Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires*

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Abstract

We revisit the issue of the high cyclicality of wages of new hires. We show that after controlling for composition effects likely involving procyclical upgrading of job match quality, the wages of new hires are no more cyclical than those of existing workers. The key implication is that the sluggish behavior of wages for existing workers is a better guide to the cyclicality of the marginal cost of labor than is the high measured cyclicality of new hires wages unadjusted for composition effects. Key to our identification is distinguishing between new hires from unemployment versus those who are job changers. We argue that to a reasonable approximation, the wages of the former provide a composition-free estimate of the wage flexibility, while the same is not true for the latter. We then develop a quantitative general equilibrium model with sticky wages via staggered contracting, on-the-job search, and heterogeneous match quality, and show that it can account for both the panel data evidence and aggregate evidence on labor market volatility.

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1 Introduction

Aggregate wage data suggests relatively little variation in real wages as compared to output and unemployment. This consideration has motivated incorporating some form of wage rigidity in quantitative macroeconomic models to help account for business cycle fluctuations, an approach that traces back to the early large scale macroeconometric models and remains prevalent in the recent small scale DSGE models.¹ Such considerations have also motivated the inclusion of wage rigidity in search and matching models of the labor market in the tradition of Diamond, Mortensen and Pissarides. Most notably, Shimer (2005) and Hall (2005) show that the incorporation of wage rigidity greatly improves the ability of search and matching models to account for unemployment fluctuations.²

An influential paper by Pissarides (2009), however, argues that the aggregate data may not provide the relevant measure of wage stickiness: What matters for employment adjustment is the present discounted value of wages of new hires, which needs to be disentangled from aggregate measures of wages. In this regard, there is a volume of panel data evidence beginning with Bils (1985) that finds that entry wages of new hires are substantially more cyclical than the wages of existing workers. Further, it is then possible to account for the inertia in existing workers wages by appealing to wage smoothing that stems from an implicit contracting arrangement (e.g., Beaudry and DiNardo, 1991). Pissarides then interprets the findings in this literature as evidence for a high degree of contractual wage flexibility among new hires, which in turn implies a high degree of flexibility in the marginal cost of labor. The net effect is to call into question efforts to incorporate wage rigidity into macroeconomic models.

In this paper, we revisit new hire wage cyclicality and the associated implications for aggregate unemployment fluctuations. We argue that the interpretation of new hire wage cyclicality as direct evidence of wage flexibility ignores confounding cyclical variation in wages that is due to workers moving to better job matches during expansions. As we make clear, failing to control for this composition effect on wage changes leads to significant upward bias in the measure of the procyclicality of the marginal cost of labor. We then adopt a novel empirical strategy to separate contractual wage flexibility from cyclical match quality. We find that after controlling for composition effects, the wages of new hires are no more flexible than those of existing workers. A key implication, which we make precise, is that the low variability of existing workers’ wages provides a better guide to the cyclicality

² Gertler and Trigari (2009), Hall and Milgrom (2008), Blanchard and Gali (2010), and Christiano, Eichenbaum and Trabandt (2016) build on this approach and model the wage setting mechanism in greater detail.
of the marginal cost of labor than does the high volatility of new hire wages (unadjusted for composition). We then develop a quantitative macroeconomic model that is able to account for both the aggregate and panel data evidence.

Key to our identification of composition effects is the distinction between new hires who are job changers versus those coming from unemployment. We argue based on both theory and evidence that procyclical upgrading of job match quality is predominant among job changers. The main reason that a worker with a job moves is to improve the match and the opportunity for these workers to upgrade is procyclical. Thus, by failing to control for wage changes reflecting changes in match quality, estimates of the wage cyclicity of job changers overstate true wage flexibility. By contrast, under a standard assumption in the literature about the quality distribution of available jobs, upgrading of match quality is acyclical for workers coming from unemployment. In our view, further, the baseline assumption of no cyclical upgrading for these types of workers is consistent with a reasonable reading of the evidence. Given this assumption, accordingly, the wage cyclicity of new hires from unemployment provides a reasonable composition-free estimate of new hire wage flexibility.

To develop our estimate of new hire wage flexibility, we construct a unique data set from the Survey of Income and Program Participation (SIPP) that allows us to separately estimate the wage cyclicity of new hires from unemployment versus that of those making job-to-job transitions. We first show that by pooling the two types of new hires with our data, we can replicate the typical result of the existing literature: New hire wages appear to be more flexible than the wages of continuing workers. When we estimate separate terms for both types of new hires, however, we find no evidence of excess wage cyclicity for new hires coming from unemployment, but substantial evidence of this phenomenon for workers making job-to-job transitions. We then discuss how our estimates suggest considerable sluggishness in the marginal cost of labor, consistent with the macroeconomic models that feature wage rigidity described above.

Even if one does not accept our identifying assumptions, our panel data estimates provide a new set of conditional moments that any macroeconomics model of unemployment fluctuations must confront. To illustrate, we develop a search and matching model with the following three modifications (i) staggered wage contracting, (ii) variable match quality, and (iii) on-the-job search with endogenous search intensity. We show that the model is consistent with both the aggregate data and the panel data evidence on the relative cyclicalities of the wages of new hires from unemployment versus job changers. In particular, while the wages of new hires are sticky within the model, cyclical improvements in match quality for job changers generate new hire wage cyclicity, offering the appearance of wage flexibility among new hires. All the three modifications of the model are critical for reconciling the aggregate and panel evidence.
Our results are consistent with a rich literature on earnings growth and job-to-job transitions. Beginning with Topel and Ward (1992), an extensive empirical literature has documented that a large fraction of the wage increases experienced by a given worker occur through job-to-job transitions. Such job movements can be understood as employed workers actively searching for higher paying jobs, along the lines of Burdett and Mortensen (1998). A related theoretical literature has shown that such match improvements are more easily realized during expansions than during recessions (Barlevy, 2002; Menzio and Shi, 2011). In contrast, such job-ladder models offer no systematic prediction for wage changes of workers searching from unemployment, as such workers are predicted to adopt a reservation wage strategy that is not contingent on their most recent wage. Beyond this theoretical prediction, as we will show, there is evidence to suggest that our baseline assumption that composition effects are relevant for job changers but not for new hires from unemployment is a reasonable approximation of reality.

This paper is also related to Gertler and Trigari (GT, 2009), which controls for composition effects on new hire wage cyclicality allowing for a job-person fixed effect on wages. The advantage of the current approach is that the wage cyclicality of new hires from unemployment provides a directly observable composition free measure of new hire wage flexibility.\(^3\) By distinguishing between new hires from unemployment versus job changers, further, we obtain a new set of facts that macroeconomic models of unemployment and wage dynamics must confront. We then develop such a model.

Other work with a message similar to this paper includes Hagedorn and Manovskii (2013). These authors make clever use of an indirect measure of match quality – specifically, the sum of log market tightness over different durations of a worker’s employment – to show that findings that have been previously interpreted as evidence of implicit contracts can be accounted for by composition effects. In addition to using a more direct way to control for composition effects, we differ by analyzing how estimates of excess new hire wage cyclicality can be reconciled with models of wage stickiness used to account for aggregate labor market dynamics. Also relevant are papers that use Portuguese data, including Martins, Solon and Thomas (2012) and Carneiro, Guimaraes and Portugal (2013), and German data, including Stüber (2017). Using different methods, the estimates in these papers also suggest that new hire wage cyclicality is roughly the same as that for continuing workers.\(^4\)

In terms of empirical methodology, our paper is closest to Haefke, Sonntag and van Rens (2013) who examine directly the wage cyclicality of new hires from unemployment.

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\(^3\) Gertler and Trigari (2009) also requires an additional identifying assumption: There must be recontracting of wages at some point over the worker’s observed history with the firm. Otherwise, it is not possible to distinguish the firm/worker fixed effect from an implicit contract where the wage is permanently indexed to aggregate conditions in the first period of a match.

\(^4\) However, overall real wage variation in these data exhibits greater procyclicality than in the U.S., suggesting some limits to the relevance of this evidence to U.S. labor market volatility.
They use cross-sectional data from the CPS and recover point estimates suggestive of excess wage cyclicality of new hires from unemployment, although not statistically significant. We instead use a rich, high-frequency panel data set from the SIPP. The panel aspect of our data permits sharp controls for unobserved heterogeneity and compositional effects. To this end, we find statistically significant evidence that new hires wages from unemployment are no more cyclical than for existing workers. As a corollary, we show that the excess wage cyclicality of new hires recovered by the literature is entirely driven by new hires from employment, raising the possibility that this excess cyclicality is an artifact of cyclical movements in match quality via the job ladder, as opposed to true wage flexibility. Finally, as noted earlier, we develop a macroeconomic model of labor market dynamics and show that simulated data from the model is consistent with both the aggregate and panel data evidence.

Section 2 provides the new panel data evidence. We first describe the data and the econometric methodology we use to identify a composition-free estimate of the marginal cost of labor. We then present evidence that new hire wages are no more flexible than those of existing workers, as well as a variety of robustness exercises that support this finding. Section 3 describes the model and Section 4 shows how the model generates composition bias. Section 5 presents the numerical results and also demonstrates how the model can reconcile the aggregate and panel evidence. Section 6 then discusses the implications for the cyclicality of the marginal cost of labor, including the relevance of our results for Kudlyak’s (2014) notion of the “user cost of labor”. Concluding remarks are in Section 7.

2 Data and Empirics

In this section we present evidence on the relative cyclicality of new hires’ wages using a rich data set. We first describe the data. We next revisit the results on excess cyclicality of new hire wages. We then present new evidence based on making the distinction between new hires from employment (i.e., job changers) versus new hires from unemployment. In doing so, we use a data generating model to make concrete the assumptions we use to identify a composition-free estimate of new hire wage flexibility. We conclude the section with some evidence on the robustness of our identifying assumptions.

2.1 Data

We use data from the Survey of Income and Program Participation (SIPP) from 1990 to 2012. The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. Over our sample period the
Census Bureau introduced eight panels. The starting years were 1990-1993, 1996, 2001, 2004, and 2008. The average length of time an individual stays in a sample ranges from 32 months in the early samples to 48 months in the 2008 panel.

Most key features of the SIPP are consistent across panels. Each household within a panel is interviewed every four months, a period referred to as a wave. During the first wave that a household is in the sample, the household provides retrospective information about employment history and other background information for working age individuals in the household. At the end of every wave, the household provides detailed information about activities over the time elapsed since the previous interviews, including job transitions that have occurred within the wave. Although individuals report earnings for each month of the wave, we only use reported earnings from the last month of the wave to accommodate the SIPP “seam effect.”

The SIPP has several features that make it uniquely suited for our analysis. Relative to other commonly used panel data sets, the SIPP follows many more households, follows multiple representative cohorts, and is assembled from information collected at a high frequency (e.g. surveys are every four months as opposed to annually). This high frequency structure of the data is crucial for constructing precise measurements of employment status and wages. In particular, we use job-specific earnings to generate monthly records of job-holding for each individual, allowing us to discern direct job-to-job transitions from job transitions with an intervening spell of non-employment. As the SIPP contains multiple cohorts, at each point in time the sample is always representative of the U.S. population, in contrast to other widely used panel data sets such as the NLSY.

Crucial to our approach is that the SIPP maintains consistent job IDs. Fujita and Moscarini (2017) document that, starting with the 1996 SIPP wave, a single job may be assigned multiple IDs for an identifiable subset of survey respondents. In the appendix, we develop a procedure that exploits a feature of the SIPP employment interview module that allows us to identify jobs that may have been assigned multiple IDs. We find evidence for recall employment, corroborating Fujita and Moscarini (2017)’s finding that recalls compose a significant fraction of transitions to employment from non-employment.

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5 Specifically, we find that the vast majority of earnings changes for workers continuously employed at the same job across multiple waves occur between waves, as opposed to during a wave. The “seam effect” is discussed in greater detail in the SIPP User’s Guide (U.S. Census Bureau, 2001, 1-6).

6 Starting with the 1996 panel, respondents report the start and end dates associated with a job. While our measure is highly correlated with the self-reported measure, the self-reported measure is sometimes inconsistent with self-reported activity from other waves—e.g., a worker will report a starting date that corresponds to a prior wave for which the respondent had previously reported being unemployed or employed at a different job. We use our earnings-based measure for all panels to avoid such issues of measurement error and maintain consistency in our analysis of the pre- and post-1996 data.

7 We do not include these observations as new hires in our analysis; if these workers receive wages that are only as cyclical as “stayers”, they would bias the estimation of wage cyclicality of new hires from unemployment downwards.
The appendix provides further discussion of the data and the construction of the variables we use in the estimation.

2.2 Baseline Empirical Framework

Before turning to our econometric framework, we first replicate the evidence in the literature that new hire wages are more cyclical than those of existing workers. To do so we employ a simple statistical framework to study the response of individual level wages to changes in aggregate conditions that has been popular in the literature, beginning with Bils (1985). We regress the log wage of individual \( i \) at time \( t \), \( w_{it} \), on individual level characteristics \( x_{it} \), including education, job tenure, and a time trend; the unemployment rate \( u_t \); an indicator variable \( I\{N_{it} = 1\} \) equal to one if the worker is a new hire and zero if not; and an interaction term \( I\{N_{it} = 1\} \cdot u_t \). To control for unobserved characteristics, we estimate a regression equation in first differences and fixed effects. Our measurement equation estimated in first differences reads:

\[
\Delta \log w_{it} = \Delta x_{it}^t \pi_x + \pi_u \cdot \Delta u_t + I\{N_{it} = 1\} \cdot [\pi_n + \pi_{nu} \cdot \Delta u_t] + e_{it} \tag{1}
\]

where \( \Delta \) denotes first-differences and \( e_{it} \) is random error term.\(^8\) The analog equation estimated in fixed effects is obtained by replacing \( \Delta \) with \( \Delta^m \) in (1), with \( \Delta^m \) denoting mean-differences.

The inclusion of the unemployment rate in the regression is meant to capture the influence of cyclical factors on wages, while the interaction of the new hire dummy with the unemployment rate is meant to measure the extra cyclicality of new hires wages. In particular, the coefficient \( \pi_u \) can be interpreted as the semi-elasticity of wages with respect to unemployment, while \( \pi_u + \pi_{nu} \) gives the corresponding semi-elasticity for new hires.\(^9\)

The regressions are based on triannual data, i.e. data at a four month frequency.\(^11\)

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\(^8\) Included among the many studies regressing individual level wages on some measure of unemployment as a cyclical indicator are Beaudry and DiNardo (1991); Shin (1994); Solon, Barsky, and Parker (1994); Barlevy (2001); Carneiro, Guimarães, and Portugal (2012); Deveraux (2002); Martins, Solon, and Thomas (2012); and Hagedorn and Manovskii (2013).

\(^9\) Note, our notation is “compact”, in the sense that it does not directly acknowledge that differenced wage observations might span several jobs, or that the time between wage observations might vary for new hires with a non-employment spell (i.e., we could have \( \Delta \log w_{it} = \log w_{it} - \log w_{it-1} \), where \( \tau < t - 1 \)).

\(^10\) The empirical definition of the cycle is implicit in the regression specification. In the FE estimation, the cycle is defined by deviations of the unemployment rate from its three/four-year average over the panel. In the FD specification, it corresponds to the four-month change in the unemployment rate. Given the high volatility and fast transition dynamics of unemployment, the FD specification preserves the underlying relation without removing meaningful cyclical variation.

\(^11\) While we have monthly information on earnings and job mobility, the data are collected once every four months and there is reasonable suspicion of correlated measurement error of reported earnings within waves. We follow Gottschalk (2005) in limiting our analysis to reports of earnings from the final month of each four month wave.
For comparability to Bils (1985), we only use observations for men between the ages of 20 and 60. Accordingly, unemployment is the prime age male unemployment rate. We use job-specific earnings to construct our measure of wages. In cases in which an hourly wage is directly available, we use that as our measure. In cases in which an hourly wage is not directly available, we use job-specific earnings divided by the product of job-specific hours per week and job-specific weeks per month. Wages are deflated by the monthly PCE. Finally, we define “new hires” as individuals who are in the first four months of their tenure on a job.\footnote{Note that given this definition we will only have one wage observation for a new hire since we only use the final month of a four-month wave to obtain wage data.} The appendix provides additional information on variable construction, including the individual level characteristics we use. Finally, we compute robust standard errors, clustered at the individual level.

Table 1 presents the results. They are consistent with the key findings of the literature: \( \pi_{nu} \) is statistically significant and negative (along with \( \pi_u \)), suggesting greater cyclical sensitivity of new hires’ wages. The first column presents the estimates of equation (1) using first differences and the second presents estimates using fixed effects. The results are robust across specifications. Similar to Bils (1985), we find that new hires’ wages are significantly more cyclical than those for existing workers. When estimating the equation in first differences, the semi-elasticity of new hire wages is $-1.598$, compared to $-0.460$ for continuing workers. With fixed effects, the new hire semi-elasticity is estimated to be $-1.793$, compared to $-0.147$ for continuing workers.

While we recover precise coefficient estimates of the relative wage cyclicality of new hires versus continuing workers that are consistent with earlier literature, our estimates of absolute wage cyclicality are smaller. Using annual NLSY data from 1966-1980, Bils (1985) finds a continuing worker semi-elasticity of 0.6, versus 3.0 for changers. Barlevy (2001) uses annual data from the PSID and NLSY through 1993 and recovers semi-elasticities of 2.6 and 3.0 for job changers. The differences between our estimates of wage cyclicality and those from of this earlier literature are not due to the higher frequency of our data: When we re-estimate our model using data at the annual frequency we find very similar results to our baseline triannual frequency. Another possible source of the discrepancy is the difference in sample period. Our SIPP data only goes back to 1990, which means our sample is much later than that used in the earlier work. In any case, our quantitative model will generate data consistent with the degree of wage cyclicality suggested by the evidence in Table 1.

### 2.3 Reconsidering the New Hire Effect

As we noted earlier, a popular interpretation of the results in Table 1 is that they are indicative of contractual wage flexibility for new hires, e.g. Pissarides (2009). According
to this view, the present value of wages is highly responsive to the aggregate state, but the path of wages is smoothed over the lifetime of the wage contract. Hence, the negative and significant estimate for $\pi_{nu}$ reflects that new hires receive a contract with persistently higher wages when hired during a boom. In turn, the smaller estimate for $\pi_u$ reflects that wages are insulated from aggregate conditions for the rest of the match.

We offer an alternative interpretation: Rather than indicating excess wage flexibility, the estimated new hire wage cyclicality might instead be due to “cyclical composition effects”, whereby workers in existing jobs move to better jobs at a higher rate during expansions, and a slower rate during contractions. Under this scenario, the estimated high cyclicality of new hire wages from the Bils’ equation will not reflect true excess wage flexibility.

Figure 1 illustrates how procyclical match upgrading may bias estimates of new hire wage cyclicality. The figure portrays cyclical wage variation across two jobs: a good match and a bad match. The wage in each match (solid line) is modestly cyclical around a steady state wage (dotted line). Consider, however, an expansion that facilitates the movement of workers in bad matches to good matches. There are two cyclical components of such a worker’s wage increase: (i) a modest cyclical increase in wages common to both job changers and continuing workers and (ii) the improvement in match quality. Note, from the perspective of a firm, the wages of job changers and continuing workers are equally flexible. That is, the cyclical wage increase of job-changers does not translate to a cyclical increase in the hiring costs of a firm; however, an econometrician who does not take into account the cyclical change in match quality may conclude otherwise.

To test for excess wage flexibility of new hires relative to existing workers, we make the distinction between new hires coming from other jobs versus those coming from unemployment. We first argue that cyclical selection bias works mainly through job-changers who are more likely to upgrade their match quality in booms than in recessions. This conceptual framework is consistent with (i) an empirical literature finding that job changers realize substantial wage gains from switching jobs (Topel and Ward, 1992), and (ii) a theoretical literature arguing that workers in employment are more likely to select into better matches during expansions than recessions (Barlevy, 2002).

Second, we assume that for workers coming from unemployment, there is no selection bias. Our argument is reasonable to the extent that the quality distribution of jobs available to a given individual is largely invariant to the aggregate state.\footnote{While this is a strong assumption, it is similar to Hagedorn and Manovskii (2013), whose baseline model features a wage offer distribution that is invariant to the business cycle; and considerably weaker than Kudlyak (2014), who assumes no cyclical composition in new hire wages.} In Section 2.4, we discuss the robustness of this assumption. Given this identifying restriction, the wage cyclicality of new hires from non-employment serves as a valid measure of the cyclicality of the composition-adjusted hiring wage.
To formalize our identification strategy, we consider a simple data-generating model (DGM) that can approximate the full equilibrium model that we develop later in Section 3 (as we show in Section 4). Under the approximate DGM,

$$\log w_{it} = \psi_0 + x'_{it} \psi_x + \psi_u \cdot u_t + \alpha_i + \alpha_{it} + \varepsilon_{it},$$

where $\psi_u$ gives the common wage semi-elasticity for new and continuing workers, $\alpha_i$ is a time-invariant person fixed effect, $\alpha_{it}$ is unobserved match quality for individual $i$ at a given job at time $t$, and $\varepsilon_{it}$ is an iid error term.

We next consider a process for average match quality $\bar{\alpha}_{it}$. In particular, we suppose that match quality evolves as follows:

$$\Delta \bar{\alpha}_{it} = \mathbb{I}\{EE_{it} = 1\} \cdot \psi_{EE}^{EE} + \psi_{nu}^{EE} \cdot \Delta u_t + \mathbb{I}\{ENE_{it} = 1\} \cdot \psi_{n}^{ENE},$$

where $\mathbb{I}\{EE_{it} = 1\}$ is an indicator equal to one if the worker is a new hire who makes a direct employment to employment transition and zero otherwise; and where $\mathbb{I}\{ENE_{it} = 1\}$ equals one if the worker is a new hire with an intervening spell of unemployment and zero otherwise. Equation (3) describes a process for average match quality in which workers in continuing matches experience no change in match quality, workers hired from non-employment incur a level change in match quality independent of the cycle, and job changers incur a change in match quality that also depends on the change in unemployment. As the unemployment rate falls, job-changers are more likely to move to higher-paying matches, and vice versa when the rate rises. Hence, the change in average match quality for job-changers is proportional to the change in the unemployment rate.\footnote{For the full model, Section 4 shows something similar: the change in average match quality from time $t - 1$ to time $t$ is proportional to the change in the distribution of workers across different match qualities. The change in average match quality across periods itself depends in part on the change in unemployment.}

Workers coming from unemployment, by contrast, have acyclical changes in match quality, as they are more likely to take the first job they find regardless the state of the cycle.

To see how the typical regression of the literature may yield misleading evidence of new hire wage flexibility, we first take first differences of the DGM (2), integrate over changes...
in unobserved match quality $\Delta \alpha_{it}$ over new hires, and then combine with (3)\textsuperscript{15,16}:

$$
\Delta \log w_{it} = \Delta x'_{it} \psi_x + \psi_u \cdot \Delta u_t \\
+ \mathbb{I} \{EE_{it} = 1\} \cdot [\psi_{EE} + \psi_{nu} \cdot \Delta u_t] \\
+ \mathbb{I} \{ENE_{it} = 1\} \cdot \psi_{ENE} + \Delta \varepsilon_{it}. \tag{4}
$$

As can be seen from comparing equations (1) and (4), the Bils regression is misspecified under our DGM. In particular, it imposes that the wage semi-elasticities of job changers and new hires from unemployment are equal and given by $\pi_{nu}$. By contrast, our DGM implies that the elasticity for job changers is $\psi_{EE} < 0$ and the elasticity for new hires from unemployment is zero. Accordingly, taking the Bils equation to the data will lead to an estimate $\pi_{nu} < 0$, but this will be due to the composition bias captured by $\psi_{EE}$. Indeed, as we show in the appendix, $\pi_{nu} \propto \psi_{EE}$. Thus, under our DGM, the estimates of excess new hire wage cyclical reflect composition bias rather than true flexibility.

We can test whether the data is consistent with the simple DGM by estimating a version of equation (1) that includes separate interaction terms for job changers versus new hires from unemployment:\textsuperscript{17}

$$
\Delta \log w_{it} = \Delta x'_{it} \pi_x + \pi_u \cdot \Delta u_t \\
+ \mathbb{I} \{EE_{it} = 1\} \cdot [\pi_{EE} + \pi_{nu} \cdot \Delta u_t] \\
+ \mathbb{I} \{ENE_{it} = 1\} \cdot [\pi_{ENE} + \pi_{nu} \cdot \Delta u_t] + e_{it}, \tag{5}
$$

Under the null of our DGM,

$$
\pi_{EE}^{nu} = \psi_{EE} \\
\pi_{ENE}^{nu} = 0
$$

implying that (i) the excess wage cyclical for job changers reflects composition bias and

\textsuperscript{15} Section A.6 in the appendix shows how procyclical changes in composition of workers making job-to-job changes (as in the full equilibrium model) map into procyclical changes in average match quality (as in the approximate DGM) once we integrate over changes in unobserved match quality $\Delta \alpha_{it}$ over new hires. Thus, the approximate DGM is consistent with the primary mechanism of the full model.

\textsuperscript{16} Our results are robust to using as the cyclical indicator a distributed lag of current and past unemployment rates, as would be implied by the kind of staggered wage contracting model we develop later in the paper. See section A.5 of the online appendix to this paper.

\textsuperscript{17} We differ from Haeckel et al. (2013) in two key dimensions. First, we estimate our equations in fixed-effects and first-differences to control for unobserved heterogeneity in workers; Haeckel et al. (2013) use cross-sectional data from the CPS. Second, we follow the majority of the literature in using the unemployment rate as a cyclical indicator, whereas Haeckel et al. use labor productivity. Unemployment is a valid cyclical indicator across a variety of business cycle episodes, whereas the relation between labor productivity and the cycle has proved to be less stable over time.
(ii) there is no excess wage cyclicality for new hires from unemployment. As we show, our estimates are all consistent with the null of $\pi_{nu}^{ENE} = 0$.

The fixed-effects estimator shares the same crucial properties: namely, $\pi_u$ and $\pi_{nu}^{ENE}$ are consistent estimators for $\psi_u$ and $\psi_{nu}^{ENE}$, so that we obtain composition-free estimates of the excess wage flexibility of new hires. We confirm that the fixed-effects estimator yields estimates consistent with the null that $\pi_{nu}^{ENE} = 0$.

Table 2 presents the results just mentioned, in first differences and fixed-effects. For robustness, we consider two different measures of what constitutes a new hire from non-employment. In our most narrow measure, an individual is classified as a job-changer only if the individual is recorded at a new job with no interruption in earnings. In the second measure, we also classify new hires with a single month of no earnings between jobs as job-changers, allowing for the possibility that the worker found the new job from employment but took a short break between job spells.

Across all specifications, we never recover a significant new hire effect for new hires from non-employment: the coefficient estimates for $\pi_{nu}^{ENE}$ are small in magnitude and not statistically different from zero. Thus, for new hires from unemployment, wages are no more cyclical than those for existing workers. Meanwhile, we find substantial evidence of procyclical changes in match quality for job changers. Indeed, the coefficient $\pi_{EE}^{nu}$ on the job-changer interaction term is higher than the coefficient $\pi_{nu}$ on the interaction term for the baseline regressions in Table 1, where both types of new hires are pooled together. In every case, we can reject the null hypothesis that the wage cyclicality for new hires from non-employment equals the wage cyclicality for new hires from employment at the 5% level.

### 2.4 Robustness

We interpret the lack of statistical significance of the interaction term for new hires from unemployment as evidence against new hire wage flexibility. However, an alternative inter-

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18 As we show in the appendix, fixed-effects and first-differences imply different limiting distributions for $\pi_{nu}^{EE}$ under the approximate DGM, so our estimates for $\psi_{nu}^{EE}$ in fixed effects will generally be biased. Once we turn to the full model, however, we are able to match the separate wage semi-elasticities in first differences and fixed effects.

19 A recent working paper by Hahn, Hyatt, and Janicki (2018) uses quarterly data on earnings from the LEHD for eleven states from 1996 to 2015. They find that the wage cyclicality of job-changers and new hires from non-employment are very similar. However, for three reasons, we suspect that their estimates are likely to be imprecise relative to ours: (1) they use an imputed measure of wages, whereas we work with a direct measure; (2) given that we have monthly data, we can draw a sharper distinction between job-changers and new hires from unemployment than they can from their quarterly data; (3) we have a more representative and longer sample; and (4) they are unable to control for occupation, which we show in Section 2.4 can be important for eliminating a spurious new hire effect. Another working paper by Bauer and Lochner (2017) uses German administrative data provided by the Institute for Employment Research (IAB), from 2000 to 2014. They also confirm, among other things, that it is important to control for occupational mobility within a Bils’ type regression that distinguishes between new hires from unemployment and job changers.
pretation of our results is that new hires from unemployment are subject to a countercyclical composition bias that by coincidence exactly offsets wage cyclicity due to new hire wage flexibility. Under such a scenario, we could fail to reject the null hypothesis that $\pi_{nu}^{ENE} = 0$ when new hires wages are indeed flexible.

In this section, we explore such a possibility by considering an expanded set of regressions similar to (5), but where we control for various forms of composition bias that have been emphasized in the existing literature. In doing so, we are able to isolate subsets of $ENE$ transitions that are less likely to be contaminated by a cyclical composition bias. Hence, if our previous estimates were confounded by countercyclical composition for $ENE$ workers, we should be more likely to recover a new hire effect consistent with flexible wages in these more restricted samples of $ENE$ workers. However, the results continue to consistent with the hypothesis that the wages of newly hired workers are no more flexible than of continuing workers.

First, we consider $EE$ and $ENE$ new hires who switch or remain in the same occupation or industry when moving into a new job. Indeed, the literature has emphasized inter-occupation and industry mobility as a natural candidate by which workers may “cyclically upgrade” into higher-paying jobs during an expansion, e.g. Vroman (1977). Much of the literature has emphasized the importance of cyclical upgrading both across industries and occupations, although we note that there has been little in the way of empirical work to explore separate implications for employed and unemployed workers. McLaughlin and Bils (2001) document a set of industry employment and wage patterns consistent with cyclical upgrading and show that a model of selection of workers into industries by comparative advantage can explain these patterns. Chodorow-Reich and Wieland (2018) use a model with industry-switching among employed and unemployed workers to explore the interaction of sectoral reallocation and business cycle conditions. Kambourov and Manovskii (2009) argue that occupational tenure is more important than industry or employer tenure in explaining variation in wages, suggesting that cyclical patterns in occupation switching should also be important for explaining cyclical variation in wages. Indeed, Altonji, Kahn, and Speer (2016) argue that initial occupational placement is important for explaining cross-section variation in the cost of entering the labor market during a recession (Kahn, 2010). Huckfeldt (2016) documents that occupation-downgrading is countercyclical among displaced workers, helping to account for the cyclical cost of job loss (Davis and von Wachter, 2011).\footnote{Taken together, the existing evidence on displaced workers and new entrants suggests that if anything, there should be a procyclical composition bias associated to occupation switching among $ENE$ workers. However, displaced workers are a subset of the non-employed, and the focus on wage growth precludes us to considering new entrants to the labor market. Thus, \textit{a priori}, it is not clear whether the bias for $ENE$ would also be procyclical.}
Table 3 gives the results for EE and ENE new hires where we control for a cyclical composition effect that may be due to occupation or industry switching. The semi-elasticities for EE and ENE occupation/industry non-switchers are given in rows two and three; rows four and five give the differential cyclcity for switchers. As emphasized above, we consider the semi-elasticity for ENE non-switchers to be the relevant measure of new hire wage cyclcity, as it is least likely to be contaminated by a composition bias. Moreover, to the extent that similar jobs are indeed grouped by industry or occupation, we are better able to isolate true wage cyclcity by focusing on workers who remain in the same type of job across an employment transition. As before, we interpret the EE elasticities to represent the role of cyclical self-selection of workers searching on-the-job; our estimates here allow us to evaluate whether procyclical selection into better jobs is concentrated among EE switchers or non-switchers.

