Switching-track after the Great Recession

Francesca Vinci and Omar Licandro
University of Nottingham

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Abstract
Data suggests that the level of GDP shifted to a permanently lower trend following the Great Recession for most advanced countries, and researchers have not yet reached a consensus concerning the drivers of this phenomenon. We contribute to this literature by suggesting a DSGE model with financial frictions and endogenous growth through learning-by-doing. With an aggregate AK technology, a negative shock to the capital stock has the effect of moving the economy to a lower trend. A Taylor rule policy designed to reduce the output gap may counterbalance the shock, bringing the economy back to the past trend. However, when the recession is deep and persistent, a revision of potential output measures may undo the recovering role of monetary policy, making the economy converge to a lower trend. We calibrate the model to the U.S. economy and find that GDP can fully recover from a textbook TFP shock under a standard Taylor rule, whilst large demand shocks can affect the supply side permanently. Our framework is thus consistent with episodes of economic recovery as well as episodes of no-recovery. Results rely on the observation that the measurement of U.S. potential output switched track as the Great Recession unfolded, because the severe and prolonged slump put downward pressure on estimates. As a consequence, the output gap closed following the switching-track of potential output, rather than faster GDP growth.

Keywords: Great Recession, Economic Recovery, Endogenous Growth, Hysteresis, Switching-track, Trend Shift

JEL Codes: E12, E22, E32, O41, E52
1 Introduction

The Great Recession had a profound impact on the economic performance of OECD countries. Crucially, the crisis had a persistent level effect on GDP, which remained below its pre-crisis trend for the vast majority of advanced economies as the recovery has been sluggish. Focusing on the United States, past recessions were typically characterized by an acceleration of growth in the short term, to converge to the pre-crisis level of potential output, restoring the initial trend. This time, we saw potential output \textit{switching-track} in conjunction with the crisis, as the fall in economic activity was accompanied by a downward deviation from its original path.

![GDP per capita and HP trend](image.png)

Figure 1

This fact challenged the general consensus on the distinct relationship between trend, growth and business cycles. If recessions are not followed by recoveries, downturns will affect the long run growth path, and potential output cannot be represented by a stable trend in the productive capacity of the economy. In the words of Janet Yellen:
“Are there circumstances in which changes in aggregate demand can have an appreciable, persistent effect on aggregate supply? Prior to the Great Recession, most economists would probably have answered this question with a qualified “no.” (...) This conclusion deserves to be reconsidered in light of the failure of the level of economic activity to return to its prerecession trend in most advanced economies. This post-crisis experience suggests that changes in aggregate demand may have an appreciable, persistent effect on aggregate supply—that is, on potential output.”

The observed lack of recovery looks to be consistent with learning-by-doing models of endogenous growth à la Romer (1986), in which a negative shock to the capital stock would not allow the economy to return to its previous growth trajectory, affecting the level of GDP permanently. This is because the AK nature of the aggregate technology implies constant returns to capital. However, in normal times, monetary policy interventions aimed at reducing the output gap, following a standard Taylor rule, can indeed induce a recovery.

We embed learning-by-doing technology into a DSGE model with financial frictions and simulate the Great Recession as a large shock to aggregate demand with supply effects. In our model, higher aggregate risk and lower consumer confidence reduce investment and consumption demand. The subsequent contraction in credit and rise in bankruptcies cause capital destruction and thus a permanent supply effect. We follow Christiano, Motto, and Rostagno (2014), who model a shock to capital demand through an increase in credit risk, and augment the standard framework by linking the probability of bankruptcy to the depreciation rate of capital as well as by introducing a disruption channel with obsolescence amplification effects. A surge of the bankruptcy rate induces then a depletion of the capital stock. We model monetary policy interventions as a standard Taylor rule, including inflation and the output gap, but we measure potential output as a moving average of past GDP values. This is meant to capture the observed switching-track of potential output, resulting from the unprecedented revision process of potential output estimates conducted by the Congressional

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Budget Office (Coibion et al., 2017b). Figure 2 shows that the output gap closed following the potential output switching-track process instead of faster growth, and thus the strength of the policy intervention weakened over time. In this framework, a persistent increase in risk raises the bankruptcy rate, destroying capital for a period long enough for the monetary authority to revise potential output down, inducing a potential output switching-track and, consequently, making the economy converge to a lower trend. In the model, the policy rule suggests a severe reduction of the interest rate as the shock hits, and the economy reaches the zero lower bound, which exacerbates the severity of the recession. We also show that the Taylor rule is sufficient to drive the recovery when shocks are small, by modelling normal times business cycles as TFP shocks and small demand shocks. The rest of the paper is organized as follows: Section 2 describes how the paper fits into the literature, Section 3 explains the model, Section 4 explains the calibration, Section 5 contains results for the Great Recession simulations, Section 6 describes normal times, in Section 7 we relax the AK assumption and Section 8 concludes.

![GDP per capita and Potential Output revisions](image)

Figure 2: Potential Output Switching-Track (BEA and CBO data)
2 Context and motivation

“Extreme economic events have often challenged existing views of how the economy works and exposed shortcomings in the collective knowledge of economists. (...) The financial crisis and its aftermath might well prove to be a similar sort of turning point.”

The slow recovery that followed the Great Recession caught the profession by surprise, thus many prominent researchers and policy makers have offered their contribution to the ongoing debate. Empirical work has documented a large output loss in most OECD countries (Ball, 2014). Hall (2014, 2016) focuses on the United States’ experience, and identifies the shortfalls in business capital and total factor productivity as the main drivers of the deviation of GDP from its previous trend. The consequences of the Great Recession are motivating a general re-thinking of the underlying mechanisms of growth in recent years, although evidence of the permanent effects of recessions was already available prior to the crisis. Some noteworthy empirical papers argue that financial crises tend to lead to losses in aggregate supply, across developed and developing countries (Cerra & Saxena, 2008), (Reinhart & Rogoff, 2014).

Moreover, the idea that demand-driven recessions could have permanent supply effects, i.e. hysteresis, had already been an object of debate in the 1980s, as Blanchard and Summers (1986) argued this possibility. However, this strand of literature was mostly abandoned due to the advent of the so-called Great Moderation, a period characterized by reduced macroeconomic volatility. Negative shocks were small in size and the GDP trend remained stable. Many studies have been conducted on the drivers of this phenomenon, which is considered to be the result of either good luck or good policy. The good luck hypothesis relies on the idea that the economy has not been hit by substantial shocks during that period, and it has been advocated by academics including Stock and Watson (2002) or Gambetti et al. (2008). The good policy hypothesis is supported by the work of Clarida et al. (2000) and Boivin and Giannoni (2006), who identify rule-based monetary policy as the stabilization

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driver for business cycles. Although neither of the two hypothesis has been definitively proven, the truth likely lies in the middle. In a context of small negative shocks thanks to good luck, monetary policy plays a role in stabilizing output.

As the hypothesis of output hysteresis gains new traction, the role of policy interventions must also be re-evaluated (Summers, 2014). Our paper aims to contribute to the emerging theoretical literature which employs endogenous growth models to explain the permanent effects of negative shocks, and considers policy as a possible driver of economic recovery.

Benigno and Fornaro (2015), Singh and Garga (2018) and Anzoategui et al. (2019) focus on the innovation channel for growth, and show that a fall in aggregate demand can reduce the incentive to conduct R&D, thus affecting aggregate productivity.

Recent work by Bianchi et al. (2019) Queralto (2019) and Guerron-Quintana and Jinnai (2019) investigates the connection between endogenous TFP and financial frictions to explain the lack of recovery that followed the Great Recession, and identify the unusual size and persistence of negative shocks as the main driver of the phenomenon.

Another strand of the literature focuses the attention on firms’ increasing reliance on intangible capital, which comprises all assets enhancing the productivity of the firm (Corrado et al., 2012). They point out that intangible intensive firms tend to be more credit constrained, as it is harder to pledge intangible assets as collateral. Since intangible capital accumulation is a driver of economic growth, recovery after a financial crisis is hampered by the fall in intangible capital investment. Examples include Garcia-Macia (2015) and Caggese and Perez (2017).

We draw from this literature by employing a learning-by-doing technology à la Romer (1986), where knowledge is crucial for economic growth, and its accumulation is directly linked to the capital stock. Yet, these papers focus on the ability of the economy to rebuild output following negative shocks, but they abstract from considerations regarding the underlying mechanism of growth. We take an ambitious approach, by asking whether the dynamics of the cycle can be informative as to the long run mechanics of the economy. For
this reason, our framework aims to reconcile episodes of fast economic recovery and episodes on no-recovery with an endogenous growth model. Monetary policy plays a crucial role in our model, and we focus our attention on a time span encompassing the Great Moderation, the Great Recession and its aftermath. We are interested in a period of consistent monetary policy choices, and the so-called Volcker era in the U.S. is an optimal candidate.

