Sequencing bilateral negotiations with externalities*

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Preliminary Version

Abstract

We study the optimal sequence of bilateral negotiations between one principal and two agents, whereby the agents have different bargaining power. The principal chooses whether to negotiate first with the stronger or the weaker agent. We show that the joint surplus is highest when the principal negotiates with the stronger agent first, independent of externalities between agents being positive or negative. The sequence chosen by the principal maximizes the joint surplus if there are negative externalities. Instead, if externalities are positive, the principal often prefers to negotiate with the weaker agent first. We also demonstrate that the sequence can be non-monotonic in the externalities and provide conditions for simultaneous timing to be optimal.

Keywords: bargaining, sequential negotiations, externalities, bilateral contracting, endogenous timing

JEL-codes: C72, C78, D62, L14

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1 Introduction

In many situations, a principal needs to negotiate with several agents, and the outcome of the negotiation between the principal and one agent imposes externalities on the other agents. Examples include the following situations:

1. Vertical relations between a supplier and retailers who compete in the consumer market. Externalities between the retailers are negative, if they sell substitutes, but positive if they sell complements.

2. A seller of a product contracts with R&D firms (e.g., research labs) to improve the product's quality. Again, externalities between R&D firms can be negative (e.g., because research labs provide similar quality improvements) or positive (e.g., because one improvement makes the other more effective).

3. An entrepreneur negotiates with venture capitalist firms, business angels, but also distribution partners to bring a new product to the market. The negotiation partners either benefit from each other, as in case of borrowers benefiting when the entrepreneur reaches an agreement with a distributor (positive externalities), or compete against each other, as in case of different venture capitalist firms (negative externalities).

A salient feature in these settings is that the principal often bargains with each agent bilaterally (e.g., because it is too costly to bring all agents together). An important strategic choice of the principal is then the sequence in which these negotiations are conducted. A key variable driving this choice is the bargaining power of an agent. Specifically, the question arises if the principal prefers to bargain first with a strong agent and later with a weak agent or if the reverse order is optimal. In this paper, we study this choice and analyze if the sequence chosen by the principal corresponds to the efficient sequence. We determine if and how the choice and its efficiency consequences depend on the externalities between agents.

We consider a stylized model with transferable utility where a principal bargains with two agents who differ in their bargaining power. Bargaining is modeled as random proposer take-it-or-leave-it bargaining.[1] The principal chooses with which agent to bargain first. We focus on the case where negotiations are over binding contracts that fix a vector of quantities and a transfer, and do not condition on any actions taken in the other negotiation. While there is, in general, an incentive to renegotiate a contract signed in the first negotiation (or to reopen failed negotiations) after the principal has come to an agreement with the second

[1]Since the game has transferable utility, results are equivalent if negotiations were modeled according to an asymmetric Nash Bargaining Solution. Therefore, our model can also be interpreted in this way.

agent, in practice, requirements of time or significant legal costs often make renegotiation difficult. We focus on the case where no renegotiation is possible.

We study which sequence of negotiations maximizes the payoff of the principal, and which maximizes welfare (defined as the joint surplus of all three players). To trace out the effect of unequal bargaining power, we derive our main results under the assumption that agents are symmetric except for bargaining power. To keep the model as simple as possible, an agent's bargaining power is modeled as the probability of making the offer. We assume that there are no externalities on the nontraders, that is, if the principal fails to come to an agreement with one agent, then this agent's payoff is independent of the outcome of the negotiation with the other agent. This assumption seems to be natural in the examples given above.

We first show that welfare is maximized if the principal bargains first with the agent who has higher bargaining power. This result holds under very general assumptions on the payoff functions and is independent of externalities between agents being positive or negative. The intuition is easiest to grasp in the extreme case in which one agent has no bargaining power. When bargaining with this agent in the second stage, the principal obtains the full surplus. Therefore, he will take the externalities that arise from the negotiation in the first stage fully into account, and there is no distortion in the first stage. As a consequence, joint surplus is higher when the principal negotiates first with the agent who has some bargaining power. We show that this insight carries over to the case in which both agents have positive bargaining power but one of them is the stronger bargainer, as long as both agents are symmetric but for bargaining power.

We then look at the sequence chosen by the principal. We find that the principal chooses the surplus maximizing sequence if externalities are negative, but may choose an inefficient timing when externalities are positive. There are two main effects which drive our results. First, the payoff of the agent with whom the principal bargains first depends, because of the externalities, on the bargaining outcome in the second negotiation, which will be anticipated during the first negotiation. Therefore, if the principal is the proposer in the first negotiation, the transfer she can demand from the agent also depends on the anticipated externalities. In contrast, when the agent is the proposer in the first negotiation, the principal is negotiated down to her outside option, which is the payoff she can achieve by rejecting the offer of the first agent and negotiating only with the remaining agent, and does not include any externality. When the principal bargains with the weak agent first, there is a high probability that she is the proposer in the first stage and hence will bear the externality herself. Therefore, bargaining with a weak agent in the first stage is attractive for the principal when externalities are positive. Conversely, bargaining with the strong agent first is attractive if the externalities are negative.

We call this the *anticipated externality effect.* It completely determines the principal's choice of timing in a setting where the bilateral negotiations are about a monetary transfer and a binary decision, such as whether or not the agent participates in a joint project, and participation is always optimal. In this setting, the principal prefers to bargain with the stronger agent first when externalities are negative, and with the weaker agent first with positive externalities.

Typically, however, bargaining is not merely about zero/one decisions. In vertical relations, for example, various quantities can be sold from the upstream firm to the downstream firms. Then a second and somewhat more subtle effect arises. As explained above, when the principal bargains with the stronger agent first, it is likely that the principal is negotiated down to her outside option, which is equal to her payoff when she bargains only with the weaker agent. In such a negotiation with the weaker agent, it is likely that the principal makes the offer, and thereby obtains the maximum surplus achievable with just one agent. In contrast, when the principal bargains with the weaker agent first, it is likely that she proposes in the first negotiation, but is the responder in the second negotiation. This leaves her with her second-stage outside option. It is also the joint surplus of the principal together with one agent, but it is not maximum of this surplus, because the decisions taken in the first stage do not in general maximize the principal's second-stage outside option.

Because of this *outside-option* effect, the principal prefers to bargain with the stronger agent first when there are no externalities, and the two bilateral bargaining problems interact through the principal's payoff function. For example, this is the case in vertical relations where the downstream firms sell to different markets, and the cost of producing the goods sold to one downstream firm depend on the quantities sold to the other. Similarly, the principal prefers to bargain with the stronger agent first when externalities are negative, since both effects point in the same direction.

With positive externalities, however, the two effects oppose each other, and in general the principal may prefer either timing. We show that the principal prefers to bargain with the weaker agent first, resulting in an inefficient timing, when the externalities are positive but 'small' and the principal's payoff function is additively separable. Moreover, the principal also prefers to bargain with the weaker agent first when there are strong positive externalities, but equilibrium quantities bounded, and therefore the importance of the outside option effect is limited.

Finally, we consider simultaneous negotiations. We show that for negative externalities, the sequential timing in which the principal bargains with the stronger agent first, dominates the simultaneous timing. The same holds true for large positive externalities due to the efficiency considerations described above. However, with positive externalities, the simultaneous timing becomes optimal for the principal. The intuition is rooted in the fact that with simultaneous bargaining agents cannot observe the outcome in the other negotiation. Each agent suppose that an agreement will be reached there (as is true on the equilibrium path). With sequential negotiations, the agent bargaining at the second stage can observe if the bargainers in the first stage failed to reach an agreement. With positive externalities this implies that the principal when being selected as the proposer, can extract more surplus from the agent in the simultaneous timing. Although disagreement does not happen on the equilibrium path, this effect increases the outside option of the principal.

Related literature. Our paper relates to a growing literature on one-to-many negotiations. Stole and Zwiebel (1996), Cai (2000), and Bagwell and Staiger (2010) study one-to-many negotiations in different situations, such as bargaining between a firm and several workers, an buyer and multiple sellers, or between countries, respectively. These papers focus on an exogenously given bargaining sequence.

Several recent papers analyze the sequencing of negotiations. Noe and Wang (2004) consider a situation in which the principal can keep the order of negotiations confidential, and determine conditions for efficient equilibria to exist.[2] Agents are symmetric in their model. Marx and Shaffer (2007, 2010) study a buyer who bargains with two sellers, and allow for contracts conditioning on the quantity supplied by both sellers. The cost function of a seller depends only on own quantity, implying that there are no direct externalities.[3] They show that in this situation, the payoff of a seller can be decreasing in own bargaining power. Krasteva and Yildirim (2012b) analyze a model in which a buyer negotiates with two sellers supplying complementary products and the buyer's valuation for the stand-alone products are uncertain. They show that the optimal sequence depends on the extent of complementarity and the difference in bargaining power. Xiao (2015) endogenizes the bargaining order in the model of Cai (2000), in which a buyer negotiates with several sellers who own perfectly complementary goods. He shows that the buyer wants to negotiate with small sellers first.[4]

Another strand of the literature analyzes simultaneous versus sequential negotiations. Horn and Wolinsky (1988) study the situation of a union bargaining over wages with two competing firms, and find that sequential bargaining is always preferred for the union.[5] Marshall and Merlo (2004) consider pattern bargaining (i.e., the first agreement sets the

[2] Krasteva and Yildirim (2012a) provide a complementary analysis and e.g., distinguish between exploding and non-exploding offers.

[3] Raskovich (2007) also considers the case wthout direct externalities and focuses on private contracts between buyer-seller pairs.

[4] Sequencing has also been studied in the literature on agenda formation (e.g. Winter 1997, Inderst 2000). However, sequencing here refers to the order of different issues.

[5] See Banerji (2002) for a related analysis.

pattern for all subsequent negotiations) and demonstrate how it affects the optimal structure of negotiations. Guo and Iyer (2013) analyze a supplier selling through two competing retailers and allow for renegotiation. They demonstrate that the optimal sequencing choice of the supplier depends on the size difference between buyers.

The literature that is connected closest to our paper is the one on contracting with externalities. In most of this literature, one side has all the bargaining power. For example, the seminal papers by Segal (1999, 2003) analyze the *offer game* where the principal has all the bargaining power. In this context, Möller (2007) studies the principal's choice of simultaneous versus sequential offers. He focuses on the impact of early negotiations on the outside option of the agents who bargain later and shows that if externalities are declining in the amount of trade, simultaneous contracting is optimal for the principal. Genicot and Rey (2006) also analyze contracting over time and demonstrate how the principal extract most surplus from agents by combining simultaneous and sequential offers. Instead, Bernheim and Whinston (1986) study the *bidding game* where the agents make the offers. Contrary to these papers, we consider a situation with intermediate bargaining power and demonstrate how the bargaining power affect the optimal negotiation sequence.

Galasso (2008) combines the offer and the bidding game in a sequential bargaining model along the lines of Rubinstein (1982), thereby allowing both sides to have bargaining power. He focuses on negative externalities between agents and shows that the principal's payoff can be decreasing in his bargaining power. In contrast our paper, he does not analyze sequencing of negotiations.

2 The Model

Assumptions. There are three players: a principal (A, "she") and two agents (B and C). A and B negotiate over a decision $b \in \mathcal{B} \subset \mathbb{R}_+^{n_b}$, with $0 \in \mathcal{B}$ and $n_B \in \mathbb{N}_+$, and a monetary transfer $t_B \in \mathbb{R}$ from B to A. Similarly, A and C negotiate over a decision $c \in \mathcal{C} \subset \mathbb{R}_+^{n_C}$, $0 \in \mathcal{C}$, $n_C \in \mathbb{N}_+$, and a transfer $t_C \in \mathbb{R}$. The payoff of the principal is $u_A(b, c) + t_B + t_C$, the payoffs of the agents are $u_B(b, c) - t_B$ and $u_C(b, c) - t_C$, respectively.

