

# MEASURE THEORY

**Instructor: Tiziano De Angelis**

The course introduces to the theory of Lebesgue integration. After defining appropriate structures of sets such as algebras, semi- and sigma-algebras, we provide the notion of Lebesgue-Stieltjes measures, and their construction from semi-algebras to sigma-algebras. We then constructively define integrals with respect to a Lebesgue-Stieltjes measure and state their most relevant properties. The relationship with classical Riemann integration and the connection with probability theory are also examined.

## **Exam**

Written exam at the end of the course.

## **Course Outline**

- classes of subsets: algebras, semi-algebras, sigma-algebras, monotone classes
- measures: definition and properties, finite- and sigma-additivity; construction of measures on sigma-algebras; completions of measures; Lebesgue-Stieltjes measures
- measurable functions
- Lebesgue-Stieltjes integrals: construction and properties
- convergence theorems
- null-measure sets and properties holding almost everywhere
- comparison with Riemann integration
- Radon-Nikodym's theorem

## **Textbooks**

Detailed lecture notes will be provided. Reference books are:

- Folland, G.B. (1999). Real analysis. Wiley