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Corporate Policies and the Term Structure of Risk*

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Abstract

We build a dynamic corporate finance model with heterogeneity in the pricing and in the firm's exposure to aggregate risks of various persistence. All else equal, we show that if long-term (persistent) shocks have a higher market price than short-term (temporary) shocks, firms shorten the horizon of corporate policies, favoring payouts over investment. In the cross section, this effect is stronger for firms more exposed to long-term shocks, but can be reversed for firms more exposed to short-term shocks. Our analysis is extended to embed time variation in risk prices over the business cycle, motivated by recent evidence on the term structure of equity.

Keywords: Temporary vs. permanent shocks; Pricing of aggregate risk; Horizon of corporate policies

JEL Classification Numbers: G12; G31; G32

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1 Introduction

Over the last decade, asset pricing studies have uncovered novel stylized facts regarding the pricing of risks across the horizon, by focusing on a variety of financial assets (see, e.g., [van Binsbergen and Kojen, 2017](#); [Backus, Boyarchenko, and Chernov, 2018](#)). Within this strand, special attention has been given to claims to equity indexes, with the goal to understand whether equity markets mostly remunerate long-term risks—as suggested by leading asset pricing models (e.g., [Campbell and Cochrane, 1999](#); [Bansal and Yaron, 2004](#))—or short-term ones—as advanced by the recent literature on the term structure of equity pioneered by [van Binsbergen, Brandt, and Kojen \(2012\)](#). This line of research has provided key implications for discount rates applied to risky payoffs at different horizons, and has underscored the pivotal role played by risks of various persistence (and their pricing) in explaining empirical regularities of equilibrium risk premia.¹

In spite of these significant asset pricing advancements, the corporate finance literature has made little headway to understand how the relative pricing of risks of different persistence affects firm decisions. By shaping the return required by the investors and, hence, the firm cost of capital, heterogeneous market risk prices should impact the present value of cash flows expected at different horizons. Through this channel, market risk prices should play a key role in firm's financial and real decisions, potentially influencing the firm's incentives to focus on short-term objectives (e.g., current earnings and payouts) or long-term performance (e.g., firm growth through investment). Yet, these aspects have been overlooked by previous corporate finance models, either because agents are assumed to be risk-neutral or because they abstract from heterogeneity in the persistence, exposure, or pricing of aggregate (undiversifiable) risks.

The goal of this paper is to start filling this gap. To this end, we build a dynamic corporate finance model in which we introduce heterogeneity in the pricing of aggregate

¹See, for instance, [Gormsen \(2020\)](#), [Marfé \(2017\)](#), [Breugem, Colonnello, Marfé, and Zucchi \(2020\)](#), and [Bansal, Miller, Song, and Yaron \(2020\)](#).

risks of different persistence and in the firm's exposure to these risks. Specifically, we study a firm operating in an economy with two undiversifiable risks: long-term, persistent shocks—capturing permanent changes in aggregate output, consumers' tastes, or technology—and short-term, temporary shocks—representing transitory fluctuations in demand, political uncertainty, or geopolitical tensions. The firm's cash flows are imperfectly correlated with the two aggregate risks. Following [Décamps, Gryglewicz, Morellec, and Villeneuve \(2017\)](#), we assume that long-term shocks have a persistent effect on firm's productivity, whereas short-term shocks are transitory and potentially expose the firm to operating losses. To cover these losses, the firm can use retained earnings or external financing, which yet entails issuance costs.

The key novelty of our framework is the explicit focus on the relative prices of short-term versus long-term risks, to which we refer as the term structure of risk prices. Specifically, the term structure of risk prices is upward-sloping (or increasing) if long-term shocks have a higher market price than short-term shocks. Conversely, the term structure of risk prices is downward-sloping (or decreasing) if long-term shocks have a lower market price than short-term shocks. The term structure is flat if long-term and short-term shocks have the same market price. In this framework, we investigate the effect of the slope of risk prices on corporate policies, focusing on firm's optimal investment (and disinvestment), payout, cash retention, financing, and liquidation policies. Notably, we show that the relative magnitude of risk prices affects the horizon of corporate policies—i.e., the optimal balance between short-term objectives (e.g., payouts to shareholders) and long-term goals (e.g., growth through investment).

We start by analyzing the case in which the firm is symmetrically exposed to short-term and long-term aggregate risks. We show that when the term structure of risk prices is upward-sloping, the horizon of corporate policies shortens compared to the case in which the term structure is flat. Specifically, the firm invests less and pays out dividends more often. Also, the firm is more likely to disinvest productive assets when financially

constrained, at the cost of harming long-term profitability. In addition, the firm reduces the optimal size of its precautionary cash-to-asset ratio and of equity issuances. Conversely, when the term structure of risk prices is downward-sloping, the firm extends the horizon of corporate policies. In this case, we show that the firm favors growth over payouts to shareholders. Furthermore, the firm keeps a larger precautionary cash-to-asset ratio, increases the size of equity issuances, and is less likely to engage in asset sales when financial constraints are tight. Whereas corporate short- and long-termism have been largely investigated through the lens of managerial incentives and optimal contracting, we provide an explanation based on the pricing of aggregate risks of varying persistence.

We then relax the assumption of symmetric exposure to short- and long-term risks. Namely, we investigate how the firm-specific exposure to aggregate risks affects the corporate policies' response to the slope of risk prices. Our analysis demonstrates that the results derived under the assumption of symmetric risk exposure are quantitatively stronger when firms are more exposed to aggregate long-term shocks than to short-term ones. In this case, an upward-sloping term structure of risk prices triggers a sharper shortening in the horizon of corporate policies, leading to a greater emphasis on payouts and a stronger reduction in investment. Instead, if a firm is relatively more exposed to short-term aggregate shocks, it can exhibit the opposite pattern. That is, our analysis provides guidance to empirical work, by identifying characteristics that trigger mixed responses to the slope of market risk prices in the cross section of firms.

Overall, our analysis demonstrates that ignoring heterogeneity in market risk prices induces distortions in corporate investment and financial policies, whose severity varies in the cross section of firms depending on their exposure to aggregate risks. Our paper then relates to the literature studying the potential fallacies incurred by firms in determining the discount rates used in corporate decisions. In this strand, [Krueger, Landier, and Thesmar \(2015\)](#) show that firms use a single discount rate to value all of their projects, consistent with the survey evidence in [Graham and Harvey \(2001\)](#). As a result, these firms

fail to account for project-specific risk, which triggers investment distortions by leading to overestimate riskier project and to underestimate safer ones. Exploiting the survey conducted by the Association for Financial Professionals, [Jacobs and Shivdasani \(2012\)](#) reveal that the firms' assumptions in computing discount rates are quite heterogeneous. They report that companies seldom adjust their estimates of the equity market premium, and typically disagree on the time period over which the firm's exposure to aggregate risk should be gauged. Our paper warns that failing to correctly account for the pricing and exposure to undiversifiable risks leads firms to distort the optimal balance between short-term and long-term objectives. Also, by illustrating how the relative prices of short-term and long-term risks shape the term structure of equity risk premia²—which informs about the relation between discount rates and horizon—our analysis helps understand how both the timing and pricing of risk affects corporate decisions and real activity.

Finally, we extend our analysis to embed time variation in risk prices. Empirically, time variation in the level and slope of market risk prices has been inferred from the dynamics of the term structure of equity over the business cycle (e.g., [van Binsbergen, Hueskes, Koijen, and Vrugt, 2013](#); [Gormsen, 2020](#); [Bansal et al., 2020](#); [Giglio, Kelly, and Kozak, 2020](#)). We realistically assume that risk prices are higher in recessions than in expansions, and study the effect of variation in the slope of the term structure of risk prices. We show that the time-variation in the level and in the slope of risk prices play two distinct effects on corporate decisions. If the term structure of risk prices is increasing (respectively, decreasing) in expansion (recession), time variation in level and in slope of risk prices have countervailing effect on the firm's optimal investment rate. In fact, firms have greater incentives to invest when risk prices are relatively low in expansion, but the increasing slope moderates this effect. Conversely, time-variation in the level and slope have the same directional effect if the slope of the firm-relevant term structure of risk prices is decreasing (respectively, increasing) in expansion (recession).

²Whereas for tractability we refer to term structure of risk prices in most of our analysis, Appendix [A.3](#) shows how it maps to the term structure of risk premia implied by the equity market.

Related literature Our paper is motivated by the asset pricing literature studying the pricing and timing of risk (see, e.g., [Bansal, Dittmar, and Lundblad, 2005](#); [Lettau and Wachter, 2007](#); [Hansen, Heaton, and Li, 2008](#); [Da, 2009](#)). The recent literature on the term structure of equity—pioneered by [van Binsbergen, Brandt, and Kojien \(2012\)](#)—has spurred a deep rethinking of the risks that drive asset prices, especially the weight of long-term risks (such as persistent changes in expected growth) and short-term risks (such as transitory fluctuations like seasonal demand shifts). Relatedly, [Bansal et al. \(2020\)](#), [Gormsen \(2020\)](#), and [Breugem et al. \(2020\)](#) empirically and theoretically stress the need to account for multiple sources of risk—featuring heterogeneous persistence and market prices—to rationalize risk premia across the horizons.³ In this paper, we investigate the corporate finance implications of this line of research. Specifically, we analyze how the relative magnitude of market risk prices associated with shocks of different persistence (as well as their time-variation) affects firm’s real and financial decisions.

A growing strand in corporate finance aims at studying the tension that corporations face between incentivizing long-term growth versus maximizing their short-term profitability. The early literature ([Stein, 1988, 1989](#); [Aghion and Stein, 2008](#)) has emphasized that stock market pressure leads managers to boost short-term earnings at the expense of long-term performance. More recent literature, in turn, has focused on identifying the conditions under which short-termism can be efficient through the lens of agency conflicts, see [Hackbarth, Rivera, and Wong \(2019\)](#) and [Gryglewicz, Mayer, and Morellec \(2020\)](#).⁴ Our paper sheds a novel perspective on this topic by showing that the horizon of corporate policies is affected by the firm’s exposure and pricing of aggregate, undiversifiable risk of various persistence.

Our paper also contributes to the corporate finance works that embed both temporary and permanent shocks in understanding firm decisions, e.g., [Gorbenko and Strebulaev](#)

³Heterogeneous and horizon-dependent market prices of risk can also be understood through dynamically-inconsistent risk preferences (e.g., [Andries, Eisenbach, and Schmalz, 2019](#); [Lazarus, 2019](#)).

⁴[Edmans, Fang, and Lewellen \(2017\)](#) and [Ladika and Sautner \(2019\)](#) empirically investigate managerial incentives via vesting duration and analyze the consequences on corporate investment.

(2010), Décamps et al. (2017), or Gryglewicz, Mancini, Morellec, Schroth, and Valta (2020). These contributions merge two important theoretical frameworks. In the first, cash flows are governed by a geometric Brownian motion, so shocks are permanent in nature, as in the seminal contributions by Brennan and Schwartz (1985) and Leland (1994). In the second, cash flows are driven by an arithmetic Brownian motion, so shocks are essentially transitory, see Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), and Della Seta, Morellec, and Zucchi (2020). We advance this strand by acknowledging that short-term shocks and long-term shocks may differ in their market price, and we investigate the ensuing implications for corporate policies.

The paper is organized as follows. Section 2 presents our dynamic model. Section 3 derives the model solution, and Section 4 analyzes the model implications. Section 5 extends the model to nest time variation in the term structure of risk prices. Section 6 concludes. Technical developments and proofs are gathered in the Appendix.

2 The model

Time is continuous, and the horizon is infinite. There is a constant risk-free rate denoted by r . We consider an economy characterized by two sources of aggregate, undiversifiable risk: long-term shocks and short-term shocks. Long-term shocks affect the economy permanently—i.e., they can be interpreted as persistent changes in the composition of aggregate output, technology, or consumers' tastes—and are driven by a standard Brownian motion denoted by $d\tilde{W}_t$ under the physical probability measure. Short-term shocks affect the economy only temporarily—i.e., they can be interpreted as shocks capturing transitory risks in the economy, like seasonal fluctuations in demand, political uncertainty, or geopolitical tensions—and are driven by a standard Brownian motion denoted by $d\tilde{B}_t$ under the physical measure. We assume that the two shocks are independent. Investors are risk-averse, so we need to distinguish between physical and risk-neutral measures.

The stochastic discount factor reflects the two sources of aggregate risks in the econ-

omy, which are therefore priced. The dynamics of the stochastic discount factor, denoted by ξ_t , follow a geometric Brownian motion:⁵

$$\frac{d\xi_t}{\xi_t} = -r dt - \eta_A d\tilde{W}_t - \eta_Y d\tilde{B}_t. \quad (1)$$

The parameters η_A and η_Y are the prices of risk associated with long-term and short-term aggregate risks, respectively—i.e., they describe the market’s remuneration for undiversifiable risks that have long-term, persistent impact or short-term, transitory impact.

We refer to the relative magnitude of η_A versus η_Y by defining the slope of the term structure of risk prices. Specifically, we refer to the case in which the market price of long-term shocks is larger than the market price of short-term shocks (i.e., $\eta_A > \eta_Y$) as characterized by an upward-sloping (or increasing) term structure of risk prices. Conversely, we refer to the case in which the market price of short-term shocks is higher than that of long-term shocks (i.e., $\eta_A < \eta_Y$) as characterized by a downward-sloping (or decreasing) term structure of risk prices. Finally, we refer to the case in which short-term and long-term shocks have the same market price (i.e., $\eta_A = \eta_Y$) as having a flat term structure of risk prices. In Appendix A.3, we illustrate how the relative prices of short-term and long-term risks shape the slope of the term structure of equity risk premia.⁶

We consider the optimization problem of a firm operating in this economy. We assume that the firm generates a continuous stream of cash flows, which are subject to both the sources of aggregate risks. Realistically, we assume that the firm’s shocks are imperfectly correlated with aggregate shocks. We assume that the firm’s long-term shocks affect the size of the firm’s productive assets and are correlated with the aggregate long-term shocks by a factor $\rho_A \geq 0$. We describe these shocks through a standard Brownian motion \hat{W}_t under the physical measure. Conversely, short-term shocks affect the firm’s profitability

⁵The stochastic discount factor can be generated by a consumption-based asset pricing model. We follow Bolton, Chen, and Wang (2013) and take it as given, to focus on corporate policies.