3 The model

We build a New Keynesian model of endogenous growth by introducing a learning-by-doing technology à la Romer (1986) in a Dynamic Stochastic General Equilibrium model with financial frictions. We build on the financial accelerator framework introduced by Bernanke, Gertler, and Gilchrist (1998) and draw from Christiano et al. (2014) for risk shocks. Households gain utility from consumption, dis-utility from labour, and are subject to confidence shocks, i.e. reductions in their marginal utility of consumption. The production sector is comprised of a labour union, final good producers, intermediate good producers, capital producers and entrepreneurs. The latter face financial frictions, since they need to fund part of their capital expenditure with external resources, exposing themselves to the possibility of bankruptcy. We model a risk shock as an increase in the probability of bankruptcy for entrepreneurs. The financial sector acts as an intermediary between the production side and households, by providing loans to entrepreneurs and selling bonds to savers. The monetary authority sets the nominal interest rate according to a Taylor rule, factoring in the deviation of inflation from its target and the output gap. We also assume that potential output is measured as an average of past GDP values, so that large negative shocks lead to downward revisions.

The following sections describe the characteristics of all agents in the model.
3.1 Labour packer

Households, indexed by $\ell \in (0,1)$, supply differentiated labour input to a labour packing firm, i.e. a union, which then supplies an homogeneous labour input $h^p_t$ to the production sector. The bundling technology is

$$h^p_t = \left( \int_0^1 h_t(\ell)^{\frac{\varepsilon}{\varepsilon-1}} d\ell \right)^{\frac{\varepsilon-1}{\varepsilon}}. \quad (1)$$

The parameter $\varepsilon > 1$ represents the elasticity of substitution across types of labour, which are assumed to be gross substitutes. Labour demand by the labour packer is:

$$h_t(\ell) = \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} h^p_t, \quad (2)$$

where $W_t(\ell)$ is the wage of variety $\ell$. The aggregate wage index is

$$W_t^{1-\varepsilon} = \int_0^1 W_t(\ell)^{1-\varepsilon} d\ell. \quad (3)$$

Thus labour supply is:

$$h_t = \int_0^1 \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} h^p_t d\ell. \quad (4)$$

Defining $\int_0^1 \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} d\ell$ as a measure of wage dispersion across varieties implies that if the latter is $> 1$, then aggregate labour used in production will be smaller than aggregate labour supply. However, in this paper we will only consider symmetric equilibria.

3.2 Households

There is a continuum of households of measure one. The $\ell$-type household consumes, supplies labour, buys bonds and owns firms subject to wage frictions à la Rotemberg (1982); when

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3We choose this specification for price and wage frictions because they are better suited in the context of large shocks compared to the standard Calvo style. The letter imply a constant probability of resetting prices, which does not account for size effects. For a detailed discussion see Reiff and Karadi (2012).
changing its wage, it incurs in a cost assumed to be proportional to aggregate output. Since \( \ell \)-type households are assumed to be identical, differing only on the type of labor they offer, labor market equilibrium is symmetric. We will then omit index \( \ell \) to simplify notation.

A representative household, offering a particular type of labor, maximises utility subject to its budget constraints

\[
\max_{c_t, w_t, B_{t+1}} u_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \epsilon_t^c \left( \log \left( c_{t+j} - \chi c_{t-1+j} \right) - \frac{\psi h_{t+j}^{1+\nu}}{1+\nu} \right) \right] \tag{5}
\]

s.t. \( B_{t+1} + P_t c_t = R_{t-1} B_t + P_t w_t h_t - \frac{\chi}{2} \left( \frac{w_t}{w_{t-1}} - 1 \right)^2 P_t Y_t + D_t - \tau_t, \tag{6} \)

and the labor demand (2), where \( c_t \) is consumption and \( h_t \) is labor, \( \beta \in (0, 1) \) is the time discount factor, \( \chi > 0 \) regulates the degree of habit formation, \( \nu > 0 \) is the inverse of labour supply elasticity, \( \psi > 0 \) is a parameter regulating labour hours in steady state and \( \chi \omega > 0 \) regulates price frictions. \( B_{t+1} \) represent riskless one period nominal bonds, purchased at time \( t \), earning the risk-less nominal interest rate \( R_t. \) \(^4\) \( w_t \) is the real wage rate, \( P_t \) is the price of the final good, \( D_t \) are profits redistributed to households by firms and \( \tau_t \) are taxes. \( \epsilon_t^c \) is a confidence shock, affecting the marginal utility of consumption, which follows an AR(1) process of the form:

\[
\log(\epsilon_t^c) = \rho_c \log(\epsilon_{t-1}^c) + \epsilon_{c,t} \tag{7}
\]

where \( \rho_c \in (0, 1) \) and \( \epsilon_{c,t} \sim N(0, \sigma_c^2) \). The FOCs for \( c_t \) and \( B_{t+1} \) in real terms are:

\[
\lambda_t = (c_t - \chi c_{t-1})^{-1} \epsilon_t^c + \beta \chi E_t (c_{t+1} - \chi c_t)^{-1} \epsilon_{t+1}^c \tag{8}
\]

\[
\lambda_t = \beta E_t R_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{9}
\]

where \( \lambda_t \) is the Lagrange multiplier associated to the budget constraint and \( \pi_t = \frac{P_t}{P_{t-1}}. \)

\(^4\)All nominal variables in this paper are defined in an arbitrary numeraire.
The FOC for $w_t$ gives the wage Phillips curve:

$$w_t = \frac{\varepsilon}{\varepsilon - 1} \psi \frac{\nu}{\lambda_t} + E_t \left[ \beta \lambda_{t+1} \Omega_{t+1} Y_{t+1} \frac{1}{h_t} \right] - \Omega_t \pi_t \frac{Y_t}{h_t}$$

(10)

$$\Omega_t = \chi \frac{\pi_w^w}{\varepsilon - 1} (\pi_{t+1}^w - 1) \pi_t^w$$

(11)

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

(12)

where $\varepsilon$ is the degree of substitution across labour types. The RHS of (10) shows that wages depend on the wage markup, the marginal rate of substitution and expectations. The expectation term implies that the labour supply is forward looking, and therefore will be less sensitive to contemporaneous shocks. Equation (12) is an identity to pin down the equilibrium.

### 3.3 Final good sector

The final good $Y_t$ is produced under perfect competition and can be turned into consumption or investment, as well as used to cover the costs associated with financial, price and wage frictions. Production uses intermediate goods as inputs according to the following technology

$$Y_t = \left( \int_0^1 y_t(i) \frac{\theta^1}{\pi^1} di \right)^{\frac{\theta}{\theta^1}}$$

(13)

with the elasticity of substitution $\theta > 1$. The associated demand function for intermediate goods is

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t$$

(14)

with aggregate price index

$$P_t = \left( \int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$
3.4 Intermediate good sector

Each intermediate firm $i, i \in (0,1)$ operates under monopolistic competition. It employs capital services and labour by the mean of the following technology

$$y_t(i) = a_t k_t(i)^{\alpha} h_t(i)^{1-\alpha} K_t^\eta, \quad \alpha \in (0,1) \quad (16)$$

where $K_t$ represents a measure of knowledge, freely available to all firms and acquired through learning-by-doing. We assume that $K_t = k_t$, where $k_t = \int_0^1 k_t(i) di$. This implies that $K$ is a pure externality that comes from the aggregate level of capital employed in the economy, and $0 \leq \eta \leq 1 - \alpha$ represents the strength of the spillovers. $a_t$ is an aggregate productivity shock, following

$$\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t}, \quad \rho_a \in (0,1) \text{ and } \epsilon_{a,t} \sim N(0, \sigma^2).$$

Solving the firms’s problem we find labour and capital services demand

$$w_t = (1-\alpha) s_t \frac{y_t(i)}{h_t(i)} \quad (17)$$

$$r^k_t = \alpha s_t \frac{y_t(i)}{k_t(i)}, \quad (18)$$

where $s_t$ is the real marginal cost function

$$s_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{a_t K_t^\eta} w_t^{1-\alpha} (r^k_t)^{\alpha}. \quad (19)$$

Intermediate good producers are monopolistically competitive and they face an adjustment cost when changing prices. Following Rotemberg (1982), the adjustment cost increases with the magnitude of the change in prices and the size of the economy. It is given by

$$\frac{\phi_p}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t, \quad (20)$$
where $\phi_p \geq 0$ is a measure of price rigidities.