Negotiations are bilateral, and the order is chosen by A. Within each stage, there is random proposer take-it-or-leave-it bargaining.[6] Bargaining power is modelled as the probability of making the offer: B proposes with probability $\beta \in [0, 1]$, C proposes with $\gamma \in [0, 1]$. Without loss of generality, assume that $\beta \geq \gamma$; that is, B is the stronger bargainer among the agents. As it is the objective of the paper to analyze which agent the principal will approach

[6]Alternatively, one can think of the outcome of each negotiation as given by the asymmetric Nash bargaining solution (see, for example, Muthoo 1999). All our results then continue to hold.

first, we follow the literature on sequencing decisions and rule out renegotiation.[7]

The timing of the game is as follows. In stage 0, A chooses whether to bargain with B first (timing BC) or with C first (timing CB). In timing BC, in stage 1, A bargains with B. With probability β, B proposes a contract $(b, t_B) \in \mathcal{B} \times \mathbb{R}$, and A either accepts or rejects. With probability $1-\beta$, A proposes, and B then accepts or rejects. If A and B reach an agreement on a contract (b, t_B), the decision b is implemented and the transfer t_B is made. In case of rejection, $b = t_B = 0$. In $t = 2$, C observers the outcome of stage 1. Then A and C bargain. With probability γ, C proposes a contract $(c, t_C) \in \mathcal{C} \times \mathbb{R}$; with probability $1-\gamma$, A proposes. If they reach an agreement on a contract (c, t_C), the decision c is implemented and the transfer t_C is paid. Otherwise, $c = t_C = 0$. *Timing* CB is similar, except that A bargains with C in stage 1 and with B in stage 2.

In our bargaining game, the principal negotiates with one agent at a time. This is a very relevant situation in reality because negotiations often require physical presence of the principal and it is too costly to communicate to all agents at the same time.[8] However, there can be circumstances in which the principal can delegate the negotiations, which gives rise to the possibility of simultaneous negotiations. We will consider this case in Section 5. Moreover, we assume that the contract negotiated in stage 1 cannot condition on any actions chosen in stage 2, because of exogenous legal constraints, or other reasons for incomplete contracting. For example, if A is an upstream firm serving two retailers B and C, a contract between A and B that conditions on c might be in conflict with competition law. As noted by Möller (2007), in practice, contingent contracts are rare, and hard to enforce.

We assume that there are no externalities on the nontraders: $u_B(0, c)$ is constant in c, and $u_C(b, 0)$ is constant in b. Moreover, we normalize the utility functions such that $u_A(0, 0) = u_B(0, c) = u_C(b, 0) = 0$.

We say that b has negative (no, positive) externalities on C if $u_C(b, c) \leq (=, \geq)\, u_C(0, c)$ for $b > 0$. As b can be a vector, $b > 0$ means that $b_i \geq 0$ for all $i = 1, ..., n_B$ and $b_i > 0$ for at least one $i = 1, ..., n_B$. Similarly, c has negative (no, positive) externalities on B if $u_B(b, c) \leq (=, \geq)\, u_C(b, 0)$ for $c > 0$. Finally, there are negative (no, positive) externalities if b has negative (no, positive) externalities on C, and c has negative (no, positive) externalities on B.

Moreover, we say that b has strictly negative (strictly positive) externalities on C if $u_C(b, c) < (>)\, u_C(0, c)$ for $b > 0$ whenever $c > 0$, and (ii) c has strictly negative (strictly positive) externalities on B if $u_B(b, c) < (>) u_B(b, 0)$ for $c > 0$ whenever $b > 0$.

[7] See Möller (2007) or Montez (2014), among others, for reasons why renegotiation is often not possible.

[8] Due to this reason, many recent studies on bargaining such as Cai (2000), Noe and Wang (2004), and Krasteva and Yildirim (2012a,b) analyze sequential negotiations.

To isolate the impact of differences in bargaining power, our main results assume some degree of symmetry between players B and C. We say that *agents are symmetric except for bargaining power* if $\mathcal{B} = \mathcal{C}$ and for all $(b, c) \in \mathcal{B}^2$, (i) u_A is a symmetric function, i.e. $u_A(b, c) = u_A(c, b)$, and (ii) $u_C(c, b) = u_B(b, c)$. Note that under symmetry, b (c) has negative externalities on C (B) if and only if there are negative externalities, and similarly for positive externalities.

Define the joint surplus of all three players as $S(b, c) := \sum_{i \in \{A,B,C\}} u_i(b, c)$. We impose the tie-breaking rule that, if A is indifferent, but surplus is strictly higher in one of the timings, A selects the surplus-maximizing timing.

Example with a Supplier and Retailers. To give an interpretation, b and c could be quantities of goods, sold by a supplier A for fixed amounts of money t_B and t_C to retailers B and C. If retailers compete in quantities in the downstream market (and face no costs, for simplicity), the retailers' payoff functions u_B and u_C are $u_B = bp_B(b, c)$ and $u_C = cp_C(c, b)$, respectively, where p_B and p_C are the prices of the products which depend on the quantities of both retailers. If retailers sell substitutes, then the price p_B is falling in c (and, similarly, p_C is falling in b). By contrast, if products are complements, prices are increasing in the quantity of the other retailer. The function u_A describes the supplier's production costs and could be $u_A = -k(b, c)$.

Our setting can also accommodate strategic interaction after stage 2. For example, b and c could describe the unit prices constituting the variable parts of a two-part tariff, whereas t_B and t_C constitute the fixed parts. To be consistent with our interpretation that no agreement corresponds to a decision of zero, one should use an inverse scaling where the unit price to (say) agent B is equal to $1/b$ so that a decision $b = 0$ means that all goods are infinitely costly for B. After stage 2, retailers set their downstream prices, given the per-unit wholesale prices $1/b \equiv b'$ and $1/c \equiv c'$. Their utility function can then be written as $u_B = (p_B(b', c') - b')\, q_B(p_B(b', c'), p_C(c', b'))$ and $u_C = (p_C(c', b') - c')\, q_C(p_C(c', b'), p_B(b', c'))$. The function u_A summarizes the production cost of the supplier and the variable payments and could be written as $u_A = -k(q_B, q_C) + b'q_B + c'q_C$, with $q_B = q_B(p_B(b', c'), p_C(c', b'))$ and $q_C = q_C(p_C(c', b'), p_B(b', c'))$.

Preliminaries. Since within each stage there is take-it-or-leave-it-bargaining with transferable utility, the decisions reached in the stage maximize the joint expected surplus of the two bargaining players. Moreover, whoever proposes chooses the transfer such that the other player is just willing to accept.

Consider timing BC (timing CB can be analyzed similarly). In stage 2, the decision

b and transfer t_B are already fixed. The decision reached in stage 2 maximizes the joint surplus of A and C, given b. We assume that, for any b, there exists a unique

$$c^* (b) := \arg\max_{c\in\mathcal{C}} \{u_A (b, c) + u_C (b, c)\}.$$

Existence is ensured when (i) the sets $\mathcal{B}$ and $\mathcal{C}$ are finite, or (ii) the payoff functions u_i $(i = A, B, C)$ are continuous on $\mathcal{B} \times \mathcal{C}$ and the sets $\mathcal{B}$ and $\mathcal{C}$ are compact. A sufficient condition for uniqueness of decisions in case (ii) is that $u_A (b, c) + u_B (b, c)$ is strictly quasiconcave in b, and $u_A (b, c) + u_C (b, c)$ is strictly quasiconcave in c.

The expected payoff of A in stage 2 of timing BC is

$$(1 - \gamma) (u_A (b, c^* (b)) + u_C (b, c^* (b))) + \gamma u_A (b, 0) + t_B.$$

When $b = t_B = 0$, the expected payoff of A in stage 2 is

$$O_A^{BC} = (1 - \gamma) \max_{c\in\mathcal{C}} \{u_A (0, c) + u_C (0, c)\}.$$

This is the expected utility of A when the first stage negotiation with B fails; it therefore is the outside option of A in the first stage.

In the first stage of timing BC, the joint surplus of A and B consists of player B 's payoff, and the expected payoff of A in stage 2:

$$S_{AB}^{BC} (b) := u_B (b, c^* (b)) + (1 - \gamma) (u_A (b, c^* (b)) + u_C (b, c^* (b))) + \gamma u_A (b, 0). \quad (1)$$

In any equilibrium of timing BC, A and B reach a decision $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC} (b)$,[9] and the expected payoff of A is

$$U_A^{BC} = (1 - \beta) S_{AB}^{BC} \left(b^{BC}\right) + \beta O_A^{BC}.$$

In case that there exists several $b \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC} (b)$, note that they all lead to the same payoffs for A and B. In case they lead to a different joint surplus, we assume that a decision that maximizes $S (b, c^* (b))$ is selected. Therefore, the surplus in any equilibrium of timing BC is unique, even if the first-stage decisions are not unique. We impose the corresponding assumptions on timing CB, and denote the equilibrium first-stage decision in timing CB by c^{CB}.

[9] Existence of a maximum of $S_{AB}^{BC} (b)$ is ensured under the conditions discussed above (in case (ii), $c^* (b)$ is continuous by the Maximum Theorem, thus $S_{AB}^{BC} (b)$ is continuous, and a solution to $\max_{b\in\mathcal{B}} S_{AB}^{BC} (b)$ exists by the Weierstrass Theorem).

3 The surplus-maximizing sequence

The first-best surplus is

$$S^{FB} = \max_{b \in \mathcal{B},\, c \in \mathcal{C}} \left\{ u_A(b,c) + u_B(b,c) + u_C(b,c) \right\}.$$

There are two reasons why, in general, the equilibrium decisions are not surplus maximizing. The first is that the negotiation in the second stage maximizes the surplus of the two players involved, but does not take into account the effect of the decision on the agent with whom A has already signed a contract. This effect works through the externality of c on B in timing BC (and through the externality of b on C in timing CB). In the supplier-retailers example, agreeing on a larger quantity in the second-stage negotiation has a negative effect on the agent with whom A bargained first.

The second reason why equilibrium decisions are not maximizing industry surplus is because A only receives a fraction of the surplus in the second-stage negotiation. This implies that, in the first stage, A does only partially consider the second-stage surplus. Therefore, first-stage decisions may be distorted away from the surplus-maximizing outcome. This effect works through two channels. First, through the externality of b on C in timing BC (and through the externality of c on B in timing CB). In the example above, if A signs a contract with a large quantity in the first stage, the surplus A and her negotiation partner can achieve in the second stage is lower due to the negative externalities of the decisions. Second, through interaction of b and c in A's utility function. This occurs because the agent with whom A bargains in the second stage, extracts A's utility with some probability. In our example, suppose that A has a convex cost function and the negotiation sequence is BC. Then, the first-stage decision b might be chosen too high from the point of view of the joint surplus because with some probability A will not be the proposer in the second stage, implying that C has to bear this higher cost.

Remark 1 illustrates that these two effects are indeed the *only* reasons for inefficiencies. It shows that the equilibrium decisions maximize joint surplus in timing BC if $\gamma = 0$ (which shuts down the latter effect because A receives the full surplus in the negotiation with C) and c has no externality on B (which shuts down the former effect). Denote the joint surplus in timing BC by S^{BC}, and in timing CB by S^{CB}.

Remark 1 *Suppose that* $1 \geq \beta > \gamma = 0$, *and* c *has no externalities on* B. *Then* $S^{BC} = S^{FB} \geq S^{CB}$.

Proof. Consider timing BC. In the second stage, the decision reached is

$$\begin{aligned} c^*(b) &= \arg\max_{c\in\mathcal{C}} \{u_A(b,c) + u_C(b,c)\} \\ &= \arg\max_{c\in\mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\} \\ &= \arg\max_{c\in\mathcal{C}} S(b,c), \end{aligned}$$

since $u_B(b,c)$ is independent of c, b is predetermined from the first stage, and adding a constant does not change the location of the maximum. In the first stage, the decision maximizes the joint surplus $S_{AB}^{BC}(b)$ of A and B. Since $\gamma = 0$, $S_{AB}^{BC}(b) = S(b, c^*(b))$. Therefore, $S^{BC} = \max_{b\in\mathcal{B}} S(b, c^*(b)) = S^{FB} \geq S^{CB}$. ■

The next proposition shows that the insight derived in the remark also applies if C has some bargaining power (i.e., $\gamma > 0$) and agents are symmetric but for bargaining power.