⁶Following the asset pricing literature, we simply assume that the dynamics of the payout of the equity market index are also driven by the permanent and transitory aggregate risks and, then, we compute the dividend strip risk premium across the horizon.

and are correlated with aggregate short-term shocks by a factor $\rho_Y \geq 0$. We describe these shocks via the standard Brownian motion \hat{B}_t under the physical measure. We assume that the Brownian motions \hat{W}_t and \hat{B}_t are orthogonal.

We denote the firm's productive assets by A_t . We assume that the dynamics of A_t are governed by the following dynamics:

$$dA_t = (\mu A_t + L_t)dt + \sigma_A A_t d\hat{W}_t = (\mu + l_t)A_t dt + \sigma_A A_t d\hat{W}_t. \quad (2)$$

In this equation, μ and $\sigma_A > 0$ are constant parameters. We do not impose restrictions on the sign of the parameter μ —if negative, it represents the depreciation rate of the firm's productive assets. Furthermore, $l_t = L_t/A_t$ represents the firm's investment rate, which is endogenously chosen to maximize firm value. Following Bolton, Chen, and Wang (2011, henceforth BCW), we assume that the price of productive assets is normalized to one, and that investment entails a quadratic adjustment cost given by the following expression:

$$G(L, A) = g(l)A = \frac{\kappa l^2}{2}A, \quad (3)$$

where κ is a positive constant.

Similar to Décamps et al. (2017), we assume that the firm's operating cash flows dX_t are proportional to its productive assets. Operating cash flows are governed by the following dynamics:

$$dX_t = A_t dY_t = A_t \left(\alpha dt + \sigma_Y d\hat{B}_t \right). \quad (4)$$

In this equation, dY_t is an arithmetic Brownian motion, and $\alpha > 0$ and $\sigma_Y > 0$ are positive constants denoting the associated drift and volatility, respectively. Notably, shocks associated with dY_t are short-lived and affect the firm's cash flows only temporarily. In the extreme case in which the firm's asset size A_t is constant, our framework nests the model of Décamps et al. (2011) as a special case.

Absent short-term shocks, the firm's cash flows are always positive (i.e., given by

$\alpha A_t dt$) because so is A_t . In turn, short-term shocks can give rise to operating losses, which can be covered by raising external financing or by using retained earnings. We allow management to save earnings inside the firm and denote by M_t the firm's cash reserves at any time $t > 0$. Cash reserves earn a rate of return $r - \lambda$ inside the firm, where λ represents the opportunity cost of cash. The firm can increase its cash reserves by raising external financing. Raising external funds is costly and entails a proportional cost p as well as fixed cost fA_t , which means that the firm gets Ψ when raising the amount $(1 + p)\Psi + fA$ from investors. Following BCW (2011, 2013) and [Décamps et al. \(2017\)](#), the fixed cost scales with firm size, to ensure that the firm does not grow out of its fixed cost of equity issuance. Cash reserves satisfy the following dynamics:

$$dM_t = (r - \lambda)M_t dt + A_t dY_t - (l_t + g(l)) A_t dt + dH_t - d\Phi_t - dU_t \quad (5)$$

where H_t , Φ_t , and U_t are non-decreasing processes that represent the cumulative gross equity financing, the cumulative issuance costs of equity financing, and the cumulative payout to shareholders. This equation illustrates that cash reserves increase with the interest on cash (the first term on the right-hand side), with the firm's earnings (the second term), and external financing (the fourth term), whereas it decreases with investment-related costs (the third term), issuance costs (the fifth term), and payouts to shareholders (the last term).

The firm can be forced into default if its cash reserves reach zero following a series of negative shocks and it is not possible/optimal to raise fresh funds. As BCW (2011), we assume that the liquidation value of risky assets, denoted by ℓ_τ , is a fraction of asset value. That is, $\ell_\tau \equiv \phi A_\tau$, where $\phi \in [0, 1)$ represents the recovery rate of assets (equivalently, $1 - \phi$ represents a haircut related to default costs) and τ denotes the firm's time of default.

Management chooses the firm's payout (U_t), financing (H_t), investment (l_t), and default (τ) policies to maximize shareholder value. Because agents in the economy (and, therefore, the firm's investors) are risk averse, we need to distinguish between physical

and risk-neutral measures using the stochastic discount factor in equation (1). That is, management solves

$$V(A, M) = \sup_{U, H, l, \tau} E^Q \left[\int_0^\tau e^{-rt} (dU_t - dH_t) + e^{-r\tau} \ell_\tau \right],$$

subject to equation (5), where the expectation is taken under the risk-neutral probability measure. In this equation, the first term in the square brackets represents the flow of dividends accruing to incumbent shareholders, net of the claim of new shareholders. The second term represents the present value of the cash flow to shareholders in default.

3 Model solution

Using the stochastic discount factor in equation (1), we pin down the dynamics of the relevant stochastic processes under the risk-neutral probability measure. Denoting by dW_t the risk-neutral counterpart of $d\hat{W}_t$, standard arguments yield the risk-neutral dynamics of the firm's productive assets, given by the following equation:

$$dA_t = (\mu + l_t - \sigma_A \rho_A \eta_A) A_t dt + \sigma_A A_t dW_t.$$

Similarly, we denote the risk-neutral counterpart of $d\hat{B}_t$ by dB_t and obtain the risk-neutral dynamics of the firm's operating revenues:

$$dY_t = (\alpha - \sigma_Y \rho_Y \eta_Y) dt + \sigma_Y dB_t.$$

Using these dynamics, we start by solving the model under the assumption that the firm does not face financing frictions, in which case the firm has no incentives to hoard cash. We then move to the case with financing frictions in Section 3.2, in which cash retention, payout, and financing policies are non-trivial.

3.1 The case with no financing frictions

Absent financing frictions, the firm has no incentives to keep cash reserves because any funding need can be covered by raising new equity at no cost nor delay. Thus, it is optimal for the firm to pay out any cash flow exceeding its operating needs to shareholders. In this environment, the optimal investment rate satisfies (see Appendix A.1):

$$l^* = (r - \mu + \sigma_A \rho_A \eta_A) - \sqrt{(r - \mu + \sigma_A \rho_A \eta_A)^2 - 2(\alpha - \sigma_Y \rho_Y \eta_Y - r + \mu - \sigma_A \rho_A \eta_A) / \kappa} \quad (6)$$

where the inequality $(r - \mu + \sigma_A \rho_A \eta_A)^2 - 2(\alpha - \sigma_Y \rho_Y \eta_Y - r + \mu - \sigma_A \rho_A \eta_A) > 0$ guarantees that the investment policy is well defined. Notably, the investment rate l^* is positive if the inequality

$$\alpha - \sigma_Y \rho_Y \eta_Y - r + \mu - \sigma_A \rho_A \eta_A > 0 \quad (7)$$

holds, i.e., if the risk-adjusted productivity $(\alpha - \sigma_Y \rho_Y \eta_Y)$ is greater than the risk-adjusted required return $r - \mu + \sigma_A \rho_A \eta_A$. In the following, we focus on cases in which this inequality is satisfied.⁷

To isolate the effect of the relative magnitude of market risk prices on corporate investment, we start by considering the case in which the firm has symmetric exposure to the two sources of aggregate risks, i.e., the equality $\sigma_A \rho_A = \sigma_Y \rho_Y$ holds. Calculations reported in Appendix A.1 relate the firm's optimal investment rate to the slope of the term structure of market risk prices. We report our results in the next proposition.

Proposition 1 *If the firm is symmetrically exposed to the two sources of aggregate shocks (i.e., $\sigma_A \rho_A = \sigma_Y \rho_Y$), investment decreases with the slope of the term structure of risk prices. All else equal, the firm's optimal investment rate is greater (respectively, smaller) if the term structure of risk prices is downward-sloping $\eta_A < \eta_Y$ (upward-sloping $\eta_A > \eta_Y$)*

⁷The above expression nests the optimal investment rate if investors are actually risk neutral, which satisfies $l^* = (r - \mu) - \sqrt{(r - \mu)^2 - 2(\alpha - r + \mu) / \kappa}$ as in BCW (2011). In this setup, the argument of the square root needs to be positive for the problem to be well-defined, and $\alpha - r + \mu > 0$ to guarantee that investment is positive.

compared to the flat case $\eta_A = \eta_Y$.

Proposition 1 demonstrates that if the firm is symmetrically exposed to short- and long-term aggregate shocks, an upward-sloping term structure of risk prices leads to underinvestment compared to the flat or, even more so, the downward-sloping case.

Next, we relax the assumption of symmetric exposure to short-term and long-term shocks. This is the case if the firm's correlation with aggregate short-term shocks differs from the correlation with long-term shocks. Proposition 2 formalizes that the firm's exposure to aggregate shocks is key to pin down the sensitivity of investment to the slope of the term structure of risk prices.

Proposition 2 *If the firm is more exposed to aggregate long-term shocks than to short-term shocks (i.e., $\sigma_A \rho_A \geq \sigma_Y \rho_Y$), then the optimal investment rate decreases with the slope of the term structure of risk prices. If, instead, the firm's exposure to aggregate short-term shocks is sufficiently larger than the exposure to aggregate long-term shocks (i.e., $\sigma_A \rho_A < \sigma_Y \rho_Y$ holds), investment can become increasing with the slope of the term structure.*

Proposition 2 illustrates how the firm's exposure to aggregate shocks provides a source of cross-sectional heterogeneity in the observed corporate responses to the slope of market risk prices. Specifically, if the firm is more exposed to aggregate long-term shocks, then investment decreases with the slope of the term structure, consistent with Proposition 1. This result is reversed—and investment becomes increasing with the slope of the term structure—if the firm's exposure to aggregate short-term shocks is sufficiently larger than its exposure to long-term shocks. In this case, the greater price of long-term risk associated with an upward-sloping term structure is more than offset by the firm's weak exposure to such risk, thus investment does not decrease with the slope of the term structure. We investigate this result further in Section 4.2.

Whereas the firm's financing and payout policy is trivial if the firm faces no financing frictions, we next investigate the joint interaction between firm's investment, savings, payout, and financing policies in the presence of financing frictions.

3.2 Firm policies in the presence of financing frictions

In the following, we assume that the firm faces financing frictions, as described in Section 2. In this case, the firm has incentives to hold precautionary cash reserves. Consider first the firm's cash retention and payout policies. Because both external financing and liquidation are costly, it is optimal for the firm to delay equity issuance or liquidation decisions until cash reserves are depleted. When the cash reserves are depleted, the firm issues new equity if financing is not too costly (below we define conditions that warrant the optimality of refinancing versus liquidation). Otherwise, the firm enters default as it does not have funds to cover operating losses and continue operations. Notably, the benefit of holding cash decreases with cash reserves, whereas the opportunity cost of cash is constant. Thus, we conjecture that there is a target cash level M^* , at which costs and benefits are equalized. Above this target level, it is optimal to pay out all the excess cash to shareholders. Below this level, the firm retains earnings in the cash reserves.

Standard arguments yield that firm value satisfies the following Hamilton-Jacobi-Bellman (HJB) equation in the cash retention region $[0, M^*]$:

$$rV(a, m) = \max_i (\mu + l - \sigma_A \rho_A \eta_A) a V_a + \left[\left(\alpha - \sigma_Y \rho_Y \eta_Y - l - \frac{\kappa}{2} l^2 \right) a + (r - \lambda) m \right] V_m + \frac{1}{2} a^2 (\sigma_A^2 V_{aa} + \sigma_Y^2 V_{mm}). \quad (8)$$

The left-hand side of this equation represents the return required by investors under the risk-neutral measure. The right-hand side is the expected change in firm value on an infinitesimal time interval. Specifically, the first term on the right-hand side represents the effect of changes in asset size on firm value, whereas the second term represents the

effect of changes in cash reserves. Notably, both of these terms depend on the market prices of risk η_A and η_Y . The last term on the right-hand side represents the effect of (asset and cash flow) volatility.