Using the demand function for intermediate goods and assuming symmetry, the first order condition of the optimization problem yields the New Keynesian Phillips curve:

$$
(1 - \theta) + \theta s_t - \pi_t \phi_p (\pi_t - 1) + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1} \phi_p (\pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right] = 0. \quad (21)
$$

### 3.5 Capital producers

There is a unit mass of identical perfectly competitive capital producers. Each period $t$, the representative capital producer buys undepreciated capital $(1 - \delta_t) k_t$ at real price $\hat{q}_t = \frac{q_t}{P_t}$, uses it to produce new capital $k_{t+1}$ by combining it with investment $i_t$, and then sell $k_{t+1}$ to entrepreneurs at the same price. The evolution law of raw capital reads

$$
k_{t+1} = i_t \left(1 - S \left(\frac{i_t}{i_{t-1}}\right)\right) + (1 - \delta_t) k_t. \quad (22)
$$

As in Christiano et al. (2014), investment is subject to the adjustment cost function $S \left(\frac{i_t}{i_{t-1}}\right)$. This assumption helps reducing the volatility of investment and tames inflation as the reaction of investment to shocks is smoother.

Capital producers maximise their flow of profits subject to the evolution law of capital above. The FOC reads

$$
1 = \hat{q}_t \left(-S' \left(\frac{i_t}{i_{t-1}}\right) \left(\frac{i_t}{i_{t-1}}\right) + 1 - S \left(\frac{i_t}{i_{t-1}}\right)\right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \hat{q}_{t+1} S' \left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2. \quad (23)
$$
### 3.6 Entrepreneurs and Financial Intermediation

This section closely follows Christiano et al. (2014), with a few differences designed to generate an endogenous depreciation rate depending on the fraction of firms going bankrupt. There is a unit mass of perfectly competitive entrepreneurs. At the end of any period $t$, each entrepreneur has a net worth $N$, $N > 0$\(^5\). At equilibrium, net worth is distributed $f_t(N)$ across entrepreneurs, with total net worth

$$\hat{N}_{t+1} = \int_0^\infty N f_t(N) \, dN. \quad (24)$$

At the end of period $t$, entrepreneurs use their net worth $N$ and loans $B_{t+1}^N$, $B_{t+1}^N \geq 0$, to acquire capital $k_{t+1}^N$ from capital producers.\(^6\) They pay price $q_t$ for any capital unit they buy such that

$$q_t k_{t+1}^N = N + B_{t+1}^N. \quad (25)$$

All capital is allocated to entrepreneurs:

$$k_{t+1} = \int_0^\infty k_{t+1}^N f_t(N) \, dN. \quad (26)$$

As shown below, like in Christiano et al. (2014), the equilibrium debt-to-net-worth ratio does not depend on $N$, making loans and capital proportional to it. At $t+1$, entrepreneurs use capital $k_{t+1}^N$ to produce capital services $\omega k_{t+1}^N$ that they sell to intermediate firms at price $r_{t+1}^k P_{t+1}$, where $\omega$ is an entrepreneur specific productivity shock and $P_{t+1}$ is the aggregate price index. The idiosyncratic productivity $\omega$ is assumed to be i.i.d. across time and firms, drawn from the c.d.f. $F_t(\omega)$, log normally distributed, with unit mean and standard deviation $\sigma_t$.

At $t+1$, after production takes place, successful entrepreneurs sell their undepreciated

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\(^5\)As for financial frictions, the model mainly draws from Bernanke et al. (1998), so we assume that the entrepreneur employs its own net worth $N$ as well as loans from financial intermediaries, i.e. mutual funds, to finance his venture.

\(^6\)As usual, for an arbitrary asset $X$, $X_{t+1}$ refers to the amount of this asset transferred from $t$ to $t+1$. 

capital \((1 - \delta_t)\) back to capital producers at price \(q_{t+1}\). The depreciation rate of capital is 
\[
\hat{\delta}_t = \delta \left( \frac{F_{t-2}(\bar{\omega}_{t-1})}{F(\bar{\omega})} \right)^{\alpha_{\omega}} \quad \alpha_{\omega} \geq 0 \quad \text{where} \ \delta \in (0, 1) \quad \text{is a parameter.}
\]
This specification includes spillovers from entrepreneurs that went bankrupt in the previous period. It is a simplified way of representing the negative network effects on capital value associated with the disruptions in the production process generated by bankruptcy. A larger bankruptcy rate will make the capital of surviving firm less productive in the future due to the destroyed links. When the probability of bankruptcy is at its steady state value, the impact of spillovers is normalized to 1. The gross return to a unit of capital for an \(\omega\)-type successful entrepreneur is then

\[
R^k_{t+1} = \frac{r^k_{t+1} P_{t+1} + (1 - \hat{\delta}_{t+1}) q_{t+1} - 1}{q_t}. \quad (27)
\]

An entrepreneur with net worth \(N\) obtains a loan \(B^N_{t+1}\) from mutual funds at the interest factor \(Z_{t+1}\). The interest factor is contingent on the state of the economy in \(t + 1\) and, as shown below, the equilibrium interest factor is independent of \(N\). For this reason, index \(N\) is omitted. On top of aggregate risks, the debt contract has to take into account the presence of idiosyncratic risk, since entrepreneurs facing low realizations of the shock \(\omega\) may be unable to repay the loan, and go bankrupt. Conversely, entrepreneurs with sufficiently high returns on their capital will repay their loans and make positive cash flow. For a given state contingent interest factor \(Z_{t+1}\), let us define \(\bar{\omega}_{t+1}\) as the state contingent productivity \(\omega\) that zeroes the entrepreneur’s cash flow at \(t + 1\), i.e.,

\[
\Pi^N_{t+1}(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} R^k_{t+1} q_{t+1} k^N_{t+1} - B^N_{t+1} Z_{t+1} = 0, \quad (28)
\]

where \(\Pi^N_{t+1}(\omega)\) represents the entrepreneur’s cash flow.\(^7\) If \(\omega > \bar{\omega}_{t+1}\), then \(\Pi^N_{t+1}(\omega) > 0\), whilst if \(\omega < \bar{\omega}_{t+1}\), the entrepreneur goes bankrupt.

In case of bankruptcy, the mutual fund pays a monitoring cost \(\mu\), \(\mu \in (0, 1)\), to appropriate the payments generated by the capital services provided to intermediate firms as well as the

\(^7\)Like \(Z_{t+1}\), \(\bar{\omega}_{t+1}\) is independent of \(N\) at equilibrium. For this reason, index \(N\) is omitted.
capital stock, which is then liquidated, subject to physical depreciation and obsolescence. The parameter \( \kappa \), \( \kappa \in (\delta, 1) \) is meant to capture both, obsolescence being measured by the difference \( \kappa - \delta \). Moreover, we assume that spillovers affect bankrupt entrepreneurs too, so that: \( \hat{\kappa}_t = \kappa \left( \frac{F_{t-2}(\bar{\omega}_{t-1})}{F(\bar{\omega})} \right)^{a_\omega} \). The return to capital, for bankrupt \( \omega \)-type capital, is then:

\[
R^F_{t+1} = \frac{\hat{\kappa}_t^{k} P_{t+1} + (1 - \hat{\kappa}_t)q_{t+1}}{q_t}
\]

which in this case is appropriated by mutual funds.

The latter issue bonds to households at the risk-less factor \( R_t \) to raise the resources needed to finance entrepreneurs. They also receive a transfer from the Monetary Authority, who learns the distribution of \( \omega \) from mutual funds. The transfer is meant as a tax on household’s profits from entrepreneurial activity, and is thus proportional to the return to capital of successful entrepreneurs. The parameter \( \xi \) regulates the size of the transfer. This is a measure implemented to relax the cash constraint, aiming to promote credit provision in bad times, and will assure steady state values in line with U.S. data. For simplicity, let us assume that mutual funds specialize in entrepreneurs with net worth \( N \) and operate under perfect competition. Since the interest factor \( Z_{t+1} \) is state contingent, at each state of nature a zero profit condition holds, i.e.:

\[
(1 - \mu)q_t k_t^{N} \int_0^{\bar{\omega}_{t+1}} \omega R^F_{t+1} dF_t(\omega) + (1 - F_t(\bar{\omega}_{t+1}))B_t^{N} Z_{t+1} + \xi \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R^k_{t+1} q_t k_t^{N} = B_t^{N} R_t.
\]

Divide both sides by \( R^k_{t+1} \), use (28) and define leverage \( L_t = \frac{N+B_t^{N}}{N} \) to get

\[
(1 - \mu) \frac{R^F_{t+1}}{R^k_{t+1}} \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) + \bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + \xi \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) = \frac{R_t}{R^k_{t+1}} L_t - 1.
\]

The zero profit condition above can then be used to find an expression for leverage

\[
L_t = \left( 1 - \frac{R^k_{t+1}}{R_t} \left( \bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1}) \right) \right)^{-1}, \tag{29}
\]
where

\[
H_t(\bar{\omega}_{t+1}) = \frac{R_{k+1}^F}{R_{k+1}^t} \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega),
\]

\[
G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega).
\]

This is a measure of the spread between the average return of bankrupt entrepreneurs and the return on the loan mutual funds collect from successful entrepreneurs.