Proposition 1 *(i) S^{BC} is decreasing in γ and constant in β. Similarly, S^{CB} is decreasing in β and constant in γ. (ii) Suppose that agents are symmetric except for bargaining power, and $1 \geq \beta > \gamma \geq 0$. Then $S^{BC} \geq S^{CB}$.*

Proof. See Appendix 8.1. ■

The Proposition shows that, under symmetry, surplus is higher when the principal bargains with the stronger agent first, irrespective of whether externalities are negative or positive.[10] The intuition is rooted in the effect that first-stage decisions do not maximize the joint surplus, as the distortions of these decisions is different in both timings. If the principal negotiates with the weaker agent in the second stage, she receives a larger share of the surplus in this stage. Therefore, the utility of the agent with whom the principal bargains in the second stage is taken into account to a larger extent in the first stage negotiation. This effect leads to a larger distortion when the bargaining power of the agent with whom the principal negotiates in stage 2 increases. By contrast, when agents are symmetric except for bargaining power, the effect that second-stage decisions ignore the utility of the agent with whom the principal bargained first, plays out similarly in the two timings. As a consequence, the joint surplus is higher in case the principal bargains with the weaker player in the second stage. This explains our main insight that the welfare-optimal bargaining sequence is BC independent of the externalities. As we proceed to show, the sequence preferred by the principal depends on the nature of externalities.

[10] Interestingly, it also does not matter whether the principal has more or less bargaining power than the agents, or one of them. Whenever $\beta \geq \gamma$, $W^{BC} \geq W^{CB}$, no matter whether the principal's bargaining power is higher or lower than the agents' bargaining power.

We finally note that while part (i) of Proposition 1 does not need symmetry, part (ii) does. In fact, if agents were asymmetric, industry surplus can be higher in timing CB than in timing BC.

4 The sequence preferred by the principal

We start this section by considering the special case in which $\beta = 1$, that is, B has all bargaining power. This case shows in a particularly transparent way how the externalities affect the principal's preference over the bargaining sequences.

Let U_A^{BC} $\left(U_A^{CB}\right)$ denote the expected payoff of A in timing BC (CB).

Remark 2 *Suppose that $\beta = 1$, $\gamma \in [0,1)$. If b has negative (no, positive) externalities on C, then $U_A^{BC} \geq U_A^{CB}$ $\left(U_A^{BC} = U_A^{CB},\ U_A^{BC} \leq U_A^{CB}\right)$. Moreover, when externalities are strictly negative (strictly positive) and equilibrium decisions in timing CB are not zero, then $U_A^{BC} > U_A^{CB}$ $\left(U_A^{BC} < U_A^{CB}\right)$.*

Proof. Since $\beta = 1$, $U_A^{BC} = O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\}$. In contrast, in timing CB, $U_A^{CB} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}$ where

$$b^*(c) = \arg\max_{b\in\mathcal{B}}\{u_A(b,c) + u_B(b,c)\}.$$

Therefore,

$$U_A^{BC} - U_A^{CB} = (1-\gamma)\left(\max_{c\in\mathcal{C}}\{(u_A(0,c) + u_C(0,c))\} - \max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}\right)$$

When there are negative externalities of b on C, then $u_C(0,c) \geq u_C(b,c)$ for all b, c. Hence $U_A^{BC} \geq U_A^{CB}$. Moreover, when externalities are strictly negative and $c \neq 0 \neq b^*(c)$, then $U_A^{BC} > U_A^{CB}$. The results on positive and no externalities can be established similarly. ■

The remark shows that for $\beta = 1$, the principal's preference is solely driven by the externality of b on C. The externality of c on B does not matter for the principal, because B has all the bargaining power and thus fully bears the externality himself. Why is the externality of b on C crucial for the principal's choice of the order of negotiation? When she bargains with C in the first stage of timing CB, they will anticipate the decision $b^*(c)$ taken in the second stage. Therefore, in case A proposes in the first stage, C will be willing to pay up to $u_C(b^*(c),c)$ to the principal, an amount that depends on the externality of b on C. In contrast, in timing BC, B will drive the principal down to her outside option in the first stage. This first-stage outside option depends on the surplus that A and C can achieve together. If A rejects in the first stage and consequently $b = t_B = 0$, the outside option is free from any external effect from b on C. The principal prefers timing CB if externalities are positive, because in this timing she can (with positive probability) gain the positive external effect of b on C for herself. In contrast, she prefers timing BC, which insulates her from the externality, when externalities are negative. Finally, if there are no externalities, the principal is indifferent.

Remarks 1 and 2 have a straightforward implication for the efficiency of equilibrium timing in the case where B has all the bargaining power and C has no bargaining power.

Remark 3 *Suppose that $\beta = 1$, $\gamma = 0$, and c has no externalities on B. The equilibrium timing is efficient if b has negative externalities or no externalities on C. If b has positive externalities on C, the equilibrium timing is inefficient, unless the principal is indifferent between the two timings.*

Proof. By Remark 1, $S^{BC} \geq S^{CB}$. Suppose that b has negative externalities, or no externalities, on C. By Remark 2, $U_A^{BC} \geq U_A^{CB}$. Moreover, we assumed that if $U_A^{BC} = U_A^{CB}$ but $S^{BC} > S^{CB}$, A selects the timing BC. It follows that the equilibrium timing is surplus maximizing. Now suppose that b has positive externalities on C. By Remark 2, $U_A^{BC} \leq U_A^{CB}$. Thus, if the principal is not indifferent between the timings, $U_A^{BC} < U_A^{CB}$. ■

We now turn to the analysis of the case in which the bargaining power of both agents is strictly below 1. In particular, we are interested whether the conclusions of Remark 2 need to be modified if $\beta < 1$. To isolate the effect of differing bargaining power, we focus our analysis on the symmetric case, that is, agents are symmetric but for bargaining power. The symmetry of the agents has two implications that will be used frequently below. First, the second-stage decision ensuing after any first-stage decision $x \in \mathcal{B} = \mathcal{C}$ is the same in both sequences:

$$\arg\max_{c \in \mathcal{C}} \{u_A(x, c) + u_C(x, c)\} = \arg\max_{b \in \mathcal{B}} \{u_A(b, x) + u_C(b, x)\} =: f(x) \tag{2}$$

Equation (2) shows the equilibrium second-stage decision as a function of the first-stage decision x; under symmetry, it is the same function f in both timings. Second, symmetry implies that

$$\max_{c \in \mathcal{C}} \{u_A(0, c) + u_C(0, c)\} = \max_{b \in \mathcal{B}} \{u_A(b, 0) + u_B(b, 0)\}. \tag{3}$$

This is the joint surplus that A and one agent can achieve together, given that the other agent does not participate. Under symmetry, it does not depend on the identity of the agent. For future reference, we call (3) the *one-agent surplus*.

Our first main result concerns negative externalities.

Proposition 2 *Assume that the agents are symmetric except for bargaining power, and* $1 > \beta > \gamma$*. If externalities are negative, then* $U_A^{BC} \geq U_A^{CB}$*, with strict inequality if externalities are strictly negative and equilibrium decisions are not zero.*

Proof. See Appendix 8.2. ■

In contrast to the result that timing BC maximizes industry surplus (Proposition 1ii), Proposition 2 is *not* driven by the fact that the first stage decision internalizes more of the second agent's utility in timing BC. Indeed, within each timing, the equilibrium first stage decision is optimal for A: either A proposes in the first stage and proposes what is best for her, or A responds in the first stage and her utility is determined by her outside option, which is independent of the first-stage decision.

Since the first-stage decision is optimal for A, the expected payoff of A in equilibrium of timing BC is at least as high as it would be if, hypothetically, in timing BC the equilibrium first-stage decision of the other timing, c^{CB}, was chosen. The proof of Proposition 2 shows that even this lower bound on A 's payoff in timing BC is higher than her payoff in CB. To understand the economics behind this, suppose for a moment that first-stage decisions were equal, say $b^{BC} = c^{CB} = x$, and consider the principal's payoff in the following cases (see Table 1).

- Suppose that A has proposed in both stages. Then she receives the whole surplus S, which is the same in the two timings by symmetry and since by assumption first-stage decisions are equal. Moreover, this case has the same probability in both timings; thus, it cancels when comparing the timings.

- Now consider the payoff of A in case that A has proposed in stage 1 and responded in stage 2. This is equal to her outside option in the second stage, given the first stage decision x has been implemented and the first stage transfer has been paid:

$$u_A(x, 0) + u_B(x, f(x)) \tag{4}$$

  Because (4) is the outside option of the principal in the second stage, conditional on having proposed in the first stage, we call (4) the *conditional second-stage outside option.*

Table 1: Payoff of A, given $b^{BC} = c^{CB} = x$

	Timing BC	Timing CB
A proposes in both stages Probability	$S(x, f(x))$ $(1-\beta)(1-\gamma)$	$S(f(x), x)$ $(1-\beta)(1-\gamma)$
A proposes in stage 1, responds in stage 2 Probability	$u_A(x,0) + u_B(x, f(x))$ $(1-\beta)\gamma$	$u_A(0,x) + u_C(f(x), x)$ $(1-\gamma)\beta$
A responds in stage 1 Probability	$(1-\gamma)\max_c\{u_A(0,c) + u_C(0,c)\}$ β	$(1-\beta)\max_b\{u_A(b,0) + u_B(b,0)\}$ γ

- Finally, suppose that A responds in the first stage. Then her expected payoff equals her first-stage outside option, which is equal to the probability that A proposes in the second stage times the one-agent surplus (3).

By symmetry, the expected payoff of A can in both timings be written as a weighted sum of the surplus, the conditional second-stage outside option (4), and the one-agent surplus (3). In timing BC, the one-agent surplus has more weight because A bargains with the stronger agent first. Vice versa, in CB the conditional second-stage outside option has more weight. Therefore, a comparison of these expressions is key for understanding the principal's preferred sequence.

The one-agent surplus differs in two respects from the conditional second-stage outside option:

1. The *anticipated externality effect*: In the one-agent surplus, the other agent's decision is fixed at zero, and no externality needs to be taken into consideration. In contrast, the conditional second-stage outside option includes the externality, as can be seen in the term $u_B(x, f(x))$ (respectively, $u_B(f(x), x)$) in Table 1. The reason is that it includes the transfer from the agent with whom A has bargained in stage 1. In fact, this agent anticipates that A will reach an agreement with the other agent in the second stage and will therefore take the decision $f(x)$ into account.

2. The *outside option effect*: As can be seen from (3), in the one-agent surplus, the two bargainers maximize the joint utility. By contrast, the first-stage decision x does not, in general, maximize the conditional second-stage outside option because it maximizes the joint utility of the bargainers in the first stage, under the condition that there will be reached an agreement in the second stage. Therefore, when there are no externalities, the one-agent surplus is greater than the conditional second-stage outside option.

With negative externalities, these two effects point in the same direction, which implies that $\max_c\{u_A(0,c)+u_C(0,c)\} > u_A(0,x)+u_B(f(x),x)$, and due to symmetry, this is equivalent to $\max_b\{u_A(b,0)+u_B(b,0)\} > u_A(x,0)+u_C(x,f(x))$. Because $\beta > \gamma$, the payoff in the third line of Table 1 will be reached with a larger probability in timing BC than in timing CB, and this payoff is larger than the payoff in the second line of Table 1. As a consequence, the principal prefers to bargain with the stronger agent first.

We now turn to the case in which there are no externalities between agents. As demonstrated in Remark 2, if $\beta = 1$, then the principal is indifferent between the two timings. The reason is that, when $\beta = 1$, the outside option effect is zero: the equilibrium decision in the first stage of timing CB maximizes A 's conditional second-stage outside option because A anticipates that she will (with probability 1) respond in the second stage, and, thus, be negotiated down to her outside option. However, this is no longer true if $\beta < 1$. Even without externalities, the two bargaining problems are not independent of each other because the decisions b and c interact through the principal's payoff function. Although the anticipated externality effect is not at work, the outside option effect is for $\beta < 1$. As the the one-agent surplus is higher than the conditional second-stage outside option, and A obtains this surplus with a higher probability in timing BC than in timing CB, she prefers the former. Moreover, our next result shows this preference is strict if either the first-stage decisions do not coincide with the decision that maximizes the second-stage outside option, or the first-stage decisions differ across timings.

