ISSN 2279-9362

Collegio Carlo Alberto

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No. 577
December 2018

Carlo Alberto Notebooks

www.carloalberto.org/research/working-papers

Life-Cycle Portfolios, Unemployment and Human Capital Loss*

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Abstract

The recent Great Recession highlighted that long-term unemployment spells may entail persistent losses in workers' human capital. This paper extends the life-cycle model of savings and portfolio choice with unemployment risk, by allowing the possibility of permanent reductions in expected earnings following long-term unemployment. The optimal risky portfolio share becomes flat in age due to the resolution of uncertainty about future returns to human capital that occurs as the worker ages. This may help explaining the observed relatively flat, or only moderately increasing, risky share of investors during working life, and have important consequences for the design of optimal life-cycle portfolios by investment funds.

Keywords: life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D15, E21, G11

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Declaration of interest: none.

*We thank Jonathan Berk, Antoine Bommier, Margherita Borella, Marie Brière, Laurent Calvet, Claudio Campanale, Andrea Colciago, Frank DeJong, Bernard Dumas, Jordi Gali, Stefano Giglio, Jerome Glachant, Francisco Gomes, Luigi Guiso, Michael Haliassos, Tullio Jappelli, Christian Julliard, Dirk Krueger, Giovanni Mastrobuoni, Elisa Luciano, Sidney Ludvigson, Marco Pagano, Patrizio Tirelli, Ernesto Villanueva, Gianluca Violante and Bas Werker for very useful comments and suggestions. We thank participants to the NETSPAR International Workshop on Pensions, 2016, to the RiskForum2016 (Institut Louis Bachelier), to the to the CEPR Workshop on Household Finance, 2016, to the RES 2017 and to the EEA 2017 annual conferences. We are grateful to CINTIA-Italy for funding.

1 Introduction

Several findings in the macro-labor literature indicate that long-term unemployment may lead to a loss of human capital. In this paper, we embed the possibility of entering long-term unemployment with permanent consequences on human capital in a life-cycle model of consumption and portfolio choice. We model working life careers as a three-state Markov chain driving the transitions between employment, short-term and long-term unemployment states, as in Bremus and Kuzin (2014), calibrated to broadly match recently observed U.S. labor market features. Importantly, we allow for human capital loss during unemployment. When unemployed, individuals receive benefits but simultaneously experience a cut in the permanent component of labor income which captures diminished future income prospects. This represents the observed permanent earning losses (Arulampalam, Booth and Taylor, 2000; Arulampalam, 2001; Schmieder, von Wachter and Bender, 2016) due to skill loss during long-term unemployment (Neal, 1995; Edin and Gustavsson, 2008).

Potential losses of human capital considerably lower the optimal portfolio share invested in stocks with respect to the case of no unemployment risk. Importantly, optimal stock investment is no longer decreasing with age but remains remarkably flat over the whole working life, in line with the evidence on U.S. portfolios (Amkeriks and Zeldes, 2004). On the contrary, traditional life-cycle models imply that households should reduce exposure to risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The reason is that human capital provides a hedge against shocks to stock returns, making financial risk bearing more attractive. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is large relative to accumulated financial wealth, and then gradually falling until retirement as human capital decreases relative to financial wealth. This model implication is embodied in the popular financial advice of a stock exposure steadily decreasing with age, the so-called “age rule”. In our model with human capital loss, such effect is instead moderated by the resolution of uncertainty concerning labor and pension income, as the worker safely comes close to retirement age. Since the risk of long term unemployment falls together with human capital as retirement approaches, the resolution of uncertainty compensates the hedge effect and the optimal investment in stocks is relatively flat over the life-cycle.

Optimal risky portfolios are highly heterogeneous in models without human capital loss. On the contrary, the permanent consequences of long-term unemployment shrink the heterogeneity of optimal portfolio choices across agents characterized by different employment histories. In the face of possible human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible adverse labor market outcomes. Optimal early consumption consequently falls, becoming higher during both late working life and retirement years. The working-year responses to unemployment risk, including the flat age profile in stock investment, are remarkably robust to changes in preferences on the intertemporal correlation of shocks. In fact, allowing for Epstein-Zin preferences only causes slower wealth decumulation and less risk taking during retirement years. Similarly, an increase in the correlation between stock returns and labor income shocks leaves the flat shape of optimal equity investment during working age unaltered, only increasing the portfolio share allocated to the riskfree asset. Thus, it is the human capital loss the first order determinant of the optimal financial risk-taking at different ages.

The above results obtain in calibrations to U.S. data: in particular, the implied unconditional probabilities of being short-run unemployed (3.78%) and long-run unemployed (1.72%) are set at the levels observed in the U.S. after the Great Recession. The human capital loss amounts to some 25% of all future expected earnings only in the occurrence of a long-term unemployment spell, in a calibration that captures the relatively slow re-employment process experienced by U.S. workers. We select the magnitude of the human capital loss during long-term unemployment considering both the total loss of human capital for the fraction of workers abandoning the labor force, and the partial loss for those who are able to find a job. Our results go through even when the human capital loss parameter is reduced as far as 15% of future expected earnings, and when the probability of moving into long-term unemployment from an initial unemployment state is reduced by half (from 0.15 to 0.075).

Previous life-cycle models with unemployment and self-insurance leave the observed age pattern of stock holding during working life largely unexplained. Some versions of the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labor income: in these models unemployment risk affects income only during the unemployment spell with no consequences on subsequent earnings ability

(Cocco, Gomes and Maenhout, 2005) even when unemployment is persistent (Bremus and Kuzin, 2014). With no permanent consequence on subsequent earnings ability, the stock holding is still counterfactually decreasing in age till retirement although, on average, lower than what obtained without unemployment risk. Thus, it is the possibility of human capital loss entailed by long–term unemployment -rather than unemployment per se - that restrains risk-taking by the young and middle-aged workers.

Several papers already investigate alternative hypotheses that may deliver the relatively flat stock profile observed in the data, departing from the pattern implied by traditional life-cycle models. Some of this prior research already relate the flattening of the age profile of stock investment to the resolution of uncertainty over working life. Hubener, Maurer and Mitchell (2016) point to the time-varying risk of changing family status during working age due to marriage, fertility and divorce, which affects consumption both directly and through labor supply. In Bagliano, Fugazza and Nicodano (2014), such flattening crucially depends on the presence of both another risky asset, besides equities, and a positive correlation between stock returns and permanent labor income shocks. Moreover, it only appears when risk aversion or the variance of labor income shocks are higher than in the baseline calibration of Cocco Gomes and Maenhout (2005). Most importantly, Chang, Hong and Karabarbounis (2018) introduce labor market uncertainty into an otherwise standard life-cycle model. They show that the interaction between unemployment risk, occupational uncertainty and gradual learning about earnings ability generates a moderately increasing age profile of stock investment, with an average portfolio risky share (conditional on participation) substantially lower than in a typical life-cycle setting. Our model complements and strengthens their main conclusions by exploring the effects of an additional dimension of age-dependent labor market uncertainty, namely the risk of permanent human capital losses due to long-term unemployment, yielding an average optimal stock share below 60 percent and remarkably flat during working life. Notably, as in Chang, Hong and Karabarbounis (2018), our results are achieved under the assumptions of a moderate degree of risk aversion and the absence of positive correlation between labor income and stock market returns.

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 3 and discuss our main results in Section 4. Various robustness checks are presented in Section 5. Section 6 concludes the paper.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The investor starts working at age t_0 and retires with certainty at age $t_0 + K$. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t (with $p_{t_0-1} = 1$). Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0}\left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k}\right)\left(p_{t_0+j-1}\frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + (1-p_{t_0+j-1})\, b\frac{(X_{it_0+j}/b)^{1-\gamma}}{1-\gamma}\right)\right] \tag{1}$$

where C_{it} is the level of consumption at time t, X_{it} is the amount of wealth the investor leaves as a bequest to her heirs after her death, $b \geq 0$ is a parameter capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter.

2.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labor supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term (u_1) and long-term (u_2) unemployment. Individual labor market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} = \begin{pmatrix} \pi_{ee} & 1-\pi_{ee} & 0 \\ \pi_{u_1e} & 0 & 1-\pi_{u_1e} \\ \pi_{u_2e} & 0 & 1-\pi_{u_2e} \end{pmatrix} \tag{2}$$

where $\pi_{nm} = \text{Prob}\left(s_{t+1} = n | s_t = m\right)$ with $n, m = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability $\pi_{eu_1} = 1 - \pi_{ee}$. Since she must experience short-term unemployment prior to becoming long-term unemployed, we set the probability of directly entering long-term unemployment at zero, $\pi_{eu_2} = 0$. Conditional on being short-term unemployed at t ($s_t = u_1$), she exits unemployment ($s_{t+1} = e$) with probability $\pi_{u_1 e}$ or becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1 u_2} = 1 - \pi_{u_1 e}$; consequently, we set $\pi_{u_1 u_1} = 0$. Finally, if she is long-term unemployed at t ($s_t = u_2$), she is re-employed in the following period ($s_{t+1} = e$) with probability $\pi_{u_2 e}$ and remains unemployed with probability $\pi_{u_2 u_2} = 1 - \pi_{u_2 e}$.

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labor income driven by permanent and transitory shocks. In each working period, labor income Y_{it} is generated by the following process:

$$Y_{it} = H_{it} U_{it} \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_{it} = F\left(t, \mathbf{Z}_{it}\right) P_{it}$ represents the permanent income component. In particular, $F\left(t, \mathbf{Z}_{it}\right) \equiv F_{it}$ denotes the deterministic trend component that depends on age (t) and a vector of individual characteristics ($\mathbf{Z}_{it}$) such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$N_{it} = \log P_{it} = \log P_{it-1} + \omega_{it} \tag{4}$$

where ω_{it} is distributed as $N(0, \sigma_\omega^2)$. U_{it} denotes the transitory stochastic component and $\varepsilon_{it} = \log(U_{it})$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_{it}.