From our estimates in fixed effects, we find evidence that ENE occupation/industry switchers have more cyclical wages than ENE stayers, suggesting that the composition bias introduced by occupation/industry switching is procyclical rather than countercyclical. Among EE workers, we find evidence of a stronger procyclical composition effect for non-switchers than switchers, but both are estimated to be procyclical.\footnote{We speculate the presence of important asymmetries between switchers and non-switchers among the employed and unemployed. For example, if within-industry or within-occupation hiring standards are countercyclical (as documented for occupation by Hershbein and Kahn, 2018), an unemployed worker during a recession might be unable to find a job in his previous occupation or industry and be forced to search for a job in a worse one. This is a procyclical composition effect that would be absent for job-changers, perhaps accounting for sign differences in the “switcher” coefficient for EE and ENE workers.}

For both first-differences and fixed-effects, however, our findings on ENE wage cyclcity are largely unchanged: we recover point-estimates for $\gamma_{nE}$ that are positive, not significant, and for the most part, close to zero.

Next, we consider composition effects due to unobserved characteristics that are correlated with unemployment duration. While unemployment duration has been identified as crucial for understanding re-employment wage outcomes (see Schmieder, von Wachter, and Bender, 2015), there are multiple reasons why unemployment duration might matter for wages, and the overall cyclcity of the associated composition bias is ambiguous. For example, Ljundgvist and Sargent (1998) argue that workers who experience longer durations of unemployment are subject to greater human capital loss, and hence lower re-employment wages. Therefore, longer unemployment durations during a recession could introduce a procyclical bias. The literature has also identified stigma effects associated to the duration of the unemployment spell.\footnote{For example, Kroft, Lange, and Notowidigdo (2013) find that workers of longer unemployment durations are less likely to be called for an interview relative to a worker with a shorter duration of unemployment who is otherwise identical.} This could generate a countercyclical composition bias à la Mortensen and Pissarides (1994): if workers with long unemployment spells must be of
higher ex-post match quality to compensate for stigma effects, the average long-duration worker hired during a recession should be of higher quality than during an expansion; in contrast workers with short unemployment durations are less likely to be affected by countercyclical hiring standards.\footnote{To be clear, Mortensen and Pissarides (1994) abstracts from endogenous hiring standards by assuming that all matches are created with the highest match productivity. This assumption can be relaxed, as in Barlevy (2002).}

Tables 4 and 5 contain results for $EE$ and $ENE$ new hires where we control for duration of non-employment, by first differences and fixed-effects. We estimate separate wage semi-elasticities for new hires from non-employment with non-employment durations less than or equal to $\tau$ months and new hires from non-employment with durations greater than $\tau$ months, with the two groups identified by the indicator variables $ENE$ and $LTU$. For analogous reasons as before, we consider the $ENE$ coefficient as the primary coefficient of interest. We estimate separate regressions for $\tau = 9, 8, 7, 6, \text{ and } 5$. For each value of $\tau$, we estimate the regression equation without controlling for occupation or industry switchers, and controlling for occupation switchers. In Tables B.2 and B.3 of the appendix, we estimate the same regressions, but controlling for industry switchers.

Once again, our results are unchanged. When we isolate shorter duration workers without controlling for occupation or industry switching, the $ENE$ coefficient is negative and larger in magnitude, but never statistically significant. Additionally, once we control for occupation and industry switching, the coefficient estimates for $ENE$ tend to switch signs and become positive. In the first-differences estimation, the differential effect for short-duration $ENE$ occupation or industry switchers is negative (consistent with a procyclical composition effect) but not statistically different from zero. In our fixed-effects estimation, however, these coefficients become statistically significant.\footnote{As we discuss in the appendix in the context of the approximate DGM, the fixed-effects and first-differences estimators do not generally yield identical coefficient estimates.} Our estimates suggest that controls for occupation and industry may be important when estimating true $ENE$ wage cyclicality across different groups of workers.

Overall, our estimates from this section are consistent with those from of our baseline regressions: That is, even after controlling for composition bias of $ENE$ workers based on observables, we find no evidence of new hire wage flexibility. While these exercises may not definitely rule out composition bias for these types of workers, our results in this section suggest that this possibility is less likely. At a minimum, further, our results provide a set of conditional moments that any model of unemployment and wage dynamics must satisfy. That is, within such a model, only the wages of job changers should exhibit excess cyclicality. New hire wages from unemployment should be no more cyclical than those of existing workers.
Finally, note that our approach of isolating subsets of ENE workers who are unlikely to have been effected by composition bias implicitly requires that cyclical composition be related to observable characteristics of the worker prior to the match. Suppose instead that the relevant composition bias for ENE workers arises from selection on ex-post match characteristics à la Mortensen and Pissarides (1994), but without respect to ex-ante characteristics of the worker. Under such a scenario, where the bias is presumed to be uniform, we would be unable to find a subgroup of ENE workers unaffected by the bias. However, a literature evaluating the quantitative importance of selection on the basis of ex-post characteristics suggests that it must play only a minor quantitative role for generating cyclical bias in the average wage growth of new hires from unemployment. In particular, a countercyclical bias in estimates of new hire wage cyclicity could emerge if workers hired from unemployment during a recession are systematically hired into better matches, so that wage declines for new hires from unemployment will be understated during a recession. But Barlevy (2002) evaluates exactly such a scenario within an MP model and finds it to have little quantitative power in explaining cyclical changes in match quality. On the other hand, if better matches are destroyed during recessions, so that workers hired from unemployment come from higher wage matches, this type of selection has the potential to overstate the true extent of wage declines during contractions. Not only this second mechanism makes the overall cyclical impact of selection on the basis of ex-post match characteristics theoretically ambiguous, but Mueller (2017) shows that a plausibly calibrated MP model with endogenous job destruction can only generate compositional shifts that are tiny in magnitude. Hence, we view composition bias arising from selection on ex-post match characteristics to be of negligible quantitative importance.

Even if one does not accept our evidence about composition effects, our panel data evidence provides a new set of conditional moments that can be used to discipline macroeconomic models of unemployment fluctuations. In the next section, accordingly, we develop a model that can account for not only the aggregate evidence but also our cross-sectional evidence of the relative cyclicalities of job changers wages versus those of new hires from unemployment.

3 Model

We model employment fluctuations using a variant of the Diamond, Mortensen, and Pissarides search and matching framework. Our starting point is a simple real business cycle
model with search and matching in the labor market, similar to Merz (1995) and Andolfatto (1996).

We make two main changes to the Merz/Andolfatto framework. First we allow for staggered wage contracting with wage contracts determined by Nash bargaining, as in GT (2009). Second, we allow for both variable match quality and on-the-job search with variable search intensity. These features will generate procyclical job ladder effects, in the spirit of Barlevy (2002) and Menzio and Shi (2011). As we will show, both these variants will be critical for accounting for both the macro and micro evidence on unemployment and wage dynamics.

Below we describe the labor market of the model conditional. We defer to the appendix a description of the full general equilibrium.

### 3.1 Search, Vacancies, and Matching

There is a continuum of firms and a continuum of workers, each of measure unity. Workers within a firm are either good matches or bad matches. A bad match has a productivity level that is only a fraction $\phi$ of that of a good match, where $\phi \in (0, 1)$. Let $n_t$ be the number of good matches within a firm that are working during period $t$ and $b_t$ the number of bad matches. Then the firm's effective labor force $l_t$ is the following composite of good and bad matches:

$$l_t = n_t + \phi b_t.$$  \hspace{1cm} (6)

Firms post vacancies to hire workers. Firms with vacancies and workers looking for jobs meet randomly (i.e., there is no directed search). The quality of a match is only revealed once a worker and a firm meet. Match quality is idiosyncratic. A match is good with probability $\xi$ and bad with complementary probability $1 - \xi$. Hence, the outcome of a match depends neither on ex-ante characteristics of the firm or the worker. Whether or not a meeting becomes a match depends on the realization of match quality and the employment status of the searching worker.

Let $\bar{n}_t = \int_i n_t di$ and $\bar{b}_t = \int_i b_t di$ be the total number of workers who are good matches and who are bad matches, respectively, where firms are indexed by $i$. The total number of unemployed workers $\bar{u}_t$ is then given by

$$\bar{u}_t = 1 - \bar{n}_t - \bar{b}_t.$$  \hspace{1cm} (7)

We assume that each unemployed worker searches with a fixed intensity, normalized at unity. Under our parameterization, it will be optimal for a worker searching from unemployment to accept both good and matches.

There are two ways a worker leaves a match. First there is an exogenous separation
probability $1 - \nu$, which means the worker becomes unemployed at the beginning of the subsequent period. Second, if the match is not destroyed, which occurs with probability $\nu$, the worker will search on the job. If another match is found and accepted, the worker goes to the new firm within the period. Otherwise the worker remains with the firm for another period.

Absent other considerations, the only reason for an employed worker to search is to find a job with improved match quality.\textsuperscript{26} In our setting, the only workers who can improve match quality are those currently in bad matches. We allow such workers to search with variable intensity $\varsigma_b$. As has been noted in the literature, however, not all job transitions involve positive wage changes (see Tjaden and Wellshmied, 2014). Accordingly, we suppose that workers in good matches may occasionally leave for idiosyncratic reasons, e.g. locational constraints.\textsuperscript{27} We assume that these workers search with fixed intensity $\varsigma_n$ and accept good or bad matches. This is equivalent to a reallocation shock whereby workers in good matches are forced to search on the job with probability $\varsigma_n$. It is also similar to a reallocation shock à la Moscarini and Postel-Vinay (2016) that moves employed workers to another job drawn randomly from the available ones.\textsuperscript{28} Not only are job-to-job changes with a reduction in wages an empirical regularity, but their level and cyclicality is key for understanding the wage cyclicality of job changers via composition effects, as we show later.\textsuperscript{29}

\textsuperscript{26}Strictly speaking, with staggered wage contracting, workers may want to search to find a job of the same quality if their wages are (i) sufficiently below the norm and are (ii) not likely to be renegotiated for some time. However, because the likelihood a worker is in this situation in our model is extremely small due to the transitory nature on average of wage differentials due to staggered contracting, expected gains from lateral movements will be tiny: A small moving cost would suffice to rule them out. Hence, we abstract from lateral movements. In the appendix, we quantify gains from lateral movements and show that they are indeed tiny.

\textsuperscript{27}For similar reasons, structural econometric models formulated to assess the contribution of on-the-job search to wage dispersion in a stationary setting often include a channel for exogenous, non-economic job-to-job transitions with wage drops. Examples include Jolivet, Postel-Vinay, and Robin (2006) and Lentz and Mortensen (2012).

\textsuperscript{28}For the sake of analytical simplicity, we only consider a reallocation shock for good workers. We have also worked out a version of the model where workers in bad matches are also exposed to a reallocation shock. This does not have any noticeable implication on the quantitative results. The reason is that, under our calibration strategy, the two versions of the model are close to be observationally equivalent.

\textsuperscript{29}Several facts involving the distribution of wages for workers making job-to-job transitions are consistent with our modeling assumptions. While in our data the average wage changes of job-changers is modest – plus 4.5 percent, from the first column of Table 2 – the conditional wage changes are considerably larger in magnitude, equal to plus 30 percent for the 53 percent share of workers realizing wage gains and minus 23 percent for the 47 percent share of workers realizing wage losses. Hence, movements up and down the job-ladder involve large gains and losses, making our two-quality modeling assumption a reasonable one. Moreover, workers making match-improving job-to-job changes leave systematically lower-paying jobs. We recover the log wage residuals from a simple Mincer wage regression of log wages on observables. The average log wage residual on the prior job for job-changers moving to a higher-paying job is $-0.249$, indicating that wage-improving job-changers are strongly selected from the population of workers earning lower wages than would be predicted by observable characteristics. This form of selection is consistent with a notion of “active search”, whereby the workers with the most to gain have greater incentive to invest effort in potentially costly search. Meanwhile, the average wage residual of job-changers realizing a decrease in wages is more centered
We derive the total efficiency units of search $\bar{s}_t$ as a sum of searchers weighted by the search intensity of their respective type:

$$\bar{s}_t = \bar{u}_t + \nu(\bar{b}_t + \bar{n}_t).$$

The first term reflects the effective amount of search by the unemployed (who each have search intensity normalized to unity). The second term is effect search of the employed, allowing for the difference in search intensity of bad and good matches. As we will show, the search intensity of bad matches on the job will be procyclical. Furthermore, the cyclical sensitivity of the efforts of workers in bad matches to find better jobs will ultimately be the source of procyclical movements in match quality and new hire wages.

The aggregate number of matches $\bar{m}_t$ is a function of the efficiency weighted number of searchers $\bar{s}_t$ and the number of vacancies $\bar{v}_t$, as follows:

$$\bar{m}_t = \sigma_m \bar{s}_t^{\sigma} \bar{v}_t^{1-\sigma},$$

where $\sigma$ is the elasticity of matches to units of search effort and $\sigma_m$ reflects the efficiency of the matching process.

The probability $p_t$ a unit of search activity leads to a match is:

$$p_t = \frac{\bar{m}_t}{\bar{s}_t}.$$  

(10)

The probability the match is good $p^n_t$ and the probability it is bad $p^b_t$ are given by:

$$p^n_t = \xi p_t, \quad (1 - \xi)p_t, \quad (11)$$

$$(12)$$

The probability for a firm that posting a vacancy leads to a match $q^m_t$ is given by

$$q^m_t = \frac{\bar{m}_t}{\bar{v}_t}.$$  

(13)

Not all matches lead to hires, however, as workers in bad matches only accept good matches. Hires also vary by quality. The probability $q^n_t$ a vacancy leads to a good quality hire and around zero, with an average log wage residual of 0.055. This is consistent with the often idiosyncratic reasons for job changes such as family reasons, where a job transfer is motivated by reasons unrelated to pay.
the probability $q^b_t$ it leads to a bad quality one are given by

\begin{equation}
q^n_t = \xi q^m_t, \tag{14}
\end{equation}

\begin{equation}
q^b_t = (1 - \xi) \left(1 - \frac{\nu \beta_t}{s_t}\right) q^m_t. \tag{15}
\end{equation}

Since all workers accept good matches, $q^n_t$ is simply the product of the probability of a match being good conditional on a match, $\xi$, and the probability of a match, $q^m_t$. By contrast, since workers in bad matches do not make lateral movements, to compute $q^b_t$ we must net out the fraction of searchers who search on-the-job from bad matches, $\nu \beta_t/s_t$.

Finally, we can express the expected number of workers in efficiency units of labor that a firm can expect to hire from posting a vacancy, $q_t$, as

\begin{equation}
q_t = q^n_t + \phi q^b_t. \tag{16}
\end{equation}

It follows that the total number of new hires in efficiency units is simply $q_t v_t$.

### 3.2 Firms

Firms add labor through a search and matching process that we describe shortly. Labor in efficiency units $l_t$ is the quality adjusted sum of good and bad matches in the firm (see equation (6)). The current value of $l_t$ is a predetermined state.

It is convenient to define $\gamma_t \equiv b_t/n_t$ as the ratio of bad-to-good matches in the firm. We can then express $l_t$ as the following multiple of $n_t$:

\begin{equation}
l_t = n_t + \phi b_t = (1 + \phi \gamma_t)n_t, \tag{17}
\end{equation}

where as before, $\phi \in (0, 1)$ is the productivity of a bad match relative to a good one. The ratio of bad-to-good matches $\gamma_t$ is also a predetermined state for the firm.

The evolution of $l_t$ depends on the dynamics of both $n_t$ and $b_t$. Letting $\rho_i^t$ be the probability of retaining a worker in a match of type $i = n, b$, we can express the evolution of $n_t$ and $b_t$ as follows:

\begin{equation}
n_{t+1} = \rho^n_t n_t + q^n_t v_t, \tag{18}
\end{equation}

\begin{equation}
b_{t+1} = \rho^b_t b_t + q^b_t v_t, \tag{19}
\end{equation}

where $q^i_t v_t$ is the quantity of type $i$ matches and where equations (14) and (15) define $q^n_t$ and $q^b_t$. The probability of retaining a worker, in turn, is the product of the job survival probability $\nu$ and the probability the worker does not leave for a job elsewhere, giving the
following expressions for good and bad matches:

\[ \rho_t^n = \nu(1 - \gamma_t), \tag{20} \]
\[ \rho_t^b = \nu(1 - \phi_t), \tag{21} \]

where workers in bad matches searching on-the-job only accept good matches, while workers in good matches subject to the reallocation shock move to both good and bad matches.

It follows from equations (17), (20) and (21) that we can express the survival probability of a unit of labor in efficiency units, \( \rho_t \), as the following convex combination of \( \rho_t^n \) and \( \rho_t^b \):

\[ \rho_t = \frac{\rho_t^n + \phi_t \gamma_t}{1 + \phi_t}, \tag{22} \]

where we once again use \( \gamma_t \) to denote the ratio of bad-to-good matches.

The hiring rate in efficiency units of labor, \( \zeta_t \), is ratio of new hires in efficiency units \( q_t v_t \) to the existing stock, \( l_t \)

\[ \zeta_t = \frac{q_t v_t}{l_t}, \tag{23} \]

where the expected number of efficiency weighted new hires per vacancy \( q_t \) is given by equation (16). The evolution of \( l_t \) is then given by:

\[ l_{t+1} = (\rho_t + \zeta_t) l_t. \tag{24} \]

It is useful to define \( \gamma_t^h \equiv (q_t^h v_t) / (q_t^n v_t) = q_t^b / q_t^n \) as the ratio bad-to-good matches among new hires. Then, making use of equations (16), (17), (18), (19) and (23) to characterize how the ratio of bad-to-good matches across all workers \( \gamma_t \) evolves over time, we obtain:

\[ \gamma_{t+1} = \frac{\rho_t^b \gamma_t + q_t^h v_t / n_t}{\rho_t^n + q_t^n v_t / n_t} = \frac{\gamma_t \left[ \frac{\gamma_t^h}{1 + \phi_t} \rho_t^b + \frac{\gamma_t^i}{1 + \phi_t^h} \zeta_t \right]}{\frac{1}{1 + \phi_t} \rho_t^n + \frac{1}{1 + \phi_t^h} \zeta_t}, \tag{25} \]

where \( 1/(1 + \phi_t) \) is the share of good matches among incumbent workers and \( 1/(1 + \phi_t^h) \) is the share of good matches among new hires and where \( \gamma_t/(1 + \phi_t) \) and \( \gamma_t^h/(1 + \phi_t^h) \) are the complementary shares of bad matches.

We now turn to the firm’s decision problem. Let \( a_t \) be the productivity of an efficiency unit of labor, \( \Lambda_{t,t+1} \) be the firm’s stochastic discount factor and \( w_t \) be the wage per efficiency unit of labor. Assume that labor recruiting costs are quadratic in the hiring rate for labor in efficiency units, \( \zeta_t \), and homogeneous in the existing stock \( l_t \). Then the firm’s decision problem is to choose the hiring rate \( \zeta_t \) to maximize the firm’s value (the discounted stream of profits net recruiting costs) subject to the equations that govern the laws of motion for
labor in efficiency units $l_t$ and the ratio of bad-to-good matches within the firm $\gamma_t$, given the expected paths of wages.

Since the firm’s value $F_t(l_t, \gamma_t, w_t) = F_t$ is homogeneous in $l_t$, we express the value of each firm per efficiency unit of labor $J_t(\gamma_t, w_t) \equiv J_t = F_t/l_t$ as

$$J_t = \max_{\kappa_t} \{ a_t - \frac{\kappa_t}{2} \gamma_t^2 - w_t + (\rho_t + \kappa_t) E_t \{ L_{t+1}J_{t+1} \} \}, \quad (26)$$

subject to equation (25), given the values of firm-level states for labor composition and contract wage $(\gamma_t, w_t)$ and the aggregate state vector.\(^\text{30}\) For the time being, we take the firm’s expected wage path as given. In Section 3.4 we describe how wages are determined for both good and bad workers.

The first order condition for hiring is

$$\kappa_t = E_t \left\{ \Lambda_{t, t+1} \left[ J_{t+1} + (\rho_t + \kappa_t) \left[ \frac{\partial J_{t+1}}{\partial \gamma_{t+1}} \right] \frac{\partial J_{t+1}}{\partial w_{t+1}} \frac{\partial w_{t+1}}{\partial \gamma_{t+1}} \right] \frac{\partial \gamma_{t+1}}{\partial \kappa_t} \right\}. \quad (27)$$

The expression on the left is the marginal cost of adding worker, and the expression on the right is the discounted marginal benefit. The first term on the right-hand side of (27) is standard: it reflects the marginal benefit of adding a unit of efficiency labor. The second term reflects a “composition effect” of hiring. While the firm pays the same recruitment costs for bad and good workers (in quality adjusted units), bad workers have separate survival rates within the firm due to their particular incentive to search on-the-job. The composition term reflects the effect of hiring on period-ahead composition, and the implied effect on the value of a unit of labor quality to the firm.\(^\text{31}\)

### 3.3 Workers

We next construct value functions for unemployed workers, workers in bad matches, and workers in good matches. These value functions will be relevant for wage determination, as we discuss in the next section. Importantly, they will also be relevant for the choice of search intensity by workers in bad matches who are looking to upgrade.

We begin with an unemployed worker: Let $U_t$ be the value of unemployment, $V^n_t$ the value of a good match, $V^b_t$ the value of a bad match, and $u_B$ the flow benefit of unemployment. Then, the value of a worker in unemployment satisfies

$$U_t = u_B + E_t \left\{ \Lambda_{t, t+1} \left[ p^n_t V^n_{t+1} + p^b_t V^b_{t+1} + (1 - p_t) U_{t+1} \right] \right\}, \quad (28)$$

\(^{30}\) The firm’s decision problem is formulated according to the following intra-period timing protocol: (i) realization of aggregate and firm-level shocks, (ii) wage bargaining and production, (iii) realization of match-level separation shocks, and (iv) search and matching.

\(^{31}\) Under our calibration, the effect will be zero, up to a first order. See the appendix for details.
where \( p_t^b = \xi p_t, \) \( p_t^b = (1 - \xi)p_t, \) \( p_t \) is given by (10), and where \( V_{t+1}^n \) and \( V_{t+1}^b \) are the average values of good and bad matches at time \( t + 1.\)

For workers that begin the period employed, we suppose that the cost of searching as a function of search intensity is given by

\[
c(s_{it}) = \frac{s_0}{1 + \eta_k} s_{it}^{1+\eta_k}
\]

where \( i = b, n. \) As we discussed earlier, workers in bad matches search on the job with variable intensity \( s_{it} \). We assume that workers in bad matches search with variable intensity \( s_{it} \). Since \( s_{it} \) is given by (10), and where \( \eta_k \) is the expected gain from a lateral move is quantitatively trivial and can be ruled out almost surely with a small moving cost, as we show in the appendix.

Let \( w_{it} \) be the wage of a type \( i \) worker, \( i = b, n. \) The value of a worker in a bad match

\[
V_t^b(\gamma_t, w_t) \equiv V_t^b
\]

is given by

\[
V_t^b = \max_{s_{it}} \left\{ w_{bt} - \nu c(s_{bt}) + E_t \left\{ \Lambda_{t,t+1} \left[ \nu (1 - s_{bt} p_t^n) V_{t+1}^b 
+ \nu s_{bt} p_t^n \bar{V}_{t+1}^n + (1 - \nu) U_{t+1} \right] \right\} \right\}
\]

(29)

The flow value is the wage \( w_{bt} \) net the expected costs of search. If the worker “survives” within the firm, which occurs with probability \( \nu \), he searches with variable intensity \( s_{bt}. \) The first term in the continuation value is the value of continuing in the match, which occurs with probability \( \nu (1 - s_{bt} p_t^n). \) The second term reflects the value of switching to a good match, which occurs with probability \( \nu s_{bt} p_t^n. \) The final term reflects the value of being separated into unemployment.

A worker in the bad match chooses the optimal search intensity \( s_{bt} \) according to (29), satisfying

\[
s_0 \nu s_{bt} = E_t \left\{ \Lambda_{t,t+1} p_t^n (\bar{V}_{t+1}^n - V_{t+1}^b) \right\}
\]

(30)

Search intensity varies positively with the product of the likelihood of finding a good match, \( p_t^n \), and the net gain of doing so, i.e. the difference between the value of good and bad matches. One can see from equation (30) how the model can generate procyclical search.
intensity by workers in bad matches. The probability of finding a good match will be highly procyclical and the net gain roughly acyclical. Thus, the expected marginal gain from search will be highly procyclical, leading to procyclical search intensity.

The value of a worker in a good match $V^n_t(\gamma_t, w_t) \equiv V^g_t$ is similar to the value function for a bad match.

$$V^n_t = w_n - \nu c(\zeta_n) + E_t \left\{ \Lambda_{t,t+1} \left[ \nu(1 - \zeta_n p_t)V^n_{t+1} + \nu \zeta_n \left( p^n_t V^n_{t+1} + p^b_t V^b_{t+1} \right) + (1 - \nu)U_{t+1} \right] \right\} \tag{31}$$

As we discussed earlier, a worker in a good match who receives a reallocation shock may wind up moving to a bad match.

In the absence of direct evidence of the broader relation of job quality and match retention, we assume that the retention rates of good and bad matches are identical on average (implying that, in the steady state, $\zeta_{gb} = \zeta_n$). As we show in the appendix, this assumption will also be important for maintaining tractability of the firm’s and workers’ problem.\textsuperscript{35}

### 3.4 Nash Wage

As in GT, workers and firms divide the joint match surplus via staggered Nash bargaining. For simplicity, we assume that the firm bargains with good workers for a wage. Bad workers then receive the fraction $\phi$ of the wage for good workers, corresponding to their relative productivity. Thus if $w_t$ is the wage for a good match within the firm, then $\phi w_t$ is the wage for a bad match. It follows that $w_t$ corresponds to the wage per unit of labor quality. We note that this simple rule for determining wages for workers in bad matches approximates the optimum that would come from direct bargaining.\textsuperscript{36}

Our assumptions are equivalent to having the good workers and firms bargain over the wage per unit of labor quality $w_t$. For the firm, the relevant surplus per worker is $J_t$, as shown in equation (26) of Section 3.2. For good workers, the relevant surplus is the difference between the value of a good match and unemployment:

$$H_t = V^n_t - U_t \tag{32}$$

\textsuperscript{35}Two studies of job tenure and match quality over the business cycle are Bowlus (1995) and Mustre-del-Rio (2017). We note that our model is consistent with their findings on the cyclicality of job tenure as a function of the aggregate state at match formation. In particular, Mustre-del-Rio shows that workers hired from non-employment who subsequently make a job-to-job transition have shorter tenure during expansions, consistent with the prediction of our model.

\textsuperscript{36}This simple rule differs slightly due mainly to differences in duration of good and bad matches with firms. The gain from imposing this simple rule is that we need only characterize the evolution of a single type of wage. Importantly, in bargaining with good workers, firms also take account of the implied costs of hiring bad workers.
As in GT, the expected duration of a wage contract is set exogenously. At each period, a firm faces a fixed probability $1 - \lambda$ of renegotiating the wage. With complementary probability, the wage from the previous period is retained. The expected duration of a wage contract is then $1/(1 - \lambda)$.

Workers hired in between contracting periods receive the prevailing firm wage per unit of labor quality $w_t$. Thus in the model there is no new hire effect: Adjusting for relative productivity the wages of new hires are the same as for existing workers.

Let $w_t^*$ denote the wage per unit of labor quality of a firm renegotiating its wage contract in the current period. The wage $w_t^*$ is chosen to maximize the Nash product of a unit of labor quality to a firm and a worker in a good match, given by

$$H_t^* J_t^{1-\eta}$$

subject to

$$w_{t+1} = \begin{cases} w_t & \text{with probability } \lambda \\ w_{t+1}^* & \text{with probability } 1 - \lambda \end{cases}$$

where $w_{t+1}^*$ is the wage chosen in the next period if the parties are able to re-bargain and where $\eta$ is the households relative bargaining power.

Let $H_t^* \equiv H_t(\gamma_t, w_t^*)$ and $J_t^* \equiv J_t(\gamma_t, w_t^*)$ (where $H_t \equiv H_t(\gamma_t, w_t)$ and $J_t \equiv J_t(\gamma_t, w_t)$). Then the first order condition for $w_t^*$ is given by

$$\chi_t^* J_t^* = (1 - \chi_t^*) H_t^*$$

where

$$\chi_t^* = \frac{\eta}{\eta + (1 - \eta) \mu_t^*/\epsilon_t^*}$$

with

$$\epsilon_t^* = \frac{\partial H_t^*}{\partial w_t^*} \quad \text{and} \quad \mu_t^* = \frac{\partial J_t^*}{\partial w_t^*}$$

Equation (35) is a variation of the conventional sharing rule, where the relative weight $\chi_t$ depends not only on the worker’s bargaining power $\eta$, but also on the differential firm/worker horizon, reflected by the term $\mu_t/\epsilon_t$ as discussed in GT.

---

37 We use the Calvo formulation of staggered contracting for convenience, since it does not require keeping track of the distribution of remaining time on the contracts. We expect very similar results from using Taylor contracting, where contracts are of a fixed duration. An advantage with Taylor contracting is that wages are less likely to fall out of the bargaining set, since with Calvo a small fraction of firms may not adjust wages for a long time. Nonetheless, given that the broad insights from Calvo and Taylor contracting are very similar, we stick with the simpler Calvo formulation.

38 We suppress the dependence of $w^*$ and similar objects on the firm's composition in the notation.

39 Recall, $\gamma_t$ gives the ratio of bad-to-good workers within a firm.

40 Intuitively, when valuing the contract wage stream, the firm has a longer horizon than the worker because it cares about the effect of the current wage contract on payments not only to the existing workforce,
Under multi-period bargaining, the outcome depends on how the new wage settlement affects the relative surpluses, $J^*_t$ and $H^*_t$, in subsequent periods where the contract is expected to remain in effect. The net effect, as shown in GT, is that up a first order approximation the contract wage will be an expected distributed lead of the target wages that would arise under period-by-period Nash bargaining, where the weights on the target for period $t + i$ depend on the likelihood the contract remains operative, $\lambda^i$.

To a first order, we can express the evolution of average wages $\bar{w}_t$ as

$$\bar{w}_t = (1 - \lambda)\bar{w}^*_t + \lambda\bar{w}_{t-1}$$

where $1 - \lambda$ is the fraction of firms that are renegotiating and $\lambda$ is the fraction that are not and where the average wage and the average contract wage per unit of labor quality are defined by

$$\bar{w}_t = \int_{w,\gamma} w dG_t(\gamma, w)$$

$$\bar{w}^*_t = \int_{w,\gamma} w^*_t(\gamma) dG_t(\gamma, w)$$

with $G_t(\gamma, w)$ denoting the time $t$ fraction of units of labor quality employed at firms with wage less than or equal to $w$ and ratio of bad-to-good workers less than or equal to $\gamma$. (See the appendix for details.)

