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Dissecting the Yield curve: the international evidence

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Dissecting the yield curve: The international evidence

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Abstract

Nominal yields can be expressed as the sum of an expectation, term premium, and convexity component, and in turn of their real and inflation counterparts. We extract these terms from the yield curve of the U.S., Euro Area, U.K., and Japan using a term structure model that explicitly captures the interrelation between yield factors and macroeconomic conditions while allowing for aggregate stochastic volatility. We find that the bulk of yield dynamics comes from short rate expectations. Term premia vary over time and increase with maturity, but account for a smaller fraction of yield level and variance than previously documented. Over time, we observe a sustained decline in short real rate expectations and significant convexity effects. With regard to yield comovement, the U.S. and U.K. generate the strongest spillovers at the long-end of the yield curve, in particular through term premia, whereas the Japanese market is the least connected.

Keywords: Term structure, term premia, yield volatility, macro factors, comovement

JEL classification: G12, E43, E44, C58

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Introduction

The last two decades have been characterized by pronounced variation in interest rates across maturities, in the cross section of countries, as well as over time. A number of shocks, such as the global financial crisis and the European sovereign debt crisis, and central banks' adoption of unconventional monetary policies of unprecedented size have undoubtedly modified investors' economic outlook and risk appetite. At the same time, the secular decline in nominal rates in most international markets must originate from changes in expectations and risk compensation for their inflation or real side, or both, with clearly different policy implications. Disentangling the relative contribution of these effects in the term structure of yields is relevant for the transmission of monetary policy as well as for investors' and firms' saving and investment decisions.

In this paper, we develop a no-arbitrage term structure model that explicitly relates yield factors, macroeconomic conditions, and aggregate volatility, while still allowing for affine closed-form expressions for both nominal and real yields. The model enables us to identify which component is mostly responsible for movements in the yield curve of four major bond markets. To the best of our knowledge, we are the first to simultaneously incorporate these model ingredients to study yield dynamics on a panel of countries.

Our modelling framework consists of five latent state variables. We express the instantaneous real interest rate as an affine function of two variables, which capture its "slope" and "level", in the spirit of Abrahams et al. (2016). Next, we deviate from a standard yield-only approach (see, among others, Cochrane and Piazzesi, 2005, 2008; Durham, 2013; Adrian et al., 2013) by combining two features. The first feature is macroeconomic information. A number of studies show that macroeconomic data, in the form of either macro indicators or forward-looking survey expectations thereof, help capturing business-cycle variation in bond risk premia that is missed out by yields.[1] We thus introduce two latent factors and explicitly link them to the conditional mean of the price and output process, in order to

[1]For the use of macro indicators, see Ludvigson and Ng (2009), Bikbov and Chernov (2010), Joslin et al. (2014), and Kopp and Williams (2018) for the U.S., and Hördahl and Tristani (2012) and Garcia and Werner (2016) for the Euro Area. For the use of either macro or yield survey data, see Chun (2010), Kim and Wright (2005), Kim and Orphanides (2012), Crump et al. (2018), and Kaminska et al. (2018).

capture market expectations about future inflation and real growth and thus time variation in the economic outlook.

The second ingredient is a stochastic volatility factor that triggers heteroskedasticity in the opportunity set. Among others, Cieslak and Povala (2016), Feldhütter et al. (2018) and Berardi et al. (2020) emphasize the role of stochastic volatility for time-varying risk premia. For the U.S., Adrian and Wu (2009), Haubrich et al. (2012), and Berardi and Plazzi (2019) document the existence of a positive relation between volatility and inflation risk premia. We draw from this literature by assuming that this factor enters the conditional volatility of all the other variables, and risk premia.

Through the model, we break down nominal yields into investors' expectations of the short rate over the bond maturity and a term premium component, which represents the compensation for holding a long-term bond relative to a string of short-term bonds and thus depends on interest rate uncertainty. In turn, we split the first component into the sum of expected inflation and expected short-term real rates over the term of the bond. The term premium can also be expressed as the sum of a real term premium and an inflation risk premium, which captures the compensation for bearing the risk of unanticipated changes in future inflation perceived by investors holding nominal bonds. Moreover, we separately identify an additional (time-varying) convexity term, which originates from stochastic volatility and is typically lumped into either of the previous two components in conditionally homoskedastic models (Kim and Wright, 2005; Adrian et al., 2013)

We use the model to study the yield curve of four currency areas – the U.S., Euro Area, U.K., and Japan – over the January, 1999 to December, 2018 period. The model is estimated by maximum likelihood, with an approximate Kalman filter algorithm to infer the unobserved state variables. We require the model to match data on: nominal yields, as proxied by interest rate swap rates; breakeven (i.e., inflation) swap rates, which we adjust for the impact of liquidity following a similar procedure to Bekaert and Ermolov (2020); surveys of professional forecasts for CPI growth and real GDP growth rates; realized as well as swaption-implied nominal yield variance. This rich information structure allows us to

exploit the model cross-sectional restrictions efficiently.

The estimates reveal a few important regularities across countries, with some nuances. First, short rate expectations under the physical measure are, on average, by far the largest component of nominal yields. Term premia are small at short maturities, but their role increases at the long end. However, even at the 10-year maturity they account for at most one-sixth of the overall yield. For Japan, average term premia are negative and relatively large, but still do not exceed (minus) a third of the expectation component. Finally, convexity matters on average only at the long end. Dissecting the yields further, we find that inflation expectations are relatively flat, while real short rate expectations play a greater role at long maturities. The average inflation risk premium is about 15bps for the Euro Area and 30bps for the U.S. and the U.K. The real term premium is of comparable size to the inflation risk premium for the U.S., but only a few basis points for the Euro Area and the U.K. For Japan, instead, both the inflation and real risk premium are negative, and increasingly so with maturity.

Turning to the time-series behavior of the components, we observe a sustained decline in short rate expectations over the period, with a rebound in recent years for the U.S. and U.K. This pattern is mostly on account of a downward trend in real rate expectations in all four markets since 2001, whereas inflation expectations have remained stable or just slightly decreased for the U.S. and Euro Area. Nominal term premia are around 100bps in the early part of the sample, experienced a surge (for Japan, in negative territory) during the global financial crisis, and have decreased since then towards zero and below. Much of the movement has origin in the behavior of real term premia, which are estimated to be negative for all markets but the U.S. by the end of the sample. Finally, we detect economically relevant convexity effects that spike during turbulent periods and persist thereafter, mostly coming from the real side. For example, during the financial crisis, U.S. convexity is (minus) 100bps, or about the same size as term premia. Therefore, isolating this component changes our inference on the other terms, and in particular risk premia.

When looking at the decomposition of the second moment (i.e., variance) of yields, we

find that short rate expectations account for the largest fraction (between 55% and 72%) of yield fluctuations and, therefore, explain their decline over the sample.

This result is in contradiction with evidence by Wright (2011), Jotikasthira et al. (2015) and Mönch (2019). We first make sure our conclusions are not due to obvious model miss-specifications. Specifically, we verify that the model delivers economically modest fitting errors and reasonable model-implied maximal Sharpe Ratios (Duffee, 2010). For risk premia, we show that the model matches the degree of deviations from the Expectations Hypothesis in the data. Indeed, the fitted and model-implied Campbell and Shiller (1991) slope coefficients turn increasingly negative at long horizons, in line with the estimates from actual yields.

Next, we estimate Wright (2011) Gaussian five-factor affine model on our sample period and dataset. We replicate his conclusion that the model-implied term premium accounts for most of the volatility of the 10-year nominal yield (an exception is the Euro Area, for which it is almost at par with short rate expectations). Thus, our results are not driven by the particular sample period or input series. By comparing our term premia with those from Wright (2011) model, we note that while the average level is relatively similar, the volatility of its series is much higher than that from our model.

We thus drill down into the model features that are at the root of these differences. To this end, we estimate three competing models on our data: a Gaussian four-factor model of the Kim and Wright (2005) type (i.e., without stochastic volatility and output growth); a gaussian four-factor model with macro factors (but without stochastic volatility); and a stochastic volatility four-factor model (without output growth). We find that simultaneously incorporating macroeconomic information and stochastic volatility reduces the standard deviation in the nominal term premium estimation and shifts the lion's share of yield fluctuations away from term premia.

Another relevant output of the model is the implied long-run "equilibrium" real interest rate, i.e., the average expected real short rate over a five-year period starting five years ahead. Our approach combines features of the extant financial and macro approaches to the equilibrium rate (see Laubach and Williams, 2003; Christensen and Rudebusch, 2019), as it

is based on a no-arbitrage model integrated with macro factors. We find that all markets feature a marked decline towards zero in their long-run rate. However, while for the U.S., Euro Area and Japan the series has rebounded in recent years and reached 1% at the end of the sample, U.K. equilibrium real rates have remained negative since 2015.

In the last part of the paper, we examine yield and state variables co-movement in the cross-section of markets.[2] A few studies have documented the presence of common global factors and spillovers among bond markets of the major industrialized countries (see, e.g., Barr and Priestley, 2004; Dahlquist and Hasseltoft, 2013; Jotikasthira et al., 2015). We show that the model is able to match the extent of observed yield correlations across markets, which we trace back to the international factor structure in the latent state vectors. At the same time, however, even combining a two-factor global structure in the state variables with the model estimates delivers implied correlations that are far from the observed ones. We also analyse the presence of lead-lag effects in yields and their components. Consistent with the literature that posits a role of U.S. shocks as predictors and leading indicators in international markets (see, e.g., Rapach et al., 2013; Brusa et al., 2020; Caporin et al., 2020), we find that the U.S. is the strongest "exporter" of shocks to the long-end of the nominal yield curve, in particular through term premia. The U.K. has also a comparably high role, while the Japanese market generates the least amount of international spillovers.

The rest of the paper is structured as follows. Section 1 outlines the term structure model, and works out the yield decomposition. Section 2 describes the data. Section 3 presents model diagnostic and our main empirical results on yield components. Section 4 reports the analysis of cross-country comovement. Finally, section 5 offers concluding remarks.

1 The model

In this section, we outline our term structure model. Our framework consists of five latent state variables, and falls in the class of affine models that simultaneously deliver closed-form

[2]Compare Bekaert and Ermolov (2020) for a recent paper that studies international yield comovement.

expression for the term structure of both nominal and real yields (see, inter alia, Abrahams et al., 2016; Kaminska et al., 2018). A variable is meant to capture aggregate uncertainty, and drives conditional heteroskedasticity in the opportunity set. We further link two variables to the price and output process in order to identify them as macro factors, in the spirit of Ang and Piazzesi (2003), Wright (2011) and Joslin et al. (2014). Section 1.1 describes the data-generating-process and the specification of risk premia. Next, Section 1.2 presents the implied no-arbitrage term structure for nominal and real rates, and decomposes the resulting term structure of nominal yields.

1.1 State variables and the macroeconomy

We posit that the economy is driven by five latent factors, which are collected in the state vector $X = (v\ \mu\ \pi\ s\ \ell)'$. The variance factor, denoted v, is responsible for time variation in the conditional volatility of all the other variables. The instantaneous conditional mean of output growth, μ, and of the inflation rate, π, capture respectively investors' expectations of real and nominal price growth. Finally, we model the instantaneous real interest rate r as an affine function of the two variables s and ℓ, or $r = s + \ell$. This specification shares similarities with Abrahams et al. (2016), where the first two principal components of TIPS yields are used to model real interest rates. As a preview, the Kalman filter estimates of the model reveal that s is strongly correlated with the slope of the term structure of real interest rates, computed as the real short term rate minus the real long-term rate, and ℓ with the long-term real interest rate. Therefore, in what follows we refer to s and ℓ as the "slope" and "level" factor of real rates, respectively.

Under the physical probability measure $\mathbb{P}$, we let the vector X evolve according to the following Ito process:

$$dX_t = K(\Theta - X_t)dt + \Sigma\sqrt{S_t}dz_t \tag{1}$$

where: z_t is a vector of independent Brownian motions, K is the (5×5) matrix of mean-reversion coefficients and Θ is the (5×1) vector of long-term expectations. Σ is a (5×5) lower triangular matrix obtained as $\Sigma = (\Xi\Omega\Xi')^{\frac{1}{2}}$, where Ξ is a (5×5) diagonal matrix of

volatility coefficients and Ω a (5×5) correlation matrix. Matrix S_t is diagonal (5×5) with the element in position (i, i) given by $[S_t]_{ii} = \beta_i' X_t$, with β_i denoting the i-th column of the (5×5) matrix β which has ones in the first row and zeros elsewhere. To comply with the Dai and Singleton (2000) admissibility constraint, we impose that the off-diagonal terms of the first row of matrix K are null. In sum, we model the first factor v as a square-root process that enters the diffusion term of the other four, conditionally Gaussian factors. In addition, v can provide information on the expected level of the other state variables through the first column of the mean-reversion matrix K. The four variables $(\mu\, \pi\, s\, \ell)$ potentially interact with each other through both the drift term as well as correlation in their innovations.

We characterize the dynamics under the risk-adjusted probability measure $\mathbb{Q}$ by using the "essentially affine" specification of the instantaneous market price of risk of Duffee (2002):

$$\Psi_t = \sqrt{S_t^-}\ (\Lambda_0 + \Lambda_1\, X_t)\ , \tag{2}$$

where S_t^- denotes the inverse of S_t, Λ_0 is a (5×1) vector of constant risk premia, and Λ_1 is (5×5). This specification allows risk premia to vary over time with X_t, and to potentially turn negative. Moreover, it implies that the dynamics of X_t under $\mathbb{Q}$ is also affine, with the same diffusion term as in (1) and risk-adjusted drift $(\widetilde{K}\widetilde{\Theta} - \widetilde{K} X_t)$, being $\widetilde{K} = K + \Sigma\Lambda_1$ and $\widetilde{K}\widetilde{\Theta} = K\Theta - \Sigma\Lambda_0$.

