How Integrated Are Corporate Bond and Stock Markets?*

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Abstract

In this paper, I study the degree of market integration between US corporate bonds and stocks of the corresponding issuing firms, accounting for their characteristics. I find that short-selling constraints are essential restrictions to optimal Sharpe ratio portfolios that yield admissible portfolio positions and implied pricing errors within quoted bid-ask spreads. My empirical evidence suggests that larger firms, with more liquid corporate bonds and stocks, appear to be more integrated. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a greater extent of integration between their debt and equity securities.

Keywords: stochastic discount factor, corporate bonds, stocks, market integration, firm characteristics.

JEL Classification: G11, G12, G14

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1 Introduction

Canonical models in asset pricing often implicitly assume that markets are integrated, such that there is no arbitrage across markets. For example, the Merton (1974) model posits that the firm value process is divided between debt and equity investors, who hold exposures to the same source of risk. Following a contingent claim approach, debt and equity should be priced using the same stochastic discount factor (SDF). Put differently, this framework implies strict restrictions for the cross-market risk premia and dictates the way bonds and stocks should be priced cross-sectionally, irrespective of the firm-characteristics driving the returns. In reality, however, there are several financial frictions preventing markets from being fully integrated, including financial constraints, such as short-selling or transaction costs, that may lead to limits to arbitrage. Thus, shedding light on the underlying extent of market integration is crucial for refining our insights regarding the pricing of different asset classes, as well as for assessing the empirical success of asset pricing models implicitly assuming perfect integration across different markets. Moreover, recent evidence suggests that investors' behaviour may be driven by different firm specific or asset specific characteristics (see, e.g. Koijen, Richmond, and Yogo (2019), among others). Therefore, in this paper, I address the question of how integrated corporate bond and stock markets are, by incorporating in the analysis different frictions and considering various firm characteristics.

The extent of asset market integration is of particular importance for investment and financing decisions, not only for institutional investors and corporations, but also for individual investors. In absence of perfect risk sharing, access to cross-markets, both domestically and internationally, can prove beneficial in terms of portfolio diversification. Additionally, if debt and equity markets are integrated, investors can hedge across these markets. Still, studying and quantifying market integration is hindered by the lack of a well-accepted measure.[1] There are two aspects worth emphasizing. First, there can be evidence of segmentation within an asset class. For instance, a test of market integration for a particular stock which is cross-listed is equivalent to examining whether the law of one price holds across the two exchanges. Second, markets can be segmented across different assets. In this case, testing for integration is consistent with absence of arbitrage opportunities between the two markets. This follows from the fact that assets that entail identical risks should have prices that yield the same expected returns. The focus of this paper is on the second instance, since the analysis involves two different classes of assets. Still, if there exists a common risk factor for both debt and equity markets, it should be priced consistently across them. Analogously, there exists a common SDF jointly pricing corporate bonds and stocks.

To study the underlying degree of integration between corporate bonds and stock markets, I adopt a model-free approach. This methodology has the advantage of overcoming the typical model misspecification critique, since the main limitation when using factor models for example is the

[1] Several notions of market integration have been proposed in the literature. They can be summarized as pertaining to either a parametric, or to a nonparametric framework. I review these different approaches in the related literature section.

hurdle to disentangle the joint hypothesis test of market integration and correct model specification. For instance, in a Capital Asset Pricing Model (CAPM) world, market integration implies that the prices of risk associated with the market factor are identical for corporate bond and stock returns. However, if the factor model is misspecified, one might incorrectly reject market integration, as the prices of risk would tend to differ across various markets. Using instead a model-free approach, I show that the level of integration between the corporate bonds and stock markets is higher than previously thought, at least unconditionally.

Intuitively, the idea is to take the stance of investors who are Sharpe ratio maximizers. The metric used to quantify the degree of integration is given by the cross-market pricing implications of the optimal Sharpe ratio portfolios in the respective market. Whenever investors have access to both bonds and stocks, and they can freely form portfolios, the corresponding markets are going to be perfectly integrated. Consequently, there should be no cross-market pricing discrepancies. On the other hand, differences in cross-market pricing, or pricing errors, capture departures from perfect market integration. However, in order to assess whether these pricing discrepancies lead to actual arbitrage opportunities, trading costs need to be taken into account. For this reason, in the empirical exercise, I am going to compare the size of the pricing errors implied by optimal Sharpe ratio portfolios with a measure of transaction costs, given by the quoted bid-ask spreads in the corporate bonds and stocks markets. Using the universe of US firms, for both corporate bonds and stock returns, I form portfolios based on firm level characteristics, for the following reasons. First, portfolios reduce idiosyncratic volatility. Second, it is relevant to assess whether investors specialize in different types of assets, or are more interested in the firm itself, and have a preference for a particular firm. Specifically, for both corporate bond and stock returns, I form portfolios based on size, value, leverage, credit rating, duration, stock momentum, asset growth, profitability and liquidity. More importantly, analyzing the optimal portfolio positions associated with different firm characteristics sheds light on the underlying trading motifs of investors.

Hansen and Jagannathan (1991) have documented that constructing optimal Sharpe ratio portfolios is equivalent to minimizing the variance of the SDF. The reason I focus on this type of SDF is twofold. First, the minimum variance SDF bounds the maximal attainable Sharpe ratio in the economy. Second, due to the equivalent (dual) relation between minimum variance SDF and the mean-variance portfolio frontier, it admits a closed-form representation in the return space. Hence, the minimum variance SDF is going to be a linear combination of the traded returns and can be easily interpreted as the optimal Sharpe ratio portfolio. Finally, this SDF and its associated optimal portfolio would be the relevant objects of study to measure cross-pricing relations and thus quantify the underlying degree of integration, especially in presence of frictions. Essentially, I am measuring the pricing error of a return in one market, implied by the pricing rule of the other market.

Under the assumption of frictionless markets, I show that it is always possible to construct empirically a common minimum variance SDF that prices both corporate bonds and stocks. However, upon a closer look at the associated optimal Sharpe ratio portfolio, this entails large and often negative positions, suggesting that in practice, if investors face leverage or short-selling constraints, it would be hard to construct. In the real world, markets are not free of frictions and an agent's choice set includes a subset of assets that she takes into consideration or is allowed to hold. Constraints in the choice set may arise due to investment mandates, certain benchmarks, or information frictions that restrain an investor's ability to examine a large universe of assets (Merton (1987)). My contribution is to study the degree of market integration in an extended framework, that incorporates frictions in the form of short-selling constraints when deriving the minimum variance SDFs. I focus on this type of restriction in order to obtain a portfolio that agents present in the corporate bond and/or stock market can readily form. Regarding the stock market, there have been instances in which regulators prohibited short-selling all together, such as during the recent financial crisis. On the other hand, trading in the corporate bond market is over-the-counter (OTC). Typically, short-selling bonds entails a significant cost, and in illiquid corporate bond markets, it might simply not be possible (see, e.g., Asquith, Au, Covert, and Pathak (2013), Blanco, Brennan, and Marsh (2005) and Bai and Collin-Dufresne (2019)). Moreover, regulators might forbid certain type of institutional investors, such as insurance companies and pension funds, to engage in short-selling activities.

Consequently, I investigate what are the optimal portfolio implications and cross-market pricing relations when short-selling constraints are imposed. When taking into account short-selling constraints, I show that the wealth is typically going to be allocated just in one portfolio of stocks and bonds, that maximizes the Sharpe ratio. The cross-market pricing errors implied by the constrained minimum variance SDFs are between 10 and 50 basis points (b.p.) per month, and hence within the observed bid-ask spreads. This evidence suggests that even when accounting for short-selling constraints, markets exhibit a high degree of integration, as no profitable arbitrage opportunities arise. Hence, the degree of market integration is better than previously thought. Still, the pricing errors will differ across portfolio sorts, with bond characteristics such as credit rating and duration yielding smaller stock mispricing, whereas valuation ratios for stock portfolios implying a lower mispricing for bond portfolios. In particular, larger firms, with more liquid corporate bonds and stocks, appear to be more integrated. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a higher extent of integration between their debt and equity securities.

A natural question arising is whether the constrained model-free extracted SDFs perform better than existing factor models for bonds and stocks. As competing models, I consider the Fama and French (1993) 3 Factor Model, proxying for market, size and value and the bond factor model of Bai, Bali, and Wen (2019) capturing exposure to the market, downside risk, credit and liquidity factors, as well as a combination of the two models. To discriminate between the various models, I use two criteria: (i) the size of the implied pricing errors and (ii) the R-Square of a generalized least squared (GLS) regression of returns on the proposed factors. The reason I focus on GLS R-Square rather than the OLS one follows from the critique of Lewellen, Nagel, and Shanken (2010), that apparently strong explanatory power in fact provides rather weak support for a model, especially if the test assets are the ones formed on size and book-to-market portfolios. A horse race between factor models and the

SDFs suggests the latter yield on average smaller pricing errors, and can have a higher GLS R-Square. Therefore, the constrained SDFs not only exhibit a better pricing performance, but can be interpreted economically as optimal portfolios of traded assets. Finally, to rationalize the risk factors embedded in the model-free SDFs, I show that the SDFs incorporating short-selling constraints appear to be closely linked to the market factor and the intermediary risk factor of He, Kelly, and Manela (2017).

1.1 Related Literature

This paper contributes to two strands of literature. First, it is related to the large body of studies examining market integration. Extant literature has focused on two approaches when studying market integration. The first relies on a parametric specification for asset pricing models, usually expressed in terms of factor models. The underlying idea in this framework is that there should be a small number of priced factors driving the behavior of risk premia. Market integration will then imply that the corresponding prices of risk of each factor are equal across markets. An extensive body of literature studies the cross-sectional drivers of stock returns. Corporate bonds have received limited attention, even though they represent a significant portion of the financial markets.[2] Recent advances suggest that a factor model accounting for market risk, downside risk, credit, and liquidity risk performs well in explaining the cross-section of bond returns (Bai, Bali, and Wen (2019)). Still, existing factor models explaining stock returns do not perform well in fitting the cross-section of corporate bond returns. Possible explanations for these discrepancies seem to suggest different risk factors for the two asset classes (Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017)), or potential market frictions leading to differences in prices, such as financial constraints and limits to arbitrage (Shleifer and Vishny (1997), He and Krishnamurthy (2013), Kapadia and Pu (2012)). In general, the main limitation arising in such parametric tests lies in specifying the correct asset pricing model, as a rejection of market integration can simply occur from potential misspecifications.[3]

The second is a nonparametric approach, pioneered by Chen and Knez (1995), which tests the degree of integration by minimizing a distance between the SDFs of the two markets under scrutiny. Whenever this distance is zero, the two SDFs coincide and thus markets are integrated. Departures from zero reflect deviations from perfect integration and quantify the associated mispricing. This methodology is better suited for testing the law of one price across markets, rather than absence of arbitrage opportunities. In this paper, I adopt a nonparametric approach in order to study the degree of market integration between corporate bonds and stocks, by examining the optimal portfolio implications and cross-market pricing relations of Sharpe ratio investors. I contribute to this literature

[2] According to the Federal Reserve Flow of Funds, as of the fourth quarter of 2018, the total amount outstanding of corporate bonds amounts to \$13.5 trillion, compared to the US equity market of \$35 trillion.

[3] Gagliardini, Ossola, and Scaillet (2019) propose a diagnostic criterion for approximate factor structure in large cross-sectional equity datasets. An asset pricing factor model is correctly specified for a set of returns whenever the associated error terms are only weakly cross-sectionally correlated. Otherwise, the criterion indicates the number of potential omitted factors. This methodology is constructed for individual equities, i.e. for a large number of test assets, and it may yield imprecise results when considering portfolios as test assets. Moreover, even if the model is correctly specified from a statistical point of view, mispricing might not be entirely ruled out.

by studying the degree of market integration in an extended framework, that incorporates frictions in the form of short-selling constraints when deriving the SDFs. I focus on this type of restriction in order to obtain a portfolio that agents present in the corporate bond and/or stock market can readily form.

The degree of market integration has been initially studied in an international framework, where the idea is that prices of global risk should be identical across countries. Following an international CAPM model, the only priced risk should be the systematic risk relative to the world market (Stehle (1977), Errunza and Losq (1985), Jorion and Schwartz (1986)). International markets appear to be mildly segmented, as different countries have different prices of risk associated with the global factor. Dumas and Solnik (1995) test a conditional version of the international CAPM, while including an additional factor capturing foreign-exchange risk premia and find no evidence of segmentation between currency markets and stock markets. Bekaert and Harvey (1995) introduce a measure of time-varying market integration stemming from a regime-switching model. Their approach allows conditionally expected returns in any country to be driven by their covariance with a world market portfolio and by the variance of the market portfolio in the respective country. De Roon, Nijman, and Werker (2001) provide evidence that incorporating short-sale constraints and/or transaction costs leads to vanishing diversification benefits of US investors from emerging markets, highlighting the importance of accounting for trading costs when computing the risk premia.

More recently, Pukthuanthong and Roll (2009) propose as a measure of global market integration the R-Square from time-series regressions of country aggregate equity indices on common global factors. Accordingly, an R-Square equal to one implies perfect integration. Translated into an SDF approach, there exists a common SDF, correctly pricing the market portfolio in each country. Hau (2011) provides a test of international market integration that overcomes a benchmark model specification by designing an event study surrounding the MSCI index redefinition that took place in December 2010. The findings support the view of a global, rather than local, market benchmark as being appropriate for the MSCI stocks and rejects market segmentation even between developed and emerging markets. Using individual stocks, Chaieb, Langlois, and Scaillet (2018) estimate international factor models with time-varying factor exposures and find an excess country market factor which is priced, implying that markets are segmented at both world and regional levels. Sandulescu, Trojani, and Vedolin (2019) show that market segmentation is a key ingredient to address salient features of international asset returns, while keeping cross-country correlations at moderate levels. Koijen, Richmond, and Yogo (2019) document that a small set of six firm characteristics related to risk, productivity, and profitability explains most of the variation in a panel of firm-level valuation ratios across countries and determines investors' demand functions.

Overall, the evidence suggests that despite the progressive removals of trade and legal barriers, and the fact that international integration has improved over time, markets are still fairly segmented. I contribute to this literature by using a model-free approach to quantify the extent of market integration. Intuitively, I am measuring the pricing error of a return in one market, implied by the pricing rule of the other market. Furthermore, the methodology employed is general and can accommodate conditioning information in order to allow for the degree of market integration to change through time. The focus of my study is directed towards local, rather than international, market integration. Quantifying the extent of integration at the local level is a natural starting point, since if these markets exhibit segmentation, it is likely that it will spill over to international markets. In this context, understanding what type of firm features leads to more integrated markets is key, as it may explain investors' incentives to trade certain classes of assets.