Proposition 3 *Assume agents are symmetric except for bargaining power, there are no externalities, and* $1 > \beta > \gamma$. *Then* $U_A^{BC} \geq U_A^{CB}$. *Moreover, the inequality is strict if either* $b^{BC} = c^{CB} \neq f(0)$, *or* $b^{BC} \neq c^{CB}$ *i.e. first-stage decisions in the two timing differ from each other. A sufficient condition for the latter is that (i) equilibrium first-stage and second-stage decisions are interior, (ii)* u_A, u_B, u_C *and* $c^*(b)$ *are differentiable, and (iii) whenever* $c \neq c'$, *then for any* $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ *there exists some* $i = 1,...,n_B$, *such that*

$$\frac{\partial}{\partial b_i} u_A\left(b^{BC}, c\right) \neq \frac{\partial}{\partial b_i} u_A\left(b^{BC}, c'\right). \tag{5}$$

Proof. See Appendix 8.3 ■

Conditions (i)-(iii) are used to ensure that the first-stage decisions in the two timings problems differ from each other.[11] We point out that (iii) will be satisfied in many economic applications. A sufficient condition for (iii) is that the marginal returns to some b_i are strictly monotone (increasing or decreasing) in c.[12] It is satisfied in our example with a supplier and two retailers, when A has strictly increasing marginal costs. Assumption (iii) rules out the case of an additively separable u_A where there is no interaction between the bargaining problems. Assumption (iii) alone is not sufficient to rule out the possibility that first-stage decisions might be identical in the two timings, be it because they occur at a boundary of the feasible set, or because the payoff functions are not differentiable; assumptions (i) and (ii) serve to rule these possibilities out.[13]

We now turn to the case of positive externalities. As shown above, with $\beta = 1$, the principal unambiguously prefers timing CB. This is no longer true if $\beta < 1$. In fact, both timings BC and CB can emerge in equilibrium, since the anticipated externality effect and the outside option effect work in opposite directions. Positive externalities incline the principal towards CB, but as above, the outside option effect favors BC.

When the positive externalities override the outside option effect, A prefers timing CB. A simple example is when all decisions are binary. Suppose that $\mathcal{B} = \mathcal{C} = \{0, 1\}$, with interpretation that $b = 1$ ($c = 1$) indicates that B (C) participates in a joint project, or the sale of some indivisible object between A and B (C). Moreover, suppose that participation is optimal in every subgame of every timing. Then the equilibrium first stage decision is to participate, and participation also maximizes the conditional second-stage outside option (formally, $b^{BC} = c^{CB} = f(0) = 1$). Thus the outside option effect is zero, and the principal's preferences are pinned down by the externalities: if externalities are strictly positive (absent, negative), A strictly prefers CB (is indifferent between timings, strictly prefers BC).

In general, however, the principal's preference may go either way with positive externalities. Although the anticipated externality effect favors timing CB, it is clear from above that, when u_A is not additively separable, the principal may well have a strict preference for

[11] More generally, the proof of Proposition 3 shows that, if there are no externalities, and $b^{BC} = c^{CB}$, and c^{CB} maximizes $(u_A(0, c) + u_C(0, c))$, then $U_A^{BC} = U_A^{CB}$. This is the case in Krasteva and Yildirim (2012a) in the benchmark case with commonly known valuations.

[12] This sufficient condition, however, rules out some economically interesting cases covered by (iii). For example, (iii) is also satisfied when $u_A(b, c) = -\sum_{i=1}^{n} (b_i + c_i)^2$. Here, there is no single good i such that the marginal returns to b_i are strictly monotone in c. Moreover, (iii) assumes that marginal returns are unequal, not that they are monotone.

[13] Similarly, Edlin and Shannon (1998) rely on interiority and differentiability assumptions for strictly monotone comparative statics.

BC. After all, under the conditions given in Proposition 3, she strictly prefer BC when externalities are absent. Therefore, she may also strictly prefer BC when the externalities are positive but sufficiently small.

On the other hand, we show next that when u_A is additively separable, the problem is sufficiently smooth, and equilibrium decisions are strictly positive, the principal strictly prefers CB when externalities are positive but small. To make this precise, we start by giving more structure to the utility function by considering the case of "parametric externalities".

Case of parametric externalities. *The utility functions of B and C are parametrized by $k \in \mathbb{R}$ and written $u_B(b,c,k)$ and $u_C(b,c,k)$. k parametrizes the importance of externalities in the following sense: (1) u_A is constant in k; (2) if $k=0$ there are no externalities, thus $u_B(b,c;0)$ is constant in c; (3) k has no effect on u_B when $c=0$;*[14] *(4) for all $b>0$, all c and $c'>c$, $u_B(b,c';k)-u_B(b,c;k)$ is strictly increasing in k. Since $u_B(b,c';0)=u_B(b,c;0)$, it follows that, for all $k>0$, $u_B(b,c';k)>u_B(b,c';0)$.*

We employ the slightly stronger assumption[15] *that u_B is differentiable in k and*

$$\frac{\partial u_B(b,c;k)}{\partial k} > 0, \tag{6}$$

whenever $b>0$ and $c>0$.[16]

In the following Proposition, let $c^*(b,k) := \arg\max_{c\in\mathcal{C}}(u_A(b,c)+u_C(b,c,k))$ denote the second -stage decision in timing BC, and define $b^*(c,k)$ similarly.

Proposition 4 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power, $1>\beta>\gamma$, u_A is additively separable, and (i) u_i $(i=A,B,C)$ is C^1 in (b,c,k), (ii) $c^*(b,k)$ is interior and C^1 in (b,k), and (iii) $\mathcal{B}=\mathcal{C}$ is compact. Then, there exists exists a $\hat{k}>0$ such that $U_A^{BC}<U_A^{CB}$ for all $k\in\left(0,\hat{k}\right)$.*

Proof. See Appendix 8.4. ■

The proof of Proposition 4 relies on a version of the envelope theorem that does not assume differentiability of the first stage decision. Here we describe the intuition for Proposition 4 under the additional assumption that the first-stage decisions are interior and differentiable in k. The key is again a comparison of the conditional second-stage outside option (4) with the one-agent surplus (3).

[14] This assumption is motivated from the idea that k should parametrize externalities *and nothing else.*

[15] The issue is that $u_B(b,c',k)-u(b,c,k)$ could have a zero derivative with respect to k on sets of measure zero.

[16] We note that if $u_B(b,c;k)$ is differentiable in c, then condition *(4)* together with $\partial u_B(b,c;k)/\partial k>0$ implies $\partial^2 u_B(b,c;k)/(\partial c\partial k)>0$.

As explained in the discussion after Proposition 2, within each timing, the first-stage decision is optimal for the principal. Therefore, the principal's payoff of in timing CB is bounded below by the payoff she would obtain if, in the first stage of this timing CB, the first-stage decision of the other timing b^{BC} would be chosen. This lower bound is, in turn, strictly higher than the principal's equilibrium payoff in timing BC, if the conditional second-stage outside option (4), evaluated at $x = b^{BC}$, is strictly higher than the one-agent surplus (3). At $k = 0$, these two are equal since the bargaining problems do not interact (as u_A is additive separable). Consider what happens when k is marginally increased, so that there are small positive externalities. The one-agent surplus is unaffected. By contrast, k affects the conditional second-stage outside option directly, holding decisions constant, and indirectly by changing decisions. The indirect effects are second order when evaluated at $k = 0$: since the bargaining problems do not interact, b^{BC} maximizes $u_A(b, 0) + u_B(b, 0)$. Therefore, a small change of b has no effect on the conditional second-stage outside option. Moreover, a small change of c also has no first-order effect because there are no externalities at $k = 0$. Only the direct effect remains, and it is strictly positive since decisions are strictly positive by assumption. It follows that, for small positive externalities, the conditional second-stage outside option is strictly larger than the one-agent surplus. Hence the principal strictly prefers to bargain with the weaker agent first.

Proposition 4 focused on the small externalities. The question remains if the principal also prefers the timing CB if externalities are positive but large. Our next result shows that CB is optimal when the positive externalities grow beyond all bounds, but the decisions remain bounded, so that the anticipated externality effect eventually dominates the outside option effect.

Proposition 5 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power and the functions u_i, $i = A, B, C$, are continuous. Moreover, suppose that (i) the first-stage decision in BC, b^{BC}, converges to a finite and strictly positive limit $\bar{b} \in (0, \infty)$ as $k \to \infty$, and (ii) the equilibrium second-stage decision in timing BC, $f(b^{BC})$, converges to a strictly positive limit $\bar{c} > 0$, and (iii) $u_B(b, 0, k)$ is independent of k for all b, and (iv) for all (b, c) with $b > 0$ and $c > 0$,*

$$u_B(b, c, k) \to \infty$$

as $k \to \infty$. Then for sufficiently high k, A strictly prefers CB.

Proof. See Appendix 8.5. ■

In general, however, the principal's preference is not unambiguous for large positive externalities ($k \to \infty$). This can illustrated in the following example.

Example 1 *Assume symmetry, $\mathcal{B} = \mathcal{C} = \mathbb{R}_+$, and that all utility functions are additively separable with*

$$\begin{aligned} u_A &= -v(b) - v(c), \\ u_B &= g(b) + kc, \quad \text{if } b \neq 0. \end{aligned}$$

By symmetry,

$$u_C = g(c) + kb, \quad \text{if } c \neq 0.$$

We assume that the functions v and g are strictly increasing and differentiable, $v(0) = g(0) = 0$, $g'(0) > v'(0)$, $v'(b)$ is finite for all $b \in (0, \infty)$ with $\lim_{b\to\infty} v'(b) = \infty$, and $g''(b) \leq 0 < v''(b)$ for all b.

In this example, the first-stage decisions go to infinity; hence, Proposition 5 does not apply. The principal's preferred timing for large positive externalities depends on the limit behavior of her cost function v, because this determines the relative importance of the anticipated externality effect versus the outside option effect.

Remark 4 *Consider example 1. (i) If*

$$\lim_{x\to\infty} \frac{v'(x)}{v(x)} = 0, \tag{7}$$

there exists $\hat{k}$ such that for $k > \hat{k}$, the principal strictly prefers timing BC over CB. (ii) If

$$\lim_{x\to\infty} \frac{v'(x)}{v(x)} = \infty, \tag{8}$$

there exists $\hat{k}$ such that for $k > \hat{k}$, the principal strictly prefers timing CB over BC.

Proof. See Appendix 8.6. ■

In case (i), the outside option effect grows faster and eventually dominates the anticipated externality effect when externalities become large. Hence the principal strictly prefers timing BC for large positive externalities. Note that case (i) applies, for example, whenever v is a polynomial function. In contrast, in case (ii), the anticipated externality effect grows faster, and the principal prefers CB for large positive externalities. Note that case (ii) applies, for example, when $v(b) = \exp(h(b)) - 1$, where h is a strictly increasing and strictly convex function. Here, the cost function is highly convex, which slows down the growth of the first-stage decisions, and hence the growth of the outside option effect, as k gets large. Finally, if neither (7) nor (8) holds, then the comparison of the one-agent surplus with the conditional second-stage outside option does not suffice to pin down the principal's preferred timing, and one must also take into account the different decisions taken in the first stage of the two timings.

Example with a supplier and two retailers in Cournot competition To demonstrate the effect of the externalities on the optimal timing, consider the example outlined in Section 2, in which A is a supplier contracting with two retailers, B and C. Suppose that retailers compete in quantities and that their utility functions are $u_B(b,c) = (1 - b + kc)b$ and $u_C(b,c) = (1 - c + kb)c$, respectively, with $k \in [-1, 1]$. Therefore, if $k = -1$, the retailers sell perfect substitutes, whereas if $k = 1$, the two goods are perfect complements. If $k = 0$, the profit functions are independent of each other and there are no externalities. The supplier's utility function is $u_A(b,c) = -y(b+c) - x(b+c)^2/2$, with $0 \leq y \leq 1$ to ensure that, in equilibrium, $b, c > 0$, and $x \geq 0$. This implies that the supplier's cost function has a linear and a (weakly) convex term. For $x = 0$, u_A is additive-separable.