Notice that the loan contract \((Z_{t+1}, B^N_{t+1})\) can be also written as a contract on \((\bar{\omega}_{t+1}, L_t)\). Any pair \((\bar{\omega}_{t+1}, L_t)\) that satisfies (29) is a \((t + 1)\)-state contingent contract offered to entrepreneurs. As it will become clear below, at equilibrium, the conditions of the loan contract \((\bar{\omega}_{t+1}, L_t)\) are the same for all entrepreneurs irrespective of their net worth \(N\). On one side, for a given net worth \(N\), choosing loan \(B^N\) is equivalent to choosing leverage \(L\). On the other side, setting the nominal interest rate \(Z\) determines the cutoff productivity \(\bar{\omega}\).

At time \(t + 1\), for any realization of the aggregate shocks, the debt contract \((\bar{\omega}_{t+1}, L_t)\) for an entrepreneur with net worth \(N\) is expected to generate the cash flow

\[
\int_{\bar{\omega}_{t+1}}^{\infty} \Pi^N_{t+1}(\omega) dF(\omega) = \left(1 - G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}(1 - F_t(\bar{\omega}_{t+1}))\right) R_{k+1}^t L_t N. \tag{30}
\]

Where \(1 - \Gamma_t(\bar{\omega}_{t+1})\) represents the share of net worth the entrepreneur gets, conditional on risk, and:

\[
\Gamma_t(\bar{\omega}_{t+1}) = [1 - F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1}). \tag{31}
\]

For any state of nature in \(t + 1\), the entrepreneur chooses the contract that maximizes expected profit, which is equivalent to: \(^8\)

\[
\max_{\bar{\omega}_{t+1}} \frac{1 - G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}(1 - F_t(\bar{\omega}_{t+1}))}{1 - \frac{R_{k+1}^F}{R_{k+1}^t} \left(\bar{\omega}_{t+1}(1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\right)} = \max_{\bar{\omega}_{t+1}} L_t(\bar{\omega}_{t+1}) (1 - \Gamma_t(\bar{\omega}_{t+1})). \tag{32}
\]

\(^8\)Use (29) to substitute for \(L_t\) in (30), then divide by \(R_{k+1}^t N\) to get (32).
Thus the FOC pinning down the equilibrium $\bar{\omega}_{t+1}$ reads:

$$
1 - F_t(\bar{\omega}_{t+1}) = \frac{R_{k+1}^t [1 - F_t(\bar{\omega}_{t+1}) - G_t(\bar{\omega}_{t+1})(1 - \frac{R_{k+1}^t}{R^t_k} (1 - \mu) - \xi)]}{1 - \frac{R_{k+1}^t}{R^t_k} (\bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1}))}.
$$

(33)

This shows that the loss of accepting a higher threshold for entrepreneurs equals the benefit of higher leverage. The LHS is the elasticity of the share the entrepreneur keeps w.r.t. $\bar{\omega}_{t+1}$, whilst the RHS is the elasticity of leverage w.r.t. $\bar{\omega}_{t+1}$. Also, notice that this specification implies that the elasticity of leverage is affected by credit subsidies.

It is easy to see that the equilibrium $\bar{\omega}_{t+1}$ does not depend on $N$. From (29), leverage does not depend on it either. Consequently, from (28), mutual funds set the same $(t + 1)$-state contingent interest factor $Z_{t+1}$ irrespective of net worth.

Let us assume the standard deviation of $F_t(\omega)$ follows

$$
\log \left( \frac{\sigma_t}{\bar{\sigma}} \right) = \rho_\sigma \log \left( \frac{\sigma_{t-1}}{\bar{\sigma}} \right) + \epsilon_{\sigma,t},
$$

with $\bar{\sigma} > 0$, $\rho_\sigma \in (0, 1)$ and the risk shock $\epsilon_{\sigma,t}$ being i.i.d. Notice that a higher value of $\sigma_t$ implies a higher probability of drawing a low value of $\omega$. As the variance of the shock increases, the tails of the distribution get thicker, increasing the probability of tail events and modifying the threshold for $\bar{\omega}$, i.e. increasing the probability of bankruptcy.

Let us finally assume that at the end of period $t + 1$ (after entrepreneurs pay back to mutual funds their period $t$ debt) a fraction $(1 - \gamma)$ of successful entrepreneur’s cash flow $\Pi_{t+1}^N(\omega)$ gets transferred to households, $\gamma \in (0, 1)$. Moreover, each entrepreneur receives a transfer $P_{t+1} w^e k_{t+1}^N$ from households, $w^e \in (0, 1)$, as a form of insurance, to compensate for risk taking, and assuring that bankrupt entrepreneurs will have a strictly positive net worth allowing them to buy some capital the following period.

---

9 The FOC also shows that our framework reduces to the standard model in Christiano et al. (2014) in the case $R^k = R^F$ and $\xi = 0$. 

16
Since liquidating the capital of failed entrepreneurs generates physical capital losses, the depreciation of capital is endogenous and depends on the fraction of entrepreneurs going bankrupt. For entrepreneurs with net worth $N$, the aggregate depreciation rate at time $t$ reads

$$1 - \delta_t = (1 - \hat{\kappa}_t) F_{t-1}(\bar{\omega}_t) + (1 - \hat{\delta}_t)(1 - F_{t-1}(\bar{\omega}_t)).$$

The intuition is that when bankruptcy happens some capital (tangible or intangible) is destroyed in the process, reducing the overall value of capital. The externality is because bankruptcy generates some negative disruptive spillovers, like some specialized machines not being reallocated as effectively as in normal times, or some important network links being destroyed, or knowledge embedded in intangible capital being lost.

### 3.7 Aggregate economy

The quantity of capital produced by capital producers must be equal to the capital purchased by entrepreneurs:

$$k_{t+1} = \int_0^\infty k_{t+1}^N f_t(N) dN. \quad (34)$$

And using $L_t$:

$$q_k k_{t+1} = \frac{1}{1 - \frac{K_{t+1}}{K_t}\left(\bar{\omega}_{t+1}(1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\right)} \bar{N}_{t+1}. \quad (35)$$

This condition shows that the level of capital in the economy will depend on aggregate net worth, but also on financial conditions.

All firms face the same wage and capital cost, therefore, by symmetry:

$$\frac{k_t(j)}{h_t(j)} = \frac{k_t}{h_t}. \quad (36)$$

Also, market clearing for capital and labour implies $\int_0^1 k_t(j) dj = k_t$ and $\int_0^1 h_t(j) dj = h_t$. 

17
Firms face the same costs and can change their price in every period, so they will choose the same price and produce the same quantity. Therefore $P_t(i) = P_t$ and $Y_t(j) = Y_t$, hence aggregate production in this economy is:

$$Y_t = a_t k_t^{\alpha + \eta} h_t^{1-\alpha}. \quad (37)$$

Therefore, if $\eta = 1 - \alpha$ the technology presents an AK structure.

Aggregate profits of all entrepreneurs at the end of time $t$ are $[1 - \Gamma_{t-1}(\omega_t)] R_t^k q_{t-1} k_t$, so that aggregate net worth at $t + 1$ is:

$$\bar{N}_{t+1} = \gamma [1 - \Gamma_{t-1}(\omega_t)] R_t^k q_{t-1} k_t + P_t w^c k_t \quad (38)$$

Using the aggregate production function and $K_t = k_t$:

$$r_t^k = \alpha s_t a_t k_t^{\alpha + \eta - 1} h_t^{1-\alpha} \quad (39)$$

$$w_t = (1 - \alpha) s_t a_t k_t^{\alpha + \eta} h_t^{-\alpha} \quad (40)$$

$$s_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \frac{1}{a_t k_t^\eta} w_t^{1-\alpha} (r_t^k)^\alpha \quad (41)$$

All bonds held by households must be equal to the amount of loans in aggregate, and transfers to mutual funds must equal taxes:

$$q_t k_{t+1} - \bar{N}_{t+1} = B_{t+1}$$

$$\xi \int_0^{\omega_{t+1}} \omega dF_t(\omega) R_t^k q_{t+1} k_{t+1}^N = \tau_t$$