The second strand of literature is fairly recent and studies the degree of integration between corporate bonds and stock markets. The existence of differences in expected returns for the two asset classes is typically attributed to different risk factors (Fama and French (1993), Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017)), or the presence of heterogeneous investors (Dumas (1989), Chien, Cole, and Lustig (2011)). In particular, different classes of investors hold these two assets: stocks are readily available and mainly held by households, whereas institutional investors, such as insurance companies, pension funds or mutual funds dominate the corporate bond market. Despite the existence of different risk factors or different types of investors, I argue that this does not necessarily imply that stock and bond markets are segmented. In fact, if there exists an agent able to trade in the two markets, it suffices to render them integrated. For instance, Ma (2018) documents that non-financial firms act as cross-market arbitrageurs in their debt and equity securities.

In related work, Choi and Kim (2018) test market integration between corporate bonds and stock markets by examining the relationship between risk premia in the two markets. Fitting different factor models to corporate bond returns and stocks, they find that the prices of risk tend to differ across the two markets, suggesting a limited degree of market integration. I complement this study by examining portfolios of corporate bonds and stocks formed on a larger set of anomaly variables, as well as sorts following from implications of structural models. To have a higher cross-sectional dispersion, I consider additionally portfolios derived based on double sorts. My findings suggest that corporate bonds and stock markets exhibit a high degree of integration within characteristic sorts, even after incorporating short-selling constraints. Moreover, since I employ a nonparametric approach, this circumvents the issue of possible misspecifications, while the derived SDFs admit an economic interpretation in terms of optimal Sharpe ratio portfolios.

Finally, there are several market frictions that might segment corporate bonds and stock markets, including financial constraints and limits to arbitrage (Shleifer and Vishny (1997), Duarte, Longstaff, and Yu (2006), He and Krishnamurthy (2013)), or slow moving capital (Duffie (2010), Greenwood, Hanson, and Liao (2018)). Kapadia and Pu (2012) document short-horizon pricing discrepancies across firms' credit and equity markets, pointing to a lack of integration between the two markets, stemming from limits to arbitrage. Analyzing the credit default swap (CDS)-equity markets, the authors propose a statistical measure of market integration, using the concordance of price changes in the two markets. Specifically, their definition of market integration is based on the frequency of instances when price discrepancies, i.e. arbitrage opportunities, arise for a pair of stock prices and CDS spreads. Different from this study, I focus on returns for corporate bonds, which cover a larger part of the universe of US firms, since CDS do not span all corporate bonds. I quantify the level of market integration by analyzing the cross-market pricing relations of optimal Sharpe ratio portfolios of corporate bonds and stocks formed based on firm-level characteristics. Accounting for short-selling constraints, I find that the pricing discrepancies across portfolios sorted on the same characteristics are typically contained inside the quoted bid-ask spreads, suggesting an absence of profitable arbitrage opportunity.

The rest of the paper is organized as follows. Section 2 provides the theoretical framework for studying market integration and model-free SDFs. Section 3 describes the potential determinants driving corporate bond and stock returns. Section 4 presents the data and the main empirical findings. Section 5 contains robustness checks. Section 6 concludes the paper.

2 Market Integration and Stochastic Discount Factors

Using a model-free approach, this paper studies the degree of market integration across and within different asset classes, with an emphasis on corporate bonds and stocks of the same firm, accounting for various characteristics. Additionally, an analysis of the properties of minimum variance stochastic discount factors (SDFs) is provided. The goal is to document the extent of market integration and pricing performance implied by different SDFs in presence of market frictions, such as short-selling constraints.

Consider an economy populated by square integrable random variables X defined on an L^2 space, with corresponding norm $||X|| \equiv \sqrt{\mathbf{E}[X^2]}$. In this economy, investors can trade in a set of N assets consisting of corporate bonds and stocks of the issuing firms. The corresponding gross returns are gathered in a vector $\mathbf{R}$ of length N. Let M denote a generic stochastic discount factor (SDF) pricing return vector $\mathbf{R}$. Investors can construct portfolios of the form:

$$\mathcal{P} = \{R_p : R_p = \mathbf{R}'\theta, \quad \theta \in C \subseteq \mathbb{R}^N\}, \tag{1}$$

with θ being the vector of portfolio weights.

Case 1. Under the assumption of *frictionless* markets, the no-arbitrage condition implies the following Euler equation:

$$\mathbb{E}[M\mathbf{R}] = \mathbf{1}, \tag{2}$$

with $\mathbf{1}$ denoting the vector of ones of appropriate size.

The set of admissible SDFs is defined as:

$$\mathcal{M} = \{M \in L^2 : M \geq 0, \quad \mathbb{E}[M\mathbf{R}] = 1\}. \tag{3}$$

Accordingly, an admissible SDF is a random variable with finite second moment, that has to be nonnegative and has to price correctly the set of available returns. Moreover, investors can form portfolios freely, i.e. $C = \mathbb{R}^N$.

To define a market integration measure, assume there are two markets, denoted by B and S, with corresponding SDFs M_B and M_S, for which the law of one price holds separately. This assumption does not require pricing consistency across the two markets, but whenever two markets are perfectly integrated, they will assign identical prices to identical returns. Different asset classes, or assets belonging to the same class but divided into different characteristics can form the two markets. For the purpose of the present study, the two markets considered are the one pertaining to the corporate bonds and the remaining one to the stocks of the issuing firms.

Let $\mathcal{M}_B$ and $\mathcal{M}_S$ be the sets of admissible SDFs for markets B and S, pricing returns $\mathbf{R}_B$ and $\mathbf{R}_S$, respectively. Since integration implies that pricing across markets should be as close as possible, a natural way to define a measure of integration is to minimize the distance between the SDFs in the two markets. To this end, I build upon the approach of Hansen and Jagannathan (1997) (HJ, henceforth). The idea is to minimize the distance between a set of admissible SDFs $\mathcal{M}_S$ and a candidate SDF $y \in \mathcal{M}_B$. Specifically, the problem reads:

$$\delta(y, S) = \min_{M_S \in \mathcal{M}_S} ||y - M_S||^2, \tag{4}$$

with $|| \cdot ||^2$ denoting the squared L^2 norm. Intuitively, this measure is going to reflect how differently the two markets assign prices to assets. Naturally, markets are going to be integrated whenever the two SDFs will coincide. The greater the cross-market pricing differences, the stronger the evidence of market segmentation between the two markets considered. In the data, integration between two markets can be tested by estimating this distance directly. Notice that this test is independent of any parametric specifications for the underlying asset pricing models. In summary, the integration measure $\delta(\cdot, \cdot) = 0$ can be interpreted as a benchmark cross-market relation. In all other circumstances, it provides an estimate of the maximal pricing error between the two markets, given by $\sqrt{\delta(\cdot, \cdot)}$. Put differently, it represents the maximal arbitrage opportunity from buying a return in one market and simultaneously selling it in the other.

There are two interconnected issues arising: (i) the SDF is not observable and (ii) how to optimally pick the candidate SDF y. Since the goal is not to rely on any parametric specifications, I derive a well-established SDF in the literature, the minimum variance SDF of Hansen and Jagannathan (1991), which bounds the maximal attainable Sharpe ratio in the economy. It follows that due to the equivalent representation of minimum variance SDFs in the return space, in terms of optimal portfolios, one can derive the SDFs directly from observable prices. In other words, the minimum variance SDF problem can be recast equivalently through (Korsaye, Quaini, and Trojani (2018)):[4]

$$\begin{aligned} M^* &= \min_{M \in \mathcal{M}} \frac{1}{2} \mathbb{E}[M^2] \\ R_p^* &= \max_{\theta \in \mathbb{R}^N} \mathbb{E}[\mathbf{1}'\theta - (\mathbf{R}'\theta)^2/2] \quad \textit{s.t.} \quad \mathbf{R}'\theta \geq 0. \end{aligned} \tag{5}$$

[4]The approach can be generalized to accommodate a large family of dispersion measures, such that higher-order moments of returns are taken into account.

Accordingly, solving the optimization problem expressed in terms of returns and portfolio weights yields closed-form solutions for model-free SDFs that can be estimated directly from the data. This powerful insight will set the ground for the empirical analysis section of this study.

Specifically, the minimum variance SDF reads:

$$M^* = \mathbf{R}'\theta^* = R_p^*, \tag{6}$$

with $\theta \in \mathbb{R}^N$ being the optimal vector of portfolio weights that delivers the maximal attainable Sharpe ratio in the economy. Since M^* is linear in the set of returns $\mathbf{R}$ available to investors, it is also the only tradable SDF in the set $\mathcal{M}$.

Finally, the problem in (4) quantifies the maximal pricing error across the two markets. Using the pricing rule of one market to price the returns in the other market yields the actual cross-market pricing performance. To determine how well do bond minimum variance SDFs price stock returns and vice versa, I compute the following pricing error:

$$PE_j = \mathbb{E}[M_i^* \mathbf{R}_j] - \mathbf{1}, \tag{7}$$

with PE_j denoting the pricing error for returns in market $j = B, S$ implied by the SDF in market $i = B, S$ and $i \neq j$. Whenever the implied pricing errors are zero, it follows that the minimum variance SDF can be used to price both sets of returns, consistent with absence of arbitrage opportunities across markets. Notice however that in practice, differences in cross-market pricing, or pricing errors, should be compared with trading costs in order to assess whether they lead to profitable arbitrage opportunities.

Case 2. In presence of market *frictions*, the constrained SDF, denoted by $\widetilde{M}$, will yield an error when pricing gross returns $\mathbf{R}$. In particular, the pricing relation is going to be sublinear (see, e.g. Luttmer (1996)):

$$\mathbb{E}[\widetilde{M}\mathbf{R}] \leq \mathbf{1}, \tag{8}$$

with the inequality holding componentwise. Market frictions, such as short-selling constraints, that might be enforced by some regulatory institution, or transaction costs, can be incorporated by adjusting either the set of possible weights, or the returns. In particular, in presence of short-selling constraints, $C = \mathbb{R}_+^N$, the nonnegative part of $\mathbb{R}^N$. With transaction costs, the set of returns will be augmented to reflect the long and short positions at the corresponding bid and ask prices, i.e. the dimension of the return vector becomes $2N$.

The set of admissible SDFs in presence of market frictions is given by:

$$\widetilde{\mathcal{M}} = \{\widetilde{M} \in L^2 : \widetilde{M} \geq 0, \quad \mathbb{E}[\widetilde{M}\mathbf{R}] \leq 1\}. \tag{9}$$

The duality relation between optimal portfolio of returns and minimum variance SDFs holds also in presence of different market frictions. The solution is obtained by including the additional restrictions in Equation (5), i.e. $M \in \widetilde{\mathcal{M}}$, or, equivalently, $\theta \in \mathbb{R}^N_+$. Specifically, the minimum variance SDF is still going to be a linear function of the returns for which the short selling constraints are not binding.

To study the degree of market integration in presence of financial frictions, while being consistent with a model-free approach, I employ the HJ distance previously described. Put differently, my goal is to identify whether there is a common SDF for corporate bond and stock returns, after accounting for market frictions such as short-selling constraints. To this end, I quantify the distance between a constrained SDF in one market and the set of admissible SDFs in the other market, according to Equation (4). In this case, the candidate SDF y will be derived consistently with the frictions and the HJ distance will provide an estimate of the maximal pricing error made when pricing returns in market S. Moreover, cross-market pricing relation should satisfy the relation in (8). Intuitively, since market integration implies that similar returns should be assigned similar prices, the pricing error implied by the restricted common SDF should be economically small.

Definition 1. In presence of frictions, if the following relations hold, markets are integrated:

$$\begin{aligned} \sqrt{\delta(\widetilde{M}_B, S)} &\leq \tau_S \\ \sqrt{\delta(B, \widetilde{M}_S)} &\leq \tau_B, \end{aligned} \tag{10}$$

with τ_S (τ_B) denoting the corresponding bid-ask spread in market S (B).

Hence, in the empirical application, I compare the entailed pricing errors with the size of the bid-ask spreads. The view is that if the pricing errors are within the quoted bid-ask spreads, there are no arbitrage opportunities across markets.

Since returns on corporate bonds and stocks can be driven by different firm-level characteristics, which affect investors' trading motifs, I outline in the subsequent section what are the potential determinants, as well as their implications on the underlying degree of integration.

3 Potential Determinants of Corporate Bond and Stock Returns

In the following, I describe what are the potential factors driving expected returns for corporate bonds and stocks. The factors are chosen to reflect both firm-level and asset specific characteristics. In addition, I consider well-established variables, or anomalies, documented in the literature to price the cross section of stock returns. The idea is to examine whether they have the same implications also for bond returns, since a contingent claim approach would imply a strong link between the risk premia on the two asset classes. Moreover, these variables will constitute the building blocks of different portfolio sorts for stocks and bonds, which will represent the test assets for constructing the SDFs and quantifying the underlying degree of integration. These variables will be relevant particularly in presence of different market frictions, such as short-selling constraints, when investors will likely form their optimal portfolios by taking into account certain firm characteristics. I consider the following variables.

Credit Ratings. The underlying credit riskiness of firms reflect their ability to pay back their debt and provide an estimate of the probability of default. Depending on their credit riskiness profile, different classes of investors will hold different assets (Collin-Dufresne and Goldstein (2001)). For the corporate bond market in particular, asset-class-sensitive institutional investors tend to segment the markets between investment-grade (safe) and high-yield (junk) bonds (Chen, Lookman, Schürhoff, and Seppi (2014)). Consequently, different market participants can interpret these two types of bonds as distinct asset classes, including passive and active asset managers, asset owners in general and even regulators, who might prohibit certain types of institutional investors from holding high-yield bonds and prevent them from engaging in short-selling activities. As a typical example, pension funds and insurance companies hold mainly investment-grade bonds, whereas hedge funds have a preference for riskier high-yield bonds. Portfolio managers usually view high-yield corporate bonds as exhibiting a behavior similar to stocks, due to their sensitivity to the market conditions, especially during bad times. For instance, using high-yield bond data, Ronen and Hotchkiss (1999) provide evidence that bond and stock returns react jointly to common factors. Moreover, in frictionless markets, the high-yield bond can be replicated using the risk-free interest rate and a put option on the firm value (Merton (1974)). It is worthwhile to mention that the focus of this study is on credit ratings, as opposed to credit default swaps (CDSs), because the latter do not span all corporate bonds and will bias the selection towards large issuing companies. Hence, it is interesting to examine the implications regarding the underlying degree of market integration of sorting corporate bonds and stocks into portfolios based on credit ratings. Notice however that in practice, it suffices that one investor is able to trade both corporate bonds and stocks in order for the markets to be integrated. If adjusting the price discrepancies across the two markets would incur high transaction costs, then these differences might persist.

Duration. The average period of time until the cash flows are received can represent another dimension along which the extent of market integration might vary. In particular, according to the preferred-habitat theory, introduced by Culbertson (1957) and refined by Vayanos and Vila (2009), which is a variant of the market segmentation theory, different investors prefer one maturity length over another. For instance, pension funds have a preference for longer maturities than mutual funds, in order to match their obligations. In general, an investor will buy a bond with a different maturity than his preferred one, only for a premium. Regarding the stock side, Lettau and Wachter (2007) develop a dynamic model in which firms are perceived as long-lived assets discriminated by the timing of their cash-flows. Using the corresponding book-to-market (BM) ratio, the authors show that growth firms are high-duration assets, similar to long-term bonds, whereas value firms are low-duration assets. Analyzing bond and stocks' portfolios of different duration represents thus an appealing feature to study the level of market integration.