Solving for the optimal quantities and the respective utilities in both timings, we obtain that the utility of the principal in timing BC is

$$U_A^{BC} = \frac{(1-\beta)(1-y)^2\left[\gamma x^2 + 2(2(1+k) - \gamma k)(2+x) + k^2\right]}{2(2+x)\left[\gamma x^2 + 2(2(1+k) - \gamma k)x + 4 - (3-\gamma)k^2\right]} + \frac{\beta(1-\gamma)(1-y)^2}{2(2+x)}, \tag{9}$$

whereas the principal's utility in timing CB is

$$U_A^{CB} = \frac{(1-\gamma)(1-y)^2\left[\beta x^2 + 2(2(1+k) - \beta k)(2+x) + k^2\right]}{2(2+x)\left[\beta x^2 + 2(2(1+k) - \beta k)x + 4 - (3-\beta)k^2\right]} + \frac{\gamma(1-\beta)(1-y)^2}{2(2+x)}. \tag{10}$$

Comparing U_A^{BC} with U_A^{CB}, it is easily checked that for all $k \leq 0$ and $x \geq 0$, the principal prefers timing BC over CB, strictly so if k is strictly negative and/or x is strictly positive, following Propositions 2 and 3. For $k > 0$ and $x = 0$, the principal prefers timing CB for k close to 0 (as stated by Proposition 4). Instead, for $k = 1$, we obtain $\operatorname{sign}\left\{U_A^{BC} - U_A^{CB}\right\} = \operatorname{sign}\left\{9(1 + \beta\gamma) - 16(\beta + \gamma)\right\}$, which implies that the principal prefers timing BC if $\beta \leq (9 - 16\gamma)/(16 - 9\gamma)$. In fact, one can show that if the latter inequality holds, there is always a unique threshold value for k between 0 and 1, such that the principal prefers timing CB for k below this threshold and timing BC for k above this threshold. This confirms the finding above that either timing can be optimal if externalities are positive and large.

To conclude this section, we summarize the implications for the efficiency, in terms of joint surplus of all parties, of the sequence chosen by the principal: When agents are symmetric except for bargaining power, the equilibrium timing maximizes this surplus when there are negative or no externalities, but equilibrium timing may be inefficient if externalities are positive.

5 Simultaneous Negotiations

We so far focused on the optimal timing of sequential negotiations. Under some circumstances, simultaneous bilateral negotiations with the two agents are also possible. In this section, we analyze whether the principal may prefer simultaneous to sequential negotiations.

We start by determining the outcome with simultaneous negotiations. Since the principal bargains bilaterally with each agent but cannot divide himself, a natural way for simultaneous negotiations is that the principal delegates the negotiations to two delegates who act on his behalf. Each delegate maximizes the bilateral profit in the negotiation he is involved in, given his belief about the outcome in the second negotiation. This implies that no information exchange between the two delegates is possible. In particular, the delegates cannot exchange the information on who is the proposer in each bargaining game. If this were the case, the mechanisms at work are then very different to the ones identified in our analysis of sequential bargaining, which makes the comparison between the to scenarios difficult. In addition, the negotiations then do no longer correspond to the Nash Bargaining Solution. This is an undesirable feature as it is natural to consider the outcome of simultaneous negotiations as the outcome of two Nash bargaining procedures. In fact, a large literature on negotiations (e.g., Horn and Wolinsky, 1988, and Marshall and Merlo, 2004) considers this scenario, which is often denoted by Nash-in-Nash conjectures (Collard-Wexler et al., 2017).

Therefore, we analyze a situation in which the two pairs of bargainers are negotiating at the same time and do not observe what is happening in the other negotiation. In the negotiation between A and B, the solution $b^\star$ is given by

$$b^\star(c) := \arg\max_{b\in\mathcal{B}} \{u_A(b,c) + u_B(b,c)\},$$

where c is the belief about the outcome in the other negotiation. Similarly, in the negotiation between A and C, $c^\star$ is given by

$$c^\star(b) := \arg\max_{b\in\mathcal{B}} \{u_A(b,c) + u_C(b,c)\},$$

where b is the belief about the outcome in the other negotiation. In equilibrium, beliefs are correct. There may exist multiple equilibria in the simultaneous game. We do not impose a selection rule on the equilibria in this case. As we will explain below, all results of the propositions in this section hold independent of the selected equilibrium.

Turning to the transfers, if A's delegate is drawn as the proposer in the negotiation with B, she sets $t_B = u_B(b^\star, c^\star)$. Similarly, in the negotiation with C, she sets $t_C = u_C(b^\star, c^\star)$. By contrast, if B is selected as the proposer in the negotiation with A, he offers $t_B = -u_A(b^\star, c^\star) + u_A(0, c^\star)$. This occurs because the principal (or her delegate) obtains as an outside option $u_A(0, c^\star)$ when rejecting B's contract. By the same argument, if C is selected as the proposer in the negotiation with A, he sets $t_C = -u_A(b^\star, c^\star) + u_A(b^\star, 0)$.

The payoff of the principal can then be written as

$$(1-\beta)(1-\gamma)\{u_A(b^\star, c^\star) + u_B(b^\star, c^\star) + u_C(b^\star, c^\star)\} + (1-\beta)\gamma\{u_A(b^\star, 0) + u_B(b^\star, c^\star)\} \quad (11)$$

$$+\beta(1-\gamma)\{u_A(0, c^\star) + u_C(b^\star, c^\star)\} + \beta\gamma\{u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)\}.$$

We can now compare the principal's payoff in the simultaneous timing with the one in the sequential timing. As above, we start with the case of negative externalities. We focus on the timing BC because we know from Proposition 2 that this timing dominates timing CB in case of negative externalities.

Proposition 6 *Suppose externalities are negative and that u_A is weakly super-modular. The principal prefers timing BC to the simultaneous timing; moreover, the preference is strict if externalities are strictly negative or u_A is strictly super-modular, and equilibrium decisions are not zero.*

Proof. See Appendix 8.7. ■

The intuition behind this result is driven by three effects. The first one is related to the intuition given in the section on the surplus-maximizing sequence. In the sequential timing BC, the two bargainers take the utility of agent C partially into account because the principal receives a share of it. By contrast, in the simultaneous timing the delegate of the principal and agent B do not consider the utility of agent C when negotiating with respect to b because a change in b will not affect the outcome in the negotiation between the principal and agent C. As a consequence, in the simultaneous timing, the decision made by A and B is further away from the welfare-optimal decision, implying that the overall cake is lower with simultaneous timing.

The second effect, which is inherent in the simultaneous timing, is rooted in the fact that the bargainers in each negotiation cannot observe the outcome of the other negotiation (because negotiations take place simultaneously). In particular, agent C cannot observe if an agreement was reached between A and B. She supposes (correctly so on the equilibrium path) that the decision in the other negotiation was $b^\star > 0$. In the sequential timing BC, agent C instead observes if there was an agreement in the negotiation between A and B. This difference affects the expected transfer that A obtains. Specifically, in case A and B failed to reach an agreement, the principal can demand a transfer from C that equals C's utility given that $b = b^\star$ in the simultaneous timing, whereas in the sequential timing he can demand a transfer from C that equals C's utility given that $b = 0$. With negative externalities, that latter is higher than the former, thereby favoring the sequential timing.

The third effect, which is also inherent in the simultaneous timing, is that it now matters how b and c interact in u_A (i.e., whether u_A is super-modular or sub-modular). Even if both agents are selected as the proposers in the respective negotiation, A's payoff is not necessarily zero. The reason is that in the negotiation with, say, agent B, the outside option of the principal's delegate is to reject B's offer and obtain a utility of $u_A(0, c^\star)$. Therefore, the agent will claim a payment from the principal equal to $u_A(b^\star, c^\star) - u_A(0, c^\star)$. This implies that the principal's payoff in case she responds in both negotiations is $u_A(b^\star, 0) + u_A(0, c^\star) - u_A(b^\star, c^\star)$. If u_A is super-modular in b and c, A's payoff is therefore negative. This effect works again favor of the sequential timing.

Let us now turn to the case without externalities. We obtain the following proposition:

Proposition 7 *Assume agents are symmetric except for bargaining power and that there are no externalities.*
(i) If u_A is super-modular in b and c, timing BC is preferred over the simultaneous timing.
(ii) If u_A is sub-modular in b and c, then timing BC is preferred over the simultaneous timing for γ close to 0, whereas the simultaneous timing if preferred over BC for γ close to 1.

Proof. See Appendix 8.8. ■

Without externalities but interaction of b and c in the principal's utility function, the first effect described after Proposition 6 is still present. This works in favor of timing BC. In addition, the third effect is present as well, which favors timing BC if u_A is super-modular but timing CB is u_A is sub-modular. The result of Proposition 7 therefore depends on the bargaining power of the agents. If the principal has a lot of bargaining power (i.e., β and γ or relatively small), the latter effect has only little bite. The principal then prefers the sequential timing. By contrast, if the agents have a high bargaining power (i.e., β and γ are relatively large), the effect is particularly strong, and the principal favors the simultaneous timing.

The result is case of sub-modularity can be illustrated with the help of the supplier-retailers example considered in the previous section. Recall that for all $x > 0$, u_A is sub-modular. If $k = 0$ (i.e., there are no externalities), the utility of the principal in timing BC is

$$\frac{(1-\alpha)^2\left[(1-\beta\gamma)(4+\gamma x^2)+4(2-\beta\gamma)\right]}{2(2+x)(4(1+x)+\gamma x^2)}. \tag{12}$$

Instead, in the simultaneous timing, the utility of the principal is

$$\frac{(1-\alpha)^2\left[4(1+x)-(\beta+\gamma)(2+x)\right]}{8(1+x)^2}. \tag{13}$$

Subtracting (13) from (12) and letting $\gamma \to 0$ yields $((1-y)^2\beta x^2) / (8(2+x)(1+x)^2)$, which is strictly positive. Hence, the sequential timing is preferred over the simultaneous one. By contrast, if $\gamma \to 1$, which implies that also $\beta \to 1$, the difference between (12) and (13), becomes $-((1-y)^2 x) / (4(1+x)^2)$, which is strictly negative. It is easy to show that there is a unique threshold value of γ, such that the simultaneous timing is preferred for γ above this threshold.

Finally, we turn to the case of positive externalities and again derive a result for small positive externalities.

Proposition 8 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power, $1 > \beta > \gamma$, u_A is additively separable, and (i) u_i $(i = A, B, C)$ is C^1 in (b, c, k), (ii) $c^*(b, k)$ is interior and C^1 in (b, k), and (iii) $\mathcal{B} = \mathcal{C}$ is compact. Then, there exists a $\hat{k} > 0$ such that $U_A^{sim} > U_A^{CB}$ for all $k \in \left(0, \hat{k}\right)$.*

Proof. See Appendix 8.9. ■

The result demonstrates that the simultaneous timing can dominate the sequential timing even if u_A is additive-separable. The result is perhaps not obvious at first glance because the simultaneous timing has the disadvantage that it does not allow the principal to commit to one of the decision variables. However, there is a clear intuition why the simultaneous timing can be optimal if externalities are positive, which rests on the non-observability of outcomes.

In any sequential timing, the bargainers in the second stage know the outcome of the first stage. If there was no agreement reached in the first stage, the principal, when being selected as the proposer, can extract from the agent in the second stage an amount that equals her payoff, given that the decision variable of the first stage is 0. By contrast, in the simultaneous timing, an agent does not observe the outcome of the other bargaining game and supposes that an agreement was reached there. If externalities are positive, this implies that the principal can demand more from the agent. Although rejections do not happen on the equilibrium path, this effect increases the outside option of the principal.

Finally, we can illustrate the result with positive externalities in our example with a supplier and two retailers. The utility of the principal in the simultaneous timing is given by

$$U_A^{sim} = \frac{(1-\alpha)^2\,[4(1+x) - (\beta+\gamma)(2+x)]}{2(2(1+x)-k)^2}. \tag{14}$$

Comparing (14) with the principal's utility from timings BC and CB given by (9) and (10), respectively, we obtain that for any $k \in (0, 1]$, there exists a threshold $\hat{k} < 1$, such that $U_A^{sim} > U_A^{CB}$ if and only if $k < \hat{k}$. Moreover, if $\beta \leq (9-16\gamma)/(16-9\gamma)$, which implies that the timing BC is more profitable for the principal than the timing CB whenever k is large enough, $\hat{k}$ is below the threshold at which the timing BC is preferred by the principal. This implies that, when externalities are positive, three different timings may be optimal for the principal, dependent on the level of k: if k is small, the principal chooses the simultaneous timing, for intermediate values of k, she chooses the timing CB, and for large values of k (i.e., k close to 1), she chooses the timing BC.