### 4 New Hire Wages, Match Quality and Job-to-Job Flows

We now show how cyclical composition allows our framework to generate the appearance of excess new hire wage cyclicality, even though within the model new hire wages are no more flexible than those of existing workers. In doing so, we demonstrate how our model framework developed in Section 3 maps into the canonical Bils’ regression from the empirical literature. To do so we derive an expression for the average wage growth of job changers that permits us to interpret estimates of job changer wage cyclicality from the data. In the process, we relate the full model to the approximate DGM that we developed in Section 2 to interpret our reduced form empirical results.

Let $\bar{g}^w_t$ denote the average wage growth of continuing workers, $\bar{g}^{EE}_t$ the average wage growth of new hires who are job changers, and $\Delta \bar{a}^{EE}_t$ the component of $\bar{g}^{EE}_t$ due compositional effects (i.e. changes in match quality across jobs). Further, let $\delta_{BG,t}$ be the share of flows moving from bad to good matches out of total job flows at time $t$ and $\delta_{GB,t}$ the but also to the new workers who enter under the terms of the existing contract. A worker, on the other hand, only cares about wages during his or her tenure at the firm. While the horizon effect is interesting from a theoretical perspective, GT shows that it is quantitatively miniscule, implying $\chi_t$ very close to $\eta$. 
share moving from good to bad matches. Then we can express the average wage growth for job-changers as the following:

\[ \bar{g}_t^{EE} = \bar{g}_t^w + \bar{\alpha}_t^{EE} \]  \hspace{1cm} (39)

with

\[ \bar{g}_t^w = \log \bar{w}_t - \log \bar{w}_{t-1} \]  \hspace{1cm} (40)

and

\[ \Delta \bar{\alpha}_t^{EE} = (\log \phi) (\delta_{BG,t-1} - \delta_{GB,t-1}) \]  \hspace{1cm} (41)

As in the approximate DGM, we see from equation (39) that the wage growth of job-changers is additively separable in two components: a common component equal to average wage growth of continuing workers, as described in equation (40); and a separate component \( \Delta \bar{\alpha}_t^{EE} \), described in equation (41), that serves as the model analogue for the average change in match quality for job-changers under equation (3) of the approximate DGM.

Equation (41) indicates that a fraction \( \delta_{BG,t-1} \) of job-changers realizes wage gains of \( (\log \phi) \) at \( t \), a fraction \( \delta_{GB,t-1} \) realizes wage losses of \( (\log \phi) \) at \( t \), and the remaining fraction realizes no change at all. Absent the composition effect (i.e. if \( \phi = 1 \)), average wage growth for job changers would look no different than for continuing workers. With the composition effect present, however, cyclical variation of match quality will enhance the relative cyclicality of job changers wages, as we discuss next.

We first note that, absent cyclical fluctuations, the average change in match quality is given by the term \( -\log(\phi) (\bar{\delta}_{BG} - \bar{\delta}_{GB}) \), where \( \bar{z}_t \) denotes the steady state value of a variable \( z_t \). This term is analogous to the coefficient \( \psi_n^{EE} \) in equation (3) of the approximate DGM, which describes the non-cyclical component of changes in match quality for job-changers.

The change in average match quality for job changers also has a cyclical component whose dynamic behavior is function of the dynamics of the flow shares \( \delta_{BG,t} \) and \( \delta_{GB,t} \). As might be expected, this is where the mapping of the model to the approximate DGM becomes more complicated. Whereas the change in average match quality for job changers is proportional to the change in the unemployment rate in the approximate DGM, in the full model the dependence will be expressed more generally in terms of changes in variables.

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41 The model includes two types of job-to-job movers: those who search with variable search intensity from bad matches and those in good matches who are forced to search for non economic reasons, i.e., who are subject to a reallocation shock. Since workers in bad matches searching on the job only accept good matches, the first type of job changers leads only to bad-to-good flows, \( \nu_{\bar{o}t} \xi_{pb} \). The second type of job changers instead leads to both good-to-bad and good-to-good flows, \( \nu_{\bar{c}t} (1 - \xi) p_{\bar{n}t} \) and \( \nu_{\bar{c}t} \xi_{p\bar{n}t} \). Importantly, job-to-job changes with either no appreciable change in wages or with a reduction in wages are important not only for matching empirical evidence, but also for understanding the wage cyclicality of job changers via composition effects. Later, we use empirical moments on the level and cyclicality of the share of bad-to-good flows out of total job flows to discipline the calibration of the model.
describing the composition of match quality in the aggregate labor market. This relation emerges from the search decision of workers in bad matches.

Specifically, loglinearizing the compositional component of job changers wage growth, after substituting the expressions for the flow shares $\delta_{BG,t-1}$ and $\delta_{GB,t-1}$ (see the appendix for details), gives:

$$\Delta \tilde{\alpha}_{t}^{EE} = \alpha_1 \tilde{s}_{bt-1} + \alpha_2 \tilde{\gamma}_{t-1},$$

(42)

where $\tilde{z}_t$ denotes log deviations of variable $z_t$ from steady state and where the parameters $\alpha_1$ and $\alpha_2$ are positive and functions of model primitives. As we have discussed, the search intensity by workers in bad matches, $\tilde{s}_{bt-1}$, is highly procyclical, leading to $\delta_{BG,t-1}$ being procyclical and $\delta_{GB,t-1}$ countercyclical. The dynamics of the shares, and thus the dynamics of changes in match quality, also depend on the relative stocks of bad and good matches available to make a job-to-job transition, as measured by $\tilde{\gamma}_{t-1}$. As we show in the next section, while the aggregate ratio of bad-to-good matches $\tilde{\gamma}_{t-1}$ is countercyclical, the net quantitative effect leads to procyclical composition. At the beginning of a boom, search intensity $\tilde{s}_{bt-1}$ increases, generating a positive change in average match quality. Only in the latter phase of an expansion does the reduction of workers in bad matches available to make a bad-to-good change overshadow the effect on composition of the increase in search intensity among workers in bad matches. In order to make the connection between the cyclical component of average match quality in the full model and the cyclical component in the approximate DGM, we next show that equation (42) implies a relation between the change in match quality and the change in the unemployment rate that well approximates the exact proportional relation imposed in the DGM.

To do so, we first note that the dynamic evolution of the ratio of bad to good matches, $\tilde{\gamma}_t$, implies the following relation between search intensity by workers in bad matches, $\tilde{s}_{bt-1}$, and the change in $\tilde{\gamma}_t$ relative to both $\tilde{\gamma}_{t-1}$ and $\tilde{\gamma}^h_{t-1}$ (where the latter is the ratio of bad to good matches among new hires at $t-1$):

$$(\nu - \tilde{\rho}) \tilde{s}_{bt-1} = -\rho \left( \tilde{\gamma}_t - \tilde{\gamma}_{t-1} \right) - (1 - \tilde{\rho}) \left( \tilde{\gamma}_t - \tilde{\gamma}^h_{t-1} \right).$$

(43)

We then note that simply using the definition of the unemployment rate we can write the change in the ratio of bad-to-good matches from $t-1$ to $t$ as a linearly increasing function of the change in the unemployment rate from $t-1$ to $t$:

$$\tilde{\gamma}_t - \tilde{\gamma}_{t-1} = \frac{\tilde{u}}{n} (\tilde{u}_t - \tilde{u}_{t-1}) + \frac{1 - \tilde{u}}{n} \left( \tilde{b}_t - \tilde{b}_{t-1} \right).$$

(44)

It is easy to see that combining equation (42) with equations (43) and (44) gives an expression that relates $\Delta \tilde{\alpha}_{t}^{EE}$ to $\Delta u_t$, consistently with the DGM. However, the relation
is not exact, due to the presence of other terms related to the composition of the labor force. Further, the dynamics of these terms will be jointly determined with the dynamics of unemployment, implying that neither the set of estimated coefficients from the canonical regression nor those of our new regression equation permit a clear structural interpretation. Hence, we shift to a quantitative focus.

Specifically, in the next section, we show that the net effect of procyclical search intensity $\bar{z}_{bt-1}$ and countercyclical bad-to-good match quality $\bar{\gamma}_{t-1}$, as described in (42), is that $\Delta \alpha_t^{EE}$ is procyclical. That is, the composition effect for job changers enhances measured wage growth during expansions and weakens it during recessions. In this way the model can replicate the kind of cyclical movements in match quality that can lead to estimates of new hire wage cyclicality that suffer from the kind of composition bias we discussed in Section 2. We demonstrate this concretely in the next section by showing that data generated from the model will generate estimates of a new hire effect on wages for job changers, even as new hires’ wages are no more flexible than those of existing workers.

5 Results

In this section we present some simulations to show how the model can capture both the aggregate evidence on unemployment fluctuations and wage rigidity and the panel data evidence on the relative cyclicality of new hires’ versus continuing workers’ wages. We first describe the calibration before turning to the results.

5.1 Calibration

We adopt a monthly calibration. There are 16 parameters in the model for which we must select values. We calibrate 9 of the parameters using external sources. Five of the externally calibrated parameters are common to the macroeconomics literature: the discount factor, $\beta$; the capital depreciation rate, $\delta$; the “share” of labor in the Cobb-Douglas production technology, $\zeta$; and the autoregressive parameter and standard deviation for the total factor productivity process, $\rho_z$ and $\sigma_z$ (see Section C.1 in the appendix). Our parameter choices are standard: $\beta = 0.99^{1/3}$, $\delta = 0.025/3$, $\zeta = 1/3$, $\rho_z = 0.95^{1/3}$, and $\sigma_z = 0.007$.42,43

Four more parameters are specific to the search literature. We assume a Cobb-Douglas matching function, and our choice of the matching function elasticity with respect to

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42 Note that, in contrast to the frictionless labor market model, the term $\zeta$ does not necessarily correspond to the labor share, since the labor share will in general depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below $\zeta$, here we simply follow convention by setting $\zeta = 2/3$.

43 The parameter $\sigma_z$ is chosen to target the standard deviation of output.
searchers, \( \sigma \), is 0.4, guided by the estimates from Blanchard and Diamond (1989). We set the worker’s bargaining power \( \eta \) to 0.5, as in GT. We normalize the matching function constant, \( \sigma_m \), to 1.0. We choose \( \lambda \) to target the average frequency of wage changes. Taylor (1999) argues that medium to large-size firms adjust wages roughly once every year; this is validated by findings from microdata by Gottschalk (2005), who concludes that wages are adjusted roughly every year. We set \( \lambda = 11/12 \), implying an average duration between negotiations of twelve months. The parameter values are given in Table 6.

The remaining seven parameters are jointly calibrated to match model-relevant moments measuring average transition probabilities, individual-level wage dynamics, and the value of leisure. We calibrate the inverse productivity premium, \( \phi \); the probability that a new match is good, \( \xi \); the elasticity of the search cost, \( \eta_\kappa \); the hiring cost parameter, \( \kappa \); the separation probability, \( (1 - \nu) \); the scale parameter of the search cost, \( \varsigma_0 \); and the flow value of unemployment, \( u_B \), to match seven moments: the average wage change of workers making EE transitions; the average share of bad-to-good flows out of total job flows; the cyclicality of the share of bad-to-good flows; the average UE probability; the average EU probability; the average EE probability; and the relative value of non-work. Although there is not a one-to-one mapping of parameters to moments, there is a sense in which the identification of particular parameters are more informed by certain moments than others. We use this informal mapping to provide a heuristic argument of how the various parameters are identified.

We calibrate \( \phi \) to target the average wage change of workers making direct job-to-job transitions in our data, 4.5 percent (see Table 2, column 1); holding everything constant, a higher \( \phi \) implies a smaller (positive) average percentage wage increase for job changers. We recover \( \phi = 0.79 \). We calibrate \( \xi \) to match the average share of job transitions involving positive wage changes out of total job flows in our data, 0.527. Holding fixed the targeted transition probabilities, a lower \( \xi \) corresponds to a higher steady state value of bad-to-good workers \( \tilde{\gamma} \), and hence a higher average share of bad-to-good flows. We recover \( \xi = 0.285 \).

We calibrate \( \eta_\kappa \) to match the cyclicality of the share of bad-to-good flows, measured as the coefficient from a regression of the bad-to-good flow share on unemployment;\(^{44}\) a higher \( \eta_\kappa \) corresponds to a lower elasticity of search intensity to changes in the expected gain from

\[ I \{ BG_{it} = 1 \} = \pi_c + \Delta x_{it}^a \pi_s + \eta_{BG} \cdot u_t + \varepsilon_{it} \]

The coefficient \( \eta_{BG} \) tells how the share of workers that improve wages varies with the unemployment rate. Our point estimate of \( \eta_{BG} \) is \( -1.40 \), significant at the one percent level. The estimate suggests that if the unemployment rate increases by one percentage point, the share of workers that are upgrading their jobs drops by 0.014 percentage points, consistent with a procyclical share of job-changers moving to jobs with better pay.

\( ^{44} \) We first create an indicator variable, \( I \{ BG_{it} = 1 \} \) which takes on a value of one if a worker who changes jobs receives a pay increase and zero otherwise. We then regress the indicator on a first difference of individual characteristics and the unemployment rate, as follows:

We calibrate \( \phi \) to target the average wage change of workers making direct job-to-job transitions in our data, 4.5 percent (see Table 2, column 1); holding everything constant, a higher \( \phi \) implies a smaller (positive) average percentage wage increase for job changers. We recover \( \phi = 0.79 \). We calibrate \( \xi \) to match the average share of job transitions involving positive wage changes out of total job flows in our data, 0.527. Holding fixed the targeted transition probabilities, a lower \( \xi \) corresponds to a higher steady state value of bad-to-good workers \( \tilde{\gamma} \), and hence a higher average share of bad-to-good flows. We recover \( \xi = 0.285 \).

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on-the-job search (see equation (30)), and thus, other things equal, to a lower cyclicality of bad-to-good flows. We obtain $\eta_k = 1.085$.  

We calibrate the separation probability $(1 - \nu)$ to match the empirical EU probability of 0.025. The hiring cost parameter $\kappa$ determines the resources that firms place into recruiting, and hence, influences the probability that a worker finds a job. We set the steady state job finding probability $\tilde{p}$ to match the monthly UE transition probability, 0.42; and then calibrate $\kappa$ to be consistent with $\tilde{p}$. We restrict $\varsigma_0 = \xi \varsigma_b$ to have on average equal retention rates for workers in good and bad matches and note that a higher search cost implies a lower EE probability. We calibrate $\varsigma_0$ to match an EE probability of 0.025; we obtain $\varsigma_0 = 1.65$. 

We interpret the flow value of unemployment $u_B$ as capturing both unemployment insurance and utility of leisure. We calibrate $u_B$ to target a relative value of nonwork to work activity $\bar{u}_T$ equal to 0.71 as in Hall and Milgrom (2008). In our setting, the relative value of nonwork activities satisfies

$$
\bar{u}_T = \frac{u_B + \nu c(\varsigma_0)}{\bar{a} + (\kappa/2) \bar{z}^2},
$$

where $\bar{a} = (1 - \zeta) \bar{y}/\bar{l}$. Note that the value of nonwork includes saved search costs from on-the-job search and the value of work includes saved vacancy posting costs. Finally, when taking the model to the data, we assume that workers in less productive matches receive a period surplus proportional to that of more productive matches by a factor $\phi$, through a lower disutility of labor. In doing so, we ensure that workers in bad matchers always receive positive surplus from employment.

The full list of parameter values and targeted moments are given in Table 7. Having fully calibrated the model, we now evaluate whether it provides an accurate description of aggregate and individual-level dynamics. We first test the ability of the model to match the cyclical properties of aggregate unemployment and wages. Second, we assess the ability of the model to generate the correct relative cyclicality in wage growth for job changers versus continuing workers.

### 5.2 Model Simulations of Aggregate and Panel Data Evidence

We first explore whether the model provides a reasonable description of labor market volatility. In particular, we compare the model implications to quarterly U.S. data from 1964:1 to 2013:2. We take quarterly averages for monthly series in the data. Given that the model is

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45 This is close to a quadratic search cost function parameterization and similar to Lise (2013) and Christensen et al. (2005).

46 The values for the EU, EE and UE probabilities are from Lise and Robin (2017).

47 This is similar to Postel-Vinay and Robin (2002) and others, who assume that the worker surplus is linear in the idiosyncratic productivity of the worker.
calibrated to a monthly frequency, we take quarterly averages of the model simulated data series.

We measure output $y$ as real output in the nonfarm business sector. The wage $w$ is average per worker earnings of production and non-supervisory employees in the private sector, deflated with the PCE. Total employment $n + b$ is measured as all employees in the nonfarm business sector. Unemployment $u$ is civilian unemployment 16 years and older. Vacancies $v$ are a composite help-wanted index computed by Barnichon (2010) combining print and online help-wanted advertising. The data and model output are detrended with an HP filter with the conventional smoothing parameter.

To explore how the model works to capture the aggregate data, we first compute impulse responses to a one percent shock to productivity. To highlight the role of staggered wage contracting, we compute the model generated output for the staggered case and the flexible wage case. The model with wage rigidity produces an enhanced response of output and the various labor market variables, relative to the flexible wage case. This result is standard in the literature dating back to Shimer (2005) and Hall (2005) and in close keeping with Gertler and Trigari (2009), who use a similar model of staggered wage contracting, but without variable match quality or on-the-job search with endogenous search intensity. Our results confirm that these additional model elements do not alter the main implications of wage rigidity for aggregate dynamics. Given these basic features, we then compute a variety of business cycle moments obtained from stochastic simulation obtained from feeding in a random sequence of productivity shocks.

The impulse responses to a one percent increase in productivity are plotted in Figure 2. The solid line is the response of the baseline model with staggered wage contracting and the dashed line is the model with period-by-period Nash bargaining. Under period-by-period contracting, the model implications are reminiscent of those of the standard Nash bargaining model discussed by Hall (2005) and Shimer (2005). Wages immediately increase following a technology shock, whereas employment, unemployment, and vacancy posting respond only gradually and moderately. In the case with staggered contracting, the pattern is reversed: wages adjust gradually and only modestly, whereas there are greater changes in employment and unemployment. These are to a great extent the result of larger increases in vacancies and the job-finding probability under staggered bargaining. Additionally, we see that for both period-by-period and staggered bargaining, the stock of workers in good matches increases while the stock of workers in bad matches decreases; however, the quantitative magnitude of the change is greater for the economy with staggered bargaining.

Table 8 compares the various business cycle statistics and measures of labor market volatility generated by the model with the data. The top panel gives the empirical standard deviations, autocorrelations, and correlations with output of wages, employment, un-
employment, and vacancies. All standard deviations are normalized relative to output. The bottom panels compute the same statistics using the model. We simulate the model for recontracting on average every four quarters and continuous recontracting.

Overall, the model does a reasonable job of accounting for the relative volatility of unemployment (4.52 in the model versus 5.74 in the data) and for wages (0.47 versus 0.48). As is common in the literature, the model underestimates the volatility of employment; here, the absence of a labor force participation margin is relevant. Consistent with Shimer (2005) and Hall (2005), the wage inertia induced by staggered contracting is critical for the ability of the model to account for the volatility of unemployment. This result is robust to allowing for on-the-job search and procyclical match quality.

We next turn to the model’s ability to account for the panel data evidence. We simulate the model to generate a panel for unemployment rates and wages of new hires and continuing workers.\footnote{We generate a simulated panel of six three-month waves from our model to replicate our sample from the SIPP. Note, although the simulated panel will match the frequency of EE, EU, and UE transitions, the simulated panel will not generically match the frequency of workers who ever report being a new hire while they are followed by the SIPP. This is due to a multitude of reasons, including sample exclusion restrictions that we make for our empirical analysis and lifecycle patterns that are beyond the scope of the model. To correct for this discrepancy, we employ sampling weights for the simulated data so that the simulated panel matches the fraction of workers who are ever EE, ever ENE, and never ENE or EE in the SIPP.} We then use the simulated data to perform two validation checks. First, we run the regression in equation (1), where we estimate a single term for new hire wage cyclicality. Second, we compute the coefficients in equation (5), where we allow separate terms for new hires from employment and non-employment. Both equations are estimated on model simulated data using both a first-differences and a fixed-effects estimator and compared to the SIPP estimates.

Results for the first exercise are given in Table 9, where we compare the results from the SIPP panel data (the first column for first differences and the fourth column for fixed effects) with those obtained from data from our model with wage contracts fixed for four quarters on average (the second and fifth columns), and flexible wages (the third and sixth columns). When estimated in first differences, the calibrated model with staggered contracting generates (untargeted) wage semi-elasticities remarkably similar to the coefficient estimates from the SIPP, for both continuing workers (−0.51 in the model versus −0.46 in the data) and new hires (−1.00 versus −1.14). The estimated excess wage cyclicity for new hires, however, is an artifact of cyclical composition bias, as wages for new hires in the model are no more flexible than wages of continuing workers (as illustrated by the structural equations we have developed in Section 4). The model also replicates the patterns in the data when using a fixed-effects estimator, though the quantitative match between the model and the data is less precise.48

In columns three and six, we explore the implications of period-by-period Nash bargain-
ing for wage determination. Although the model generates a new hire effect, the estimated wage elasticities are too large. Thus, to account for the panel data estimates it is necessary to have not only procyclical movements in new hires’ match quality but also some degree of wage inertia as, for example, produced by staggered multi-period contracting.

Table 10 gives results for the second exercise, where we estimate separate terms for new hires from unemployment and employment. The results show that the excess wage cyclicality of new hires in the model is driven by those coming from employment. The coefficient for workers making a direct employment-to-employment transition that we estimate from model simulated data is $-1.71$ against $-1.88$ estimated in the SIPP data when using first differences, and $-1.14$ against $-1.98$ when using fixed effects. For new hires from unemployment, the measure of excess wage flexibility moves close to zero. For first differences, the estimate is $-0.44$ (and not significant) for the SIPP versus $-0.42$ for the model. With fixed effects it is $-0.34$ (and not significant) versus $-0.35$.49

Figures 3 and 4 illustrate how compositional effects influence wage dynamics. We repeat the experiment of a one percent increase in TFP. Figure 3 then reports impulse responses for labor in efficiency units, good matches, bad matches and job flows between good and bad matches. In the wake of the boom, labor quality increases. Underlying this increase is a rise in good matches and a net fall in bad matches. The rise in good matches is due in part to good matches being hired out of unemployment. But it is mostly due to an increase in the job flow share of workers moving from bad to good matches and a decline in the reverse flow share, as the two bottom left panels indicates. This pattern in the net flows also leads to a net decline in bad matches.50

Figure 4 then decomposes the response of job changers’ wage growth into the part due to the growth of contracts wages and the part due to compositional effects, using the loglinear versions of equations (39), (40), and (41). The solid line in the top panel is total new hires’ wage growth, the dashed line is the component due to composition, and the dashed line is the component due to average contract wage growth. As the figure illustrates, most of the wage response of new hires that are job changers is due to compositional effects. The bottom panel then relates the compositional effect mainly to the increase in the share of

\footnote{49 In the numerical results we thus recover a small indirect composition effect that lends additional cyclicity to the wage growth of new hires from unemployment. At the peak of an expansion, after unemployment has begun to return to its higher steady state level, the slow-moving average match quality is still improving. At this point, when unemployment is fast increasing, new hires from unemployment will have had higher wages on their last job, implying larger-than-average wage reductions upon re-employment. This explains the slight negative correlation between wage growth across jobs and the change in unemployment for new hires from unemployment. Note, however, that the ENE coefficient from the model is small in magnitude and falls within a one standard error confidence band of the SIPP estimates reported in Table 10. This applies to both first differences and fixed effects.}

\footnote{50 In gross term there are bad matches due to workers being hired from unemployment; however, the behavior of the job-to-job flows swamps this effect.}
job flows moving from bad to good matches.

6 The Marginal Cost of Labor and Composition Bias

In models with long-term firm-worker relations, the firm’s hiring decision depends on the present value of wages a new hire is expected to receive, along with hiring costs. Within this class of models, Kudlyak (2014) derives an expression for the wage component of the marginal cost of labor in terms of current and future wages, i.e. the “user cost” of labor $ucl_t$:

$$ucl_t = w_{t,t} + E_t \left\{ \sum_{s=1}^{\infty} (\beta \rho)^s (w_{t,t+s} - w_{t+1,t+s}) \right\}$$

where $w_{t,t+s}$ is the wage paid at $t+s$ to a worker hired in $t$, $\rho$ is the worker survival rate, and $\beta$ is the discount factor. The user cost is the sum of two components: (i) the current contract wage $w_{t,t} \equiv w_t$, and (ii) a measure of history dependence in future wages $f_t$, expressed as the difference between the discounted stream of wages paid from $t+1$ to a worker hired in $t$ and the discounted stream to be paid to an identical worker hired in $t+1$. In our model — or any model of contracting whereby workers of the same characteristics within the same firm receive the same wage — $f_t$ is equal to zero since wages are independent of the hiring date, controlling for worker/firm fundamentals. Thus in our framework the user cost of labor is simply equal to the contract wage.$^{51}$ However, if wages are permanently indexed to the time that a worker is hired à la Pissarides (2009), history dependence captured by $f_t$ will make the user cost of labor more cyclical than the hiring wage $w_t$.

Kudlyak (2014) and Basu and House (2017) estimate the cyclicality of the user cost of labor relative to the wages of continuing workers and new hires.$^{52}$ Their key identifying assumption is that match quality and separation rates are invariant to the cycle. As we now argue, such an identifying assumption will conflate procyclical composition as evidence of history dependence à la Pissarides. To do so, we contrast the true user cost in equation (45) with the measured user cost $ucl^m_t$ constructed from ex-post wages by Kudlyak (2014) and Basu and House (2017):

$$ucl^m_t = w_{t,t}^{m} + E_t \sum_{s=1}^{\infty} (\beta \rho)^s (w_{t,t+s}^{m} - w_{t+1,t+s}^{m})$$

$^{51}$This property can be described as “equal treatment” within a firm. Note, it holds trivially in a setting with period-by-period Nash bargaining à la Shimer (2005) and Hagedorn and Manovskii (2008).

$^{52}$If the wages of new hires are no more cyclical than of continuing workers, the cyclical of $f_t$ is zero, and the cyclical of the user cost is equal to the cyclical of the contract wage.
where $w_{t,t+s}$ is the average measured wage of workers at time $t + s$ who still occupy the job that they were hired into at time $t$, $\rho^m_t$ is the average measured retention rate, and $f^m_t$ is the measured history dependent component of the user cost. Whereas the user cost reflects the marginal cost of labor in efficiency units, the measured user cost is not adjusted for composition. If the distribution of job quality among new hires and retention rates are acyclical, as assumed by Kudlyak (2014), the cyclicity of the true user cost and the measured user cost will be the same. But under our model, where the true user cost $ucl_t$ is simply equal to the contract wage $w_t$, the measured user cost will be equal to the contract wage plus a compositional term $\tilde{c}^{ucl}_t$, up to a first order. Because $\tilde{c}^{ucl}$ is highly procyclical, its presence will bias upward the estimate of the cyclicity of the user cost.

We express the loglinearized measured user cost of labor in our model $\tilde{ucl}_t$ as the sum of the true user cost $\tilde{ucl}_t$ and the compositional component $\tilde{c}^{ucl}_t$:  

\begin{equation}
\tilde{ucl}_t = \tilde{ucl}_t + \tilde{c}^{ucl}_t = \tilde{w}_t + \tilde{c}^{ucl}_t.
\end{equation}

The composition component captures both the bias in the measure in the current wage and also the bias in the measured history dependence component $f^m_t$. It can be be expressed as

\begin{equation}
\tilde{c}^{ucl}_t = -\Psi \left( \gamma_t^h + \frac{\tilde{\beta} \tilde{\rho}^b}{1 - \tilde{\rho}^g} \left[ (\tilde{\rho}_t^b - \tilde{\rho}_t^g) + (\tilde{\gamma}_t^h - \tilde{\gamma}_{t-1}^h) \right] \right)
\end{equation}

where $\gamma_t^h$ is the bad-to-good ratio of matches among new hires at time $t$, $\rho_t^g$ is the retention rate of good matches, $\rho_t^b$ is the retention rate of bad matches, $\tilde{\rho}$ is the steady-state retention rate, and $\Psi > 0$ is a function of model primitives.

The first term in the expression for $\tilde{c}^{ucl}_t$ reflects how procyclical match quality introduces upward bias in the measure of the cyclicity of the hiring wage at $t$, $w_{t,t}$: $\tilde{c}^{ucl}_t$ varies inversely with $\gamma_{t-1}^h$, which is countercyclical. The latter two terms reflect the impact of composition bias on the measured history dependent component of the user cost, $f^m_t$. The first term in brackets reflects a discount rate effect due to cyclical variability in the relative retention rates of workers in good and bad matches. Given that workers in bad matches are expected to leave the firm at a higher rate during expansions (and that the retention rate of workers in bad matches is more cyclical than of workers in good matches), this effect provides another source of procyclical composition bias. Finally, the second term in brackets captures how composition bias affects the future stream of wages. Given that the improvement in the composition of match quality of new hires is sufficiently cyclical and persistent, as captured by $-(\tilde{\gamma}_t^h - \tilde{\gamma}_{t-1}^h)$, this last term provides a third source of procyclical composition bias.

\begin{footnotesize}
\footnotesize
53 See the appendix for details.
\footnotesize
54 Recall, workers are hired at the end of the period and begin as new hires the subsequent period. Hence, the ratio of bad-to-good matches among new hires at time $t$ is given by $\gamma_t^h$.
\end{footnotesize}
In sum, while the true user cost $ucl_t$ of our model is simply equal to the contract wage $w_t$, the measured user cost will demonstrate considerable cyclicality due to the unmeasured compositional effects. This measured cyclicality however does not reflect true flexibility in the user cost. To illustrate, we accordingly use our baseline model of Section 3 to simulate a time series for both the true user cost of labor, given by the hiring wage $w_t$; and the measured user cost of labor, given by the sum of hiring wage $w_t$ and the compositional component $c_{ucl}^t$. We calculate the semi-elasticity to unemployment of both measures and obtain −0.66 for the true user cost and −2.49 for the measured one (unadjusted for composition). Hence, the semi-elasticity of the measured user cost is almost 4 times as large as the semi-elasticity of the true user cost. This higher measured elasticity is entirely due to composition bias.

While one could attempt to formulate an alternative empirical representation for the measured user cost of labor that is closer to the true user cost, the main finding of our paper — that the composition-corrected wage cyclicality of new hires is approximately equal to that of continuing workers — implies that the cyclicality of the wage component of the marginal cost of labor can be read directly from the wage cyclicality of workers in continuing matches.

7 Concluding Remarks

We present panel data evidence suggesting that the excess cyclicality of new hires’ wages relative to existing workers may be an artifact of compositional effects in the labor force that have not been sufficiently accounted for in the existing literature. We then use our results to draw inferences about the true flexibility of the marginal cost of labor. Key to our identification is that to a reasonable approximation, the wages of new hires from unemployment provide a composition free estimate of new hire wage flexibility. By contrast, the wages of new hires who are job changers, which account for the overall cyclicality of new hire wages, appears to be driven entirely by procyclical job upgrading and not true wage flexibility.