Size. Smaller firms require on average a higher expected return than larger firms (Banz (1981), Fama and French (1992)). Market capitalization of firms might thus drive the behavior of investors for trading. Since larger firms entail a lower expected return on their stocks and are less likely to default, in the data their bonds and stocks might be more closely integrated than those of small firms.

Value. Value firms are generally trading at a lower price than their fundamentals, as measured by their large BM ratios. Consequently, they have a higher expected return than growth firms (Fama and French (1992)). On the other hand, enhancing firm growth prospects may imply a lower likelihood of defaulting. Following Pastor and Veronesi (2003), the BM ratio can be used to proxy for future profitability. Therefore, the BM influences the underlying value of the firm, driving the trading incentives of investors and might have an impact on the degree of integration between corporate bond and stock markets.

Leverage. Firms that exhibit higher leverage ratios are more indebted and typically closer to default (Merton (1974)). As such, for highly leveraged firms, the expected return on bonds should be larger. Since leverage affects the firm value, it might also influence the extent of market integration between corporate bonds and stocks. Leverage is computed as the ratio between book value of debt and the sum between the book value of debt and the market value of equity.

Stock Momentum. Jegadeesh and Titman (1993) document that past stock winners tend to outperform past losers over short to medium horizons, a phenomenon referred to as momentum. Higher stock momentum thus imply a higher valuation of the firm, which could potentially decrease the probability of default. In addition, Avramov, Chordia, Jostova, and Philipov (2007) provide strong evidence of momentum among low credit quality firms, suggesting that high-yield bonds are likely more affected. The return stock momentum is computed as the cumulative 12 months return skipping the most recent one.

Asset Growth. Firms with improved asset growth are farther away from default. The findings of Cooper, Gulen, and Schill (2008) suggest a strong relation between asset growth and stock returns. In line with the aforementioned study, asset growth is measured as the percentage change in total assets.

Profitability. Firms with higher profitability are likely to be distant from the boundary triggering the default and to have on average higher expected returns (Fama and French (2008)). Profitability is derived as the ratio of equity income to book equity.

Liquidity. Pástor and Stambaugh (2003) provide evidence that liquidity is an important factor explaining the cross section of stock returns. Similarly, liquidity appears to drive also the returns on corporate bonds (Chen, Lesmond, and Wei (2007)). Liquidity represents thus a meaningful dimension on which to characterize not only the behavior of returns, but also the implied degree of market integration. In particular, the two classes of assets trade on different platforms, and corporate bonds are more expensive to trade than stocks, i.e. on average an order of magnitude higher, as measured by their bid-ask spreads. Accounting for trading restrictions will be particularly relevant for illiquid bonds, as these are precisely the ones difficult to short-sell (Blanco, Brennan, and Marsh (2005)). In the empirical analysis, I measure liquidity using the percent quoted spread, computed as the ratio between the bid-ask spread and the mid price (see, e.g. Fong, Holden, and Trzcinka (2017) for a comparison of liquidity proxies).

4 Empirical Analysis

In this section, I describe the data sources and the construction of the variables of interest. Further, I estimate the degree of integration between corporate bond and stock markets by considering different portfolios sorted on firm-level characteristics. The reason I focus on portfolio sorts is twofold. First, deriving SDFs that correctly price a large cross-section of returns might not be feasible using standard techniques. For instance, to provide a comparison between the performance of model-free SDFs and existing factor models, I will employ the Fama MacBeth procedure, which cannot handle a large number of test assets. Second, constructing sorts is beneficial because it provides a simple picture of how average returns vary across a given characteristic. Moreover, aggregating returns into portfolios eliminates noise and delivers more precise estimates, while improving efficiency. In addition, I use value-weighted returns when sorting stocks or corporate bonds on variables of interest, in order to avoid the estimates to be driven by small companies. Next, I provide comparative statistics for the derived minimum variance SDFs introduced in the theory section and examine their time series properties. Further, I supply a horse race between the pricing performance of model-free extracted SDFs in presence of market frictions in the form of short-selling constraints and existing stock and bond factor models. Finally, to rationalize the risk factors embedded in model-free extracted SDFs, I explore their link with the market factor and an intermediary risk factor.

4.1 Data

The data for corporate bonds and the associated stock of the issuing firms is obtained by merging four different databases. The sample comprises the entire universe of US corporate bonds satisfying selection criteria commonly used in the literature. FINRA's TRACE (Trade Reporting and Compliance Engine) database reports the most comprehensive data regarding corporate bonds transactions, including the price and the trading volume. In particular, it provides information regarding over-the-counter (OTC) secondary market transactions by all market participants, who are mainly large institutional investors, such as insurance companies, pension funds and mutual funds.

Using the TRACE database to study corporate bond returns is appealing because it reports actual transaction prices from dealers, which enables a natural framework for examining the economic forces of the market. On the other hand, databases that report matrix-based quotes, i.e. a price derived by a rule of thumb based on bond characteristics in absence of a dealer quote, although they might have a longer sample period, they can obscure the market mechanisms. These serve more as indicative quotations, and not as an obligation on price and quantity (Elton, Gruber, Agrawal, and Mann (2001)).

The sample period spans January 2005 to December 2017, with a monthly frequency. Since corporate bonds are not as liquid as stocks, it is very likely that for many of them no transactions are encountered at a daily or higher frequency. The beginning of the sample period corresponds to the date when reporting to TRACE became mandatory. This circumvents issues with regards to possible biases arising in the preceding period, dating back to July 2002 (the launch date), when companies were reporting to TRACE based on a voluntary basis. Corporate bond issue characteristics, such as bond type, issue and maturity date, the offering amount, the coupon rate, as well as information about the issuing firm, including the credit ratings, are retrieved from Mergent FISD. Moreover, the WRDS Bond Returns database merges the two aforementioned databases and provides information about corporate bonds monthly returns, as well as the link with equity issues for every firm, through the CRSP database.

The filters used follow the standard literature (see, e.g., Bai, Bali, and Wen (2019)). As a first step, cleaning the data requires eliminating the three most common errors encountered in the trade reports filled by dealers, i.e. cancellations, corrections and reversals (Dick-Nielsen (2009)). The bond type has to fulfill the following criteria: (i) coupon type is fixed or zero, (ii) not under Rule 144A, (iii) the bond type is senior, either debenture, medium term note, or medium term note zero, i.e. no convertible or assets backed bonds are considered. Further minor corrections include filtering for missing data, such as missing price, volume or data ranges.

Additionally, the following filters are considered: (i) remove bonds maturing in less than a year, since these are the ones that are going to be delisted from indices,[5] (ii) remove bonds that have missing credit ratings or duration, (iii) focus on bonds having a trading volume higher than $\$10,000$[6] and (iv) remove bonds that do not have corresponding stock data in CRSP and Compustat. Finally, to avoid bonds that show up just for several months and then disappear from TRACE, I require the bonds to be in the TRACE data for at least one full year.

Lastly, equity data for the issuing firms regarding price and shares outstanding is retrieved from CRSP, whereas firm fundamentals are obtained from Compustat.

4.1.1 Variable Definitions

In this subsection, I illustrate how the variables of interest used in the empirical analysis are constructed.

For each corporate bond $i = 1, ..., I$, the monthly gross return is computed as follows:

$$R_{i,t+1} = \frac{P_{i,t+1} + C_{i,t+1} + AI_{i,t+1}}{P_{i,t} + AI_{i,t}}, \tag{11}$$

where $P_{i,t+1}$ is the price, $C_{i,t+1}$ is the coupon payment and $AI_{i,t+1}$ the accrued interest of bond i at time $t + 1$.

[5] Since trading corporate bonds is mainly performed over-the-counter (OTC), the main class of investors are institutional ones, such as insurance companies, pension funds, mutual funds and to a lesser extent, hedge funds. Insurance companies and pension funds typically follow long-term, buy-and-hold strategies, so they do not rebalance their portfolios often. As such, they tend to discard the bonds having a maturity less than a year.

[6] Bessembinder, Maxwell, and Venkataraman (2006) remove transactions with trading volume less than $\$100,000$, to reflect trades from institutional investors.

Similarly, the gross return of stock $j = 1, ..., J$ reads:

$$R_{j,t+1} = \frac{P_{j,t+1} + D_{j,t+1}}{P_{j,t}}, \tag{12}$$

where $P_{j,t+1}$ is the price and $D_{j,t+1}$ is the dividend payment of stock j at time $t+1$.

The liquidity proxy, or percent quoted spread, for stock $j = 1, ..., J$ is derived as:

$$L_{j,t+1} = \frac{P^A_{j,t+1} - P^B_{j,t+1}}{(P^A_{j,t+1} + P^B_{j,t+1})/2}, \tag{13}$$

where P^A (P^B) denotes the ask (bid) price.

The stock return momentum used to construct sorts is computed as:

$$R^{MOM}_{j,t} = \prod_{s-13}^{s-1} R_{j,s}. \tag{14}$$

The leverage of a firm is defined as the ratio between the book value of debt and the sum between the book value of debt and common equity. The profitability of a firm is derived as the ratio between operating profit and book equity. Operating profit is equal to revenue minus costs of goods sold, interest expense and selling, general and administrative expenses. The size of a firm is computed as the market value of equity, whereas the value as the ratio between the book value of equity and market value of equity. Asset growth reflects the annual percentage change in total assets.

After applying the corresponding filters, the data consists of $443,206$ bond-month observations and $93,253$ unique stock-month observations. Provided that a company can have multiple bonds outstanding at a given point in time, the number of bond observations is naturally higher. To illustrate, throughout the sample period considered, there are in total $1,636$ firms and $11,743$ bonds. The construction of portfolio sorts proceeds as follows. For deciles based on size, book-to-market, leverage, asset growth and profitability, the sorts are done once a year in June and monthly value-weighted returns are calculated from July through June the following year (Fama and French (1993)). For momentum, the deciles sorts are constructed based on the cumulative 12-month returns, excluding the most recent month. For liquidity, the deciles sorts are derived based on the previous month transaction cost incurred, measured as the percent quoted bid-ask spread. For credit rating and duration, the quantile sorts are formed based on observations in the preceding month. A higher numerical score for the credit rating indicates a riskier corporate bond. To proxy for the overall credit rating of the stock, I take the average credit ratings of the issuing firms' outstanding bonds.

Finally, the methodology yields 10 portfolios of corporate bonds and of stocks using value-weighted returns sorted on size, value, momentum, leverage, asset growth, profitability, liquidity and 5 portfolios sorted on credit rating and duration. The reason I use quantiles for credit rating and duration rather than a higher granularity is because bonds are highly concentrated across this two dimensions. In particular, 80% of the bonds in the sample are investment grade, the rest being high-yield bonds.

4.1.2 Summary Statistics

The summary statistics regarding corporate bond and stock returns, as well as firm-level characteristics are reported in Table 1. It follows that, on average, the representative bond has a monthly return of 0.544% and is an investment grade bond, having a numerical rating of 8.37 (BBB) and a duration of 6.48 years. The average firm is a growth firm, mid-cap, with a market capitalization of about 13 billion, a leverage of 0.34 and a book-to-market ratio of 0.7. Moreover, it has on average a profitability of 12% and an annual asset percentage growth of 2.8%. The corresponding momentum return over the past 12 months yields about 13% on average, however it exhibits substantial cross-section variation, with a median of 6%. The average percent quoted bid-ask spread for individual stocks is about 8 b.p., whereas the one for bonds is one order of magnitude higher, around 0.7%.

Table 1. Summary Statistics

This table reports the mean, median and the standard deviation of the returns, numerical rating and duration for bonds and stocks. The credit rating is expressed in conventional numerical scores, and can take values between 1 (AAA rating) and 21 (C rating). Higher numerical score means higher credit risk. Numerical ratings of 10 or below (BBB- or better) are considered investment grade, and ratings of 11 or higher (BB + or worse) are labeled high yield. Duration is expressed in years. Size is computed as the natural logarithm of market capitalization. Value is the book-to-market equity value. Leverage is the ratio between book value of debt and the book value of debt plus common equity. Momentum is the cumulative return over the previous 12 months skipping the most recent month. Asset growth represents the percentage change in total assets. Profitability is the ratio between operating profit and book equity. Stock (bond) bid-ask spread is the ratio between the bid-ask spread and the mid price. Data is monthly and runs from January 2005 to December 2017.

	Mean	Median	Std
Bond Return	0.005	0.004	0.030
Stock Return	0.010	0.010	0.102
Credit Rating	8.367	8	3.334
Duration	6.484	5.295	4.036
Size	9.477	9.653	1.797
Value	0.703	0.550	1.533
Leverage	0.337	0.312	0.179
Momentum	0.134	0.059	0.686
Asset Growth	0.028	0.005	0.112
Profitability	0.122	0.064	1.843
Stock Bid-Ask Spread	0.0008	0.0003	0.0034
Bond Bid-Ask Spread	0.0069	0.0048	0.0074

Tables 2 and 3 report the average value-weighted portfolio returns for the different characteristics sorts considered. Panel A outlines the bond returns, whereas Panel B the stock returns. Specifically, Table 2 reports the portfolio returns formed on quantile credit rating and duration sorts. The safest bonds are in the lowest quantile, as measured by their credit rating number.[7] Naturally, they exhibit a lower return. High-yield bonds in the top quantile require a return double in size the one in the bottom quantile. A similar picture emerges for the stock returns, which increase in the credit ratings. Finally, bonds in the bottom quantile duration have lower returns than the ones in the top quantile, suggesting that investors typically require a higher return to hold longer maturity bonds. The same evidence is encountered also for the stock returns.

Table 2. Portfolio returns based on quantile sorts

This table reports the value-weighted returns of portfolios formed on credit rating and duration. Panel A reports the bond returns, whereas Panel B reports the stock returns. Data is monthly and runs from January 2005 to December 2017.

