A Class of Non-Parametric Deformed Exponential Manifolds modelled on Sobolev Space

Nigel J. Newton

The talk will develop Sobolev variants of the non-parametric statistical manifolds constructed in [2, 3], in which the sample space is R^d. Details can be found in [1]. The manifolds are modelled on a variety of weighted Sobolev spaces, including Hilbert-Sobolev spaces and mixed-norm spaces. They are based on the deformed logarithm $\log_d y = y - 1 + \log y$, whose inverse has linear growth. Unusually for the Sobolev context, and as a consequence of its linear growth, this 'lifts' to a nonlinear superposition (Nemytskii) operator that acts continuously on a particular class of mixed-norm model spaces, and on the fixed-norm space $W^{2,1}(\mu)$; i.e. it maps each of these spaces continuously into itself. Some of the results make essential use of log-Sobolev embedding. Each manifold contains a smoothly embedded submanifold of *probability* measures, the properties of which are naturally expressed in the context of this embedding.

The manifolds in [2, 3] are constructed with the Boltzmann-Gibbs-Shannon entropy in mind, and as such support the Fisher-Rao metric as a weak Riemannian metric. However, the construction in [1] can be applied to any non-parametric manifold based on a deformed exponential with particular properties, primarily the boundedness of its derivatives, and this includes the Kaniadakis exponential with parameter $\kappa = 1$.

[1] Newton, N.J, A Class of Non-Parametric Statistical Manifolds modelled on Sobolev Space, arXiv:1808.06451 (2018).

[2] Newton, N.J.: An infinite-dimensional statistical manifold modelled on Hilbert space, J. Functional Analysis, 263, 1661--1681 (2012).

[3] Newton, N.J.: Infinite-dimensional statistical manifolds based on a balanced chart, Bernoulli, 22, 711-731 (2016).