We reinforce the idea that the observed excess cyclicality of new hire wages could reflect compositions effects by developing a model of unemployment that can account for both the macro and micro data. Within the model, new hires receive the same wage as existing workers with the same fundamental characteristics (i.e., productivity, outside option). Due to this “equal treatment” of workers, there is not true excess flexibility of new hire wages. However, as we find in our estimates from panel data, new hire wages appear to be more cyclical due to the procyclicality of job quality in new matches that stems from workers

\footnote{Consistently with Kudlyak’s estimates, the cyclicality of the measured user cost implied by our model is higher than the cyclicality of new hire wages, not adjusted for composition ($= −1.44$).}
changing jobs.

Indeed within our model, where workers receive “equal treatment”, the user cost of labor is simply the current wage. Since new hires and existing workers receive the same (productivity adjusted) wage, our analysis suggests that the sluggish behavior of existing workers wages may be a better guide to the true flexibility of the marginal cost of labor than the observed high cyclicality of new hires wages unadjusted for composition. What all this suggests is that it is reasonable for macroeconomists to continue to make use of wage rigidity to account for economic fluctuations.

Finally, our model of unemployment fluctuations with staggered wage contracting differs from much of the DSGE literature in allowing a channel for procyclical job-to-job transitions. For many purposes, it may be fine to abstract from this additional channel. However in major recessions like the recent one, a slowdown in job reallocation is potentially an important factor for explaining the overall slowdown of the recovery. A recent study by Haltiwanger et al. (2018) provides evidence that the rate of job-to-job transitions has not recovered relative to the overall job-finding rate in the current recovery. Our model provides a hint about how the slowdown in job reallocation might feedback into other economic activity, by reducing overall total factor productivity. The latter can be thought of as a sullying effect of recessions along the lines of Barlevy (2002). It might be interesting to explore this issue in more detail in subsequent research.
References


Hahn, Joyce K., Henry R. Hyatt, and Hubert P. Janicki. 2018. “Job Ladders and Growth in Earnings, Hours, and Wages.”


Table 1: “Canonical regression” à la Bils (1985) and the new hire effect

<table>
<thead>
<tr>
<th></th>
<th>First differences (1)</th>
<th>Fixed-effects (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate</td>
<td>-0.460***</td>
<td>-0.147**</td>
</tr>
<tr>
<td></td>
<td>(0.0967)</td>
<td>(0.0605)</td>
</tr>
<tr>
<td>Unemp. rate · Ι(new)</td>
<td>-1.138**</td>
<td>-1.646***</td>
</tr>
<tr>
<td></td>
<td>(0.4609)</td>
<td>(0.3265)</td>
</tr>
<tr>
<td>Ι(new)</td>
<td>0.010**</td>
<td>-0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.0040)</td>
<td>(0.0017)</td>
</tr>
</tbody>
</table>

| No. observations     | 321,402                | 378,669           |
| No. individuals      | 57,267                 | 57,267            |
| No. new hires        | 14,667                 | 18,091            |

* p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table 2: Job changers (EE) vs. new hires from unemployment (ENE)

<table>
<thead>
<tr>
<th></th>
<th>First differences</th>
<th>Fixed-effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>UR</td>
<td>−0.425***</td>
<td>−0.420***</td>
</tr>
<tr>
<td></td>
<td>(0.0967)</td>
<td>(0.0966)</td>
</tr>
<tr>
<td>UR · I(EE)</td>
<td>−1.878***</td>
<td>−1.674***</td>
</tr>
<tr>
<td></td>
<td>(0.6800)</td>
<td>(0.6223)</td>
</tr>
<tr>
<td>UR · I(ENE)</td>
<td>−0.439</td>
<td>−0.549</td>
</tr>
<tr>
<td></td>
<td>(0.6639)</td>
<td>(0.7346)</td>
</tr>
<tr>
<td>I(EE)</td>
<td>0.045***</td>
<td>0.038***</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0046)</td>
</tr>
<tr>
<td>I(ENE)</td>
<td>−0.047***</td>
<td>−0.065***</td>
</tr>
<tr>
<td></td>
<td>(0.0066)</td>
<td>(0.0074)</td>
</tr>
</tbody>
</table>

\[ P(\pi_{\text{EE}}^{n_{\text{nu}}} = \pi_{\text{ENE}}^{n_{\text{nu}}}) \]

0.126       0.238 

Unemp. spell for ENE | 0+ | 1+ | 0+ | 1+ |

No. observations | 318,768 | 318,768 | 375,649 | 375,649 |
No. individuals  | 56,881  | 56,881  | 56,881  | 56,881  |
No. EE new hires | 8,714   | 10,124  | 9,857   | 10,124  |
No. ENE new hires| 5,331   | 3,921   | 6,437   | 4,858   |

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table 3: EE & ENE: controls for occupation and industry switchers

<table>
<thead>
<tr>
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<th>First differences</th>
<th>Fixed effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>UR</td>
<td>−0.423***</td>
<td>−0.423***</td>
</tr>
<tr>
<td></td>
<td>(0.0967)</td>
<td>(0.0967)</td>
</tr>
<tr>
<td>UR · 1(EE)</td>
<td>−2.659***</td>
<td>−2.194**</td>
</tr>
<tr>
<td></td>
<td>(0.9380)</td>
<td>(0.8818)</td>
</tr>
<tr>
<td>UR · 1(ENE)</td>
<td>0.090</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>(1.0007)</td>
<td>(0.9329)</td>
</tr>
<tr>
<td>UR · 1(EE &amp; switcher)</td>
<td>1.331</td>
<td>0.613</td>
</tr>
<tr>
<td></td>
<td>(1.3255)</td>
<td>(1.3374)</td>
</tr>
<tr>
<td>UR · 1(ENE &amp; switcher)</td>
<td>−0.836</td>
<td>−1.403</td>
</tr>
<tr>
<td></td>
<td>(1.3060)</td>
<td>(1.2816)</td>
</tr>
<tr>
<td>1(EE)</td>
<td>0.055***</td>
<td>0.052***</td>
</tr>
<tr>
<td></td>
<td>(0.0066)</td>
<td>(0.0061)</td>
</tr>
<tr>
<td>1(ENE)</td>
<td>−0.006</td>
<td>−0.015*</td>
</tr>
<tr>
<td></td>
<td>(0.0093)</td>
<td>(0.0084)</td>
</tr>
<tr>
<td>1(EE &amp; switcher)</td>
<td>−0.017**</td>
<td>−0.012</td>
</tr>
<tr>
<td></td>
<td>(0.0086)</td>
<td>(0.0085)</td>
</tr>
<tr>
<td>1(ENE &amp; switcher)</td>
<td>−0.059***</td>
<td>−0.015*</td>
</tr>
<tr>
<td></td>
<td>(0.0121)</td>
<td>(0.0084)</td>
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\[ P(\pi_{nu}^{EE} = \pi_{nu}^{ENE}) \]

<table>
<thead>
<tr>
<th>Controls for switchers</th>
<th>Occupation</th>
<th>Industry</th>
<th>Occupation</th>
<th>Industry</th>
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<td>No. observations</td>
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<td>318,767</td>
<td>375,648</td>
<td>375,648</td>
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\* p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table 4: ENE by unemployment duration, controls for occupation switchers, first differences

<table>
<thead>
<tr>
<th></th>
<th>≤ 9 months</th>
<th>≤ 8 months</th>
<th>≤ 7 months</th>
<th>≤ 6 months</th>
<th>≤ 5 months</th>
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<tbody>
<tr>
<td>UR</td>
<td>-0.408***</td>
<td>-0.407***</td>
<td>-0.406***</td>
<td>-0.406***</td>
<td>-0.405***</td>
</tr>
<tr>
<td></td>
<td>(0.0966)</td>
<td>(0.0966)</td>
<td>(0.0966)</td>
<td>(0.0966)</td>
<td>(0.0966)</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(EE))</td>
<td>-1.864***</td>
<td>-2.658***</td>
<td>-1.863***</td>
<td>-1.862***</td>
<td>-1.861***</td>
</tr>
<tr>
<td></td>
<td>(0.6802)</td>
<td>(0.9372)</td>
<td>(0.6802)</td>
<td>(0.9371)</td>
<td>(0.6802)</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(ENE))</td>
<td>-0.463</td>
<td>-0.127</td>
<td>-0.702</td>
<td>-0.900</td>
<td>-0.623</td>
</tr>
<tr>
<td></td>
<td>(0.7504)</td>
<td>(1.0871)</td>
<td>(0.7551)</td>
<td>(1.0955)</td>
<td>(0.7090)</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(LTU))</td>
<td>-1.439</td>
<td>0.251</td>
<td>-1.079</td>
<td>-0.601</td>
<td>-0.971</td>
</tr>
<tr>
<td></td>
<td>(1.2929)</td>
<td>(2.2796)</td>
<td>(1.2556)</td>
<td>(1.4762)</td>
<td>(1.3015)</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(EE &amp; \text{sw.}))</td>
<td>...</td>
<td>1.351</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>(1.3256)</td>
<td>...</td>
<td>(1.3256)</td>
<td>...</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(ENE &amp; \text{sw.}))</td>
<td>...</td>
<td>-0.585</td>
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<td>-1.262</td>
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<tr>
<td></td>
<td>...</td>
<td>(1.4710)</td>
<td>...</td>
<td>(1.4444)</td>
<td>...</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(LTU &amp; \text{sw.}))</td>
<td>...</td>
<td>-2.038</td>
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<td></td>
<td>...</td>
<td>(2.7108)</td>
<td>...</td>
<td>(2.8692)</td>
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Occ. controls

<table>
<thead>
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<th>No</th>
<th>Yes</th>
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<tbody>
<tr>
<td>P((\pi_{nu}^{EE} = \pi_{nu}^{ENE}))</td>
<td>0.162</td>
<td>0.077</td>
<td>0.248</td>
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<td>0.322</td>
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<td>0.213</td>
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<td>318,768</td>
<td>318,767</td>
<td>318,768</td>
<td>318,767</td>
</tr>
</tbody>
</table>

* p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table 5: $ENE$ by unemployment duration, controls for occupation switchers, fixed-effects

<table>
<thead>
<tr>
<th></th>
<th>$\leq 9$ months</th>
<th>$\leq 8$ months</th>
<th>$\leq 7$ months</th>
<th>$\leq 6$ months</th>
<th>$\leq 5$ months</th>
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</thead>
<tbody>
<tr>
<td>UR</td>
<td>$-0.144^{**}$</td>
<td>$-0.144^{**}$</td>
<td>$-0.144^{**}$</td>
<td>$-0.144^{**}$</td>
<td>$-0.144^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.0609)</td>
<td>(0.0610)</td>
<td>(0.0609)</td>
<td>(0.0610)</td>
<td>(0.0610)</td>
</tr>
<tr>
<td>UR $\cdot I(EE)$</td>
<td>$-1.981^{***}$</td>
<td>$-1.981^{***}$</td>
<td>$-1.980^{***}$</td>
<td>$-1.980^{***}$</td>
<td>$-1.981^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.5046)</td>
<td>(0.5046)</td>
<td>(0.5046)</td>
<td>(0.5046)</td>
<td>(0.5046)</td>
</tr>
<tr>
<td>UR $\cdot I(ENE)$</td>
<td>$-0.248$</td>
<td>0.830</td>
<td>$-0.335$</td>
<td>0.769</td>
<td>$-0.389$</td>
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<td>(0.5802)</td>
<td>(0.7980)</td>
<td>(0.5867)</td>
<td>(0.8044)</td>
<td>(0.6024)</td>
</tr>
<tr>
<td>UR $\cdot I(LTU)$</td>
<td>$-0.162$</td>
<td>$-0.965$</td>
<td>0.222</td>
<td>$-0.834$</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td>(1.3939)</td>
<td>(1.6313)</td>
<td>(1.3145)</td>
<td>(1.5370)</td>
<td>(1.2084)</td>
</tr>
<tr>
<td>UR $\cdot I(EE &amp; sw.)$</td>
<td>$-0.463$</td>
<td>0.963</td>
<td>$-2.068^{**}$</td>
<td>$-2.316^{**}$</td>
<td>$-2.571^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.7276)</td>
<td>(0.7276)</td>
<td>(0.9896)</td>
<td>(1.0018)</td>
<td>(1.0309)</td>
</tr>
<tr>
<td>UR $\cdot I(ENE &amp; sw.)$</td>
<td>$-1.999^{***}$</td>
<td>$-2.068^{**}$</td>
<td>$-2.316^{**}$</td>
<td>$-2.571^{**}$</td>
<td>$-2.133^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.9767)</td>
<td>(0.9896)</td>
<td>(1.0018)</td>
<td>(1.0309)</td>
<td>(1.0746)</td>
</tr>
<tr>
<td>UR $\cdot I(LTU &amp; sw.)$</td>
<td>$1.042$</td>
<td>1.324</td>
<td>2.185</td>
<td>2.477</td>
<td>0.239</td>
</tr>
<tr>
<td></td>
<td>(2.1212)</td>
<td>(1.9710)</td>
<td>(1.7984)</td>
<td>(1.6922)</td>
<td>(1.4996)</td>
</tr>
<tr>
<td>Occ. controls</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>0.162</td>
<td>0.077</td>
<td>0.248</td>
<td>0.067</td>
<td>0.322</td>
</tr>
<tr>
<td></td>
<td>(2.1212)</td>
<td>(1.9710)</td>
<td>(1.7984)</td>
<td>(1.6922)</td>
<td>(1.4996)</td>
</tr>
<tr>
<td>$P(\pi_{nu}^{EE} = \pi_{nu}^{ENE})$</td>
<td>0.162</td>
<td>0.077</td>
<td>0.248</td>
<td>0.067</td>
<td>0.322</td>
</tr>
<tr>
<td>No. observations</td>
<td>375,649</td>
<td>375,649</td>
<td>375,649</td>
<td>375,649</td>
<td>375,649</td>
</tr>
</tbody>
</table>

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table 6: Externally calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>$0.997 = 0.99^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>$0.008 = 0.025/3$</td>
<td></td>
</tr>
<tr>
<td>Production function parameter</td>
<td>$\zeta$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Technology autoregressive parameter</td>
<td>$\rho_z$</td>
<td>$0.983 = 0.95^{1/3}$</td>
<td></td>
</tr>
<tr>
<td>Technology standard deviation</td>
<td>$\sigma_z$</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>Elasticity of matches to searchers</td>
<td>$\sigma$</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Bargaining power parameter</td>
<td>$\eta$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Matching function constant</td>
<td>$\sigma_m$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Renegotiation frequency</td>
<td>$\lambda$</td>
<td>$11/12$ (4 quarters)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Jointly calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Inverse productivity premium</td>
<td>0.79</td>
<td>Average E-E wage increase (4.5%)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Prob. of good match</td>
<td>0.28</td>
<td>Average wage-improv. flow share ($\delta_{BG} = 0.53$)</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>Search cost elasticity</td>
<td>1.09</td>
<td>Cyclicality of wage-improv. flow share ($\eta_{BG} = -1.40$)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Hiring cost parameter</td>
<td>71.27</td>
<td>U-E probability (0.42)</td>
</tr>
<tr>
<td>$1 - \nu$</td>
<td>Separation probability</td>
<td>0.025</td>
<td>E-U probability (0.025)</td>
</tr>
<tr>
<td>$\varsigma_0$</td>
<td>Scale parameter of search cost</td>
<td>1.65</td>
<td>E-E probability (0.025)</td>
</tr>
<tr>
<td>$u_B$</td>
<td>Flow value of unemployment</td>
<td>2.59</td>
<td>Relative value, non-work (0.71)</td>
</tr>
</tbody>
</table>
Table 8: Aggregate statistics

<table>
<thead>
<tr>
<th></th>
<th>$y$</th>
<th>$w$</th>
<th>$n+b$</th>
<th>$u$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S. Economy, 1964:1-2013:02</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.48</td>
<td>0.64</td>
<td>5.74</td>
<td>6.38</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.88</td>
<td>0.87</td>
<td>0.94</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.79</td>
<td>-0.87</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Model Economy, $\lambda = 11/12$ (4 quarters)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.47</td>
<td>0.27</td>
<td>4.52</td>
<td>9.47</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.83</td>
<td>0.95</td>
<td>0.84</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>0.65</td>
<td>0.92</td>
<td>-0.92</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Model Economy, $\lambda = \infty$ (Flex wages)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative St. Dev.</td>
<td>1.00</td>
<td>0.83</td>
<td>0.08</td>
<td>1.35</td>
<td>3.68</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.81</td>
<td>0.81</td>
<td>0.89</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td>Correlation with $y$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.88</td>
<td>-0.88</td>
<td>1.00</td>
</tr>
</tbody>
</table>
### Table 9: Wage semi-elasticities: All new hires

<table>
<thead>
<tr>
<th>Semi-elasticities of wages w.r.t. unemployment</th>
<th>First differences</th>
<th>Fixed-effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIPP</td>
<td>Model, 4Q</td>
</tr>
<tr>
<td>UR</td>
<td>-0.46 (0.097)</td>
<td>-0.51</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{new}))</td>
<td>-1.14 (0.461)</td>
<td>-1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SIPP</td>
<td>Model, 4Q</td>
</tr>
<tr>
<td>UR</td>
<td>-0.147 (0.061)</td>
<td>-0.88</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{new}))</td>
<td>-1.65 (0.326)</td>
<td>-0.89</td>
</tr>
</tbody>
</table>

### Table 10: Wage semi-elasticities: EE vs. ENE

<table>
<thead>
<tr>
<th>Semi-elasticities of wages w.r.t. unemployment</th>
<th>First differences</th>
<th>Fixed-effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIPP</td>
<td>Model, 4Q</td>
</tr>
<tr>
<td>UR</td>
<td>-0.43 (0.097)</td>
<td>-0.52</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{EE}))</td>
<td>-1.88 (0.680)</td>
<td>-1.71</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{ENE}))</td>
<td>-0.44 (0.664)</td>
<td>-0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SIPP</td>
<td>Model, 4Q</td>
</tr>
<tr>
<td>UR</td>
<td>-0.145 (0.061)</td>
<td>-0.88</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{EE}))</td>
<td>-1.98 (0.504)</td>
<td>-1.14</td>
</tr>
<tr>
<td>UR · (\mathbb{I}(\text{ENE}))</td>
<td>-0.34 (0.539)</td>
<td>-0.35</td>
</tr>
</tbody>
</table>
Figure 1: New hires from employment and cyclical composition bias

The dashed lines refer to the average wage at either a good match, \( \hat{w}^G \), or a bad match, \( \hat{w}^B \). The solid lines refer to the wage in recessions and expansions at either a good match (\( w^G \) and \( \bar{w}^G \)) or a bad match (\( w^B \) and \( \bar{w}^B \)).
Figure 2: Impulse responses to productivity shock

Response of aggregate quantities to one percent shock to total factor productivity.
Figure 3: Labor market composition and job flows

Response of employment and job flows to one percent shock to total factor productivity.
Figure 4: Wage growth and components

Wage growth of job-changers in response to one percent shock to total factor productivity. Top panel shows response of wage growth $\bar{g}_{t}^{EE}$ as sum of a component reflecting the average change in match quality of workers making $EE$ job transitions, $\Delta \bar{g}_{t}^{EE}$, and a component reflecting average contract wage growth common to all workers, $\bar{g}_{t}^{w}$. Bottom panel shows contribution of bad-to-good and good-to-bad job flows, $\delta_{BG,t-1}$ and $\delta_{GB,t-1}$, towards changes in average match quality of job-changers, $\Delta \bar{g}_{t}^{EE}$.
Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires

Supplementary Appendix

Mark Gertler*, Christopher Huckfeldt† and Antonella Trigari‡

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*New York University and NBER
†Cornell University
‡Bocconi University, CEPR and IGIER
A  An approximate data-generating model

In this appendix, after describing the canonical Bils (1985) regression, we detail the approximate DGM introduced in Section 2 of the main text and use it to discuss inference of the canonical Bils regressions, where all new hires are pooled together, and our preferred regression specifications, where we distinguish between new hires from unemployment and employment. We also discuss inference with staggered contracting, and we detail how the DGM is consistent with the theoretical model of procyclical job upgrading that we develop in Section 3 of the main text. Throughout this appendix, we develop testable predictions that we employ in the main text of the paper that would allow us to reject the approximate model. As we discuss in the main text and mention here, however, the data are fully consistent with the approximate model.

A.1  Bils (1985) regressions

Bils (1985) estimates the following regression in first differences:

\[
\Delta \log w_{it} = \Delta x_{it} \cdot \pi_x + \pi_u \cdot \Delta u_t + \pi_n \cdot I\{N_{it} = 1\} + \pi_{nu} \cdot I\{N_{it} = 1\} \cdot \Delta u_t + e_{it},
\]

which regresses individual wages at time \(t\) on a cyclical indicator taken to be the unemployment rate and allows for a different cyclicality for continuing workers and new hires. The coefficient \(\pi_u\) gives the common wage semi-elasticity for new and continuing workers, while the coefficient \(\pi_{nu}\) measures the excess semi-elasticity for new hires. We note that in the original Bils specification, the indicator function is directly applied to first-differenced unemployment (as opposed to being first-differenced itself).

An alternative way to control for unobserved permanent heterogeneity is to estimate a regression in fixed effects. The fixed-effect version of (1) is

\[
\Delta^m \log w_{it} = \Delta^m x_{it}' \cdot \pi_x + \pi_u \cdot \Delta^m u_t + \pi_n \cdot I\{N_{it} = 1\} + \pi_{nu} \cdot I\{N_{it} = 1\} \cdot \Delta^m u_t + e_{it},
\]

where \(\Delta^m\) indicates mean-differences.

A.2  An approximate DGM with a cyclical composition effect

The DGM relates individual wages to individual factors, the state of the cycle (measured by unemployment) and to job match quality. There is no true excess wage flexibility for new hires. We then model match quality as procyclical for workers making job-to-job transitions (consistent with the structural model developed later in the paper). Accordingly, we can then show that because job-changers drive cyclicality among new hires, the Bils'
regressions will generate upward biased estimates of new hire wage flexibility. Finally, we

Under the assumption that match quality is on average acyclical for new hires coming from

Under the approximate DGM,

\[
\log w_{it} = \psi_0 + x_{it}' \psi_x + \psi_u \cdot u_t + \alpha_i + \alpha_{it} + \varepsilon_{it},
\]

where \(\psi_0\) is a constant, \(\psi_x\) describes the relation of wages to observable characteristics \(x_{it}\), 

\(\psi_u\) gives the common wage semi-elasticity for new and continuing workers, \(\alpha_i\) is a time-

\(\alpha_{it}\) is unobserved match quality for individual \(i\) at a given job 

We assume average match quality \(\bar{\alpha}_{it}\) evolves as follows: Let \(\mathbb{I}\{EE_{it} = 1\}\) be an indicator 

\[
\Delta \bar{\alpha}_{it} = \mathbb{I}\{EE_{it} = 1\} \cdot \left[\psi_{n}^{EE} + \psi_{nu}^{EE} \cdot \Delta u_t\right] + \mathbb{I}\{ENE_{it} = 1\} \cdot \psi_{n}^{ENE}.
\]

Equation (4) describes a process for average match quality in which workers in continuing 

A.3 Inference with a first-difference estimator

To see how the typical regression of the literature may yield misleading evidence of new 

in unobserved match quality $\Delta \alpha_{it}$ over new hires, and then combine with (4):

$$
\Delta \log w_{it} = \Delta x_{it}' \psi_x + \psi_u \cdot \Delta u_t \\
+ \mathbb{I} \{ EE_{it} = 1 \} \cdot \left[ \psi_n^{EE} + \psi_n^{EN} \cdot \Delta u_t \right] \\
+ \mathbb{I} \{ ENE_{it} = 1 \} \cdot \psi_n^{ENE} + \Delta e_{it}.
$$

(5)

We then note that we can rewrite the Bils equation (1) in first differences as

$$
\Delta \log w_{it} = \Delta x_{it}' \pi_x + \pi_u \cdot \Delta u_t \\
+ \mathbb{I} \{ EE_{it} = 1 \} \cdot \left[ \pi_n + \pi_{nu} \cdot \Delta u_t \right] \\
+ \mathbb{I} \{ ENE_{it} = 1 \} \cdot \left[ \pi_n + \pi_{nu} \cdot \Delta u_t \right] + e_{it},
$$

(6)

distinguishing between the two types of new hires. Under our DGM, the Bils regression is misspecified. In particular, it imposes that the wage semi-elasticities of job changers and new hires from unemployment are equal and given by $\pi_{nu}$. By contrast, the DGM implies that the elasticity for job changers is $EE_{nu} < 0$ and the elasticity for new hires from unemployment is zero. Accordingly, taking the Bils equation to the data will lead to an estimate $\hat{\pi}_{nu} < 0$, but this will be due to the composition bias captured by $\psi_{nu}^{EE}$. Indeed, as we show next, $\pi_{nu} \propto \psi_{nu}^{EE}$.

In particular, from equations (1), (5) and (6), we have

$$
\hat{\pi}_{nu} = \frac{\text{Cov}^{MX}(\Delta \log w_{it}, \mathbb{I} \{ N_{it} = 1 \} \cdot \Delta u_t)}{\text{Var}^{MX}(\mathbb{I} \{ N_{it} = 1 \} \cdot \Delta u_t)} \\
= \psi_{nu}^{EE} \cdot \frac{\text{Var}^{MX}(\mathbb{I} \{ EE_{it} = 1 \} \cdot \Delta u_t)}{\text{Var}^{MX}(\mathbb{I} \{ N_{it} = 1 \} \cdot \Delta u_t)} \leq 0,
$$

where $X_{it} = \{ \Delta x_{it}, \Delta u_t, I \{ N_{it} = 1 \} \}$, $M^X = I - X' (X' X)^{-1} X'$ is a linear operator that residualizes a vector with respect to $X$, and the superscript $M^X$ denotes that we are computing covariances and variances with respect to the residualized arguments. Note, the literature takes the coefficient $\pi_{nu}$ as a measure of the excess contractual flexibility of new hire wages; but under the approximate DGM, $\hat{\pi}_{nu}$ is estimated to be non-zero solely through its proportionality to $\psi_{nu}^{EE}$.

We can then test whether the data is consistent with the simple DGM by estimating equation (6) allowing for separate interaction terms for job changers versus new hires from
unemployment:

\[
\Delta \log w_{it} = \Delta x'_{it}\pi_x + \pi_u \cdot \Delta u_t \\
+ \mathbb{I}\{EE_{it}=1\} \cdot [\pi_{nu}^{EE} + \pi_{nu}^{EE} \cdot \Delta u_t] \\
+ \mathbb{I}\{ENE_{it}=1\} \cdot [\pi_{nu}^{ENE} + \pi_{nu}^{ENE} \cdot \Delta u_t] + \epsilon_{it},
\]

(7)

Under the null of our DGM,

\[
\pi_{nu}^{EE} = \psi_{nu}^{EE} \\
\pi_{nu}^{ENE} = 0
\]

implying that (i) the excess wage cyclicality for job changers reflects composition bias and (ii) there is no excess wage cyclicality for new hires from unemployment. Of course, the data alone does not tell us the source of excess wage cyclicality for job changers. However, we can test the null that there should be no excess cyclicality for new hires from unemployment, i.e., whether \(\pi_{nu}^{ENE} = 0\). Under our identifying assumptions \(\pi_{nu}^{ENE}\) provides a composition-free estimate of the excess wage flexibility of new hires (in contrast to \(\pi_{nu}^{EE}\), which reflects cyclical composition). When we estimate equation (7) in Section 2.3 of the main text, we recover coefficients consistent with the null of \(\pi_{nu}^{ENE} = 0\).

A.4 Inference with a fixed-effects estimator

The fixed-effects estimator shares the same crucial properties: we can apply it to a subset of our sample and obtain \(\pi_u\) and \(\pi_{nu}^{ENE}\) as consistent estimators for \(\psi_u\) and \(\psi_{nu}^{ENE}\).

Similar to how we proceed in Section A.3, we first consider a fixed-effects estimator applied to the approximate DGM. We first take mean differences of equation (3), integrate over changes in unobserved match quality \(\Delta m_{it}\) and obtain

\[
\Delta^m \log w_{it} = \psi_u \cdot \Delta^m u_t + \Delta^m \tilde{\alpha}_{it} + \Delta^m \tilde{\epsilon}_{it},
\]

(8)

where \(\Delta^m \tilde{\alpha}_{it}\) denotes average mean deviations in match quality. However, it requires more algebra to express (8) in a form that is readily comparable to the canonical regression equation, as we must generate expressions for mean deviations of unobserved match quality.

For ease of exposition, we assume in the following that individuals in the sample only make at most one job-transition; i.e., individuals will experience either one \(EE\) transition, one \(ENE\) transition, or will be continuing workers for the entire sample period. The length of the sample is denoted as \(T\).

We first consider the mean-deviation of a worker who makes either a \(EE\) or a \(ENE\)
transition at period \( \tau_i \). Match quality for such newly hired worker evolves as

\[
\alpha_{it} = \begin{cases} 
\alpha_{i,0} & \text{if } t < \tau_i \\
\alpha_{i,0} + \Delta \alpha_{\tau_i} & \text{if } t \geq \tau_i 
\end{cases}
\]

(9)

where \( \Delta \alpha_{\tau_i} \) is the change in match quality experienced by the worker at time \( \tau_i \). When we average over \( T \), we obtain the fixed-effect \( \alpha_{i}^{FE} \), where

\[
\alpha_{i}^{FE} = \alpha_{i,0} + \frac{T - \tau_i + 1}{T} \cdot \Delta \alpha_{\tau_i}.
\]

(10)

Then, the associated mean-deviations of the average match quality of an EE or ENE worker who moves to a new job at period \( \tau_i \) is given by

\[
\Delta^m \alpha_{it} = \begin{cases} 
- \left( \frac{T - \tau_i + 1}{T} \right) \cdot \Delta \alpha_{\tau_i} & \text{if } t < \tau_i \\
\left( \frac{\tau_i - 1}{T} \right) \cdot \Delta \alpha_{\tau_i} & \text{if } t \geq \tau_i 
\end{cases}
\]

(11)

Integrating over unobserved match quality by types of job transition, we obtain

\[
\Delta^m \bar{\alpha}_{it} = \begin{cases} 
- \left( \frac{T - \tau_i + 1}{T} \right) \cdot \Delta \bar{\alpha}_{\tau_i} & \text{if } t < \tau_i \\
\left( \frac{\tau_i - 1}{T} \right) \cdot \Delta \bar{\alpha}_{\tau_i} & \text{if } t \geq \tau_i 
\end{cases}
\]

(12)

where, analogously to equation (4), \( \Delta \bar{\alpha}_{\tau_i} \) is given by

\[
\Delta \bar{\alpha}_{\tau_i} = \mathbb{I} \{ \text{ever } EE_i = 1 \} \cdot [\psi_{n}^{EE} + \psi_{nu}^{EE} \cdot \Delta u_{\tau_i}] + \mathbb{I} \{ \text{ever } ENE_i = 1 \} \cdot \psi_{n}^{ENE},
\]

(13)

and where \( \mathbb{I} \{ \text{ever } EE_i = 1 \} \) is an indicator variable equal to one if the worker makes one EE transition over the sample period and zero otherwise, \( \mathbb{I} \{ \text{ever } ENE_i = 1 \} \) is an indicator variable equal to one if a worker makes an ENE transition over the sample period and zero otherwise, and \( \tau_i \) is the date of the job transition of a worker who makes either an EE or ENE transition.