Equation (8) illustrates that two state variables—the size of productive assets and the level of cash reserves—enter the firm’s optimization problem. Because this problem is homogeneous of degree one in a and m , we can solve for firm value by defining cash reserves scaled by assets, $c \equiv m/a$, i.e., the firm cash-to-asset ratio. We also define the scaled value function, denoted by $v(c)$ and satisfying:

$$V(a, m) \equiv av(c). \quad (9)$$

Substituting equation (9) into equation (8) and dividing by a gives:

$$rv(c) = \max_l (\mu + l - \sigma_A \rho_A \eta_A) [v(c) - v'(c)c] + \left[\alpha - \sigma_Y \rho_Y \eta_Y - l - \frac{\kappa l^2}{2} + (r - \lambda)c \right] v'(c) + \frac{v''(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2). \quad (10)$$

Differentiating the above equation with respect to l gives the first-order condition for the firm’s investment rate, which gives (see Appendix A.2):

$$l(c) = \frac{1}{\kappa} \left(\frac{v(c)}{v'(c)} - 1 - c \right). \quad (11)$$

Plugging $l(c)$ back into equation (10) yields the following equation:

$$(r - \mu + \sigma_A \rho_A \eta_A) v(c) = [\alpha - \sigma_Y \rho_Y \eta_Y + (r - \lambda - \mu + \sigma_A \rho_A \eta_A)c] v'(c) + \frac{v''(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2) + \frac{1}{2\kappa} \frac{[v(c) - (1 + c)v'(c)]^2}{v'(c)}. \quad (12)$$

Equation (12) shows how the market prices of short-term and long-term shocks affect firm value. The first two terms on the right-hand side illustrate that a greater market price of

short-term shocks η_Y reduces firm profitability, and more so if the firm's cash flow shocks are more correlated with aggregate shocks (i.e., ρ_Y is greater) or more volatile (i.e., σ_Y is greater). In turn, the left-hand side of this equation illustrates that the market price of long-term shocks η_A leads to an increase in the return required by the investors. This effect commands a greater discount rate on future cash flows and, thus, should depress firm value. This effect is stronger if the firm's correlation with aggregate long-term shocks ρ_A is greater or if assets are more volatile (σ_A is greater).⁸

Equation (12) is solved subject to the following boundary conditions. First, in the payout region $c > C^*$, the firm pays out all the cash in excess of C^* , meaning that $v(c) = v(C^*) + c - C^*$ in this region. Subtracting $v(C^*)$ from both sides of this equation, dividing by $c - C^*$, and taking the limit as $c \rightarrow 0$ shows that the firm satisfies the following value-matching condition at C^* :

$$v'(C^*) = 1. \quad (13)$$

To ensure optimality of the target payout threshold C^* , the following super-contact condition needs to hold (see [Dumas, 1991](#)):

$$v''(C^*) = 0. \quad (14)$$

Because equity financing is costly, the firm delays refinancing or default decisions until cash reserves are depleted. If the firm raises equity when $c = 0$, then the following boundary condition holds:

$$v(0) = v(C_*) - (1 + p)C_* - f, \quad (15)$$

where C_* represents the optimal issuance size. This equation implies that the value of the

⁸Note that the last term in the square brackets on the right-hand side shows that a greater η_A also leads to an increase in the return on cash. Yet, the opportunity cost of cash, i.e., the gap between the return required by the investors and the return on cash ($r - \mu + \sigma_A \rho_A \eta_A - (r - \lambda - \mu + \sigma_A \rho_A \eta_A) = \lambda$) remains constant and equal to λ .

firm when cash reserves are depleted (the left-hand side) equals the post-issuance firm-value net of the associated financing costs (the right-hand side). The optimal issuance size C_* is endogenously determined by the following condition:

$$v'(C_*) = 1 + p, \quad (16)$$

which guarantees that the marginal benefit (the left-hand side) and cost of an equity issuance (the right-hand side) are equalized at the post-issuance cash level. Notably, the firm is better off issuing new equity than liquidating, which is the case if the following inequality

$$v(C_*) - (1 + p)C_* - f > \phi \quad (17)$$

holds. The left-hand side of this inequality represents firm value at $c = 0$ if the firm raise equity, whereas the right-hand side represents firm value in liquidation. We summarize our results in the next proposition

Proposition 3 *Firm value satisfies $V(m, a) = av(c)$, where $v(c)$ represents firm value scaled by productive assets. On the interval $[0, C^*]$, scaled firm value $v(c)$ satisfies equation (12), in which the market price of long-term shocks leads to an increase in the return required by the investors whereas the market price of short-term shocks leads to a decrease in firm profitability. Equation (12) is solved subject to the boundary conditions (13)–(16), and the optimal investment rate is given by equation (11). On the interval $[C^*, \infty)$, the firm pays out all cash above C^* and the scaled firm value is linear.*

We next turn to analyze our model, to understand how the slope of the term structure of risk prices affects investment, payout, cash retention, the size of equity issuance, and the optimality of refinancing versus default.

4 Model analysis

In this section, we analyze the model predictions. Table 1 reports our baseline parameterization. The risk-free rate is set to 0.04, and the opportunity cost of cash is equal to 0.01. Proportional and fixed financing costs are, respectively, equal to 0.06 and 0.002, in line with BCW (2013) and Décamps et al. (2017). The parameter μ is equal to -0.13 , which lies in the ballpark of the depreciation rates in BCW (2011, 2013). We set the adjustment cost parameter to 3.9, which is consistent with the estimates of Eberly, Rebelo, and Vincent (2008). The cash flow drift and volatility are equal to 0.225 and 0.12, as in BCW (2013). We follow previous models with temporary and permanent shocks (Décamps et al., 2017; Hackbarth, Rivera, and Wong, 2019; Lee and Rivera, 2020) and assume that the volatility of assets is greater than the volatility of operating cash flows.⁹

Table 1

To focus on the effect associated with the slope of the term structure of risk prices, we assume that the sum $\eta_Y + \eta_A$ is constant across the different cases (increasing, decreasing, and flat). This condition allows us to keep the level of the term structure constant and, thus, single out the effect of the slope. In particular, we assume that $\eta_Y = \eta_A = 0.2$ in the case in which the term structure is flat, $\eta_Y = 0.05 < 0.35 = \eta_A$ in the case in which the term structure is increasing, and $\eta_Y = 0.35 > 0.05 = \eta_A$ in the case in which the term structure is decreasing.

4.1 Symmetric correlation with aggregate shocks

We start by considering the case in which the firm's shocks are symmetrically correlated with short-term and long-term aggregate risks, i.e., the equality $\rho_Y = \rho_A$ holds. On top

⁹Throughout our quantitative analysis, we follow previous contributions in this strand and rule out parameterizations for which investment rates are largely negative for any cash level (i.e., depicting a “sinking ship”).

of the baseline values of σ_A and σ_Y , we also consider the case $\sigma_A = \sigma_Y$.

Figure 1

Figure 1 shows scaled firm value ($v(c)$, on the left) and the firm's investment rate ($l(c)$, on the right) over the cash retention interval $[0, C^*]$ when the term structure of risk prices is flat, increasing, and decreasing. The top panel considers the case in which $\sigma_A = \sigma_Y$, so that the equality $\sigma_A \rho_A = \sigma_Y \rho_Y$ holds and the firm is symmetrically exposed to long-term and short-term risks. This case is analogous to that analyzed in Proposition 1 for the case with no financing frictions. The figure illustrates that the firm's investment rate is the lowest if the term structure of risk prices is increasing, in which case it always lies below the flat case. Indeed, the higher market price of long-term shocks leads to an increase in the return required by the investors (as illustrated by equation (12)), which weakens the incentives to invest in productive assets. Conversely, the investment rate is the largest if the term structure of risk prices is decreasing, then exceeding the investment rate associated with the flat case. The bottom panel displays firm value and the investment rate if $\sigma_A > \sigma_Y$ (as in Table 1), in which case the effect of the slope of risk prices is qualitative similar to the top panel but quantitatively stronger. This result is consistent with Proposition 2 for the case with no financing frictions.

Figure 2

The top panel of Figure 2 further investigates the optimal investment rate by varying the magnitude of the correlations of firm's shocks with aggregate shocks, still preserving the assumption that $\rho_A = \rho_Y$.¹⁰ Similar to BCW (2011), the firm may have incentives to disinvest (i.e., sell productive assets) when its cash ratio is low to ease financial con-

¹⁰For the sake of brevity, Figure 2 focuses on the baseline parameterization with $\sigma_A > \sigma_Y$, but the results for the case $\sigma_A = \sigma_Y$ are qualitatively similar.

straints.¹¹ The figure illustrates that the firm is more likely to disinvest if the term structure of risk prices is increasing, and more so if the firm's shocks are more correlated with aggregate shocks (i.e., ρ_A and ρ_Y are greater), in which case market risk prices have a larger weight on corporate decisions. Conversely, if the term structure of risk prices is decreasing, the firm is less likely to disinvest—that is, to relax financial constraints, the firm prefers to bear the costs of equity issuance than harming long-term performance by selling productive assets. The bottom panel of Figure 2 also shows that disinvestment is less likely when the level of risk prices is lower (gauged by the sum $\eta_Y + \eta_A$), all else equal. In turn, when the level of risk prices is higher, corporate investment declines, and the firm is more likely to engage in asset sales when the cash ratio is close to zero.

Figure 3

Figure 3 analyzes these patterns by varying not only the sign but also the steepness of the slope of the term structure of risk prices. The top (respectively, bottom) panel focuses on the case in which the term structure is increasing (decreasing). The figure shows that the effects illustrated so far are stronger if the gap between the market price of short-term and long-term shocks is larger, in which case the term structure of risk prices is steeper. The top panel shows that if the term structure is increasing, a larger gap between η_A and η_Y leads to smaller investment rates, and makes the firm more willing to disinvest when the cash ratio is close to $c = 0$. The bottom panel shows that if the term structure is decreasing, a more negative slope leads to larger firm value and investment rate (notably, the firm never disinvests). Again, the effects are quantitatively greater if $\sigma_A > \sigma_Y$, as shown in the right panels.

The left panels of Figure 1 show that firm value is the lowest when the term structure is increasing, and the highest when the term structure is decreasing. This result has

¹¹When the cash ratio is low, the marginal value of cash increases, and firm value decreases. As a result, the first term in parenthesis of equation (11) declines, and l can be negative in a neighborhood of $c = 0$.

implications for the firm's decision to liquidate or raise new equity whenever cash reserves are depleted. As examined in Section 3.2, the firm finds it optimal to raise new financing at $c = 0$ (instead of liquidating) if condition (17) holds, meaning that the firm continuation value is greater than the liquidation value of assets. If firm value decreases sufficiently due to the effect of upward-sloping risk prices, then the firm may find it optimal to liquidate the first time it runs out of cash. All else equal, the firm is more likely to liquidate if the term structure of risk prices is upward-sloping.

The figures presented so far also show that the target payout threshold C^* is smaller if the term structure is increasing compared to the flat case, which means that the firm pays out cash to shareholders more often. In turn, the target payout threshold is larger if the term structure is decreasing compared to the flat case, in which case the firm's target cash ratio is larger compared to both the cases in which the term structure is flat and increasing.

Table 2

Table 2 further investigates the firm's optimal cash saving/payout policies together with its equity issuance policy. The table shows the target payout threshold C^* and the optimal issuance size C_* for a range of combinations of η_Y and η_A (again, assuming that their sum is constant) under the baseline parameterization in Table 1 (top panel) and, additionally, when assuming $\sigma_Y = \sigma_A$ (middle panel), and when $f = 0.01$ (i.e., financing frictions are more severe, bottom panel). The table confirms that the target payout level and the optimal issuance size are larger when the term structure of risk prices is decreasing compared to the flat case. Conversely, if the term structure is increasing, the target payout threshold is smaller, which means that the firm pays out cash to the investors more often. The size of equity issuances is also smaller if the term structure is increasing. Table 2 illustrates that these effects are stronger if the term structure is steeper. Moreover, comparing the top and bottom panels shows that the thresholds C^* and C_* are larger if the firm faces greater costs of equity issuance, in which case the firm

has a greater precautionary demand for cash.

The last column of Table 2 investigates the likelihood of firm (dis-)investment in relation with its cash ratio. It shows how the cash threshold above (below) which the firm's investment rate is positive (negative), denoted by C_0 , varies with the slope of the term structure.¹² The table shows that the threshold C_0 is larger if the term structure of risk prices is increasing and sufficiently steep, and more so if the firm's financial constraints are stronger (i.e., when f is greater, bottom panel). Conversely, if the gap $\eta_A - \eta_Y$ becomes more negative (meaning that the term structure is decreasing) or if the firm is less exposed to long-term shocks (i.e., if $\sigma_A = \sigma_Y$), the firm is less likely or never engages in asset sales. Coupled with the analysis in Figure 2, Table 2 confirms that an increasing term structure of risk prices not only reduces the firm's investment rate compared to the flat or decreasing cases, but also increases the likelihood that the firm engages in asset sales when its cash ratio is low.

The term structure of risk and short- vs long-termism Our analysis illustrates that the relative magnitude of market risk prices importantly affects corporate policies. Specifically, if the term structure is increasing, the firm reduces its investment in productive assets and is more likely to pay out cash to shareholders. It also reduces the size of its precautionary target cash ratio and of equity issuances. Furthermore, it is more likely to disinvest productive assets when financial constraints are binding, at the cost of harming long-term profitability. Overall, the firm favors current earnings and payouts over long-term growth through investment (and precautionary cash retention)—i.e., if the term structure of risk prices is upward-sloping, the firm shortens the horizon of corporate policies compared to the case in which the term structure is flat. Conversely, if the term structure is downward-sloping, the firm increases its investment rate (which benefits long-term profitability), delays payouts to shareholders, increases its target cash

¹²In the table, we report “n.a.” if the threshold does not lie in the interval $[0, C^*]$ —i.e., if the firm's investment rate is always positive.

ratio and the size of equity issuances, and is less likely to sell productive assets when severely financially constrained. That is, if the term structure is downward-sloping, the firm extends the horizon of corporate policies.

Notably, the correlation of firm's shocks with aggregate risks plays a key role in determining the extent to which the term structure of risk prices impacts corporate policies. Consider the extreme case in which the firm's shocks are uncorrelated with aggregate shocks (i.e., $\rho_A = \rho_Y = 0$). In this case, the term structure of risk prices has no impact on corporate decisions, and firm value satisfies the following ODE:

$$(r - \mu) v(c) = [\alpha + (r - \lambda - \mu)c] v'(c) + \frac{v''(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2) + \frac{1}{2\kappa} \frac{[v(c) - (1+c)v'(c)]^2}{v'(c)}.$$

Conversely, when firm's shocks are positively correlated with aggregate shocks, ρ_Y and ρ_A quantify the impact of risk prices on corporate policies. Equation (12) indeed illustrates that greater correlation with aggregate long-term shocks ρ_A or greater market price associated with these shocks η_A both lead to an increase in the (risk-adjusted) return required by the investors. In turn, larger firm's correlation with short-term shocks ρ_Y or a greater market price of short-term shocks η_Y both lead to a decrease in the expected (risk-adjusted) profitability of cash flows. In the next section, we better investigate the role played by the correlations ρ_Y and ρ_A .

4.2 Asymmetric correlation with aggregate shocks

We now relax the assumption that the firm's shocks are symmetrically correlated with short-term and long-term aggregate shocks and allow for the case $\rho_Y \neq \rho_A$. That is, we now vary both the prices of aggregate risks and the firm's correlation with these risks.