6 Extensions

6.1 Contract Disclosure

- So far we assumed that the decision agreed upon in the first stage is observable to the agent with whom A bargains in the second stage.
- The idea is that A can show the contract to the agent, which implies full information of the agent.
- However, it is not clear whether A indeed has the incentive to disclose the contract.
- To address this we consider an augmented game in which there is an additional stage between the first-stage and the second-stage negotiation. In this stage, A decides whether or not to disclose the contract to the agent with whom she will bargain in the second stage. Our equilibrium concept is then (weak) Perfect Bayesian Equilibrium.

Proposition 9 *In the game with endogenous contract disclosure, for any utility functions of A, B, and C, a (weak) Perfect Bayesian Equilibrium in which A discloses the first-stage contract, exists.*

Moreover, if u_A is additive-separable and the bilateral surplus in the second-stage is monotone in the first-stage decision, the unique (weak) Perfect Bayesian Equilibrium involves contract disclosure.

Proof. We will show the result for the timing BC. By an analogous argument, the result also holds for the timing CB. From our analysis of Section 2, the expected surplus that the principal obtains in the second stage (net of the first-stage payment t_B) is given by

$$(1-\gamma)\left(u_A\left(b,c^*\left(b\right)\right)+u_C\left(b,c^*\left(b\right)\right)\right)+\gamma u_A\left(b,0\right), \tag{15}$$

in case she discloses the contract between A and B to C.

We now turn to the case, in which A does not disclose the contract. As C then does not know the decision b implemented in the contract between A and B, she needs to form a belief about this decision. Let us denote this belief by b'. We will now determine A's expected payoff if C's belief equals b'.

If A is drawn to make the offer in the second stage, she will make an offer $c = c^*(b')$ and demand $t_C = u_C(b', c^*(b'))$. This implies that her payoff is $u_A(b, c^*(b'))+u_C(b', c^*(b'))$ because the real decision A and B agreed on is b, which might be different from b'. The probability to make the offer in the second stage is $1-\gamma$. Instead, with probability γ, the agent is drawn to make the offer. Given his belief, C will then make an offer t_C such that the principal is indifferent between accepting or rejecting this offer. This implies $u_A(b', c^*(b'))+t_C = u_A(b', 0)$ or $t_C = -u_A(b', c^*(b')) + u_A(b', 0)$. Therefore, when accepting the offer, A obtains

$$u_A(b, c^*(b')) - u_A(b', c^*(b')) + u_A(b', 0).$$

The principal can also reject the agent's offer, in which case her payoff equals $u_A(b, 0)$. It follows that the principal's payoff in the second stage when not disclosing the first-stage contract is

$$(1-\gamma)\left(u_A\left(b,c^*\left(b'\right)\right)+u_C\left(b',c^*\left(b'\right)\right)\right)+\gamma\max\left\{u_A\left(b,0\right), u_A(b,c^*(b'))-u_A(b',c^*(b'))+u_A(b',0)\right\}. \tag{16}$$

To support contract disclosure as a (weak) Perfect Bayesian Equilibrium, note first that if C expects A to disclose the first-stage contract, but A plays out-of-equilibrium and does not disclose, a (weak) Perfect Bayesian Equilibrium does not restrict the belief b' of C. As a consequence, suppose that if A does not disclose than b' is such that (16) is (weakly) smaller than (15).[17] In that case, a deviation by A to non-disclosure of the first-stage contract is

[17] Such a b' necessarily exists because the first-stage decision is chosen by partially considering the second-

not profitable. As this argument does not rely on the the exact form of the utility function, a (weak) Perfect Bayesian Equilibrium with contract disclosure always exists.

Consider now the case in which u_A is additive-separable. The second term in the curly brackets of (16) can then be written as

$$u_A(b,0) + u_A(0, c^*(b')) - u_A(b',0) - u_A(0, c^*(b')) + u_A(b',0) = u_A(b,0).$$

Rearranging terms, (16) is equal to

$$(1-\gamma)\left(u_A\left(0, c^*\left(b'\right)\right) + u_C\left(b', c^*\left(b'\right)\right)\right) + u_A\left(b, 0\right), \tag{17}$$

where we used the fact that due to u_A being additive-separable, $u_A(b, c^*(b')) = u_A(b,0) + u_A(0, c^*(b'))$. Similarly, we can write (15) as

$$(1-\gamma)\left(u_A\left(0, c^*\left(b\right)\right) + u_C\left(b, c^*\left(b\right)\right)\right) + u_A\left(b, 0\right). \tag{18}$$

If the second-stage surplus—i.e., $u_A\left(0, c^*(b)\right) + u_C\left(b, c^*(b)\right)$—is monotone in b, a principal who agreed with B on a contract with a decision that maximizes this second-stage surplus will always disclose as this will lead to a (weakly) higher expected profit than non-disclosure. Because the structure of A's payoff is the same in (17) as in (18), a standard unraveling argument then implies that regardless of the decision implemented in the first stage (i.e., regardless of which type b the principal is), in the unique (weak) Perfect Bayesian Equilibrium the principal will disclose the first-stage contract. ■

6.2 N Agents

7 Conclusion

This paper has studied the optimal sequence of negotiations between one principal and two agents. We have shown that their joint surplus is higher when the principal bargains with the stronger agent first, independent of externalities between agents are positive or negative. By contrast, the sequence chosen by the principal depends on the externalities. If externalities are negative, the principal chooses the surplus maximizing sequence. By contrast, with positive externalities, we identify conditions under which the equilibrium timing is to bargain with the weaker agent first. As a consequence, the equilibrium timing can be inefficient only

stage surplus. It follows that a first-stage decision that takes this surplus into account to a smaller extent leads a (weakly) lower second-stage surplus.

if externalities are positive. In addition, we also contribute to the debate if the principal prefers simultaneous or sequential bargaining. We show that simultaneous negotiations are optimal is externalities are positive but only slightly so.

In our study, we focused on the role of bargaining power and derived our main results under the assumption that agents are symmetric except for bargaining power.[18] Our analysis can therefore be extended in many dimensions. For example, agents may differ in their contribution to the total surplus instead of the bargaining power. Also, agents may be asymmetric in the externalities they exert on each other. It is interesting to analyze how these differences drive the welfare-optimal sequence and the sequence chosen by the principal. In particular, asymmetries in those other dimensions may bring in new effects that could qualify are strengthen the effects shown in the paper. We leave this for future research.

[18]Without the assumption of symmetry, we have derived some results for limiting cases of bargaining power. In particular, if one agent has all the bargaining power, the principal will negotiate with this agent first if externalities are negative but with the weaker agent first if externalities are positive.

8 Appendix

8.1 Proof of Proposition 1

The proof of part (i) of Proposition 1 uses the following Lemma:

Lemma 1 *Suppose that $w : \mathcal{B} \to \mathbb{R}$ and $v : \mathcal{B} \to \mathbb{R}$ are functions and suppose that*

$$b^* (\gamma) := \arg\max_{b \in \mathcal{B}} (1 - \gamma) w (b) + \gamma v (b)$$

exists for all $\gamma \in [0, 1]$. Then for all $\gamma_1 \in [0, 1]$ and $\gamma_0 \in [0, 1]$, $\gamma_1 > \gamma_0$ implies $w (b^* (\gamma_1)) \leq w (b^* (\gamma_0))$.

Proof. Suppose to the contrary that $w (b^* (\gamma_1)) > w (b^* (\gamma_0))$. From the definition of $b^* (\gamma)$,

$$(1 - \gamma_0) w (b^* (\gamma_0)) + \gamma_0 v (b^* (\gamma_0)) \geq (1 - \gamma_0) w (b^* (\gamma_1)) + \gamma_0 v (b^* (\gamma_1)),$$

or equivalently,

$$(1 - \gamma_0) (w (b^* (\gamma_0)) - w (b^* (\gamma_1))) \geq \gamma_0 (v (b^* (\gamma_1)) - v (b^* (\gamma_0))) \tag{19}$$

Since $w (b^* (\gamma_1)) > w (b^* (\gamma_0))$ and $1 \geq \gamma_1 > \gamma_0$, the left side of inequality (19) is strictly negative. Therefore, $v (b^* (\gamma_1)) < v (b^* (\gamma_0))$.

Similarly,

$$- (1 - \gamma_1) (w (b^* (\gamma_0)) - w (b^* (\gamma_1))) \geq -\gamma_1 (v (b^* (\gamma_1)) - v (b^* (\gamma_0))) \tag{20}$$

Adding (20) to (19) shows that

$$(\gamma_1 - \gamma_0) w (b^* (\gamma_0)) - w (b^* (\gamma_1)) \geq (\gamma_0 - \gamma_1) (v (b^* (\gamma_1)) - v (b^* (\gamma_0)))$$

This is a contradiction because the left hand side is strictly smaller than zero, and the right hand is strictly greater than zero. ■

Proof of Proposition 1. Part (i). Consider timing BC (the result concerning timing CB can be established similarly). It is evident from (1) that the equilibrium decisions $\left(b^{BC}, c^* \left(b^{BC}\right)\right)$ do not depend on β. Therefore, $S^{BC} = S \left(b^{BC}, c^* \left(b^{BC}\right)\right)$ is constant in β.

Moreover, $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$, where

$$\begin{aligned} S_{AB}^{BC}(b) &= u_B(b, c^*(b)) + (1-\gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0) \\ &= (1-\gamma) W(b, c^*(b)) + \gamma [u_A(b, 0) + u_B(b, c^*(b))] \end{aligned}$$

Applying Lemma 1 with $w(b) = S(b, c^*(b))$ and $v(b) = u_A(b, 0) + u_B(b, c^*(b))$ shows that $S\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is decreasing in γ.

Part (ii). Suppose agents are symmetric. If $\beta = \gamma$, timings BC and CB differ only in the names of the agents. Since equilibrium industry surplus is unique, $S^{BC} = S^{CB}$. Part (i) therefore implies that, if $\beta > \gamma$, $S^{BC} \geq S^{CB}$. ■

8.2 Proof of Proposition 2

The outside options of A in stage one are

$$\begin{aligned} O_A^{BC} &= (1-\gamma) \max_{c\in\mathcal{C}} \{u_A(0, c) + u_C(0, c)\}, \\ O_A^{CB} &= (1-\beta) \max_{b\in\mathcal{B}} \{u_A(b, 0) + u_B(b, 0)\}. \end{aligned}$$

By symmetry,

$$\beta O_A^{BC} - \gamma O_A^{CB} = (\beta - \gamma) \max_{c\in\mathcal{C}} \{u_A(0, c) + u_C(0, c)\}. \tag{21}$$

The surplus of A and B in timing BC as a function of b is

$$S_{AB}^{BC}(b) = (1-\gamma)(u_A(b, f(b)) + u_C(b, f(b))) + \gamma u_A(b, 0) + u_B(b, f(b)).$$

In equilibrium of timing BC, $b = b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$. Similarly, the surplus of A and C in timing CB as a function of c is

$$S_{AC}^{CB}(c) = (1-\beta)(u_A(f(c), c) + u_B(f(c), c)) + \beta u_A(0, c) + u_C(f(c), c).$$

In equilibrium of timing CB, $c = c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$. The expected payoffs of A in timing BC and CB are, respectively,

$$\begin{aligned} U_A^{BC} &= (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC} \\ U_A^{CB} &= (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) + \gamma O_A^{CB}. \end{aligned}$$

Since $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$,

$$S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right). \tag{22}$$

Moreover, by symmetry,

$$S_{AB}^{BC}\left(c^{CB}\right) = \left((1-\gamma)\left(u_A\left(f\left(c^{CB}\right), c^{CB}\right) + u_B\left(f\left(c^{CB}\right), c^{CB}\right)\right) + \gamma u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)$$

and therefore

$$(1-\beta) S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) = (\gamma-\beta)\left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right). \tag{23}$$

From (21), (22), and (23),

$$\begin{aligned} & U_A^{BC} - U_A^{CB} \\ \geq\ & (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\} - \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)\right) \quad (24) \\ \geq\ & (\beta-\gamma)\left(\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\} - \max_{c\in\mathcal{C}}\{(u_A(0,c) + u_C(f(c), c))\}\right). \end{aligned}$$

Negative externalities imply $u_C(0,c) \geq u_C(b,c)$ for all $b \geq 0$, and therefore $U_A^{BC} \geq U_A^{CB}$. Moreover, whenever externalities are strictly negative and $b > 0$, $u_C(0,c) > u_C(b,c)$ for all $c > 0$, therefore $U_A^{BC} > U_A^{CB}$.