In order to enhance the identification of the latent macro factors, we explicitly link them to the realized process for price level and output growth. We assume that the exogenously-given equilibrium price level p of the single physical good, and its real production output q follow Ito processes with time-varying drifts and volatilities:

$$\frac{dp_t}{p_t} = \pi_t dt + \gamma_p dw_{p,t} + \sigma_p \sqrt{v_t} du_{p,t} \tag{3}$$

$$\frac{dq_t}{q_t} = \mu_t dt + \xi\left(\gamma_q dw_{p,t} + \sigma_q \sqrt{v_t} du_{p,t}\right) + \sqrt{1-\xi^2}\left(\gamma_q dw_{q,t} + \sigma_q \sqrt{v_t} du_{q,t}\right), \tag{4}$$

with w_ts and u_ts being uncorrelated Brownian motions. The variables π and μ capture therefore the conditional mean of the inflation rate and real output growth, respectively.

In addition, the stochastic volatility factor generates conditional heteroscedasticity in both processes, and thus captures commonality in their volatility dynamics.

1.2 Term structure and yield components

Based on the assumptions above, the real SDF takes form

$$\frac{dM_t}{M_t} = -r_t dt - \Psi_t' dz_t \tag{5}$$

while Ito's lemma implies that the dynamics of the SDF that prices nominal assets $M_t^{\$} = M_t/p_t$ is

$$\frac{dM_t^{\$}}{M_t^{\$}} = \frac{dM_t}{M_t} - \frac{dp_t}{p_t} + \left(\frac{dp_t}{p_t}\right)^2 - \frac{dM_t}{M_t}\frac{dp_t}{p_t}\,. \tag{6}$$

By imposing the no-arbitrage condition that the drift of this process equals (minus) the equilibrium instantaneous nominal interest rate y_t, we obtain

$$y_t = \pi_t + r_t \underbrace{-\frac{1}{dt}Var_t\left(\frac{dp_t}{p_t}\right) + \frac{1}{dt}Cov_t\left(\frac{dM_t}{M_t}, \frac{dp_t}{p_t}\right)}_{irp_t} \tag{7}$$

where irp_t denotes the instantaneous inflation risk premium. Combining equations (2), (3), and (5) we can express irp_t as a linear function of the state vector X_t:

$$irp_t \;=\; -\sigma_p^2 v_t - \sigma_p\sqrt{v_t}\,(\Lambda_0 + \Lambda_1 X_t)' \left(\sqrt{S_t^-}\right)' \Gamma \equiv \varepsilon_0 + \varepsilon' X_t\,, \tag{8}$$

with $\Gamma = Corr(du_{p,t}, dz_t)$ being (5×1), which in turn entails that the instantaneous nominal interest rate y_t is affine in X_t, or $y_t = \delta_0 + \delta' X_t$. This result, together with the affine risk-neutral dynamics of the state vector, implies that the time-t equilibrium arbitrage-free price of a nominal unit discount bond with time to maturity τ has an exponentially affine closed-form solution, namely

$$F_t(\tau) = \exp\left\{A(\tau) - B'(\tau)X_t\right\}. \tag{9}$$

The nominal term structure is therefore affine in the state vector, i.e.

$$Y_t(\tau) = a(\tau) + b'(\tau)X_t, \tag{10}$$

where $a(\tau) \equiv -A(\tau)/\tau$ and $b(\tau) \equiv B(\tau)/\tau$, and $A(\tau)$ and $B(\tau)$ solve the no-arbitrage system of ordinary differential equations (Piazzesi, 2010).

The expression in (10) relates yields with the state vector through a linear function, whose coefficients embed both a risk-adjustment and the expectation of the future path of the short rate, plus a Jensen's inequality term. Most term structure models (see, e.g., Kim and Wright, 2005; Adrian et al., 2013) lump the Jensen's convexity term into either of the other two components. However, as also noted by Cieslak and Povala (2016), Rebonato and Putyatin (2018) and Berardi et al. (2020), its contribution is time-varying, and may well affect our inference especially during turbulent times.

Formally, we can write the yield on a τ-maturity zero coupon bond as the sum of the average $\mathbb{P}$-expectation of the nominal short rate ESY, a nominal term premium TPY, and the average nominal convexity CXY, all computed over τ:

$$Y_t(\tau) \quad = \quad ESY_t(\tau) + TPY_t(\tau) + CXY_t(\tau). \tag{11}$$

Appendix A provides full derivation. This decomposition serves as basis for our empirical investigation of the drivers of the yield curve in four major markets.

We can alternatively express the τ-year nominal yield as the sum of a breakeven or inflation rate $H_t(\tau)$, i.e. the yield on a zero-coupon asset that pays realized inflation over the maturity τ, and a real yield $R_t(\tau) = Y_t(\tau) - H_t(\tau)$, i.e. the yield on an inflation-protected security. The model-implied equilibrium real and breakeven rates are also affine in the state variables and could be similarly split into the sum of three components,

$$H_t(\tau) \quad = \quad ESH_t(\tau) + TPH_t(\tau) + CXH_t(\tau) \tag{12}$$

$$R_t(\tau) \quad = \quad ESR_t(\tau) + TPR_t(\tau) + CXR_t(\tau)\,. \tag{13}$$

This means that, overall, a τ-maturity nominal yield is driven by six terms, all computed over τ: the average $\mathbb{P}$-expectation of the inflation rate (ESH) and of the real rate (ESR); the inflation term premium (TPH) and the real term premium (TPR); the average convexity of inflation expectations (CXH) and of real rates (CXR). We now take the model to the data, and empirically assess the importance of each of these terms.

2 Data and preliminary statistics

We estimate the model using data on nominal yields, breakeven rates, yield volatilities, and macroeconomic expectations spanning the period from January, 1999 to December, 2018 for the following four markets: the United States, Eurozone, the United Kingdom, and Japan.[3] We provide a detailed description of data sources and transformations in Appendix B.

For nominal yields, we use the zero curve from interest rate swap rates with maturities between 2 and 10 years. Compared to yields on Government bonds, swap rates come from standardized contracts which are reset every day and are therefore not contaminated by interpolation procedures that are necessary to construct artificial zero coupon bonds. Moreover, swap rates do not carry the convenience yield that depresses Treasury rates especially during crisis periods (Du et al., 2019).

A key ingredient of the model is the simultaneous role of time-varying volatility and macroeconomic expectations in driving yields. Therefore, we exploit information on the second moment of yields and on macroeconomic forecasts. For the former, we require the model to fit the realized within-month variance of daily changes in yields at the most liquid maturities of 2, 5, and 10 years. In addition, we use the implied variance of interest rate derivative contracts to proxy for the expected variance of yields under the risk-neutral measure. To be precise, we use the implied variance in 6-month swaption contracts that give the holder the right to enter a swap with maturities of either 2, 5, or 10 years. By capturing risk-adjusted market expectations of nominal yield volatilities, these series prove very useful in identifying

[3] In what follows, we refer to them alternatively as markets or countries, even though the Eurozone is strictly speaking not a country.

the latent variance factor v and the risk premia parameters. For macro forecasts, we use the median 1-year ahead forecasts of annual CPI growth and annual real GDP growth rates, which are available on a quarterly basis.[4]

We capture breakeven rates through zero-coupon inflation swap rates, again spanning maturities between 2 and 10 years. Inflation swaps are derivative contracts where, at maturity, the protection seller pays the other party the cumulative CPI inflation over the lifespan of the contract. In exchange, the protection buyer pays a rate which is predetermined and fixed, and is known as the synthetic breakeven inflation (BEI) rate.[5] The data start on July, 2004 for all countries but Japan, for which it just begins in March, 2007.

We collect summary statistics for the dataset in Table 1. Panel A reports the mean and standard deviation of the series in each country, focusing on the 2-, 5-, and 10-year maturities for yields and breakeven rates. A few facts are noteworthy. First, the term structure of yields has been upward sloping in all countries, with the difference between the 10-year and 2-year yield being lowest at 65bps for the U.K. and highest at 128bps for the U.S. For breakeven rates, however, the term structure has been flat to downward sloping in Japan, moderately steep at 35bps in the U.K. and Euro Area, and steepest at 58bps for the U.S. Turning to realized yield volatilities, the term structure is initially increasing until the 5-year maturity, and then flattening at the long end. Their standard deviations (the "vol of vol") are instead decreasing with maturities. Somewhat similar patterns are observed for implied volatilities, which tend to be some 5–10bps higher than the realized ones. Finally, inflation and real GDP growth forecasts vary on average in the 2–3% range for the U.S. and the U.K., and are lower at around 1.5% for Euro Area. For Japan, consistent with the breakeven rates, inflation forecasts are instead much smaller at 0.5%, whereas GDP growth is predicted to be on average higher at 1.8%.

In Panel B of the table, we look at the factor structure of the data. Specifically, we report

[4]We do not have access to long-term forecasts, which are not freely available for all countries in our study. For the U.S. and Euro Area, SPF forecasts are available at the 10-year and 5-year horizon respectively. Our conclusions on the yield dissection continue to hold when using these series. In the interest of preserving the same information set in the estimation, and guarantee that the results are not driven by differences in the input series, we stick to the common 1-year forecast series.

[5]See Fleckenstein et al. (2017) for a formal definition of the zero-coupon inflation swap contract, and Haubrich et al. (2012) and Kaminska et al. (2018) for other studies that use such data in the context of term structure models.

the result of the Principal Component Analysis, PCA, on different groups of series. For each market, we report the percentage of the overall variance that is explained by the first (column 'PC1') and collectively by the second to fourth (column 'PC2–4') principal component of the correlation matrix of the corresponding group of series. The PCA is performed on the sample period over which all of them are available. Since our model is based upon the assumption of common latent factors driving yields, volatilities, and macroeconomic variables, this analysis provides us an indication whether and for which market and series such an assumption is supported by the data.

In the first row of the panel, the PCA is on the group of nominal yields and breakeven rates (across all nine maturities). We see that the first factor (PC1) accounts for a sizeable fraction of the overall variance. However, the importance of this factor varies significantly across markets, with fractions ranging from 56% for the U.K. to 92% for Euro Area. In turn, the variability that is missed by the first component is captured by the second to fourth ones, so that collectively the four factors explain above 99% of the overall variance in nominal and inflation rates across all countries. For volatilities, reported in the second row, the evidence is less diverse, with PC1 accounting about 75% is all countries but the U.S., where it peaks at 86%. Finally, the last row is for the PCA when combining all twenty-six series together (nine each for yields and breakeven rates, three each for volatilities, and two for macro factors). The importance of PC1 varies from 43% for the U.K. to 62% for Euro Area. Clearly, more than a single factor is needed to fully capture the richness in the dynamics of the entire cross-section, with the role of the other principal components being also quite diverse across markets. Such heterogeneity in factor structure highlights the potential benefits of an international study.

Before proceeding with the model estimation, we address a potential issue with the use of inflation swap rates, namely the impact of liquidity risk in their pricing. Some authors contend that these derivative contracts are much more liquid than inflation-linked securities (such as TIPS) as evidenced by nearly flat swap bid-ask spread during the crisis, see e.g. Haubrich et al. (2012). Other authors argue instead that the market for inflation-linked

bonds (and securities) is not as liquid as the nominal bond market, and such liquidity differential would impact breakeven inflation swap rates negatively (see, e.g., Pflueger and Viceira, 2016). Given that the four countries were hit at various times by market turmoils during the relatively long time span of our study, we want to avoid a potentially significant liquidity risk premium to distort our estimates. For this reason, we account for swings in liquidity by regressing the breakeven rates on measures of liquidity while controlling for inflation expectations, following the approach in D'Amico et al. (2018), Pflueger and Viceira (2016), and Bekaert and Ermolov (2020). To be precise, for each maturity and country, we regress breakeven rates on the 10-year inflation-swap spread (obtained as the breakeven rate minus the difference between the nominal yield and the yield on an inflation-linked security, all at the 10-year maturity), the country TED spread (computed as Libor minus the yield on a T-bill, at the 3-month maturity), and the 1-year survey inflation expectation defined above.[6] We normalize the resulting (maturity-specific) liquidity premium to zero at its minimum, and incorporate it in the breakeven rate (thereby, in turn, lowering the implied real rate). Details on the procedure are provided in Appendix B.

3 Empirical results

In this section, we discuss the estimation of the term structure model and the resulting series. Section 3.1 outlines the estimation method. Section 3.2 collects a battery of specification tests. In section 3.3, we present our main empirical results, which are organized around the discussion of average yield decomposition, time series patterns, and variance decomposition. Section 3.4 carries a comparison of our term premia estimates with those from alternative models. Finally, in section 3.5 we comment on our model-implied long-term equilibrium real rate.

[6] An alternative would be to model liquidity as yet another variable in the model, as, for example, in Abrahams et al. (2016). However, we opted for this "reduced form" approach to avoid increasing the dimensionality of the state vector.

3.1 Estimation method

Since the model features unobservable state variables with affine dynamics, we rely on Quasi Maximum Likelihood using the Kalman filter (see Duffee and Stanton, 2012, for a review of this methodology). This estimation approach is well suited to accommodate the different frequencies and time periods at which the series are available.[7]

The estimation is performed separately for each country using all twenty-six series: i) 2- to 10-year nominal yields; ii) 2- to 10-year breakeven rates; iii) the realized variance of nominal yield changes at the 2-, 5-, and 10-year maturities; iv) the corresponding implied variance from the 6-month swaptions; and v) real GDP and inflation forecasts (at quarterly frequency). We obtain the corresponding observation equations by adding to each model-implied expression a normally distributed and homoskedastic error.

