Mathematics for finance

Aim

The course motivates and illustrates the role and limitations of mathematics in quantitative finance. We recall basic concepts of calculus and linear algebra, within the context of financial applications. We also illustrate and generalize essential ideas in normed and inner product spaces, which are essential in both financial theory and numerical methods. Finally, we deal with optimization models and methods. Everything is illustrated from three related points of view: theory, applications, and numerical implementation in MATLAB.

Content

- *Motivations*. Basic problems in quantitative finance: portfolio optimization, risk management, asset pricing.
- *Linear algebra*. Linear spaces. Matrices, eigenvalues, quadratic forms. Semidefinite quadratic forms. Applications: mean-variance portfolio optimization; pricing by no arbitrage; complete and incomplete markets.
- *Norms, distances, and inner products*. Defining norms and inner products for vector and matrices. Projection on subspaces. Cones and norm cones; dual norms and dual cones. Applications: linear regression, regularized linear regression, representation of robust and stochastic constraints.
- *Numerical linear algebra*. Direct and iterative methods to solve systems of linear equations. Computational complexity, convergence, numerical stability, conditioning.
- *Calculus*. Continuity, differentiability, and convexity. Approximating functions by Taylor expansions. Applications: finite differences, price sensitivities, risk aversion.
- *Function approximation*. Issues with polynomial interpolation. Splines and their variants.
- *Solving nonlinear equations*. Bisection, Newton's method, optimization-based approaches.
- *Numerical integration*. Quadrature methods, Gaussian quadrature, Monte Carlo and quasi-Monte Carlo methods.
- *Stochastic calculus.* A preview of stochastic differential equations and Ito's lemma. Application to BSM model.
- *Optimization*. Unconstrained optimization in the differentiable case and generalization to nonsmooth optimization. Constrained optimization, Lagrange multipliers, Karush-Kuhn-Tucker conditions and their economic interpretation. Dynamic programming. Model building in linear, mixed-integer, and nonlinear programming. Applications: financial portfolio optimization and model calibration; statistical learning.

Bibliography

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