Output is allocated across consumption and investment, but a portion of it is transformed into intermediate goods due to the price and wage adjustment costs as well as monitoring.
\[ Y_t = c_t + i_t + \mu \int_0^{\omega_t} \omega dF(\omega) \left( R_t^{f_{t-1}} k_t + \phi_0 - 1 \right) + \frac{\phi_1}{2} (\pi_t - 1)^2 Y_t + \frac{\lambda}{2} \left( \pi_t^w - 1 \right)^2 Y_t \quad (42) \]

Aggregate demand is then:

\[ GDP_t = c_t + i_t \quad (43) \]

### 3.8 The monetary authority

The monetary authority sets the Taylor rule for the nominal interest rate, subject to the zero lower bound constraint:

\[ R_t^m = \bar{R}_m + \rho_\pi (\pi_t - \bar{\pi}_m) + \rho_y \log \left( \frac{\hat{GDP}_t}{y_p^t} \right) \quad (44) \]

\[ R_t = \max(1, R_t^m) \quad (45) \]

Where \( \bar{R}_m, \bar{\pi} \) are target values, \( \rho_\pi > 0 \) and \( \rho_y > 0 \) are policy parameters and \( y_p^t \) is a measure of potential output, which is computed as a moving average of past GDP values, stationarized by the trend growth rate of the economy \( g_z \), so that \( \hat{GDP}_t = GDP_t (1 + g_z)^t \).

\[ y_p^t = y_p^{t-1} + \rho \left( \frac{1}{n} \sum_{j=1}^{n} \hat{GDP}_{t-4-j} - y_p^{t-1} \right) \quad 0 > \rho > 1 \quad (46) \]

We define this measure to take into account the revisions carried out by central banks in the context of the Great Recession. We assume that the monetary authority does not have full information about the functioning of the economy, and it infers the underlying path from observed values of GDP. In particular, our measure takes into account the lag in potential output revisions, as we disregard the most recent 4 periods. This implies that the estimate of potential output is not very sensitive to negative shocks if they are small in size and duration. We set \( n \) equal to 10, to allow the central bank to consider a long enough period of time in the estimation of the underlying trend, but also react relatively quickly to
shocks. The Taylor rule provides stimulus to the economy, to close the output gap after a negative shock and, because of the intervention, GDP appears to fluctuate around a stable linear trend in normal times. This is consistent with the diminishing returns to capital of a Neoclassical technology, as typically observed at the firm level. However, if the shock is large and prolonged, potential output will be revised downwards and the stimulus to the economy will lose strength over time. As a result, the economy will reveal its AK structure and the recovery will fail to materialize.

4 Baseline calibration

We calibrate the model on quarterly data for the United States, selecting parameters commonly used in the literature. We target a labour share of 60.8%, which was calculated as the average value of the share of labour compensation in GDP between 2004 and 2007. We select this period to capture the state in the economy in the years preceding the Great Recession, taking into account the downward trend in the labour share and the rise in markups (Autor et al., 2017). This is achieved by by setting $\alpha = 0.24$ and assuming a price mark-up of 25%. The latter implies a value of 5 for the elasticity of substitution across intermediate good, $\theta$. We also calibrate the model to have zero inflation in steady state and a quarterly growth rate of 0.6%. This is obtained by setting labour hours in steady state to 0.2, in line with the average of annual hours worked by persons engaged between 2004 and 2007, choosing $\gamma = 0.966$ (close to Bernanke et al. (1998)), with $\delta$ set to 0.028 and $\kappa$ to 0.04. These are somewhat higher than standard values, but we make this choice because our definition of capital is wider compared to national accounts, including all categories of intangible capital, but also knowledge spillovers. Although these stocks are not fully included in GDP measures, they do affect the outcome of production, and they depreciate faster than traditional measures of capital, so we have to account for their depreciation in our calibration. Monitoring costs are set to $\mu = 0.21$, in line with Christiano et al. (2014) and within the rage suggested by Carl-
We set \( \xi = (1 - \mu)\left(1 - \frac{R_F^e}{R_F^s}\right) \) to obtain a steady state bankruptcy rate of approximately 1%. We set \( \eta = 1 - \alpha \) so that the aggregate technology is AK and the economy is on a balanced growth path, growing at the rate \( g_z \). We then stationarize equilibrium conditions using the growth rate, so that \( \tilde{c}_t = \frac{c_t}{(1+g_z)^t} \) and \( \tilde{y}_t = \frac{Y_t}{(1+g_z)^t} \), for example. For Rotemberg adjustment costs, we follow the approaches proposed by Ascari and Rossi (2012) and Born and Pfeifer (2016), calculating them as follows:

\[
\phi^p = \frac{(\theta - 1)\theta^p}{(1 - \theta^p)(1 - \beta\theta^p)} \quad (47)
\]

\[
\chi^w = \frac{(\epsilon - 1)\theta - 1}{\theta^w}(1 - \alpha)\theta^w}{(1 - \theta^w)(1 - \beta\theta^w)} \quad (48)
\]

where \( \theta^w = \theta^p = 0.75 \) represent the probabilities of not being able to reset prices and wages. \( \epsilon \) is set to 21, implying a wage mark-up of 1.05, and at the high end of values used within the literature, to capture the lower bargaining power workers enjoy in bad times. Finally, we set \( a_\omega \) to 0.15, which implies that a 1% deviation of the probability of bankruptcy from its normal value leads to an increase in the depreciation rate of approximately 0.1%.

In order to appropriately account for the size of the shock and the ZLB constraint, we solve the model non-linearly, employing a Newton-type method to solve the system of non-linear equations (Juillard, 1996).

5 The Great Recession

To simulate the Great Recession, we analyse the model response to a risk shock and a confidence shock, hitting both at the same time. We specify the shocks to target an increase in the bankruptcy probability from a baseline value of 1% to 4%, in line with the delinquency rate on commercial and industrial loans in the United States\(^{10} \), and a prolonged ZLB episode (Figure 3 and 4). This is achieved by setting the magnitude of the shocks to 15.7% for the

\(^{10}\text{See the Appendix for more details.}\)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.028</td>
<td>successful entrepreneurs’ depreciation rate</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.04</td>
<td>bankrupt entrepreneurs’ depreciation rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.24</td>
<td>power on firm capital in production</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(1-0.24, 0.6, 0.06)</td>
<td>power on knowledge spillovers in production</td>
</tr>
<tr>
<td>$\theta$</td>
<td>5</td>
<td>substitution in final good</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1</td>
<td>inverse of Frisch elasticity</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>21</td>
<td>substitution in labour input</td>
</tr>
<tr>
<td>$wh$</td>
<td>0.608</td>
<td>labour share</td>
</tr>
<tr>
<td>$\psi$</td>
<td>16.8</td>
<td>scaling parameter for labour hours</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>46.6</td>
<td>Rotemberg price adjustment parameter</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>1.5</td>
<td>policy response to inflation</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.125</td>
<td>policy response to output gap</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.966</td>
<td>fraction of profits going to entrepreneur</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.97</td>
<td>persistence of the risk shock</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.9</td>
<td>persistence of the confidence shock</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.79</td>
<td>persistence of the TFP shock</td>
</tr>
<tr>
<td>$S^*$</td>
<td>5</td>
<td>investment adjustment costs</td>
</tr>
<tr>
<td>$\chi^w$</td>
<td>141.7</td>
<td>Rotemberg wage adjustment parameter</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.21</td>
<td>monitoring costs</td>
</tr>
<tr>
<td>$w^c$</td>
<td>0.0005</td>
<td>transfer from household</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.7</td>
<td>potential output persistence</td>
</tr>
<tr>
<td>$\alpha_\omega$</td>
<td>0.15</td>
<td>strength of depreciation spillovers</td>
</tr>
</tbody>
</table>

Table 1: Quarterly calibration parameters
risk shock and 13.345% for the confidence shock. We chose $\rho_\sigma = 0.97$ and $\rho_\sigma = 0.9$, in line with Christiano et al. (2014) as well as the wider literature. Our model does not capture unconventional monetary policies implemented by the FED, nor forward guidance, so we target a shorter ZLB duration compared to federal funds rate data, in line with what a standard Taylor rule would imply.

The impulse responses in figure 10 in the Appendix show, as a reaction to the Great Recession shocks, a sharp rise in bankruptcy rates leading to faster capital depreciation and thus capital destruction. GDP falls as a consequence of capital destruction, and so do consumption and investment. As it is common in financial crises, credit declines and net worth contracts, mirroring a stock market crash. Monetary policy is in place to mitigate the shock, but the ZLB constraint limits its effectiveness and, most importantly, the severity and the length of the recession put downward pressure on potential output estimates, giving rise to a switching-track. Without a strong policy intervention, the economy moves to a lower growth path, revealing its AK nature.