	Panel A: Bond Returns				
	Low	2	3	4	High
Credit rating	0.440%	0.560%	0.530%	0.530%	0.880%
Duration	0.290%	0.380%	0.480%	0.570%	0.600%
	Panel B: Stock Returns				
	Low	2	3	4	High
Credit rating	0.470%	0.640%	0.760%	0.620%	0.840%
Duration	0.710%	0.490%	0.520%	0.580%	0.740%

From Table 3, both bonds and stocks portfolios formed on decile size sorts yield a higher return for small firms (in the Low portfolio), than for large firms (High portfolio). Growth firms exhibit a lower return for bonds, consistent with the idea that their corresponding probability of default is lower. They also have a higher average stock return, congruent with the fact that value stocks have been underperforming after the crisis. Leverage appears to be negatively related with stock returns, as the portfolios in the bottom decile have a higher average return than the one in the top decile. The average bond returns are very stable across the leverage deciles, exhibiting thus a low cross sectional dispersion. This may arise as a result of other relevant firm or asset characteristics inside the bins. To explore this possibility in a more systematic way, I will consider double sorted portfolios as a robustness check. Momentum portfolio sorts deliver low stock returns in the bottom decile and high returns in the top decile, strengthening the idea that past winners tend to outperform past losers. The opposite effect is encountered for the bond returns, where the portfolio in the bottom momentum decile generates a higher return than the one in the top decile. Firms with higher asset growth entail a lower return both for stocks and bonds, in line with the findings of Hou, Xue, and Zhang (2015). At the same time, whereas the stock returns exhibit significant cross-sectional variation, the average

[7]The credit rating is expressed in conventional numerical scores, and can take values between 1 (AAA rating) and 21 (C rating). Higher numerical score means higher credit risk. Numerical ratings of 10 or below (BBB- or better) are considered investment grade, and ratings of 11 or higher (BB + or worse) are labeled high yield.

Table 3. Portfolio returns based on decile sorts

This table reports the value-weighted returns of portfolios formed on different sorts. Panel A reports the bond returns, whereas Panel B reports the stock returns. Data is monthly and runs from January 2005 to December 2017.

	Panel A: Bond Returns									
	Low	2	3	4	5	6	7	8	9	High
Size	0.729%	0.730%	0.696%	0.701%	0.614%	0.563%	0.558%	0.557%	0.526%	0.475%
Value	0.489%	0.466%	0.484%	0.445%	0.522%	0.486%	0.505%	0.450%	0.486%	0.650%
Leverage	0.460%	0.490%	0.480%	0.460%	0.510%	0.500%	0.480%	0.460%	0.490%	0.480%
Momentum	0.614%	0.436%	0.549%	0.482%	0.455%	0.530%	0.519%	0.540%	0.495%	0.395%
Asset Growth	0.480%	0.481%	0.469%	0.486%	0.503%	0.537%	0.545%	0.512%	0.504%	0.464%
Profitability	0.546%	0.549%	0.510%	0.489%	0.443%	0.521%	0.489%	0.485%	0.471%	0.465%
Liquidity	0.310%	0.350%	0.370%	0.420%	0.480%	0.550%	0.540%	0.590%	0.570%	0.710%
	Panel B: Stock Returns									
	Low	2	3	4	5	6	7	8	9	High
Size	1.014%	0.834%	1.004%	0.943%	0.987%	0.879%	0.900%	0.886%	0.884%	0.739%
Value	1.005%	0.874%	0.768%	1.036%	0.608%	0.737%	0.763%	0.729%	0.800%	0.690%
Leverage	0.750%	0.920%	0.730%	0.810%	0.900%	0.790%	0.650%	0.580%	0.810%	0.640%
Momentum	0.409%	0.618%	0.736%	0.858%	0.813%	0.997%	0.996%	1.086%	0.939%	1.301%
Asset Growth	0.866%	0.915%	0.875%	0.552%	0.706%	0.844%	0.987%	0.991%	0.846%	0.706%
Profitability	0.337%	0.719%	0.824%	0.933%	0.791%	0.730%	0.878%	0.805%	1.029%	0.845%
Liquidity	1.080%	0.740%	0.800%	0.330%	0.740%	0.600%	0.610%	0.270%	0.290%	0.380%

return on the bonds vary less across the deciles. Similar to the leverage sorts, some other firm or bond characteristics may be obscured, such as duration or credit riskiness. This will be addressed when constructing double sorts. Profitability sorted portfolios display a higher stock return when moving from the bottom to the top decile, consistent with the idea that more profitable firms earn higher equity returns (see, e.g., Fama and French (2015), Novy-Marx (2013)). However, since more profitable firms are likely further away from the default boundary and thus safer, they should entail a lower bond return, consistent with the results in Table 3. Bonds that are more liquid, i.e. they exhibit lower bid-ask spreads, have lower average returns. The stock average returns for liquidity decile sorts display significant cross-sectional variation. The absence of a monotone relationship might be due to the fact that both the liquidity measure and the holding period return are computed at a monthly frequency, in line with evidence provided by Liu (2006). Moreover, since returns are value-weighted inside the deciles, the returns of small, illiquid stocks will be assigned a tighter share.

In summary, forming portfolios of corporate bonds and the stock of the corresponding issuer based on firm characteristics sorts should not be arbitrary, as firm features can have different effects on the returns of the two classes of assets. Specifically, when sorting on size, leverage, asset growth, credit rating and duration, average bond and stock returns tend to follow similar patterns. Value, momentum, profitability and liquidity, on the other hand, induce opposite effects for average bond and stock returns. Moreover, the dispersion across decile portfolios varies depending on the characteristic considered.

4.2 Market Integration and Minimum Variance SDFs

In this section, I estimate the degree of integration and the minimum variance SDFs introduced in the theory. I start by describing the results derived under the assumption of frictionless markets and document why this setting yields unrealistic implications. Then, I provide findings obtained in presence of different market frictions. The frictionless markets case can be regarded as a benchmark where it is relevant to examine what are the implications of the optimal Sharpe ratio portfolio, in order to infer whether in the actual markets, investors would be able to construct it.

I. The Case of Frictionless Markets Assumption

4.2.1 Minimum Variance SDFs

Testing for market integration in the data is equivalent to testing whether there exists a common SDF pricing corporate bonds and stock returns. In the following, I derive the minimum variance SDFs that price exactly the portfolio returns of bonds, stocks, or both, sorted on the different firm-level characteristics. The SDFs are estimated by replacing the expectation operator in Equation (5) with the corresponding sample average and solving the corresponding convex optimization program. The annualized volatility of these SDFs are reported in Table 4. The minimum variance bond SDFs entail a higher annualized volatility than the minimum variance stock SDFs, for portfolios sorted on credit rating, duration, size, and momentum, suggesting that they offer a higher Sharpe ratio. In contrast, stock portfolios sorted on value, leverage, asset growth and profitability yield a higher Sharpe ratio than the bond portfolios. For some characteristics sorts, especially in the case of bond portfolios, some might argue that not every type of investor would be able to generate a Sharpe ratio as high as 1.6, particularly if it involves short positions. Perhaps a hedge fund will have no issues to form such an optimal portfolio, but insurance companies or pension funds might be unable to do so.

Table 4. Volatility of SDFs

This table reports the annualized volatility of the various SDFs derived that correctly price the portfolios of returns sorted on different characteristics. Data is monthly and runs from January 2005 to December 2017.

	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
Minimum Variance Bond SDF	1.124	1.039	0.721	0.805	0.524	0.708	0.672	0.824	1.632
Minimum Variance Stock SDF	0.366	0.363	0.689	0.897	0.836	0.640	0.919	0.840	1.017
Minimum Variance Bond & Stock SDF	0.819	1.160	1.040	1.290	1.088	1.252	1.272	1.390	2.383

To have a better understanding regarding the dynamics between marginal minimum variance SDFs pricing either bonds or stocks, I report in Table 5 their correlations. Several aspects are worth mentioning. First, most of the correlations between the bond and stock SDFs are not statistically different from zero. Second, firm-level characteristics play an important role, as they have different implications for the sign of the correlation, consistent with the results reported in Tables 2 and 3. For instance, the minimum variance SDF that prices portfolios of bonds sorted on credit ratings positively co-moves with the one pricing stocks sorted on credit rating. The opposite is observed for the SDFs pricing portfolios of returns sorted on profitability, as the minimum variance SDFs exhibit a negative correlation. In summary, different firm characteristics may entail contrasting effects regarding the average returns of corporate bonds and stocks. In particular, firm-specific information that is mainly related to the mean (variance) of the firm's underlying assets induce a positive (negative) contemporaneous correlation between corporate bonds and stocks (see, e.g. Kwan (1996)).

Table 5. Correlations between Bond and Stock Minimum Variance SDFs

This table reports the correlations between minimum variance bond and stock SDFs for different portfolio sorts considered. Data is monthly and runs from January 2005 to December 2017. ***, ** and * denotes significance at the 1%, 5% and 10% level, respectively.

		Minimum Variance Bond SDF								
		Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
Stock SDF	Credit rating	0.296***	-0.199**	0.260***	0.009	0.051	0.059	0.169**	-0.050	-0.018
	Duration	0.163**	-0.115	0.105	-0.019	0.062	0.111	-0.006	-0.125	-0.085
	Size	0.020	-0.014	0.137*	-0.040	0.028	0.059	0.046	0.013	-0.070
	Value	0.063	-0.016	0.040	-0.073	0.152*	-0.158**	0.050	0.127	-0.031
	Leverage	−0.156*	-0.027	0.008	0.124	0.143*	0.131	-0.208***	-0.129	-0.042
	Momentum	0.095	-0.228***	0.079	-0.130	0.102	0.068	-0.068	0.030	-0.013
	Asset Growth	0.187**	0.067	0.088	-0.113	0.066	0.128	0.089	-0.106	0.040
	Profitability	0.068	-0.018	0.033	-0.018	0.209***	0.050	0.087	-0.250***	-0.070
	Liquidity	0.095	0.003	0.005	-0.133*	0.019	0.146*	0.068	-0.062	-0.182**

Having established a rather low correlation between the minimum variance bond and stock SDFs, it is relevant to examine the pricing performance implied by one type of SDF when applied to the other market. Indeed, return correlations entail little information regarding the underlying degree of market integration and one should examine instead the cross-pricing relations. To this end, I report in Table 6 the pricing errors implied by minimum variance SDFs, computed as $\mathbb{E}[M_B R_S] - 1$ and $\mathbb{E}[M_S R_B] - 1$, with S denoting the stocks and B the bonds. For illustrative purposes, the results are reported for the minimum variance SDFs that correctly price portfolios sorted on the same characteristic, i.e. the first column outlines the pricing error made by the bond SDF correctly pricing bond portfolios constructed on credit rating sorts when applied to stock portfolios constructed on credit rating sorts and so on. It is important to mention that the pricing performance varies when fixing one SDF and applying it to all the portfolio sorts considered in the other market, and in most of the cases, the fit worsens. For the different characteristics, the minimum variance bond SDFs yield a pricing error that varies between 2 b.p. and 90 b.p. (Panel A). The lowest pricing errors are the ones associated with credit rating portfolio sorts, whereas the highest ones are encountered for momentum and liquidity. Overall, bond valuations tend to overprice stock portfolios. In contrast, stock valuations tend to underprice bond portfolios (Panel B). Additionally, the pricing errors implied by stock SDFs for bond portfolios are higher on average (in absolute terms). It follows that under the assumption of frictionless markets, if one uses the marginal SDF derived in one market to price the assets in the remaining market, the associated pricing errors exhibit significant cross-sectional dispersion and in some cases, the mispricing is as high as 90 b.p. per month.

In order to have a sense of the maximal pricing error implied by the SDF of one market when used for the other market, I report in Table 7 the results of the Hansen and Jagannathan (1997) distance between the minimum variance bond SDFs and the set of admissible stock SDFs, as well as the distance between the minimum variance stock SDFs and the set of admissible bond SDFs. The maximal implied pricing errors obtained by fixing the minimum variance bond SDF vary between 6 b.p. for credit rating sorts and 40 b.p. for liquidity. Using the stock SDF to price bond portfolios leads on average to higher maximal pricing errors, with the exception of leverage and asset growth sorts. Specifically, the pricing errors vary between 15 b.p. for leverage sorts and 51 b.p. for liquidity sorts. Notice however that if trading costs exceed the price discrepancy, then, loosely speaking, these differences are compatible with absence of arbitrage opportunities across the two markets. Put differently, if typical transaction costs, for instance as measured by the bid-ask spreads, are in the order of magnitude of these maximal implied pricing errors, then one can use marginal SDFs to price the two markets.

Figure 1 plots the time-series of minimum variance SDFs pricing the portfolios of bonds sorted on credit rating (solid line), along with the minimum variance SDF pricing stock returns sorted on credit rating (dashed-dot line) and the minimum variance SDF pricing both bond and stock returns sorted on credit rating (dashed line). All SDFs spike during the financial crisis and the higher volatility of the bond SDF compared to the stock SDF is now apparent. Moreover, the SDF pricing both classes

Table 6. Pricing errors

This table reports the pricing errors implied by minimum variance SDFs, computed as $E[M_B R_S]-1$ and $E[M_S R_B]-1$, with S denoting the stocks and B the bonds. Panel A (B) reports the pricing errors implied by the bond (stock) SDF that correctly prices the portfolios sorted on the corresponding characteristic. Data is monthly and runs from January 2005 to December 2017.

	Panel A: Minimum Variance Bond SDF										
				Low	2	3	4	High			
	Credit rating			0.000	0.002	0.001	0.000	-0.001			
	Duration			0.005	0.002	0.001	0.002	0.004			
		Low	2	3	4	5	6	7	8	9	High
Stock portfolios	Size	0.001	0.000	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	Value	0.005	0.003	0.002	0.004	-0.001	0.001	0.002	0.001	0.001	-0.002
	Leverage	0.002	0.004	0.001	0.002	0.004	0.002	0.001	0.000	0.002	-0.000
	Momentum	0.000	0.003	0.003	0.004	0.004	0.006	0.005	0.006	0.005	0.009
	Asset Growth	0.003	0.004	0.004	0.000	0.002	0.002	0.004	0.004	0.003	0.002
	Profitability	-0.006	-0.002	0.000	0.002	0.000	0.000	0.001	0.001	0.004	0.001
	Liquidity	0.008	0.003	0.004	-0.002	0.002	-0.001	0.000	-0.002	-0.006	-0.002
	Panel B: Minimum Variance Stock SDF										
				Low	2	3	4	High			
	Credit rating			-0.004	-0.003	-0.003	-0.003	-0.000			
	Duration			-0.005	-0.004	-0.003	-0.002	-0.002			
		Low	2	3	4	5	6	7	8	9	High
Bond portfolios	Size	-0.002	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.004	-0.004
	Value	-0.004	-0.004	-0.004	-0.005	-0.003	-0.004	-0.004	-0.004	-0.004	-0.002
	Leverage	-0.006	-0.006	-0.006	-0.006	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006
	Momentum	-0.005	-0.007	-0.006	-0.007	-0.007	-0.006	-0.007	-0.007	-0.007	-0.008
	Asset Growth	-0.006	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.006	-0.006	-0.006
	Profitability	-0.006	-0.007	-0.007	-0.007	-0.008	-0.007	-0.007	-0.008	-0.007	-0.008
	Liquidity	-0.008	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006	-0.005	-0.005	-0.004

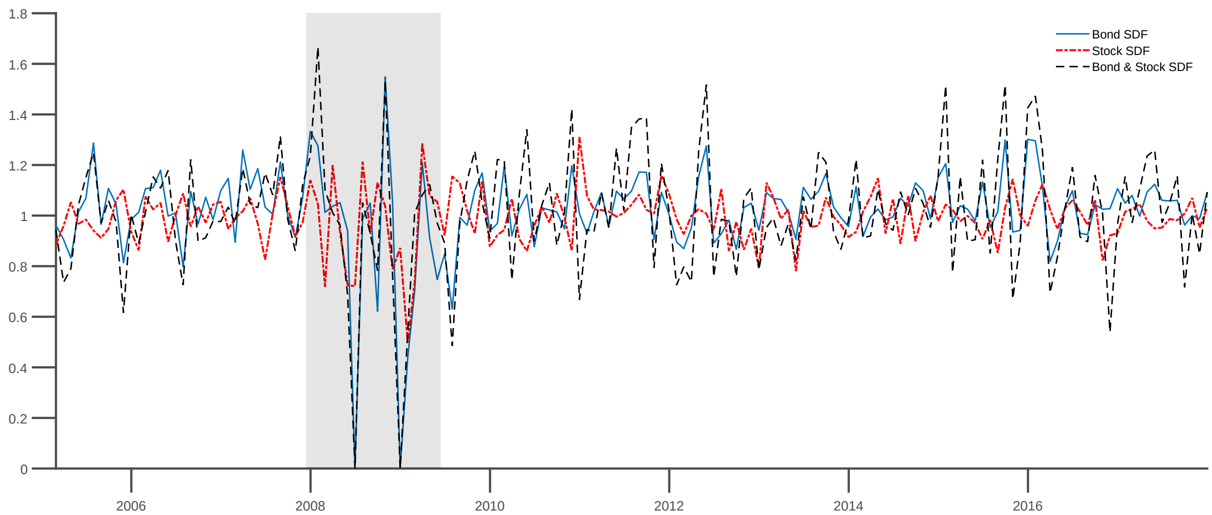
Table 7. HJ Distance of Minimum Variance SDFs

This table reports the Hansen and Jagannathan (1997) (HJ) distance between the minimum variance bond SDF, denoted by $\hat{M}_B$ and the set of admissible SDFs for the stock market S (first row), as well as the HJ distance between the minimum variance stock SDF, denoted by $\hat{M}_S$ and the set of admissible SDFs for the bond market B. The measure is reported in basis points. Data is monthly and runs from January 2005 to December 2017.

