the dividend non-negativity constraint is relaxed). Tobin's Q is computed as the market value of assets divided by the book value of assets.

	Counterfactuals: Tobin's Q								
	Large Firms			Small Firms			Private Firms		
	TO	LE	MH	TO	LE	MH	TO	LE	MH
Autarky	0.84	0.40	0.88	1.08	1.07	0.88	0.35	1.36	1.04
Baseline	1.88	1.06	1.02	2.60	3.52	1.00	0.61	1.78	3.91
Unconstrained	2.70	1.65	3.41	3.62	4.40	4.69	1.29	2.09	4.76

Table 10
Counterfactuals: Severity of Financing Constraints

The table reports values for Tobin's Q from the following counterfactual exercise. For the subsamples of large, small and private firms, data are simulated from all models under the benchmark estimated parameters for the best fitting model, namely tradeoff (TO) for large public firms, limited enforcement (LE) for small public firms, and moral hazard (MH) for private firms. The financing constraint parameters ξ in the tradeoff model, θ in the limited enforcement model, and λ in the moral hazard model are instead set to their estimated values for the corresponding subsample. Tobin's Q is computed as the market value of assets divided by the book value of assets.

	Counterfactuals: Tobin's Q		
	Large Firms	Small Firms	Private Firms
TO	1.88	2.35	2.03
LE	1.19	3.52	2.21
MH	1.02	1.00	3.91