8.3 Proof of Proposition 3

Proof. The proof of Proposition (2) also establishes that with no externalities, $U_A^{BC} \geq U_A^{CB}$. When $b^{BC} = c^{CB} \neq f(0)$, (22) holds with equality, and hence by the argument leading to (24),

$$\begin{aligned} U_A^{BC} - U_A^{CB} &= \max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\} - \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right) \\ &= u_A(0, f(0)) + u_C(0, f(0)) - \left(u_A\left(0, c^{CB}\right) + u_C\left(0, c^{CB}\right)\right) > 0, \end{aligned}$$

where the second equality uses the assumption that there are no externalities, and the inequality follows from $f(0) \neq c^{CB}$.

When $b^{BC} \neq c^{CB}$, then inequality (22) is strict. Since $\beta < 1$, it follows that $U_A^{BC} > U_A^{CB}$ when $b^{BC} \neq c^{CB}$ for any $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ and $c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$. We show that (i)-(iii) imply this is the case.

By (ii), $S_{AB}^{BC}(b)$ and $S_{AC}^{CB}(c)$ are differentiable. Since any $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ is interior by (i), it satisfies the first order condition

$$\begin{aligned} \frac{\partial S_{AB}^{BC}\left(b^{BC}\right)}{\partial b_i} &= \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC}, 0\right) \\ &\quad + (1-\gamma)\left(\sum_k \frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right)\frac{df_k\left(b^{BC}\right)}{db_i}\right) = 0. \end{aligned}$$

Since $f\left(b^{BC}\right)$ is interior by (i), and $u_A(b,c) + u_C(c)$ is differentiable by (ii), the first order condition

$$\frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right) = 0$$

holds, thus

$$\frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC}, 0\right) = 0.$$

Since $f\left(b^{BC}\right)$ is interior by (i), $f\left(b^{BC}\right) > 0$. Thus (5) implies

$$\begin{aligned} & \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC}, 0\right) \\ \neq\; & \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\beta)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \beta\frac{\partial}{\partial b_i}u_A\left(b^{BC}, 0\right) \\ =\; & \frac{\partial u_C\left(b^{BC}\right)}{\partial c_i} + (1-\beta)\frac{\partial}{\partial c_i}u_A\left(f\left(b^{BC}\right), b^{BC}\right) + \beta\frac{\partial}{\partial c_i}u_A\left(0, b^{BC}\right) \\ =\; & \frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i} \end{aligned}$$

where the first equality is from symmetry. We have shown that

$$\frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i} \neq 0.$$

Since any $c^{CB} \in \arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$ is interior by (i), it satisfies the first order condition

$$\frac{\partial S_{AC}^{CB}\left(c^{CB}\right)}{\partial c_i} = 0,$$

thus $b^{BC} \neq c^{CB}$. ■

8.4 Proof of Proposition 4

Proof. In what follows, we denote the first stage decision in timing BC, which depends on k, by $b^{BC}(k)$.

To show the result in the most concise way, we first determine in the next lemma how the social surpluses in the two timings change with k. We note that the proof of the lemma uses a version of the envelope theorem applied to the joint first-stage surplus. We cannot directly apply to standard versions of the envelope theorem (e.g., Simon and Blume 1994, Theorem 19.4) for two reasons. First, we do not assume $b^{BC}(k)$ to be differentiable in k. We solve this issue by using an envelope theorem from Milgrom and Segal (2002) that does not presuppose differentiability of the maximizer. Second, the choices in the second-stage do in general not maximize the joint surplus of those who bargain in the first stage. As in the envelope theorem for Stackelberg games (Caputo 1998), we need to take into account the effect of k on the second-stage reaction function. Under the assumptions of Proposition 4, however, at $k = 0$ the second-stage decision also maximizes the surplus of the negotiation in the first stage, therefore the corresponding terms disappear.

Lemma 2 *Under the assumptions of Proposition 4,* $S_{AB}^{BC}(k) = \max_{b \in \mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ *and* $S_{AC}^{CB}(k) = \max_{c \in \mathcal{C}} S_{AC}^{CB}(b^*(c,k), c, k)$ *are differentiable in* k *at* $k = 0$*, and*

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta - \gamma)\frac{\partial}{\partial k} u_B(b, c; k)\Bigg|_{\substack{k=0 \\ b=b^{BC}(0) \\ c=c^*\left(b^{BC},0\right)}} > 0.$$

Proof. If $k = 0$, there is no interaction between the two bargaining problems, and

$$\arg\max_b S_{AB}^{BC}(b, 0) = \arg\max_b u_A(b, 0) + u_B(b, 0, 0)$$

Our assumption that second stage decision are unique ensures that $\arg\max_b u_A(b,0) + u_B(b,0,0)$ is unique. Therefore, when $k = 0$, the first stage decision in timing BC is unique. Since $c^*(b,0)$ is interior by assumption (ii), symmetry implies that if $k = 0$, $c^*(b,0) = b^{BC}(0)$. Thus $b^{BC}(0)$ is interior. Moreover, the function $S_{AB}^{BC}(b, c^*(b,k), k)$ is continuous in b and continuously differentiable in k.

Therefore, Corollary 4 from Milgrom and Segal (2002) applies (here we use assumption (iii)), and $\max_{b\in\mathcal{B}} S_{AB}^{BC}\left(b, c^*\left(b,k\right), k\right)$ is differentiable in k at $k=0$, with

$$\begin{aligned}\frac{d}{dk}\max_{b\in\mathcal{B}} S_{AB}^{BC}\left(b, c^*\left(b,k\right), k\right)|_{k=0} \quad &= \quad \frac{\partial}{\partial k}\left(u_B\left(b, c^*\left(b;k\right);k\right) + \left(1-\gamma\right)u_C\left(b, c^*\left(b;k\right);k\right)\right)\Big|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} \\ &\quad + \sum_{i=1}^{n}\frac{\partial S_{AB}^{BC}\left(b,c,k\right)}{\partial c_i}\frac{\partial c_i^*\left(b;k\right)}{\partial k}\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}}.\end{aligned}$$

The first term of the right-hand side is the direct effect of k, keeping b and c constant, whereas the second term captures that the second-stage reaction function depends on k.

We next show that

$$\frac{\partial S_{AB}^{BC}\left(b,c,k\right)}{\partial c_i}\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0$$

for all $i = 1, ..., n$. We have

$$\frac{\partial S_{AB}^{BC}\left(b,k\right)}{\partial c_i} = \frac{\partial}{\partial c_i}u_B\left(b, c^*\left(b\right);k\right) + \left(1-\gamma\right)\frac{\partial}{\partial c_i}\left(u_A\left(b, c^*\left(b;k\right)\right) + u_C\left(b, c^*\left(b;k\right);k\right)\right).$$

Since $c^*\left(b,0\right)$ maximizes $u_A\left(b,c\right) + u_C\left(b,c;0\right)$ and is interior,

$$\frac{\partial}{\partial c_i}\left(u_A\left(b,c\right) + u_C\left(b,c,k\right)\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0.$$

Moreover, at $k=0$ there are no externalities, thus

$$\frac{\partial}{\partial c_i}u_B\left(b,c,k\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0.$$

It follows that

$$\frac{d}{dk}S_{AB}^{BC}\left(k\right) = \frac{\partial}{\partial k}\left(u_B\left(b,c;k\right) + \left(1-\gamma\right)u_C\left(b,c;k\right)\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}}.$$

Similarly,

$$\frac{d}{dk}S_{AC}^{CB}(k) = \frac{\partial}{\partial k}\left(u_C(b,c;k) + (1-\beta)\,u_B(b,c;k)\right)\Bigg|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}}.$$

By symmetry, for all x, y, and k, $u_B(x,y;k) = u_C(y,x;k)$ and thus

$$\frac{\partial}{\partial k}u_B(x,y;k) = \frac{\partial}{\partial k}u_C(y,x;k) \tag{25}$$

Moreover, symmetry implies $b^{BC}(0) = c^{CB}(0)$ and $b^*\left(c^{CB}(0);0\right) = c^*\left(b^{BC}(0),0\right)$. Evaluating (25) at $k=0$, $x = b^{BC}(0) = c^{CB}(0)$, and $y = b^*\left(c^{CB}(0);0\right) = c^*\left(b^{BC}(0),0\right)$ gives

$$\frac{\partial}{\partial k}u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}} = \frac{\partial}{\partial k}u_C(b,c;k)\Bigg|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}}.$$

Similarly, evaluating (25) at $k=0$, $x = b^*\left(c^{CB}(0);0\right) = c^*\left(b^{BC}(0),0\right)$, and $y = b^{BC}(0) = c^{CB}(0)$ gives

$$\frac{\partial}{\partial k}u_B(b,c;0)\Bigg|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}} = \frac{\partial}{\partial k}u_C\left(b^{BC}, c^*\left(b^{BC},0\right);0\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}}.$$

Therefore,

$$\frac{d}{dk}\left((1-\gamma)\,S_{AC}^{CB}(k) - (1-\beta)\,S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k}u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}},$$

which is strictly positive since by assumption $c^*(b,k) > 0$ and, as shown above, $b^{BC}(0) > 0$. ■

We can now show the result of Proposition 4. Since u_A does not depend on k, and $u_C(0,c;k)$ is independent of k,

$$O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A(0,c) + u_C(0,c;k)\right\}$$

does not depend on k. Similarly, O_A^{CB} is independent of k. They payoff of A in timings BC

and CB is

$$\begin{aligned} U_A^{BC}(k) &:= (1-\beta) S_{AB}^{BC}(k) + \beta O_A^{BC}, \\ U_A^{CB}(k) &:= (1-\gamma) S_{AC}^{CB}(k) + \gamma O_A^{CB}. \end{aligned}$$

Therefore, Lemma 2 implies that

$$\frac{\partial}{\partial k}\left(U_A^{CB}(k) - U_A^{BC}(k)\right)|_{k=0} > 0.$$

If $k=0$, the bargaining problems do not interact, and $U_A^{BC}(0) = U_A^{CB}(0)$. By continuity, it follows that for sufficiently small $k>0$, $U_A^{CB}(k) > U_A^{BC}(k)$. ■

8.5 Proof of Proposition 5

By an argument similar to the proof of Proposition 2, it is sufficient to establish that

$$u_A\left(b^{BC},0\right) + u_B\left(b^{BC}, f\left(b^{BC}\right)\right) > u_A(0,f(0)) + u_C(0,f(0)). \tag{26}$$

Here u_B and u_C are parametrized by k, and

$$\begin{aligned} & u_A\left(b^{BC},0\right) + u_B\left(b^{BC}, f\left(b^{BC}\right), k\right) - \left(u_A(0,f(0)) + u_C(0,f(0),k)\right) \\ \to\ & \lim_{k\to\infty}\left(u_A\left(\bar{b},0\right) + u_B\left(\bar{b},\bar{c},k\right) - \left(u_A(f(0),0) + u_B(f(0),0,k)\right)\right) \\ =\ & u_A\left(\bar{b},0\right) + \lim_{k\to\infty} u_B\left(\bar{b},\bar{c},k\right) - \left(u_A(f(0),0) + u_B(f(0),0,0)\right) \end{aligned}$$

The first equality holds since u_A does not depend on k, and similarly u_B is constant in k when $c=0$. Since

$$\lim_{k\to\infty} u_B\left(\bar{b},\bar{c},k\right) = \infty,$$

and the remaining terms are finite, it follows that (26) holds for sufficiently large k.

8.6 Proof of Remark 4

In timing BC, the second stage decision solves

$$\max_c -v(c) + g(c)$$

The second stage decision c^* is implicitly given by the first order condition $g'(c^*) = v'(c^*)$. Due to the separability of the utility functions, c^* neither depends on b nor on k. Similarly, the second stage decision in timing CB is $b^* = c^*$.