In our set-up, which differs from that of Bremus and Kuzin (2014), labor income received by the employed individual at time t depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labor income, H_{it}. Since the empirical evidence suggests that the longer the unemployment spell the larger is the worker's human capital depreciation (Schmieder, von Wachter and Bender, 2016), we let human capital loss increase with unemployment duration. Thus, after 1-year unemployment the permanent component H_{it} is equal to H_{it-1} eroded by a fraction Ψ_1, and after a 2-year unemployment spell the permanent component, H_{it-1}, is eroded by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. This introduces non-linearity into the expected permanent labor income. In compact form, the permanent component of labor income H_{it} evolves according to

$$H_{it} = \begin{cases} F_{it}P_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income $H_{it-1} = F_{it-1}P_{it-1}$, whereas in the long-term unemployment state ($s_t = u_2$) they receive an unemployment benefit in proportion ξ_2 of $H_{it-2} = F_{it-2}P_{it-2}$[1]. Thus, the income received during unemployment is

$$Y_{it} = \begin{cases} \xi_1 F_{it-1}P_{it-1} & \text{if } \mathrm{s}_t = u_1 \\ \xi_2 F_{it-2}P_{it-2} & \text{if } \mathrm{s}_t = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{6}$$

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labor income in the last working year:

$$Y_{it} = \lambda F\left(t, \mathbf{Z}_{it_{0+l}}\right) P_{it_{0+l}} \qquad t_0 + K < t \leq T \tag{7}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period and λ is level of the replacement rate.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real

[1] While keeping the model tractable (by allowing to get rid of one state variable F_{it}), this simplifying assumption is rather conservative in terms of the effects of long-term unemployment.

returns R_t^s, for each period. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{8}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2017).

At the beginning of each period, financial resources available to the individual for consumption and saving are given by the sum of accumulated financial wealth W_{it} and current labor income Y_{it}, which we call cash on hand $X_{it} = W_{it} + Y_{it}$. Given the chosen level of current consumption, C_{it}, next period cash on hand is given by

$$X_{it+1} = (X_{it} - C_{it})R_{it}^P + Y_{it+1} \tag{9}$$

where R_{it}^P is the investor's portfolio return:

$$R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s)\, R^f \tag{10}$$

with α_{it}^s and $(1 - \alpha_{it}^s)$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and we assume that the investor is liquidity constrained. Consequently, the amounts invested in stocks and in the riskless asset are non negative in all periods. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

2.3 Solving the life-cycle problem

In this intertemporal optimization framework, the investor maximizes the expected discounted utility over life span, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the optimization problem is written as:

$$\max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j-1} \frac{C_{it_0+j}^{1-\gamma}}{1-\gamma} + \right.\right.\right.$$
$$\left.\left.\left. + \left(1-p_{t_0+j-1}\right) b \frac{\left(X_{it_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{11}$$

$$s.t. \quad X_{it+1} = \left(X_{it} - C_{it}\right) \left(\alpha_{it}^s R_t^s + \left(1-\alpha_{it}^s\right) R^f \right) + Y_{it+1} \tag{12}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} + \beta E_t \left[p_t V_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right.\right.$$
$$\left.\left. + \left(1-p_t\right) b \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{13}$$

At each time t the value function V_{it} describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_{it}), the stochastic permanent component of income at beginning of t (P_{it}), and the labor market state $s_{it} = e$, u_1, u_2. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit:

$$V_{it}\left(X_{it}, P_{it}, s_{it}\right) = \max_{\{C_{it}\}_{t_0}^{T-1},\{\alpha_{it}^s\}_{t_0}^{T-1}} \left(\frac{C_{it}^{1-\gamma}}{1-\gamma} \right.$$
$$+ \beta \left[p_t \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \widetilde{E_t V}_{it+1}\left(X_{it+1}, P_{it+1}, s_{it+1}\right) \right.$$
$$\left.\left. + \left(1-p_t\right) b \sum_{s_{it+1}=e,u_1,u_2} \pi\left(s_{it+1}|s_{it}\right) \frac{\left(X_{it+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{14}$$

where $\widetilde{E_t V}_{it+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{it+1}, ε_{it+1}, and ν_{t+1}^s. The history dependence that we introduce in our set-up by making unemployment affect subsequent labor income prospects prevents using the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible state as (dropping for simplicity the term involving the bequest motive):

$$V_{it}(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{e,e} \\ \text{with } P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, u_1) \quad \text{with prob. } 1 - \pi_{e,e} \\ \text{with } P_{it+1} = (1 - \Psi_1) P_{it} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_1 F_{it} P_{it} \end{cases} \end{cases}$$

$$V_{it}(X_{it}, P_{it}, u_1) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_1,e} \\ \text{with } P_{it+1} = (1 - \Psi_1) P_{it-1}\, e^{\omega_{it+1}} = P_{it}\, e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-1} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, u_2) \quad \text{with prob. } 1 - \pi_{u_1,e} \\ \text{with } P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_2 F_{it-2} P_{it-2} \end{cases} \end{cases}$$

$$V_{it}(X_{it}, P_{it}, u_2) = u(C_{it}) + \beta p_t \begin{cases} \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, e) \quad \text{with prob. } \pi_{u_2,e} \\ \text{with } P_{it+1} = P_{it} e^{\omega_{it+1}} \quad \text{and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + F_{it-2} P_{it+1} e^{\varepsilon_{it+1}} \end{cases} \\ \\ \begin{cases} \widetilde{E_t V}_{it+1}(X_{it+1}, P_{it+1}, u_2) \quad \text{with prob. } 1 - \pi_{u_2,e} \\ \text{with } P_{it+1} = (1 - \Psi_2) P_{it} \text{ and} \\ X_{it+1} = (X_{it} - C_{it}) R^p_{it} + \xi_2 F_{it-2} P_{it-2} \end{cases} \end{cases} \tag{15}$$

This problem has no closed form solution; therefore, we obtain the optimal values for consumption and portfolio shares, depending on the values of each state variable at each point in time, by means of numerical techniques. To this aim, we apply a backward induction procedure starting from the last possible period of life T and computing optimal consumption and portfolio share policy rules for each possible value of the continuous state variables (X_{it}, P_{it}) by means of the standard grid search method.[2] Going backwards, for every period $t = T-1, T-2, ..., t_0$, we use the Bellman equation (14) to obtain optimal rules for consumption and portfolio shares.

3 Calibration

Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. For reference, we initially solve the model by abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005). Then, we introduce unemployment risk and consider two scenarios: (i) unemployment spells cause only temporary income losses, as in Bremus and Kuzin (2014), and (ii) unemployment has permanent consequences on the worker's earnings ability.

Across all scenarios, the agent begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the U.S. National Center for Health Statistics. With regards to preferences, we set the utility discount factor $\beta = 0.96$ (as in Cocco, Gomes and Maenhout, 2005), and the parameter capturing the strength of the bequest motive $b = 2.5$ (as in Gomes and Michaelides, 2005), which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for. Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$, much lower than the value typically adopted in the literature. The riskless (constant) interest rate is set at 0.02, with an expected equity premium μ^s fixed at 0.04. The standard deviation of the return innovations is set at $\sigma_s = 0.157$ (as in Cocco, Gomes and Maenhout, 2005). Finally, we impose a zero correlation between stock return innovations and aggregate permanent labor income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters with source references.

[2] The problem is solved over a grid of values covering the space of both the state variables and the controls in order to ensure that the obtained solution is a global optimum.

Table 1: Calibration parameters

Description	Parameter	Value	Source
Working life		20-65	
Retirement		65 -100	
Risk aversion	γ	5	
Replacement ratio	λ	0.68	OECD (2015)
Discount factor	β	0.96	Cocco et al. (2005)
Bequest Motive	b	2.5	Gomes and Michaelides (2005)
Variance of permanent shocks to labor income	σ^2_ω	0.0106	Cocco et al. (2005) on PSID
Variance of transitory shocks to labor income	σ^2_ϵ	0.0738	Cocco et al. (2005) on PSID
Riskless rate	r	2%	Cocco et al. (2005)
Excess returns on stocks	μ^s	4%	Cocco et al. (2005)
Variance of stock returns innovations	σ_s	0.025	Cocco et al. (2005)
Stock ret./permanent lab. income shock correlation	ρ_{sY}	0	Cocco et al. (2015)
Unemployment benefits			
Short-term unemployed	ξ_1	0.3	OECD (2010)
Long-term unemployed	ξ_2	0.1	OECD (2010)
Human Capital Loss			
Short-term unemployed	Ψ_1	0	
Long-term unemployed	Ψ_2	0.25	Jacobson et al. (1993a)

This table reports benchmark values of relevant parameters with source reference.

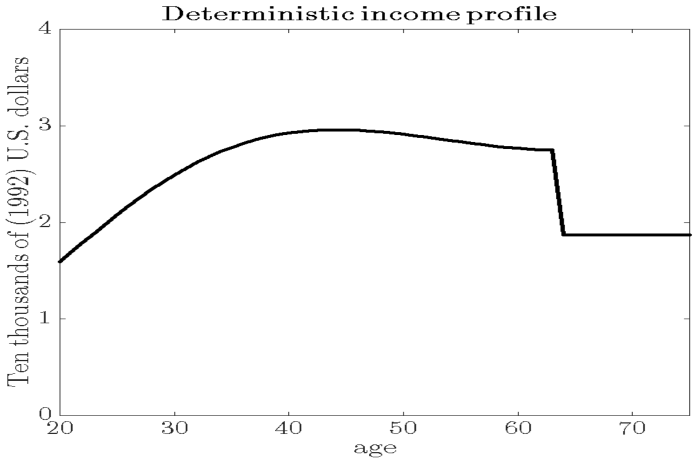
3.1 Labor income and unemployment risk

The labor income process is calibrated using the estimated parameters for U.S. households with high school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). For the high school group, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are equal to $\sigma^2_\omega = 0.0106$ and $\sigma^2_\varepsilon = 0.0738$. After retirement, income is a constant proportion λ of the final (permanent) labor income, with $\lambda = 0.68$, as the net replacement rate of total pension benefits for the average earner in the U.S. (OECD, 2015)[3]. The age-dependent trend is captured by a third-order polynomial in age

[3]For a more realistic Social Security System design and its implications on retirement, consumption and investment decisions see Hubener, Maurer and Mitchell (2016).

fitted to the age coefficients estimates in Cocco, Gomes and Maenhout (2005), delivering the typical hump-shaped profile until retirement depicted in Figure 1.