Figure 4

Figure 4 shows the firm's optimal investment rate when varying the relative magnitude

of the firm's correlation with aggregate shocks, under the assumption that the firm's volatilities assume the same value, $\sigma_A = \sigma_Y$. The analysis illustrates that as long as the inequality $\sigma_A \rho_A \geq \sigma_Y \rho_Y$ holds (which is the case for the left and the middle panels of this figure), the firm exhibits a larger investment rate if the price of long-term risks is smaller than the price of short-term risks, as in the analysis in Section 4.1. Conversely, if the inequality $\sigma_A \rho_A < \sigma_Y \rho_Y$ holds (which is the case in the right panel of this figure), the opposite pattern can arise. These results are consistent with Proposition 2 for the case with no financing frictions. In fact, the firm's exposure to aggregate risks determines the extent to which the term structure of market risk prices affects corporate decisions. In the right panel of Figure 4, the firm is more exposed to short-term aggregate shocks than to long-term ones. As a result, if the term structure is decreasing (meaning that $\eta_Y > \eta_A$), the firm is more exposed to the risk with the largest market price. This, in turn, leads to a decrease in investment (recall from Figure 2 that investment drops when the average level of the term structure is higher).

Figure 5

Figure 5 further investigates these patterns, by also varying the relative magnitude of the firm's volatility of assets and of cash flows, σ_A and σ_Y . In the top panel, we assume our baseline values for $\sigma_A > \sigma_Y$, as in previous models featuring temporary and permanent shocks (see, e.g., Décamps et al., 2017; Lee and Rivera, 2020). This figure confirms that as long as the firm is more exposed to long-term than to short-term aggregate shocks (i.e., the inequality $\sigma_A \rho_A \geq \sigma_Y \rho_Y$ holds, as in the left and middle panels), investment decreases with the slope of risk prices. Conversely, this is not the case in the last panel, in which the firm is more exposed to short-term aggregate shocks (i.e., the inequality $\sigma_A \rho_A < \sigma_Y \rho_Y$ holds). For illustration, the bottom panel considers the case in which $\sigma_A < \sigma_Y$. In this environment, the inequality $\sigma_A \rho_A \geq \sigma_Y \rho_Y$ is satisfied in the left panel only, in which case investment decreases with the slope of the term structure of risk prices.

This analysis then shows that the firm’s correlation with aggregate shocks (and, more generally, the firm’s exposure to aggregate risks, which also depends on the firm’s volatility of assets and cash flows) provides a source of cross-sectional heterogeneity in the firm’s response to the slope of risk prices. Our analysis then translates into practical guidance for empirical work, by exploiting differential responses in the cross-section of firms based on their exposure to aggregate risks of different persistence.

4.3 Risk premia

We next investigate how the relative magnitude of risk prices affects the firm’s risk premium.¹³ The conditional risk premium is given by the sum of two components:

$$\theta(c) = \underbrace{\eta_A \rho_A \sigma_A \left(1 - \frac{cv'(c)}{v(c)}\right)}_{\theta_A(c)} + \underbrace{\eta_Y \rho_Y \sigma_Y \frac{v'(c)}{v(c)}}_{\theta_Y(c)}. \quad (18)$$

The term $\theta_A(c)$ compensates investors for the exposure to long-term aggregate risk, whereas the term $\theta_Y(c)$ for the exposure to short-term risk. Both terms are a function of the cash-to-asset ratio, c . Because c varies over time, the risk premium and its components vary too.¹⁴

Figure 6

Figure 6 shows the risk premium and its components. The top panel shows the case in which the firm is symmetrically exposed to short-term and long-term shocks (i.e., $\sigma_A \rho_A = \sigma_Y \rho_Y$).¹⁵ When the term structure of risk prices is increasing (respectively, decreasing), investors require a larger compensation for long-term (short-term) risk, so $\theta_A(c)$ ($\theta_Y(c)$) is the largest. The top left panel shows that the risk premium $\theta(c)$ is on net

¹³We adopt the heuristic approach of BCW (2011, 2013) in deriving the firm’s expected return, which involves the comparison of the HJB under the physical and the risk-neutral probability measure.

¹⁴Notably, the risk premium in equation (18) differs from that in BCW (2011), in which the firm is subject to short-term shocks and, thus, only includes the θ_Y component.

¹⁵For brevity, in this figure we assume that $\sigma_A = \sigma_Y$ and just vary ρ_Y versus ρ_A .

larger if the term structure of risk prices is increasing compared to the flat and decreasing cases. The middle and bottom panels assume that the firm is asymmetrically exposed to aggregate risks. The middle panel shows the case in which the firm is more exposed to long-term shocks $\rho_A > \rho_Y$, in which the above result is quantitatively stronger. If instead the firm is more exposed to short-term shocks $\rho_A < \rho_Y$, the risk premium is larger if the term structure is decreasing. As in Section 4.2, the relative magnitude of $\sigma_A \rho_A$ versus $\sigma_Y \rho_Y$ determines the impact of the slope of risk prices on risk premia too.

In our model, investment and risk premia are endogenous, and so is their relation. Figure 4 and 6 illustrate that the firm's investment rate and risk premium are, respectively, increasing and decreasing with the cash ratio. Moreover, they move in opposite directions when varying the slope of the term structure of risk prices— depending on the firm's sensitivity to the two sources of risk, the overall risk premium as well as the investment rate can be either increasing or decreasing with the slope of the term structure of risk prices. Notably, Figure 6 shows that the two components $\theta_Y(c)$ and $\theta_A(c)$ vary in opposite directions with the slope of risk prices: The steeper the term structure of risk prices, the higher (respectively, lower) the long-term (short-term) component of the risk premium. This implies a novel endogenous relation between investment rate, the risk premium, and its components in the cross-section of firms, conditional on the term structure of risk prices.

Consider now the conditional risk premium absent financing frictions:

$$\theta^* = \underbrace{\eta_A \rho_A \sigma_A}_{\theta_A^*} + \underbrace{\eta_Y \rho_Y \sigma_Y}_{\theta_Y^*} \frac{1}{v^*}, \quad (19)$$

where $v^* = 1 + \kappa l^*$ is firm value scaled by assets absent financing frictions (see Section 3.1 and Appendix A.1). Notably, the long-term component $\theta_A(c)$ is smaller in the presence of financing frictions— $\theta_A(c)$ and θ_A^* coincide at $c = 0$, and the inequality $\theta_A(c) < \theta_A^*$ holds for any $c \in (0, C^*]$. The reason is that financing frictions lead the firm to hold cash.

Whereas productive assets are correlated with aggregate long-term risk, cash is a safe asset. Thus, the composition of productive assets and cash leads to a decline in the firm’s exposure to long-term risk compared to the case with no financing frictions (in which the firm keeps no cash). Second, the short-term component $\theta_Y(c)$ can be either greater or smaller than θ_Y^* depending on its cash ratio, being greater (smaller) than θ_Y^* if the firm’s cash ratio is small (large). On top of the aforementioned asset composition effect—which pushes the short-term premium down as the cash-to-asset ratio increases—the increased constraints associated with low cash reserves push the short-term premium above θ_Y^* .

Figure 7

Figure 7 shows the risk premium $\theta(c)$ in the presence and in the absence of financing frictions (for brevity, we focus on the baseline values of ρ_Y and ρ_A). The figure shows that, in the presence of financing frictions, the risk premium lies below that associated with the benchmark with no financing frictions if the firm’s cash ratio is sufficiently large, as cash hoarding reduces the firm’s exposure to the long-term risk. Conversely, the risk premium can be larger than in the benchmark with no financing frictions if the firm’s cash ratio is sufficiently low due to the dynamics of the short-term component, $\theta_Y(c)$.

5 Time-variation in risk prices

The analysis so far assumes that the term structure of risk prices is static. In this section, we relax this assumption and allow risk prices to vary over the business cycle. This extension is especially relevant in light of recent asset pricing studies revealing that time-variation in risk prices is necessary to capture key properties of risk premia on several financial assets (see [Cochrane, 2011](#)). For instance, [van Binsbergen et al. \(2013\)](#), [Gormsen \(2020\)](#), and [Bansal et al. \(2020\)](#) document that the level and the slope of the term structure of equity vary with economic conditions, which can be rationalized in a

general equilibrium setting with time-varying prices of risk (see [Breugem et al., 2020](#)).¹⁶

5.1 Deriving firm value

In this extension, we assume that the firm can be in two (observable) states $i = G, B$, where G denotes the good state (expansion) and B denotes the bad state (recession). Specifically, we assume that the state switches from G to B (respectively, from B to G) with probability $\pi_G dt$ (respectively, $\pi_B dt$) on any infinitesimal interval $(t, t + dt)$, similar to [BCW \(2013\)](#) or [Della Seta, Morellec, and Zucchi \(2020\)](#). When the state switches, the market prices of risk change. We respectively denote by η_{AG} and η_{AB} the price of long-term shocks in the good and in the bad state. Similarly, we denote by η_{YG} and η_{YB} the price of short-term shocks in the good and in the bad state. Because our focus is on the time-variation of risk prices, we keep other quantities—among which, financing costs, cash flow profitability, and the volatility of long-term and short-term shocks—to be invariant across the two states.

In this augmented setting, firm value as well as the optimal investment rate are state-contingent (see [Appendix A.4](#) for analytical details). Specifically, the optimal investment rate in state i (with $i = G, B$ and $i \neq j$) is given by:

$$l_i(c) = \frac{1}{\kappa} \left(\frac{v_i(c)}{v'_i(c)} - 1 - c \right) \quad (20)$$

and the valuation equations for firm value in each state satisfy:

$$\begin{aligned} (r - \mu + \sigma_A \rho_A \eta_{Ai}) v_i(c) &= [\alpha - \sigma_Y \rho_Y \eta_{Yi} + (r - \lambda - \mu + \sigma_A \rho_A \eta_{Ai})c] v'_i(c) \\ &+ \frac{v''_i(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2) + \frac{1}{2\kappa} \frac{[v_i(c) - (1+c)v'_i(c)]^2}{v'_i(c)} + \pi_i [v_j(c) - v_i(c)]. \end{aligned} \quad (21)$$

¹⁶Moreover, [Haddad, Kozak, and Santosh \(2020\)](#) and [Giglio, Kelly, and Kozak \(2020\)](#) stress that time-variation in risk prices is a critical property of the stochastic discount factor and equity term structure dynamics. Consistently, [Chernov, Lochstoer, and Lundebj \(2018\)](#) and [Favero, Ortu, Tamoni, and Yang \(2019\)](#) emphasize the importance of time-variation in risk prices by implementing tests of asset-pricing models exploiting multi-horizon returns and their predictability.

This equation is subject to boundary conditions that are similar to those in the one-state model in Section 3. In each state, there exists a target payout threshold C_i^* above which all excess cash is paid out to investors, i.e., $v_i'(C_i^*) = 1$. This target level C_i^* satisfies the super-contact condition in each state, $v_i''(C_i^*) = 0$. Suppose that the target cash level is greater in the state i , $C_i^* > C_j^*$. Then, if the state switches from i to j while the cash-to-asset ratio is in $c \in [C_j^*, C_i^*]$, the firm pays a lumpy payout equal to $c - C_j^*$, meaning that the following equality holds:

$$v(C) = v(C_j^*) + c - C_j^* \quad c \in [C_j^*, C_i^*].$$

Furthermore, when the cash ratio decreases sufficiently, the firm raises new equity.¹⁷ Denote by $\underline{C}_i \geq 0$ the issuance boundary in state i . For any $c \in [0, \underline{C}_i]$, firm value satisfies the following boundary equation:¹⁸

$$v_i(c) = v_i(C_{*i}) - (1 + p)(C_{*i} - c) - f \tag{22}$$

where C_{*i} denotes the post-issuance cash ratio that satisfies the following boundary condition $v_i'(C_{*i}) = 1 + p$. If the firm raise funds before the cash buffer is depleted in one of the two states, it must be that $v_i'(\underline{C}_i) = 1 + p$ in that state. Otherwise, the firm raise fresh financing when it reaches zero (which implies that $v_i'(0) > 1 + p$, meaning that it is not optimal to time the market in a right neighborhood of zero).

¹⁷For simplicity, we focus on the case in which liquidating is never preferred to raising fresh financing. Equivalently, the left-hand side of equation (22) is always greater than the recovery rate of assets, ϕ .

¹⁸Because the firm faces the same issuance cost in the two states, heuristic arguments imply that it should be optimal for the firm to raise funds when cash reserves are depleted. However, we cast the problem in the general case in which we allow the firm to raise financing for a positive cash ratio. As we show in Section 5.2, the firm indeed issues new equity when the cash buffer is depleted.

5.2 Implications

We consider again our baseline parameterization in Table 1 and additionally assume that the transition intensities between the two states are symmetric, to filter out any effect driven by the longer duration of one state over the other and, thus, single out the effect of time-variation in risk prices.¹⁹ Throughout our numerical implementation, we realistically assume that the market risk prices are, on average, higher in the bad state than in the good state. Under this assumption, we consider two cases. In the first, the firm-relevant term structure of risks is upward-sloping in expansion and downward sloping in recession. In the second, it is downward-sloping in expansion and upward sloping in recession. To focus on time-variation of risk prices, we fix the firm's exposure to long-term and short-term risks at their baseline values (see Table 1).

Increasing (decreasing) term structure in expansion (recession) Figure 8 shows firm value and the optimal investment rate when the term structure is increasing in expansion (the good state G) and decreasing in recession (the bad state B).

Figure 8

The top panel of Figure 8 shows that firm value and the endogenous investment rate are larger in the good state than in the bad state.²⁰ This result is the composition of two forces. In the good state, the term structure of risk prices is not only increasing, but also lower in levels compared to the bad state. As a result, while the slope of the term

¹⁹BCW (2013) assume that the transition intensity out of the good state G is 0.1—i.e., expansions on average last ten years—and the transition intensity out of bad state B is 0.5—i.e., recessions last on average two years. However, under the risk-neutral measure, the relative magnitude of these intensities reverses: the transition intensity out of the good state is 0.3 and the transition intensity out of the bad state is 0.167. We do not attach additional risk prices to state transitions and, thus, risk adjustments to transition intensities. Instead, our assumption that $\pi_G = \pi_B = 0.3$ helps us single out the effects driven by time-variation in the level and the slope of risk prices.