	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\sqrt{\delta(\hat{M}_B, S)}$	6.10	12.28	18.54	27.70	22.31	19.13	25.47	30.97	39.58
$\sqrt{\delta(B, \hat{M}_S)}$	25.84	31.25	18.39	29.36	15.01	28.87	18.48	30.28	51.40

of assets appears to follow more the dynamics of the bond SDF. Indeed, taking a closer look at the associated optimal mean-variance portfolio, the positions are higher in the bond portfolios, than in the stock portfolios. Importantly, the characteristics on which portfolios are constructed are going to impact the dynamics of the global minimum variance SDF differently. To provide an additional example, for portfolios formed on firm size, the global minimum variance SDF is going to resemble more the SDF pricing stocks only, as Figure 2 illustrates. The SDFs also spike during episodes when the Federal Reserve raised the interest rates, such as in 2016 and 2017.

Figure 1. Time-series of minimum variance SDFs



This figure plots the time-series of minimum variance SDFs pricing portfolios of bonds sorted on credit rating (solid line), along with the minimum variance SDF pricing stock returns sorted on credit rating (dashed-dot line) and the minimum variance SDF pricing both bond and stock returns sorted on credit rating (dashed line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

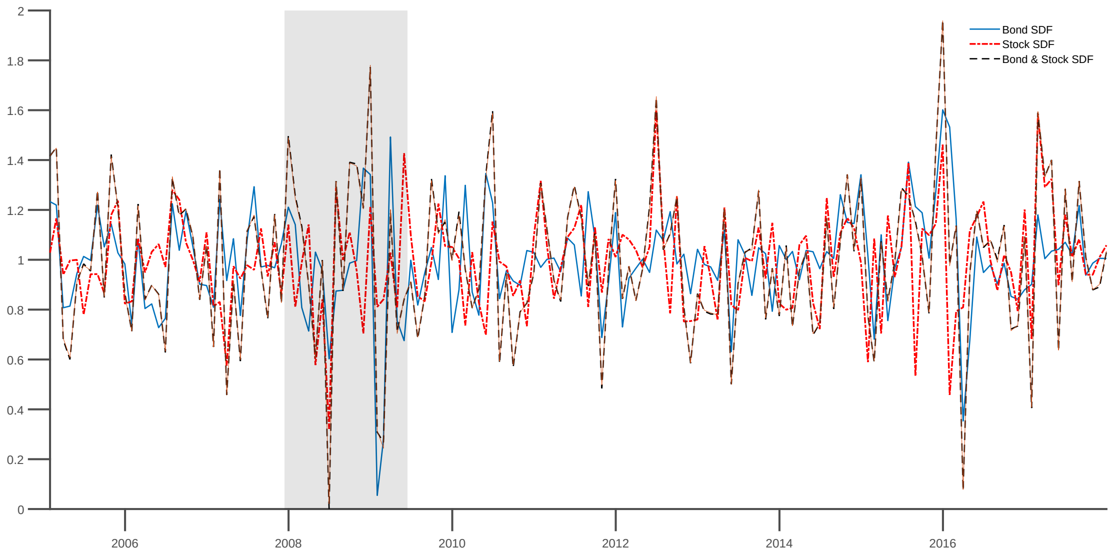
Finally, to have a better sense of the evolution between the bond and stock returns sorted into credit rating quantiles, on the one hand, and between the minimum variance SDFs pricing separately the corporate bonds and the stocks, I report in Figure 3 their rolling correlations. The rolling window size is 12 months, yielding annual correlations estimates. Both rolling correlations fluctuate during the period considered, exhibiting large swings. Nevertheless, there is on average a positive correlation between stock and bond returns sorted on credit rating, as well as a positive co-movement between their respective minimum variance SDFs. Both correlations increase during the recent financial crisis.

4.2.2 Portfolio Implications

In this section, I investigate the portfolio implications of the minimum variance SDFs derived under the assumption of frictionless markets. This analysis is useful in order to determine whether in practice the average investor would be able to form such portfolios.

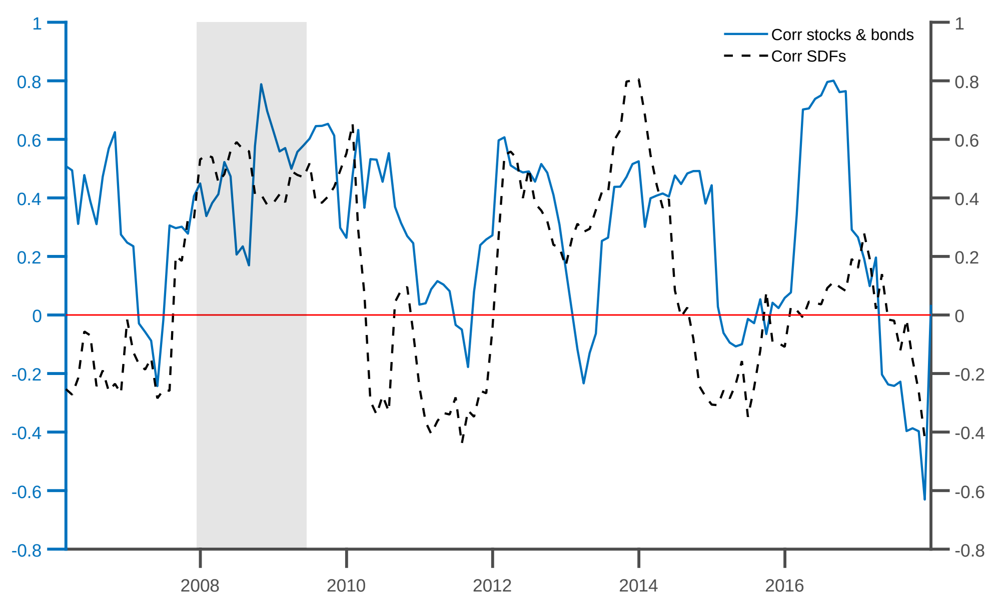
Table 8 reports the portfolio weights associated with the minimum variance SDFs that price portfolios of bond returns for the various characteristics considered (Panel A) and the weights

Figure 2. Time-series of minimum variance SDFs



This figure plots the time-series of minimum variance SDFs pricing portfolios of bonds sorted on firm size (solid line), along with the minimum variance SDF pricing stock returns sorted on firm size (dashed-dot line) and the minimum variance SDF pricing both bond and stock returns sorted on firm size (dashed line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

Figure 3. Rolling correlations



This figure plots the moving correlations between the average stock returns and the average bond returns sorted on credit rating (left axis). The right axis plots the moving correlations between the minimum variance SDFs pricing the stock returns, and the minimum variance SDF pricing bond returns, sorted on credit rating (dashed line). Data is monthly and runs from January 2005 to December 2017. The window size for the moving correlations is 12 months, producing trailing annual correlation estimates.

corresponding to stock portfolios of the issuing firms (Panel B). There are two noteworthy observations. First, the weights in the different bond portfolios are significantly higher than in the stock portfolios. In practice, this would imply that only investors able to take excessive leveraged positions could construct such a portfolio. Second, many of the bond weights carry a negative sign, yielding different short positions along some quantile or decile portfolios. In reality however, it is difficult to short-sell corporate bonds, as this activity is not costless, and in some cases, especially for highly illiquid bonds, it might simply not be possible (see, e.g., Asquith, Au, Covert, and Pathak (2013) and Blanco, Brennan, and Marsh (2005), among others). Therefore, there are significant costs associated with short-selling the bonds, and for some classes of investors, such as pension funds or insurance companies for example, regulators might even not allow them to engage in short-selling activities. To examine the cross-market pricing relation and the associated degree of market integration in a more realistic framework, I derive next the results in presence of different financial frictions, such as short-selling constraints.

II. The Case of Market Frictions Assumption

4.2.3 Minimum Variance SDFs

Since in reality financial markets are not free of frictions, and investors face different constraints, I derive additionally minimum variance SDFs while restricting short-selling in bonds, stocks, or both. The estimation procedure proceeds as before, with the additional restriction of imposing nonnegative weights. The minimum variance SDFs are still going to be a linear function of the returns, but only the ones for which the short-selling constraints are not binding. Naturally, since these returns will be a subset of all the available returns, the SDFs will have a lower volatility. Moreover, as in most cases the majority of the wealth will be allocated only to one portfolio, the SDFs constructed on various portfolio sorts will exhibit a higher correlation. In order to study the extent of market integration, i.e. whether a common SDF for portfolios of bonds and stocks exists in presence of short selling constraints, I examine the cross-market pricing performance.

The pricing errors implied by the restricted SDFs are reported in Tables 9 and 10. Notice that in presence of short-selling constraints, the constrained bond (stock) SDFs are not ensured to correctly price the bond (stock) portfolios (Table 9). Still, the implied pricing errors are small, varying from 0 to 40 b.p. for bond SDFs. Stock SDFs also imply a small pricing error, varying from 0 to 40 b.p., with the exception of one portfolio of liquidity sort, for which the pricing error is 70 b.p. Importantly, they all satisfy the pricing restriction in presence of constraints, consistent with Equation (8). It is more relevant to analyze the cross-market implied pricing errors, especially in order to determine whether a common SDF exists also when short-selling constraints are accounted for. Table 10 Panel A reports the pricing errors made when using the constrained bond SDF to price stock portfolios. Again, the bond SDF employed is formed on the same characteristics as the stock portfolios. For credit rating and duration sorts, the constrained bond SDF perform well in pricing the stock portfolios, as

Table 8. Portfolio weights

This table reports the portfolio weights implied by the minimum variance SDFs correctly pricing portfolios of bonds sorted on different firm-level characteristics (Panel A) and in the corresponding stocks of the issuing firms (Panel B). Data is monthly and runs from January 2005 to December 2017.

Panel A: Portfolio Weights in Bonds

	Low	2	3	4	High
Credit rating	44.32	-48.78	-21.52	50.36	-23.28
Duration	46.58	-12.81	-7.12	-47.44	21.88

	Low	2	3	4	5	6	7	8	9	High
Size	-1.60	0.40	-13.04	-14.48	-0.20	27.00	27.93	-18.38	-19.12	12.51
Value	-12.03	3.67	-18.47	27.70	-20.90	0.73	14.05	14.38	16.19	-24.25
Leverage	-4.43	3.67	23.94	2.48	4.87	-45.30	-13.98	22.37	0.14	7.25
Momentum	-5.09	5.59	-8.56	13.64	20.81	-18.37	14.98	-36.76	2.08	12.75
Asset Growth	19.34	-13.75	26.12	21.79	15.87	-23.79	-32.03	-0.56	-17.30	5.34
Profitability	-9.95	-36.26	9.59	14.56	36.08	-29.89	-4.86	-7.80	14.97	14.60

Panel B: Portfolio Weights in Stocks

	Low	2	3	4	High
Credit rating	2.10	0.68	-2.60	2.29	-1.48
Duration	-0.94	1.94	1.86	1.49	-3.34

	Low	2	3	4	5	6	7	8	9	High
Size	-2.59	10.13	-7.52	-1.01	-6.66	4.25	2.87	-0.82	-7.75	10.12
Value	-1.52	-2.71	4.45	-14.74	11.04	0.90	3.01	3.47	-2.90	0.05
Leverage	-2.08	1.86	-0.92	3.06	-2.65	-8.21	1.22	3.11	5.65	0.00
Momentum	-0.02	0.04	-0.30	-0.89	6.89	-3.49	-0.30	-3.92	7.47	-4.47
Asset Growth	-3.24	-0.27	-6.05	9.49	11.20	-1.25	-6.49	-3.92	-3.53	5.11
Profitability	4.55	2.55	-5.52	-6.75	0.06	7.43	-3.52	6.20	-6.48	2.51

Table 9. Pricing errors in presence of short-selling constraints

This table reports the pricing errors implied by minimum variance SDFs accounting for short-selling constraints, computed as $E[M_B R_B]-1$, or $E[M_S R_S]-1$, with B denoting the bonds and S the bonds. Panel A (B) reports the pricing errors implied by the bond (stock) SDF for bond (stock) portfolios. Data is monthly and runs from January 2005 to December 2017.