In the first stage of timing BC, the decision maximizes

$$\begin{aligned} & u_B(b, c^*) + (1-\gamma)(u_A(b, c^*) + u_C(b, c^*)) + \gamma u_A(b, 0) \\ = \; & g(b) + kc^* + (1-\gamma)(-v(b) - v(c^*) + g(c^*) + kb) - \gamma v(b). \end{aligned}$$

The first stage decision b^{BC} is given by

$$g'\left(b^{BC}\right) + (1-\gamma)k - v'\left(b^{BC}\right) = 0.$$

Similarly, c^{CB} is given by

$$g'\left(c^{CB}\right) + (1-\beta)k - v'\left(c^{CB}\right) = 0$$

Note that c^{CB} is increasing in k, with $\lim_{k\to\infty} c^{CB} = \infty$. (To see this, note that if c^{CB} converges to a finite limit $\bar{c}$, then $g'\left(c^{CB}\right) - v'\left(c^{CB}\right) \to g'(\bar{c}) - v'(\bar{c})$ which is finite, but $(1-\beta)k \to \infty$, contradicting the first order condition.)

We are now in a position to prove Part (i). As shown in the Proof of Proposition 2, A strictly prefers BC if

$$\max_{c\in\mathcal{C}} \{u_A(0, c) + u_C(0, c)\} > \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right) \tag{27}$$

(see inequality (24)). In the current setting,

$$\begin{aligned} & u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right) \\ = \; & -v\left(c^{CB}\right) + g\left(c^{CB}\right) + kb^* \\ = \; & -v\left(c^{CB}\right)\left[1 - \left(\frac{g\left(c^{CB}\right)}{v\left(c^{CB}\right)} + b^* \frac{k}{v\left(c^{CB}\right)}\right)\right] \end{aligned} \tag{28}$$

We show this diverges to minus infinity as $k \to \infty$ (and, consequently, $c^{CB} \to \infty$). Note that $v\left(c^{CB}\right) \to \infty$. In addition, we will show next that the term in brackets $[\cdot]$ converges to one.

First, the term $g(c)/v(c)$ converges to zero. Recall we assumed $g'(c) > 0 \geq g''(c)$ for all $c > 0$. If $g(c)$ converges to some finite limit, $\lim (g(c)/v(c)) = 0$. Moreover, if $\lim g\left(c^{CB}\right) =$

∞, then by L'Hopital's rule

$$\begin{aligned}
\lim \frac{g\left(c^{CB}\right)}{v\left(c^{CB}\right)} &= \lim \frac{g'\left(c^{CB}\right)}{v'\left(c^{CB}\right)} \\
&= \lim \frac{g'\left(c^{CB}\right)}{g'\left(c^{CB}\right)+(1-\beta)\,k} \\
&= \frac{1}{1+(1-\beta)\lim \frac{k}{g'\left(c^{CB}\right)}} = 0
\end{aligned}$$

The second equality uses the first order condition defining c^{CB}. The last equality follows since g is strictly increasing and concave, which implies $\lim g'\left(c^{CB}\right) \leq g'(0) < \infty$.

Second, consider the term $b^{*}k/v\left(c^{CB}\right)$. Recall b^{*} does not depend on k. By the first order condition defining c^{CB},

$$k = \frac{v'\left(c^{CB}\right) - g'\left(c^{CB}\right)}{1-\beta},$$

thus

$$\frac{k}{v\left(c^{CB}\right)} = \frac{1}{1-\beta}\left(\frac{v'\left(c^{CB}\right)}{v\left(c^{CB}\right)} - \frac{g'\left(c^{CB}\right)}{v\left(c^{CB}\right)}\right)$$

By assumption, $v'\left(c^{CB}\right)/v\left(c^{CB}\right) \to 0$. Moreover, $\lim g'\left(c^{CB}\right) < \infty = \lim v\left(c^{CB}\right)$, thus $g'\left(c^{CB}\right)/v\left(c^{CB}\right) \to 0$.

We have shown that the bracket $[\cdot]$ in (28) converges to 1. Thus

$$\lim\left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right) = -\infty$$

Therefore, for large enough k, (27) is satisfied, and the principal strictly prefers BC.

It remains to prove Part (ii). As above, it is sufficient to show that (26) holds. In the current setting,

$$\begin{aligned}
u_A\left(b^{BC}, 0\right) + u_B\left(b^{BC}, f\left(b^{BC}\right)\right) &= -v\left(b^{BC}\right) + g\left(b^{BC}\right) + kc^{*} \\
&= -v\left(b^{BC}\right)\left[1 - \left(\frac{g\left(b^{BC}\right)}{v\left(b^{BC}\right)} + \frac{kc^{*}}{v\left(b^{BC}\right)}\right)\right]
\end{aligned}$$

By the same argument as above,

$$\frac{g\left(b^{BC}\right)}{v\left(b^{BC}\right)} \to 0.$$

Moreover,

$$\frac{k}{v\left(b^{BC}\right)} = \frac{1}{1-\gamma}\frac{v'\left(b^{BC}\right) - g'\left(b^{BC}\right)}{v\left(b^{BC}\right)} \to \infty$$

since $g'/v \to 0$ as above, but $v'/v \to \infty$ by (8). Therefore,

$$\lim \left(u_A\left(b^{BC},0\right)+u_B\left(b^{BC},f\left(b^{BC}\right)\right)\right)=+\infty$$

and for large enough k, A strictly prefers CB.

8.7 Proof of Proposition 6

Proof. A's payoff in timing BC is

$$U_A^{BC}=(1-\beta)\,S_{AB}^{BC}\left(b^{BC}\right)+\beta O_A^{BC}, \tag{29}$$

with

$$S_{AB}^{BC}\left(b^{BC}\right)=u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+(1-\gamma)\left(u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right)+\gamma u_A\left(b^{BC},0\right)$$

and

$$O_A^{BC}=(1-\gamma)\max_{c\in\mathcal{C}}\left\{u_A\left(0,c\right)+u_C\left(0,c\right)\right\}.$$

Inserting the last two expressions into (29) and rearranging yields

$$(1-\beta)\left\{u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+(1-\gamma)\left(u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right)+\gamma u_A\left(b^{BC},0\right)\right\}$$

$$+\beta(1-\gamma)\left\{u_A\left(0,c^*(0)\right)+u_C\left(0,c^*(0)\right)\right\}.$$

This can be written as

$$(1-\beta)(1-\gamma)\left\{u_A\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)+u_C\left(b^{BC},c^*\left(b^{BC}\right)\right)\right\} \tag{30}$$

$$+(1-\beta)\gamma\left\{u_A(b^{BC},0)+u_B\left(b^{BC},c^*\left(b^{BC}\right)\right)\right\}+\beta(1-\gamma)\left\{u_A\left(0,c^*(0)\right)+u_C\left(0,c^*(0)\right)\right\}.$$

We now compare (30) with (11). Let us first look at the last term of (11). If u_A is weakly super-modular in b and c, then $u_A(b,c)\geq u_A(b,0)+u_A(0,c)$ for all b and c. Therefore, the last term of (11) is weakly negative.

Looking at the first and the second term of (30), it is easy to see that the structure is the same as the one of the first two terms of (11). However, the arguments are different. In (11), they are $b^\star$ and $c^\star$ or $b^\star$ and 0, whereas in (30) they are b^{BC} and $c^*\left(b^{BC}\right)$ or b^{BC} and 0. If b^{BC} were equal to $b^\star$, then $c^*\left(b^{BC}\right)$ will also be equal to $c^\star$ because the maximization problem with respect to c is then the same in the simultaneous and the sequential timing. However, b^{BC} is chosen to maximize the first two terms of (30) (i.e., taken into account the reaction of c in the second stage). Therefore, by a revealed preference argument, if b^{BC} differs from $b^\star$, the first two terms of (30) must be larger than the corresponding ones of (11). In case multiple equilibria exist with simultaneous negotiations, this argument holds for any such equilibrium.

Finally, we need to compare the last term of (30) (i.e., $\beta(1-\gamma)\{u_A(0, c^*(0)) + u_C(0, c^*(0))\}$), with the third term of (11) (i.e., $\beta(1-\gamma)\{u_A(0, c^\star) + u_C(b^\star, c^\star)\}$). Since $b^\star \geq 0$ and $c^*(0)$ maximizes $u_A(0, c) + u_C(0, c)$, it is evident that the latter term is lower then the former if externalities are negative. It follows that all terms in (11) are weakly lower than those in (30) if externalities are negative and u_A is super-modular. In addition, unless equilibrium decisions are zero, (11) is strictly lower than (30) if externalities are strictly negative and/or u_A is strictly super-modular in b and c. ■

8.8 Proof of Proposition 7

Proof. We know from Proposition 3 that timing BC is preferred over timing CB in case of no externalities. Therefore, can we focus on timing BC in our comparison with the simultaneous timing.

We again need to to compare (30) with (11). Starting with the first two terms of each expressions, the argument made in the previous proof does not depend on externalities: as b^{BC} is chosen to maximize these terms whereas b^* is not, these terms must be weakly larger in (30) than in (11). Comparing the third term of (11) with the last term of (30), as there are no externalities, the difference in the first argument of u_C in both terms is irrelevant. Therefore, the driving force in the difference between these two terms is that $c^*(0)$ maximizes $u_A(0, c) + u_C(0, c)$ but c^* does not necessarily do so. As a consequence, this term is also weakly larger in (30) than in (11). Evidently, these arguments hold independent of selected equilibrium in the simultaneous game, in case there are multiple ones. Finally, if u_A is super-modular in b and c, we know from the proof of the previous proposition, that the last term of (11) is negative. It follows that timing BC is preferred by the principal if u_A is super-modular.

We now turn to the case in which u_A is sub-modular in b and c. Suppose first that $\gamma = 0$, which implies the last term in (11) drops out. However, the arguments given above continue to hold. In particular, the difference in the first three terms between (11) and (30) is still weakly negative. Hence, timing BC is still preferred by the principal if u_A is sub-modular and $\gamma = 0$. By continuity, the result also holds in the vicinity of $\gamma = 0$.

Finally, suppose $\gamma = 1$, which implies that $\beta = 1$ (since $\gamma \leq \beta$). It is evident that (30) is equal to zero, whereas (11) equals $u_A(b^*, 0) + u_A(0, c^*) - u_A(b^*, c^*)$. But u_A being sub-modular implies $u_A(b^*, 0) + u_A(0, c^*) > u_A(b^*, c^*)$; hence, (11) is positive. Again, by continuity, the result also holds in the vicinity of $\gamma = 1$. ■

8.9 Proof of Proposition 8

Proof. From the proof of Proposition 4 we know that, when evaluated at $k = 0$,

$$\frac{d}{dk} u_A^{CB}(k) = \frac{\partial}{\partial k}\left((1-\gamma) u_C(b, c; k) + (1-\gamma)(1-\beta) u_B(b, c; k)\right)\Bigg|_{\substack{k=0 \\ b=b^*\left(c^{CB}(0), 0\right) \\ c=c^{CB}(0)}}$$

Applying the same logic to (11) , we obtain

$$\frac{d}{dk} U_A^{sim}(k) = \frac{\partial}{\partial k}\Big((1-\beta)(1-\gamma)\left(u_B(b, c; k) + u_C(b, c; k)\right)$$

$$+ (1-\beta)\gamma u_B(b, c; k) + \beta(1-\gamma) u_C(b, c; k)\Big)\Bigg|_{\substack{k=0 \\ b=b^* \\ c=c^*}}$$

$$= \frac{d}{dk} U_A^{sim}(k) = \frac{\partial}{\partial k}\left((1-\gamma) u_C(b, c; k) + (1-\beta) u_B(b, c; k)\right)\Bigg|_{\substack{k=0 \\ b=b^* \\ c=c^*}}$$

Symmetry of agents, no externalities and u_A being additive separable implies $b = b^*\left(c^{CB}(0), 0\right) = b^*$ and $c^{CB}(0) = c^*$. As a consequence,

$$\frac{d}{dk}\left\{U_A^{sim}(k) - U_A^{CB}(k)\right\} = \frac{\partial}{\partial k}\left(\gamma(1-\beta) u_B(b, c; k)\right)\Bigg|_{\substack{k=0 \\ b=b^* \\ c=c^*}} > 0$$

If $k = 0$, the bargaining problems do not interact. This implies that there is a unique equilibrium in the simultaneous game, which is equivalent to the equilibrium of the sequential game and $U_A^{sim}(0) = U_A^{CB}(0)$. By continuity, it follows that for sufficiently small $k > 0$, $U_A^{sim}(k) > U_A^{CB}(k)$. ■

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