The state equations are represented by the first-order monthly VAR process implied by the continuous-time model in (1). In order to better identify the volatility coefficients we also require the model to match the cross-equation restrictions implied by the conditional variance-covariance matrix of the shocks to the state vector.

3.2 Specification tests

We collect the country-level parameter estimates in Appendix Table D.1. We observe that the variance factor v is mean reverting in all four countries, with coefficients ranging from 0.22 in the case of the Euro Area and Japan to 0.55 for the U.K. In all countries, a significant mean reversion occurs also for the conditional mean of output growth μ, expected inflation π and the slope of real rates s, whereas the long-term real rate ℓ has very low mean reversion, although the coefficients remain statistically significantly different from zero.

Summary statistics on the goodness of fit are reported in Panel A of Appendix Table D.2. The standard deviation of estimation errors in nominal yields ranges between 3bps for Japan to 8 bps for the U.K., while in the case of breakeven inflation rates the range is between 9 and 15 bps. The fit of yield volatilities is also rather good, with a standard deviation

[7]For details on the implementation, see the Appendix in Berardi and Plazzi (2019).

of estimation errors comprised between 10 and 17 bps. As regards the macro forecasts, we notice that the fit of real GDP growth is quite accurate, while inflation rate forecasts are relatively harder to match.

In order to evaluate the ability of the model to capture the dynamics of risk premia in a reasonable way, we report two complementary statistics. First, we look at the model-implied maximal Sharpe ratio, which is the Sharpe ratio that can be attained by a portfolio of bonds spanning the payoff of the implied stochastic discount factor (Duffee, 2010). Its summary statistics are collected in Panel B of Appendix Table D.2. The maximum ranges from 0.48 for Japan to 2.62 for the U.S., which reassures that the model does not deliver implausibly high Sharpe Ratios.

Second, we check whether the model can replicate the extent of deviations from the Expectations Hypothesis (EH) that is found in the data. Following Wachter (2006) and Joslin and Le (2013), we resort to Campbell and Shiller (1991) regressions as a diagnostic test for detecting departures from EH. Specifically, we first estimate regression

$$Y_{t+12}(\tau-1) - Y_t(\tau) = \phi_0 + \phi(\tau)\left[\frac{Y_t(\tau) - Y_t(1)}{\tau - 1}\right] + \epsilon_{t+12}(\tau) \tag{14}$$

on both observed and fitted (i.e. model-implied sample) yields. Moreover, following Dai and Singleton (2002), we compute the model-implied population coefficient $\phi(\tau)$, which obtains using the estimated model parameters into the expression

$$\widehat{\phi}(\tau) = (\tau-1)\left[\frac{Cov\{Y_{t+12}(\tau-1) - Y_t(\tau), Y_t(\tau) - Y_t(1)\}}{Var\{Y_t(\tau) - Y_t(1)\}}\right], \tag{15}$$

where the covariance and variance are expressed in closed form. Finally, we run a Monte Carlo simulation in which we use the estimated model parameters to generate five hundred samples of length 50,000 and, for each sample, calculate $\phi(\tau)$. As expected, the average value of the estimated ϕs is very close to the model-implied population coefficients. We take the plus/minus one-standard deviation interval around these estimates as confidence interval for the $\hat{\phi}(\tau)$ coefficients.

Figure 1 displays the corresponding estimates for maturities ranging between 2 and 10 years. We find that the coefficients obtained by running the Campbell and Shiller (1991) regression on fitted yields are perfectly aligned with those from the data. Moreover, the model-implied projection coefficients decrease with maturity, turn negative and are statistically different from one. Therefore, we conclude that the risk premia from the model vary in a way that matches the observed failure of EH. This conclusion runs contrary to the extant critique of stochastic volatility term structure models, see Dai and Singleton (2002) and Joslin and Le (2013). We conjecture that this difference stems from the ability of the model to generate less noisy term premia, as highlighted by the comparative analysis of Section 3.4.

We also look at the model ability to match short term nominal rates, which are not used in the estimation. We find that the model fares quite well at matching market rates. For example, the correlation between our series and the 6-month LIBOR rate is 0.98 for the U.S., 0.97 for the Euro Area, 0.98 for the U.K., and 0.88 for Japan. If we consider monthly changes in the series, the correlation becomes, respectively, 0.65, 0.58, 0.59, and 0.42. Moreover, both the mean and the standard deviation of the two series are comparable. In all cases, our estimated short rate is slightly more volatile than the actual 6-month LIBOR rate, but the differences are economically reasonable.[8] Finally, as a further validation test, in Appendix C we show that the estimated volatility factor from our conditionally Gaussian model shares very similar patterns with the yield volatility from a reduced-form regime-switching model.

3.3 Dissection of yields

3.3.1 Average yield components

Table 2 reports the average estimated yield components at the 2, 5, and 10-year maturities for each of the four markets in our study. The first three columns contain the decomposition of nominal yields, Eq.(11). Is it clear that, across all markets, the bulk of the level of nominal

[8] For example, in the case of the U.S. the standard deviation is 239 vs 197 bps, while in the case of the Euro Area 182 vs 168 bps.

yields (no less than 90%) comes from short rate expectations under the physical measure, $ESY(\tau)$. This component averages about 100bps for Japan and about 300bps for the other markets. Across maturities, its term structure is steepest for Japan (from 38bps to 157bps), quite steep for the U.S. and Euro Area (from 253bps to 351bps and from 216bps to 312bps, respectively) and least so for the U.K. (from 298bps to 357bps).

Turning to term premia TPY, their contribution to the level of yields is much smaller than ESY, in particular at short horizons when it accounts for no more than 5% of the average yield. The term premium estimates are positive and increasing with maturity for the U.S. (from 14bps to 60bps) and the U.K. (from 16bps to 32bps); for the Euro Area, they are essentially null at short maturities, and a meager 16ps at the long end. In contrast, average term premia in Japan are negative at all horizons and reach a significant –47 bps at the 10-year maturity. Overall, while we confirm that risk compensation plays a role at longer maturities, its effect remains modest, an exception being Japan where it enters with a negative sign and is about (minus) a third of the expectation component.

Finally, the contribution of the convexity component CXY is (by construction) negative, and increasing in absolute terms with maturity. On average, convexity plays a somewhat economically significant role at long maturities for the U.S. (-18bps, or about (minus) 5% of the average yield) and U.K. (-12bps), while it is quite modest for the Euro Area and Japan.

The remaining six columns report the average of the inflation and real components, Eq.(12)-(13). A few regularities emerge across all markets, with some differences for the Japanese market. The term structure of inflation expectations is almost flat, varying from about 300bps for U.K. to only about 80bps for Japan, although even in this case it accounts for the majority of the average level of the expected short rate. This behavior contrasts with the steep average real term structure, which implies that real short rate expectations get progressively more important at the long end. For example, for the U.S., they account for only 3% (7/253) of the average expected short rate expectations over a 2-year period, but about 30% (107/351) at the 10-year horizon. The inflation risk premium is increasing

with maturity, reaching 15bps for the Euro Area and twice as much (30bps) for the U.S. and the U.K. The real term premium is of comparable size to the inflation risk premium for the U.S., and only a few basis points for the U.K. and Euro Area. For Japan, instead, both the inflation and real risk premium are negative and increasingly such with maturity. Lastly, we note that convexity mainly comes from the real side, as the contribution of convexity in inflation is essentially null.

3.3.2 Time series of nominal yield components

Figure 2 displays the time series of the decomposed 10-year nominal yield. For the U.S. (Panel A), the estimated term premium peaks at 125bps in the first part of the sample, ranges between 50bps and 90bps for the period 2004 to 2010, and finally declines towards zero by the sample end. Short rate expectations are characterized by a sustained decline from 6% in 2000 to about 1.5% in mid-2016, followed by an upward trend towards values around 3%. The convexity term is, at times, rather substantial. It reaches −55bps between end-2001 and mid-2003, is smaller during the 2004-2006 expansionary monetary policy, and hits its minimum of −110bps during the 2008 financial crisis. In the last part of the sample, it shrinks in correspondence with the Fed's unconventional monetary policies. Thus, while the importance of convexity from the numbers in Table 2 appears modest, its magnitude compared to the level of yields (for the U.S., but also for Euro Area and U.K.) is all but negligible for prolonged periods.

Isolating the convexity effect has, in turn, important implications for the resulting term premia. To show this, we compare our estimates of the 10-year term premium with those from alternative Gaussian-type models – namely, Kim and Wright (2005) and Adrian et al. (2013) (henceforth respectively KW and ACM) – that do not explicitly consider the convexity term, but rather treat it as a constant.[9] On average, our term premium is comparable to that of KW (60bps versus 59bps) and considerably lower than that of ACM (108bps). However, the standard deviation of our estimated term premium is only 40bps against 54bps for KW and

[9] Term premium estimates for the KW and ACM models are frequently updated on their respective websites at the Federal Reserve for KW and New York Fed for ACM.

92bps for ACM.[10] Kim and Wright (2005) use expectations on short term rates at different time horizons which, as shown by Kim and Orphanides (2012), make short-rate expectations more volatile and stabilize the term premium.[11] We obtain a similar effect through the use of survey expectations on macro factors, which also allow us to disentangle the real and the inflation component of yields and to study the interrelation between macro factors and the term structure. We further investigate this feature of the model in Section 3.4 below.

It is also interesting to compare our term premium estimates with those from other types of models, such as the constant volatility models of Christensen and Rudebusch (2012) and Kopp and Williams (2018), both based on a modified Nelson and Siegel (1987) framework, which is augmented, in Kopp and Williams (2018), by two macroeconomic factors related to the unemployment rate and the inflation gap. The path of the 10-year term premium from these two models appears similar to ours, but our term premia are lower in absolute terms and much less erratic. Haubrich et al. (2012) stochastic volatility model, instead, delivers a term premium that is relatively high (at about 140bps, on average) and does not spike around turbulent times.[12]

For the Euro Area in Panel B, we again observe a decadal fall in yields which is mostly due, until 2012, to a decline in short rate expectations. However, unlike the U.S., we do not see a rebound in the last part of the period, as the expectations remain flat at about 1.6%. Convexity is some negative 20bps until late 2008, when it jumps to −45bps during the 2008 crisis and to −30bps during the European sovereign debt crisis, falling to zero thereafter. The term premium, instead, is positive and relatively large ranging between 70bps and 100bps until 2005, then it decreases to nearly zero in 2006, when it starts a steady increase up to the 1% peak in 2009. Eurozone term premia it become negative in 2012, and remain persistently so afterwards reaching values in the −60 to −100bps range which almost equal (minus) the level of yield. This pattern shares analogies with that in Cohen et al. (2018), where are reported estimates for the 10-year term premium obtained applying

[10] If we consider monthly changes in the term premium, the difference in standard deviation is even more pronounced: 9bps versus 16bps and 25bps, respectively, for KW and ACM.

[11] Li et al. (2017) show that once survey expectations of long-term yields are included in the ACM framework, ACM and KW produce approximately the same term premium estimates.

[12] It must be considered, however, that the sample period spanned by Haubrich et al. (2012) estimates is from 1982 to 2010.

the ACM model to German government bonds, the macro term structure model of Hördahl and Tristani (2014) to French government bonds, and the Bank of France model (based on Monfort et al., 2017) to OIS rates.

A similar time series behavior of term premia is observed for the U.K. in Panel C. The series is some 50-60bps in the early part of the sample, and it shrinks to few basis points in the years preceding the 2008 financial crisis, consistent with evidence by Joyce et al. (2010) and Malik and Meldrum (2016).[13] Then, we observe a run up with a peak at about 100bps followed by a decline and a rebound in 2013. By 2015, U.K. term premia are negative and remain such until the end of the sample, averaging −30bps or about half those for continental Europe. The pattern of the 10-year convexity term also mirrors quite closely the Euro Area: it reaches some −30-40bps in the early part of the sample and during the 2008 financial crisis, to then revert at zero by 2010. Hence, the importance of this terms to the level of yields is at times quite comparable to that of term premia.

Compared to the other markets, the expected short rate for Japan (Panel D) is much smaller and smooth, slowly declining from 2% in 2006 to about 1% by 2018. An almost equal contribution to the overall demise of the 10-year nominal yield is coming from the term premium component, which is around zero until 2009 when it jumps to some −70bps and remains persistently negative thereafter reaching a trough of −100bps.[14] Unlike the other markets, the convexity term is instead mostly negligible.

3.3.3 Time series of inflation and real components

Panel A of Figure 3 displays the time series of inflation rate expectations across countries. These are highest at 300-350bps and vary the most for the U.K., while are mainly below 100bps for Japan. All series are characterized by a sharp drop during the financial crisis. However, while for the U.K. and Japan the expectations have reverted back to the pre-crisis

[13] According to these and other studies (see also Guimarães, 2012; Joyce et al., 2012) a large fall occurred in the term premium, especially in its inflation component, around 1997, as a consequence of the operational independence given to the Bank of England and the application of the minimum funding requirement which induced a high demand of index-linked bonds from pension funds.

[14] Our term premium estimates are significantly lower and, conversely, our estimated short rate expectations higher than those obtained by Imakubo and Nakajima (2015) using a shadow rate model.

levels by the end of the sample, for the U.S. and especially Euro Area they remain about 50bps lower.

Panel B shows that real rate expectations have markedly declined in all four markets since 2001. U.S. expectations have wondered quite a bit over the sample, reaching a value of zero towards the end of 2011 until mid-2013 and then again in 2016, but have recovered to nearly 100bps in the last period. A similar pattern is observed for the U.K., for which however expectations have remained consistently negative since mid-2010 and have reached a minimum in 2017, ending up at −100bps by the end of the sample. Expectations for the European and Japanese market have fallen to zero by mid-2013, and have stuck around this value thereafter.