Finally, we can rewrite (12) as

\[
\Delta^m \bar{\alpha}_{it} = \left[ \mathbb{I} \{ t < \tau_i \} \cdot \left( - \frac{T - \tau_i + 1}{T} \right) + \mathbb{I} \{ t \geq \tau_i \} \cdot \left( \frac{\tau_i - 1}{T} \right) \right] \Delta \bar{\alpha}_{\tau_i}.
\]

(14)

Equations (8), (13), and (14) describe the approximate DGM in fixed effects.

We next rewrite the analogue of our baseline regression equation for wage cyclicality (7)
in fixed effects,

\[
\Delta^m \log w_{it} = \Delta^m x_{it}' \pi_x + \pi_u \cdot \Delta^m u_t + \mathbb{I}\{EE_{it} = 1\} \cdot \left[ \pi_n^{EE} + \pi_n^{EE} \cdot \Delta^m u_t \right] + \mathbb{I}\{ENE_{it} = 1\} \cdot \left[ \pi_n^{ENE} + \pi_n^{ENE} \cdot \Delta^m u_t \right] + \epsilon_{it},
\]

(15)

While it was readily apparent that the preferred regression equation in first-differences would allow consistent estimates of the parameters of the DGM, it is perhaps less clear that this is true for our fixed effects estimator. Here, we show that \(\pi_n^{ENE}\) is a consistent estimator of \(\pi_n^{ENE}\), \(\pi_u\) is a consistent estimator for \(\pi_u\) when we estimate equation (15) over a subset of the full sample, but that \(\pi_n^{EE}\) is a biased estimator for \(\pi_n^{EE}\).

A.4.1 Consistent estimator for \(\pi_n^{ENE}\) in fixed-effects

Let \(X_{it} = \{\Delta^m x_{it}, \Delta^m u_t, \mathbb{I}\{EE_{it} = 1\}, \mathbb{I}\{EE_{it} = 1\} \cdot \Delta^m u_t, \mathbb{I}\{ENE_{it} = 1\}\}\) and let \(M^X\) denote the linear operator that residualizes a vector with respect to \(X\). Then, by the Frisch-Waugh theorem,

\[
\hat{\pi}_n^{ENE} = \frac{\text{Cov}^X(\Delta^m \log w_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}
\]

(16)

\[
= \frac{\text{Cov}^X(\Delta^m x_{it}' \psi_x + \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \bar{\varepsilon}_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}
\]

\[
= 0 + \frac{\text{Cov}^X(\Delta^m \bar{\alpha}_{it}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}
\]

\[
= \frac{\text{Cov}^X(\mathbb{I}\{\text{ever } ENE_{i} = 1\}, \mathbb{I}\{t < \tau_i\} \cdot \left(-\frac{T-\tau_i+1}{T} \right) \cdot \psi_n^{ENE}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}
\]

\[
+ \frac{\text{Cov}^X(\mathbb{I}\{\text{ever } ENE_{i} = 1\}, \mathbb{I}\{t \geq \tau_i\} \cdot \left(\frac{\tau_i-1}{T} \right) \cdot \psi_n^{ENE}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}{\text{Var}^X(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t)}
\]

\[
= 0
\]

where first equality follows from equation (15) and residualization, the second equality from equation (8), the third equality from residualization, the fourth equality from equations (13) and (14), and the final equality follows from the zero covariance of \(\tau_i\) and \(\mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\).\(^1\) The estimates we obtain in Section 2.3 of the main text are consistent with the null of \(\pi_n^{ENE} = 0\).

\(^1\) Note, \(\text{Cov}(\tau_i, \Delta^m u_t) = 0\) obtains from the random initial condition for \(u_t\).
A.4.2 Consistent estimator for $\psi_u$ in fixed-effects

First, note that we cannot obtain consistent estimates of $\psi_u$ from a full sample that includes workers who ever make an EE transition. As shown in equation (13), the mean deviations of wages for ever-EE workers includes a term that is proportional to the change in the unemployment rate of the period that they make a job-transition. As the fixed-effects regression does not include this as an explanatory variable, estimates of $\psi_u$ will suffer from omitted variable bias. We can, however, derive consistent estimates of $\psi_u$ if we consider a restricted sample that omits ever-EE workers. Let $X_{it} = \{\Delta^m x_{it}, \mathbb{I}\{ENE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\}$ and let $M_X$ denote the linear operator that residualizes a vector with respect to $X$.

Then,

$$\hat{\psi}_u = \frac{\text{Cov}^X (\Delta^m \log w_{it}, \Delta^m u_t)}{\text{Var}^X (\Delta^m u_t)}$$

$$= \frac{\text{Cov}^X (\Delta x_{it}' \psi_x + \psi_u \cdot \Delta^m u_t + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, \Delta^m u_t)}{\text{Var}^X (\Delta^m u_t)}$$

$$= \psi_u + \frac{\text{Cov}^X (\Delta^m \bar{\alpha}_{it}, \Delta^m u_t)}{\text{Var}^X (\Delta^m u_t)}$$

$$= \psi_u + \frac{\text{Cov}^X (\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t < \tau_i\} \cdot \left(\frac{T-\tau_i+1}{T}\right) \cdot \hat{\psi}_{n, ENE}^E, \Delta^m u_t)}{\text{Var}^X (\Delta^m u_t)}$$

$$+ \frac{\text{Cov}^X (\mathbb{I}\{\text{ever } ENE_i = 1\} \cdot \mathbb{I}\{t \geq \tau_i\} \cdot \left(\frac{\tau_i-1}{T}\right) \cdot \hat{\psi}_{n, ENE}^E, \Delta^m u_t)}{\text{Var}^X (\Delta^m u_t)}$$

$$= \psi_u$$

where first equality follows from equation (15) and residualization, the second equality from equation (8), the fourth equality from equations (13) and (14), and the final equality follows from the zero covariance of $\tau_i$ and $\Delta^m u_t$.

A.4.3 Biased estimator for $\psi_{nu}$ in fixed-effects

Let $X_{it} = \{\Delta^m x_{it}, \Delta^m u_t, \mathbb{I}\{EE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\}, \mathbb{I}\{ENE_{it} = 1\} \cdot \Delta^m u_t\}$ and let $M_X$ denote the linear operator that residualizes a vector with respect to $X$. Without loss of
generality, assume that $\psi_n^{EE} = 0$ and $\psi_n^{ENE} = 0$. Then,

$$
\hat{\psi}_{nu}^{EE} = \frac{\text{Cov}^X(\Delta^m \log w_{it}, I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}{\text{Var}^X(I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}
= \frac{\text{Cov}^X(\Delta x_{it}^{EE} + \Delta^m u_{it} + \Delta^m \bar{\alpha}_{it} + \Delta^m \varepsilon_{it}, I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}{\text{Var}^X(I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}
= \psi_n^{EE} \cdot \frac{\text{Cov}^X(\left(\frac{T - \tau_{i} + 1}{T}\right) \cdot I\{EE_{it} = 1\} \cdot \Delta u_{\tau_{i}}, I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}{\text{Var}^X(I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}
+ \frac{\text{Cov}^X(\{t > \tau_{i}\} \cdot \left(\frac{\tau_{i} - 1}{T}\right) \cdot I\{EE_{it} = 1\} \cdot \psi_{nu}^{EE} \cdot \Delta u_{\tau_{i}}, I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}{\text{Var}^X(I\{EE_{it} = 1\} \cdot \Delta^m u_{it})}
\neq \psi_n^{EE}
\]
relation for wage cyclicality which allows for separate interactions for job changers and new hires from unemployment.

\[
\Delta \log w_{it} = \Delta x'_{it} \pi_x + \pi_u \cdot (1 - \lambda) \sum_{\tau=0}^{\infty} \lambda^\tau \Delta u_{t-\tau}
\]
\[
+ \mathds{1} \{ EE_{it} = 1 \} \cdot \left[ \pi^n_{EE} + \pi^{EE}_{nu} \cdot \Delta u_t \right]
\]
\[
+ \mathds{1} \{ ENE_{it} = 1 \} \cdot \left[ \pi^n_{ENE} + \pi^{ENE}_{nu} \cdot \Delta u_t \right] + e_{it},
\]

Note the difference with (7) is that the cyclical indicator is now a distributed lag of current and past unemployment growth rates as opposed to just the current one. In the final section of this appendix, we show that all our results are robust to this alternative formulation; that is, the estimate of \( \pi^n_{nu} \) is still zero, implying no excess wage flexibility for new hires. The estimate of \( \pi^n_{nu} \) remains negative, but this reflects cyclical composition bias under our identifying assumptions.

A.6 Average match quality with constant wage premium and cyclical shares

We detail here how the DGM is consistent with the theoretical model of procyclical job upgrading that we develop in the paper. In particular, we show we can write the procyclical growth in average match quality for job changers in equation (4) as generated by a constant wage premium across types of jobs and cyclical changes in the probability of moving across job types, as predicted by our full equilibrium model.

Suppose that there are two types of match quality, good and bad. Bad matches are paid a fraction \( \phi \) of good matches. We observe whether a worker makes a direct job-to-job transition or one with an intervening spell of nonemployment, but nothing else about the job transition. The probability of making a bad-to-good (or good-to-bad) transition depends on the cycle for job-changers, but not for new hires from unemployment.

Then, conditional on a EE transition,

\[
\Delta \alpha_{it} = \begin{cases} 
- \log \phi & \text{w/ prob. } \delta_{BG,t} = \eta_{BG,0} + \eta_{BG,1} \cdot \Delta u_t \\
\log \phi & \text{w/ prob. } \delta_{GB,t} = \eta_{GB,0} + \eta_{GB,1} \cdot \Delta u_t \\
0 & \text{otherwise}
\end{cases}
\]

where \( \delta_{BG,t} \) indicates the fraction of job-changers who make a bad-to-good transition and where

\[
\delta_{BG,t} + \delta_{GB,t} + \delta_{BB,t} + \delta_{GG,t} = 1.
\]
Conditional on a ENE transition,

\[
\Delta \alpha_{it} = \begin{cases} 
- \log \phi & \text{w/ prob. } \delta_{BNG,t} = \eta_{BNG,0} \\
\log \phi & \text{w/ prob. } \delta_{GNB,t} = \eta_{GNB,0} \\
0 & \text{otherwise}
\end{cases}
\]

(23)

where \( \delta_{BNG,t} \) is the fraction of new hires from unemployment who make a bad-to-good transition (with an intervening spell of unemployment) and where

\[
\delta_{BNG,t} + \delta_{GNB,t} + \delta_{BNB,t} + \delta_{GNG,t} = 1.
\]

(24)

If we integrate over the probability of different types of match transitions, we obtain

\[
\Delta \bar{\alpha}_{it} = \mathbb{I}\{EE_{it} = 1\} \cdot (-\log \phi) \cdot (\eta_{BG,0} - \eta_{GB,0}) \\
+ \mathbb{I}\{EE_{it} = 1\} \cdot (-\log \phi) \cdot (\eta_{BG,u} - \eta_{GB,u}) \cdot \Delta u_t \\
+ \mathbb{I}\{ENE_{it} = 1\} \cdot (-\log \phi) \cdot (\eta_{BNG,0} - \eta_{GNB,0})
\]

(25)

This is nested in (4), where

\[
\psi_{EE}^n = (-\log \phi) \cdot (\eta_{BG,0} - \eta_{GB,0}) \\
\psi_{nu}^E = (-\log \phi) \cdot (\eta_{BG,u} - \eta_{GB,u}) \\
\psi_{n}^{ENE} = (-\log \phi) \cdot (\eta_{BNG,0} - \eta_{GNB,0})
\]

A.7 Robustness to alternative approximate DGMs

Here, we consider three deviations to our preferred regression specification that are motivated by three changes to the baseline approximate DGM. As in the text, we consider the canonical regression, where all new hires are treated the same, and our preferred specification, where new hires from employment and unemployment are permitted separate wage cyclicalities. Results are given in Tables A.1 and A.2. First, we consider the case in which the average change in match quality for workers making job-to-job transitions is proportional to the level unemployment rate rather than the change in the unemployment rate. Hence, whereas the common cyclical indicator is expressed in first differences and mean deviations, the interaction term is expressed in the level HP-filtered unemployment rate. Second, we consider the case of staggered contracting, where the relevant cyclical indicator is the distributed lag of unemployment, as in Section A.5, where we set \( \lambda = 11/12. The third case we consider is a combination of the first two. As seen in Table A.1, we obtain a new hire effect à la Bils (1985) across all three specifications. We confirm in Table A.2, however, that the effect is entirely driven by new hires from employment, and there is no
evidence for excess wage cyclicality for new hires from non-employment.
Table A.1: Alternative cyclical indicators: canonical regression

<table>
<thead>
<tr>
<th></th>
<th>First differences</th>
<th>Fixed-effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>UR</strong></td>
<td>−0.519***</td>
<td>−0.193***</td>
</tr>
<tr>
<td></td>
<td>(0.0991)</td>
<td>(0.0602)</td>
</tr>
<tr>
<td><strong>UR · I(new)</strong></td>
<td>−0.279**</td>
<td>−0.183***</td>
</tr>
<tr>
<td></td>
<td>(0.1225)</td>
<td>(0.0613)</td>
</tr>
<tr>
<td><strong>I(new)</strong></td>
<td>0.291**</td>
<td>0.172***</td>
</tr>
<tr>
<td></td>
<td>(0.1236)</td>
<td>(0.0617)</td>
</tr>
</tbody>
</table>

Change to cyclical indicators?

<table>
<thead>
<tr>
<th></th>
<th>Level Inter'n</th>
<th>Distr. lag</th>
<th>Level &amp; Distr. lag</th>
<th>Level Inter'n</th>
<th>Distr. lag</th>
<th>Level &amp; Distr. lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. observations</td>
<td>321,402</td>
<td>321,402</td>
<td>321,402</td>
<td>378,669</td>
<td>378,669</td>
<td>378,669</td>
</tr>
<tr>
<td>No. individuals</td>
<td>57,267</td>
<td>57,267</td>
<td>57,267</td>
<td>57,267</td>
<td>57,267</td>
<td>57,267</td>
</tr>
<tr>
<td>No. new hires</td>
<td>14,667</td>
<td>321,402</td>
<td>321,402</td>
<td>18,091</td>
<td>18,091</td>
<td>18,091</td>
</tr>
</tbody>
</table>

* * p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table A.2: Alternative cyclical indicators: preferred specification

### A.2A. First differences

<table>
<thead>
<tr>
<th></th>
<th>Level interactions</th>
<th>Distributed lag</th>
<th>Level interactions &amp; distributed lag</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UR</strong></td>
<td>-0.481*** (0.0993)</td>
<td>-0.486*** (0.0989)</td>
<td>-1.058*** (0.1380) -1.049*** (0.1380)</td>
</tr>
<tr>
<td><strong>UR \cdot I(EE)</strong></td>
<td>-0.333*** (0.1468)</td>
<td>-0.354** (0.1455)</td>
<td>-1.658** (0.6799) -1.459** (0.6215)</td>
</tr>
<tr>
<td><strong>UR \cdot I(ENE)</strong></td>
<td>-0.204 (0.2245)</td>
<td>-0.091 (0.2460)</td>
<td>-0.163 (0.6601) 0.7316</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unemp. spell for ENE</th>
<th>0+</th>
<th>1+</th>
<th>0+</th>
<th>1+</th>
<th>0+</th>
<th>1+</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(\pi_{nu}^{EE} = \pi_{nu}^{ENE}) )</td>
<td>0.630</td>
<td>0.359</td>
<td>0.111</td>
<td>0.207</td>
<td>0.567</td>
<td>0.307</td>
</tr>
</tbody>
</table>

### A.2B. Fixed effects

<table>
<thead>
<tr>
<th></th>
<th>Interaction of levels</th>
<th>Distributed lag</th>
<th>Interaction of levels &amp; distributed lag</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UR</strong></td>
<td>-0.169*** (0.0608)</td>
<td>-0.170*** (0.0608)</td>
<td>-0.957*** (0.0704) -0.957*** (0.0704)</td>
</tr>
<tr>
<td><strong>UR \cdot I(EE)</strong></td>
<td>-0.305*** (0.0824)</td>
<td>-0.288*** (0.0806)</td>
<td>-1.599*** (0.5047) -1.555*** (0.4731)</td>
</tr>
<tr>
<td><strong>UR \cdot I(ENE)</strong></td>
<td>0.081 (0.1059)</td>
<td>0.174 (0.1106)</td>
<td>0.056 (0.5376) 0.445 (0.5936)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unemp. spell for ENE</th>
<th>0+</th>
<th>1+</th>
<th>0+</th>
<th>1+</th>
<th>0+</th>
<th>1+</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(\pi_{nu}^{EE} = \pi_{nu}^{ENE}) )</td>
<td>0.004</td>
<td>0.001</td>
<td>0.022</td>
<td>0.008</td>
<td>0.003</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Robust standard errors in parenthesis. * \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
B Empirical appendix

B.1 More on robustness of empirical results

In this section, we discuss several issues related to the robustness of our empirical findings.

B.1.1 Non-employment versus unemployment

We interpret our findings for the wage cyclicality of workers from non-employment to be relevant to understanding the wage cyclicality of workers from unemployment. The SIPP offers monthly data on search activity, but many workers do not report active search for each month of a given spell. Hence, there is no straightforward criteria by which to classify a complete non-employment spell as one of “unemployment”, which would be necessary to refine our measure of ENE transitions to one of EUE transitions.

We have explored one possibility, however, which is to classify a new hire as a new hire from unemployment (EUE) if the worker reports searching for at least a single month of her non-employment spell. Roughly 72% of new hires from non-employment in our sample report searching for at least one month of non-employment. Of those workers, they search for around 86% of the total duration of their non-employment spell. From this measure, we can define three types of new hires: EE, EUE, and EOE (new hires from OLF). We estimate a variation of our baseline specification with three interaction terms for each type of new hire. The results are reported in Table B.1. Our findings are virtually unchanged. If anything, the point estimates suggest that the wages of EUE workers are less cyclical than the wages of ENE workers, but this difference is not statistically significant.

B.1.2 More findings on cyclical selection from non-employment

In Section 2.4 of the main paper, we discuss the robustness of our findings when isolate groups of ENE workers who are less likely to be subject to some form of cyclical composition bias. In Tables 4 and 5 of the main text, we isolate workers with shorter durations of non-employment, where we alternatively do not control or control for occupation switchers. Here, we do the same, but for with industry switchers. Results are given in Tables B.2 and B.3 and entirely consistent with those presented in Tables 4 and 5.

B.2 The Survey of Income and Program Participation

Here, we discuss issues related to the Survey of Income and Program Participation as they pertain to our analysis.
B.2.1 Overview

The SIPP is administered by the U.S. Census Bureau and is designed to track a nationally representative sample of U.S. households. The SIPP is organized by panel years, where each panel year introduces a new sample of households. From 1990 to 1993, the Census Bureau would introduce a new panel on an annual basis, where each panel is administered for a period of 32 to 40 months. Hence for certain years in the early 1990s, data is available from multiple panels, each consisting from around 15,000 to 24,000 households. Starting in 1996, the Census changed the structure of the survey to follow contiguous panels. Since the redesign, new panels have been introduced in 1996, 2001, 2004, and 2008. For each of these panels, the Census has followed a larger sample of households (e.g. 40,188 in 1996) over a longer period. There are two inter-panel periods for which we have no coverage: April 2000 to February 2001, and February 2008 to August 2008. We use the SIPP sample weights for all of our analysis. For the periods where there is panel overlap, we adjust the weights according to the procedure recommended by Census (e.g., see SIPP User’s Guide, 2001, pages 8-9).

Survey respondents are interviewed every four months on activity since the previous interview, a period referred to as a wave. However, some information (including for example, employment) is available at different frequencies within a wave. For example, the SIPP provides weekly measures of employment status, monthly measures of earnings, and job identifiers are constant for the entire period of the wave. As described in the main text, we combine monthly earnings records specific to each job to discern the pattern of job flows and sources of earnings over the wave.

The SIPP has several advantages relative to other commonly used panel data sources such as the PSID or the NLSY. Relative to the PSID, the SIPP follows a larger number of households, is nationally representative, and has more frequent observations. For the purposes of this paper, the PSID also suffers the disadvantage that it is difficult to identify wage earnings with a particular job in years where multiple jobs are held. Relative to the NLSY, the SIPP follows a larger number of households, but more importantly, multiple cohorts. Relative to both surveys, the SIPP suffers the disadvantage that it follows any particular individual for a shorter overall duration. But as mentioned before, the SIPP collects rich retrospective information that gets around problems of left-censoring: in particular, we observe start dates for jobs held during the first wave but started prior to the first interview (including the 1990 to 1993 panels). We then use our earnings-based measures of job transitions to determine the following sequence of jobs spells for the rest of the sample.\(^2\)

\(^2\) For each wave, the survey contains fields for up to two jobs. The survey maintains longitudinally consistent job IDs for each individual and tracks certain job-specific characteristics at a monthly frequency,
One frequently noted problem with the SIPP is the so-called “seam effect,” described by the Census Bureau as follows:

*This effect results from the respondent tendency to project current circumstances back onto each of the 4 prior months that constitute the SIPP reference period. When that happens, any changes in respondent circumstances that occurred during that 4-month period appear to have happened in the first month of the reference period. A disproportionate number of changes appear to occur between the fourth month of one wave and the first month of the following wave, which is the “seam” between the two wave—hence the terminology.* (SIPP User’s Guide 2001, pages 1-7)

For our purposes, such an effect could potentially generate problems, for example, with aligning job-changes with changes in the unemployment rate. This sort of measurement error has the potential to bias coefficient estimates towards zero. Having experimented with different measures of the unemployment rate that should not lead to such bias, e.g. average unemployment rate over the wave, we believe the quantitative impact of such measurement error is negligible. While a downward bias would make it harder to reject the null hypothesis of no excess cyclicality of ENE wages, we recover both small negative and positive coefficient estimates for ENE workers, so we are not suspicious that such a bias could be driving our results. Moreover, given there is no ex-ante reason to believe that such measurement error should differentially effect ENE versus EE workers, the bias would not make it easier to reject that excess wage cyclicality of EE and ENE workers are the same.

A second problem from the seam effect is discussed in Gottschalk (2005), which is the presence of measurement error for the amount of earnings in non-interview months of a wave. Thus, we follow Gottschalk (2005) and others in only using the fourth month wage in our analysis.

Finally, we note that some sort of “seam bias” is present for any retrospective data. In this sense, the SIPP provides an almost-ideal survey instrument, as it offers a representative sample for multiple cohorts with a rich set of variables, but also allows the researcher to follow workers over several years with relatively high frequency interviews, limiting the duration over which the seam bias can influence survey responses.

---

including earnings. We follow the procedure detailed by Stinson (2003) to correct inconsistent job identification variables for the 1990 to 1993 panels. We use monthly earnings data within waves to determine at which job the individual is working and for what months the individual is working at each potential job. From these data, we determine within a wave whether an individual made a job transition; and whether the job transition was characterized by an intervening period of non-employment.
B.2.2 Variables and sample selection

Following Bils (1985), we only consider males between the ages of 20 and 60. We drop observations for individuals who are disabled, self-employed, serving in the armed forces, or enrolled in school full-time. We use the monthly employment status recode variable to identify and drop observations where an individual reports not working for the entire month. We drop observations where an individual works less than 10 or greater than 100 hours a week. We also drop observations where the wage is top-coded or below the minimum wage. All observations are associated with a job-specific wage. As such, we drop observations where a worker is working at multiple jobs. Such observation may either reflect a job-to-job transition or multiple job-holding; but in either case, it is difficult to determine which observation should be included in the estimation.

We use hourly wages as our measure of earnings. In some instances, SIPP includes hourly wages and total monthly earnings. In cases where the hourly wage is directly available, we use that as our measure of wages. In cases where the hourly wage is not available, we construct a measure of implied hourly wages from monthly earnings divided by the product of weeks worked and hours worked per week. For the 1990 to 1993 waves, all of these variables are job-specific. Starting with the 1996 panel, the measure for weeks worked is no longer job-specific. We instead construct a measure of weeks worked from weeks with job minus weeks absent from work. Note that the implied hourly wage measure is subject to greater measurement error at the beginning of a job, when an individual does not necessarily spend a full month working at a job. In such cases, we use the second observation as the “new hire” wage. There is no considerable change for the fixed effects regression if we do not apply this correction, but many of the coefficients are not statistically significant for the first-differences regression, including for new hires from employment. We deflate wages using a four-month average of the PCE. Covariates include four indicators for educational attainment, separate indicators for union coverage and marital status, a quadratic in job tenure, and a time trend. We use combined weights across panels, applying the method recommended by the SIPP User’s Guide (2001). We use monthly prime-aged male unemployment.

B.2.3 Identifying recalls

The SIPP maintains job-specific longitudinally consistent employment information over waves for which an individual reports non-zero employment. For such case, the SIPP maintains the same job identifier for a given job, allowing users to distinguish new jobs from “recalls” (to adopt the terminology of Fujita and Moscarini (2017). Table B.4 gives an example employment history of an individual who works at a job, spends four months
in non-employment, but returns to the same job. The SIPP correctly records that the individual returned to the job that she left.

But starting in 1996, the SIPP resets employment records for individuals who are without employment for an entire wave. If individuals return to a previously held job after spending an entire wave in non-employment, the SIPP will incorrectly record the individual as starting a new job. Hence, a single job can be given multiple job identifiers. Table B.5 gives a sample employment history of an individual who works at a job, spends an entire wave out of work, and then returns to the same job. As in the previous example, the individual spends four months not working; but because those four months happen to fall over the entirety of a wave, the job is given a new identifier when the individual returns to work. For such individuals, we could mistakenly label a recall to be a transition across separate jobs.

We exploit an additional source of information recorded by the SIPP to identify potential recalls. Every time that a distinct job identifier is associated with an individual, the survey also adds a start date. This is indicated by the box around “start date” in the third row of Table B.5. When we observe a start date that falls before the date that the SIPP purges job identifiers, we have a good indication that the “new job” is in fact a recall.

To what extent do respondents report the date that they began the job, inclusive of employment gaps, versus the date that they last began a contiguous employment spell? We note that the survey question recording start dates is explicitly designed to identify the start date to be the former of the two, as it is designed to distinguish jobs that began within the wave from jobs that began before the wave.

For example, in the 1996 panel, respondents are asked “Did [FIRST AND LAST NAME] begin [HIS HER] employment with [NAME OF EMPLOYER] at some time between [MONTH1] 1st and today?” (variable STRTJB). If individuals respond in the affirmative, they are asked about the month and day within the wave that the job began (STRTREFP). Otherwise, they are asked to give their “BEST estimate” of the year, month, and date that the job began (variables STRTMONJB, STRTJYR, STRTJMTH).

To identify potential recalls, we apply the following criterion: for individuals with an incomplete employment record – e.g. respondents who have spent a complete wave in non-employment – we consider any job with a start date prior to the period of non-employment (the date at which the SIPP purges internal employment records) as a potential recall, and we do not count the individual as a new hire.

We illustrate our criterion in Tables B.6 and B.7. In Table B.6, we observe an individual work a wave at Job A, spend an entire wave in non-employment, and then start work at

---

3 See the 1996 Panel Wave 02 Questionnaire at http://www.census.gov/content/dam/Census/programs-surveys/sipp/questionnaires/1996/SIPP%201996%20Panel%20Wave%202-%20Core%20Questionnaire.pdf
Job B in wave 3. The start date of Job B is before the “gap date”, and hence, it is more likely that Job B is the same as Job A. Hence, we do not consider the individual as a new hire at Job B. In Table B.7, we similarly observe an individual work at Job A, spend a wave in non-employment, and then work at Job B; however, the start date for job B in this instance is after the gap date, and hence, we consider the worker to be a new hire in wave 3.