²⁰In this panel, we assume that $\eta_{YG} = 0$ and $\eta_{AG} = 0.2$ in the good state so that the price of short-term shocks is smaller in the good state, whereas $\eta_{YB} = 0.6$ and $\eta_{AB} = 0.2$ so that the price of long-term shocks is smaller in the bad state.

structure should lead to smaller firm value and investment (as illustrated in the analysis in Section 4.1), the level acts as an offsetting strength (see Figure 2). Offsetting strengths also arise in the bad state. In this state, whereas we would expect greater investment and firm value resulting from the decreasing slope of the term structure, its higher level depresses these quantities.

To disentangle the effects triggered by the level and the slope of the term structure of risk prices in the two states, the bottom panel of Figure 8 considers an environment in which $\eta_{Yi} + \eta_{Ai}$ is constant across the two states (i.e., $\eta_{YG} + \eta_{AG} = \eta_{YB} + \eta_{AB}$), while preserving the above assumptions regarding the slope of the term structure. In this environment, firm value, the investment rate, and the payout threshold are greater in the bad state than in the good state, consistent with the analysis in Section 4.

Overall, comparing the two panels of Figure 8 illustrates that when the term structure is increasing (decreasing) in expansion (recession), the time-variation in the level and in the slope of the term structure have opposite qualitative effects on the firm's optimal investment rate. When risk prices are relatively low in expansion, firms should have greater incentives to invest, all else equal. However, upward-sloping market prices moderate this effect—the greater price of long-term shocks decreases the firm's optimal investment rate. Conversely, when the prices of risk are high in recession, firms reduce investment, but a downward-sloping term structure acts as a countervailing strength.

Decreasing (increasing) term structure in expansion (recession) We now consider the case in which the term structure is decreasing in expansion and increasing in recession, while continuing to assume that the average level of market prices is higher in the bad state. Figure 9 displays firm value and the optimal investment rate as a function

of the cash ratio under these assumptions.²¹

Figure 9

The top panel of Figure 9 shows that firm value and investment are greater in the good than in the bad state, similar to Figure 8. Again, this is the composition of the effect driven by time-variation in level and slope of the term structure. However, in the current case, time variation in level and slope have the same qualitative effect, as illustrated by the bottom panel of this figure (in which we assume that the sum of the prices of risk $\eta_{Yi} + \eta_{Ai}$ is constant across the two states). In fact, when prices of risk are relatively high in the bad state (recession), the firm cuts its investment rate. Furthermore, upward-sloping market prices in the bad state further reduce the firm's optimal investment rate. Conversely, in the good state (expansion), market risk prices are lower in levels and are downward-sloping, which both have a positive effect on investment.

6 Concluding Remarks

Whereas the recent asset pricing literature has largely investigated the effects of market risk prices of various persistence on equilibrium risk premia, corporate finance models typically abstract from this dimension. This paper seeks to fill this gap. Our analysis illustrates that the relative magnitude of the price of long-term versus short-term risks affects the firm's incentives to focus on short-term objectives (such as payouts to shareholders) or long-term growth. That is, while corporate short- vs. long-termism is typically analyzed through agency theory and optimal contracting, we provide an explanation based on the price of undiversifiable risk in the economy.

Our model demonstrates that when the firm is symmetrically exposed to short-term

²¹In the top panel, we assume that $\eta_{YG} = 0.2$ and $\eta_{AG} = 0$ in the good state so that the price of long-term risk is smaller in the good state, whereas $\eta_{YB} = 0.2$ and $\eta_{AB} = 0.6$ so that the price of short-term risk is smaller in the bad state.

and long-term shocks, firms exhibit short-termism in corporate policies if the term structure of risk prices is upward-sloping. In this case, firms reduce their investment and pay out dividends more often. Also, firms reduce their target cash ratio and the size of equity issuances, and are more likely to engage in asset sales when financially constrained. Conversely, if the term structure of risk prices is downward-sloping, firms exhibit greater investment rates, greater cash ratios, and pay out dividends less often. These effects are stronger for firms that are more exposed to long-term aggregate shocks, but may reverse for firms more exposed to short-term aggregate shocks. Our predictions then translate into practical guidance for empirical tests, by identifying characteristics that trigger heterogeneous responses to the term structure of risk prices. Our analysis also extends to nest time-variation in the level and slope of the term structure of risk prices. Overall, our analysis demonstrates that ignoring the term structure of risk prices induces distortions in corporate financial and real policies.

A Appendix

A.1 Proof of the results in Section 3.1

To pin down the risk-neutral dynamics of firm's productive assets and cash flows, we note that

$$d\hat{W}_t = dW_t - \rho_A \eta_A dt$$

and

$$d\hat{B}_t = dB_t - \rho_Y \eta_Y dt,$$

where \hat{W}_t and \hat{B}_t respectively denote the firm's long-term and short-term shocks under the physical measure. Using the above relations into equations (2) and (4) gives the risk-neutral dynamics of the firm's productive assets and cash flows reported in Section 3. In the following, we start by solving the model in the benchmark case with no financing frictions, and then move to the environment embedding financing frictions.

In the case with no financing frictions, the firm keeps no cash. Firm value, denoted by $V^*(a)$ in this case, satisfies the following HJB equation:

$$rV^*(a) = \max_{l^*} (\mu + l^* - \sigma_A \rho_A \eta_A) a V_a^*(a) + \frac{1}{2} a^2 \sigma_A^2 V_{aa}(a) + \left(\alpha - \sigma_Y \rho_Y \eta_Y - l - \frac{\kappa l^2}{2} \right) a. \quad (23)$$

Conjecture that $V^*(a) = av^*$, where v^* represents firm value scaled by productive assets. Substituting this expression into equation (23) yields

$$rv^* = \max_{l^*} (\mu + l^* - \sigma_A \rho_A \eta_A) v^* + \left(\alpha - \sigma_Y \rho_Y \eta_Y - l^* - \frac{\kappa (l^*)^2}{2} \right). \quad (24)$$

Maximizing this equation with respect to l^* gives the optimal investment rate reported in equation (6) in the main text. Substituting equation (6) into the above equation gives

an expression for scaled firm value in the absence of financing frictions, which is equal to

$$v^* = 1 + \kappa l^*. \quad (25)$$

We next prove Proposition 1 and Proposition 2, which relate the firm's optimal investment rate to the slope of the term structure of risk prices.

Proof of Proposition 1 This proposition focuses on the case in which the firm is symmetrically exposed to long-term and short-term aggregate shocks, i.e., $\sigma_A \rho_A = \sigma_Y \rho_Y \equiv \sigma \rho$ (in the rest of this proof, we use $\sigma \rho$ to ease the notation). To focus on the relative magnitude of η_Y versus η_A , we assume that the sum of the market prices of risk is constant and denote it by $\eta \equiv \eta_Y + \eta_A$. Namely, we assume that the market price of long-term and short-term shocks respectively satisfy $\eta_A \equiv \eta \epsilon$ and $\eta_Y \equiv \eta(1 - \epsilon)$, with $\epsilon \in [0, 1]$. If $\epsilon = \frac{1}{2}$, the term structure of market risk prices is flat. If $\epsilon \in [0, \frac{1}{2})$, the term structure is downward-sloping. If, instead, $\epsilon \in (\frac{1}{2}, 1]$, the term structure is upward-sloping. Using this notation, equation (6) boils down to:

$$l^* = (r - \mu + \sigma \rho \eta \epsilon) - \sqrt{(r - \mu + \sigma \rho \eta \epsilon)^2 - 2(\alpha - \sigma \rho \eta(1 - \epsilon) - r + \mu - \sigma \rho \eta \epsilon) / \kappa}. \quad (26)$$

Differentiating with respect to ϵ gives:

$$\frac{\partial l^*}{\partial \epsilon} = \eta \rho \sigma \left(1 - \frac{r - \mu + \sigma \rho \eta \epsilon}{\sqrt{(r - \mu + \sigma \rho \eta \epsilon)^2 - 2(\alpha - \sigma \rho \eta - r + \mu) / \kappa}} \right). \quad (27)$$

The second term in the square root of this equation is positive if equation (7) in the main text holds (guaranteeing that the investment rate is positive when the firm faces no financing frictions). Notably, if this term is positive, then equation (27) is negative. That is, if ϵ increases, investment decreases. Thus, the firm invests more if the term structure is downward sloping than if it is flat or, even more so, if it is upward sloping.

We summarize this important result below. The claim in Proposition 1 follows. \diamond

Proof of Proposition 2 This proposition focuses on the case in which the firm is asymmetrically exposed to long-term and short-term aggregate shocks, i.e., $\sigma_A \rho_A \neq \sigma_Y \rho_Y$. As in the proof of Proposition 1, we use the following notation: $\eta_A \equiv \eta \epsilon$ and $\eta_Y \equiv \eta(1 - \epsilon)$, with $\epsilon \in [0, 1]$. The first derivative of the optimal investment rate (equation (6)) with respect to ϵ satisfies:

$$\frac{\partial l^*}{\partial \epsilon} = \eta \rho_A \sigma_A \left(1 - \frac{r - \mu + \sigma_A \rho_A \eta \epsilon}{\sqrt{(r - \mu + \sigma_A \rho_A \eta \epsilon)^2 - 2(\alpha - \sigma_Y \rho_Y \eta(1 - \epsilon) - r + \mu - \sigma_A \rho_A \eta \epsilon) / \kappa}} \right) - \frac{\eta}{\kappa} \frac{(\rho_A \sigma_A - \rho_Y \sigma_Y)}{\sqrt{(r - \mu + \sigma_A \rho_A \eta \epsilon)^2 - 2(\alpha - \sigma_Y \rho_Y \eta(1 - \epsilon) - r + \mu - \sigma_A \rho_A \eta \epsilon) / \kappa}}. \quad (28)$$

The first term on the right-hand side is negative, being analogous to the term in equation (27). It then follows that if $\rho_A \sigma_A \geq \rho_Y \sigma_Y$ —i.e., if the firm is relatively more exposed to aggregate long-term shocks than to short-term aggregate shocks—then the result in Proposition 1 continues to hold, and the optimal investment rate decreases with the slope of the term structure of risk prices.

Consider now the case in which the inequality $\rho_A \sigma_A \geq \rho_Y \sigma_Y$ does not hold. Equation (28) can be rewritten as follows:

$$\frac{\partial l^*}{\partial \epsilon} = \eta \rho_A \sigma_A \left(1 - \frac{r - \mu + \sigma_A \rho_A \eta \epsilon + 1/\kappa - \rho_Y \sigma_Y / (\kappa \rho_A \sigma_A)}{\sqrt{(r - \mu + \sigma_A \rho_A \eta \epsilon + 1/\kappa)^2 - 2(\alpha - \sigma_Y \rho_Y \eta(1 - \epsilon) + 1/(2\kappa)) / \kappa}} \right). \quad (29)$$

If the second term in parenthesis is smaller than one, then $\frac{\partial l^*}{\partial \epsilon} > 0$. By calculation, we find that this is the case if the following inequality holds:

$$\rho_Y \sigma_Y \geq \rho_A \sigma_A [1 + \kappa(r - \mu + \eta \rho_A \sigma_A)] - \sqrt{\rho_A^2 \sigma_A^2 [(1 + \kappa(r - \mu + \eta \rho_A \sigma_A))^2 - 1 - 2\alpha\kappa]}. \quad (30)$$

That is, when this is the case, then $\frac{\partial l^*}{\partial \epsilon} > 0$. \diamond

A.2 Proof of the results in Section 3.2

In this section, we derive firm value in the presence of financing frictions, as characterized in Proposition 3. Equation (9) gives:

$$\begin{aligned} V_m(a, m) &= v'(c) \\ V_{mm}(a, m) &= \frac{v''(c)}{a} \\ V_a(a, m) &= v(c) - cv'(c) \\ V_{aa}(a, m) &= \frac{c^2}{a}v'(c) \end{aligned}$$

Plugging these expressions back into equation (8) yields the scaled HJB equation (see equation (10)). Maximizing equation (10) with respect to the investment rate gives the following first-order condition:

$$[v(c) - v'(c)c] - v'(c) - \kappa lv'(c) = 0.$$

Solving for this equation gives the expression for $l(c)$ reported in equation (11). Substituting $l(c)$ back into the HJB equation yields:

$$\begin{aligned} rv(c) &= \left[\mu + \frac{1}{\kappa} \left(\frac{v(c)}{v'(c)} - 1 - c \right) - \sigma_A \rho_A \eta_A \right] [v(c) - v'(c)c] + \frac{v''(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2) \\ &+ \left[\alpha - \sigma_Y \rho_Y \eta_Y - \frac{1}{\kappa} \left(\frac{v(c)}{v'(c)} - 1 - c \right) - \frac{1}{2\kappa} \left(\frac{v(c)}{v'(c)} - 1 - c \right)^2 + (r - \lambda)c \right] v'(c) \end{aligned}$$

and, by calculations, equation (12) follows. Scaled firm value $v(c)$ is then solved subject to the boundary conditions reported in Proposition 3, which are standard and can be derived using arguments similar to Décamps et al. (2017).

Consider now firm value at the target payout threshold, C^* . Using conditions (13) and (14) into equation (12) gives:

$$(r - \mu + \sigma_A \rho_A \eta_A) v(C^*) = \alpha - \sigma_Y \rho_Y \eta_Y + (r - \lambda - \mu + \sigma_A \rho_A \eta_A) C^* + \frac{1}{2\kappa} [v(C^*) - C^* - 1]^2.$$

As in Malamud and Zucchi (2019), define $w \equiv v(C^*) - C^*$ to obtain an expression for $v(C^*)$ as a function of C^* . Substituting in the equation above, we get

$$(r - \mu + \sigma_A \rho_A \eta_A) w = \alpha - \sigma_Y \rho_Y \eta_Y - \lambda C^* + \frac{1}{2\kappa} (w^2 - 2w + 1),$$

so we need to solve

$$\frac{1}{2\kappa} w^2 - \left(r - \mu + \sigma_A \rho_A \eta_A + \frac{1}{\kappa} \right) w + \alpha - \sigma_Y \rho_Y \eta_Y - \lambda C^* + \frac{1}{2\kappa} = 0$$

which gives²²

$$w = \kappa \left[r - \mu + \sigma_A \rho_A \eta_A + \frac{1}{\kappa} - \sqrt{\left(r - \mu + \sigma_A \rho_A \eta_A + \frac{1}{\kappa} \right)^2 - \frac{2}{\kappa} \left(\alpha - \sigma_Y \rho_Y \eta_Y - \lambda C^* + \frac{1}{2\kappa} \right)} \right].$$

and, thus, $v(C^*) = w + C^*$.