Figure 1: Age-Income profile for U.S. high-school educated workers



The figure reports the age-income profile derived using the calibration in Cocco et al. (2005) for high-school educated workers.

The resulting labor income process does not capture the evidence in Krueger, Cramer and Cho (2014) that the long-term unemployed experience a progressive declining re-employability over time and are more likely to exit the labor force. We use data from the Current Population Survey (CPS) to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment along with the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft, Lange, Notowidigdo and Katz (2016), the annual transition probability from employment to unemployment is 4%. Given the duration dependence and the steady decline in the annual outflow rate from unemployment to employment during the first year of unemployment (Kroft, Lange, Notowidigdo and Katz, 2016), we set the probability of leaving unemployment after the first year at 85%.

The annual transition probabilities between labor market states are chosen to match the average annual unemployment rate in the United States:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.85 & 0 & 0.15 \\ 0.33 & 0 & 0.67 \end{pmatrix} \tag{16}$$

The assumed transition matrix (16) yields unconditional probabilities of being short-run (3.8%) and long-run unemployed (1.7%) in line with respect to the 2015 overall (5.5%) and long-term (1.7%) unemployment rates in the U.S..

In our baseline calibration with "*human capital loss*" we assume a non-negligible human capital depreciation following a 2-year unemployment spell. While Ψ_1 is kept at 0, Ψ_2 is increased up to 0.25, implying a 25% loss of the individual permanent labor income component after the second year of unemployment, which captures the long-lasting effects of protracted inactivity on job careers. Well-established empirical evidence on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement may range from 30% (Couch and Placzek, 2010) to 40% of earnings (Jacobson, Lalonde and Sullivan, 1993b). Earnings losses are shown to be persistent in a range from 15% (Couch and Placzek, 2010) to about 25% (Jacobson, LaLonde and Sullivan, 1993a) of their pre-displacement levels. These estimates abstract from the effect of unemployment duration, while Cooper (2013) finds that earnings losses are larger the longer unemployment lasts. Also, based on administrative data, Jacobson, LaLonde and Sullivan (2005) estimate that average earnings losses for displaced workers amount to 43-66% of their predisplacement wage and Guevenen, Karahn, Ozkan and Song (2017) estimate that the median earnings loss after a full year nonemplyment amounts to 50% of the predisplacement wage. Overall, compared with the body of existing evidence, our choice of $\Psi_2 = 0.25$ is relatively conservative, being well in the range of available estimates of earning losses in the literature. Given the relevance of the human capital loss effect in our model, we consider a wider range of alternative values for $\Psi_2 = 0.25$ in section 4.2.1 below.

Unemployment benefits are calibrated according to the U.S. unemployment insurance system (OECD, 2010). In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks, we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a smaller value of $\xi_2 = 0.1$ for the long-term unemployed.

For comparison, we also consider a calibration of the model without unemployment risk. This "*no unemployment risk*" scenario corresponds to the standard life-cycle set up with $\pi_{ee} = 1$ and all other entries equal to zero in the transition probability matrix (2). In addition, to highlight the effects of permanent consequences of unemployment on future earnings prospects, we consider a third calibration by adding the unemployment risk embedded in the transition probability matrix (16) with no human capital loss. In this "*unemployment with no human capital loss*" scenario, unemployment has no permanent consequences on future earnings (i.e. $\Psi_1 = \Psi_2 = 0$) but entails only a cut in current income. This case closely corresponds to the set-up studied by Bremus and Kuzin (2014), who focus only on temporary effects of long-term unemployment.

4 Results

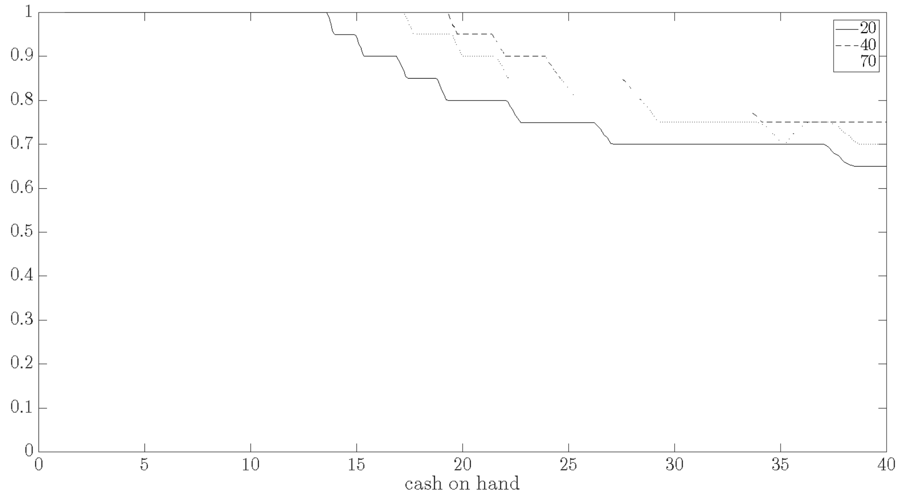
4.1 Optimal policies

Figure 2 compares investors' optimal stock shares in the standard case of "*no unemployment risk*" (panel (a)) and in our preferred scenario with "*human capital loss*" (panel (b)). In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labor income component of investors at three different ages (20, 40, and 70). In the case with no unemployment risk, standard life-cycle results are obtained. Labor income acts as an implicit risk-free asset and affects the optimal portfolio composition depending on an investor's age and wealth. For example, at age 20 the sizable implicit holding of the risk-free asset (through human capital) makes it optimal for less-wealthy investors to tilt their portfolio towards the risky financial asset. Indeed, for a wide range of wealth levels, agents optimally choose to be fully invested in stocks. The optimal stock holding decreases with financial wealth because of the relatively lower implicit investment in (risk-free) human capital.

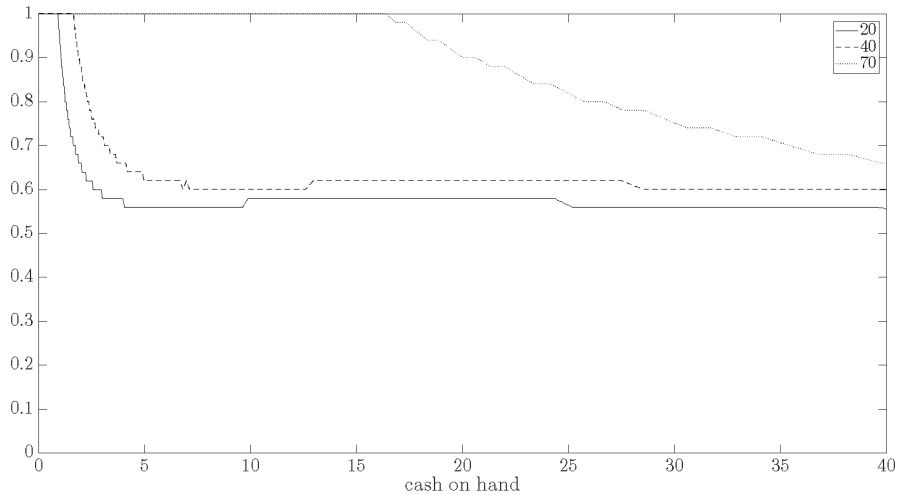
When the model is extended to allow for permanent effects of unemployment spells on labor income prospects at re-employment ("*human capital loss*"), with the parameters governing the proportional loss of permanent labor income set at Ψ_1= 0 after one year of unemployment and at Ψ_2= 0.25 after 2 years, the resulting policy functions are shifted abruptly leftward. The optimal stock share still declines with financial wealth but a 100% share of investment in stocks is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the case of no unemployment risk.

Figure 2: Policy functions

(a) No unemployment risk



(b) Unemployment risk with human capital loss



This figure shows the portfolio rules for stocks as a function of cash on hand for an average level of the stochastic permanent labor income component. The policies refer to selected ages: 20, 40, and 70. Panel (a) and (b) refer respectively to the cases with no unemployment risk and with unemployment risk and human capital loss. In the latter case, the parameters governing the human capital loss during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Cash on hand is expressed in ten thousands of U.S. dollars.

4.2 Life-Cycle Profiles

On the basis of the optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 3, panel (a), shows the average optimal stock shares plotted against age when unemployment risk is ignored and when it is accounted for. In the case of no unemployment risk (dotted line), the well-known result on the age profile of optimal stock portfolio shares is obtained. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and decreases with age to reach around 80% at retirement. When unemployment risk without human capital loss is considered (dashed line), the optimal portfolio share of stocks still declines with age, though being slightly lower at all ages, with a 100% optimal stock share only for very young investors.