We apply the gap date criterion with two small additions: first, for a subset of job dissolutions, workers report the cause of the dissolution. If the worker reports that he left the pre-gap job to take another job, we do preclude the possibility that the post-gap job is a recall to the first job. Second, if the start date at a post gap job is missing or statistically imputed, we identify the job as a potential recall and do not count the worker as a new hire.
Table B.1: EE, EUE, & ENE

<table>
<thead>
<tr>
<th></th>
<th>FD</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>UR</td>
<td>-0.423***</td>
<td>-0.419***</td>
</tr>
<tr>
<td></td>
<td>(0.0966)</td>
<td>(0.0966)</td>
</tr>
<tr>
<td>UR \cdot I(EE)</td>
<td>-1.876***</td>
<td>-1.673***</td>
</tr>
<tr>
<td></td>
<td>(0.6800)</td>
<td>(0.6224)</td>
</tr>
<tr>
<td>UR \cdot I(EUE)</td>
<td>-0.319</td>
<td>-0.480</td>
</tr>
<tr>
<td></td>
<td>(0.7312)</td>
<td>(0.7954)</td>
</tr>
<tr>
<td>UR \cdot I(EOE)</td>
<td>-0.949</td>
<td>-0.800</td>
</tr>
<tr>
<td></td>
<td>(1.6690)</td>
<td>(2.0145)</td>
</tr>
<tr>
<td>I(EE)</td>
<td>0.045***</td>
<td>0.038***</td>
</tr>
<tr>
<td></td>
<td>(0.0048)</td>
<td>(0.0046)</td>
</tr>
<tr>
<td>I(EUE)</td>
<td>-0.057***</td>
<td>-0.075***</td>
</tr>
<tr>
<td></td>
<td>(0.0071)</td>
<td>(0.0081)</td>
</tr>
<tr>
<td>I(EOE)</td>
<td>-0.010</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>(0.0152)</td>
<td>(0.0176)</td>
</tr>
</tbody>
</table>

\[ P(\pi_{nu}^{EE} = \pi_{nu}^{EUE}) \]

|                  | 0.115 | 0.233 | 0.010 | 0.004 |
|                  | 0+    | 1+    | 0+    | 1+    |
| No. observations | 318,768 | 318,768 | 375,649 | 375,649 |

* \( p < 0.10 \), ** \( p < 0.05 \), *** \( p < 0.01 \)

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table B.2: ENE by unemployment duration, controls for industry switchers, first differences

<table>
<thead>
<tr>
<th>UR</th>
<th>≤ 9 months</th>
<th>≤ 8 months</th>
<th>≤ 7 months</th>
<th>≤ 6 months</th>
<th>≤ 5 months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−0.408*** (0.0966)</td>
<td>−0.407*** (0.0966)</td>
<td>−0.406*** (0.0966)</td>
<td>−0.405*** (0.0966)</td>
<td>−0.406*** (0.0966)</td>
</tr>
<tr>
<td>UR · I(EE)</td>
<td>−1.864*** (0.6802)</td>
<td>−1.863*** (0.6802)</td>
<td>−1.862*** (0.6802)</td>
<td>−1.861*** (0.6802)</td>
<td>−1.863*** (0.6802)</td>
</tr>
<tr>
<td></td>
<td>−0.463 (0.7504)</td>
<td>0.287 (1.0417)</td>
<td>−0.702 (0.7551)</td>
<td>0.501 (0.7090)</td>
<td>−0.623 (0.7386)</td>
</tr>
<tr>
<td>UR · I(ENE)</td>
<td>−1.439 (1.2929)</td>
<td>0.448 (1.6967)</td>
<td>−1.079 (1.2556)</td>
<td>0.218 (1.6946)</td>
<td>−0.601 (1.4762)</td>
</tr>
<tr>
<td></td>
<td>−0.601 (1.4762)</td>
<td>−1.085 (2.6394)</td>
<td>−0.971 (1.3015)</td>
<td>0.027 (2.1072)</td>
<td>−0.710 (1.1779)</td>
</tr>
<tr>
<td>UR · I(LTU)</td>
<td>−1.371 (1.4649)</td>
<td>−1.720 (1.4776)</td>
<td>−2.478* (1.3813)</td>
<td>−1.700 (1.4442)</td>
<td>−2.667* (1.4642)</td>
</tr>
<tr>
<td></td>
<td>−2.616 (2.3023)</td>
<td>−1.807 (2.2727)</td>
<td>0.535 (3.1624)</td>
<td>−1.505 (2.6370)</td>
<td>0.240 (2.4647)</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Ind. controls</td>
<td>0.162</td>
<td>0.069</td>
<td>0.248</td>
<td>0.073</td>
<td>0.322</td>
</tr>
<tr>
<td>P(π_{nu}^{EE} = π_{nu}^{ENE})</td>
<td>375,649</td>
<td>375,648</td>
<td>375,649</td>
<td>375,648</td>
<td>375,649</td>
</tr>
</tbody>
</table>

* p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table B.3: ENE by unemployment duration, controls for industry switchers, fixed-effects

<table>
<thead>
<tr>
<th></th>
<th>≤ 9 months</th>
<th>≤ 8 months</th>
<th>≤ 7 months</th>
<th>≤ 6 months</th>
<th>≤ 5 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>UR</td>
<td>−0.144**</td>
<td>−0.144**</td>
<td>−0.144**</td>
<td>−0.144**</td>
<td>−0.144**</td>
</tr>
<tr>
<td></td>
<td>(0.0609)</td>
<td>(0.0609)</td>
<td>(0.0609)</td>
<td>(0.0609)</td>
<td>(0.0609)</td>
</tr>
<tr>
<td>UR · 1(EE)</td>
<td>−1.981***</td>
<td>−2.828***</td>
<td>−1.980***</td>
<td>−2.828***</td>
<td>−1.980***</td>
</tr>
<tr>
<td></td>
<td>(0.5046)</td>
<td>(0.6068)</td>
<td>(0.5046)</td>
<td>(0.6068)</td>
<td>(0.5046)</td>
</tr>
<tr>
<td>UR · 1(ENE)</td>
<td>−0.248</td>
<td>1.115</td>
<td>−0.335</td>
<td>1.052</td>
<td>−0.389</td>
</tr>
<tr>
<td></td>
<td>(0.5802)</td>
<td>(0.7253)</td>
<td>(0.5867)</td>
<td>(0.7312)</td>
<td>(0.6024)</td>
</tr>
<tr>
<td>UR · 1(LTU)</td>
<td>−0.162</td>
<td>−1.466</td>
<td>0.222</td>
<td>−0.905</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td>(1.3939)</td>
<td>(1.5197)</td>
<td>(1.3145)</td>
<td>(1.4355)</td>
<td>(1.2084)</td>
</tr>
<tr>
<td>UR · 1(EE &amp; sw.)</td>
<td>—</td>
<td>1.615**</td>
<td>—</td>
<td>1.615**</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(0.7477)</td>
<td>—</td>
<td>(0.7477)</td>
<td>—</td>
</tr>
<tr>
<td>UR · 1(ENE &amp; sw.)</td>
<td>—</td>
<td>−2.830***</td>
<td>—</td>
<td>−3.150***</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(0.9802)</td>
<td>—</td>
<td>(1.0092)</td>
<td>—</td>
</tr>
<tr>
<td>UR · 1(LTU &amp; sw.)</td>
<td>—</td>
<td>1.777</td>
<td>—</td>
<td>1.487</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(2.0951)</td>
<td>—</td>
<td>(1.9504)</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ind. controls</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(π_{EE}^{nu} = π_{ENE}^{nu})</td>
<td>0.162</td>
<td>0.069</td>
<td>0.248</td>
<td>0.073</td>
<td>0.322</td>
<td>0.040</td>
<td>0.213</td>
<td>0.064</td>
<td>0.242</td>
<td>0.023</td>
</tr>
</tbody>
</table>

* p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable: log hourly real wage. Controls for education, union coverage, marital status, a quadratic in tenure, and a linear time trend. Robust standard errors in parenthesis, clustered by individual.
Table B.4: Two separate employment spells, one job, correct IDs. Job ID preserved across contiguous employment spells because individual reports employment for each wave.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>A</td>
<td>09/95</td>
<td>M1</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>A</td>
<td>09/95</td>
<td>M2-M4</td>
<td>A</td>
</tr>
</tbody>
</table>

Table B.5: Two separate employment spells, one job, incorrect IDs. Job ID information is lost when individual spends an entire wave without employment. At wave 3, the job is incorrectly coded as being a new job and the start date is asked again.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
</tr>
</tbody>
</table>
Table B.6: Two separate employment spells, “gap date” falls after reported job start date for job “B”. Rule out wave 3 job as “new hire”.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
<th>Gap date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
<td>05/96</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
<td>05/96</td>
</tr>
</tbody>
</table>

Table B.7: Two separate employment spells, “gap date” is prior to reported job start date for job “B”. Count wave 3 job as “new hire”.

<table>
<thead>
<tr>
<th>Wave</th>
<th>Time Period</th>
<th>Recorded Job ID</th>
<th>Recorded Start Date</th>
<th>Employment within wave</th>
<th>Actual Job ID</th>
<th>Gap date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01/96-04/96</td>
<td>A</td>
<td>09/95</td>
<td>M1-M4</td>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>05/96-08/96</td>
<td>–</td>
<td>–</td>
<td>none</td>
<td>–</td>
<td>05/96</td>
</tr>
<tr>
<td>3</td>
<td>09/96-12/96</td>
<td>B</td>
<td>08/96</td>
<td>M1-M4</td>
<td>A</td>
<td>05/96</td>
</tr>
</tbody>
</table>
C Model appendix

In this appendix we first close the model by describing firms’ capital renting decision and the households’ consumption/saving decision (which in turn determine labor productivity and the discount factor), and we define the recursive equilibrium. We then proceed to derive the log-linear equations that describe the first-order dynamics of the model. Most of the derivations are standard or are similar to those in Gertler and Trigari (GT, 2009). We thus focus on non-standard or new derivations. In the second section, we discuss first order-approximations related to the three key decisions in the model: hiring, search intensity and bargaining. In the third section, we derive log-linear expressions for the wage growth of job changers and the shares of job-to-job flows. In the fourth section, we describe the steady state of the labor market and in the fifth section we show how our calibration strategy permits to pin down the key labor market parameters associated with the composition effect.

In the sixth section, we derive the log-linear expression for the measured user cost of labor that we discuss in Section 6 of the main text. In the seventh section, we consider lateral job match movements: We show that up to a first order the gains from lateral movements are negligible, which justifies our ruling out this possibility in analyzing the worker’s decision problem. In the last section, we tie up a final loose end and define the operator mapping the distribution function from period $t$ to period $t + 1$.

C.1 Closing the model

C.1.1 Firms: capital renting and labor productivity

Firms produce output $y_t$ using capital and labor according to a Cobb-Douglas production technology:

$$y_t = z_t k_t^\zeta l_t^{1-\zeta},$$

(26)

where $k_t$ is capital, $l_t$ labor in efficiency units and $z_t$ is total factor productivity. Capital is perfectly mobile. Firms rent capital on a period by period basis. As described in the main text, firms add labor through a search and matching process.

The firm’s decision problem is to choose capital $k_t$ and the hiring rate $\zeta_t$ to maximize the discounted stream of profits net recruiting costs, subject to the equations that govern the laws of motion for labor in efficiency units $l_t$ and the quality mix of labor $\gamma_t$, and given the expected paths of rental rates $r_t$ and wages $w_t$. We express the value of each firm $F_t(l_t, \gamma_t, w_t) \equiv F_t$ as

$$F_t = \max_{k_t, \zeta_t} \left\{ z_t k_t^\zeta l_t^{1-\zeta} - \frac{K}{2} \zeta_t^2 l_t - w_t l_t - r_t k_t + E_t \{ \Lambda_{t,t+1} F_{t+1} \} \right\},$$
subject to dynamic equations for \( l_t \) and \( \gamma_t \), and given the values of the firm level states \((l_t, \gamma_t, w_t)\) and the aggregate state vector.

Given constant returns and perfectly mobile capital, the firm’s value \( F_t \) is homogeneous in \( l_t \). The net effect is that each firm’s choice of the capital/labor ratio and the hiring rate is independent of its size. Let \( J_t \) be the firm value per efficiency unit of labor and let \( \tilde{k}_t \equiv k_t/l_t \) be its capital labor ratio. Then

\[
F_t = J_t \cdot l_t, \tag{27}
\]

with \( J_t \equiv J_t(\gamma_t, w_t) \) given by

\[
J_t = \max_{\tilde{k}_t} \{ z_t \tilde{k}_t^\zeta - \frac{\kappa}{2} \tilde{k}_t^2 - w_t - r_t \tilde{k}_t + (\rho_t + \zeta_t)E_t \{ A_{t,t+1}J_{t+1} \} \}, \tag{28}
\]

subject to the law of motion for \( \gamma_t \).

The first order condition for capital renting is

\[
r_t = \zeta \tilde{k}_t^{\zeta - 1}. \tag{29}\]

Given Cobb-Douglas production technology and perfect mobility of capital, \( \tilde{k}_t \) does not vary across firms.

Substituting yields the expression for \( J_t \) that appears in the main text, given by

\[
J_t = \max_{\zeta_t} \{ a_t - \frac{\kappa}{2} \zeta_t^2 - w_t + (\rho_t + \zeta_t)E_t \{ A_{t,t+1}J_{t+1} \} \}, \tag{30}
\]

where \( a_t \) denotes the current marginal product of labor (i.e., \( a_t = (1 - \zeta)/l_t \)), which is independent of the firm.

### C.1.2 Households: consumption and saving

We adopt the representative family construct, following Merz (1995) and Andolfatto (1996), allowing for perfect consumption insurance. There is a measure of families on the unit interval, each with a measure one of workers. Before making allocating resources to per-capita consumption and savings, the family pools all wage and unemployment income. Additionally, the family owns diversified stakes in firms that pay out profits. The household can then assign consumption \( \bar{c}_t \) to members and save in the form of capital \( k_t \), which is rented to firms at rate \( r_t \) and depreciates at the rate \( \delta \).

Let \( \Omega_t \) be the value of the representative household. Then,

\[
\Omega_t = \max_{\bar{c}_t, k_{t+1}} \{ \log(\bar{c}_t) + \beta E_t \Omega_{t+1} \} \tag{31}
\]
subject to
\[
\bar{c}_t + \bar{k}_{t+1} + \frac{\xi_0}{1 + \eta_c} \left\{ \nu_{\bar{n}n}^{1+\eta_c} \bar{n}_t + \nu_{\bar{k}bt}^{1+\eta_c} \bar{b}_t \right\} \\
= \bar{w}_t \bar{n}_t + \phi \bar{w}_t \bar{b}_t + (1 - \bar{n}_t - \bar{b}_t) u_B + (1 - \delta + r_t) \bar{k}_t + T_t + \Pi_t, \tag{32}
\]
and
\[
\bar{n}_{t+1} = \bar{\rho}_n^n \bar{n}_t + \xi_j \bar{s}_t \\
\bar{b}_{t+1} = \bar{\rho}_b^b \bar{b}_t + \xi_j^m \bar{s}_t 	ag{33}
\]
where \(\Pi_t\) are the profits from the household’s ownership holdings in firms and \(T_t\) are lump sum transfers from the government.\(^4\)

The first-order condition from the household’s savings problem gives
\[
1 = (1 - \delta + r_t) E_t \{ \Lambda_{t,t+1} \} 	ag{35}
\]
where \(\Lambda_{t,t+1} \equiv \beta \bar{c}_t / \bar{c}_{t+1}\).

### C.1.3 Resource constraint, government policy, and equilibrium

The resource constraint states that the total resource allocation towards consumption, investment, vacancy posting costs, and search costs is equal to aggregate output:
\[
\bar{y}_t = \bar{c}_t + \bar{k}_{t+1} - (1 - \delta) \bar{k}_t \\
+ \frac{\kappa}{2} \int \bar{x}_t^2 \bar{l}_t d\bar{i} + \frac{\xi_0}{1 + \eta_c} \left\{ \nu_{\bar{n}n}^{1+\eta_c} \bar{n}_t + \nu_{\bar{k}bt}^{1+\eta_c} \bar{b}_t \right\} 	ag{36}
\]

The government funds unemployment benefits through lump-sum transfers:
\[
T_t + (1 - \bar{n}_t - \bar{b}_t) u_B = 0. \tag{38}
\]

A recursive equilibrium is a solution for (i) a set of functions \(\{ J_t, V^n_t, V^b_t, U_t \} \); (ii) the contract wage \(w^*_t\); (iii) the hiring rate \(\bar{z}_t\); (iv) the subsequent period’s wage rate \(w_{t+1}\); (v)\(^4\) Chodorow-Reich and Karabarbounis (2016) show the introduction of utility from leisure can greatly increase the difficulty of generating sufficient unemployment volatility when the model is calibrated to match the estimated cyclicality of the opportunity cost of employment. For simplicity we do not include utility from leisure, but in ongoing work we show that our model with staggered wage contracting is robust to this critique. Further, we model the cost of search as pecuniary rather than utility. Having a utility cost of search may dampen the procyclicality of search intensity because recessions are times when workers value additional consumption relative to leisure. We leave the quantitative exploration of this mechanism for future research.

\(^4\) Chodorow-Reich and Karabarbounis (2016) show the introduction of utility from leisure can greatly increase the difficulty of generating sufficient unemployment volatility when the model is calibrated to match the estimated cyclicality of the opportunity cost of employment. For simplicity we do not include utility from leisure, but in ongoing work we show that our model with staggered wage contracting is robust to this critique. Further, we model the cost of search as pecuniary rather than utility. Having a utility cost of search may dampen the procyclicality of search intensity because recessions are times when workers value additional consumption relative to leisure. We leave the quantitative exploration of this mechanism for future research.
the search intensity of a worker in a bad match \( \varsigma_{bt} \); (vi) the rental rate on capital \( r_t \); (vii) the average wage, the average contract wage, the average search intensity of workers in bad matches and the average hiring rate, \( \bar{\tilde{w}}_t, \tilde{w}^*_t, \varsigma_{bt} \) and \( \tilde{\varsigma}_t \); (viii) the capital labor ratio \( \tilde{k}_t \); (ix) the average consumption and capital, \( \bar{c}_t \) and \( \bar{k}_{t+1} \); (x) the average employment in good and bad matches, \( \bar{n}_t \) and \( \bar{b}_t \); (xi) the density function of composition and wages across workers \( dG_t(\gamma, w) \); and (xii) a transition function \( Q_{t,t+1} \). The solution is such that (i) \( w_t^* \) satisfies the Nash bargaining condition; (ii) \( \tilde{w}_t \) satisfies the hiring condition; (iii) \( w_{t+1} \) is given by the Calvo process for wages; (iv) \( \varsigma_{bt} \) satisfies the first-order condition for search intensity of workers in bad matches; (v) \( r_t \) satisfies the first-order condition for capital renting; (vi) \( \tilde{w}_t = \int_{w,\gamma} wdG_t(\gamma, w) \), \( \tilde{w}^*_t = \int_{w,\gamma} w^*_t(\gamma) dG_t(\gamma, w) \), \( \varsigma_{bt} = \int_{w,\gamma} \varsigma_{bt}(\gamma, w) dG_t(\gamma, w) \) and \( \tilde{\varsigma}_t = \int_{w,\gamma} \tilde{\varsigma}_t(\gamma, w) dG_t(\gamma, w) \); (vii) the rental market for capital clears, \( \tilde{k}_t = \tilde{k}_t / (\bar{n}_t + \phi \bar{b}_t) \); (viii) \( \tilde{c}_t \) and \( \tilde{k}_{t+1} \) solve the household problem; (ix) \( \bar{n}_t \) and \( \bar{b}_t \) evolve according to (33) and (34); (x) the evolution of \( G_t \) is consistent with \( Q_{t,t+1} \); (xi) \( Q_{t,t+1} \) is defined in section C.7 of the appendix.

C.2 Hiring, search intensity and staggered Nash bargaining

Relative to GT, where the only firm-specific state variable was wages, here we must also keep track of composition. As might be expected, there is a non-trivial interplay between composition and wages at the firm level. Composition is inherited from the previous period and influences the wage through the Nash wage bargain; the wage influences next-period composition through hiring and search intensity. We introduce the restriction that good and bad matches have the same steady state job survival rates, which lends considerable analytic and computational tractability to the analysis.

We first state a set of results that will simplify the derivation of the log-linear equations. These include a set of steady state results and approximations of period-ahead firm and worker surpluses at renegotiating firms. We establish how these properties are used to show that the "composition effect" in hiring – wherein firms vary the hiring rate to vary next-period composition – is zero up to a first order. We also briefly discuss the absence of a composition effect in search intensity. We then go over the relevant equations for determining the Nash contract wage: the worker and firm surpluses and the Nash first order condition. We then derive recursive log-linear expressions for the average firm and worker surpluses making use of the surplus approximations previously stated. In doing that, we also derive expressions for the average hiring rate, search intensity, and retention rate at a renegotiating firm. Finally, we prove the steady-state results and the surplus approximations that we invoke for deriving recursive log-linear expressions for the average worker and firm surplus and for linearizing the composition term in hiring.
C.2.1 Some useful results

Let \( \tilde{z} \) denote the steady state of variable \( z_t \). Assume that \( \zeta_n = \xi b, \) so that \( \tilde{\rho}_n = \tilde{\rho}_b = \tilde{\rho} \) (i.e., retention rates of good and bad workers are the same in steady state). Then we obtain the following steady state results:

\[
\frac{\partial \gamma_{t+1}}{\partial x_t} \bigg|_{ss} = 0 \quad (39)
\]

\[
\frac{\partial w_t^* (\gamma_t)}{\partial \gamma_t} \bigg|_{ss} = 0 \quad (40)
\]

\[
\frac{\partial J_t (\gamma_t, w_t)}{\partial \gamma_t} \bigg|_{ss} = \frac{\partial H_t (\gamma_t, w_t)}{\partial \gamma_t} \bigg|_{ss} = 0 \quad (41)
\]

Let \( \bar{z}_t \equiv \int z_t (\gamma, w) dG_t (\gamma, w) \) denote the time \( t \) average of a firm-specific variable \( z_t (\gamma_t, w_t) \) and note that, up to a first order, \( \bar{z}_t = z_t (\bar{\gamma}_t, \bar{w}_t) \). Let \( \tilde{z}_t \) denote the log deviation of a variable \( z_t \) from its steady state value \( \tilde{z} \). The following approximations for the period-ahead firms and worker surpluses hold:

\[
\tilde{H}_{t+1} (\gamma_{t+1}, w^*_t (\gamma_{t+1})) = \tilde{H}_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) = \tilde{H}_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) + \eta_{H w} (\bar{w}^*_t - \bar{w}^*_t) \quad (42)
\]

\[
\tilde{J}_{t+1} (\gamma_{t+1}, w^*_t (\gamma_{t+1})) = \tilde{J}_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) = \tilde{J}_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) + \eta_{J w} (\bar{w}^*_t - \bar{w}^*_t) \quad (43)
\]

\[
\tilde{H}^b_{t+1} (\gamma_{t+1}, w^*_t (\gamma_{t+1})) = \tilde{H}^b_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) = \tilde{H}^b_{t+1} (\bar{\gamma}_{t+1}, \bar{w}^*_t) + \eta_{H^b w} (\bar{w}^*_t - \bar{w}^*_t) \quad (44)
\]

where \( \eta_{H w}, \eta_{J w}, \) and \( \eta_{H^b w} \) are the steady state elasticities of the worker surplus in good matches, \( H \), the firm surplus, \( J \), and the worker surplus in bad matches, \( H^b \), with respect to the wage.

We use results (39)-(44) to prove that the “composition effect” of hiring is zero up to a first order and to solve for the average contract wage up to a first order, which in turn requires deriving recursive loglinear equations of the firm and worker surpluses. We will invoke these results in the following subsections and then prove them at the end of the section.
C.2.2 Hiring

In the main text, we derive the first order condition for hiring, \( \kappa_t \). Given that next period wage equals this period wage \( w_t \) with probability \( \lambda \) and next period contract wage \( w_{t+1}^* (\gamma_{t+1}) \) with probability \( 1 - \lambda \), we can write the hiring condition at a firm with composition \( \gamma_t \) and wage \( w_t \) as follows:

\[
\kappa_t (\gamma_t, w_t) = E_t \{ \Lambda_{t,t+1} \left[ \lambda J_{t+1} (\gamma_{t+1}, w_t) + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1})) \right] \} + \omega_t (\gamma_t, w_t)
\]

where the second term represents a composition term in hiring:

\[
\omega_t (\gamma_t, w_t) = [p_t (\gamma_t) + \kappa_t (\gamma_t, w_t)] \times E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial J_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial J_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right] \right\}
\]

The firm cares about period-ahead composition for the implied period-ahead retention rate of a unit of labor quality (represented by the first two terms in square brackets) and through possible effects of period-ahead firm composition on future renegotiated wages (the third term).

Since we will prove that \( \partial J / \partial \gamma \), \( \partial w^* (\gamma) / \partial \gamma \), \( \partial \gamma' / \partial \kappa \) are all equal to 0 in the steady state, it follows that up to a first order the composition term \( \omega_t (\gamma_t, w_t) = 0 \).

C.2.3 Search intensity

In the main text, we also derive the first order condition for search intensity, \( \varsigma_{bt} \). Similarly to hiring, we can write the search intensity condition at a firm with composition \( \gamma_t \) and wage \( w_t \) as

\[
\varsigma_{bt} (\gamma_t, w_t) = p_t^b E_t \left\{ \Lambda_{t,t+1} \left[ \bar{H}_{t+1} - \lambda H_{t+1}^b (\gamma_{t+1}, w_t) - (1 - \lambda) H_{t+1}^b (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1})) \right] \right\}
\]

One difference with respect to the hiring condition is that no composition term in search intensity is present. A worker deciding on her own search intensity does not internalize the effect her choice has on the average search intensity of workers employed in bad matches and thus on period-ahead firm composition. This happens because the firm employs a continuum of workers and each worker behaves atomistically.
C.2.4 Staggered Nash bargaining

Consider the problem of a firm and its workers employed in good matches renegotiating a new contract wage, \( w_t^* (\gamma_t) \). For any composition \( \gamma_t \), we can write the surplus of workers in good matches \( H_t (\gamma_t, w_t^* (\gamma_t)) \) as

\[
H_t (\gamma_t, w_t^* (\gamma_t)) = w_t^* (\gamma_t) - u_B - \nu c(\varsigma_n) + E_t \left\{ \Lambda_{t,t+1} \left[ \nu \varsigma_n p_t \bar{H}_t^a - p_t \bar{H}_t^a \right] \right\} \\
+ \nu (1 - \varsigma_n p_t) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda H_{t+1} (\gamma_{t+1}, w_t^* (\gamma_t)) \right] \\
+ (1 - \lambda) H_{t+1} (\gamma_{t+1}, w_t^* (\gamma_{t+1})) \right\}
\]

with

\[
\bar{H}_t^a \equiv \xi \bar{H}_t + (1 - \xi) \bar{H}_t^b \\
\bar{H}_t \equiv V_t^n - U_t \\
\bar{H}_t^b \equiv V_t^b - U_t
\]

Similarly, we can write firm surplus \( J_t (\gamma_t, w_t^* (\gamma_t)) \) as

\[
J_t (\gamma_t, w_t^* (\gamma_t)) = a_t - w_t^* (\gamma_t) - \frac{\kappa}{2} z_t (\gamma_t, w_t^* (\gamma_t))^2 \\
+ [p_t (\gamma_t) + z_t (\gamma_t, w_t^* (\gamma_t))] \times \\
E_t \left\{ \Lambda_{t,t+1} \left[ \lambda J_{t+1} (\gamma_{t+1}, w_t^* (\gamma_t)) \right] + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w_t^* (\gamma_{t+1})) \right\}
\]

with

\[
\kappa z_t (\gamma_t, w_t^* (\gamma_t)) = E_t \left\{ \Lambda_{t,t+1} \left[ \lambda J_{t+1} (\gamma_{t+1}, w_t^* (\gamma_t)) \right] + (1 - \lambda) J_{t+1} (\gamma_{t+1}, w_t^* (\gamma_{t+1})) \right\}
\]

and

\[
a_t \equiv (1 - \zeta) \bar{H}_t^a
\]

In the main text, we write the Nash bargaining condition:

\[
\chi_t (\gamma_t, w_t^* (\gamma_t)) J_t (\gamma_t, w_t^* (\gamma_t)) = (1 - \chi_t (\gamma_t, w_t^* (\gamma_t))) H_t (\gamma_t, w_t^* (\gamma_t))
\]

where

\[
\chi_t (\gamma_t, w_t^* (\gamma_t)) = \frac{\eta}{\eta + (1 - \eta) \mu_t (\gamma_t, w_t^* (\gamma_t)) / \epsilon_t (\gamma_t, w_t^* (\gamma_t))}
\]

with

\[
\epsilon_t (\gamma_t, w_t^* (\gamma_t)) = \frac{\partial H_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} \quad \text{and} \quad \mu_t (\gamma_t, w_t^* (\gamma_t)) = \frac{\partial J_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)}
\]

32
As we discuss in the text, the Nash condition is a variation of the conventional sharing rule, where the relative weight $\chi$ depends not only on the worker’s bargaining power $\eta$, but also on the differential firm/worker horizon, reflected by the term $\mu/\epsilon$. While the horizon effect is interesting from a theoretical perspective, GT shows that it is not quantitatively important. Since this greatly enhances model tractability, in the quantitative analysis we abstract from the horizon effect by having the relative weight $\chi$ fixed at $\eta$, thus effectively working with the following simplified sharing rule:

$$\eta J_t (\gamma_t, w^*_t (\gamma_t)) = (1 - \eta) H_t (\gamma_t, w^*_t (\gamma_t))$$

To solve for the average contract wage, $\bar{w}^*_t$, we now derive loglinear recursive expressions for both the average surplus of workers in good matches, $H_t (\gamma_t, \bar{w}^*_t)$, and the average firm surplus, $J_t (\gamma_t, \bar{w}^*_t)$, at a renegotiating firm. As it will become clear, we will also need expressions for renegotiating firms of the average hiring rate, $\alpha_t (\gamma_t, \bar{w}^*_t)$, the average retention rate, $\rho_t (\gamma_t, \bar{w}^*_t)$, the average search intensity, $s_{bt} (\gamma_t, \bar{w}^*_t)$, and the average worker surplus in bad matches, $H^b_t (\gamma_t, \bar{w}^*_t)$.

### C.2.4.1 Average surplus of workers in good matches at a renegotiating firm

We start deriving a loglinear recursive expression for the average surplus of workers in good matches at a renegotiating firm.

The nonlinear expression for $H_t (\gamma_t, \bar{w}^*_t)$ is given by

$$H_t (\gamma_t, \bar{w}^*_t) = w^*_t (\gamma_t) - u_B - \nu c (s_n) + E_t \left\{ \Lambda_{t,t+1} \left[ \nu s_n p_t \bar{H}^a_t + p_t \bar{H}^a_t \right] \right\} + \nu (1 - s_n p_t) \lambda E_t \left\{ \Lambda_{t,t+1} H_{t+1} (\gamma_{t+1}, \bar{w}^*_t) \right\} + \nu (1 - s_n p_t) (1 - \lambda) E_t \left\{ \Lambda_{t,t+1} H_{t+1} (\gamma_{t+1}, w^*_t (\gamma_{t+1})) \right\}$$

where $\gamma_{t+1}$ denotes next period composition at the firm and $w^*_t (\gamma_{t+1})$ next period contract wage in case of renegotiation.

Loglinearizing and rearranging, we obtain:

$$\hat{H}_t (\gamma_t, \bar{w}^*_t) = \left( \bar{w}/\bar{H} \right) \bar{w}^*_t + (\nu - \bar{\rho}) \beta \left( \bar{H}^a/\bar{H} \right) E_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{\bar{H}}^a_t \right\} - \bar{\rho} \beta \left( \bar{H}^a/\bar{H} \right) E_t \left\{ \hat{\bar{\nu}} + \hat{\Lambda}_{t,t+1} + \hat{\bar{H}}^a_t \right\} - (\nu - \bar{\rho}) \beta \left( 1 - \left( \bar{H}^a/\bar{H} \right) \right) \hat{\nu}_t + \bar{\rho} \beta E_t \left\{ \hat{\Lambda}_{t,t+1} + \lambda \hat{\bar{H}}_{t+1} (\gamma_{t+1}, \bar{w}^*_t) + (1 - \lambda) \hat{\bar{H}}_{t+1} (\gamma_{t+1}, w^*_t (\gamma_{t+1})) \right\}$$

Using now the approximation in equation (42) to substitute out terms in the last line.
and rearranging, we obtain a recursive loglinear expression for $H_t (\tilde{\gamma}_t, \tilde{w}_t^*)$, as follows:

$$
\tilde{H}_t (\tilde{\gamma}_t, \tilde{w}_t^*) = \left( \tilde{w} / \tilde{H} \right) \left[ \tilde{w}_t^* + \tilde{\rho} \lambda \tilde{\beta} \tilde{\varepsilon} E_t \{ \tilde{w}_t^* - \tilde{w}_{t+1}^* \} \right] + (\nu - \tilde{\rho}) \beta \left( \tilde{H}^a / \tilde{H} \right) E_t \left\{ \tilde{\Lambda}_{t,t+1} + \tilde{H}_{t+1}^a \right\} - \tilde{\rho} \beta \left( \tilde{H}^a / \tilde{H} \right) E_t \left\{ \tilde{\rho}_t + \tilde{\Lambda}_{t,t+1} + \tilde{H}_{t+1}^a \right\} - (\nu - \tilde{\rho}) \beta (1 - \left( \tilde{H}^a / \tilde{H} \right) ) \tilde{\rho}_t + \tilde{\rho} \beta E_t \left\{ \tilde{\Lambda}_{t,t+1} + \tilde{H}_{t+1} (\tilde{\gamma}_{t+1}, \tilde{w}_{t+1}^*) \right\}
$$

where $\tilde{\varepsilon} \equiv \partial H / \partial w$.

### C.2.4.2 Average surplus of firms at a renegotiating firm

Here we derive a loglinear recursive expression for the average firm surplus at renegotiating firms.