A.3 Risk prices and the term structure of equity

In this Appendix, we link the relative prices of short-term and long-term risks to the slope of the term structure of equity risk premia. In Section 2, we assume that the stochastic discount factor is described by equation (1), where long-term and short-term risks feature market prices denoted by η_A and η_Y respectively. Additionally, we now assume that there

²²We choose the solution that is continuous in the limit in which the second term in the square bracket tends to zero.

is an equity market index in the economy. The payout of the equity market index, denoted by D_t , satisfies the following dynamics:

$$d \log D_t = (\bar{D} + D_{At})dt + dD_{Yt}, \quad (31)$$

where the component D_{At} is subject to the long-term aggregate risk, whereas the component D_{Yt} is subject to the short-term aggregate risk. Namely, we assume that D_{At} follows:

$$dD_{At} = -\lambda_A D_{At}dt + s_A d\tilde{W}_t,$$

and D_{Yt} follows:

$$dD_{Yt} = -\lambda_Y D_{Yt}dt + s_Y d\tilde{B}_t.$$

The component D_{At} induces a unit root and, thus, its fluctuations accumulate over time and capture permanent risk. Instead, the component D_{Yt} produces stationary fluctuations and, thus, captures transitory risk. The joint dynamics of the stochastic discount factor and the payout of the equity market are inspired by the recent asset pricing literature on the term structure of equity (e.g., [Breugem et al., 2020](#)).

We compute the term structure of equity risk premia, i.e., the instantaneous risk premium on the dividend strip as a function of maturity. The dividend strip price with maturity τ is the present value of the market dividend paid out at the horizon τ and has exponential closed form:

$$\begin{aligned} P(t, \tau) &= \mathbb{E}_t \left[\frac{\xi_{t+\tau}}{\xi_t} D_{t+\tau} \right] \\ &= \xi_t^{-1} \exp [p_0(\tau) + p_\xi(\tau) \log \xi_t + p_D(\tau) \log D_t + p_A(\tau) D_{At} + p_Y(\tau) D_{Yt}], \end{aligned}$$

where the coefficients satisfy the following HJB equation:

$$0 = p'_0(\tau) + p'_\xi(\tau) \log \xi_t + p'_D(\tau) \log D_t + p'_A(\tau) D_{At} + p'_Y(\tau) D_{Yt}$$

$$\begin{aligned}
& + p_\xi(\tau)(-r - \eta_A^2/2 - \eta_Y^2/2) + p_\xi(\tau)^2(\eta_A^2 + \eta_Y^2)/2 + p_D(\tau)(\bar{D} + D_{At} - \lambda_Y D_{Yt}) \\
& + p_D(\tau)^2 s_Y^2/2 + p_A(\tau)(-\lambda_A D_{At}) + p_A(\tau)^2 s_A^2/2 + p_Y(\tau)(-\lambda_Y D_{Yt}) + p_Y(\tau)^2 s_Y^2/2 \\
& + p_\xi(\tau)p_D(\tau)(-\eta_Y s_Y) + p_\xi(\tau)p_A(\tau)(-\eta_A s_A) + p_\xi(\tau)p_Y(\tau)(-\eta_Y s_Y) + p_D(\tau)p_Y(\tau)s_Y^2,
\end{aligned}$$

with initial conditions $p_\xi(0) = p_D(0) = 1$ and $p_A(0) = p_Y(0) = 0$. It turns out that

$$\begin{aligned}
p_0(\tau) &= (\bar{D} - r)\tau + \frac{s_A^2}{4\lambda_A^3} (4e^{-\lambda_A\tau} + 2\lambda_A\tau - 3 - e^{-2\lambda_A\tau}) \\
&\quad + \frac{s_Y^2}{4\lambda_Y} (1 - e^{-2\lambda_Y\tau}) + \frac{s_Y\eta_Y}{\lambda_Y} (e^{-\lambda_Y\tau} - 1), \\
p_\xi(\tau) &= 1, \\
p_D(\tau) &= 1, \\
p_A(\tau) &= \frac{1 - e^{-\lambda_A\tau}}{\lambda_A}, \\
p_Y(\tau) &= e^{-\lambda_Y\tau} - 1.
\end{aligned}$$

Thus, the dividend strip prices can be simply written as

$$P(t, \tau) = D_t \exp [p_0(\tau) + p_A(\tau)D_{At} + p_Y(\tau)D_{Yt}].$$

In turn, we derive the risk premium as a function of the horizon:

$$\begin{aligned}
rp(t, \tau) &= -\frac{1}{dt} \left\langle \frac{d\xi_t}{\xi_t}, \frac{dP(t, \tau)}{P(t, \tau)} \right\rangle \\
&= p_A(\tau)s_A\eta_A + [1 + p_Y(\tau)]s_Y\eta_Y \\
&= s_A\eta_A \frac{(1 - e^{-\lambda_A\tau})}{\lambda_A} + s_Y\eta_Y e^{-\lambda_Y\tau}. \tag{32}
\end{aligned}$$

The first term on the right hand side of equation (32) induces an upward-sloping effect with the horizon, whereas the second term induces a downward-sloping effect. As a result, the slope of the term structure of equity can be either positive or negative. The larger

(smaller) the market price of long-term risk relative to the market price of short-term risk, the stronger the upward-sloping (downward-sloping) effect. By calculations, the slope of the risk premium is given by:

$$\partial_{\tau} rp(t, \tau) = e^{-\lambda_A \tau} s_A \eta_A - e^{-\lambda_Y \tau} \lambda_Y s_Y \eta_Y. \quad (33)$$

Notably, the slope depends on the market prices of risk together with the exposure of the market dividend (i.e., the payout of the equity index) to the source of risk s_A and s_Y (scaled by the rate at which these shocks dissipate—i.e., the speed of mean reversion). In our example, the sign of the slope only depends on the sign of $\eta_A - \eta_Y$ when $s_A/s_Y = \lambda_Y$ and $\lambda_A = \lambda_Y$, otherwise both the prices of risks and the exposures to risks affect its sign.²³ The relevance of both prices of risk and exposures is consistent with our firm-level analysis, where the impact of relative risk pricing depends on the firm’s exposure to such risks.

Overall, by mapping the relative prices of risks into the slope of the term structure of equity premia, we view our analysis as a first step towards a more comprehensive understanding of how the term structure of equity and its dynamics affect real activity.

A.4 Proof of the results in Section 5

In this section, we derive firm value when the term structure varies with the business cycle. Standard arguments imply that, in each state $i = G, B$ (with $i \neq j$), firm value satisfies the following HJB equation:

$$\begin{aligned} rV_i(a, m) = \max_{l_i} & (\mu + l_i - \sigma_A \rho_A \eta_{Ai}) a V_{ia} + \left[\left(\alpha - \sigma_Y \rho_Y \eta_{Yi} - l_i - \frac{\kappa}{2} l_i^2 \right) a + (r - \lambda) m \right] V_{im} \\ & + \frac{1}{2} a^2 (\sigma_A^2 V_{iaa} + \sigma_Y^2 V_{imm}) + \pi_i (V_j(a, m) - V_i(a, m)). \end{aligned} \quad (34)$$

²³Note that in general equilibrium η_A and η_Y are endogenous and naturally increase with s_A and s_Y respectively (see, e.g., Breugem et al., 2020). It turns out that the relative price of the two risks is a first order determinant of the slope of the equity risk premium term structure.

Let us define the scaled value function in each state $v_i(c)$, with

$$V_i(a, m) \equiv av_i(c). \quad (35)$$

Substituting equation (35) into equation (34) and dividing by a , we get

$$\begin{aligned} rv_i(c) = \max_{l_i} (\mu + l_i - \sigma_{AP} \rho_A \eta_{Ai}) [v_i(c) - v'_i(c)c] + \left[\alpha - \sigma_Y \rho_Y \eta_{Yi} - l_i - \frac{\kappa l_i^2}{2} + (r - \lambda)c \right] v'_i(c) \\ + \frac{v''_i(c)}{2} (\sigma_A^2 c^2 + \sigma_Y^2) + \pi_i (v_j(c) - v_i(c)). \end{aligned} \quad (36)$$

Differentiating the above equation with respect to l_i gives the first-order condition for the firm's investment rate

$$[v_i(c) - v'_i(c)c] - v'_i(c) - \kappa l_i v'_i(c) = 0.$$

Solving this equation gives the optimal investment rate reported in equation (20). Plugging $l_i(c)$ back into equation (36) yields the system of ordinary differential equations reported in equation (21).

In each state, at the target cash level C_i^* , firm value satisfies

$$\begin{aligned} (r - \mu + \sigma_{AP} \rho_A \eta_{Ai}) v_i(C_i^*) = [\alpha - \sigma_Y \rho_Y \eta_{Yi} + (r - \lambda - \mu + \sigma_{AP} \rho_A \eta_{Ai}) C_i^*] \\ + \frac{1}{2\kappa} [v_i(C_i^*) - (1 + C_i^*)]^2 + \pi_i [v_j(C_i^*) - v_i(C_i^*)], \end{aligned}$$

which boils down to

$$\begin{aligned} (r - \mu + \sigma_{AP} \rho_A \eta_{Ai} + \pi_i) w_i = [\alpha - \sigma_Y \rho_Y \eta_{Yi} - (\pi_i + \lambda) C_i^*] \\ + \frac{1}{2\kappa} [w_i^2 - 2w_i + 1] + \pi_i v_j(C_i^*). \end{aligned}$$

where we have defined $w_i \equiv v_i(C_i^*) - C_i^*$. By calculations, we get

$$\frac{1}{2\kappa}w_i^2 - \left(r - \mu + \sigma_{AA}\rho_A\eta_{A_i} + \pi_i + \frac{1}{\kappa}\right)w_i + \alpha - \sigma_Y\rho_Y\eta_{Y_i} - (\pi_i + \lambda)C_i^* + \frac{1}{2\kappa} + \pi_iv_j(C_i^*) = 0,$$

which gives

$$w_i = \kappa \left[B_i - \sqrt{B_i^2 - \frac{2}{\kappa} \left(\alpha - \sigma_Y\rho_Y\eta_{Y_i} - (\pi_i + \lambda)C_i^* + \frac{1}{2\kappa} + \pi_iv_j(C_i^*) \right)} \right]$$

where we have defined $B_i = r - \mu + \sigma_{AA}\rho_A\eta_{A_i} + \pi_i + \frac{1}{\kappa}$.

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Table 1: BASELINE PARAMETERS.

Parameter	Description	Value
r	Risk-free rate	0.04
λ	Opportunity cost of cash	0.01
μ	Growth (depreciation) rate of assets	-0.13
κ	Adjustment cost coefficient	3.90
α	Mean cash flow rate	0.225
σ_A	Asset volatility	0.18
σ_Y	Cash flow volatility	0.12
ϕ	Recovery rate in liquidation	0.90
ρ_A	Firm's correlation with aggregate long-term shocks	0.35
ρ_Y	Firm's correlation with aggregate short-term shocks	0.35
p	Proportional financing cost	0.06
f	Fixed financing cost	0.002

Table 2: REFINANCING, PAYOUTS, INVESTMENT, AND CASH MANAGEMENT. The table reports the optimal issuance size C_* , the target cash level C^* , and the threshold C_0 below which the firm disinvests as a function of the gap between the market price of long-term and short-term shocks, which in turn determines whether the slope of the term structure of market risk prices is increasing (denoted as I), flat (denoted as F), or decreasing (denoted as D). The top panel focuses on the baseline parameterization, the middle panel shows the case $\sigma_A = \sigma_Y = 0.12$, and the bottom panel assumes that $f = 0.01$.

Slope	$\eta_A - \eta_Y$	C_*	C^*	C_0
Baseline				
I	0.40	0.0377	0.171	0.007
I	0.20	0.0388	0.176	0.002
F	0.00	0.0399	0.182	n.a.
D	-0.20	0.0413	0.189	n.a.
D	-0.40	0.0432	0.198	n.a.
$\sigma_Y = \sigma_A = 0.12$				
I	0.40	0.0398	0.180	n.a.
I	0.20	0.0406	0.184	n.a.
F	0.00	0.0413	0.187	n.a.
D	-0.20	0.0421	0.191	n.a.
D	-0.40	0.0433	0.196	n.a.
$f = 0.01$				
I	0.40	0.067	0.200	0.038
I	0.20	0.069	0.206	0.035
F	0.00	0.072	0.213	0.031
D	-0.20	0.074	0.222	0.027
D	-0.40	0.078	0.232	0.022