In Panel C of the figure, we see that the 10-year inflation risk premium for the U.S., the U.K. and the Euro Area shares very similar dynamics. It is mostly positive, volatile in the early years of the sample, it spikes during the financial crisis, and is relatively flat over the last four years of the sample, with a decline towards zero for the Euro Area at the end of the sovereign debt crisis. For the U.S., the path of the implied 5-10 year forward inflation risk premium (not reported) is similar to the one in Abrahams et al. (2016), and their average over the common sample is approximately the same (around 70bps). For the U.K., our estimated premium is quite persistent but remains positive, in contrast to the highly volatile and often negative series obtained in Kaminska et al. (2018). For Japan, instead, the premium is mainly negative at about −50bps and much more erratic, reaching a trough at the onset of the financial crisis.

Finally, real term premia (Panel D) are characterized by a distinct downward trend over the sample, which mimics but does not fully coincide in the time series and across markets with that in short rate expectations. The decline is especially pronounced after the financial crisis, and real term premia enter in negative territory for all markets starting in 2012. By 2018, real term premia are negative 50–70bps for all markets but the U.S., whose premium has reverted back to zero in recent years. These negative real term premia are consistent with a power utility consumption-based asset pricing model, where persistent shocks to

consumption growth make long-term inflation-indexed bonds a desirable hedge (Campbell et al., 2009).

3.3.4 Variance decomposition

In Table 3 we report the decomposition of the variance of yields that are implied by the time series reported above.[15] At the 2-year maturity, short rate expectations account for the vast majority of yield variance (from 91% in the case of Japan to 97% for the U.K.). The term premium accounts, on average, for only a meager 5% of the variability of the 2-year yield, whereas the contribution of the convexity component is essentially null. At the 10-year maturity, the impact of the term premium component increases significantly, with noticeable differences across countries. In particular, it contributes for 29% of the variability of the 10-year nominal yield of the U.S, 36% and 38% in the case of U.K. and the Euro Area, respectively, and for a large 44% in the case of Japan. Nevertheless, short rate expectations remain by far the most important element in driving the dynamics of long term yields, with values ranging from 55% for Japan to 72% for the U.S.

When breaking down the components into their inflation and real counterparts, we observe that most of the variability comes from the real side, both at short and long maturities. For example, real short rate expectations and the real term premium account, respectively, for 46% and 29% of the variability of the 10-year yield averaging across the four markets. The average contribution of inflation expectations and risk premium for the variability of the 10-year yield is, in total, around 25%. Inflation expectations are relevant in the case of the U.K. (26%), while the role of the inflation risk premium is particularly pronounced for Japan (14%).

3.4 Term premium: the source of difference

The finding that expected short rates are mostly responsible for driving bond yields and their decline over the sample runs contrary to previous international evidence by Wright

[15] As the components are not orthogonal, the decomposition is obtained akin to the computation of a component-VaR.

(2011), Jotikasthira et al. (2015) and Mönch (2019). This conclusion, in turn, originates from different estimated term premia. In this section, we drill down into the reason for such differences.

We first verify that our result is not specific to the sample and dataset we used in the empirical work. To this end, we fit the Wright (2011) model (hereafter, WR) to our data. Similar to ours, WR is a five-factor affine term structure model with macro factors. However, unlike our model, WR does not feature stochastic volatility, and the factors are represented by the first three principal components of yields and two macro variables, proxied by the exponentially weighted moving average of quarterly inflation and GDP growth. We extract the three principal components from our yield curves, but, in order to facilitate the comparison with our estimates, we use as macro factors the 1-year ahead survey forecasts of inflation and GDP growth (see Appendix B.1).[16]

Panel A of Table 4 compares the mean and the standard deviation of the 10-year short rate expectations and term premium obtained from the estimation of WR with those provided by our model. We find that, apart from Japan, the average level of the components is relatively similar for the two models, especially for the U.S. and the Euro Area. Instead, the standard deviation of the components is significantly different, as in WR the volatility of short rate expectations is much lower and the volatility of the term premium much higher than that estimated by our stochastic volatility model. Panel B of the table shows that, consistent with the evidence in Wright (2011), the term premium in WR accounts for most of the volatility of the 10-year yield in all countries but the Euro Area. This evidence reassures us that our results are not driven by the particular sample period or input series.

To trace the source of the difference back to our model features, notably the joint inclusion of macro expectations and stochastic volatility, we next carry out a "horse race" experiment. We do this by benchmarking our estimates against those from three competing models: a gaussian four-factor model of the Kim and Wright (2005) type (i.e., without stochastic volatility and macro factor μ); a gaussian four-factor model with macro factors (but without

[16] As a robustness check, we replicate the estimation using the macro factors calculated as in Wright (2011) and find that the results for the estimated term premia and short rate expectations are very similar.

stochastic volatility); and a stochastic volatility four-factor model (without macro factor μ). For the sake of brevity, we limit our analysis to the U.S. market, and estimate the models over the period July, 2004 to December, 2018, where the begin date is dictated by the availability of breakeven rates.

In the gaussian four-factor model of the KW type (which we label KW4) the state variables are represented by the expected inflation rate π and, similarly to Kim and Wright (2005), three latent factors. We fit both nominal yields and breakeven inflation rates, with maturity between 2 and 10 years. In addition, as in Kim and Wright (2005), we fit a survey forecast for the average short rate over a 10-year time horizon.[17]

The gaussian four-factor model with macro factors (GM4) includes as unobservable factors the conditional mean of output growth μ, the expected inflation rate π, the slope of the real term structure s and the long-term real interest rate ℓ. We require the model to fit nominal yields and breakeven inflation rates, with maturity between 2 and 10 years, and 1-year ahead survey forecasts for real GDP growth and CPI inflation rate.

Finally, the stochastic volatility four-factor model (SV4) assumes that the four unobservable factors are given by the variance factor v, the expected inflation rate π, the slope of the real term structure s and the long-term real interest rate ℓ. We fit the model using nominal yields and breakeven inflation rates, with maturity between 2 and 10 years, and realized variance and implied variance of yields with maturity 2, 5 and 10 years.

We plot the resulting 10-year nominal term premium in Appendix Figure D.1. We note that the standard deviation of the term premium is 61 bps for the KW4 model, 42 bps for both GM4 and SV4, and 31 bps for our model.[18] Furthermore, when we decompose the variance of monthly changes in the 10-year yield we find that KW4 – as noted in section 3.3 for KW and ACM – explains the variability of yields mainly as a term premium effect, while GM4, SV4 and our model provide a different interpretation, with changes in expected future short rate as the main driver of the variability of the 10-year yield. In particular, the percentage

[17] As a proxy for this forecast, we use the median value for the 10-Year Bill Return (BILL10) from the Survey of Professional Forecasters of the Federal Reserve Bank of Philadelphia. These forecasts are available on an annual basis.

[18] If we separate the inflation and the real component of the 10-year term premium, we find that in all the models the real term premium is much more volatile than the inflation risk premium, with the latter being particularly smooth in models with stochastic volatility.

of the total variability of the 10-year yield explained by the term premium component is equal to 67% for KW4, 43% for GM4, 32% for SV4 and 27% for our model. Overall, these findings show that taking simultaneously into account macroeconomic information and time-varying volatility reduces the noise in the nominal term premium estimation and shifts the lion's share of yield fluctuations away from term premia, in contradiction with the previous (mainly gaussian-based) evidence.

3.5 Equilibrium real rate

In this section, we comment on the long-run "equilibrium" real interest rate that is implied by the model, defined as the average expected real short rate over a five-year period starting five years ahead. This rate is a proxy for the wicksellian "natural" rate, which represents the long-run real interest rate consistent with a closed output gap and stationary inflation.

In the literature, the estimation of the equilibrium real interest rate has been carried out through either a macroeconomic approach, which is based on the relationship between output, inflation and interest rates (see, among others, Laubach and Williams, 2003), or a financial approach, which relies only on data for the yield curve (see, e.g., Christensen and Rudebusch, 2019). Macro-based measures of the equilibrium rate have been obtained using the Kalman filter or similar filtering techniques in order to disentangle the real interest rate trend and cycle, as in Bauer and Rudebusch (2017). The financial approach extracts instead the equilibrium real rate using filtering techniques or model closed forms to decompose nominal rates. An advantage of using market-based measures is that they are forward-looking and available at high frequencies. On the other hand, they implicitly assume that market prices contain all the relevant information on macro expectations.

A comparison of the estimates from the above-mentioned studies reveals that the two approaches generate substantially different equilibrium rates. Hence, considerable uncertainty remains about the "correct" level. Our approach provides a bridge between the financial and the macro approaches to the modeling of the equilibrium real rate, as it is based on a finance-based no-arbitrage model integrated with macro factors. In addition, and different

from previous methodologies, our estimates are consistent with both the first and second moment of interest rates since we allow for stochastic volatility.

Figure 4 displays (blue solid line) the implied estimates of the five- to ten-year real forward rate for the four markets in our study. All series are characterized by a marked decline over the sample period. Average rates are highest for the Euro Area and U.S. at around 1.5%, followed by Japan (1.2%) and U.K. (0.9%). Perhaps surprisingly, rates to the U.K. have been nearly zero starting mid-2012, and consistently negative since 2015, while those for Japan and the Euro Area always remain above 20-30 basis points. For the U.S. and U.K. we can compare our series to a "model-free" benchmark, namely the five- to ten-year real forward rate that is implicit in the market price of inflation-protected securities. To be precise, the dash dotted line in Panel A displays the forward rate from the U.S. TIPS yield curve calculated by Gurkaynak et al. (2008). The two series move remarkably closely together (correlation is 0.96), with the TIPS line being some 50-60 basis points higher on average, which is most likely on account of TIPS illiquidity in the early part of the sample. For the U.K. in Panel C, we instead show the forward rate computed from the yield curve of U.K. Index-Linked Gilts published by the Bank of England. Again, the correlation with our model implied equilibrium rate is high at 0.89, although significant differences appear in the 2001-2004 and 2009-2012 period. Both rates turn negative by 2015, with the Gilts series exhibiting an even more extreme drop to nearly -200 basis points.

4 Analysis of comovement

In this section, we exploit more forcefully the cross-sectional dimension of our study by examining cross-country comovement in yields. In Section 4.1, we investigate the drivers of yield correlations, while in Section 4.2 we look at interdependence and spillovers.

4.1 Yield correlation and factor structure

As a first step in our analysis of comovement, we ask whether our model is capable of matching the extent of yield correlation in the data, or whether instead significant excess yield correlation is left out. To this end, Panel A of Figure 5 displays the scatter plot of correlations in observed nominal yields (ρ_y, Y-axis) against those in fitted nominal yields ($\rho_{\widehat{y}}$, X-axis). Each correlation is computed between yields of the same maturity but different country, for a total of 54 observations (9 maturities, 6 combinations out of 4 countries). The estimates from a linear fit are also displayed (with standard errors in parentheses) along with the 45-degree line. From the plot, it is clear that the model does remarkably well at capturing yield correlations, with an R-squared of 0.99, a slope coefficient very close to one and an intercept term that, albeit statistically significant, is economically very small at -0.05.

Since (fitted) yields are linear combinations of the latent state variables, international bond yield comovement depends on the correlation structure of the countries' state vectors and on the country- and maturity-specific loadings $B(\tau)$. Even if we do not explicitly model cross-country relations for tractability purposes, the estimated state variables might and indeed do follow common patterns in the cross section. We examine the strength of such linkages by performing a principal component analysis (PCA) of each variable across countries, which is reported in Panel A of Table 5.[19] We observe that the factor structure is particularly pronounced in the variance factor v, whose first principal component PC1 accounts for nearly 70% of the overall variance. Upon inspection of the corresponding eigenvectors (not reported for brevity), we see that this factor loads equally on the U.S., Euro Area and U.K., whereas movements in the Japanese market are almost entirely captured by the second factor, PC2. We find a similar pattern in μ, and also in π and s, although the importance of the first principal component is lower. For ℓ, instead, a markedly different structure is observed, as PC1 loads on Japan and even more so on the U.K. and U.S., while

[19] Clearly, the overall comovement (say, between yields to U.S. and Euro Area) depends not only on the variables' own cross-sectional correlation (say, the correlation between v_t^{US} and v_t^{EU}) but also on the off-diagonal elements of the correlation matrix (say, the correlation between v_t^{US} and μ_t^{EU}). We focus on the former given that they are the dominant effects, and the most intuitive from a factor perspective.

PC2 carries most of the weight on the Euro Area. Hence, it appears that comovement in the level of real rates behave much differently in the cross-section compared to volatility.

Spurred by this evidence, we drill down into its implications for the modelling of international comovement. We regard the full model as "unconstrained", as it allows for country-specific state variables, and test how a "constrained" version that assumes an international factor structure in the state variables fares in matching observed yield correlations. To isolate the importance of each ingredient of our model, we do this separately for: the instantaneous variance v; for the macro expectations μ and π; and for the instantaneous real rate components s and ℓ.

We begin by imposing a one-factor structure in Panel B of Figure 5. For v, this is done as follows. We first compute the first principal component of v across countries, call it $PC1^v$. Next, we recompute fitted yields using the model estimated coefficients but replacing each country i's v_t^i series with $(L1_i^v \times PC1^v)$, where $L1_i^v$ is the country element in the first eigenvector of the PCA for v. For μ and π (and analogously for s and ℓ), we replace simultaneously a country i's μ_t^i and π_t^i series with their one-factor analogue – that is, $(L1_i^\mu \times PC1^\mu)$ and $(L1_i^\pi \times PC1^\pi)$ respectively – and use them along with the other domestic variables to reconstruct fitted bond yields. The leftmost plot in the panel shows that a one-factor assumption in v is largely rejected by the data, with a nearly null R-squared. Much but not all of this performance is due to correlations with Japanese yields. A similar picture emerges for the drivers of instantaneous real rates s and ℓ, in the rightmost panel, for which the goodness of fit is a modest 0.01, although in this case the missed correlations are mainly those with Euro Area yields. Finally, the middle plot shows that assuming a single factor in macro expectations captures about half of the variance in observed correlations, but typically underestimates a wealth of them as testified by the 0.52 slope coefficient.