The nonlinear expression for $J_t (\tilde{\gamma}_t, \tilde{w}_t^*)$ is given by

$$
J_t (\tilde{\gamma}_t, \tilde{w}_t^*) = a_t - \tilde{w}_t^* + \frac{\kappa}{2} \sigma_t (\tilde{\gamma}_t, \tilde{w}_t^*)^2 + \rho_t (\tilde{\gamma}_t, \tilde{w}_t^*) \lambda E_t \left\{ \Lambda_{t,t+1} J_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^*) \right\} + \rho_t (\tilde{\gamma}_t, \tilde{w}_t^*) (1 - \lambda) E_t \left\{ \Lambda_{t,t+1} J_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* (\gamma_{t+1})) \right\}
$$

where, as for $H_t (\tilde{\gamma}_t, \tilde{w}_t^*)$, $\gamma_{t+1}$ denotes next period composition and $w_{t+1}^* (\gamma_{t+1})$ next period contract wage, and where we have dropped the term $\omega_t (\tilde{\gamma}_t, \tilde{w}_t^*)$ that is zero up to a first order.

Loglinearizing and rearranging, we obtain:

$$
\tilde{J}_t (\tilde{\gamma}_t, \tilde{w}_t^*) = \left( \tilde{a} / \tilde{J} \right) \tilde{a}_t - \left( \tilde{w} / \tilde{J} \right) \tilde{w}_t^* + (1 - \tilde{\rho}) \beta \tilde{\varepsilon}_t (\tilde{\gamma}_t, \tilde{w}_t^*) + \tilde{\rho} \beta \lambda E_t \left\{ \tilde{\rho}_t (\tilde{\gamma}_t, \tilde{w}_t^*) + \tilde{\Lambda}_{t,t+1} + \tilde{J}_{t+1} (\gamma_{t+1}, \tilde{w}_t^*) \right\} + \tilde{\rho} \beta (1 - \lambda) E_t \left\{ \tilde{\rho}_t (\tilde{\gamma}_t, \tilde{w}_t^*) + \tilde{\Lambda}_{t,t+1} + \tilde{J}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* (\gamma_{t+1})) \right\}
$$

Using now the approximation in equation (43) to substitute out terms in the last two lines and rearranging, we obtain a recursive loglinear expression for $J_t (\tilde{\gamma}_t, \tilde{w}_t^*)$, as follows:

$$
\tilde{J}_t (\tilde{\gamma}_t, \tilde{w}_t^*) = \left( \tilde{a} / \tilde{J} \right) \tilde{a}_t - \left( \tilde{w} / \tilde{J} \right) E_t \left\{ \tilde{w}_t^* + \tilde{\rho} \lambda \beta \tilde{\mu} (\tilde{w}_t^* - \tilde{w}_{t+1}^*) \right\} + (1 - \tilde{\rho}) \beta \tilde{\varepsilon}_t (\tilde{\gamma}_t, \tilde{w}_t^*) + \tilde{\rho} \beta E_t \left\{ \tilde{\rho}_t (\tilde{\gamma}_t, \tilde{w}_t^*) + \tilde{\Lambda}_{t,t+1} + \tilde{J}_t (\gamma_{t+1}, \tilde{w}_t^*) \right\}
$$

where $\tilde{\mu} \equiv \partial J / \partial w$.

We note that $\tilde{J}_t (\tilde{\gamma}_t, \tilde{w}_t^*)$ depends on $\tilde{\varepsilon}_t (\tilde{\gamma}_t, \tilde{w}_t^*)$ and $\tilde{\rho}_t (\tilde{\gamma}_t, \tilde{w}_t^*)$. The latter, in turn,
depends on $\zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*)$ since

$$\hat{p}_t(\tilde{\gamma}_t, \tilde{w}_t^*) = -\frac{\nu - \tilde{p}}{\tilde{\rho}} (\hat{p}_t + \zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*))$$

We thus proceed to derive loglinear expressions for $z_t(\tilde{\gamma}_t, \tilde{w}_t^*)$ and $\zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*)$.

C.2.4.3 Average hiring rate at a renegotiating firm The nonlinear expression for $z_t(\tilde{\gamma}_t, \tilde{w}_t^*)$ is given by

$$\kappa z_t(\tilde{\gamma}_t, \tilde{w}_t^*) = E_t\{\Lambda_{t,t+1} \left[ \lambda J_{t+1} (\gamma_{t+1}, \bar{w}_{t+1}) + (1 - \lambda) J_{t+1} (\gamma_{t+1}, \bar{w}_{t+1}^*) \right] \}$$

where, as before, we have dropped the term $\omega_t(\tilde{\gamma}_t, \tilde{w}_t^*)$ that is zero up to a first order.

Loglinearizing and rearranging, we obtain:

$$\tilde{z}_t (\tilde{\gamma}_t, \tilde{w}_t^*) = -\left( \bar{w}/\bar{J} \right) \lambda \bar{\mu} (\tilde{w}_t^* - \tilde{w}_{t+1}) + E_t \left\{ \hat{\Lambda}_{t,t+1} + \hat{J}_t (\tilde{\gamma}_{t+1}, \tilde{w}_{t+1}^*) \right\}$$

Using again the approximation in equation (43), we obtain

$$\zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*) = \frac{1}{\gamma} \left( \tilde{p}_t + \hat{\Lambda}_{t,t+1} + \left( \bar{H}/(\bar{H} - \bar{H}^b) \right) \hat{H}_{t+1} \right)$$

C.2.4.4 Average search intensity at a renegotiating firm The nonlinear expression for $\zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*)$ is given by

$$\zeta_{bt}(\tilde{\gamma}_t, \tilde{w}_t^*) = E_t \left\{ \tilde{p}_t + \hat{\Lambda}_{t,t+1} + \left( \bar{H}/(\bar{H} - \bar{H}^b) \right) \hat{H}_{t+1} \right\}$$

and the loglinear version is given by

$$\eta_{\zeta_{bt}}(\tilde{\gamma}_t, \tilde{w}_t^*) = E_t \left\{ \tilde{p}_t + \hat{\Lambda}_{t,t+1} + \left( \bar{H}/(\bar{H} - \bar{H}^b) \right) \hat{H}_{t+1} \right\}$$

Using now the approximation in equation (44), we obtain

$$\eta_{\zeta_{bt}}(\tilde{\gamma}_t, \tilde{w}_t^*) = -\lambda \left( \bar{w}/(\bar{H} - \bar{H}^b) \right) \tilde{z}_t \left\{ \tilde{w}_t^* - \tilde{w}_{t+1} \right\} + E_t \left\{ \tilde{p}_t + \hat{\Lambda}_{t,t+1} + 1/\left( \bar{H}/(\bar{H} - \bar{H}^b) \right) \right\} \left( \hat{H}_{t+1} - \bar{H}^b \hat{H}_{t+1} (\tilde{\gamma}_{t+1}, \tilde{w}_{t+1}^*) \right)$$

where $\tilde{z}_t = \partial H^b/\partial w$. 35
Since \( \zeta_{bt} (\tilde{\gamma}, \tilde{w}^*_t) \) depends on \( \tilde{H}^b_t (\tilde{\gamma}_{t+1}, \tilde{w}^*_{t+1}) \), we finally derive a recursive loglinear expression for \( H^b_t (\tilde{\gamma}_{t+1}, \tilde{w}^*_{t+1}) \).

**C.2.4.5 Average surplus of workers in bad matches at a renegotiating firm**

The nonlinear expression for \( H^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) \) is given by

\[
H^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) = \phi \tilde{w}^*_t (\tilde{\gamma}_t) - u_B - \nu c (\zeta_{bt} (\tilde{\gamma}_t, \tilde{w}^*_t))
\]

Using again equation (44) and rearranging, we obtain a recursive loglinear expression for \( H^b_t (\tilde{\gamma}_{t+1}, \tilde{w}^*_{t+1}) \), as follows:

\[
\tilde{H}^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) = \left( \tilde{w} / \tilde{H} \right) \phi \tilde{w}^*_t (\tilde{\gamma}_t) - \left( \nu \zeta_{bt} / \tilde{H}^b \right) \zeta_{bt} (\tilde{\gamma}_t, \tilde{w}^*_t)
\]

Loglinearizing and rearranging, we obtain:

\[
\tilde{H}^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) = \left( \tilde{w} / \tilde{H} \right) E_t \left\{ \phi \tilde{w}^*_t (\tilde{\gamma}_t) + \zeta_{bt} (\tilde{\gamma}_t, \tilde{w}^*_t) \right\}
\]

Using again equation (44) and rearranging, we obtain a recursive loglinear expression for \( H^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) \), as follows:

\[
\tilde{H}^b_t (\tilde{\gamma}_t, \tilde{w}^*_t) = \left( \tilde{w} / \tilde{H} \right) E_t \left\{ \phi \tilde{w}^*_t (\tilde{\gamma}_t) + \zeta_{bt} (\tilde{\gamma}_t, \tilde{w}^*_t) \right\}
\]

where \( \tilde{\varepsilon} \equiv \partial H / \partial w \).
C.2.5 Derivation of steady-state results

We now derive the steady-state results invoked at the beginning of the appendix. In particular, we show that $\partial w_t^* (\gamma_t) / \partial \gamma_t$ equals zero in the steady state. In doing that, we also show that $\partial \gamma_{t+1} / \partial \delta_t$, $\partial J_t (\gamma_t, w_t^*) / \partial \gamma_t$, $\partial H_t (\gamma_t, w_t^*) / \partial \gamma_t$, $\partial \delta_{bt} (\gamma_t, w_t^*) / \partial \gamma_t$, and $\partial \rho_t (\gamma_t, w_t^*) / \partial \gamma_t$ are all equal to zero in the steady state.

More precisely, we solve a system of steady-state equations in the partial derivatives of several variables with respect to the two firm-level state variables: current composition and current wage. We show that in the steady state the outcome of the current wage bargain is independent of the firm’s current composition, as long as the relevant parties believe that the same is true of the outcome of future wage bargains.\(^5\)

In what follows, we first derive the relevant equations belonging to the system; evaluate them at steady state; and finally show that the solution is such that $\partial w_t^* (\gamma_t) / \partial \gamma_t \big|_{ss} = 0$.

C.2.5.1 Effect of composition on contract wage

Consider a renegotiating firm with composition $\gamma_t$ and contract wage $w_t^* (\gamma_t)$. Define

$$ F_t (\gamma_t, w_t^* (\gamma_t)) \equiv \eta J_t (\gamma_t, w_t^* (\gamma_t)) - (1 - \eta) H_t (\gamma_t, w_t^* (\gamma_t)) $$

Since $F_t (\gamma_t, w_t^* (\gamma_t)) = 0$ by the surplus sharing condition, we have

$$ \frac{\partial w_t^* (\gamma_t)}{\partial \gamma_t} = - \frac{\partial F_t (\gamma_t, w_t^* (\gamma_t)) / \partial \gamma_t}{\partial F_t (\gamma_t, w_t^* (\gamma_t)) / \partial w_t^* (\gamma_t)} $$

where

$$ \frac{\partial F_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t} = \eta \frac{\partial J_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t} - (1 - \eta) \frac{\partial H_t (\gamma_t, w_t^* (\gamma_t))}{\partial \gamma_t} $$

$$ \frac{\partial F_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} = \eta \frac{\partial J_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} - (1 - \eta) \frac{\partial H_t (\gamma_t, w_t^* (\gamma_t))}{\partial w_t^* (\gamma_t)} $$

Evaluating at steady state:

$$ \frac{\partial w^* (\gamma)}{\partial \gamma} = - \frac{\eta \partial J / \partial \gamma - (1 - \eta) \partial H / \partial \gamma}{\eta \partial J / \partial w - (1 - \eta) \partial H / \partial w} \quad (S1) $$

which gives us the first equation of the system.

C.2.5.2 Effect of composition and wages on worker surplus in good matches

We then obtain expressions for $\partial H / \partial \gamma$ and $\partial H / \partial w$. For any given composition $\gamma_t$ and wage

\(^5\) While in principle there may be other self-fulfilling solutions in which firm’s composition matters to current wages in the steady state simply because the parties believe it will matter in the future, we believe that our fundamentals-based solution is most natural, given the environment.
\(w_t\), the worker surplus in a good match is

\[
H_t(\gamma_t, w_t) = w_t - u_B - \nu c(s_t)
\]

\[
+ E_t \{ \Lambda_{t,t+1} \left[ \nu s_t p_t H_t^{a} - p_t H_{t+1}^{a} \right] \}
+ \nu (1 - s_t p_t) E_t \{ \Lambda_{t,t+1} \left[ \lambda H_{t+1} (\gamma_{t+1}, w_t) \right]
+ (1 - \lambda) H_{t+1} (\gamma_{t+1}, w_{t+1} (\gamma_{t+1})) \}
\]

We can write \(\partial H_t(\gamma_t, w_t) / \partial \gamma_t\) as follows

\[
\frac{\partial H_t(\gamma_t, w_t)}{\partial \gamma_t} = \nu (1 - s_t p_t) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right]
+ (1 - \lambda) \frac{\partial H_{t+1} (\gamma_{t+1}, w_{t+1} (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right\} 
\]

Evaluating at steady state gives

\[
\frac{\partial H}{\partial \gamma} \left( 1 - \rho \beta \frac{d\gamma'}{d\gamma} \right) = \rho \beta (1 - \lambda) \frac{\partial H}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{d\gamma}
\]

(S2)

We now turn to \(\partial H / \partial w\). We can write \(\partial H_t(\gamma_t, w_t) / \partial w_t\) as follows

\[
\frac{\partial H_t(\gamma_t, w_t)}{\partial w_t} = 1 + \nu (1 - s_t p_t) E_t \left\{ \Lambda_{t,t+1} \frac{\partial H_{t+1} (\gamma_{t+1}, w_t)}{\partial w_t} \right\}
+ \nu (1 - s_t p_t) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_{t+1} (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right]
+ (1 - \lambda) \frac{\partial H_{t+1} (\gamma_{t+1}, w_{t+1} (\gamma_{t+1}))}{\partial \gamma_{t+1}} \right\} 
\]

Evaluating at steady state gives the third equation of the system:

\[
\frac{\partial H}{\partial w} \left( 1 - \rho \beta \lambda - \rho \beta (1 - \lambda) \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{d\gamma} \right) = 1 + \rho \beta \frac{\partial H}{\partial \gamma} \frac{d\gamma'}{d\gamma}
\]

(S3)
C.2.5.3 Effect of composition and wages on firm surplus  We now turn to $\partial J/\partial \gamma$ and $\partial J/\partial w$. For any given composition $\gamma_t$ and wage $w_t$, the firm surplus is given by

$$J_t(\gamma_t, w_t) = a_t - w_t - \frac{\kappa}{2} z_t(\gamma_t, w_t)^2$$

$$+ \left[ \rho_t(\gamma_t, w_t) + z_t(\gamma_t, w_t) \right] \times$$

$$E_t \left\{ \Lambda_{t,t+1} \left[ \lambda J_{t+1}(\gamma_{t+1}, w_t) \right.$$ 

$$+ (1 - \lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}(\gamma_{t+1})) \right] \right\}$$

which gives $\partial J_t(\gamma_t, w_t) / \partial \gamma_t$ as follows

$$\frac{\partial J_t(\gamma_t, w_t)}{\partial \gamma_t} = \frac{d \rho_t}{d \gamma_t} \times E_t \left\{ \Lambda_{t,t+1} \left[ \lambda J_{t+1}(\gamma_{t+1}, w_t) + (1 - \lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}(\gamma_{t+1})) \right] \right\}$$

$$+ \left[ \rho_t(\gamma_t, w_t) + z_t(\gamma_t, w_t) \right] \times$$

$$E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \right.$$ 

$$+ (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}(\gamma_{t+1}))}{\partial \gamma_{t+1}} \right.$$ 

$$+ (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1}(\gamma_{t+1}))}{\partial w_{t+1}(\gamma_{t+1})} \frac{\partial w_{t+1}(\gamma_{t+1})}{\partial \gamma_{t+1}} \right \} \times \frac{d \gamma_{t+1}}{d \gamma_t}$$

where $J_t(\gamma_t, w_t)$ is maximized with respect to $z_t(\gamma_t, w_t)$, so that in taking the derivative with respect to $\gamma_t$, we can hold $z_t(\gamma_t, w_t)$ fixed at its optimal value.

Evaluating at steady state gives

$$\frac{\partial J}{\partial \gamma} \left( 1 - \beta \frac{d \gamma'}{d \gamma} \right) = \frac{d \rho}{d \gamma} \beta J + \beta (1 - \lambda) \frac{\partial J}{\partial w} \frac{d w'(\gamma)}{d \gamma} \quad (S4)$$

We can then write $\partial J_t(\gamma_t, w_t) / \partial w_t$ as

$$\frac{\partial J_t(\gamma_t, w_t)}{\partial w_t} = -1 + \left[ \rho_t(\gamma_t, w_t) + z_t(\gamma_t, w_t) \right] \lambda E_t \left\{ \Lambda_{t,t+1} \frac{\partial J_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t} \right\}$$

$$+ \varphi_t(\gamma_t, w_t)$$

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\[ \varphi_t(\gamma_t, w_t) = [\rho_t(\gamma_t, w_t) + \varphi_t(\gamma_t, w_t)] E_t \left\{ \Lambda_{t+1} \left[ \lambda \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1})}{\partial \gamma_{t+1}} \right] \right. \\
+ (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1})}{\partial \gamma_{t+1}} \\
\left. + (1 - \lambda) \frac{\partial J_{t+1}(\gamma_{t+1}, w_{t+1})}{\partial w_{t+1}^*} \frac{\partial w_{t+1}^*}{\partial \gamma_{t+1}} \right\} E_t \left( \xi_t + (1 - \lambda) J_{t+1}(\gamma_{t+1}, w_{t+1}) \right) \]

where we have used the envelope condition.

Evaluating at steady state gives

\[ \frac{\partial J}{\partial w} \left( 1 - \lambda \beta - \beta (1 - \lambda) \frac{\partial w^*}{\partial \gamma} \frac{d\gamma^*}{dw} \right) = -1 + \beta \frac{\partial J}{\partial \gamma} \frac{d\gamma}{dw} + \frac{d\rho}{dw} \beta J \]  

(S5)

**C.2.5.4 Effect of composition and wages on the retention rate**  Here we derive expressions for \(d\rho/d\gamma\) and \(d\rho/dw\). For any given composition \(\gamma_t\) and wage \(w_t\), the average retention rate is

\[ \rho_t(\gamma_t, w_t) = \frac{\rho_t^p + \phi \gamma_t \rho_t^b(\gamma_t, w_t)}{1 + \phi \gamma_t} \]

where

\[ \rho_t^b(\gamma_t, w_t) = \nu (1 - \zeta_{bt}(\gamma_t, w_t) \rho_t^p) \]

The derivative of \(\rho_t\) with respect to \(\gamma_t\) is

\[ \frac{d\rho_t}{d\gamma_t} = \frac{\partial \rho_t}{\partial \gamma_t} + \frac{\partial \rho_t}{\partial \gamma_t} \frac{\partial \gamma_t}{\partial \gamma_t} \]

and the derivative with respect to \(w_t\) is

\[ \frac{d\rho_t}{dw_t} = \frac{\partial \rho_t}{\partial w_t} + \frac{\partial \rho_t}{\partial w_t} \frac{\partial \gamma_t}{\partial w_t} \]

Let us consider each relevant term. We have

\[ \frac{\partial \rho_t}{\partial \gamma_t} = \frac{\phi \rho_t^b(1 + \phi \gamma_t) - \phi (\rho_t^p + \phi \gamma_t \rho_t^b)}{(1 + \phi \gamma_t)^2} = \frac{\phi (\rho_t^b - \rho_t^p)}{(1 + \phi \gamma_t)^2} \]

\[ \frac{\partial \rho_t}{\partial \rho_t^b} = \frac{\phi \gamma_t}{1 + \phi \gamma_t} \]
\[
\frac{\partial p_t^b}{\partial s_{bt}} = -\nu p_t^n \\
\frac{\partial p_t}{\partial \bar{w}_t} = 0
\]

Evaluating terms at steady state, using \( \bar{\rho}^b = \bar{\rho}^n \), and substituting, we obtain

\[
\frac{dp}{d\gamma} = -\frac{\phi \gamma}{1 + \phi \gamma} \nu p_t^n \frac{\partial s_b}{\partial \gamma} \\
\frac{dp}{dw} = -\frac{\phi \gamma}{1 + \phi \gamma} \nu p_t^n \frac{\partial s_b}{\partial w} \tag{S6}
\]

\[
\frac{dp}{d\gamma} = \rho_t^b / (1 + \phi \gamma_t) + s_t (\gamma_t, w_t) \gamma_t^h / (1 + \phi \gamma_t^h) \\
\rho_t^n / (1 + \phi \gamma_t) + s_t (\gamma_t, w_t) / (1 + \phi \gamma_t^h)
\]

where

\[
\rho_t^b (\gamma_t, w_t) = \nu (1 - s_{bt} (\gamma_t, w_t) p_t^n)
\]

The derivative of \( \gamma_{t+1} \) with respect to \( \gamma_t \) is

\[
\frac{d\gamma_{t+1}}{d\gamma_t} = \frac{\partial \gamma_{t+1}}{\partial \gamma_t} + \frac{\partial \gamma_{t+1}}{\partial \gamma_t} \frac{\partial \gamma_t}{\partial \gamma_t} + \frac{\partial \gamma_{t+1}}{\partial \gamma_t} \frac{\partial p_t^b}{\partial s_{bt}} \frac{\partial s_{bt}}{\partial \gamma_t}
\]

and the derivative with respect to \( w_t \) is

\[
\frac{d\gamma_{t+1}}{dw_t} = \frac{\partial \gamma_{t+1}}{\partial w_t} + \frac{\partial \gamma_{t+1}}{\partial s_t} \frac{\partial \gamma_t}{\partial w_t} + \frac{\partial \gamma_{t+1}}{\partial p_t^b} \frac{\partial s_{bt}}{\partial w_t}
\]

Let us consider each relevant term at a time. We have

\[
\frac{\partial \gamma_{t+1}}{\partial \gamma_t} = \frac{\rho_t^b}{(1 + \phi \gamma_t^h)} \left[ \frac{\rho_t^n / (1 + \phi \gamma_t) + s_t (1 + \phi \gamma_t^h)}{(1 + \phi \gamma_t)^2} \right] + \frac{\rho_t^b}{(1 + \phi \gamma_t) + s_t (1 + \phi \gamma_t^h)} \left[ \gamma_t / (1 + \phi \gamma_t) \right] \\
\frac{\partial \gamma_{t+1}}{\partial s_t} = \frac{(1 + \phi \gamma_t) (1 + \phi \gamma_t^h)}{[\rho_t^b (1 + \phi \gamma_t^h) + s_t (1 + \phi \gamma_t)]^2} \left( \gamma_t^h / \rho_t^b - \gamma_t / \rho_t^b \right)
\]

\[
\frac{\partial \gamma_{t+1}}{\partial \rho_t^b} = \frac{1}{\rho_t^b / (1 + \phi \gamma_t) + s_t (1 + \phi \gamma_t) / (1 + \phi \gamma_t)} \gamma_t \\
\frac{\partial \gamma_{t+1}}{\partial s_{bt}} = -\nu p_t^n
\]
Evaluating terms at steady state, using $\bar{p}^b = \bar{p}^n$ and $\bar{\gamma} = \bar{\gamma}^h$, we obtain
\[
\frac{\partial \gamma_{t+1}}{\partial w_t} = 0
\]

Substituting, we obtain
\[
\frac{d\gamma'}{d\gamma} = \rho - \gamma \nu p^n \frac{\partial \kappa_b}{\partial \gamma} \tag{S8}
\]

\[
\frac{d\gamma'}{dw} = -\gamma \nu p^n \frac{\partial \kappa_b}{\partial w} \tag{S9}
\]

### C.2.5.6 Effect of composition and wages on search intensity

From the expressions in the previous sub-section, we need $\partial \kappa_b / \partial \gamma$ and $\partial \kappa_b / \partial w$. For any given composition $\gamma_t$ and wage $w_t$, search intensity is given by
\[
\kappa_0 \kappa_b (\gamma_t, w_t) = E_t \{ \Lambda_{t,t+1} p^n_t [ \bar{H}_{t+1} - \lambda H_{t+1}^b (\gamma_{t+1}, w_t) - (1 - \lambda) H_{t+1}^b (\gamma_{t+1}, w_{t+1} (\gamma_{t+1})) ] \}
\]

We can write $\partial \kappa_b (\gamma_t, w_t) / \partial \gamma_t$ as follows
\[
\frac{\partial \kappa_b (\gamma_t, w_t)}{\partial \gamma_t} = - \left( \frac{\bar{s}_b^{1-\eta_s}}{\kappa_0 \kappa_b} \right) E_t \left\{ \Lambda_{t,t+1} p^n_t \left[ \lambda \frac{\partial H_{t+1}^b (\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial H_{t+1}^b (\gamma_{t+1}, w_{t+1} (\gamma_{t+1}))}{\partial w_{t+1} (\gamma_{t+1})} \right] \frac{d\gamma_{t+1}}{d\gamma_t} \right\}
\]

Letting
\[
\tau \equiv \frac{s_b^{1-\eta_s} \beta p^n}{\kappa_0 \kappa_b}
\]

and evaluating at the steady state, we obtain
\[
\frac{\partial \kappa_b}{\partial \gamma} = -\tau \left( \frac{\partial H_{t+1}^b}{\partial \gamma} + (1 - \lambda) \frac{\partial H_{t+1}^b}{\partial w} \frac{\partial w^* (\gamma)}{\partial \gamma} \right) \frac{d\gamma'}{d\gamma} \quad \tag{S10}
\]
We also have \( \frac{\partial \kappa_{bt}(\gamma_t, w_t)}{\partial w_t} \) as follows

\[
\frac{\partial \kappa_{bt}}{\partial w_t} = - \left( \frac{1-\eta_c}{\eta_c \gamma_0} \right) E_t \left\{ \Lambda_{t,t+1} p_t^n \lambda \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t} \right\} - \left( \frac{1-\eta_c}{\eta_c \gamma_0} \right) E_t \left\{ \Lambda_{t,t+1} p_t^n \left[ \lambda \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_{t+1}^*)}{\partial \gamma_{t+1}} \right] \right\} + (1 - \lambda) \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_{t+1}^*)}{\partial w_{t+1}} \frac{\partial w_{t+1}^*}{\partial w_t} \times \frac{d\gamma_{t+1}}{dw_t}
\]

Evaluating at steady state gives

\[
\frac{\partial \kappa_b}{\partial w} = -\tau \lambda \frac{\partial H^b}{\partial w} - \tau \left( \frac{\partial H^b}{\partial \gamma} + (1 - \lambda) \frac{\partial H^b}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \right) \frac{d\gamma'}{dw} \tag{S11}
\]

C.2.5.7 Effect of composition and wages on worker surplus in bad matches

Finally, we derive expressions for \( \frac{\partial H^b}{\partial \gamma} \) and \( \frac{\partial H^b}{\partial \gamma} \). For any given composition \( \gamma_t \) and wage \( w_t \), the worker surplus in a bad match is

\[
H_t^b(\gamma_t, w_t) = \phi w_t - u_B - \nu c(\kappa_{bt}) + E_t \left\{ \Lambda_{t,t+1} \left[ \nu \kappa_{bt} p_t^n \bar{H}_{t+1} - p_t \bar{H}_{t+1}^a \right] + \nu (1 - \kappa_{bt} p_t^n) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \bar{H}_{t+1}^b(\gamma_{t+1}, w_t) + (1 - \lambda) \bar{H}_{t+1}^b(\gamma_{t+1}, w_{t+1}^*) \right] \right\}
\]

We can write \( \frac{\partial H_t^b(\gamma_t, w_t)}{\partial \gamma_t} \) as follows

\[
\frac{\partial H_t^b(\gamma_t, w_t)}{\partial \gamma_t} = \nu (1 - \kappa_{bt} p_t^n) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H_t^b(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} + (1 - \lambda) \frac{\partial H_t^b(\gamma_{t+1}, w_{t+1}^*)}{\partial \gamma_{t+1}} \right] \right\} + (1 - \lambda) \frac{\partial H_t^b(\gamma_{t+1}, w_{t+1}^*)}{\partial w_{t+1}} \frac{\partial w_{t+1}^*}{\partial \gamma_{t+1}} \times \frac{d\gamma_{t+1}}{d\gamma_t}
\]

where \( H_t^b(\gamma_t, w_t) \) is maximized with respect to \( \kappa_{bt}(\gamma_t, w_t) \), so that in taking the derivative with respect to \( \gamma_t \), we can hold \( \kappa_{bt}(\gamma_t, w_t) \) fixed at its optimal value.

Evaluating at steady state gives

\[
\frac{\partial H^b}{\partial \gamma} \left( 1 - \rho \beta \frac{d\gamma'}{d\gamma} \right) = \rho \beta (1 - \lambda) \frac{\partial H^b}{\partial w} \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{d\gamma} \tag{S12}
\]

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We then have $\frac{\partial H^b_t(\gamma_t, w_t)}{\partial w_t}$ as

$$
\frac{\partial H^b_t(\gamma_t, w_t)}{\partial w_t} = \phi + \nu (1 - \varsigma_b p_t^\theta) E_t \left\{ \Lambda_{t,t+1} \lambda \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_t)}{\partial w_t} \right\} \\
+ \nu (1 - \varsigma_b p_t^\theta) E_t \left\{ \Lambda_{t,t+1} \left[ \lambda \frac{\partial H^b_{t+1}(\gamma_{t+1}, w_t)}{\partial \gamma_{t+1}} \\
+ (1 - \lambda) \frac{\partial H^b_{t+1}(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1}))}{\partial \gamma_{t+1}} \right] \\
+ (1 - \lambda) \frac{\partial H^b_{t+1}(\gamma_{t+1}, w^*_{t+1}(\gamma_{t+1}))}{\partial w^*_{t+1}(\gamma_{t+1})} \frac{\partial w^*_{t+1}(\gamma_{t+1})}{\partial \gamma_{t+1}} \right\} \times \frac{d\gamma_{t+1}}{dw_t}
$$

where we have used the envelope condition.