Panel A: Minimum Variance Bond SDF

		Low	2	3	4	High
Bond portfolios	Credit rating	0.000	0.001	0.001	0.001	0.004
	Duration	0.000	0.001	0.002	0.003	0.003

		Low	2	3	4	5	6	7	8	9	High
Bond portfolios	Size	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.000
	Value	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.002
	Leverage	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	Momentum	0.002	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	Asset Growth	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000
	Profitability	0.001	0.001	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000
	Liquidity	0.000	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.004

Panel B: Minimum Variance Stock SDF

		Low	2	3	4	High
Stock portfolios	Credit rating	0.000	0.001	0.002	0.001	0.003
	Duration	0.003	0.000	0.000	0.001	0.002

		Low	2	3	4	5	6	7	8	9	High
Stock portfolios	Size	0.004	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.002	0.000
	Value	0.004	0.002	0.001	0.004	0.000	0.001	0.001	0.001	0.002	0.001
	Leverage	0.001	0.003	0.001	0.002	0.003	0.002	0.000	0.000	0.002	0.001
	Momentum	0.000	0.001	0.000	0.001	0.000	0.001	0.001	0.002	0.000	0.003
	Asset Growth	0.003	0.003	0.003	0.000	0.001	0.003	0.004	0.004	0.003	0.001
	Profitability	0.000	0.002	0.004	0.004	0.003	0.002	0.004	0.002	0.005	0.003
	Liquidity	0.007	0.004	0.004	0.000	0.004	0.003	0.003	0.000	0.000	0.000

the implied pricing errors vary from 2 b.p. to 45 b.p. A similar pattern is observed for the remaining firm characteristics sorts. It follows that, in absence of short-selling, minimum variance bond SDFs based on portfolios that are rebalanced monthly, yield an average monthly pricing error of 30 b.p. Provided that these implied pricing error are within the quoted bid-ask spreads for stock portfolios, the constrained bond SDF can be used to price the returns in the stock market. For instance, analyzing a series of anomalies, Novy-Marx (2013) estimate transaction costs between 20 and 65 b.p. for stock portfolios. In reality, the actual transaction costs incurred by individual investors can be even higher, especially if taking into account monitoring costs.

Table 10. Cross-market pricing errors in presence of short-selling constraints

This table reports the pricing errors implied by minimum variance SDFs accounting for short-selling constraints, computed as $E[M_B R_S] - 1$ and $E[M_S R_B] - 1$, with S denoting the stocks and B the bonds. Panel A (B) reports the pricing errors implied by the bond (stock) SDF for stock (bond) portfolios. Data is monthly and runs from January 2005 to December 2017.

	Panel A: Minimum Variance Bond SDF										
				Low	2	3	4	High			
	Credit rating			0.000	0.002	0.003	0.002	0.004			
	Duration			0.004	0.002	0.002	0.003	0.005			
		Low	2	3	4	5	6	7	8	9	High
Stock portfolios	Size	0.005	0.004	0.005	0.005	0.005	0.004	0.004	0.004	0.004	0.003
	Value	0.006	0.004	0.003	0.006	0.002	0.003	0.003	0.003	0.003	0.002
	Leverage	0.003	0.005	0.003	0.004	0.004	0.003	0.002	0.001	0.004	0.002
	Momentum	0.000	0.002	0.003	0.005	0.004	0.006	0.006	0.007	0.005	0.009
	Asset Growth	0.004	0.004	0.004	0.001	0.002	0.004	0.005	0.005	0.004	0.002
	Profitability	-0.001	0.003	0.004	0.005	0.003	0.003	0.004	0.004	0.006	0.004
	Liquidity	0.008	0.004	0.005	0.000	0.004	0.003	0.003	0.000	0.000	0.001
	Panel B: Minimum Variance Stock SDF										
				Low	2	3	4	High			
	Credit rating			-0.003	-0.001	-0.002	-0.002	0.002			
	Duration			-0.004	-0.003	-0.002	-0.001	-0.001			
		Low	2	3	4	5	6	7	8	9	High
Bond portfolios	Size	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003	-0.003	-0.003	-0.004
	Value	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.001
	Leverage	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003
	Momentum	-0.004	-0.006	-0.005	-0.006	-0.006	-0.006	-0.006	-0.006	-0.006	-0.007
	Asset Growth	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002	-0.002	-0.003
	Profitability	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003
	Liquidity	-0.002	-0.002	-0.002	-0.001	-0.001	0.000	0.000	0.001	0.001	0.002

Turning to the performance of stock SDFs with short-selling constraints for bond portfolios, they also appear to do well in terms of pricing. In particular, the implied pricing errors are always within quoted bid-ask spreads, especially since bonds have higher transaction costs than stocks. In practice, the costs associated with trading bonds can be even higher, if one takes into account the potential price impact.

Overall, the evidence suggests that using the constrained SDF in one market performs reasonably well in pricing the portfolio of returns in the other market, at least in the sense that the implied pricing errors are similar in magnitude with typically bid-ask spreads. To obtain an estimate of the maximal implied pricing errors across markets in presence of short-selling constraints, I derive the HJ distance and report the results in Table 11. Accordingly, the associated pricing errors fluctuate between 10 and 30 b.p. for bond constrained SDFs, implying that the cross-pricing can be actually improved by incorporating some type of constraints. Intuitively, this is equivalent with avoiding the overfitting in sample (for the bond portfolios), while enhancing the fit out-of-sample (for stock portfolios). The constrained stock portfolios yield pricing errors that vary between 15 b.p. for leverage sorts and 47 b.p. for liquidity sorts.

Table 11. HJ Distance of Constrained Minimum Variance SDFs

This table reports the Hansen and Jagannathan (1997) (HJ) distance between the minimum variance bond SDF with no short-selling constraints, denoted by $\hat{M}_B$ and the set of admissible SDFs for the stock market S (first row), as well as the HJ distance between the minimum variance stock SDF with no short-selling constraints, denoted by $\hat{M}_S$ and the set of admissible SDFs for the bond market B. The measure is reported in basis points. Data is monthly and runs from January 2005 to December 2017.

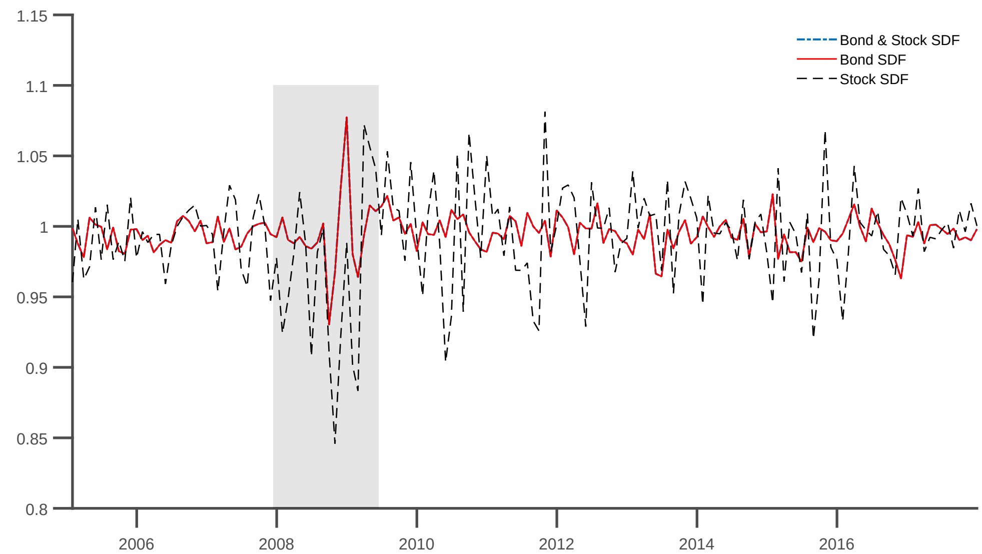
	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\sqrt{\delta(\hat{M}_B, S)}$	9.81	9.98	19.52	25.93	18.39	17.99	26.25	24.05	29.20
$\sqrt{\delta(B, \hat{M}_S)}$	28.57	30.57	21.13	27.33	14.49	27.97	19.77	26.30	46.64

Ultimately, to illustrate the properties of the constrained SDFs, I plot in Figure 4 the time-series of the minimum variance SDF pricing both stock and bond portfolios sorted on firm size, while restricting short-selling in bonds (dashed-dotted line), the minimum variance bond SDF with short-selling constraints (solid line) and the minimum variance stock SDF with short-selling constraints (dashed line). The following observations are worth emphasizing. First, the minimum variance SDFs in presence of short-selling constraints exhibit a significantly lower dispersion. Second, the minimum variance SDF that prices both stock and bond portfolios sorted on size, consistently with no short-selling allowed, follows closely the dynamics of the marginal bond SDF.

4.2.4 Portfolio Implications

Since financial constraints impact the form of the optimal SDF in the underlying market, I examine next the portfolio implications that emerge in such cases. The corresponding portfolio weights are reported in Table 12. In presence of bond short-selling constraints, all resources will be allocated only in the quantile or decile portfolio that maximizes the Sharpe ratio (Panel A). With respect to the stock short-selling constraints, there are instances when the resources will be divided across two or three portfolios. Overall, the portfolio implications derived when imposing bond and/or stock short-selling constraints appear to be more realistic, in the sense that investors would be able to readily form such

Figure 4. Time-series of minimum variance SDFs in presence of short-selling constraints



This figure plots the time-series of minimum variance SDFs pricing portfolios of both bond and stock returns sorted on firm size (dashed-dotted line), along with the marginal minimum variance SDFs, in presence of short-selling constraints. The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

portfolios in practice.

The evidence provided in the case of markets with frictions in the form of short-selling constraints suggests that it is still possible to construct a common SDF pricing both corporate bonds and stocks reasonably, at least consistently with the typical bid-ask spreads. Moreover, taking a closer look at the associated optimal portfolio, it seems plausible to argue that agents can devise such an object. However, a potential concern regarding the cross-market pricing performance stems from the fact that some of the decile portfolios do not exhibit sufficient cross-sectional dispersion. In addition, the bond constrained SDF is relatively smooth. I examine this fact more systematically in the robustness section, where I construct double sorted portfolios based on firm-level characteristics. I show that results are consistent if I use single or double sorts.

4.3 Constrained Minimum Variance SDFs vs. Factor Models

A natural question arising is whether the constrained model-free extracted SDFs perform better than existing factor models for bonds and stocks. Therefore, in this subsection, I evaluate the pricing performance of existing factor models and provide a comparison with respect to the minimum variance constrained SDFs. As competing models, I consider the Fama and French (1993) 3 Factor Model, proxying for market, size and value and the bond factor model of Bai, Bali, and Wen (2019) capturing exposure to the market, downside risk, credit and liquidity factors, as well as a combination of the two models. As test assets, I use double sorted portfolios based on quantile firm characteristics, for the following reasons: (i) double sorting will potentially deliver more cross-sectional variation

Table 12. Portfolio weights in presence of short-selling constraints

This table reports the portfolio weights implied by minimum variance SDFs pricing portfolios sorted on different firm-level characteristics in presence of short-selling constraints. Panel A (B) reports results for the bond (stock) portfolios. Data is monthly and runs from January 2005 to December 2017.

	Panel A: Portfolio Weights in Bonds									
			Low	2	3	4	High			
Credit rating			0.99	0.00	0.00	0.00	0.00			
Duration			0.99	0.00	0.00	0.00	0.00			
	Low	2	3	4	5	6	7	8	9	High
Size	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99
Value	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
Leverage	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Momentum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99
Asset Growth	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99
Profitability	0.00	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00
Liquidity	0.99	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00

	Panel B: Portfolio Weights in Stocks									
			Low	2	3	4	High			
Credit rating			0.99	0.00	0.00	0.00	0.00			
Duration			0.00	0.35	0.63	0.00	0.00			
	Low	2	3	4	5	6	7	8	9	High
Size	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99
Value	0.00	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00
Leverage	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.99	0.00	0.00
Momentum	0.54	0.00	0.02	0.00	0.42	0.00	0.00	0.00	0.00	0.00
Asset Growth	0.00	0.00	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
Profitability	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Liquidity	0.00	0.00	0.00	0.73	0.00	0.00	0.00	0.11	0.15	0.00

between the portfolios and (ii) this approach will yield 25 test assets, which is the benchmark used in testing asset pricing models.

I consider the following regression:

$$R_{it} = \alpha_i + \beta_i' F_t + \varepsilon_{it}, \tag{15}$$

where R_{it} is the return on the test assets $i = 1, ..., 25$ at time t and F_t is the vector containing the corresponding factors at time t. For the Fama French 3 factor model, $F = [MKT \quad SMB \quad HML]'$, capturing the market, size and value factors. For the bond factor model, $F = [MKT \quad DRF \quad CRF \quad LRF]'$, proxying for market, downside risk, credit risk and liquidity risk. I also consider the case when the stock and bond factors are stacked. For the model-free extracted global SDF accounting for short selling constraints, $F = [\hat{M}]'$.

I estimate the factor models using as test assets the different double sorted portfolios by employing the Fama MacBeth approach. The methodology proceeds in two steps. First, I run time-series regressions to obtain the betas, or loadings on the corresponding factors. Second, I run cross-sectional regressions on the estimated betas from the first step in order to retrieve the price of risk of the factors. When performing this two-stage regression, I adjust the standard errors to account for errors-in-variables, since the betas are estimated in the first step, heteroskedasticity, as the variance of residuals is not constant, and autocorrelation in error terms.

To discriminate between the various models, I use two criteria: (i) the size of the implied pricing errors, as measured by the HJ distance and (ii) the R-Square of a generalized least squared (GLS) regression of returns on the proposed factors. The reason I focus on GLS R-Square rather than the OLS one follows from the critique of Lewellen, Nagel, and Shanken (2010), that apparently strong explanatory power in fact provides quite weak support for a model, especially if the test assets are the ones formed on size and book-to-market portfolios. Another advantage is that the GLS R-Square has a meaningful economic interpretation in terms of the relative mean-variance efficiency of a model's factor–mimicking portfolios. In other words, the GLS R-Square reflects a factor's proximity to the mean-variance boundary, whereas the OLS R-Square does not have in general a direct relation to the factor's location in mean-variance space.

The size of the implied pricing errors for the various models considered, as measured by the HJ distance is reported in Table 13. Specifically, for the corresponding bond test assets, I compute the pricing errors implied by the minimum variance SDF with short-selling constraints, as well as the pricing errors implied by the bond factor model. All the estimates are statistically significant at the 1% level. The model-free SDF entails lower pricing errors than the bond factor model. When considering stocks as test assets, I compare the performance of the minimum variance SDF with short-selling constraints with the Fama French 3 factor model. Again, all the estimates are strongly statistically significant at conventional levels. Overall, the SDFs accounting for short-selling constraints tend to deliver an enhanced pricing performance relative to factor models.

Table 13. Pricing Errors Implied by Factor Models

This table reports the pricing error implied by factor models, as well as the one implied by the minimum variance SDF with short-selling constraints, computed using the Hansen and Jagannathan (1997) (HJ) distance. The measure is reported in basis points. Heterokedasticity robust standard errors are reported in brackets. Data is monthly and runs from January 2005 to December 2017.