In Panel C of Figure 5, we repeat the same experiment for a two-factor structure.[20] For aggregate uncertainty v and macro expectations this structure raises the R-squared to about 0.70, and the points line up more closely with the 45-degree line. For the real rate, instead,

[20] For v, it means we compute the first two principal components $PC1^v$ and $PC2^v$, and construct fitted yields using the full model estimates but replacing v_t^i with $\left(L1_i^v \times PC1^v + L2_i^v \times PC2^v\right)$, where $L2_i^v$ is now the element pertaining to country i in the second eigenvector for v. We proceed similarly for the other variables.

the two-factor assumption is still not enough, as the R-squared remains a modest 0.09, with several country pair correlations largely mis-estimated. In all, this analysis highlights differences in the relative importance of global versus local drivers of the yield curve, and shows the extent to which a low-dimensional factor structure gets us closer to match the comovement in the data.

Finally, we look at comovement in the three components of nominal yields, Eq.(11). Specifically, Panel B of Table 5 reports the cross-country correlation matrix in changes in short rate expectations, term premium and convexity at the 2-, 5- and 10-year maturities. The correlations between the U.S., U.K. and Euro Area are in the 0.67-0.75 range for short rate expectations, in the 0.44-0.59 range but increasing with maturity for the term premium, and around 0.50 for the convexity term. For Japan, the correlations with the other countries are considerably smaller, about 0.30 for short rate expectations and term premium and in the 0–0.20 range for convexity. When looking at correlations in the inflation and real components in Appendix Table D.3, we find that those in the real part are on average higher, which is consistent with the evidence in Bekaert and Ermolov (2020). However, this result does not generalize to all country pairs, especially those involving the U.K.

4.2 Connectedness and spillovers

We expand our examination of cross-country relations by looking at international spillovers (i.e., lead-lag effects) in yields and their components. Since these are affine functions of the state vector, the spillovers would arise from a country state vector (or combinations thereof) having predictive power for another country.[21]

Our analysis is motivated by the ongoing debate about the special role of U.S. shocks in international markets. Lagged U.S. stock market returns significantly predict non-U.S. returns, while the converse does not hold (Rapach et al., 2013). FOMC announcements induce shifts in non-US equity risk premia, possibly because the Fed leads other central

[21] We could, in principle, more coherently detect predictability by modelling the dynamics of the full (20×1) state vector and then carrying a joint estimation that pools data from all countries. While theoretically appealing, the increase in noise and computational challenge from estimating the very large number of parameters that such a full system commands would likely smash our inference.

banks in implementing new monetary policies (Brusa et al., 2020). Moreover, Fed policy announcements strongly affect international market comovement in the equity and even sovereign CDS markets (Caporin et al., 2020). In contrast, the Japanese equity market deviates systematically from the predictability patterns observed in other developed countries (see Andersen et al., 2020, and references therein). Our setting provides a natural framework to investigate whether these conclusions extend to the yield curve and its components.

We quantify the extent of interdependence using the Diebold and Yılmaz (2014) measure of market connectedness. The measure is computed based on a VAR(1) of monthly changes in the model-estimated 10-year nominal yield and its components. In the VAR, we pool data across countries for a given series over the period from May, 2004 to December, 2018. We then look at the total directional connectedness from a given country to all the others. In addition, we report the overall level of connectedness, which can be regarded as a measure of market "contagion" or price discovery.

Panel C of Table 5 contains the results. We find that the U.S. is overall the strongest "exporter" of shocks to the long-end of the nominal yield curve, in particular through term premia. Interestingly, the U.K. market has also a comparably high role. The Euro Area follows next, with a significant impact on other countries' convexity (i.e., it "exports uncertainty"), and much less on term premia. Finally, the Japanese market is by far the least responsible of spillovers to other markets. From the overall indicator, we also note that the convexity terms display the lowest degree of cross-market predictability.[22] We thus conclude that, consistent with the literature above, U.S. (and U.K.) shocks are leading indicators of movements in international yield curves.

[22] We draw similar conclusions when we look at spillovers in breakeven rates, real rates, and in their components. In particular, we observe that the real and the inflation components exhibit approximately the same level of connectedness. Moreover, we find that the U.S. plays a significant role on other markets' real term premia, while the Euro Area is the main "exporter" of shocks to long-term inflation expectations.

5 Conclusion

What drives the yield curve? Why do interest rates move over time, and in the cross-section of countries? In this paper, we contribute to our understanding of these fundamental questions by using a model that ties together unobservable yield factors, timely macroeconomic forecasts, and aggregate volatility. We require this model to match the time-series and cross-section of nominal and inflation-linked yields to learn about the relative importance of the real and inflation components. In addition, we incorporate in the estimation information about the realized and implied volatility of yields, which allows us to both capture second moments fluctuations and better identify risk premia.

We fit the model to data from the U.S., Euro Area, U.K., and Japan. We confirm prior evidence that term premia vary over time and can turn negative, but find that short rate expectations are by far the dominant driver of nominal yield dynamics. Over the sample, we observe a marked decline in short real rate expectations, and term premia that turn negative in recent years. We also separately identify the role of time-varying volatility in the expression for yields, and document a convexity term that may be quite large during periods of high aggregate uncertainty.

Exploiting the panel dimension of our study, we investigate the strength of yield comovement and international spillovers. We find relatively strong comovement for all components of yields, and document that the U.S. is the main driver of shocks to long-term yields, mostly through term premia.

Appendix

A Yield decomposition

In this appendix, we derive the expression for the decomposition of a τ-maturity yield following Berardi et al. (2020). From the partial differential equation whose solution provides the time t equilibrium price of the unit zero coupon bond in (9), we can write the time t instantaneous forward rate for date $t+\tau$, $f_t(\tau) = \frac{1}{F_t(\tau)}\frac{\partial F_t(\tau)}{\partial t}$, as

$$f_t(\tau) = y_t + B'(\tau)K(\Theta - X_t) - B'(\tau)\Sigma(\Lambda_0 + \Lambda_1 X_t) - \frac{1}{2}B'(\tau)\left(\Sigma S_t \Sigma'\right)B(\tau). \tag{A.1}$$

Rearranging terms, this expression for the instantaneous forward rate can be written as the sum of four components, i.e., the instantaneous (i) expectation of the short rate at $t+\tau$ under the $\mathbb{P}$ measure, (ii) expected excess return on τ-maturity bond, (iii) convexity, and (iv) a "duration adjustment":

$$f_t(\tau) = \mathrm{E}^{\mathbb{P}}\left[y_t(\tau)\right] + e_t(\tau) + c_t(\tau) + d_t(\tau), \tag{A.2}$$

where

$$\mathrm{E}^{\mathbb{P}}\left[y_t(\tau)\right] = y_t + \overline{B}^{\mathbb{P}\prime}(\tau)K(\Theta - X_t), \tag{A.3}$$

$$e_t(\tau) = -B'(\tau)\Sigma(\Lambda_0 + \Lambda_1 X_t), \tag{A.4}$$

$$c_t(\tau) = -\frac{1}{2}B'(\tau)\left(\Sigma S_t \Sigma'\right)B(\tau), \tag{A.5}$$

$$d_t(\tau) = \left[B'(\tau) - \overline{B}^{\mathbb{P}\prime}(\tau)\right]K(\Theta - X_t). \tag{A.6}$$

The term $\overline{B}^{\mathbb{P}}$ is the expression for the $B(\tau)$ coefficient under the $\mathbb{P}$ measure in a Gaussian framework, $\overline{B}^{\mathbb{P}\prime}(\tau) = \delta' K^{-1}(\mathbf{I} - e^{-K\tau})$. The difference $(B'(\tau) - \overline{B}^{\mathbb{P}\prime}(\tau))$ can be interpreted as an adjustment in the $B(\tau)$ "duration" coefficient due to both stochastic volatility and the essentially affine specification, which in fact goes to zero if $S_t = \mathbf{I}$ and $\Lambda_1 = \mathbf{0}$.

We define the difference between the instantaneous forward rate and short rate expectation in equation (A.2), net of the convexity effect, as the "forward term premium": $FTP_t(\tau) = e_t(\tau) + d_t(\tau)$. Taking the integral of both sides of this expression and dividing by τ, we obtain an expression for the yield term premium: $TPY_t(\tau) = \frac{1}{\tau}\int_t^{t+\tau} FTP_t(u)du$. Similarly, we define the expectation of the average short rate between t and $t+\tau$ as $ESY_t(\tau) = \frac{1}{\tau}\int_t^{t+\tau} \mathrm{E}^{\mathbb{P}}\left[y_t(u)du\right]$ and the average convexity between t and $t+\tau$ as $CXY_t(\tau) = \frac{1}{\tau}\int_t^{t+\tau} c_t(u)du$. The yield on a τ maturity zero coupon bond is thus the sum of three components, see Eq.(11).

B Data description and transformation

B.1 Data source

For nominal yields, we use interest rate swap rates with maturities between 2 and 10 years. The data source is Bloomberg. The tickers of the series are USSW (US), EUSA (Euro Area), BPSW (UK), and JYSW (Japan). In the construction of the data set, we also use Libor rates for the 6-month maturity, with tickers: US0006M (US), EUR006M (Euro Area), BP0006M (UK), and JY0006M (Japan). Swap rates are the yields that determine the semi-annual payments in the fixed leg of the swap contract until maturity. For the purpose of the model estimation, we convert them into zero-coupon bond yields using the standard methodology, see Hull (2018, Section 7.6).

As a measure of implied volatility, we use the implied variance in 6-month swaption contracts that give the holder the right to enter a swap with maturities of either 2, 5, or 10 years. The source is Bloomberg (tickers USSV0A (US), EUSV0A (Euro Area), and JYSV0A (Japan)) and Datastream (ticker ICUK6M (UK)).

For macro forecasts, we use the median 1-year ahead forecasts of annual CPI growth and annual real GDP growth rates, which are available on a quarterly basis. Specifically, we use: the Philadelphia Survey of Professional Forecasters for the U.S.; the ECB Survey of Professional Forecasters for the Eurozone; the Bank's Monetary Policy Committee survey for the U.K.; and the JCER ESP Forecast for Japan.[23]

We capture breakeven rates through zero-coupon inflation swap rates, again spanning maturities between 2 and 10 years. The data is from Bloomberg and starts from July, 2004 for all countries but Japan, for which it just begins in March, 2007. The tickers are USSWI (US), EUSWI (Euro Area), BPSWIT (UK), and JYSWIT (Japan).

B.2 Liquidity premium of inflation swaps

We isolate the effect of a liquidity premium in breakeven (inflation swap) rates through the following procedure. Let $H_t(\tau)$ be the time t τ-year zero-coupon breakeven rate for a given country. We regress $H_t(\tau)$ on the following three country-specific variables:

- $X_{1,t} \equiv (H_t(10) - (Y_t(10) - IL_t(10))$, the "basis" or inflation swap spread at the 10-year maturity, where $Y_t(10)$ is the 10-year nominal rate and $IL_t(10)$ is the yield on a 10-year inflation-linked security;[24]
- $X_{2,t} \equiv Libor_{3m,t} - Tbill_{3m,t}$, the TED spread for that country;
- $X_{3,t}$ is the 1-year ahead CPI inflation expectation from survey data (Federal Reserve Bank

[23] For Japan, the series are average (not median) forecasts, and are available over the April, 2004 to March, 2017 period. We are thankful to the Japan Center for Economic Research for sharing their data with us.

[24] For the U.S., we use the 10-year yield on TIPS from Gurkaynak et al. (2008); for the U.K., we use the yield on the 10-year inflation-linked Gilt obtained from the Bank of England website; for Europe, we use the yield on Thomson Reuter's German Inflation Linked Generic Government Bond (mnemonic GEIL10Y); for Japan, we use Thomson Reuter's redemption yield on Japan Government Inflation Linked (mnemonic TRJPI10).

of Philadelphia Survey of Professional Forecasters).

For each $H_t(\tau)$, we estimate the model

$$H_t(\tau) = a + b_1 X_{1,t} + b_2 X_{2,t} + b_3 X_{3,t} + e_{\tau,t}. \tag{B.1}$$

We then define the liquidity premium as

$$LP_t(\tau) = (\widehat{b}_1 X_{1,t} + \widehat{b}_2 X_{2,t}) - min\{(\widehat{b}_1 X_{1,t} + \widehat{b}_2 X_{2,t})\}. \tag{B.2}$$

We then add this liquidity premium to the inflation swap spreads, so that the implied real rate $R_t(\tau)$ is effectively diminished by $LP_t(\tau)$.

Figure B.1 displays the time series of the average (across maturities) liquidity premium, as defined in B.2, with underneath summary statistics. As we can see, the premium averages between 20 bps and 50 bps, with distinct spikes during the 2008 financial crisis and for the Euro Area also during the 2011-2012 sovereign debt crisis.

C Comparison with regime-switching approach

To capture swings in volatility, allowing for jumps or regime changes are natural candidate modelling approaches. A direct comparison with our conditionally Gaussian model is not as straightforward, especially since the introduction of these features for the scope of simultaneously fitting nominal yields, breakeven rates and yield volatilities would entail a very large parameter space.[25] As a way to offer a cue in this direction, we estimate a two-state model for changes in the 10-year U.S. nominal yield with either constant probabilities (as in Hamilton, 1989) or time-varying probabilities (as in Perez-Quiros and Timmermann, 2000), where the conditioning set consists of a lag, the implied 10-year T-note variance and 1-year GDP survey expectation. Appendix Figure C.1 displays the resulting conditional standard deviations together with our estimated U.S. latent volatility factor (i.e., $\sqrt{v_t}$). As we can see, the two series spike around the same time periods, with positive correlations at 0.33 to 0.67 respectively. We take this evidence as reassuring that our conditionally Gaussian model does share similar dynamics with those from a (reduced-form) regime-switching approach. A proper comparison would, of course, need a fully-fledged regime-switching model that ensures estimates consistent with no-arbitrage pricing of both nominal and real yields. We leave this task for future research.