Evaluating at steady state gives the last equation of the system

$$
\frac{\partial H^b}{\partial w} \left( 1 - \rho \beta \lambda - \rho \beta (1 - \lambda) \frac{\partial w^*(\gamma)}{\partial \gamma} \frac{d\gamma'}{dw} \right) = \phi + \rho \beta \frac{\partial H^b}{\partial \gamma} \frac{d\gamma'}{dw} \tag{S13}
$$

**C.2.5.8 System and solution** We have a system of 13 equations, given by equations (S1)-(S13), in 13 unknowns, given by

$$
\begin{align*}
\frac{\partial w^*(\gamma)}{\partial \gamma} \quad \frac{\partial H}{\partial \gamma} \quad \frac{\partial J}{\partial \gamma} \quad \frac{\partial J}{\partial w} \quad \frac{d\rho}{\partial \gamma} \quad \frac{d\rho}{\partial w} \quad \frac{d\gamma'}{\partial \gamma} \quad \frac{d\gamma'}{\partial w} \quad \frac{\partial \varsigma_b}{\partial \gamma} \quad \frac{\partial \varsigma_b}{\partial \gamma} \quad \frac{\partial H^b}{\partial \gamma} \quad \frac{\partial H^b}{\partial w}
\end{align*}
$$

It is easy to see that $\frac{\partial w^*(\gamma)}{\partial \gamma} = 0$ is a solution to the system. That is, in the steady state, the outcome of the current wage bargain is independent of current composition, as long as workers and firms believe that the same is true of the outcome of future wage
bargains. The full solution to the system is as follows:

\[
\begin{align*}
\frac{\partial w^*(-(\gamma))}{\partial \gamma} &= \frac{\partial H}{\partial \gamma} = \frac{\partial J}{\partial \gamma} = \frac{d \rho}{\partial \gamma} = \frac{\partial s_b}{\partial \gamma} = \frac{\partial H^b}{\partial \gamma} = 0 \\
\frac{\partial w}{\partial H} &= \frac{1 - \rho \beta \lambda}{\partial \gamma} \\
\frac{\partial J}{\partial w} &= 1 - \frac{\partial w}{\partial \gamma} \beta J \\
\frac{d \rho}{\partial w} &= 1 - \frac{\partial \gamma}{\partial \gamma} \lambda \beta \\
\frac{d w}{d \gamma'} &= \frac{1}{1 + \frac{\partial \gamma}{\partial \gamma'} \nu p^w \partial s_b} \\
\frac{d \gamma'}{d \gamma'} &= \rho \\
\frac{d \gamma'}{d \gamma'} &= -\frac{\gamma \nu p^w}{\partial s_b} \\
\frac{d w}{\partial s_b} &= \frac{\tau \lambda \phi}{\partial w} \\
\frac{\partial w}{\partial H^b} &= \frac{1 - \rho \beta \lambda}{\phi} \\
\frac{\partial \gamma}{\partial H} &= 1 - \frac{\rho \beta \lambda}{\phi}
\end{align*}
\]

C.2.6 Derivation of surplus approximations

We now prove the first-order approximations for the period-ahead surpluses at renegotiating firms stated in equations (42), (43) and (44) in the first sub-section. To do that, we will use the steady state results just proved.

Specifically, we derive the approximation in equation (42), given by

\[
\hat{H}_{t+1} \left( \gamma_{t+1}, w^*_{t+1} (\gamma_{t+1}) \right) = \hat{H}_{t+1} \left( \gamma_{t+1}, \hat{w}_{t+1}^* \right) = \hat{H}_{t+1} \left( \gamma_{t+1}, \hat{w}_{t+1}^* \right) + \eta_{Hw} \left( \hat{w}_{t}^* - \hat{w}_{t+1}^* \right)
\]

Similar derivations apply to equations (43) and (44).

Consider the worker surplus in good matches, \( H \). It is a function of the two firm-specific states, composition and wage, and the aggregate state. Note that the aggregate state not appear explicitly as an argument of \( H \) as it is captured by our notation with the time index on the function \( H \).

Denoting the aggregate state at time \( t \) with \( s_t \), we can then loglinearize the worker surplus at time \( t \) for any wage, \( w_t \), and any composition, \( \gamma_t \), as follows:

\[
\hat{H}_t(\gamma_t, w_t) = \eta_{H \gamma} \gamma_t + \eta_{Hw} \hat{w}_t + \eta_{Hs} s_t
\]  

where \( \eta_{H \gamma}, \eta_{Hw}, \) and \( \eta_{Hs} \) are the steady state elasticities of \( H \) with respect to composition, wage and aggregate state.
Using the approximation in (45), we can then write:

\[
\begin{align*}
\tilde{H}_{t+1} (\gamma_{t+1}, w_{t+1}^* ) &= \tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) + \eta_{H\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) + \eta_{Hw} (\tilde{w}_{t+1}^* (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) \\
\tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) &= \tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) + \eta_{H\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) + \eta_{Hw} (\tilde{w}_{t+1}^* - \tilde{\tilde{w}}_{t+1}^* ) 
\end{align*}
\] (46)

where the terms in the aggregate state \( \tilde{s}_{t+1} \) cancel out.

We also have the following approximation:

\[
\tilde{w}_{t+1}^* (\gamma_{t+1}) - \tilde{\tilde{w}}_{t+1}^* = \eta_{w\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) 
\] (47)
where \( \eta_{w\gamma} \) is the steady state elasticity of the contract wage with respect to composition and terms in the aggregate state \( \tilde{s}_{t+1} \) cancel out.

Substituting (47) in the first line of (46) we obtain:

\[
\begin{align*}
\tilde{H}_{t+1} (\gamma_{t+1}, w_{t+1}^* (\gamma_{t+1}) ) &= \tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) + \eta_{H\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) + \eta_{Hw} \eta_{w\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) \\
\tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) &= \tilde{H}_{t+1} (\gamma_{t+1}, \tilde{w}_{t+1}^* ) + \eta_{H\gamma} (\gamma_{t+1} - \tilde{\gamma}_{t+1} ) + \eta_{Hw} (\tilde{w}_{t+1}^* - \tilde{\tilde{w}}_{t+1}^* ) 
\end{align*}
\] (48)

The final step to derive equation (42) is to use \( \eta_{H\gamma} = \eta_{w\gamma} = 0 \), in turn resulting from \( \partial H / \partial \gamma = \partial w^* / \partial \gamma = 0 \) in the steady state, a result we have just proved. Substituting \( \eta_{H\gamma} = \eta_{w\gamma} = 0 \) in (48), we finally obtain equation (42).

Equations (43) and (44) are obtained with similar steps, using in this case \( \eta_{J\gamma} = \eta_{H^b\gamma} = \eta_{w\gamma} = 0 \), in turn due to \( \partial J / \partial \gamma = \partial H^b / \partial \gamma = \partial w^* / \partial \gamma = 0 \) in the steady state.

### C.3 Wage growth of job changers

In this section, we derive expressions for the flow shares of the various types of job-to-job flows; and we derive an expression for the average wage growth of job changers.

#### C.3.1 Job-to-job flows

The model includes two types of job-to-job movers: those who search with variable search intensity from bad matches and those in good matches who are forced to search for non economic reasons, i.e., who are subject to a reallocation shock. Since workers in bad matches searching on the job only accept good matches, the first type of job changers leads only to bad-to-good flows. The second type of job changers instead leads to both good-to-bad and
good-to-good flows. We have the following job-to-job flows:

\[
\begin{align*}
\text{Bad to good:} & \quad \nu \tilde{c}_{bt} \xi p_t \tilde{b}_t \\
\text{Good to bad:} & \quad \nu \varsigma_n (1 - \xi) p_t \tilde{n}_t \\
\text{Good to good:} & \quad \nu \varsigma_n \xi p_t \tilde{n}_t
\end{align*}
\]

Summing over the flows we obtain total job flows as:

\[
\nu (\tilde{c}_{bt} \xi \tilde{b}_t + \varsigma_n \tilde{n}_t) p_t
\]

The shares of flows over total flows then are defined as:

\[
\begin{align*}
\delta_{BG,t} &= \frac{\tilde{c}_{bt} \xi \tilde{b}_t}{\tilde{c}_{bt} \xi \tilde{b}_t + \varsigma_n} \\
\delta_{GB,t} &= \frac{\varsigma_n (1 - \xi)}{\tilde{c}_{bt} \xi \tilde{b}_t + \varsigma_n} \\
\delta_{GG,t} &= \frac{\varsigma_n \xi}{\tilde{c}_{bt} \xi \tilde{b}_t + \varsigma_n}
\end{align*}
\]

### C.3.2 Average wage growth of job changers

Let \( \bar{g}_t^w \) denote the average wage growth of continuing workers and \( \bar{g}_t^{EE} \) the average wage growth of workers making and employment-to-employment transition.

Up to a first order, \( \bar{g}_t^{EE} \) can be written as:

\[
\bar{g}_t^{EE} = \delta_{BG,t-1} \log \left( \frac{\bar{w}_t}{\bar{w}_{t-1}} \right) + \delta_{GB,t-1} \log \left( \frac{\phi \bar{w}_t}{\bar{w}_{t-1}} \right) + \delta_{GG,t-1} \log \left( \frac{\bar{w}_t}{\bar{w}_{t-1}} \right)
\]

Simplifying, we obtain:

\[
\bar{g}_t^{EE} = \bar{g}_t^w + \Delta \bar{g}_t^{EE}
\]

with

\[
\bar{g}_t^w = \log \left( \frac{\bar{w}_t}{\bar{w}_{t-1}} \right)
\]

and

\[
\Delta \bar{g}_t^{EE} = (-\log \phi) \left( \delta_{BG,t-1} - \delta_{GB,t-1} \right)
\]

Thus, average wage growth of new hires that are job changers equals average wage growth of continuing workers plus a composition component measuring the change in match quality among job changers. The composition component equals 0 if match quality is homogeneous (\( \phi = 1 \)).
Loglinearizing the average gross wage growth of job changers, we obtain:

$$\tilde{g}_{EE_t} = \tilde{g}_{EE_t} + \frac{1}{1 + \Delta \tilde{\alpha}_{EE}} \tilde{g}_w + \frac{\Delta \tilde{\alpha}_{EE}}{1 + \Delta \tilde{\alpha}_{EE}} \Delta \tilde{\alpha}_{EE}$$

Loglinearizing the compositional effect, we obtain:

$$\Delta \tilde{\alpha}_{EE_t} = \frac{1}{\delta_{BG} - \delta_{GB}} \left( \tilde{\delta}_{BG} \tilde{\alpha}_{BG,t-1} - \tilde{\delta}_{GB} \tilde{\alpha}_{GB,t-1} \right)$$

with

$$\tilde{\delta}_{BG,t} = \frac{1}{1 + \tilde{\gamma}} \tilde{\gamma}_{t} + \left( 1 - \tilde{\delta}_{BG} \right) \tilde{\alpha}_{mt}$$

$$\tilde{\delta}_{GB,t} = - \frac{\tilde{\gamma}}{1 + \tilde{\gamma}} \tilde{\gamma}_{t} - \tilde{\delta}_{BG} \tilde{\alpha}_{mt}$$

Rearranging, we find the expression relating the composition effect to variable search intensity of workers in bad matches and firm average composition:

$$\Delta \tilde{\alpha}_{EE_t} = \frac{\tilde{\delta}_{BG} + \tilde{\gamma} \tilde{\delta}_{GB}}{(1 + \tilde{\gamma}) \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right)} \tilde{\gamma}_{t-1} + \frac{1 - \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right)}{\delta_{BG} \delta_{GB} \tilde{\alpha}_{mt-1}}$$

C.4 Steady state

Here we describe the steady state for the labor market, conditional on the steady-state marginal product of labor, $\tilde{a}$, which is determined as in the conventional neoclassical growth model and is independent of the labor market equilibrium. The key labor market variables are the hiring rate, $\tilde{z}$, the search intensity of workers in bad matches, $\tilde{z}_b$, the wage, $\tilde{w}$, employment in good and bad matches, $\tilde{n}$ and $\tilde{b}$, the retention rates for good and bad matches, $\tilde{\rho}^n$ and $\tilde{\rho}^b$, the retention rate per unit of labor quality, $\tilde{\rho}$, the job-finding probability, $\tilde{p}$, searchers, $\tilde{s}$, unemployment, $\tilde{u}$, composition, $\tilde{\gamma}$, and vacancies, $\tilde{v}$. Further, relevant to the composition effect described in the previous section, are the average wage growth of job changers, $\tilde{y}_{EE}$, the average flow share of bad-to-good transitions, $\tilde{\delta}_{BG}$, and the average flow share of good-to-bad transitions, $\tilde{\delta}_{GB}$.

Equations (49)-(58) below determine $\tilde{z}$, $\tilde{z}_b$, $\tilde{w}$, $\tilde{n}$, $\tilde{b}$, $\tilde{\rho}^n$, $\tilde{\rho}^b$, $\tilde{\rho}$, $\tilde{p}$, and $\tilde{s}$. First, there are the three key behavioral relations: the hiring condition, the search intensity condition and the condition for the wage bargain:

$$\kappa \tilde{z} = \beta \tilde{J} \quad (49)$$

$$\varsigma_0 \tilde{z}_b = \beta \tilde{\rho} \xi \left( \tilde{H} - \tilde{H}^b \right) \quad (50)$$

$$\eta \tilde{H} = (1 - \eta) \tilde{J} \quad (51)$$
where the firm and worker surpluses are given by

$$\tilde{J} = \tilde{a} - \tilde{w} + \frac{\kappa}{2} \tilde{\varepsilon}^2 + \tilde{\rho} \beta \tilde{J}$$

$$\tilde{H} = \tilde{w} - u_B - \nu \xi_0 \frac{\tilde{\varepsilon}^n}{1 + \eta_c} + \beta \left[ \tilde{\rho}^n \tilde{H} - \tilde{\rho} \tilde{H}^a + (\nu - \tilde{\rho}^n) \tilde{H}^a \right]$$

$$\tilde{H}^b = \phi \tilde{w} - u_B - \nu \xi_0 \frac{\tilde{\varepsilon}^b}{1 + \eta_c} + \beta \left[ \tilde{\rho}^b \tilde{H}^b - \tilde{\rho} \tilde{H}^a + (\nu - \tilde{\rho}^b) \tilde{H}^a \right]$$

with

$$\tilde{H}^a = \xi \tilde{H} + (1 - \xi) \tilde{H}^b$$

Second, in the steady state, hiring equals separations for both good and bad matches:

$$\tilde{\rho} (1 - \xi) \left( \tilde{s} - \nu \xi_b \tilde{b} \right) = (1 - \tilde{\rho}^b) \tilde{b}$$

where the survival rates are given by

$$\tilde{\rho}^n = \nu (1 - \xi_n \tilde{p})$$

$$\tilde{\rho}^b = \nu (1 - \xi_b \tilde{p} \xi)$$

and searchers are given by

$$\tilde{s} = \left( 1 - \tilde{n} - \tilde{b} \right) + \nu \xi_b \tilde{b} + \nu \xi_n \tilde{n}$$

Third, since firms employ labor quality units by hiring workers in both good and bad matches, in the steady state the hiring rate in units of labor quality equals the separation rate per unit of labor quality:

$$\tilde{\varepsilon} = 1 - \tilde{\rho}$$

where

$$\tilde{\rho} = \frac{\tilde{\rho}^n \tilde{n} + \phi \tilde{\rho}^b \tilde{b}}{\tilde{n} + \phi \tilde{b}}$$

Unemployment, $\tilde{u}$, composition, $\tilde{\gamma}$, and vacancies, $\tilde{v}$, are then pinned down by equations (59)-(61), given by:

$$1 = \tilde{u} + \tilde{n} + \tilde{b}$$

$$\tilde{\gamma} = \frac{\tilde{b}}{\tilde{n}}$$
Finally, $\tilde{g}^{EE}$, $\tilde{\delta}_{BG}$, and $\tilde{\delta}_{GB}$ are determined by equations (62)-(64) as follows:

\[
\tilde{g}^{EE} = -\log \phi \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right) 
\]

\[
\tilde{\delta}_{BG} = \frac{\tilde{\gamma}_{b} \xi}{\gamma_{b} \xi + s_n} 
\]

\[
\tilde{\delta}_{GB} = \frac{s_n (1 - \xi)}{\tilde{\gamma}_{b} \xi + s_n} 
\]

C.5 Calibration strategy

In this section we discuss how our calibration strategy leads to identification of the key model parameters, in particular those driving the composition effect. The steady state for the labor market described in the previous section includes 12 parameters: $\beta$, $\sigma_m$, $\sigma$, $\eta$, $\nu$, $s_n$, $\phi$, $\xi$, $u_B$, $s_0$, $\eta_c$, $\kappa$. As we discuss in the main text, the first 4 parameters are either normalized or calibrated using external sources. The remaining 8 parameters are calibrated to target model-relevant moments: the steady state wage change of workers making a direct job-to-job transition, $\tilde{g}^{EE}$; the steady state value of the share of bad-to-good flows out of total job flows, $\tilde{\delta}_{BG}$, and its cyclicality, $\eta_{\delta_{BG}, \rho}^{t \omega}$; the steady state probabilities of making an unemployment to employment transition, $p^{UE}$, an employment to unemployment transition, $p^{EU}$, and an employment to employment transition, $p^{EE}$; and the relative value of non-work to work, $\bar{u}_T$. Finally, as we discussed in the main text, we impose a steady state restriction, whereby retention rates in good and bad matches are equal in the steady state, $\tilde{\rho}_n = \tilde{\rho}_b$.

The calibration strategy leads to a unique solution for the steady state values and the internally calibrated parameters. Here we show how the key parameter values relate to the targets.

First, the separation rate into unemployment, $1 - \nu$, is simply given by:

\[
1 - \nu = p^{EU} 
\]

Second, to explain how $\xi$ is determined, we start by noting that, given the steady state restriction, the average flow share of bad-to-good only depends on the relative number of workers in bad and good matches, as follows:

\[
\tilde{\delta}_{BG} = \frac{\tilde{\gamma}}{1 + \tilde{\gamma}} 
\]

This pins down $\tilde{\gamma}$, given a target for $\tilde{\delta}_{BG}$. Combining then equations (52)-(56) and (58)-(60
We derive a relation between $\xi$, $\tilde{\gamma}$, and the targets, $p^{EE}$ and $p^{EU}$, given by:

$$\tilde{\gamma} = \frac{(1 - \xi) \left( p^{EE} + p^{EU} \right)}{p^{EE} + \xi p^{EU}}$$

This relation uniquely pins down $\xi$. The inverse productivity parameter, $\phi$, is then computed using the expression for the average wage growth of job changers:

$$\tilde{g}^{EE} = -\log \phi \left( \tilde{\delta}_{BG} - \tilde{\delta}_{GB} \right)$$

given the target for $\tilde{g}^{EE}$, the target for $\tilde{\delta}_{BG}$ and the average good-to-bad flow share $\tilde{\delta}_{GB}$, in turn computed as:

$$\tilde{\delta}_{GB} = \frac{1 - \xi}{1 + \tilde{\gamma}}$$

The parameter $\zeta_n$ is pinned down, for given targets for the transition probabilities, by the relation:

$$p^{EE} = (1 - p^{EU}) p^{UE} \zeta_n$$

where the probability that a worker in a good match makes an job-to-job transition, $p^{EE}$, equals the probability of surviving within the match, $(1 - p^{EU})$, times the job finding rate, $p^{UE}$, times the search intensity, $\zeta_n$. The parameters $u_B, \zeta_0$ and $\kappa$ are computed solving a subset of the steady state system, equations (49)-(51), given a target for $\bar{u}_T$ and a value for $\eta_\kappa$ (and other computed parameters and steady state values). Finally, the key parameter $\eta_{BG}$, driving the elasticity of search intensity for workers in bad matches to the gain of making a bad-to-good transition, is set to implement the target $\eta_{BG}$.

### C.6 The measured user cost of labor

In this section, we derive the loglinear version of the measured user cost of labor, $uc^{lm}_t$, introduced in Section 6, and given by:

$$uc^{lm}_t = w^{m}_{t,t} + E_t \left\{ \sum_{s=1}^{\infty} (\beta \rho^m)^s \left( w^{m}_{t,t+s} - w^{m}_{t+1,t+s} \right) \right\}, \quad (65)$$

where $w^{m}_{t,t+s}$ denotes the average measured wage of workers at $t+s$ who still occupy the job that they were hired into at time $t$ and $\rho^m$ the average measured retention rate, equal in our model to $\tilde{\rho}$.

Let $\gamma^h_{t-1}$ denote the composition of new hires in $t$ and $\gamma^h_{t-1,t+s}$ the composition of workers at $t+s$ who still occupy the job they were hired into at time $t$. The latter can be expressed

\[ \text{Recall workers are hired at the end of the period and begin as new hires the subsequent period. Hence, the ratio of bad-to-good matches among new hires at time } t \text{ is given by } \gamma^h_{t-1,}. \]
recursively as follows:

\[
\gamma_{t-1,t+s}^h = \begin{cases} 
\gamma_{t-1}^h & \text{if } s = 0 \\
\gamma_{t-1,t+s}^h \frac{\rho_{t+s}^h}{\rho_{t+s}^n} & \text{if } s > 0
\end{cases}
\]  

(66)

where \( \rho_{t+s}^b \) and \( \rho_{t+s}^n \) are the retention rates at time \( t+s \) of workers employed in bad and good matches.

Then, \( w_{t,t+s}^m \) can be written as:

\[
w_{t,t+s}^m = \frac{1 + \phi \gamma_{t-1,t+s}^h}{1 + \gamma_{t-1,t+s}^h} w_{t+s},
\]

(67)

where \( w_{t+s} \) is the contract wage at time \( t+s \).

Substituting (67) in (65), loglinearizing and simplifying yields:

\[
\widehat{uc}_{t}^m = \widehat{w}_{t} + \widehat{c}_{t}^{ucld},
\]

(68)

where the measure user cost, \( \widehat{uc}_{t}^m \), is the sum of two terms: the true user cost of labor, \( \widehat{w}_{t} \), and a compositional component, \( \widehat{c}_{t}^{ucld} \), in turn given by:

\[
\widehat{c}_{t}^{ucld} = -\Psi \left( \frac{\gamma_{t-1}^h}{1 - \rho} \left[ \left( \frac{\rho^b_{t} - \rho^n_{t}}{1 - \rho^b_{t}} \right) + \left( \gamma_{t}^h - \gamma_{t-1}^h \right) \right] \right),
\]

(69)

with

\[
\Psi = \frac{(1 - \phi) \gamma_{t}^h}{(1 + \gamma_{t}^h) (1 + \phi \gamma_{t}^h)}.
\]

(70)

C.7 Lateral movements

A key maintained hypothesis in our analysis is that workers and firms can expect that workers searching on-the-job will not want to voluntarily make lateral movements, that is, to voluntarily move from a job of a given quality to another job of the same quality. As we discuss shortly, this hypothesis simplifies how workers employed in bad matches form expectations when choosing search intensity. At the same time, it also justifies our ruling out in the model of variable search intensity by workers in good matches (since it eliminates any motive for moving to improve the pecuniary gain).

Here we demonstrate that this condition holds to a reasonable approximation. Put differently, under our parameterization, expected gains from making a lateral movement are negligible. As a consequence, the introduction of a small moving cost would suffice to rule them out. Intuitively, gains from moving to a same-quality job can only come from
temporary dispersion in wages associated to infrequent bargaining. In particular, in presence
of a small moving cost, workers can expect to be willing to make a lateral movement only if
their wages are (i) substantially below the average and (ii) are not likely to be renegotiated
for sometime. However, the likelihood a worker is in this situation in our model is of trivial
quantitative importance, due to the transitory nature on average of wage differentials due
to staggered contracting.\footnote{Ranking firms by contract wage, our model implies a period 0.07 percent wage gain for workers making
a lateral movement from 10\textsuperscript{th} to 90\textsuperscript{th} percentile firm. This calculation does not account for the transitory
nature of wage gains from lateral movements, implying a far lower permanent wage improvement associated
with such a job transition relative to a transition with improvement in match quality. We discuss this in
more detail below.}

To start, consider a general framework where lateral movements are allowed for. Let
$V_{t+1}^{s,i}$ be the value of on-the-job search at $t + 1$, conditional on finding a job at $t$, for $i = n, b$.

For a worker employed in bad match, the expected value of on-the-job search, conditional
on a match, can be written as the sum of three terms:

$$E_t \{ V_{t+1}^{s,b} \} = E_t \{ V_{t+1}^b \} + \xi E_t \{ \bar{V}_{t+1}^n - V_{t+1}^b \} + (1 - \xi) L_t^b$$

(71)

The first term is the expected value of continuing with the same job, $E_t \{ V_{t+1}^b \}$; the second
term is the expected gain if the new match is good, in which case the worker makes a
bad-to-good move; the third term is the expected gain from moving to another bad match,
i.e., the expected gain from a lateral move, with $L_t^b$ to be defined shortly. The problem
faced by workers in bad matches optimally choosing search intensity that we formulate in
the main text corresponds to setting $L_t^b = 0$. This greatly simplifies the solution to the
search intensity problem.

For a worker employed in a good match, the value of on-the-job search is the sum of
only two terms (since the worker will not voluntarily move to a bad match):

$$E_t \{ V_{t+1}^{s,n} \} = E_t \{ V_{t+1}^n \} + \xi L_t^n$$

(72)

The first term is the expected value of continuing with the same job, $E_t \{ V_{t+1}^n \}$; the second
term is the expected gain from moving to another good match, that is, the expected gain
from a lateral move. Clearly, with $L_t^n = 0$, workers in good matches have no incentive to
search on-the-job as they cannot improve on their current status. Accordingly, we rule out
an optimal choice for variable search intensity for workers in good matches.

We now demonstrate that $L_t^i = 0$, for $i = n, b$, holds to a reasonable approximation.

Consider the lateral movement term, $L_t^i$, that is, the expected gain from making a lateral
movement, conditional on finding a job of the same quality, for $i = n, b$. This can be written
\[ L_t = E_t \left\{ \int_{\gamma,w} \max \left\{ V^i_{t+1}(\gamma',w') - V^i_{t+1}(\gamma,w) \right\} \ dF_t(\gamma,w) \right\} \] (73)

where \( dF_t(\gamma,w) \equiv \frac{\partial \mu(\gamma,w)}{\partial \gamma} dG_t(\gamma,w) \) and where the expression takes into account that a worker will make a lateral move only if the value at the new match, \( V^i_{t+1}(\gamma',w') \), exceeds the value at the current match, \( V^i_{t+1}(\gamma,w) \).

Through an involved series of derivations\(^8\), it can be shown that a first-order approximation of the lateral movement term around the steady state is given by

\[ L_t = \lambda \frac{\partial \tilde{V}^i}{\partial \tilde{w}} \int_{\tilde{w} > \tilde{w}} (w - \tilde{w}) dF_t(\gamma,w) \] (74)

where \( \tilde{w} \) is the steady state wage and \( \partial \tilde{V}^i / \partial \tilde{w} \) is the steady state gain from a higher wage. Intuitively, up to a first order, \( L_t \) equals the average gain at time \( t \) of moving from a match of a given quality to a match of the same quality but higher wage than the average, given the distribution of wages (per unit of labor quality) at time \( t \). The wage dispersion term, \( \int_{\tilde{w} > \tilde{w}} (w - \tilde{w}) dF_t(\gamma,w) \), is weighted by the likelihood next period that the wage at the new match is not renegotiated, \( \lambda \), and by the steady state lifetime utility gain, \( \partial \tilde{V}^i / \partial \tilde{w} \).\(^9\)

Using expression (74), we then compute the lateral movement term. The difficult object to construct is a counterpart for \( dF_t(\gamma,w) \). We proceed as follows. First, we simulate a time series for the average contract wage from the model. Second, we pick an integer \( T \) large enough so that the probability that a firm has not renegotiated its wage for \( T \) period is approximately 0. We treat \( T \) as a truncation point, that is, we assume that the probability at \( t \) that \( w_t = w^*_{t-T-h} \) for \( h > 0 \) is zero. Once the truncation point is determined, the probability that a firm has a wage \( w_t = w^*_{t-h} \) is given by\(^{10}\)

\[ \Pr \left( w_t = w^*_{t-h} \right) = \begin{cases} \lambda^h (1 - \lambda) & \text{if } h < T \\ \lambda^h & \text{if } h = T \end{cases} \] (75)

After computing the approximate wage distribution, we calculate \( L_t \) as follows

\[ \lambda \frac{\partial \tilde{V}^i}{\partial \tilde{w}} \left\{ \sum_{h=0}^{T} \max \left\{ w^*_{t-h} - \tilde{w}, 0 \right\} \Pr \left( w_t = w^*_{t-h} \right) \right\} \] (76)

\(^8\) The derivations are available upon request.

\(^9\) The utility gain \( \partial \tilde{V}^i / \partial \tilde{w} \) equals \( \phi / (1 - \rho \beta \lambda) \) for \( i = b \) and \( 1 / (1 - \rho \beta \lambda) \) for \( i = n \) and accounts for the expected duration of a match×contract.

\(^{10}\) We note that in constructing the relevant distribution, we ignore the fact that firm hiring rates will vary in the cross section with wages. However, given that (i) hiring rates are decreasing in the wage and (ii) the gain from a lateral movement is increasing in the wage at the new firm, we are constructing an upper bound for the average gain from a lateral move.
We then take the average over time and express it as a percentage of the steady state lifetime value $V^i$. We obtain the following result: the expected percent gain from a lateral movement is tiny. Precisely, it equals 0.032 percent for workers in bad matches and 0.040 percent for workers in good matches.

Importantly, wage dispersion generated by staggered bargaining is negligible not only in absolute terms, but also relative to the dispersion generated by differences in match quality. To see this, we compare the expected percent wage gain from an improvement in match quality, given by $(1 - \phi)/\phi$, to the expected percent wage gain from a lateral movement, given by

$$\left(\frac{1 - \beta\lambda}{1 - \rho\beta\lambda}\right) \lambda \int_{w > \bar{w}} \left(\frac{w - \bar{w}}{\bar{w}}\right) dF_t(\gamma, w)$$

(77)

where the latter is corrected by an adjustment term, $(1 - \beta\lambda)/(1 - \rho\beta\lambda)$, capturing the fact that while gains from lateral movements last over the employment duration as long as the wage is not recontracted, gains from improved match quality last over the entire employment duration. Taking averages over time, the first equals 0.2696, while the second equals 0.0101.

Finally, we note that while we assume that workers searching on-the-job from bad matches expect they will not want to make a lateral move, they will still run into other matches of the same quality but with higher wages. We similarly rule out the possibility that they move to those new matches. To address this point, we compute measures of the distribution of ex-post potential wage gains at time $t$. Given random search, the probability that a worker with wage $w = w^*_t - h$ finds a firm with wage $w' = w'^*_t - j$ equals $\Pr(w = w^*_t - h) \cdot \Pr(w' = w'^*_t - j)$. We compute the 75th, 90th, 95th and 99th percentiles wage gains from lateral moves at each $t$, expressed as a percent wage gain corrected as above by the adjustment term, and take averages over time. We obtain the following numbers: the 75th percentile is 0.0051; the 90th percentile is 0.0110; the 95th percentile is 0.0145 and the 99th percentile is 0.0196. Compared to the gains from moving to a better quality match, these are small numbers.

C.8 Transition function

We now define the law of motion for the distribution function, $G_t$. Let $C$ and $W$ be the sets of possible compositions and wages. Define the Cartesian product of the worker/firm state space to be $S \equiv C \times W$ with $\sigma$-algebra $\Sigma$ with typical subset $S = (C \times W)$. Define the transition function $Q_{s, s'}((\gamma, w), C \times W)$ as the probability that an individual retained or hired by a firm characterized by $(\gamma, w)$ transits to the set $C \times W$ next period when the
aggregate state transits from \( s \) to \( s' \). Then \( Q_{s,s'} \) satisfies

\[
Q_{t,t+1}((\gamma_t, w_t), C \times W)) = I(\gamma_{t+1}(\gamma_t, w_t) \in C) \\
\times \left[ (1 - \lambda)I(w_t^* (\gamma_{t+1}(\gamma_t, w_t)) \in W) \frac{\mathcal{Z}_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\mathcal{Z}_t + \bar{\rho}_t} \\
+ \lambda I(w_t \in W) \frac{\mathcal{Z}_t(\gamma_t, w_t) + \rho_t(\gamma_t, w_t)}{\mathcal{Z}_t + \bar{\rho}_t} \right]
\]

where \( I(\cdot) \) is the indicator function. Then,

\[
G_{t+1}(C \times W) = \int_{(\gamma_t, w_t) \in C \times W} Q_{t,t+1}((\gamma_t, w_t), C \times W))dG_t(\gamma_t, w_t).
\]