	Bonds		Stocks	
	SDF	BF	SDF	FF3
Credit rating & Duration	71.73	75.54	51.76	60.42
	[12.9]	[14.6]	[7.32]	[8.52]
Credit rating & Liquidity	51.60	50.99	57.92	56.55
	[9.6]	[10.7]	[7.45]	[6.42]
Credit rating & Size	52.36	54.07	45.72	44.21
	[10.3]	[11.5]	[8.8]	[8.2]
Duration & Value	57.82	61.50	40.88	43.47
	[11.1]	[11.1]	[9.4]	[10.2]
Duration & Liquidity	51.99	49.14	44.80	45.29
	[10.7]	[10.9]	[9.7]	[9.7]
Leverage & Profitability	35.65	35.06	45.54	45.73
	[9.2]	[8.9]	[7.9]	[7.9]
Size & Leverage	44.14	43.05	49.98	51.04
	[9.9]	[9.6]	[7.9]	[7.6]
Size & Liquidity	37.52	33.43	58.78	61.21
	[10.1]	[10.2]	[8.9]	[10.2]
Size & Profitability	43.55	49.89	36.63	30.70
	[7.5]	[7.8]	[7.8]	[7.7]
Size & Value	36.06	36.94	32.63	30.22
	[9.3]	[8.9]	[7.4]	[7.8]

The associated GLS R-Square from fitting the Fama French 3 factor model, the bond factor model, or a combination of both, along with fitting the global minimum variance constrained SDF are reported in Table 14. The test assets considered are the 25 portfolios obtained from double sorts, for stocks, bonds, or both. None of the factors considered provide a perfect fit, as measured by the GLS R-Square. Nevertheless, the best fit on average is obtained for portfolios of stocks. The Fama French 3 factor model delivers GLS R-Square ranging from 4 to 35%. Importantly, the global minimum variance SDF provides a better fit than the Fama French factor, bond factor, or a combination of both for some instances, as highlighted in Table 14. This evidence is meaningful especially in the view that the SDF is basically a one factor model. For the bond portfolios, the best fit tends to be given by the bond factor model. Still, it is relevant to stress that stock and bond factor models are constructed by assuming that investors can build long and short positions in some decile sorted portfolios. Besides the potential short-selling constraints, it is likely that some of these portfolios are not tradable in practice, especially if they contain small illiquid stocks or bonds. Finally, when considering instead both stocks and bonds portfolios as test assets, the fit is rather poor, regardless of the factors used.

Table 14. GLS R-Square

This table reports the GLS R-Square obtained by running the following regression using Fama MacBeth procedure:

$$R_{it} = \alpha_i + \beta_i' F_t + \varepsilon_{it},$$

where R_{it} is the return on the test assets i at time t and F_t is the vector containing the corresponding factors at time t. For the Fama French 3 factor model (FF3), $F = [MKT \quad SMB \quad HML]'$, capturing the market, size and value factors. For the bond factor model (BF), $F = [MKT \quad DRF \quad CRF \quad LRF]'$, proxying for market, downside risk, credit risk and liquidity risk. For the model-free extracted global SDF accounting for short selling constraints, $F = [\hat{M}]$. Results are reported for stock, bond and stock and bond portfolios. Data is monthly and runs from January 2005 to December 2017.

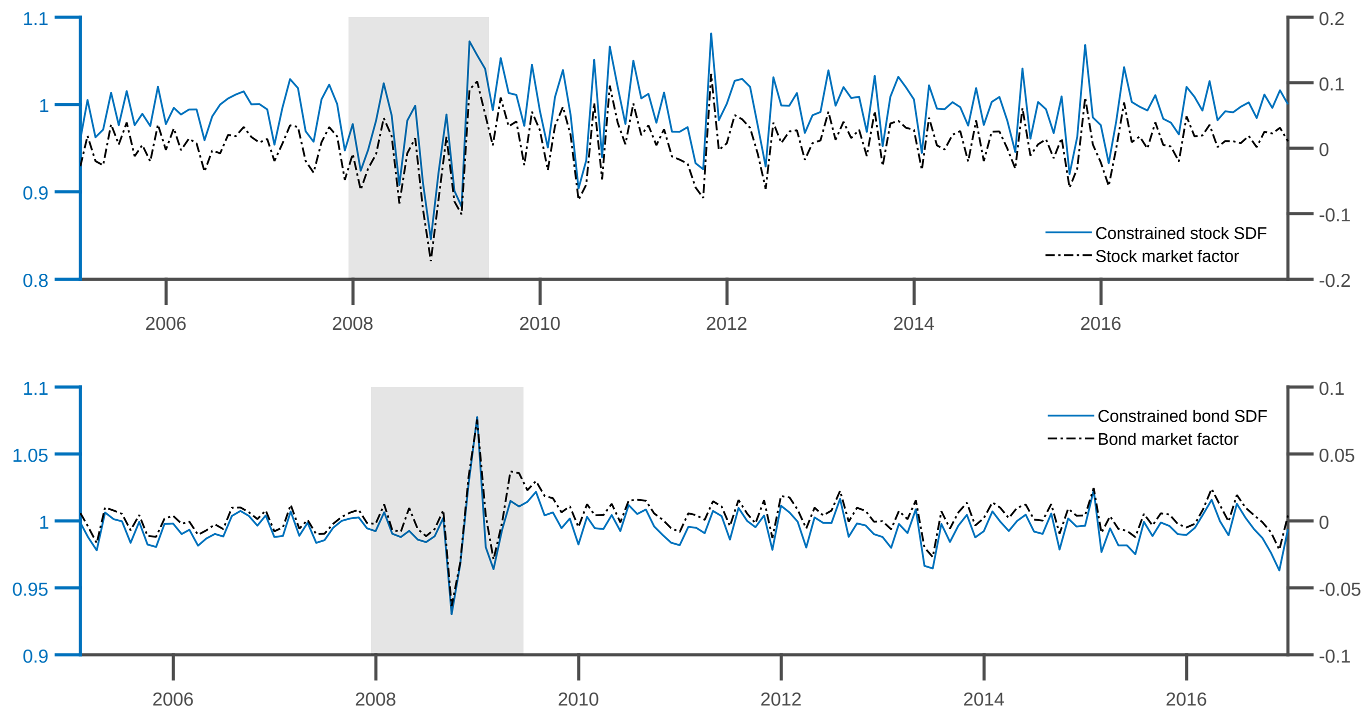
	Stocks				Bonds				Stocks & Bonds			
	FF3	BF	FF3 + BF	SDF	FF3	BF	FF3 + BF	SDF	FF3	BF	FF3 + BF	SDF
Credit rating & Duration	12.4%	18.6%	28.1%	14.6%	2.4%	9.3%	14.1%	1.1%	1.2%	2.9%	3.5%	0.0%
Credit rating & Liquidity	22.4%	13.7%	33.0%	**34.5%**	5.7%	9.9%	17.0%	0.0%	1.4%	1.7%	3.6%	0.2%
Credit rating & Size	10.1%	5.9%	34.0%	8.5%	5.0%	20.4%	11.2%	0.1%	0.9%	4.6%	5.7%	0.2%
Duration & Value	10.5%	19.3%	23.1%	6.4%	5.1%	11.4%	27.7%	0.5%	3.4%	4.6%	7.8%	0.2%
Duration & Liquidity	7.3%	8.1%	18.9%	**13.7%**	5.7%	7.1%	31.4%	0.0%	8.6%	5.7%	13.1%	0.0%
Leverage & Profitability	5.3%	5.8%	18.4%	4.2%	4.7%	26.1%	15.0%	0.6%	5.0%	1.8%	5.5%	0.6%
Size & Leverage	4.0%	2.3%	7.8%	**4.2%**	3.3%	8.5%	7.6%	0.1%	1.4%	2.0%	3.4%	0.7%
Size & Liquidity	13.8%	13.9%	30.6%	**14.8%**	12.7%	8.4%	31.1%	0.1%	4.2%	0.9%	5.1%	0.5%
Size & Profitability	35.1%	3.9%	47.4%	12.1%	5.7%	6.8%	24.0%	0.2%	6.9%	1.3%	7.7%	0.3%
Size & Value	15.4%	2.0%	25.7%	5.3%	20.3%	18.3%	22.7%	0.3%	2.6%	1.3%	2.7%	0.1%

In summary, a horse race between factor models and the SDFs suggest the latter yield on average smaller pricing errors, as measured by the corresponding HJ distance and can yield a higher GLS R-Square in some cases. Overall, the constrained SDFs not only exhibit a better pricing performance, but can be interpreted economically as optimal Sharpe ratio portfolios of traded assets.

4.4 Minimum Variance SDFs and Risk Factors

To understand what are the risk factors embedded in the model-free extracted SDFs, I investigate their relationship with the factor models. The unconstrained minimum variance SDFs seem to be largely unrelated with the Fama French 3 factor model or the bond factor model.[8] The SDFs accounting for short selling constraints, on the other hand, are typically explained by factor models, with most of the contribution coming from the market factor. This result is intuitive, as the market portfolio itself consists only of long positions and is efficient in a mean-variance sense. In fact, as Figure 5 illustrates, both the constrained minimum variance stock SDF (top panel) and the constrained bond SDF (bottom panel) strongly positively co-move with the market factor.

Figure 5. Constrained Minimum Variance SDFs and Market Factor



This figure plots the time-series of short-selling constrained minimum variance SDFs pricing portfolios of stock (bond) returns sorted on firm size in top (bottom) panel, along with the market factor of Fama and French (1993) and the bond market factor of Bai, Bali, and Wen (2019) (dashed-dotted line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017 (2016) for stocks (bonds).

Still, there remains a part which is unexplained by these factors. Motivated by the growing literature documenting the substantial role of financial institutions in determining equilibrium asset prices, I explore additionally the relationship between the minimum variance SDFs and intermediaries. The intermediary asset pricing literature posits that the SDF of the financial intermediary, rather than the one of the household, prices financial assets, in particular sophisticated securities that are harder to trade in practice (see, e.g., Adrian, Etula, and Muir (2014) and Haddad

[8]In interest of space, I do not report the results.

and Muir (2018), among others). Since the model-free extracted SDFs contain information about the relevant risk factors, it is natural to compare them to the intermediary risk factor. To this end, I use the intermediary capital risk factor of He, Kelly, and Manela (2017), which captures the shocks to the equity capital ratio of financial intermediaries (Primary Dealer counterparties of the New York Federal Reserve). Intuitively, whenever an intermediary faces a negative shock to its equity capital, its risk-bearing capacity is affected. Hence, it is likely that a strong link between constrained, rather than unconstrained, minimum variance SDF and the intermediary risk factor prevails. The top panel of Figure 6 plots the time-series of short-selling constrained minimum variance stock SDFs, along with the intermediary risk factor.[9] When short-selling in stocks is restricted, the associated SDF strongly positively co-moves with the intermediary risk factor, especially during the recent financial crisis. This result is intuitive, as according to the portfolio weights in Table 12, all the wealth is allocated to the portfolio containing the largest firms. Indeed, upon a closer inspection, the firms in the top decile are also highly leveraged and they are among the primary dealers of the NY FED.[10] The short-selling constrained bond SDF, on the other hand, features a weak co-movement with the intermediary risk factor (bottom panel).

To test the relationship between minimum variance SDFs and intermediaries more systematically, I run the following regressions:

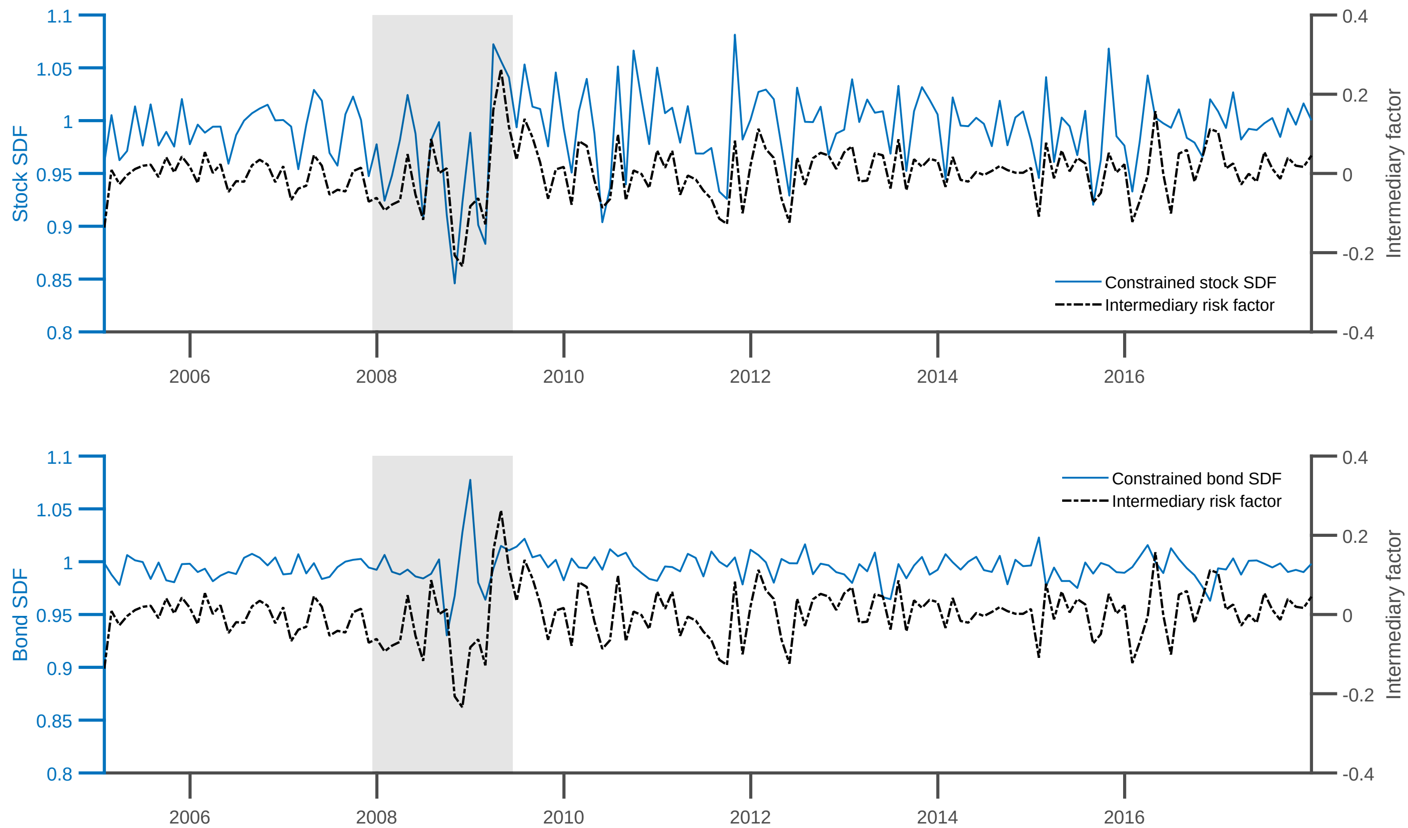
$$M_{it} = \alpha_i + \beta_i \kappa_t + \epsilon_{it}, \tag{16}$$

with $i = B, S$ denoting the bond or stock and κ the intermediary risk factor. Results are reported in Table 15. The test assets are the double-sorted portfolios. The constrained bond SDF exhibits no apparent relation with the intermediary risk factor. Only the SDFs pricing portfolios sorted on duration and value, and leverage and profitability returns load significantly and positively on the intermediary risk factor. Interestingly, the unconstrained bond SDF loads instead negatively on the intermediary risk factor, and significantly for portfolios sorted on credit rating, size, leverage and value. Overall, the explanatory power is rather low. A different picture emerges for the stock SDF, especially the constrained one: the estimates for the intermediary risk factor are positive and statistically significant at the 1% level. Moreover, the associated R^2 is high, ranging from 42 to 61%. The unconstrained stock SDFs also have a positive loading on the intermediary risk factor, but the explanatory power is lower. Altogether, the SDFs accounting for stock short-selling constraints appear to capture to a great extent the intermediary risk factor.

