

Mathematics for finance

Aim

The course motivates and illustrates the role and limitations of mathematics in quantitative finance. We recall basic concepts of calculus and linear algebra, within the context of financial applications. We also illustrate and generalize essential ideas in normed and inner product spaces, which are essential in both financial theory and numerical methods. Everything is illustrated from three related points of view: theory, applications, and numerical implementation in MATLAB.

Content

- *Motivations*. Basic problems in quantitative finance: portfolio optimization, risk management, asset pricing.
- *Linear algebra*. Linear spaces. Matrices, eigenvalues, quadratic forms. Semidefinite quadratic forms. Applications: mean-variance portfolio optimization; pricing by no arbitrage; complete and incomplete markets.
- *Norms, distances, and inner products*. Defining norms and inner products for vector and matrices. Projection on subspaces. Applications: linear regression, regularized linear regression.
- *Numerical linear algebra*. Direct and iterative methods to solve systems of linear equations. Computational complexity, convergence, numerical stability, conditioning.
- *Calculus*. Continuity, differentiability, and convexity. Approximating functions by Taylor expansions. Applications: finite differences, price sensitivities, risk aversion.
- *Function approximation*. Issues with polynomial interpolation. Splines and their variants.
- *Solving nonlinear equations*. Bisection, Newton's method, optimization-based approaches.
- *Numerical integration*. Quadrature methods, Gaussian quadrature, Monte Carlo and quasi-Monte Carlo methods.
- *Stochastic calculus*. A preview of stochastic differential equations and Ito's lemma. Application to BSM model.

Bibliography

- P. Brandimarte. *Numerical Methods in Finance and Economics: A MATLAB-Based Introduction (2nd ed)*. Wiley, 2006.
- P. Brandimarte. *Quantitative Methods: An Introduction for Business Management*. Wiley, 2011.
- P. Brandimarte. *An Introduction to Financial Markets: A Quantitative Approach*. Wiley, 2018.
- C.P. Simon, L.E. Blume. *Mathematics for Economists*. W.W. Norton & Company, 1994.