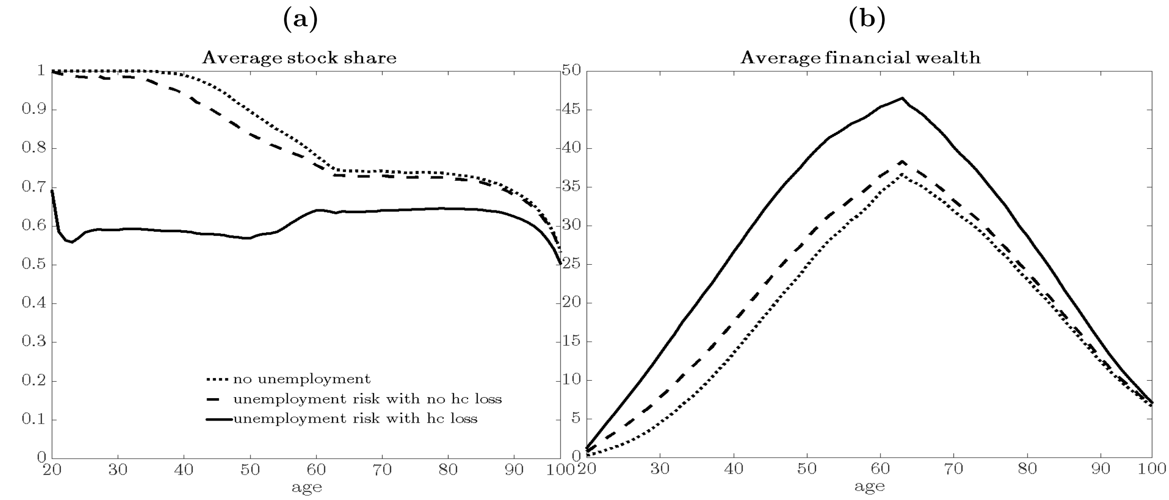
However, when long-term unemployment implies a rare but large skill loss (solid line), the optimal stock investment is sizably reduced at any age and almost flat, at around 55-65%. The risk of permanently losing a substantial portion of future labor income prospects reduces the level of human capital and increases its riskiness. Because this effect is particularly relevant for younger workers, it induces a lower optimal stock investment conditional on financial wealth especially when young. Consequently, the age profile remains remarkably flat over the whole working life.[4] These results highlight that possible long-run consequences of unemployment significantly dampen the incentive to invest in stocks, under standard calibrations, whereas unemployment persistence, with only temporary income losses as in Bremus and Kuzin (2014), has almost no effect on the age profile of optimal portfolio composition.

The reduction in the optimal portfolio share allocated to stocks is due to higher wealth accumulation, in turn induced by larger precautionary savings.[5] Panel (b) of Figure 3 displays the average financial wealth accumulated over the life cycle for the three scenarios considered. In the face of possible, albeit rare, human capital depreciation, individuals accumulate substantially more financial wealth during working life to buffer possible disastrous labor market outcomes. Optimal consumption when young consequently falls, but it is much higher during both late working years and retirement years.

[4] The relatively low investment in stocks during retirement is due to the presence of a positive bequest motive, common to all parameterization considered in this paper.

[5] Love (2006) shows that higher unemployment insurance benefits reduce calibrated contributions to pension funds by the young, suggesting that precautionary savings when young is due to unemployment risk.

Figure 3: Life-cycle profiles of stock share and financial wealth

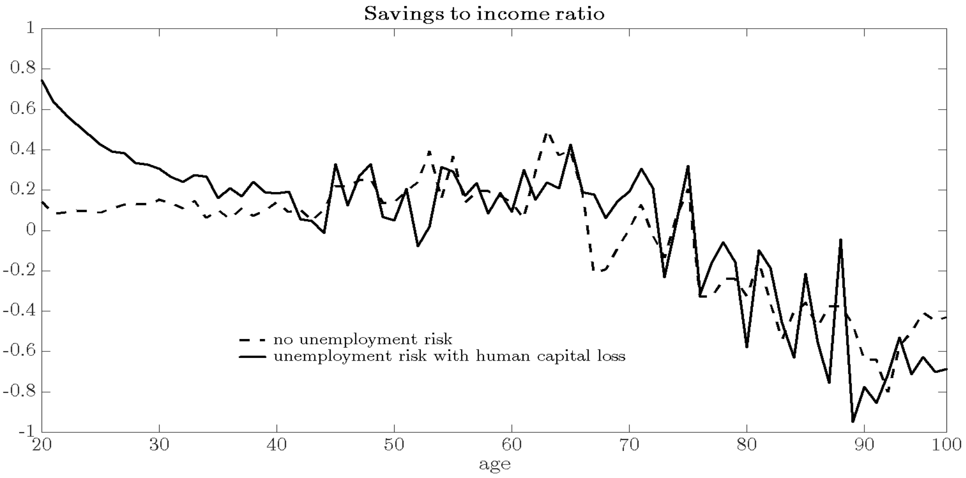


This figure displays the mean simulated stock investment and financial wealth accumulation life-cycle profiles. Age ranges from 20 to 100. The three cases correspond to no unemployment risk (dotted line); unemployment risk with no human capital loss (dashed line); unemployment risk with human capital loss (solid line). In the latter case, the parameters governing the human capital loss during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

Figure 4 displays the life-cycle profile of the ratio between savings and total (financial plus labor) income, comparing the case without unemployment risk to the one with unemployment and human capital loss. When the worker is 20 years old, the average propensity to save is especially high in the latter case, reaching 0.8 compared with less than 0.2 when unemployment risk is absent. Such propensity monotonically decreases in age, converging to the known pattern when the worker is in her forties. The figure clearly depicts the impact on savings of the resolution of uncertainty as individuals age.

Consistent with these predictions, data on Norwegian households show that they accumulate additional savings and shift toward safe assets in the years prior to unemployment and deplete savings after the job loss (Basten, Fagereng and Telle, 2016). Importantly, our results imply that labor market institutions targeted to long-term unemployment affect both risk taking in the equity market and precautionary saving. The expectation of a higher benefit may mitigate the adverse impact of long term unemployment on human capital, reducing the need for cautious investing and saving during working life. The variation of institutions across countries may thus generate different life-cycle patterns in equity investing. In this light, the decreasing stock holdings in Norwegian data (Fagereng, Gottlieb and Guiso, 2017) may be a consequence of higher long-term unemployment benefits with respect to the U.S..

Figure 4: Life-cycle profiles of savings rate



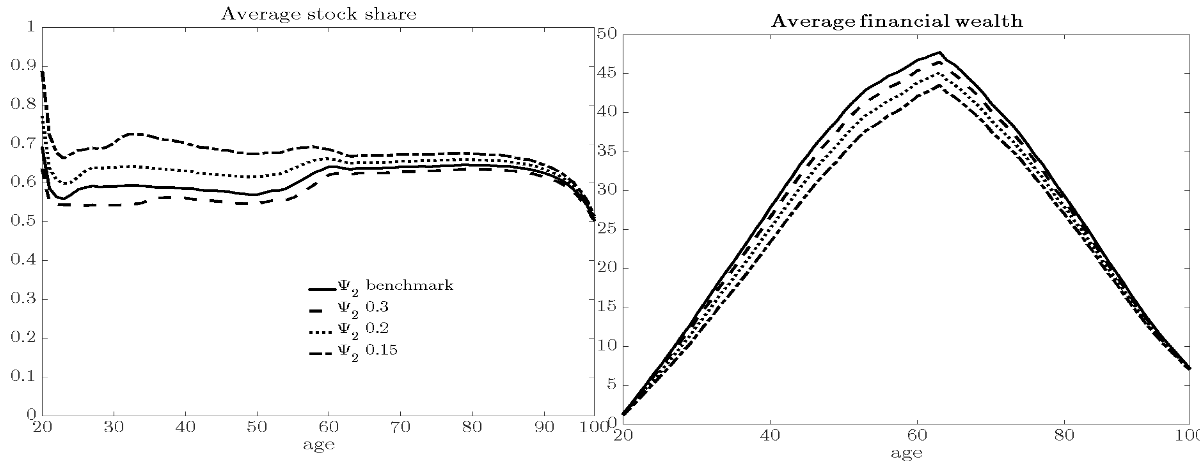
This figure displays the savings dynamics for individuals of age 20 to 100, relative to total income (i.e. labor income plus financial income). The two cases correspond to no unemployment risk (dotted line) and unemployment risk with human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$ (solid line).

4.2.1 Human capital loss intensity and labor income growth rate

We now check the sensitivity of life-cycle profiles with respect to the magnitude of the human capital loss effect due to long-term unemployment, captured by the parameter Ψ_2 (set equal to 0.25 in our baseline calibration) and to the age-income profile.

Human capital loss intensity. Since, as discussed in Section 3, available estimates of earnings losses due to long-term unemployment are as low as 15% of the level of pre-displacement earnings, Figure 5 shows the results of an experiment with the human capital loss parameter Ψ_2 in the range $0.15 - 0.30$. Even when the parameter falls to 0.15, the flattening in the portfolio share of stocks is maintained over working life around an average of 70%.

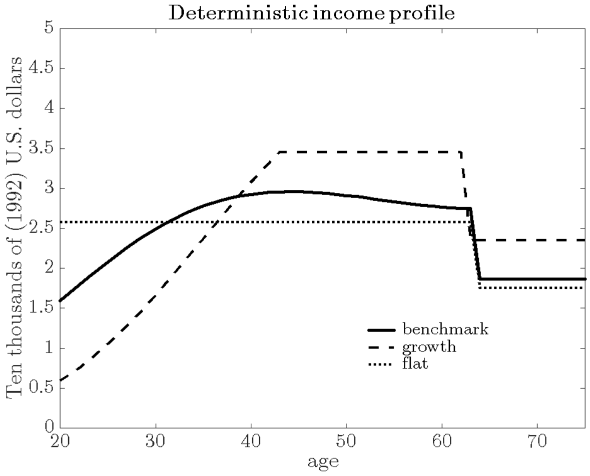
Figure 5: Sensitivity to human capital loss



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Alternative values of human capital loss during unemployment are considered: Ψ_2 ranging from 0.15 to 0.30, and in all cases $\Psi_1 = 0$. Financial wealth is expressed in ten thousands of U.S. dollars.