[25] For example, Feldhütter et al. (2018) report that the Markov-switching model in Dai et al. (2007) would feature fifty-six parameters, more than twice those of linear and nonlinear Gaussian models with the same number (three) of factors. Other models, such as those in Bansal et al. (2004) and Bansal and Zhou (2002), are subject to similar issues. The estimation of such a large parameter vector would clearly prove overly demanding.

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Table 1: **Summary statistics and factor analysis**

Panel A reports mean and standard deviation (in basis points) for the time series of nominal yields (i.e. interest swap rates), breakeven rates, realized and implied volatilities of yield changes, all at the 2-, 5-, and 10-year maturity. The last rows are for the median 1-year forecasts of inflation rate and real GDP growth. Panel B reports the result of a Principal Component Analysis on different groups of series. We show the percentage of the overall variance that is explained by the first (column 'PC1') and collectively by the second to fourth (column 'PC2–4') principal component of the correlation matrix of the corresponding group of series. The time period is January, 1999 to December, 2018 for yields, while breakeven rates start in July, 2004.

Panel A: Summary Statistics

		US		Euro Area		UK		Japan	
Series	maturity	Avg	Std	Avg	Std	Avg	Std	Avg	Std
Nominal Yields	2-year	264.2	198.5	215.8	175.4	316.5	215.8	35.4	31.4
	5-year	332.6	178.5	262.9	174.8	352.9	195.7	64.5	45.5
	10-year	392.5	160.6	319.2	166.4	381.8	167.1	117.2	60.3
Breakeven rates	2-year	184.4	82.6	150.1	61.2	278.9	67.4	35.7	97.7
	5-year	215.8	50.0	165.2	50.3	293.1	39.8	33.5	75.3
	10-year	242.2	35.0	187.5	40.5	312.6	27.4	33.2	59.3
Realized Yield Volatilities	2-year	71.4	43.0	49.4	30.2	57.5	28.8	13.5	11.0
	5-year	90.4	40.6	60.4	27.0	67.8	24.6	26.9	18.2
	10-year	91.5	37.5	60.8	23.8	68.7	21.4	38.4	21.3
Implied Yield Volatilities	2-year	76.6	38.2	71.0	26.0	73.0	29.3	19.7	14.5
	5-year	94.9	35.9	77.1	23.1	77.5	20.5	34.8	21.9
	10-year	96.8	33.1	66.7	22.9	77.3	16.3	47.9	19.6
Inflation rate forecast	1-year	219.9	24.1	161.9	30.1	208.5	50.8	48.0	77.3
Real GDP growth forecast	1-year	261.2	80.0	153.3	92.4	221.8	75.9	180.6	60.3

Panel B: Factor Analysis

		US		Euro Area		UK		Japan	
Series	Maturity	PC1	PC2–4	PC1	PC2–4	PC1	PC2–4	PC1	PC2–4
Nom. Yields and Bkv. Rates	All	75.8	24.0	92.2	7.7	56.4	43.4	66.1	33.2
Yield Volatilities	All	86.1	13.1	75.9	22.2	74.5	24.0	72.6	25.5
All series	All	55.2	40.3	62.0	32.7	43.1	49.1	58.2	36.8

Table 2: **Dissection of yields**

This table reports the average estimated components of nominal yields as defined in Eq.(11)–(13) at the 2-, 5-, and 10-year maturity for the four markets in our study over the sample period January, 1999 to December, 2018. In the case of Japan, the sample starts in April, 2004. Entries are expressed in basis points.

Maturity (years)	Expected short rate ESY	Term premium TPY	Convexity CXY	Expected inflation rate ESH	Expected real short rate ESR	Inflation risk premium TPH	Real term premium TPR	Convexity inflation CXH	Convexity real CXR
Panel A: United States									
2	253	14	-1	246	7	3	11	0	-1
5	308	30	-4	248	60	12	18	-1	-3
10	351	60	-18	244	107	27	33	-2	-16
Panel B: Euro Area									
2	216	0	0	187	29	4	-4	0	0
5	261	6	-2	190	71	9	-3	0	-2
10	312	16	-8	193	119	15	1	-1	-7
Panel C: United Kingdom									
2	298	16	0	284	14	10	6	0	0
5	325	25	-2	292	33	21	4	0	-2
10	357	32	-12	296	61	31	1	-1	-11
Panel D: Japan									
2	38	-2	0	77	-39	3	-5	0	0
5	86	-16	-1	80	6	-1	-15	0	-1
10	157	-47	-5	94	63	-17	-30	-1	-4

Table 3: **Variance decomposition of yield components**

This table reports the variance decomposition of yields at the 2- and 10-year maturity for the four markets in our study over the sample period January, 1999 to December, 2018. In the case of Japan, the sample starts in April, 2004. Entries are expressed in percentage terms.

	2-year				10-year			
Component	US	Euro Area	UK	Japan	US	Euro Area	UK	Japan
Expected short rate	95.0	95.7	97.0	91.0	71.9	63.9	64.8	55.1
Term premium	5.2	4.4	3.1	8.9	29.0	37.6	35.7	44.0
Convexity	-0.2	-0.1	-0.1	0.1	-0.9	-1.5	-0.5	0.9
Expected real short rate	79.8	73.4	63.1	11.6	55.2	48.4	38.5	40.4
Expected inflation rate	15.2	22.3	33.9	79.4	16.7	15.5	26.3	14.7
Real term premium	5.0	3.8	2.3	0.0	24.2	31.8	29.1	29.6
Inflation risk premium	0.2	0.6	0.8	8.9	4.8	5.8	6.6	14.4
Real rate convexity	-0.2	-0.1	0	0.1	-0.8	-1.4	-0.4	0.7
Inflation rate convexity	0.0	0.0	-0.1	0	-0.1	-0.1	-0.1	0.2

Table 4: **Comparison with Wright (2011) gaussian model**

Panel A reports statistics (average and standard deviation, in basis points) on the 10-year nominal yield and its components yield estimated using either our model ('Model') or the Wright (2011) gaussian model for the four markets in our study over the sample period January, 1999 to December, 2018. In the case of Japan, the sample starts in April, 2004. Panel B reports the corresponding variance decomposition (in percentage terms) from the Wright model.

Panel A: Dissection of the 10-year yield

		Avg				Std			
		US	Euro Area	UK	Japan	US	Euro Area	UK	Japan
Yield	Model	396	322	380	105	153	162	155	60
	Wright	397	322	379	107	158	172	158	59
Expected short rate	Model	351	312	357	157	120	108	128	39
	Wright	343	302	333	198	77	87	84	26
Term premium	Model	60	16	32	-47	40	59	37	24
	Wright	54	20	46	-92	85	88	75	34

Panel B: Variance decomposition of the 10-year yield with Wright model

	US	Euro Area	UK	Japan
Expected short rate	46.3	56.4	46.0	34.1
Term premium	53.7	43.6	54.0	65.9

Table 5: **Analysis of nominal yield comovement**

Panel A reports the percentage of the overall variance that is explained by the four principal components of the correlation matrix of each state variables across the four markets in our study. Panel B reports the correlation matrix at the 2-, 5-, and 10-year maturity of changes in the three components of nominal yields as defined in Eq.(11). Panel C reports the Diebold and Yılmaz (2014) connectedness measures based on VAR(1) estimation of monthly changes in 10-year maturity fitted nominal yields and its components. In the VAR, we pool data across countries for a given variable over the period from May, 2004 to December, 2018. The table reports the total directional connectedness from a given country to all the others, and in the 'Overall' row the total level of connectedness.

Panel A: Factor structure in state variables

Variable	PC1	PC2	PC3	PC4
v	68.07	23.85	4.42	3.66
μ	66.76	19.47	9.97	3.80
π	48.79	34.47	9.78	6.95
s	58.37	20.05	14.19	7.39
ℓ	53.94	27.50	15.71	2.84

Panel B: Correlation in nominal yield components

Maturity		ESY			TPY			CXY		
		US	EA	UK	US	EA	UK	US	EA	UK
	EA	0.68			0.57			0.53		
2-year	UK	0.67	0.69		0.44	0.59		0.40	0.57	
	JP	0.38	0.35	0.25	0.29	0.24	0.18	0.17	0.12	0.17
	EA	0.73			0.57			0.53		
5-year	UK	0.72	0.75		0.50	0.63		0.41	0.57	
	JP	0.37	0.35	0.29	0.31	0.10	0.21	0.16	0.14	0.17
	EA	0.67			0.58			0.53		
10-year	UK	0.74	0.75		0.59	0.58		0.40	0.57	
	JP	0.27	0.19	0.20	0.31	0.15	0.35	-0.01	0.18	0.20

Panel C: Connectedness in 10-year nominal yield and components

Country	Y	ESY	TPY	CXY
US	0.65	0.55	0.59	0.39
Euro Area	0.60	0.51	0.35	0.45
UK	0.64	0.61	0.57	0.39
Japan	0.29	0.07	0.17	0.08
Overall	0.54	0.44	0.42	0.33

FIGURE 1: **Deviations from the Expectations Hypothesis**

The figure displays the regression coefficients $\phi(\tau)$ of the Campbell and Shiller (1991) regression of Eq.(14) for the four markets in our study over the sample period January, 1999 to December, 2018. We report the coefficients estimated by running the regression on either actual yields or fitted yields, and the model-implied population coefficient generated by Eq.(15). The dotted lines are the one–standard deviation upper and lower bounds for the model-implied population coefficient. These are obtained by running a Monte Carlo simulation in which the estimated model parameters are used to generate five hundred samples of length 50,000 and calculate the regression coefficients for each of them. Panel A plots the coefficients for the U.S., Panel B for the Euro Area, Panel C for the U.K. and Panel D for Japan (sample period starts in April 2004).

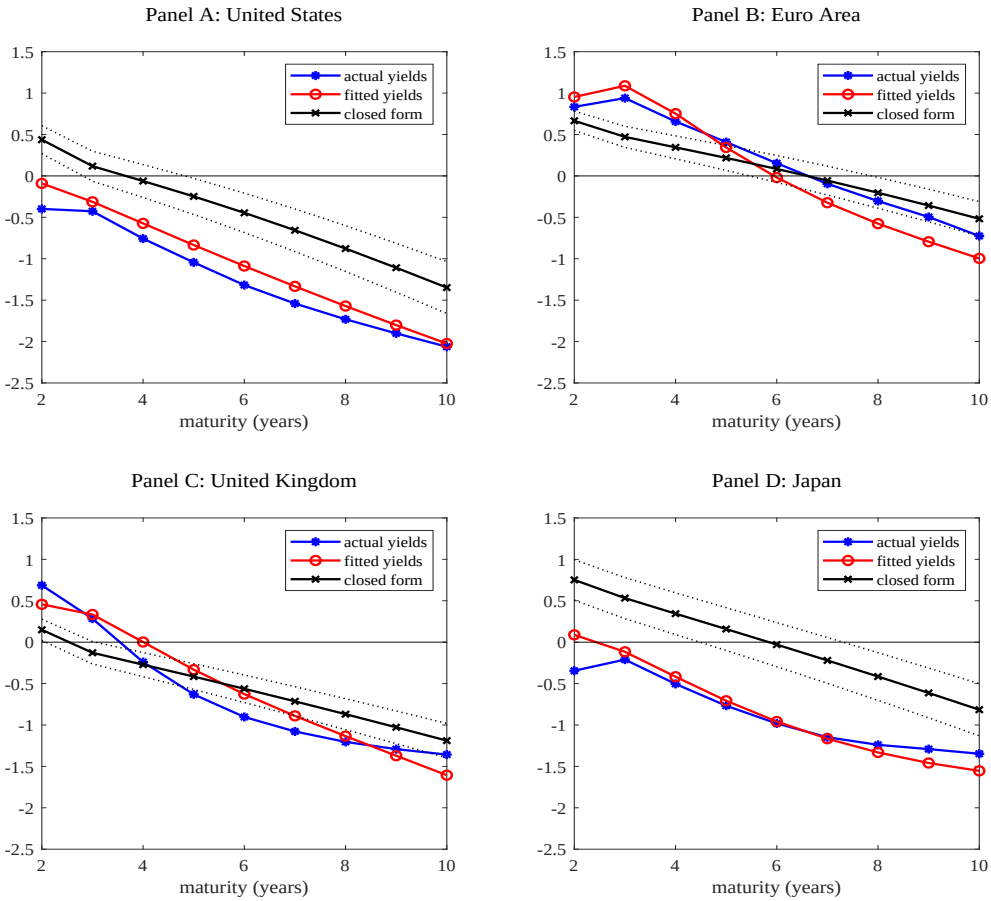


FIGURE 2: **Components of 10-year nominal yield**

The figure displays the time series of 10-year nominal yield and its estimated components of Eq.(11) for the four markets in our study over the sample period January, 1999 (April, 2004 for Japan) to December, 2018.