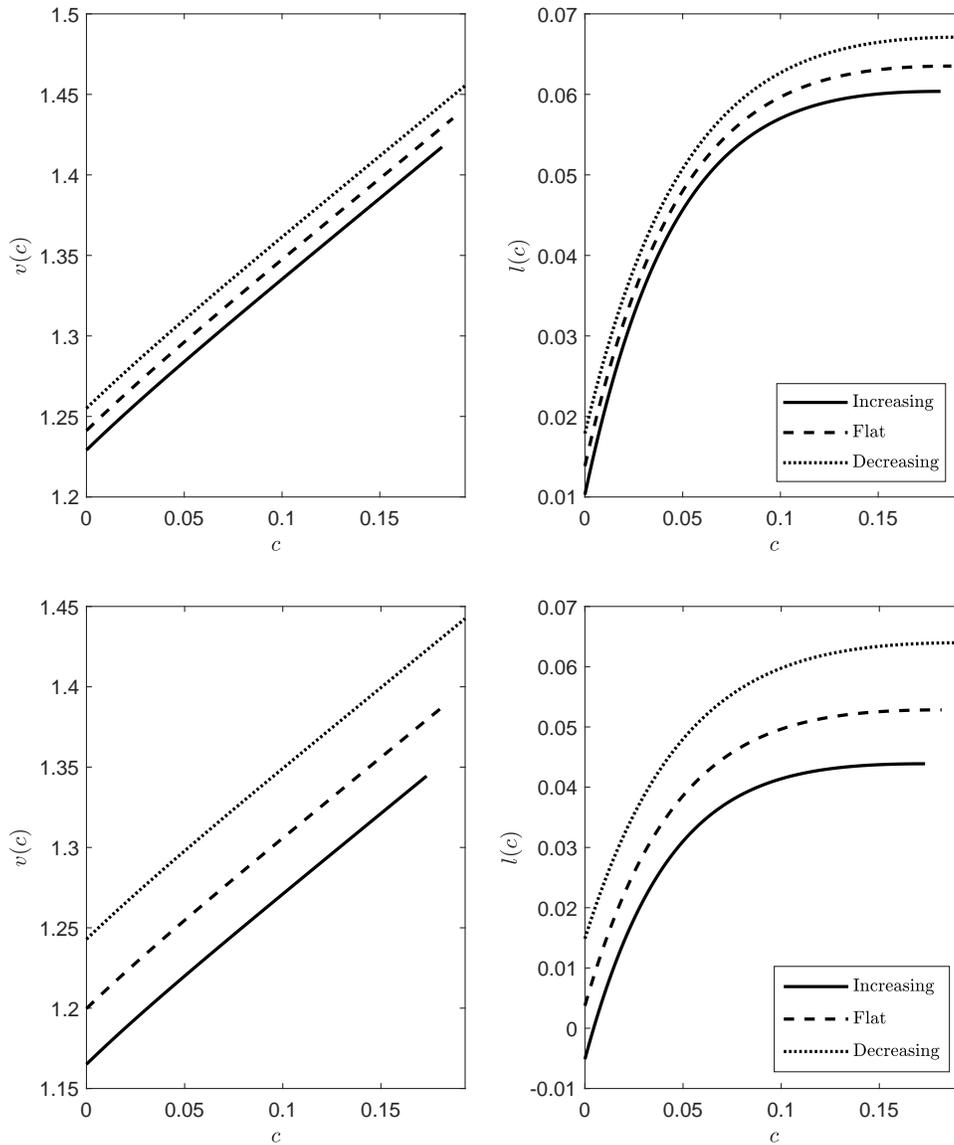


Figure 1: FIRM VALUE AND OPTIMAL INVESTMENT RATE. The figure shows the scaled firm value and the optimal investment rate as a function of the cash ratio $c \in [0, C^*]$ when varying the slope of the term structure of market risk prices. The increasing case features $\eta_Y = 0.05 < \eta_A = 0.35$, the flat case features $\eta_Y = \eta_A = 0.2$, and the decreasing case features $\eta_Y = 0.35 > \eta_A = 0.05$. The top panel focuses on the case $\sigma_A = \sigma_Y = 0.12$, whereas the bottom panel uses our baseline parameterization ($\sigma_A = 0.18 > \sigma_Y = 0.12$).

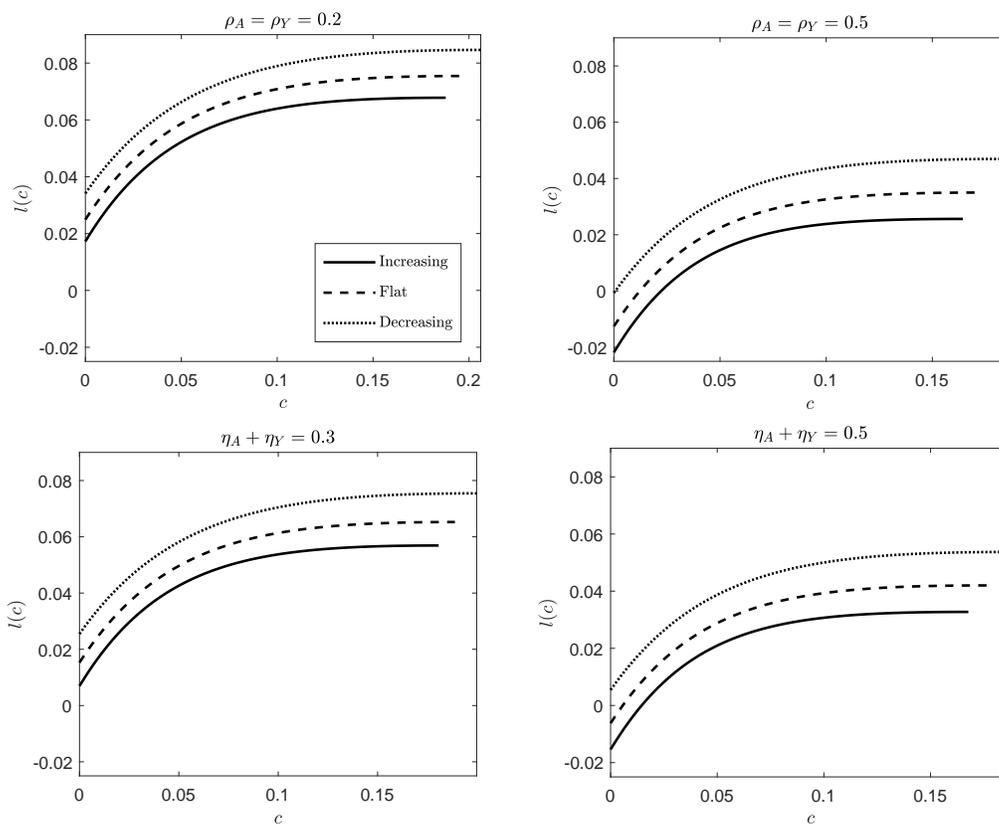


Figure 2: VARYING CORRELATION WITH AGGREGATE SHOCKS AND LEVEL OF MARKET PRICES. The figure shows the firm's optimal investment rate as a function of the cash ratio $c \in [0, C^*]$ when the slope of the term structure of market risk prices is increasing, flat, and decreasing. In the top panel, we vary the firm's correlation with aggregate shocks. In the bottom panel, we vary the level of the term structure of risk prices, i.e., the sum $\eta_A + \eta_Y$. We focus on the baseline $\sigma_A = 0.18 > \sigma_Y = 0.12$.

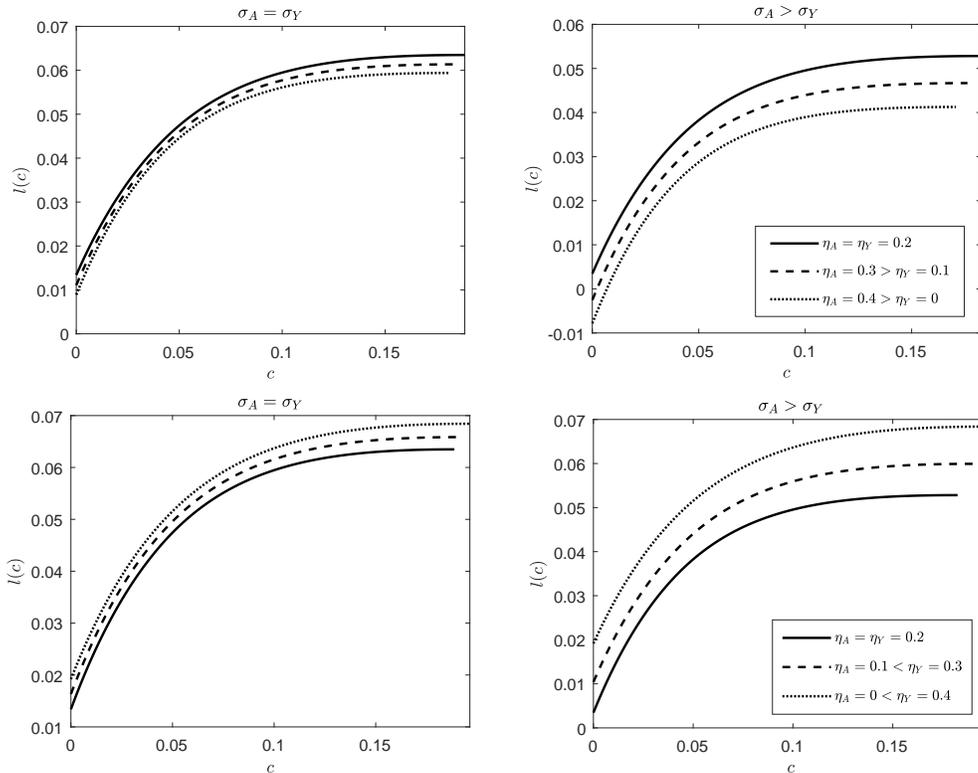


Figure 3: VARYING THE STEEPNESS OF THE SLOPE. The top panel shows the firm's optimal investment rate as a function of the cash ratio $c \in [0, C^*]$ when the term structure of risk prices is flat (solid line) and increasing (dashed and dotted lines). The bottom panel shows firm value and investment rate when the term structure is flat (solid line) and decreasing (dashed and dotted lines). The left panel depicts the case $\sigma_A = \sigma_Y$, whereas the right panel depicts the baseline parameterization in which $\sigma_A > \sigma_Y$.

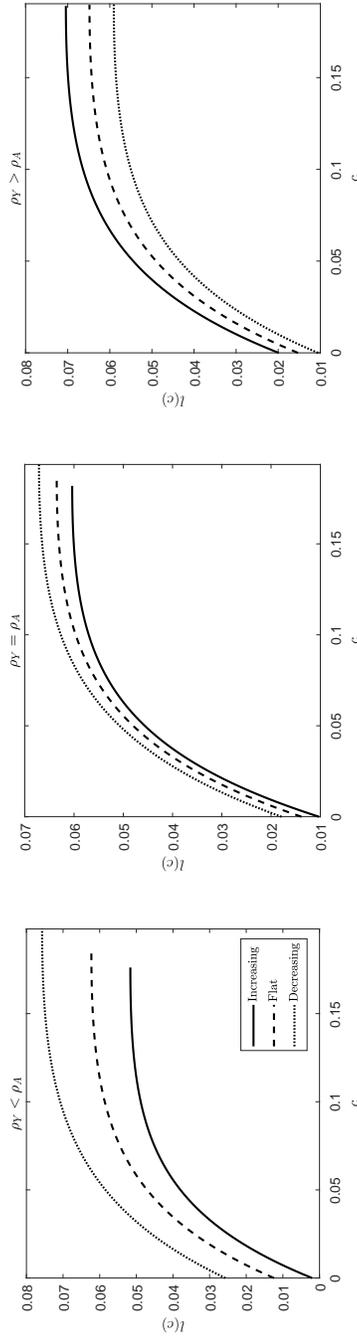


Figure 4: ASYMMETRIC CORRELATION WITH AGGREGATE SHORT-TERM AND LONG-TERM SHOCKS. The figure represents the optimal investment rate as a function of the cash ratio $c \in [0, C^*]$. The left panel represents the case in which $\rho_Y = 0.25 < \rho_A = 0.45$, the middle panel represents the case in which $\rho_Y = \rho_A = 0.35$ (as in the baseline), and the right panel represents the case in which $\rho_Y = 0.45 > \rho_A = 0.25$. In this figure, we assume that $\sigma_A = \sigma_Y = 0.12$.

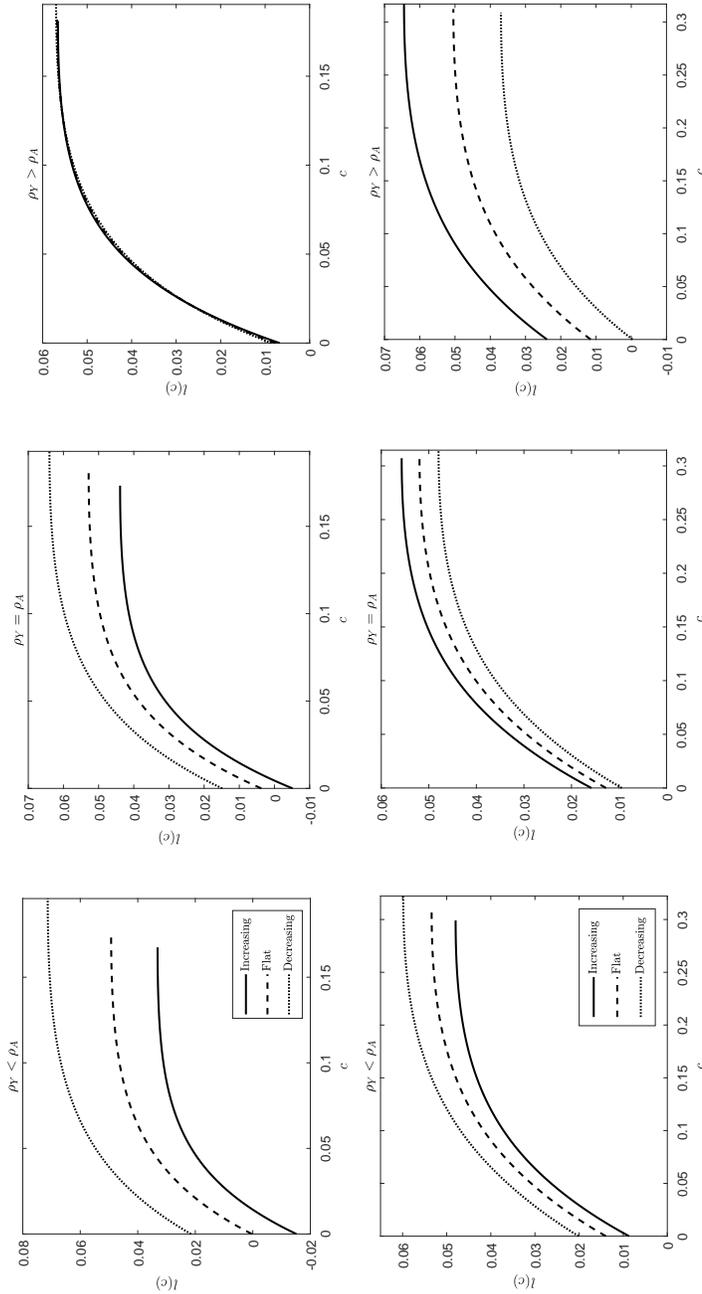


Figure 5: ASYMMETRIC EXPOSURE TO SHORT-TERM AND LONG-TERM SHOCKS. The figure represents the optimal investment rate as a function of the cash ratio $c \in [0, C^*]$. The left panels represent the case in which $\rho_Y = 0.25 < \rho_A = 0.45$, the middle panels represent the case in which $\rho_Y = \rho_A = 0.35$ (as in the baseline), and the right panels represent the case in which $\rho_Y = 0.45 > \rho_A = 0.25$. The top panel represents the case $\sigma_A = 0.18 > \sigma_Y = 0.12$ (as in the baseline parameterization), whereas the bottom panel represents the case $\sigma_A = 0.12 < \sigma_Y = 0.18$.