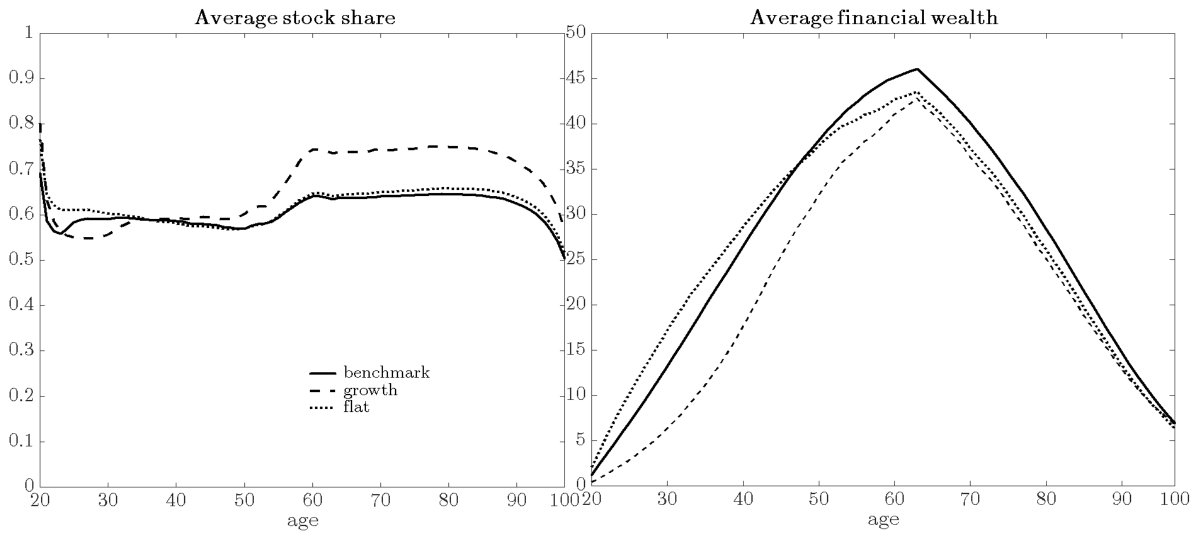
Shape of the income profile. To highlight the role of the risk of human capital loss in determining a flattening of stock investing over the life cycle, we next consider different deterministic income profiles, shown in Figure 6. In particular, we focus on two alternative profiles (one flat, and one steeply rising until mid-working life) that imply the same present discounted value of income at age 20 as the hump-shaped profile used in our benchmark calibration. Figure 7 displays the resulting portfolio stock shares and financial wealth accumulation paths. The finding of a relatively flat age pattern of the risky share over working life is robust. When deterministic labor income is flat, both wealth accumulation and optimal stock investment follow very closely the pattern obtained in the hump-shaped case. In the event of a steeper labor income profile, with larger earnings occurring in the later part of working years, the young investor faces a greater risk of human capital loss, that makes him to invest less in stocks than in the benchmark case. The risky share is then moderately adjusted upwards along working life, as the gradual resolution of uncertainty concerning labor and pension income compensates the hedging effect. The portfolio rebalancing toward stocks becomes more pronounced in the final part of working activity, since the risk of human capital loss due to long-run unemployment is reduced and a certain (and relatively high) pension income is coming closer.

Figure 6: Alternative age-income profiles



This figure displays hypothetical stylized age-income profiles that imply the same present discounted value of income at age 20. The benchmark is the labor income profile of U.S. high-school educated workers estimated in Cocco et al. (2005).

Figure 7: Life-cycle profiles with unemployment and human capital loss: different income profiles



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100, in case of different deterministic age-profiles of labor income. The benchmark computed considering the labor income profile for U.S. high school educated workers estimated in Cocco et al. (2005). In all cases the human capital loss is considered ($\Psi_1 = 0$ and $\Psi_2 = 0.25$). Financial wealth is expressed in ten thousands of U.S. dollars.

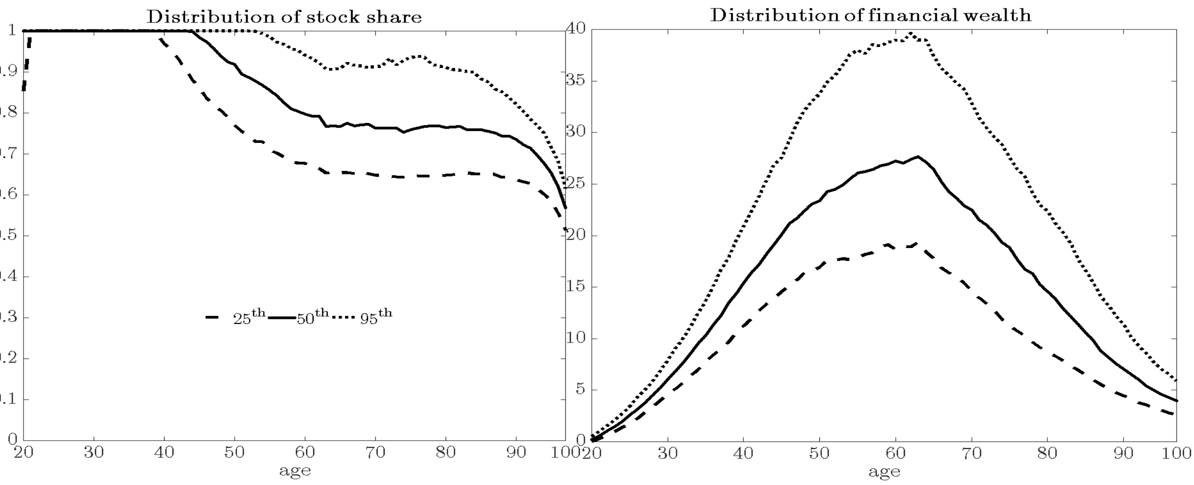
4.2.2 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but non-negligible human capital depreciation, workers on average invest about 55% of their financial wealth in stocks. This average pattern may hide considerable differences across agents. The present section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

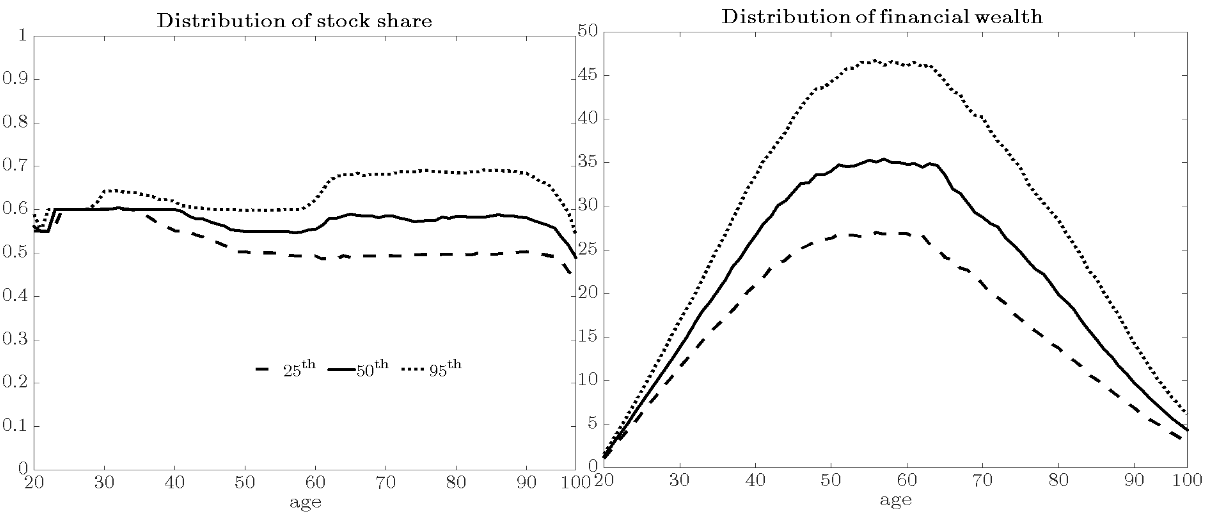
The case of no unemployment risk is displayed in panel (a) of Figure 8, which shows the 25^{th}, 50^{th} and 75^{th} percentiles of the distributions. Both the optimal stock share and the stock of accumulated financial wealth are highly heterogeneous across workers as well as retirees. The exception is young workers as they tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions displayed in Figure 2 panel (a), relating optimal stock shares to the amount of available cash on hand. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices. At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to be fully invested in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation. This situation pushes investors on the steeper portion of their policy functions and determines a gradual increase in the heterogeneity of optimal risky portfolio shares during their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions; as a consequence, the dispersion of optimal shares tends to persist.

Figure 8: Life-cycle percentile profiles

(a) No unemployment risk



(b) Unemployment risk with human capital loss



This figure displays the distribution (25^{th}, 50^{th} and 75^{th} percentiles) of simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100 in the case of no unemployment risk (panel (a)) and of unemployment risk with human capital loss (panel (b)). The parameters governing the human capital loss during short-term and long-term unemployment spells are $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

Panel (b) of Figure 8 displays the life-cycle distribution of stock share and financial wealth with unemployment risk and human capital loss. Compared with the case of no unemployment risk, the distribution of optimal stock shares is much less heterogeneous over the whole life cycle. In particular, heterogeneity shrinks during working life even for young workers, given the high human capital risk they bear at the beginning of their careers. In case of unemployment risk, policy functions are relatively flat (panel (b) of Figure 2) implying that even large differences in the level of accumulated wealth result in homogenous asset allocation choices. Then, as in the previous case, the shape of heterogeneity of stock shares and accumulated financial wealth over the life cycle is due to different realizations of background risk.

5 Robustness and Other Challenges

This section sheds additional light on the strength of our results, that radically depart from the accepted wisdom concerning optimal life-cycle behavior during working years.