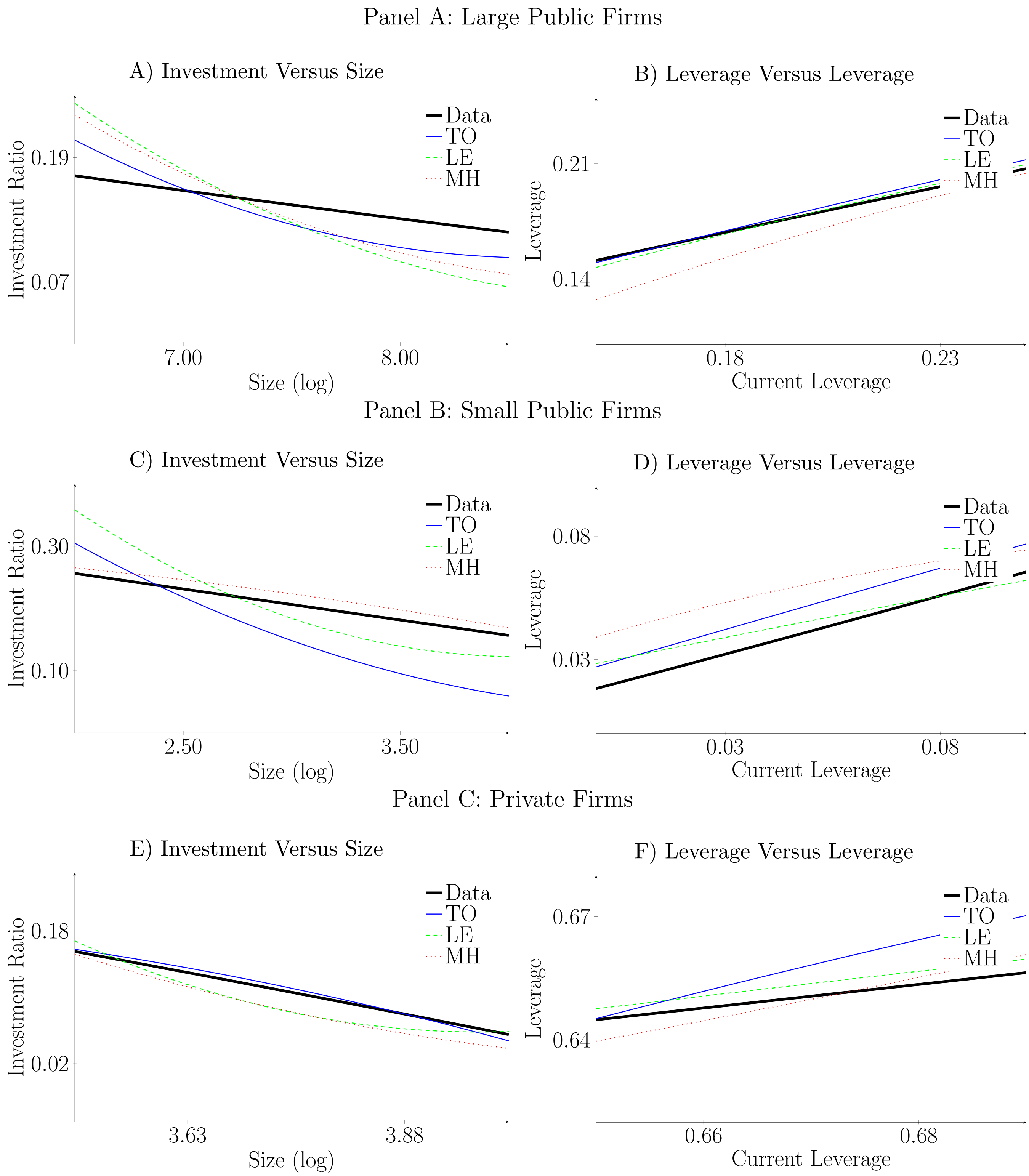

Fig. 1. Model Validation. The figure depicts the relation between the predicted empirical policy functions for the sample of firms described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample. Model parameters are set to their estimated values. 'Data' refers to the real data, 'TO' to the tradeoff model, 'LE' to the limited enforcement model, and 'MH' to the moral hazard model.

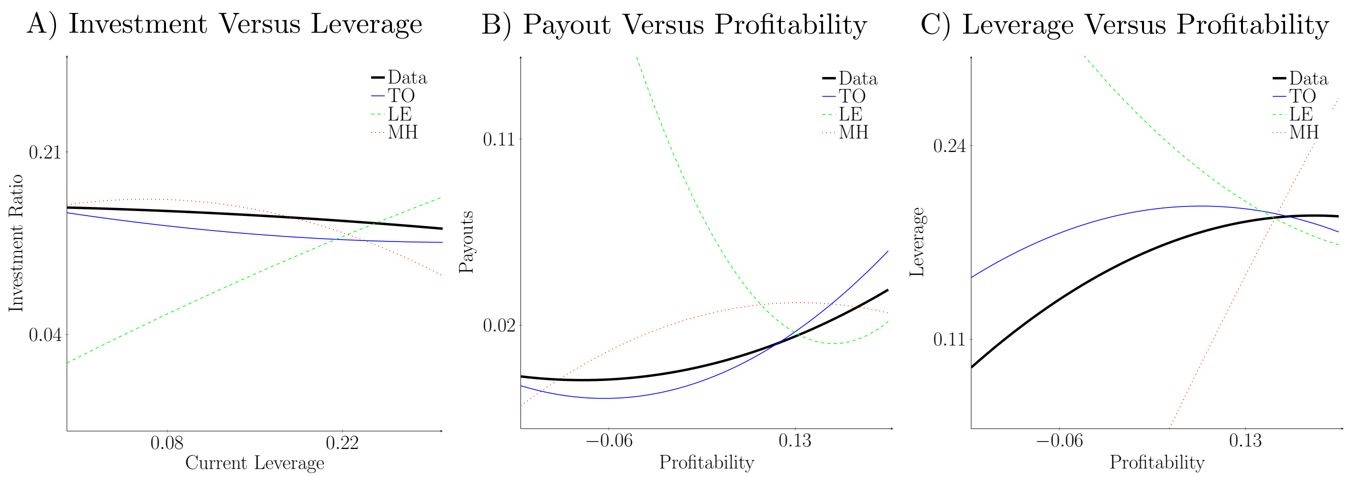

Fig. 2. Model Identification–Large Public Firms. The figure depicts the relation between the predicted empirical policy functions for the sample of large Compustat firms described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample. Model parameters are set to their estimated values. 'Data' refers to the real data, 'TO' to the tradeoff model, 'LE' to the limited enforcement model, and 'MH' to the moral hazard model.