In order to compare the simulations with key data dynamics in the U.S., we follow the approach of Christiano, Eichenbaum, and Trabandt (2015) to estimate data targets. We measure the deviation of variables from the path they would have followed, had the recession not

---

11Note that we are not specifying the standard deviation of shocks $\sigma_c$ and $\sigma_\sigma$, because we are focusing on the co-movement across variables, and we are not attempting to reproduce the volatility of data.
happened, by calculating the percentage difference from a linear trend, fitted on past data. In order to select time intervals, we aim to follow the CBO’s methodology for projections of potential output as closely as possible. ‘Typically, a trend is considered to extend from at least one previous business cycle through the most recent quarter of data (because the peak of the current cycle is not known at the time of a forecast).’\textsuperscript{12} They consider full peak-to-peak business cycles, so we build our estimates by including data from the business cycles peaks preceding the Great Recession, up to the period before the downturn. We construct min-max targets by considering the intervals $[x : 2008Q2]$, with $x = (1990Q3, 2001Q1)$. Details on data sources can be found in the Appendix.

Figure 5 shows that our model fits the data quite well, especially for GDP and consumption. The simulation does not capture the depth of the total shortfall in investment, which was driven by the the real estate market. Nonetheless, the model can account for

\textsuperscript{12}‘Revisions of CBO’s potential output since 2007’, February 2014 report.
approximately a third of the initial loss, and results are in line with private non-residential investment data. As for the nominal interest rate and credit, the model fits the data well at the beginning of the sample, and performs worse for later periods. This is likely because we are not modelling unconventional monetary policies, which kept the Federal Funds Rate low and provided credit easing.

In order to compare the switching-track of potential output in our model and in the data, we construct a measure for the potential output target, employing data from the estimates published by the CBO at the beginning of each year, and thus capturing all revisions. Then, we de-trend the resulting series and GDP data by the 2008 potential output projection published by the CBO. The results are represented in Figure 6, which clearly shows that the model economy replicates quite well the switching-track of potential output in the data. The estimated output gap shrinks with time, thus reducing its impact on the Taylor rule. Our simulations are in line with the U.S experience, and our specification can also reproduce the timing of output gap dynamics observed in the data.  

In Figure 18 in the Appendix, we compare the results of our baseline calibration with the case in which the central bank’s measure of potential output is much less sensitive to past observed data. In this case, the policy remains much stronger, and the negative level effects on GDP are considerably reduced, showing the policy’s potential effect on recovery in the model.

In order to evaluate the impact of the policy, we calculate consumption equivalent welfare losses, by comparing the path of utility following the of the Great Recession shocks to the steady state. We conduct the analysis for 50 periods and 150 periods, to capture medium/long run dynamics. Moreover, we calculate welfare for different intensities of po-

---

13 The deviation of GDP and potential output in the data in Figures 5 and in Figure 6 are different in magnitude. The main reason is that the CBO’s 2008 projection of potential output included data up to 2007Q4, a business cycle peak, whilst our targets include 2008Q1 and 2008Q2, making our estimates of the gap more conservative. Moreover, the comprehensive revision of the national income and product accounts (NIPAs), conducted in 2013, led to the inclusion of new investment categories and thus an increase in GDP value. We adjusted the potential output measures published, following the methodology indicated by the CBO in their 2014 report, but a small margin of error remains.
potential output estimate revision. The intuition is that the central bank will revise potential output to some extent after such a sizeable and persistent shock, so it is interesting to compare the implications of different speeds of downward revision. The Baseline column in Table 2 gives the welfare losses produced by the shocks when output gap is revised following the baseline revision rule. The other two columns do the same for alternative revision rules. Comparing columns, we conclude (150 periods) that stronger revision rules generate larger losses. Likely because output and consumption converge to lower levels. However, larger levels of long term consumption are achieved through larger levels of short term investment and thus lower short term consumption. Our baseline specification implies a smaller welfare loss compared to the case of faster revision, which implies that a weaker policy intervention in the model wouldn’t have been optimal.

Figure 16 in the appendix highlights the role of the ZLB constraint in our results, showing that the level effect on GDP would have been less severe had the constraint not been binding.
<table>
<thead>
<tr>
<th>Intensity of revision</th>
<th>Faster than baseline</th>
<th>Baseline</th>
<th>Slower than baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>0.7</td>
<td>0.04</td>
</tr>
<tr>
<td>50 periods</td>
<td>-7.0%</td>
<td>-6.5%</td>
<td>-7.4%</td>
</tr>
<tr>
<td>150 periods</td>
<td>-7.3%</td>
<td>-6.8%</td>
<td>-6.6%</td>
</tr>
</tbody>
</table>

Table 2: Welfare losses compared to steady state for different intensity of potential output revision

We also show that the risk shock is the main driver of these results (Figure 17), but the economy does not reach the ZLB in this scenario, as the severe supply effect of the shock drives inflation up.

6 The Great Moderation: Normal times

6.1 TFP shocks

In order to demonstrate that our model can also be consistent with fast recovery episodes, we consider a textbook 1% TFP shock. We solve the model with perturbation methods and hold the value of potential output constant, as is common practice in DSGEs. We do not update the potential output measure because the revision procedure was the first of its kind in the U.S. experience, as documented by Coibion, Gorodnichenko, and Ulate (2017a).

In normal times, “Recessions typically have little effect on historical estimates of potential output because the methodology aims to exclude cyclical effects” CBO (2014).

We plot simulation results for our baseline calibration as well as for a Taylor rule with no weight on the output gap. Figure 7 shows that small shocks have permanent effects on GDP when $\rho_y = 0$, whilst the economy recovers when $\rho_y = 0.125$. The policy in place sustains investment and savings through the return to capital and the real interest rate, thus creating the conditions for recovery. The persistence of the shock is in line with the literature, and was selected to illustrate a case where the economy fully recovers to the previous steady state. In order to compute welfare, we run the simulation with second-order perturbation methods, and we find that the policy intervention unequivocally improves welfare (Table 3).
Figure 7: Effects of a 1% negative TFP shock

<table>
<thead>
<tr>
<th>Weight on the output gap</th>
<th>$\rho_y = 0$</th>
<th>$\rho_y = 0.125$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 periods</td>
<td>-0.3%</td>
<td>-0.2%</td>
</tr>
<tr>
<td>150 periods</td>
<td>-0.2%</td>
<td>-0.1%</td>
</tr>
</tbody>
</table>

Table 3: Welfare losses of TFP shock compared to steady state for different weights on the output gap in the Taylor rule.

6.2 Demand shocks

In this section, we aim to represent demand shocks in normal times, which we characterize as periods subject to shocks that are not as severe and persistent as during the Great Recession. We opt for a 1% increase in risk combined with a 0.9% confidence shock for illustration purposes and we reduce the persistence of shocks to 0.7. This is close to pre-Great Recession estimates on confidence shocks documented by Angeletos, Collard, and Dellas (2018) and with the analysis of Christiano et al. (2015), who illustrate the increased
persistence of financial wedges during the Great Recession compared to previous downturns. Figure 8 shows that monetary policy drives the recovery in this case as well. In our baseline calibration, we choose strong depreciation spillovers, but it is unlikely for these effects to be linear in the size of shocks. As we reduce the parameter $a_\omega$ in Figure 9, we see that the model can generate a full recovery even with demand shocks.
7 What if we relax the AK assumption?

In the previous sections we assumed $\eta = 1 - \alpha$, which implies an AK technology in aggregate, but is such a strong assumption necessary to generate our results? To test this, we simulate the model for lower values of $\eta$\textsuperscript{14}. We pick $\eta = 0.06$ to get a quasi standard Cobb-Douglas technology. Figure 19 in the Appendix shows that when we subject this version of the model to comparable shocks to simulate the Great Recession and we keep potential output constant to reproduce a standard DSGE, the model generates a full recovery. This proves that the AK technology, and not the nature of the shocks, is the source of persistence in this model.

We then try an intermediate case, setting $\eta = 0.6$ to get closer to an AK model, and allowing potential output revisions as in our baseline. Figure 10 shows that the shocks result in a recession of reduced depth and persistence. Although this version of the model could be a good representation of GDP dynamics in the presence of additional shocks, we prefer our

\textsuperscript{14}Note that in this case the model does not grow endogenously anymore, and since we do not add any trend growth component this implies $g_c = 0$. 