A first robustness check concerns the sensitivity of our results to a lower probability of experiencing long-term unemployment. In performing such analysis, we also allow for an asymmetric reduction in the probability of long-term unemployment with respect to workers' age. Recent data from U.S. labor market statistics indeed show that the composition of long-term unemployment is shifting towards the elderly. In 2015 the overall and the long-term unemployment rates in U.S. were about 5.5% and 1.7%, respectively, with the share of long-term unemployment in the overall unemployment rate differing widely among age groups: from 20% among young workers (16-24 years old), to 35% among prime age workers (25-55), and up to 41% among older workers (over 55).

A second check regards the modelling of the link between unemployment risk during working life and retirement income, so as to make sure that our results do not depend on long-term unemployment occurring during the very last working years, which heavily reduces retirement income. The robustness to alternative values of the pension replacement ratio and of long-term unemplyment benefit provisions is also assessed.

Further, since the power utility function used in our life-cycle model implies that the worker is indifferent to intertemporal correlation of consumption shocks (Bommier, 2007), we adopt Epstein-Zin preferences to investigate whether positive correlation aversion boosts the impact of unemployment with human capital loss. A similar motivation leads us to analyse the sensitivity of the equity-investment profile to positive correlation between stock returns and labor income shocks.

Finally, we evaluate the robustness of our main results to alternative calibrations of key preference parameters (the subjective discount factor and the degree of risk aversion) and address some logical challenges related to our analysis, including its general equilibrium implications.

5.1 Age-dependent unemployment risk

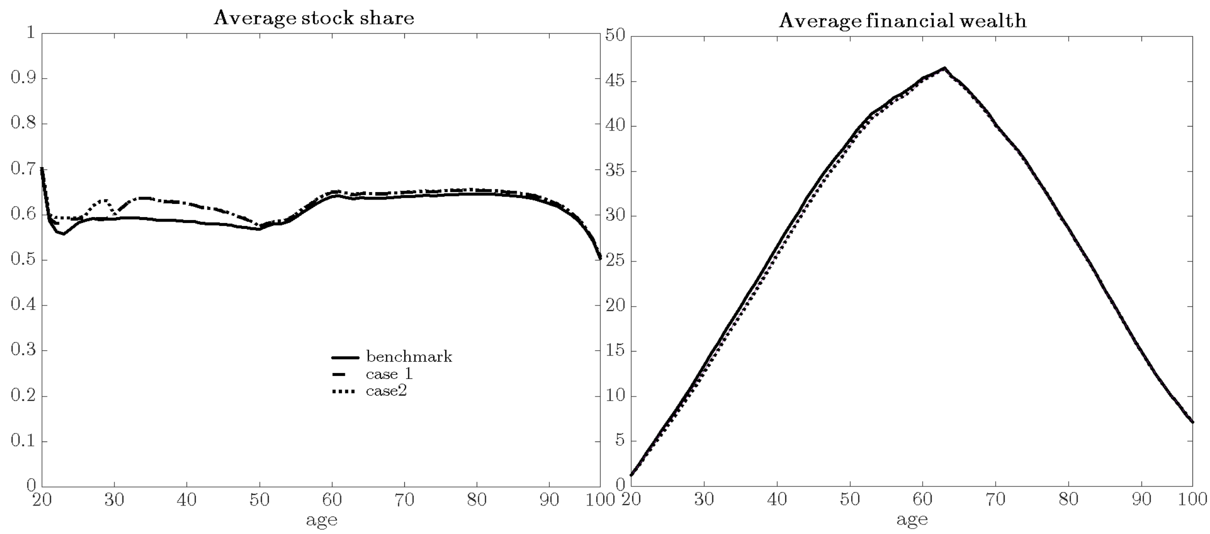
In this section, we calibrate our model with human capital loss, allowing for both a smaller and age-dependent long-term unemployment risk. We change the transition probability from short-term to long-term unemployment, denoted as $\pi_{u_1 u_2}$ in the following transition probabilities matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 1-\pi_{u_1 u_2} & 0 & \pi_{u_1 u_2} \\ 0.33 & 0 & 0.67 \end{pmatrix}$$

with respect to the baseline calibration in (16) where $\pi_{u_1 u_2} = 0.15$ irrespective of the worker's age. We consider two cases. In "*case 1*", the probability of entering long-term unemployment is reduced by one third (from 0.15 to 0.10) only for workers younger than 50 years old. In "*case 2*", we further reduce the probability of entering long-term unemployment for very young workers, setting $\pi_{u_1 u_2} = 0.075$ for workers less than 30 years old. In all scenarios, transition probabilities are rather conservative implying steady-state long-term unemployment rates lower than the actual one. For reference, in the baseline case, the steady-state long-term unemployment rate is 1.7%, while it is 1.1% and 0.8%, in case 1 and 2, respectively.

Figure 9 reports the life-cycle profiles for the optimal conditional stock holding and financial wealth accumulation when long-term unemployment risk is age-dependent. Compared with the baseline case, the age profile of stock investment is only slightly modified. A lower long-term unemployment risk at young ages implies a moderately higher stock share during prime age but it does not significantly alter investors' behavior later over the working life and during retirement. In addition, it has virtually no effect on wealth accumulation.

Figure 9: Life-cycle profiles with unemployment and human capital loss: age-dependent long-term unemployment risk



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The probability of entering long-term unemployment for an unemployed worker is set to 0.15 in the benchmark case, to 0.10 only for workers younger than 50 in Case 1, to 0.075 for workers younger than 30 in Case 2. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

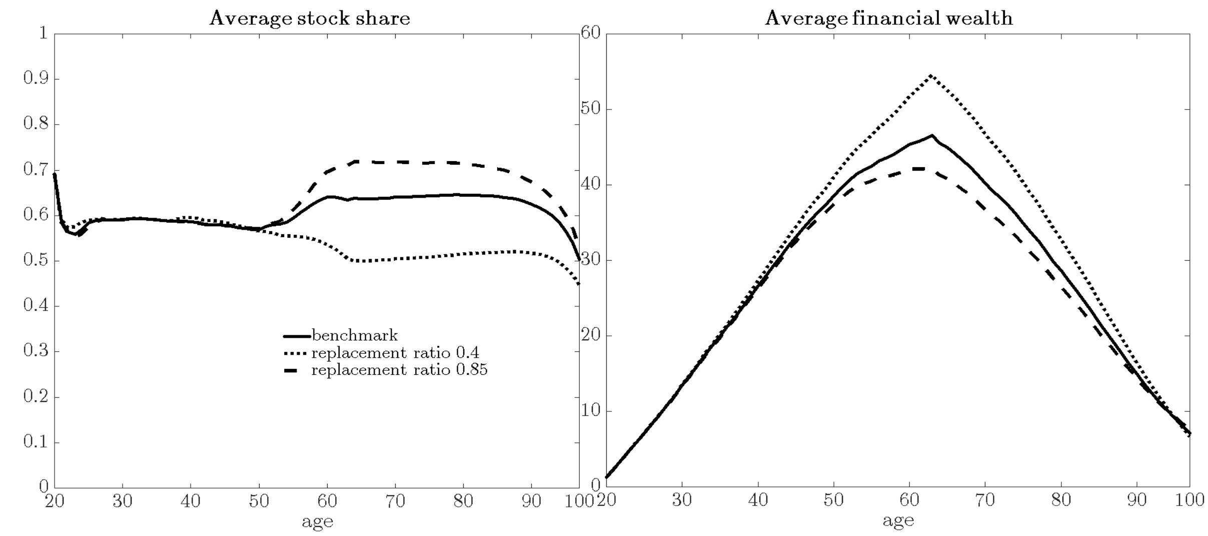
5.2 Unemployment risk and retirement income

In our model, pension benefits are a fixed proportion of the last labor income earned prior to retirement age. Such income is especially sensitive to human capital loss due to the occurrence of long-term unemployment in years just before retirement. Thus, we analyse whether our results are robust to changes in modelling the link between long-term unemployment at old ages and subsequent pension provisions. In addition, we consider different values of social security and long-term unemployment benefit replacement rates.

Timing of long-term unemployment. To begin with, we assume no human capital loss in the event that unemployment occurs in the years immediately before retirement. The flattening of the optimal stock share profile carries over to this setting, suggesting that it is not an artifact of how we model pension income. In a second check, we take the solution of our original model (calibrated in the case of unemployment risk with human capital loss) and focus on simulated life-cycle profiles for two selected groups of agents. The first group includes workers who have experienced just one long-term unemployment spell of 5 years over the entire working life at the beginning of their job career (i.e., before the age of 35), whereas the other group contains workers who have experienced just one long-term unemployment spell of 5 years over their entire working life but at the end of their career (i.e., after the age of 60). We find that in both cases, average life-cycle stock share profiles exhibit the flattening property. This experiment confirms that the flattening is due to the riskier nature of human capital, together with the resolution of uncertainty during working age, and it is not affected by specific assumptions on the determinants of pension income.

Social security and unemployment benefit replacement rates. In addition, we consider two extreme values for the pension benefits replacement rates, 40% and 85%, to reflect the wide range of Social Security Systems' generosity around the world (OECD, 2015). Figure 10 shows that if the pension replacement rate falls to 40% (rises to 85%), anticipating relatively low (high) pension incomes, the consumer will need to accumulate more (less) financial wealth to smooth consumption over retirement. This only slightly affects the optimal share of stocks over working life while it lowers (increases) it during retirement, due to the lower (larger) amount of human capital embedded in pension benefits.