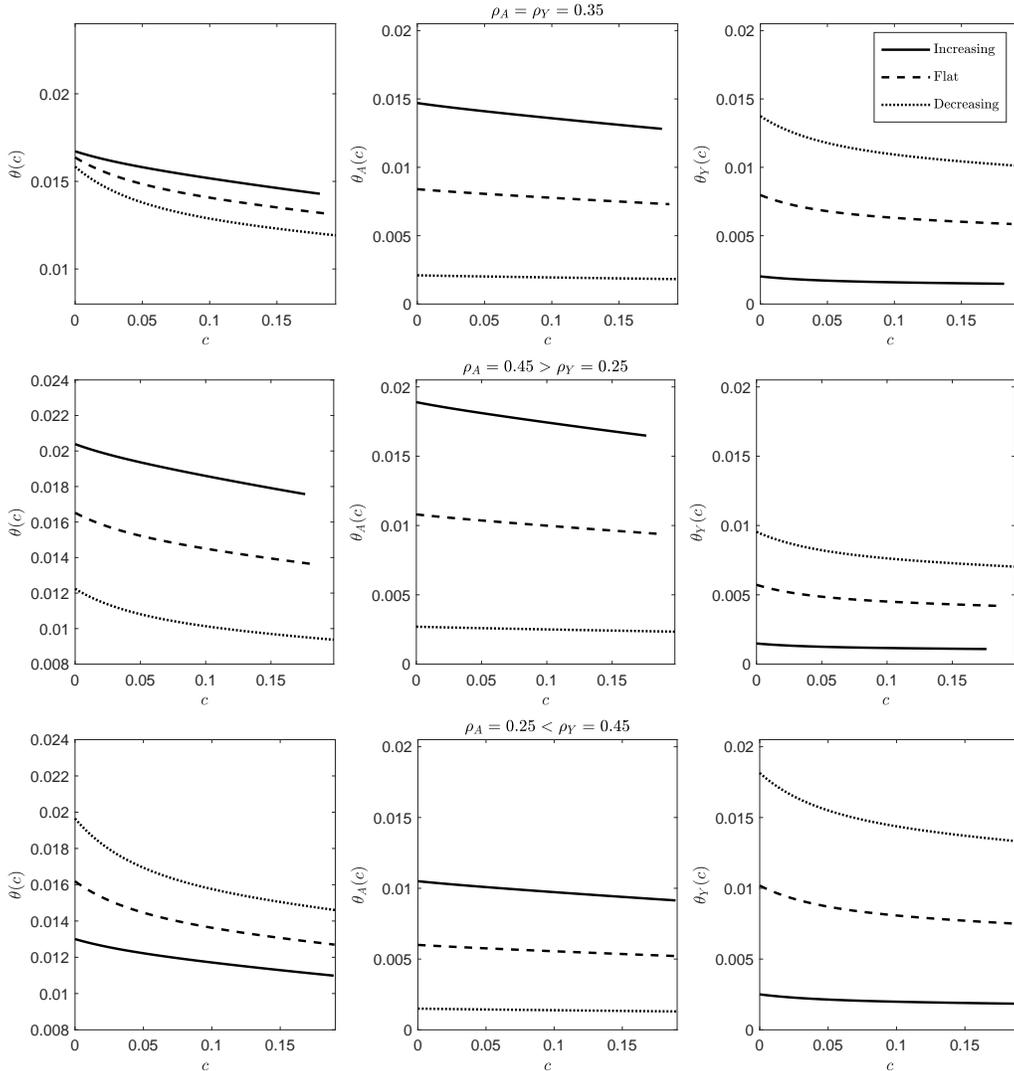


Figure 6: THE COMPOSITION OF RISK PREMIUM. The figure represents the firm's risk premium ($\theta(c)$, left panel) and its components ($\theta_A(c)$ and $\theta_Y(c)$, middle and right panels, respectively) as a function of the cash ratio $c \in [0, C^*]$. The increasing case features $\eta_Y = 0.05 < \eta_A = 0.35$, the flat case features $\eta_Y = \eta_A = 0.2$, and the decreasing case features $\eta_Y = 0.35 > \eta_A = 0.05$. The top panel represents the case in which the firm is equally exposed to short-term and long-term shocks ($\rho_A = \rho_Y = 0.35$, as in the baseline), the middle panel represents the case in which the firm is more exposed to short-term risk ($\rho_A = 0.25 < \rho_Y = 0.45$), whereas the bottom panel represents the case in which the firm is more exposed to long-term risk ($\rho_A = 0.45 > \rho_Y = 0.25$). We assume that $\sigma_A = \sigma_Y = 0.12$.

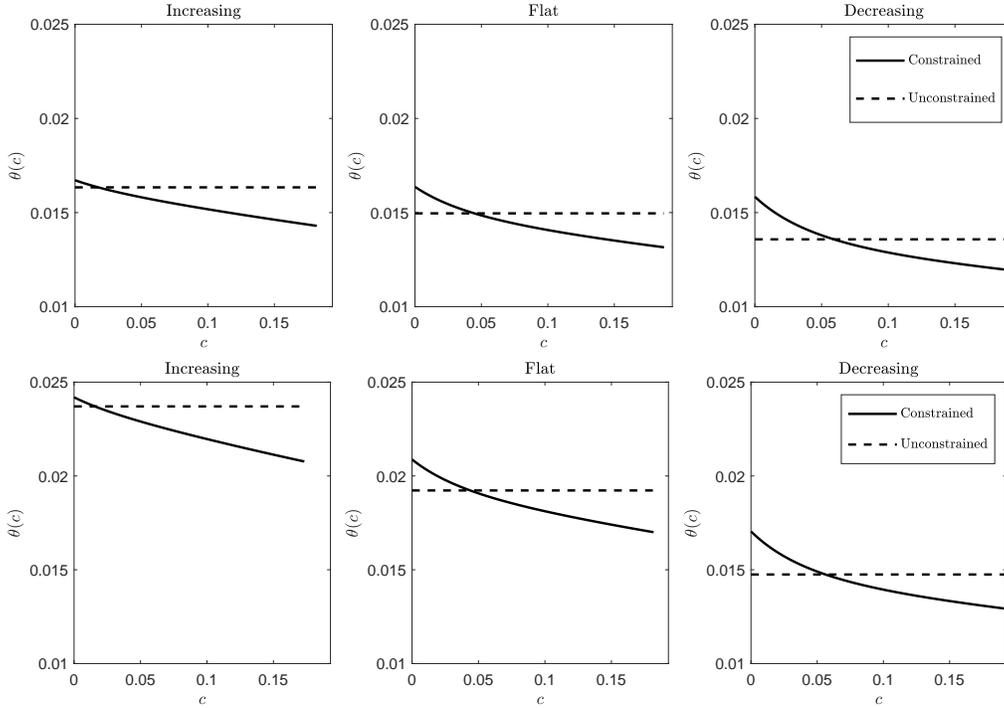


Figure 7: RISK PREMIUM WITH AND WITHOUT FINANCING FRICTIONS. The figure represents the firm's conditional risk premium ($\theta(c)$) as a function of the cash ratio $c \in [0, C^*]$. The increasing case features $\eta_Y = 0.05 < \eta_A = 0.35$, the flat case features $\eta_Y = \eta_A = 0.2$, and the decreasing case features $\eta_Y = 0.35 > \eta_A = 0.05$. The top panel represents the case $\sigma_A = \sigma_Y = 0.12$, whereas the bottom panel represents the case $\sigma_A = 0.18 > \sigma_Y = 0.12$ (as in the baseline parameterization).

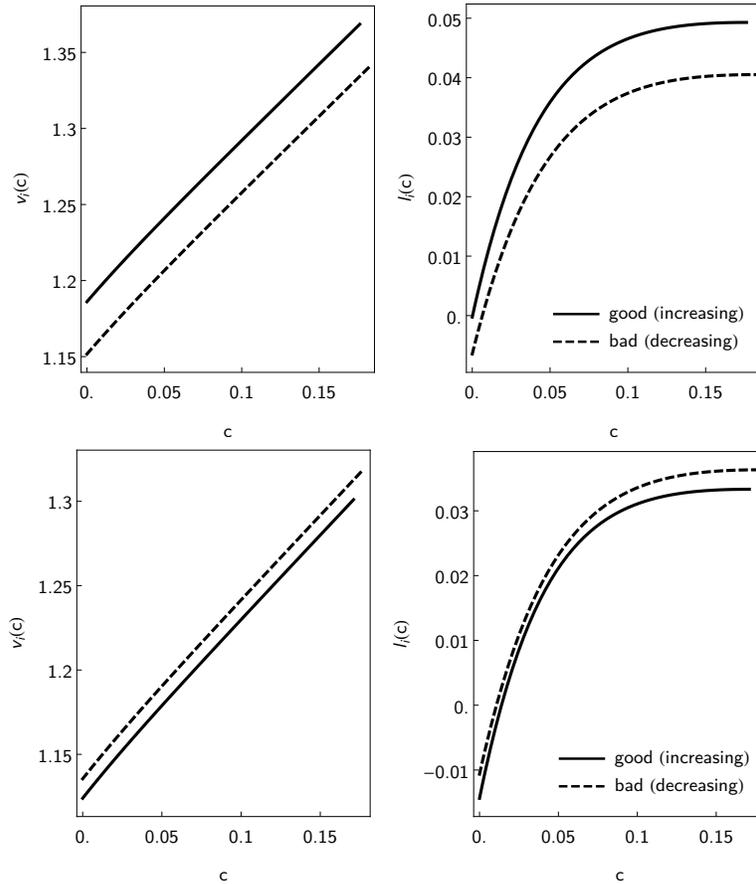


Figure 8: TIME-VARIATION IN RISK PRICES: DOWNWARD SLOPING IN RECESSION. The left panels show scaled firm value, and the right panels show the optimal investment rates as a function of the cash ratio. In the top panel, we set $\eta_{YG} < \eta_{AG} = \eta_{AY} < \eta_{YB}$ (specifically, $\eta_{AG} = \eta_{AB} = 0.2$, $\eta_{YG} = 0$, and $\eta_{YB} = 0.6$). In the bottom panel, we still assume that the bad (respectively, good) state is characterized by a downward-sloping (upward-sloping) term structure, but additionally impose that $\eta_{YG} + \eta_{AG} = \eta_{YB} + \eta_{AB} = 0.6$. The solid line depicts the G state, whereas the dashed line depicts the B state.

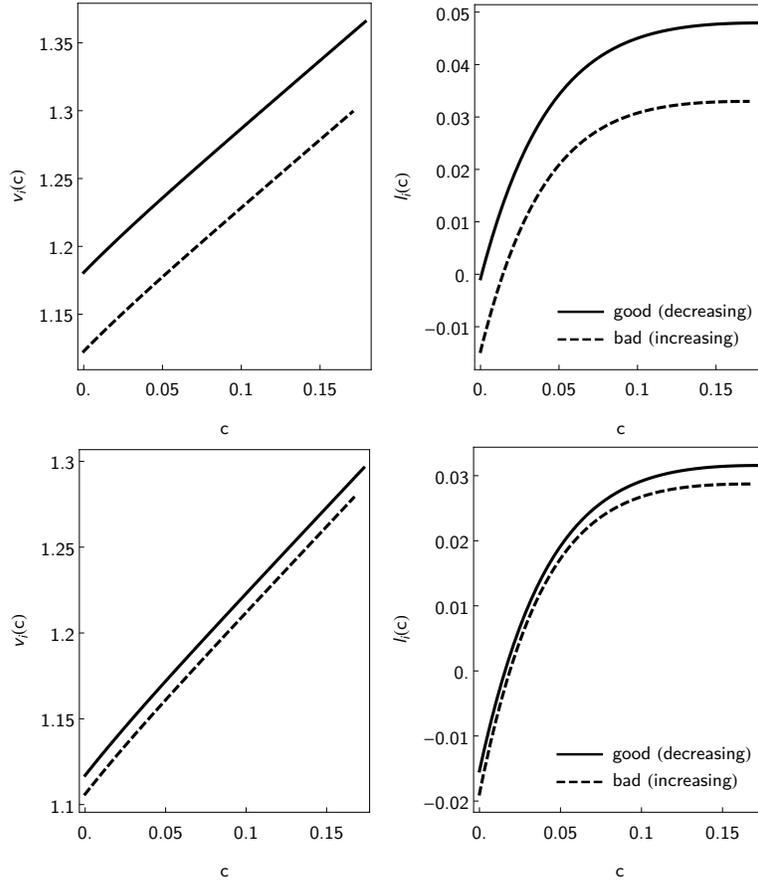


Figure 9: TIME-VARIATION IN RISK PRICES: DOWNWARD SLOPING IN EXPANSION. The left panels show scaled firm value, and the right panels show the optimal investment rates as a function of the cash ratio. In the top panels, we set $\eta_{AG} < \eta_{YG} = \eta_{YB} < \eta_{AB}$ (specifically, $\eta_{YG} = \eta_{YB} = 0.2$, $\eta_{AG} = 0$, and $\eta_{AB} = 0.6$). In the bottom panel, we still assume that the good (respectively, bad) state is characterized by a downward-sloping (upward-sloping) term structure, but additionally impose that $\eta_{YG} + \eta_{AG} = \eta_{YB} + \eta_{AB} = 0.6$. The solid line depicts the G state, whereas the dashed line depicts the B state.