[9] For illustrative purposes, Figure 6 plots the SDF pricing portfolios of returns sorted on firm size. Plots are similar for other characteristics-sorts.

[10] The list of companies present in the sample that coincide with the primary dealers of the NY FED is as follows: Citigroup, Goldman Sachs, Morgan Stanley, Bank of America, J.P. Morgan, Wells Fargo.

Figure 6. Constrained Minimum Variance SDFs and Intermediary Risk Factor



This figure plots the time-series of short-selling constrained minimum variance SDFs pricing portfolios of stock (bond) returns sorted on firm size in top (bottom) panel, along with the intermediary risk factor of He, Kelly, and Manela (2017) (dashed-dotted line). The gray bar represents the NBER recession period. Data is monthly and runs from January 2005 to December 2017.

Table 15. Minimum Variance SDFs and Intermediary Risk Factor

This table reports the estimated coefficients from regressing the minimum variance SDFs on the intermediary risk factor (κ): $M_{it} = \alpha_i + \beta_i \kappa_t + \epsilon_{it}$, with $i = B, S$ denoting the bond or stock. The constrained SDFs account for short-selling constraints, whereas the unconstrained one is unrestricted. Both types of SDFs price different double-sorted portfolios. Data is monthly and runs from January 2005 to December 2017. Standard errors are reported in brackets. ***, ** and * denotes significance at the 1%, 5% and 10% level, respectively.

		Bond SDF		**Stock SDF**	
		Constrained	Unconstrained	Constrained	Unconstrained
Credit rating & Duration	α	0.997^{***}	0.999^{***}	0.994^{***}	0.987^{***}
		[0.000]	[0.057]	[0.002]	[0.002]
	β	0.003	-1.595^{*}	0.599^{***}	1.169^{*}
		[0.007]	[0.873]	[0.040]	[0.040]
	R^2	0.07%	1.49%	58.80%	1.45%
Credit rating & Liquidity	α	0.995^{***}	0.996^{***}	0.995^{***}	0.977^{***}
		[0.001]	[0.040]	[0.002]	[0.044]
	β	-0.021	-0.887	0.586^{***}	3.807^{***}
		[0.002]	[0.615]	[0.040]	[0.675]
	R^2	0.09%	0.70%	58.20%	16.70%
Credit rating & Size	α	0.995^{***}	0.997^{***}	0.994^{***}	0.991^{***}
		[0.001]	[0.038]	[0.002]	[0.040]
	β	0.001	-1.484^{**}	0.452^{***}	0.273
		[0.015]	[0.589]	[0.042]	[0.575]
	R^2	0.00%	3.35%	41.70%	0.15%
Duration & Value	α	0.997^{***}	0.997^{***}	0.992^{***}	0.988^{***}
		[0.001]	[0.046]	[0.002]	[0.033]
	β	0.022^{**}	0.06	0.65^{***}	0.916^{*}
		[0.010]	[0.709]	[0.041]	[0.510]
	R^2	2.40%	0.00%	61.30%	1.42%
Duration & Liquidity	α	0.997^{***}	0.998^{***}	0.994^{***}	0.987^{***}
		[0.001]	[0.041]	[0.003]	[0.037]
	β	0.001	-0.627	0.628^{***}	1.244^{**}
		[0.006]	[0.630]	[0.049]	[0.564]
	R^2	0.0%	0.6%	51.4%	2.5%
Leverage & Profitability	α	0.995^{***}	0.995^{***}	0.994^{***}	0.991^{***}
		[0.001]	[0.029]	[0.003]	[0.037]
	β	0.051^{***}	-0.434	0.811^{***}	0.921
		[0.019]	[0.443]	[0.058]	[0.560]
	R^2	3.64%	0.62%	55.20%	1.09%
Size & Leverage	α	0.995^{***}	0.995^{***}	0.993^{***}	0.989^{***}
		[0.001]	[0.034]	[0.003]	[0.040]
	β	0.001	-0.89^{*}	0.862^{***}	1.224^{**}
		[0.018]	[0.513]	[0.057]	[0.613]
	R^2	0.0%	1.3%	59.3%	1.9%
Size & Liquidity	α	0.995^{***}	0.994^{***}	0.995^{***}	0.984^{***}
		[0.001]	[0.029]	[0.003]	[0.046]
	β	0.021	-0.487	0.534^{***}	2.797^{***}
		[0.018]	[0.436]	[0.048]	[0.699]
	R^2	0.23%	0.16%	43.30%	8.81%
Size & Liquidity	α	0.995^{***}	0.995^{***}	0.993^{***}	0.988^{***}
		[0.001]	[0.035]	[0.003]	[0.029]
	β	-0.01	-0.59	0.714^{***}	1.489^{***}
		[0.016]	[0.536]	[0.054]	[0.450]
	R^2	0.2%	0.1%	52.9%	6.0%
Size & Value	α	0.995^{***}	0.995^{***}	0.994^{***}	0.991^{***}
		[0.001]	[0.027]	[0.002]	[0.026]
	β	0.017	-0.696^{*}	0.488^{***}	0.758^{*}
		[0.016]	[0.411]	[0.037]	[0.391]
	R^2	0.10%	1.19%	51.90%	1.74%

5 Robustness

In this section, I perform two robustness checks to my main analysis. First, I study the implications of constructing double sorted portfolios. Second, I assess the impact of introducing an additional type of friction, in which the attainable Sharpe ratio in the economy is restricted.

5.1 Double Sorting

In this subsection, I redo the analysis for double sorted portfolios of corporate bond and stock returns, based on different firm characteristics quantiles. The reason is twofold. First, double sorting will potentially deliver more cross-sectional variation between the portfolios. This would be particularly relevant for the bond returns, for which some other characteristic might be obscured when building univariate sorts. For instance, when sorting bond returns based on a firm characteristic, such as profitability, one might obtain flat average returns across portfolios only because a bond feature, such as credit rating or duration, is not accounted for. Double sorting will explicitly rule out such instances. Second, to provide a comparison in terms of pricing performance of my model-free extracted SDFs and existing asset pricing models, it is appropriate to have a larger cross-section. Consequently, by forming double sorts based on quantile firm characteristics, the number of test assets will equal 25, which is the representative one used in testing asset pricing models.

Previous evidence from univariate sorts guides the way in which double sorts should be constructed. In particular, since firm-level characteristics appear to matter distinctly for average bond and stock returns, the sorting should not be arbitrary. I consider thus the following double sorts. Credit rating and duration, since riskier bonds and those with a longer maturity require a higher expected returns. Similarly, firms with higher credit risk have a larger average stock return. Credit rating and liquidity, since low credit quality firms are likely to be more affected by illiquidity. Credit rating and size, as smaller firms are more likely to default. Duration and value, because they both capture investors' preferences in terms of maturity length. Specifically, growth firms are high-duration assets, similar to long-term bonds. Duration and liquidity, as short-term bonds tend to be traded more frequently than long-term ones, that are typically buy-and-hold strategies. Leverage and profitability, since they both drive the underlying value of the firm. In particular, highly leveraged firms that are also profitable, might require a lower expected return. Size and leverage, for the reason that larger firms that are also significantly leveraged can entail a higher expected return. Size and liquidity, to grasp whether investors have a preference for different firm sizes, once liquidity is accounted for. Size and profitability, as larger profitable firms can also command a higher expected return. Size and value, to proxy for the typical test assets used in evaluating different asset pricing models.

Since in the real world markets are not free of frictions, I derive the minimum variance SDFs pricing double sorted portfolios of bonds and/or stocks, in presence of short-selling constraints. In interest of space, I report in Table 16 the average pricing error implied by the constrained SDFs.

The general conclusion from this evidence is that even after increasing the level of granularity of

Table 16. Average Pricing Errors for Double Sorted Portfolios

This table reports the average pricing errors implied by minimum variance SDFs for double sorted portfolios accounting for short-selling constraints, computed as $E[M_i R_i] - 1$, with $i = B, S$ denoting the bonds and stocks. Data is monthly and runs from January 2005 to December 2017.

	Bond SDF		**Stock SDF**	
	Bond Portfolios	Stock Portfolios	Bond Portfolios	Stock Portfolios
Credit rating & Duration	0.003	0.005	0.001	0.004
Credit rating & Liquidity	0.002	0.003	0.002	0.005
Credit rating & Size	0.002	0.004	0.001	0.005
Duration & Value	0.002	0.004	-0.002	0.002
Duration & Liquidity	0.002	0.003	0.001	0.004
Leverage & Profitability	0.001	0.003	-0.001	0.004
Size & Leverage	0.001	0.004	-0.001	0.004
Size & Liquidity	0.001	0.003	0.001	0.005
Size & Profitability	0.001	0.003	0.000	0.004
Size & Value	0.001	0.004	0.000	0.004

Table 17. HJ Distance of Constrained Minimum Variance SDFs

This table reports the Hansen and Jagannathan (1997) (HJ) distance between the minimum variance bond SDF with short-selling constraints, denoted by $\hat{M}_B$ and the set of admissible SDFs for the stock market S, as well as the HJ distance between the minimum variance stock SDF with short-selling constraints, denoted by $\hat{M}_S$ and the set of admissible SDFs for the bond market B, for double sorted portfolios. The measure is reported in basis points. Data is monthly and runs from January 2005 to December 2017.

	$\sqrt{\delta(\hat{M}_B, S)}$	$\sqrt{\delta(B, \hat{M}_S)}$
Credit rating & Duration	51.40	68.45
Credit rating & Liquidity	58.10	48.72
Credit rating & Size	46.64	46.78
Duration & Value	41.13	56.45
Duration & Liquidity	45.89	51.12
Leverage & Profitability	46.02	36.66
Size & Leverage	50.83	43.30
Size & Liquidity	59.36	36.38
Size & Profitability	38.38	44.42
Size & Value	32.67	35.13

the portfolios, the pricing performance of minimum variance SDF is reasonable, in the sense that it can be sustained by the usual bid-ask spreads observed in the corporate bond and stock markets. However, it is worthwhile to mention that the pricing errors for the double sorted portfolios exhibit a higher cross-sectional dispersion and it is very likely that if one would study individual bonds and stocks rather than portfolios, the associated pricing errors will be too high to be motivated by transaction costs. Put differently, the size and the similarity of the portfolios can also improve the degree of integration between the two markets. Indeed, computing the maximal implied pricing errors as measured by the HJ Distance suggest that they naturally increase when considering double sorts rather than univariate sorts, fluctuating between 30 and 70 basis points per month (see, e.g. Table 17).

5.2 Market Frictions Restricting the Attainable Sharpe Ratio

An additional friction analyzed consists of restricting the attainable Sharpe ratio in the economy. Following the seminal work of Cochrane and Saa-Requejo (2000), who introduce good-deal bounds in order to rule out high Sharpe ratios, I impose an upper bound on the volatility of the SDF. Intuitively, this SDF constraint implies that returns featuring low variance must have small prices. This is especially the case for bond returns. For consistency, I examine the asset pricing implications also when restricting the stocks' Sharpe ratios.

A natural question arising is what is an economic reasonable restriction for the Sharpe ratio available in the economy? There is also a trade-off with respect to the Sharpe ratio that investors actually desire, in order to induce them to trade. For instance, it is likely that investors will require a higher Sharpe ratio than the historical market one, of 0.5 on an annual basis. To be consistent with market valuations, I use an upper volatility bound for the stock and bond portfolios that is equal to 70% of the Sharpe ratio of the global minimum variance SDF.[11]

To study the cross-market pricing implications between corporate bonds and stocks when the attainable Sharpe ratio is restricted, I report in Table 18 the corresponding HJ distance. When an upper volatility bound is imposed for the various bond SDFs, the maximal implied pricing error with respect to the stock market varies from 9.8 basis points for credit rating portfolios to 38.87 basis points for liquidity portfolios. When instead the Sharpe ratio of stock portfolios is constrained, the pricing error is higher, ranging from 14.31 basis points for leverage portfolios to 51.49 for liquidity sorted portfolios. Overall, the pricing errors implied by the SDFs preventing good-deal bounds appear to be within the typical incurred bid-ask spreads when trading corporate bonds and stocks.

[11] Results are similar when varying the proportion considered for the global Sharpe ratio.

Table 18. HJ Distance in Presence of Good-Deal Bounds

This table reports the HJ distance between the minimum variance bond SDF derived when good-deal bounds are imposed, denoted by $\hat{M}_B$ and the set of admissible SDFs for the stock market S, as well as the HJ distance between the minimum variance stock SDF derived when good-deal bounds are imposed, denoted by $\hat{M}_S$ and the set of admissible SDFs for the bond market B. The restricted SDFs reflect an upper volatility bound for the stock and bond portfolios that is equal to 70% of the Sharpe ratio of the global minimum variance SDF. All the integration measures are reported in basis points. Data is monthly and runs from January 2005 to December 2017.

	Credit rating	Duration	Size	Value	Leverage	Momentum	Asset Growth	Profitability	Liquidity
$\delta(\hat{M}_B, S)$	9.815	10.885	18.447	26.527	22.353	18.335	25.498	27.558	38.872
$\delta(B, \hat{M}_S)$	25.844	31.246	19.025	27.768	14.312	28.669	18.442	28.178	51.493

6 Conclusion

Shedding light on the underlying extent of market integration is crucial for refining our insights regarding the pricing of different asset classes, as well as for assessing the empirical success of canonical asset pricing models implicitly assuming perfect integration across different markets. However, studying the degree of market integration is a complex issue. Its complexity is twofold. First, there can be evidence of market segmentation not only across different asset classes, but also within an asset class. Second, considering different asset characteristics can give rise to different magnitudes of cross-market pricing errors.

Using a model-free approach that circumvents potential issues arising from the joint hypothesis test of market integration and correct model specification, I show that under the assumption of frictionless markets, empirically, there always exists a common SDF jointly pricing corporate bonds and stocks in the US market. However, the optimal portfolio associated with this SDF entails large and often negative positions, suggesting that it may be costly to construct in practice, especially in presence of financial or short-selling constraints. Since in the real world investors face various restrictions when designing their optimal portfolios, I study the implications of different market frictions. For instance, I find that introducing short-selling constraints generates cross-market implied pricing errors sustainable with quoted bid-ask spreads. The results suggest that larger firms, with more liquid bonds and stocks appear to be more integrated, especially in presence of short-selling constraints. Similarly, firms that are more leveraged, have a higher asset growth and profitability feature a higher degree of integration between their debt and equity securities.

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