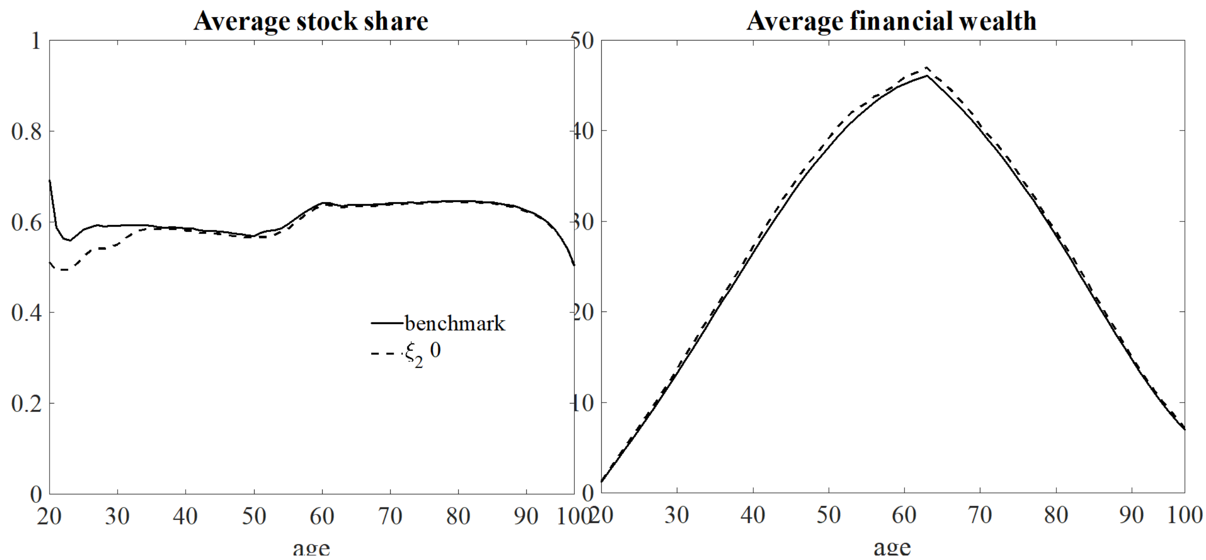
Figure 10: Life-cycle profiles with unemployment and human capital loss: pension benefits replacement rate



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The net replacement rate of total pension benefits ranges from 0.4 to 0.85. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

Finally, we assess the robustness of our benchmark results to a different value of the long-term unemployment benefit provision, setting $\xi_2 = 0$; in this case, only short-term unemployed workers receive benefits. Figure 11 shows that the portfolio risky share maintains a relatively flat age profile, with a moderately increasing pattern over the earlier part of the investor's working life.

Figure 11: Life-cycle profiles with unemployment and human capital loss: unemployment benefits replacement rate



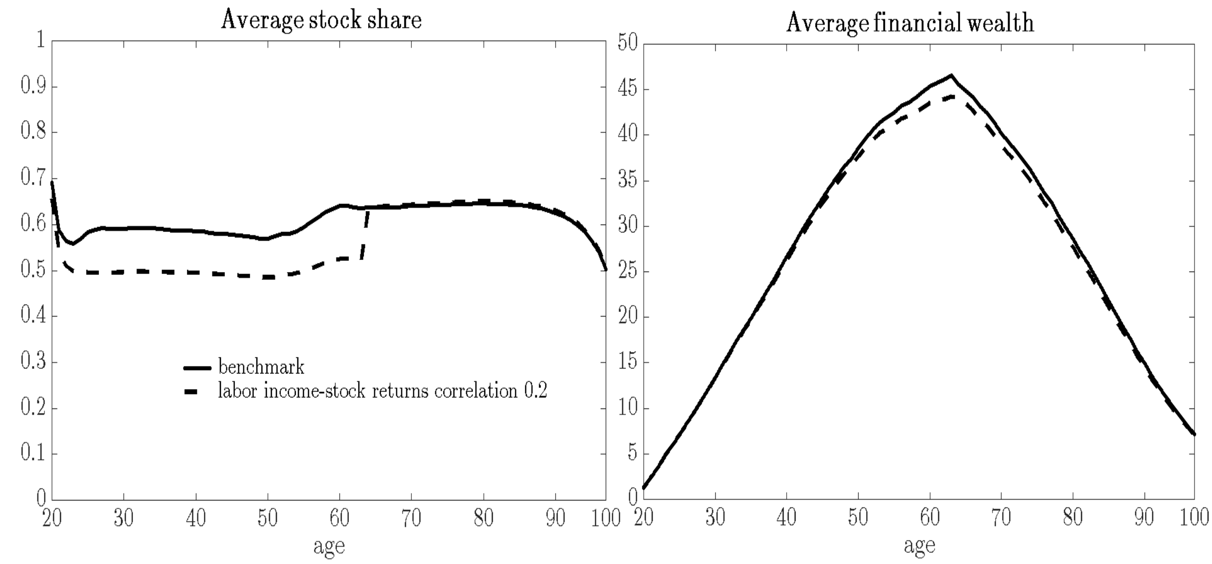
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. The replacement rate of long-term unemployment benefits (ξ_2) takes values 0.1 (benchmark) and 0. Human capital loss: $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

5.3 Labor income-stock return correlation and preference specification

In this section, we first allow for a positive correlation between stock return innovations and the innovations in permanent labor income ($\rho_{sY} > 0$), on top of human capital loss. Bagliano, Fugazza and Nicodano (2014) show that a realistically small correlation has large effects on life-cycle choices when it interacts with a higher variance of the permanent component of labor income shocks. One may therefore expect a similar effect in the presence of the risk of human capital loss. Empirical estimates of the stock return-labor income correlation differ widely, even when we restrict the scope to the U.S. economy. Cocco, Gomes and Maenhout (2005) report estimated values not significantly different from zero across various education groups, in line with Heaton and Lucas (2000), whose estimates range from -0.07 to 0.14 . However, Campbell, Cocco, Gomes and Maenhout (2001) find higher values, ranging from 0.33 for households with no high school education to 0.52 for college graduates. In the simulations below, we adopt an intermediate positive value of $\rho_{sY} = 0.2$.

Figure 12 shows optimal portfolio shares of stocks and the pattern of financial wealth accumulation with no correlation and with a positive correlation between labor income shocks and stock returns. While the shape of life-cycle profiles is relatively unaffected, the average stock share is lower at all ages. In case of positive correlation, labor income is closer to an implicit holding of stocks, reducing the incentive to invest in stocks at all ages. More specifically, in comparison with the case of no correlation, such investors are relatively more exposed to stock market risk and will prefer to offset such risk by holding a lower fraction of their financial portfolio in stocks. The stock share remains substantially flat over the whole working life, displaying limited variability around a level of about 50%. At the retirement age of 65, human capital becomes riskless since pension income is certain and therefore uncorrelated with stock return innovations. Thus investors rebalance their portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the case of no correlation. Further, the relative increase in human capital risk due to a positive correlation does not substantially alter the pattern of financial wealth accumulation.

Figure 12: Life-cycle profiles with unemployment and human capital loss: positive correlation between labor income and stock returns

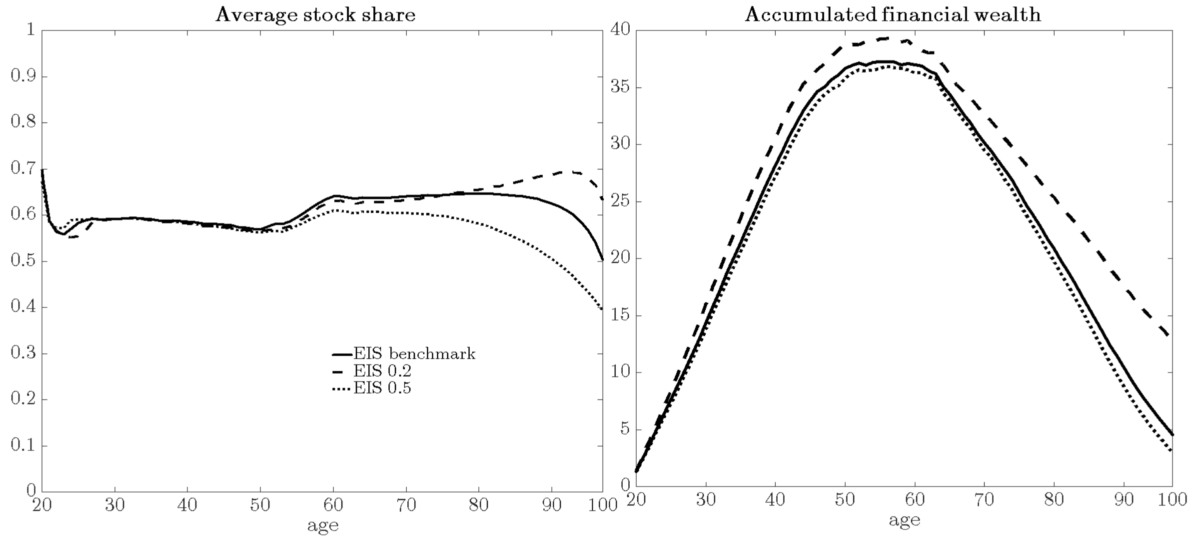


This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Positive correlation between labor income shocks and innovation to stock returns: $\rho_{sY} = 0.2$. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

The second experiment implements a change in preferences that allows for intertemporal correlation aversion (Bommier, 2007). With a power utility function, the worker is indifferent to positive or negative intertemporal correlation of consumption (shocks). With Epstein-Zin preferences, the worker is averse to positive correlation when the coefficient of relative risk aversion is greater than the inverse of the elasticity of intertemporal substitution (EIS). Adopting a recursive (Epstein-Zin) formulation for preferences and keeping the risk aversion parameter constant ($\gamma = 5$), we simulate the model with positive (EIS=0.5) and negative (EIS=0.1) correlation aversion, comparing the results with our baseline case of indifference (i.e., power utility, EIS=0.2). Figure 13 shows that aversion to positive correlation has a negligible effect during working years, while it causes a slower wealth decumulation and less risk taking during the retirement period, especially as death approaches. This finding is consistent with the known property that higher mortality risk magnifies the effects of intertemporal correlation aversion (Bommier, 2013).