Figure 9: Effects of a small demand shock, with different intensity of depreciation spillovers
baseline specification, as it captures key aspects of the Great Recession in a parsimonious way. Simulations for normal times in Figure 11 and 12 also show that the intensity of knowledge spillovers affects the severity of shocks as well as the speed of recovery, even with monetary policy in place.
Figure 11: Small demand shock for different intensities of knowledge spillovers

Figure 12: TFP shock for different intensities of knowledge spillovers
8 Conclusion

Our paper contributes to the literature by showing that an endogenous growth model can reproduce GDP dynamics well, once the role of policy is taken into account. We argue that an AK model can be a good representation of aggregate GDP because of the growing importance of intangible capital, human capital and knowledge spillovers for production in OECD countries, as documented by Haskel and Westlake (2018). "Because intangible investments, on average, behave differently from tangible investments, we might reasonably expect an economy dominated by intangibles to behave differently too."\(^{15}\) We believe it is possible that the economy has evolved from a Neoclassical structure to an AK structure due to technological advances, but we leave the evaluation of this hypothesis to future work. Finally, this paper does not fully address the role of unconventional monetary policies implemented following the financial crisis. It would be interesting to explore this aspect, by evaluating further the impact of central bank credit policies on the recovery process in our model, following the example of Gertler and Karadi (2011).

References


\(^{15}\)Page 10 (Haskel & Westlake, 2018).


Gourio, F., Messer, T., & Siemer, M. (2016, May). Firm Entry and Macroeconomic Dynam-


**Appendix**

**1.1 Baseline model**

Note: All variables are expressed in real terms, but we do not add hats in order to keep notation simple.

\[
\lambda_t = (c_t - \chi c_{t-1})^{-1} \epsilon_t^c - \beta \chi E_t(c_{t+1} - \chi c_t)^{-1} \epsilon_{t+1}^c
\]  

\[
u_t^k = \alpha s_t a_t h_t^{\alpha + \eta - 1} h_t^{1 - \alpha}
\]  

\[
w_t = (1 - \alpha) s_t a_t (h_t)^{\alpha + \eta} h_t^{-\alpha}
\]  

\[
\lambda_t = \beta R_t E_t [\lambda_{t+1} \pi_{t+1}^{-1}]
\]
\[
\Pi_{t+1}^k = \frac{\pi_{t+1} (r_{t+1}^k + (1 - \hat{\delta}_{t+1}) \hat{q}_{t+1})}{\hat{q}_t}
\]
\[
\Pi_{t+1}^{E} = \frac{\pi_{t+1} (r_{t+1}^k + (1 - \hat{\kappa}_{t+1}) \hat{q}_{t+1})}{\hat{q}_t}
\]
\[
1 - \delta_t = (1 - \hat{\kappa}_t) F_{t-1}(\hat{\omega}_t) + (1 - \hat{\delta}_t) (1 - F_{t-1}(\hat{\omega}_t)).
\]
\[
\hat{\kappa}_t = \kappa \left( \frac{F_{t-2}(\hat{\omega}_{t-1})}{F(\hat{\omega})} \right)^{a_\omega}
\]
\[
\hat{\delta}_t = \delta \left( \frac{F_{t-2}(\hat{\omega}_{t-1})}{F(\hat{\omega})} \right)^{a_\omega}
\]
\[
k_{t+1} = i_t \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) + (1 - \delta_t) k_t
\]
\[
\log \left( \frac{\sigma_t}{\sigma} \right) = \rho_a \log \left( \frac{\sigma_{t-1}}{\sigma} \right) + \epsilon_{a_t}
\]
\[
\log(\alpha_t) = \rho_a \log(\alpha_{t-1}) + \epsilon_{a_t}
\]
\[
\log(\epsilon_t) = \rho_a \log(\epsilon_t) + \epsilon_{c_t}
\]
\[
\frac{1 - F_t(\hat{\omega}_{t+1})}{1 - \Gamma_t(\hat{\omega}_{t+1})} = \frac{\frac{\Pi_{t+1}^k}{\Pi_t} \bar{\omega}_{t+1} + \frac{\bar{\omega}_{t+1}}{1 - F_t(\bar{\omega}_{t+1})} (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})}{1 - \frac{\Pi_{t+1}^k}{\Pi_t} \bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})}
\]
\[
\tilde{N}_{t+1} = \gamma [1 - \Gamma_{t-1}(\bar{\omega}_t)] \Pi_t^k \frac{\hat{q}_{t-1} - k_t}{\pi_t} + w^c k_t
\]
\[
\hat{q}_t k_{t+1} = \frac{1}{\frac{\Pi_{t+1}^k}{\Pi_t} \bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})} \tilde{N}_{t+1}
\]
\[
(1 - \theta) + \theta s_t - \pi_t \phi_p (\pi_t - 1) + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1} Y_{t+1} \frac{1}{h_t} \right] - \Omega_t \pi_t Y_t = 0
\]
\[
w_t = \frac{\epsilon}{\epsilon - 1} \psi \frac{Y_{t+1}^c}{\lambda_t} + E_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \pi_{t+1} \phi_p (\pi_{t+1} - 1) \frac{1}{h_t} \right] - \Omega_t \pi_t Y_t
\]
\[
\Omega_t = \frac{\chi}{\epsilon - 1} (\frac{\pi_t}{\pi_t^w} - 1) \pi_t^w
\]
\[
\frac{w_t}{w_{t-1}} = \frac{\pi_t}{\pi_t^w}
\]
\[
S \left( \frac{i_t}{i_{t-1}} \right) = \left[ \exp \left( \left( \sqrt{\frac{S_t}{2}} - \frac{i_t}{i_{t-1}} - 1 \right) \right) + \exp \left( -\sqrt{\frac{S_t}{2}} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) \right] - 2
\]
\[
1 = \hat{q}_t \left( -S \left( \frac{i_t}{i_{t-1}} \right) \left( \frac{i_t}{i_{t-1}} - 1 \right) + 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \hat{q}_{t+1} S \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2
\]
\[ S' \left( \frac{i_t}{i_{t-1}} \right) = \sqrt{\frac{S''}{2}} \left[ \exp \left( \left( \sqrt{\frac{S''}{2}} \right) \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) - \exp \left( \left( -\sqrt{\frac{S''}{2}} \right) \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) \right] \] (71)

\[ S' \left( \frac{i_{t+1}}{i_t} \right) = \sqrt{\frac{S''}{2}} \left[ \exp \left( \left( \sqrt{\frac{S''}{2}} \right) \left( \frac{i_{t+1}}{i_t} - 1 \right) \right) - \exp \left( \left( -\sqrt{\frac{S''}{2}} \right) \left( \frac{i_{t+1}}{i_t} - 1 \right) \right) \right] \] (72)

\[ Y_t = a_t k_t^{\alpha+n} (h_t)^{1-\alpha} \] (73)

\[ Y_t = c_t + i_t + \mu \int_0^{\tilde{\omega}_t} \omega dF(\omega) R_t^{F} \frac{\tilde{q}_{t-1}}{\pi_t} k_t + \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t + \frac{\chi^w}{2} (\pi_t^w - 1)^2 Y_t \] (74)

\[ GDP_t = c_t + i_t \] (75)

\[ R_t^m = \bar{R}^m + \rho_\pi (\pi_t - \bar{\pi}^m) + \rho_\delta log \left( \frac{GDP_t}{y_t^p} \right) \] (76)

\[ R_t = \max(1, R_t^m) \] (77)

\[ y_t^p = y_{t-1}^p + \rho \left( \frac{1}{n} \sum_{j=1}^{n} GDP_{t-4-j} - y_{t-1}^p \right) \] (78)

1.2 The stationarized AK case

\[ \tilde{\lambda}_t = \left( \tilde{c}_t - \frac{\chi}{1+g_z} \tilde{c}_{t-1} \right)^{-1} \epsilon_t^c - \beta \chi E_t (\tilde{c}_{t+1} (1+g_z) - \chi \tilde{c}_t)^{-1} \epsilon_{t+1} \] (79)

\[ r_t^k = \alpha s_t a_t h^{1-\alpha} \] (80)

\[ \tilde{\omega}_t = (1-\alpha) s_t a_t k_t h^{-\alpha} \] (81)

\[ \tilde{\lambda}_t = \frac{\beta}{(1+g_z)} R_t E_t [\tilde{\lambda}_{t+1} \pi_t^{-1}] \] (82)

\[ log \left( \frac{\sigma_t}{\sigma} \right) = \rho_\sigma log \left( \frac{\sigma_{t-1}}{\sigma} \right) + \epsilon_{\sigma} \] (83)

\[ log(\epsilon_t^c) = \rho_c log(\epsilon_{t-1}^c) + \epsilon_{c,t} \] (84)

\[ \tilde{y}_t = a_t \tilde{k}_t (h)^{1-\alpha} \] (85)

\[ log(\alpha_t) = \rho_\alpha log(\alpha_{t-1}) + \epsilon_{\alpha,t} \] (86)

\[ R_t^{k} = \frac{\pi_{t+1} \left( r_t^{k} + (1-\delta_t+1) \tilde{q}_{t+1} \right)}{\tilde{q}_t} \] (87)