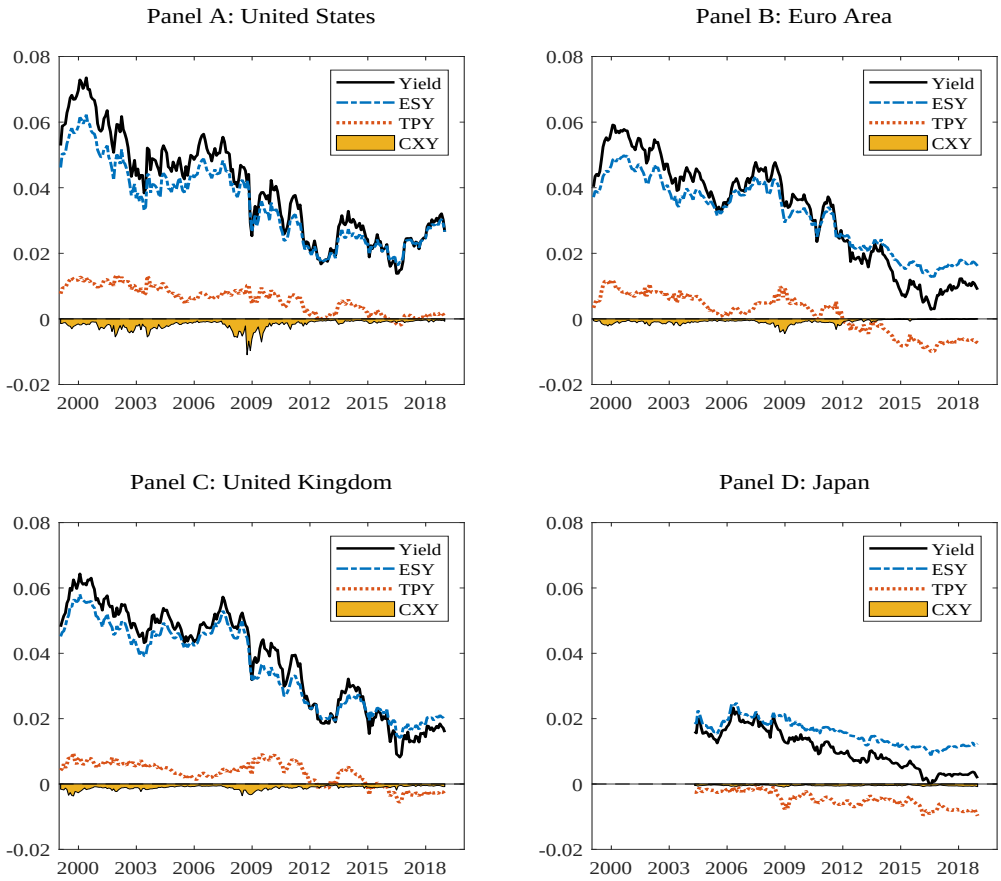


FIGURE 3: **Inflation and real components**

The figure displays the time series of the components of the 10-year breakeven and real rate. Panel (Panel B and D) of Eq.(11) for the four markets in our study over the sample period January, 1999 (April, 2004 for Japan) to December, 2018. Panel A and C report, respectively, the time series of the 10-year inflation short rate expectations and inflation risk premia. Panel B and panel D report, respectively, the time series of the 10-year real short rate expectations and real term premia.

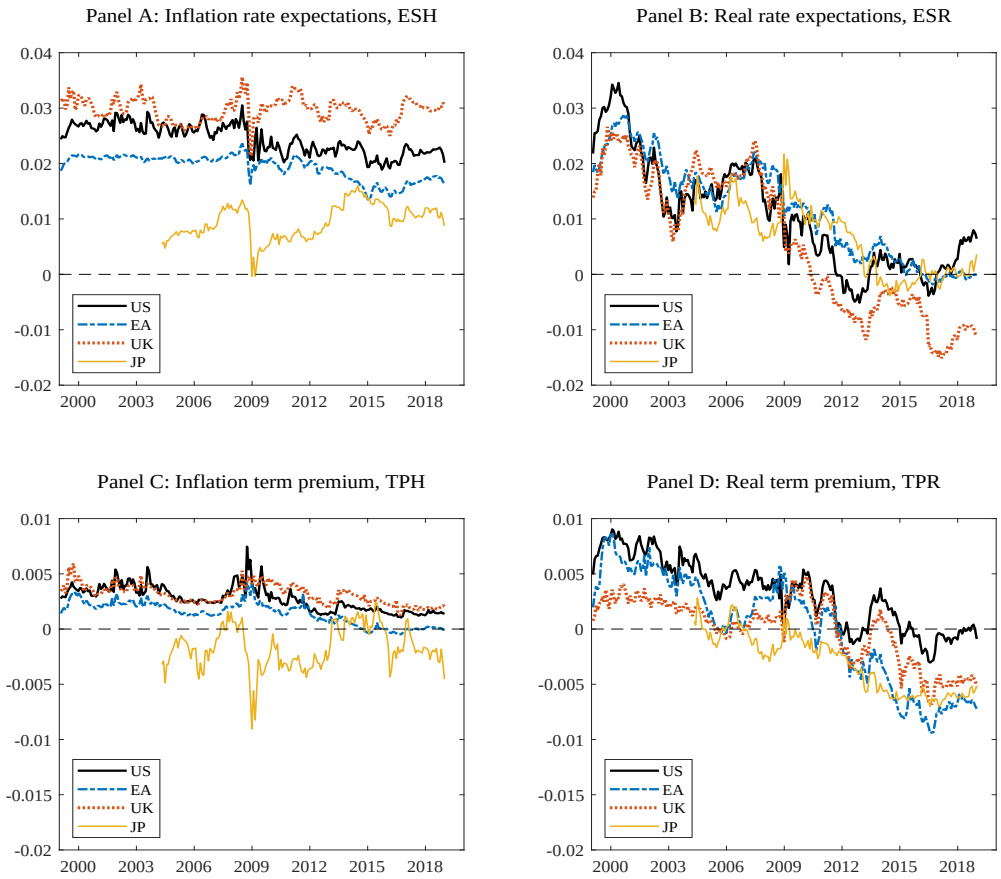


FIGURE 4: **Equilibrium real rate**

The figure displays the time series of the model-implied long-run equilibrium real rate for the four markets in our study, computed as the average expected short rate over a five-year period starting five years ahead, over the sample period January, 1999 to December, 2018. In Panel A for the US we also display the forward rate from the Gurkaynak et al. (2008) TIPS yield curve. In Panel C for the U.K. we also display the forward rate from the U.K. Index-Linked Gilts yield curve published by the Bank of England.

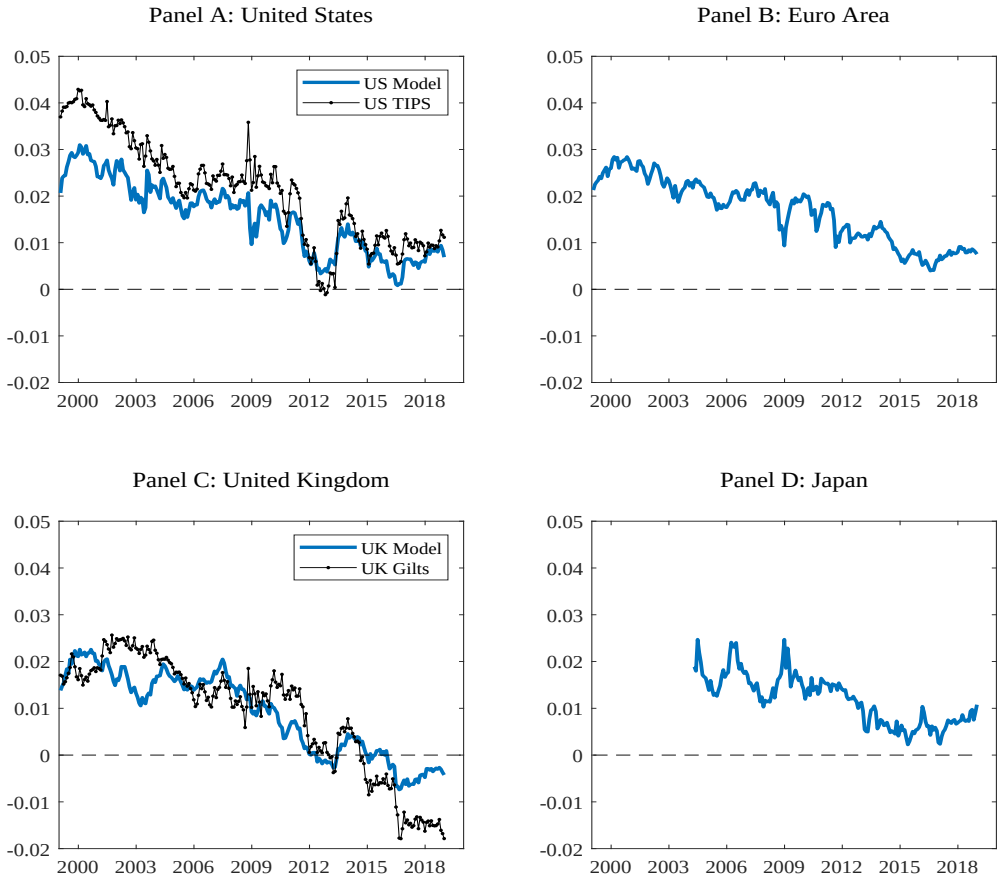
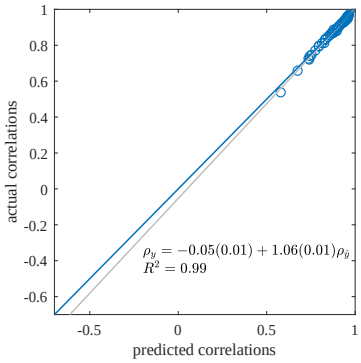


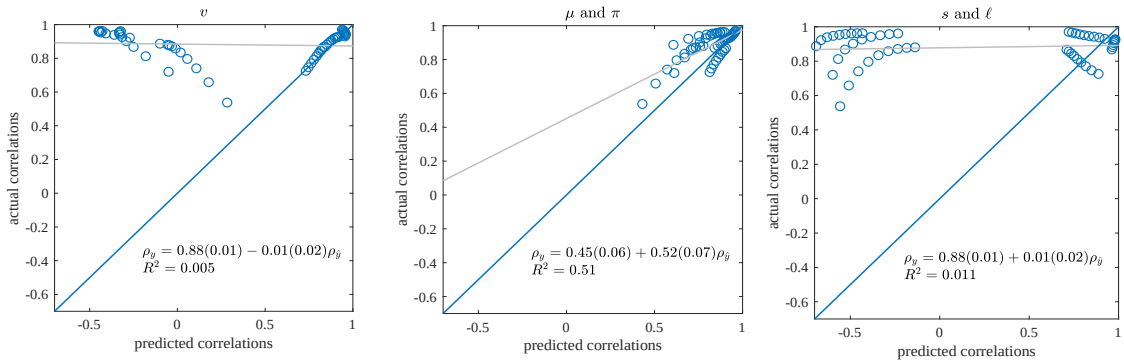
FIGURE 5: **Model predicted vs actual correlations**

The figure displays the scatter plot of correlations in observed nominal yields (ρ_y, Y-axis) against those in fitted nominal yields ($\rho_{\widehat{y}}$, X-axis) for the four markets in our study. Each correlation is computed between a pair of yields with the same maturity from two distinct countries, for a total of 54 observations (9 maturities, 6 combinations out of 4 markets). In panel A, the fitted correlations are based on the estimates from the full model of Section 1. In Panel B and C, the fitted correlations are obtained by imposing respectively a one-factor or two-factor structure in either v (left plot), μ and π (middle plot), or s and ℓ (right plot). In the plots, we report the estimates from a linear fit, with standard errors in parentheses.

Panel A: Full model



Panel B: Constrained 1-factor model



Panel C: Constrained 2-factor model

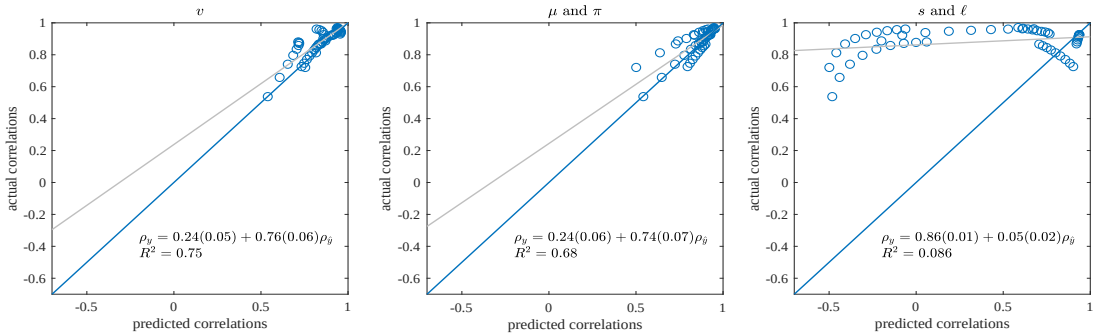
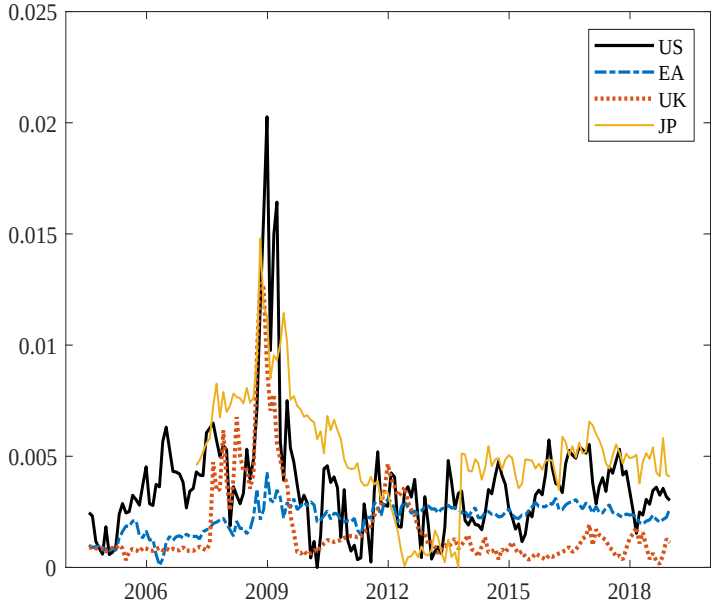


FIGURE B.1: **Liquidity premium in breakeven rates**

The figure displays the time series of the average (across maturities) liquidity premium, as defined in equation B.2, with underneath summary statistics, for the four markets in our study over the sample period January, 1999 to December, 2018.