Overall, the preceding experiments confirm the robustness of the flattening of the life-cycle profile to changes in both hedging opportunities in the stock market and to the intertemporal elasticity of substitution, pointing to the dominance of the human capital loss effect.

Figure 13: Life-cycle profiles with unemployment and human capital loss: recursive preferences



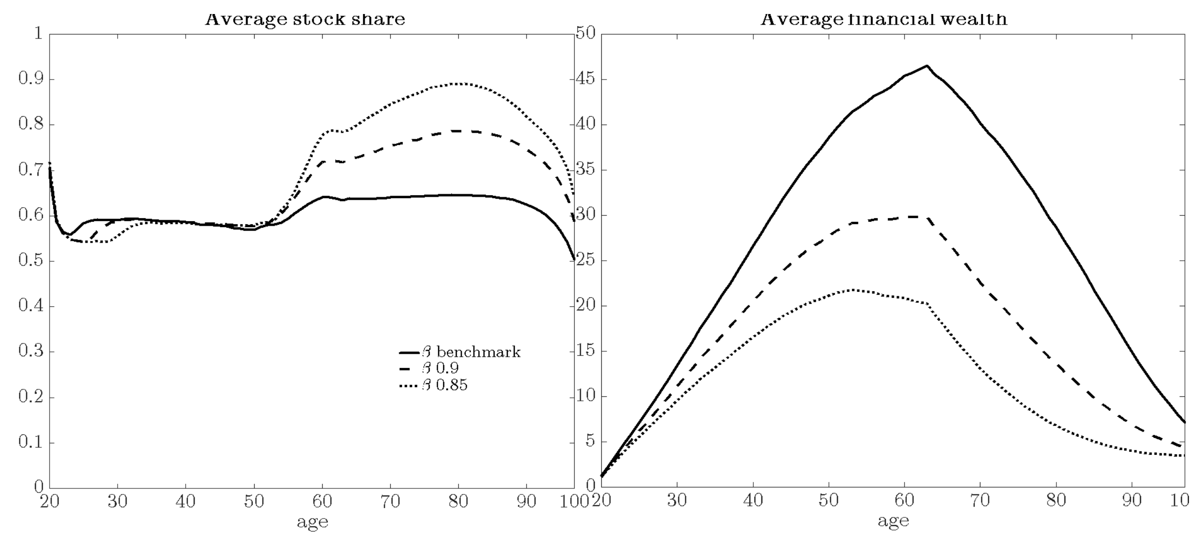
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Preferences over consumption are recursive, represented by an Epstein Zin utility function. Elasticity of intertemporal substitution varies from 0.1 to 0.5. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

5.4 Other preference parameters

Finally, we assess the robustness of our results to alternative formulations of key preference parameters, namely the subjective discount factor and the degree of risk aversion and report results in Figures 14 and 15.

Subjective discount factor. Individuals with a low subjective discount factor value current consumption relatively more than future consumption in comparison with individuals with a high discount factor. As shown in Figure 14, this leads, *ceteris paribus*, to a lower accumulation of financial wealth, and to a negligible effect on the optimal stock investing during working life. During retirement, the absence of human capital risk combined with the slower wealth decumulation induces an increase in the financial risk exposure. In addition, a discount factor equal to 0.85 implies, in our model, a wealth-to-income ratio in line with the value of 4.2 observed in recent U.S. data.[6] As in Fagereng, Gottlieb and Guiso (2017), this value is lower than the one calibrated in the standard life-cycle consumption and portfolio models but coherent with models of buffer stock savings (Deaton, 1991).

Figure 14: Life-cycle profiles with unemployment and human capital loss: discount rate



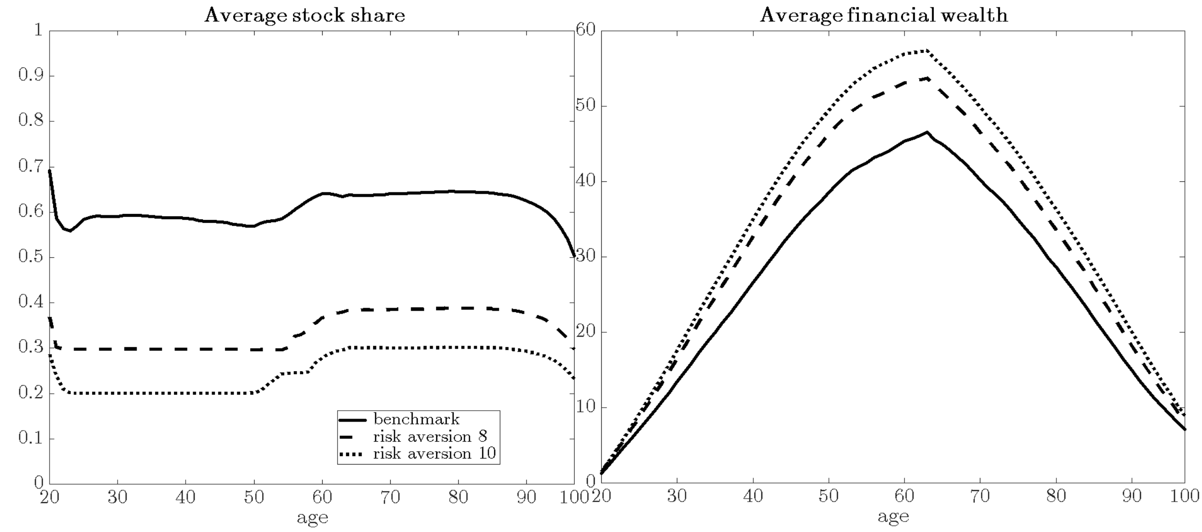
This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Different values of β are considered: 0.96 (benchmark case), 0.9 and 0.85. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

Risk aversion. The critical parameter in the CRRA utility function is the risk aversion coefficient γ, that we set equal to the relatively low value of 5 in our benchmark calibration. Here we assess the sensitivity of our results to different values of γ. Not surprisingly,

[6]Survey of Consumer Finances, 2013.

the investment in risky assets over the life cycle depends importantly on relative risk aversion. In particular, the risk of experiencing the permanent consequences of long-term unemployment induces more risk averse investors to invest a substantially lower fraction of their financial wealth in equity without affecting the flat age profile (Figure 15).

Figure 15: Life-cycle profiles with unemployment and human capital loss: relative risk aversion



This figure displays the average simulated stock investment and financial wealth accumulation life-cycle profiles for individuals of age 20 to 100. Preferences over consumption are CRRA, with relative risk aversion of 5 (benchmark), 8 and 10. Human capital loss: $\Psi_1 = 0$ and $\Psi_2 = 0.25$. Financial wealth is expressed in ten thousands of U.S. dollars.

5.5 Additional challenges

Positive work on life-cycle consumption and saving shows how to calibrate all the key parameters to match the empirical evidence. On the one hand, considering a sufficiently high subjective discount factor prevents households from accumulating an excessive amount of wealth. It also reduces the stock market participation cost needed to match the observed non-participation patterns at a realistic level (Fagereng, Gottlieb and Guiso, 2017). On the other hand, the introduction of subsistence consumption levels avoids counterfactually high saving rates, especially early in life when individuals are more likely to be liquidity constrained (Hubbard, Skinner and Zeldes, 1995; DeNardi, French and Jones, 2010).

Even though we do not attempt to explicitly compare our results to data on portfolio choice, our set-up generates a more empirically plausible life-cycle profile of portfolio choice with respect to models that abstract from possible permanent losses following long-term unemployment. In our calibrations, we focus on income profiles of workers with a high school education. Our results also apply to college-educated workers given that the long-term unemployment risk uniformly affects both high school and college-educated individuals (Mayer, 2014).

Finally, our model obtains the optimal reduction in stock holdings over working life in partial equilibrium. The combination of individuals' life cycle saving behavior and population age structure in an overlapping generation framework may reinforce our results through asset pricing (Poterba, 2001). A relatively large middle-age cohort depresses expected stock returns, strengthening the reduction in optimal risk taking by the younger generation in response to long-term unemployment risk.

6 Conclusions

As the recent Great Recession episode highlighted, long-term unemployment spells may persistently damage workers' human capital. Against this backdrop, this paper investigates the effects of human capital loss on life-cycle savings and portfolio choice. This methodological innovation delivers new insights. Even a small probability of experiencing human capital loss due to long-term unemployment can generate optimal conditional stock shares more in line with those observed in the data. Because of the possibility of human capital loss, young workers face higher uncertainty concerning future income and social security pension levels than older ones. At the same time, young workers with continuous careers have larger human capital than older workers. When a highly unlikely unemployment spell may potentially lead to considerable human capital loss, the first effect offsets the second and the optimal investment in stocks is relatively flat over the life cycle. This result departs from the implications of previous models and highlights the importance of human capital loss in shaping life-cycle portfolios.

Our calibrations also suggest an alternative, more balanced, design for target-date investment funds that would fit different kinds of workers, given the limited heterogeneity in life-cycle optimal investment policies induced by the threat of human capital losses. More generally, our analysis implies that the pattern of risk-taking at different ages in target-date funds should be related to the share of uninsured long-term unemployment risk, and that important differences should be observed in the life-cycle profile of household portfolios, both across cohorts and across countries, in response to the extent of long-term unemployment insurance. For example, our results are consistent with the decreasing age profile of the conditional shock share in a country such as Norway, where the net replacement rate for the long-term unemployed has traditionally been fairly high (Fagereng, Gottlieb and

Guiso, 2017).

As a final consideration, we acknowledge that there exist sources of possible human capital loss (such as illness, accidents, personal bankruptcy) other than long-term unemployment, as well as other partial insurance vehicles. The optimal flat asset allocation will extend to such scenarios to the degree that those additional shocks remain partially uninsured by additional hedges and that they have worse consequences the earlier in life they hit the worker.

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