\[ R_t^{F} = \frac{\pi_{t+1} \left( r_t^{F} + (1-\tilde{\kappa}_t+1) \tilde{q}_{t+1} \right)}{\tilde{q}_t} \] (88)

\[ 1 - \delta_t = (1-\tilde{\kappa}_t) F_{t-1}(\tilde{\omega}_t) + (1-\tilde{\delta}_t) (1-F_{t-1}(\tilde{\omega}_t)) \] (89)

\[ \tilde{\kappa}_t = \kappa \left( \frac{F_{t-2}(\tilde{\omega}_{t-1})}{F(\tilde{\omega})} \right)^{\omega} \] (90)
\[
\hat{\delta}_t = \delta \left( \frac{F_{t-2}(\overline{\omega}_{t-1})}{F(\overline{\omega})} \right)^{\alpha_0}
\]

\[
1 - F_t(\overline{\omega}_{t+1}) = \frac{R_{t+1}^k}{R_t} \left[ 1 - F_t(\overline{\omega}_{t+1}) - G_t'(\overline{\omega}_{t+1}) \left( 1 - \frac{R_{t+1}^p}{R_{t+1}^k} (1 - \mu) - \xi \right) \right]
\]

\[
1 - \Gamma_t(\overline{\omega}_{t+1}) = \frac{R_{t+1}^k}{R_t} \left( \overline{\omega}_{t+1} (1 - F_t(\overline{\omega}_{t+1})) + (1 - \mu) H_t(\overline{\omega}_{t+1}) + \xi G_t(\overline{\omega}_{t+1}) \right).
\]

\[
\tilde{N}_{t+1} = (\gamma [1 - \Gamma_{t-1}(\overline{\omega}_t)] R_t^k \frac{q_{t-1}}{\pi_t} \tilde{k}_t + \tilde{w} \tilde{c}_t) \frac{1}{1 + g_z}
\]

\[
\hat{q}_t \tilde{k}_{t+1} = \frac{1}{1 - \frac{R_{t+1}^k}{R_t} \left( \overline{\omega}_{t+1} (1 - F_t(\overline{\omega}_{t+1})) + (1 - \mu) H_t(\overline{\omega}_{t+1}) + \xi G_t(\overline{\omega}_{t+1}) \right)} \tilde{N}_{t+1}
\]

\[
\tilde{y}_t = \tilde{c}_t + \tilde{i}_t + \frac{\phi_p}{2} (\pi_t - 1)^2 \tilde{y}_t + \frac{\chi^w}{2} (\pi_t - 1)^2 \tilde{y}_t + \mu \int_{0}^{\omega_t} \omega dF(\omega) R_t^F \frac{q_{t-1}}{\pi_t} \tilde{k}_t
\]

\[
G \tilde{D} P_t = \tilde{c}_t + \tilde{i}_t
\]

\[
\tilde{S} \left( \frac{\tilde{i}_t}{i_{t-1}} \right) = \exp \left[ \left( \sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_t}{i_{t-1}} (1 + g_z) - g_z - 1 \right) \right] + \exp \left[ \left( -\sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_t}{i_{t-1}} (1 + g_z) - g_z - 1 \right) \right] - 2
\]

\[
\tilde{k}_{t+1} = \left( \tilde{i}_t \left( 1 - S \left( \frac{\tilde{i}_t}{i_{t-1}} \right) \right) \right) + \left( 1 - \delta_t \tilde{k}_t \right) \frac{1}{1 + g_z}
\]

\[
1 = \hat{q}_t \left( -S' \left( \frac{\tilde{i}_t (1 + g_z)}{\tilde{i}_{t-1}} \right) \left( \frac{\tilde{i}_t (1 + g_z)}{\tilde{i}_{t-1}} \right) \right) + \left( 1 - S \left( \frac{\tilde{i}_t (1 + g_z)}{\tilde{i}_{t-1}} \right) \right) + \\
+ \frac{\beta}{1 + g_z} E_t \frac{\tilde{c}_{t+1} S'}{\lambda_t} \hat{q}_{t+1} S' \left( \frac{\tilde{i}_{t+1} (1 + g_z)}{\tilde{i}_t} \right) \left( \frac{\tilde{i}_{t+1} (1 + g_z)}{\tilde{i}_t} \right)^2
\]

\[
S' \left( \frac{\tilde{i}_t (1 + g_z)}{i_{t-1}} \right) = \sqrt{\frac{S''}{2}} \left[ \exp \left( \left( \sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_t}{i_{t-1}} (1 + g_z) - g_z - 1 \right) \right) \right] - \exp \left( \left( -\sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_t}{i_{t-1}} (1 + g_z) - g_z - 1 \right) \right)
\]
\[ S' \left( \frac{\tilde{i}_{t+1}(1 + g_z)}{\tilde{i}_t} \right) = \]
\[ = \sqrt{\frac{S''}{2}} \left[ \exp \left( \left( \sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_{t+1}(1 + g_z)}{\tilde{i}_t} - 1 \right) \right) - \exp \left( \left( -\sqrt{\frac{S''}{2}} \right) \left( \frac{\tilde{i}_{t+1}(1 + g_z)}{\tilde{i}_t} - 1 \right) \right) \right] \]

(101)

\[(1 - \theta) + \theta s_t - \pi_t \phi_p (\pi_t - 1) + \beta E_t \left[ \frac{\bar{\lambda}_{t+1}}{\bar{\lambda}_t} \pi_{t+1} \phi_p (\pi_{t+1} - 1) \bar{y}_{t+1} \right] = 0 \quad (102)\]

\[\bar{\omega}_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi h'_t \epsilon'_t}{\lambda_t} + E_t \left[ \beta \frac{\bar{\lambda}_{t+1}}{\bar{\lambda}_t} \Omega_{t+1} \bar{Y}_{t+1} \frac{1}{h_t} \right] - \Omega_t \pi_t \frac{\bar{Y}_t}{h_t} \]

(103)

\[\Omega_t = \frac{\chi^w}{\epsilon_w - 1} (\pi^w_t - 1) \pi^w_t \]

(104)

\[\frac{\bar{\omega}_t}{\bar{\omega}_{t-1}} = \frac{\pi^w_t}{\pi_t} \]

(105)

\[R^m_t = \bar{R}^m + \rho_x (\pi_t - \bar{\pi}^m) + \rho_y \log \left( \frac{GDP_t}{y_t^p} \right) \]

(106)

\[R_t = \max(1, R^m_t) \]

(107)

\[\bar{y}_t^p = \bar{y}_{t-1}^p + \rho \left( \frac{1}{n} \sum_{j=1}^{n} GDP_{t-1-j} - y_{t-1} \right) \]

(108)
2 Data sources for simulation targets

All data was retrieved from the FRED database. GDP is Real Gross Domestic Product, consumption is Real Personal Consumption Expenditures, investment is Real Gross Private Domestic Investment and private non-residential investment is Private Nonresidential Fixed Investment, divided by its deflator (retrieved from BEA). All variables are expressed in billions of chained 2012 dollars, quarterly, seasonally adjusted annual rates and in per capita terms. The bankruptcy probability is the Delinquency Rate on Commercial and Industrial Loans, for all commercial banks, quarterly percentage rate, seasonally adjusted. Credit is Total Credit to the Non-Financial Corporations, adjusted for breaks, in billions of US Dollars, divided by the GDP deflator. The nominal interest rate is the Effective Federal Funds Rate (quarterly averages).

3 Bankruptcy, Firm exit and entry evidence

In the model we assume that delinquency on loans corresponds to bankruptcy for the firm. This is a simplifying assumption, but is justified by the fact that both the delinquency rate on loans and business bankruptcy filings spiked in the U.S. during the Great Recession. In addition, firms’ exit was not accompanied by entry, conversely to what happened in previous periods. This is likely an important differentiating factor between the Great Recession and past downturns, as research indicates that start-up entry can be a driver of innovation and TFP growth, see for example Gourio, Messer, and Siemer (2016). Figure 8 and 9 illustrate.
Figure 13: Taken from Leon-Ledesma and Orrillo (2016)

Figure 14: U.S. Census Bureau’s Business Dynamics Statistics program, Firm Characteristics Data Tables, “Firm Age” table, calculated as firms at year zero minus firm deaths by year.
Figure 15: The Great Recession (baseline): quarterly model simulations
Figure 16: The Great Recession: quarterly model simulations, without the ZLB constraint
Figure 17: The risk shock: quarterly model simulations
Figure 18: The Great Recession for different values of potential output revision intensity
Figure 19: The Great Recession shocks with a quasi standard Cobb-Douglas technology and no potential output revisions