Summary statistics

	US	Euro Area	UK	Japan
Avg.	0.0037	0.0022	0.0018	0.0050
Std	0.0027	0.0007	0.0020	0.0025
Min	0.0000	0.0001	0.0002	0.0000
Max	0.0203	0.0043	0.0128	0.0148

Figure C.1: **Comparison with Markov-switching model**

This figure displays the time series of the square root of our estimated variance factor $\sqrt{v_t}$ along with the yield conditional standard deviation from estimating either a constant probability two-state Markov-switching model (top panel) or a time-varying parameter two-state Markov-switching model (bottom panel), where the conditioning set consists of a lag, the implied 10-year T-note variance and 1-year GDP survey expectation.

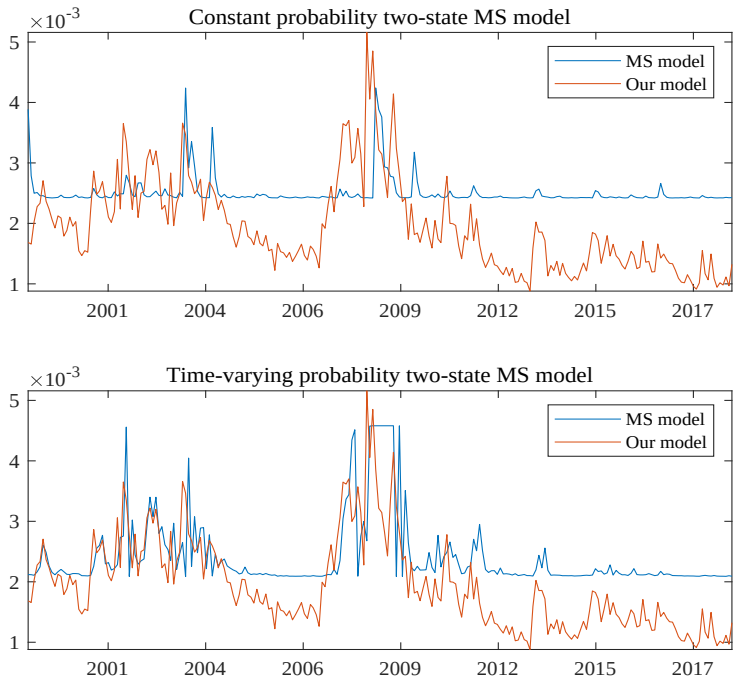


Table D.1: **Parameter Estimates**

This table reports maximum likelihood estimates of model parameters. The coefficients are ordered as $[v; \mu; \pi; s; \ell]$. ** indicates statistical significance at 5% level and * at 10% level based on bootstrapped standard errors.

Parameter	US		Euro Area		UK		Japan	
κ_{11}	0.3571	**	0.2278	**	0.5531	**	0.2215	**
κ_{22}	0.4189	**	0.4182	**	0.4332	**	0.4237	**
κ_{33}	0.3982	**	0.5411	**	0.3197	**	0.2296	**
κ_{44}	0.3260	**	0.3105	**	0.1473	**	0.2493	**
κ_{55}	0.0661	**	0.0765	**	0.0906	**	0.1137	**
κ_{12}	0		0		0		0	
κ_{13}	0		0		0		0	
κ_{14}	0		0		0		0	
κ_{15}	0		0		0		0	
κ_{21}	-0.0061	**	-0.0062	**	-0.0035	**	0.0020	*
κ_{23}	0.0013	*	0.0003	*	0.0019	**	-0.0194	*
κ_{24}	-0.0005	*	-0.0009	**	-0.0003	**	0.0053	*
κ_{25}	-0.0131	*	-0.0139	**	-0.0138	**	-0.0012	**
κ_{31}	-0.0030	*	-0.0077	**	-0.0085	**	0.0405	**
κ_{32}	-0.0106	*	-0.0049	**	-0.0061	**	-0.0052	*
κ_{34}	0.0208	**	0.0123	**	0.0171	*	-0.0138	*
κ_{35}	-0.1038	*	-0.0845	**	-0.1068	**	-0.0070	*
κ_{41}	0.0045	*	-0.0159	**	0.0002	**	0.0235	*
κ_{42}	0.0046	**	0.0041	**	0.0028	**	0.0264	*
κ_{43}	-0.0192	*	-0.0141	*	-0.0195	**	0.0095	*
κ_{45}	-0.0775	**	-0.0790	**	-0.1105	**	-0.0066	*
κ_{51}	-0.0025	**	-0.0012	**	0.0010	*	0.0162	**
κ_{52}	0.0260	**	0.0328	**	0.0264	**	0.0027	**
κ_{53}	-0.0323	**	-0.0361	**	-0.0198	**	-0.0030	**
κ_{54}	0.0024	*	0.0024	**	0.0006	**	0.0002	**
θ_1	0.0329	*	0.0346	**	0.0435	*	0.0021	**
θ_2	0.0408	**	0.0231	**	0.0418	**	0.0246	**
θ_3	0.0207	**	0.0204	**	0.0291	**	0.0159	**
θ_4	0.0020	**	0.0223	**	0.0059	**	0.0070	**
θ_5	0.0121	**	0.0231	**	0.0178	**	0.0185	**
σ_1	0.0573	**	0.0362	**	0.0356	**	0.0666	**
σ_2	0.0152	**	0.0097	**	0.0128	**	0.0187	**
σ_3	0.0090	**	0.0082	**	0.0049	**	0.0105	**
σ_4	0.0059	**	0.0096	**	0.0071	**	0.0152	**
σ_5	0.0098	**	0.0082	**	0.0073	**	0.0127	**
σ_p	0.0511	**	0.0597	**	0.0534	**	0.0073	**
σ_q	0.0536	**	0.0441	**	0.0422	**	0.0157	**
ρ_{p1}	0.0156	*	0.0111	*	0.0270	*	0.0000	
ρ_{p2}	0.0143	*	0.0177	*	0.0071	*	0.0000	
ρ_{p3}	0.6390	*	0.7043	*	0.6866	*	0.7365	
ρ_{p4}	0.0042	*	0.0048	*	0.0038	*	0.0000	
ρ_{p5}	0.0227	*	0.0244	*	0.0297		0.0000	
ρ_{12}	0.1178		0.0984		0.1716	*	-0.0112	*
ρ_{13}	-0.0232		-0.0119	*	-0.0089	*	0.0083	*
ρ_{14}	0.1344	*	0.0692	*	0.1017	*	0.0036	*
ρ_{15}	-0.0225	*	-0.0295	**	-0.0235	*	0.0000	
ρ_{23}	0.0209	*	0.0287	*	0.0225		0.0039	
ρ_{24}	-0.6862	**	0.0001		-0.6696	**	0.0229	
ρ_{25}	0.0205		0.0288	*	0.0310		0.0000	
ρ_{34}	-0.4928	**	-0.5246	*	-0.2692	*	-0.0504	
ρ_{35}	0.0113	*	0.0096		0.0107		0.0000	
ρ_{45}	0.0196		0.0216		0.0118	*	0.0000	
λ_{01}	0.0006	*	0.0002	*	0.0005	*	-0.0031	*
λ_{02}	0.0076	*	0.0075	*	0.0079	*	-0.0045	**
λ_{03}	-0.0055	**	-0.0030	**	-0.0022	*	0.0065	*
λ_{04}	-0.0041	*	-0.0038	*	-0.0036	**	0.0040	*
λ_{05}	-0.0005	**	0.0019	*	-0.0045	*	-0.0044	*
λ_{11}	-5.0765	**	-5.2955	**	-7.1053	**	-3.2815	**
λ_{22}	-5.5034	**	-4.5820	**	-6.4352	**	-2.9373	**
λ_{33}	-1.1238	**	-3.5162	**	-2.0828	**	-9.7179	**
λ_{44}	-3.5574	**	-4.6834	*	-4.3303	**	-3.5908	**
λ_{55}	-7.2705	**	-10.7201	**	-13.3681	**	-5.4298	**
λ_{12}	-0.0002	**	0.0000	*	-0.0001	**	-0.0029	*
λ_{13}	0.0017	*	0.0022	*	0.0012	**	0.0005	**
λ_{14}	0.0034	**	0.0039	*	0.0017	**	-0.0063	*
λ_{15}	0.0030	**	0.0006	**	0.0031	**	0.0001	**
λ_{21}	0.0057	**	0.0043	**	0.0067	**	0.0029	*
λ_{23}	0.0080	*	0.0067	**	0.0037	*	-0.0004	**
λ_{24}	0.0087		0.0118	*	0.0037	**	0.0010	**
λ_{25}	0.0111	*	0.0142	**	0.0067	**	0.0012	**
λ_{31}	0.0030	*	0.0033	**	0.0145	*	-0.0002	**
λ_{32}	0.0045	*	0.0054	**	0.0039	**	-0.0127	**
λ_{34}	0.0071	*	0.0036	**	0.0060	*	0.0033	**
λ_{35}	0.0016	**	0.0010	**	0.0023	**	0.0001	**
λ_{41}	0.0083	**	0.0107	**	0.0099	*	0.0005	**
λ_{42}	0.0067	**	0.0062	**	0.0051	**	-0.0029	*
λ_{43}	0.0025	*	0.0019	**	0.0032	**	0.0070	*
λ_{45}	0.0044	**	0.0063	*	0.0041	**	0.0005	*
λ_{51}	0.0100	*	0.0119	**	0.0102	**	-0.0003	**
λ_{52}	0.0035	*	0.0034	**	0.0032	**	0.0003	**
λ_{53}	0.0059	*	0.0071	**	0.0092	*	0.0006	**
λ_{54}	0.0030	*	0.0027	*	0.0031	*	0.0002	**

Table D.2: **Goodness of fit**

This table reports summary statistics for the fitting errors. Panel A contains the average (across maturities) of the standard deviation of fitting errors for: nominal yields and breakeven rates with maturity ranging between 2 and 10 years; realized and implied yield volatilities with maturity 2, 5 and 10 years; and survey forecasts of the 1-year ahead inflation rate and GDP growth rate. Values are expressed in basis points. Panel B reports mean, standard deviation and maximum value of the estimated maximal Sharpe ratio.

Panel A: Fitting errors

Variable	US	Euro Area	UK	Japan
Nominal Yields	7.8	6.2	8.3	2.7
Breakeven Rates	15.0	9.5	13.2	11.5
Realized Yield Volatilities	17.5	12.9	15.0	16.6
Implied Yield Volatilities	12.7	10.8	10.8	16.6
Inflation Rate Forecast	42.9	24.2	38.8	31.9
Real GDP Growth Forecast	3.5	3.7	4.3	1.6

Panel B: Maximal Sharpe Ratio

Statistic	US	Euro Area	UK	Japan
Mean	0.99	0.89	1.26	0.30
Standard Deviation	0.38	0.31	0.32	0.05
Maximum	2.62	2.20	2.33	0.48

Table D.3: **Correlation in breakeven and real rate components**

Panel A reports the correlation matrix at the 2-, 5-, and 10-year maturity of changes in the three components of breakeven rates, as defined in Eq.(12). Panel B reports analogous correlations for the three components of real rates, as defined in Eq.(13).

Panel A: Correlation in breakeven rate components

		ESH			*TPH*			*CXH*		
		US	EA	UK	US	EA	UK	US	EA	UK
2-year	EA	0.45			0.11			0.53		
	UK	0.35	0.58		-0.31	0.13		0.42	0.56	
	JP	-0.17	-0.01	0.21	-0.05	0.09	0.02	0.16	0.14	0.17
5-year	EA	0.48			0.53			0.53		
	UK	0.40	0.56		0.43	0.55		0.40	0.57	
	JP	-0.12	0.05	0.27	0.04	-0.04	-0.01	0.16	0.14	0.16
10-year	EA	0.48			0.53			0.53		
	UK	0.42	0.56		0.41	0.57		0.40	0.57	
	JP	-0.07	0.13	0.32	-0.04	-0.05	0.06	0.16	0.14	0.17

Panel B: Correlation in real rate components

		ESR			*TPR*			*CXR*		
		US	EA	UK	US	EA	UK	US	EA	UK
2-year	EA	0.46			0.54			0.53		
	UK	0.35	0.48		0.44	0.62		0.40	0.57	
	JP	-0.25	-0.15	0.08	-0.04	0.08	0.03	0.18	0.12	0.16
5-year	EA	0.61			0.47			0.53		
	UK	0.47	0.58		0.64	0.52		0.40	0.57	
	JP	-0.17	-0.03	0.16	0.09	0.13	-0.05	-0.01	0.18	0.20
10-year	EA	0.61			0.47			0.53		
	UK	0.47	0.58		0.64	0.52		0.40	0.57	
	JP	-0.17	-0.03	0.16	0.09	0.13	-0.05	-0.01	0.18	0.20

FIGURE D.1: **Term premium from alternative affine models**

The figure displays the time series of the estimated 10-year nominal term premium from four models: a gaussian four-factor model of the Kim and Wright (2005) type, GM4; a gaussian four-factor model with macro factors but without stochastic volatility, SV4; a stochastic volatility four-factor model without macro factor, SV4; our model featuring both macro factor and stochastic volatility, MSV.