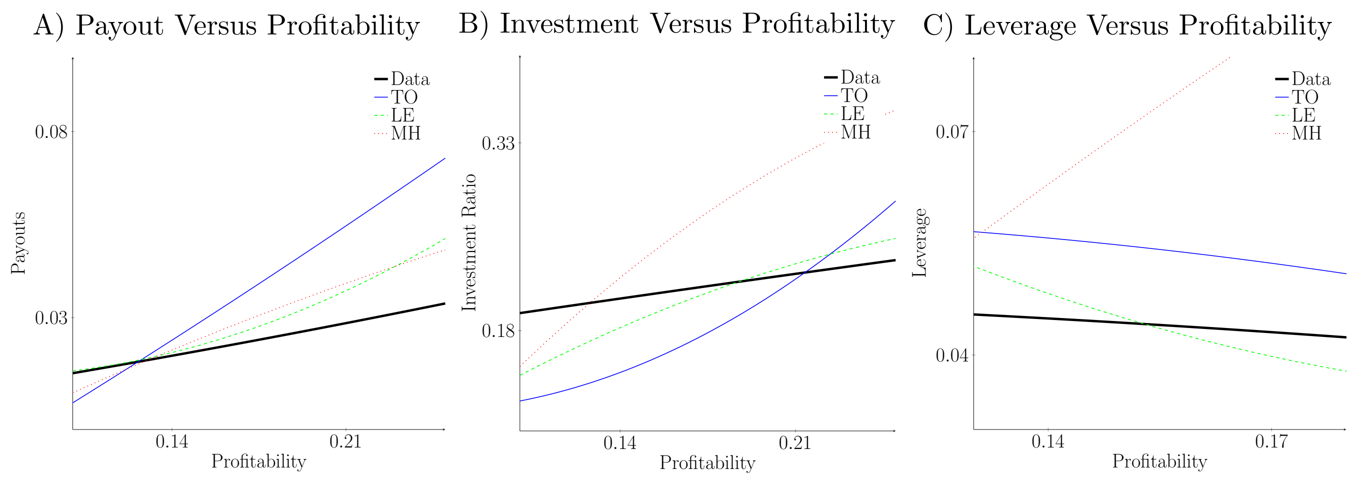

Fig. 3. Model Identification–Small Public Firms. The figure depicts the relation between the predicted empirical policy functions for the sample of small Compustat firms described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample. Model parameters are set to their estimated values. 'Data' refers to the real data, 'TO' to the tradeoff model, 'LE' to the limited enforcement model, and 'MH' to the moral hazard model.

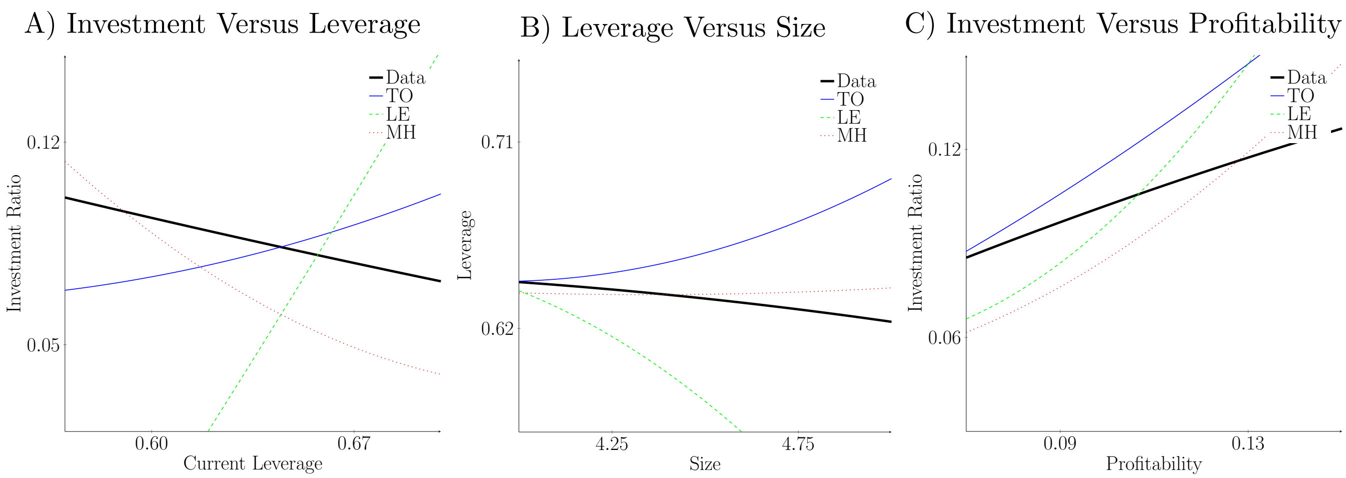

Fig. 4. Model Identification–Private Firms. The figure depicts the relation between the predicted empirical policy functions for the sample of private firms described in Section 4. Empirical policy functions are constructed as described in Section 3, where the transformed state variables that are not plotted are fixed to their average value in the sample. Model parameters are set to their estimated values. 'Data' refers to the real data, 'TO' to the tradeoff model, 'LE' to the limited enforcement model, and 'MH' to the moral hazard model.