Recapitalization, Bailout, and Long-run Welfare in a Dynamic Model of Banking*

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Abstract

We study the trade-off between the short-run costs and the long-run benefits of bank bailouts. In the model, banks leverage, thanks to their cost advantage at monitoring firms, but hold precautionary capital buffers to avoid costly equity issuance. Banks' recapitalization is suboptimal because they do not internalize the externalities of the banking sector's relative size on their individual leverage capacity and firms' investments. Systematic bailouts can improve the allocation efficiency in bad states, in which banks' leverage is persistently constrained and investments are low. In the long run, bailouts accelerate the economy's recovery by fostering growth, thereby reducing endogenous risk. (JEL D51, G21)

Keywords: Banks; bailout; dynamic complementarity; general equilibrium; financial frictions; welfare

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1 Introduction

Over the last two decades, several countries have been confronted with situations of severe distress in their banking sector. In most cases, they faced an impaired supply of financial services to the real economy, and far-reaching bank recapitalization decisions had to be taken. What followed was a major overhaul of a significant proportion of those countries' banks, often financed by public money (bailout).[1] In more recent years, particularly after the sub prime financial crisis, this has resulted in public discontent against the policy of "privatizing profits and socializing losses".

On this subject, it is by now common knowledge that bailouts can prompt excessive risk-taking by the institutions that shall be virtually rescued (Hryckiewicz, 2014). Also, there is evidence that open-ended liquidity support and repeated recapitalization may hinder their post-crisis recovery process (Honohan and Klingebiel, 2000). Yet other studies argue that in the commitment to bail out insolvent banks, the so-called "value effect" outweighs the moral hazard externality of the policy (Cordella and Levy-Yeyati, 2003; Sarin and Summers, 2016), and that bailouts may have positive effects in reducing undiversifiable contagion risk across banks (Parigi et al., 2000).[2] As a result, the public and the academic debate struggle to characterize the relative convenience of different recapitalization regimes: should the cost of banks' distress be a burden to tax payers? Are individual banks' recapitalization decisions socially optimal? And if not, can bailouts be beneficial?

While the problem of moral hazard has been extensively discussed (see for example Repullo, 2000; Gorton and Huang, 2004; Farhi and Tirole, 2012, among others), there remains a deficit in the proper treatment of the long-run spillover effects of bank bailouts on the macroeconomic dynamics, and their implications for social welfare in the theoretical banking literature. This paper aims at filling the gap.

Building on the seminal work of Brunnermeier and Sannikov (2014), we derive a continuous-

[1] In the EU alone, no less than 114 banks benefited from government support during the period 2007-2012. Over the same time span, the European Commission (2019) reports that about 3% of its 2012 GDP has been provided by member states as new capital to ailing banks.

[2] In the same spirit, Gropp et al. (2010) argue that there is no evidence that public guarantees increase the protected banks' risk-taking, except for banks that have outright public ownership. Likewise, Lambrecht and Tse (2019) propose a theoretical framework in which, even without considering the role of bailouts in containing systemic risk, banks create the most value net of any recapitalization costs under bailout regimes.

time dynamic general equilibrium model of a productive economy in which banks leverage to provide households with cost-efficient firms' monitoring and liquidity services, but hold precautionary equity buffers against negative systematic shocks. Banks' activities can improve the economy's equilibrium allocation, but also channel endogenous risk in the form of "stationary instability" through the pecuniary externalities of their precautionary motif. Similarly to Klimenko et al. (2016), we enhance our framework by allowing banks to issue equity at a fixed cost à la Løkka and Zervos (2008). Moreover, we consider government's role in imposing a state-contingent tax that re-allocates resources from households' to banks' net worth in bad states (bailout).

We characterize the economy's competitive equilibrium and show that: i) the relative size of the *aggregate* banking sector has a positive pecuniary externality on *individual* banks' leverage capacity by mitigating their precautionary motif; ii) due to their cost-efficient monitoring, additional banks' leverage capacity can assist by improving the economy's allocative efficiency and, through a second pecuniary externality, by boosting firms' investments; and iii) banks' recapitalization strategies are suboptimal because they do not internalize the effect of their individual decisions on the banking sector's relative size, that is, the joint effect of pecuniary externalities i) and ii) on the long-run macroeconomic dynamics.

In line with empirical evidence (e.g. Hoggarth et al., 2002; Bernanke, 2009; Barucci et al., 2019), our first set of results suggests that a bailout regime that internalizes banks' externalities can be beneficial when their leverage capacity is structurally constrained, firms' investments are persistently low, and therefore the economy is trapped in a long-lasting slow-growth regime.[3] We complement the theoretical banking literature by describing the mechanism through which bailouts entail a trade-off between their short-run costs and their long-run benefits. Moreover, we characterize the trade-off as a dynamic complementarity, that is, bailing out banks' equity in bad states raises firms' investments productivity (and thus banks' own valuation) at subsequent stages, thereby accelerating the transition towards good states while mitigating the endogenous risk of "stationary instability".[4]

[3]Evidence that bailouts may help to avoid allocation inefficiencies, for example when there are too many banks to liquidate, can be also found in Caprio and Klingebiel (1996) and Acharya and Yorulmazer (2007).

[4]In this respect, we show that optimal bailouts exist as long as the expected value of the future gains in terms of banks' value (similarly to the "value effect" described in Cordella and Levy-Yeyati, 2003) overtakes the loss in households' "bail-out-able" net worth in bad states.

The first important feature of the model is that, due to the assumption of homogeneity across banks, their individual recapitalization decisions are always synchronous. As a result, the competitive equilibrium outcome is suboptimal because banks face equity shortage all at once, and their supply of cost-efficient monitoring and liquidity services is structurally constrained by their precautionary motif. A bailout recapitalization regime reinforces banks' capital buffers in states of distress and thus alleviates the negative effects of systematic equity shortage on their leverage capacity. This result squares nicely with a recent study by Beck et al. (2020) showing that systematic risk increases more for economies whose banks are regulated by a more comprehensive resolution regime after negative system-wide shocks.

The second key aspect of the model concerns the relationship between bailouts and the macroeconomic dynamics. When the banking sector's relative size (capitalization) is scarce, bailouts have the important role of reallocating resources from less to more productive agents (from households to banks). This systematically increases the price of capital and encourages firms' investments through a Tobin's q relationship, therefore accelerating (and stabilizing) the transition from bad to good states. In the long run, bailouts reduce the likelihood of additional recapitalization due to high volatility-leverage and increase that of banks' dividend payouts. This happens because they allow banks to rapidly rebuild their capital buffers through the positive feedback loop of structurally higher growth rates. These results are fundamentally in line with two recent studies by Berger et al. (2016) and Homar and van Wijnbergen (2017) showing that bailouts may reduce systemic risk through a capital cushion channel, and that early interventions preserve the functions of the financial system while mitigating the macroeconomic consequences of a crisis.

In summary, our theoretical results suggest that, even in the simple case in which all economic actors are homogeneous and subject to a common source of risk, bank bailouts entail a non-trivial trade-off between their short- and long-run effects. The trade-off takes place through the banking sector's pecuniary externalities on the macroeconomic dynamics. It can be characterized by the dynamic complementarity between the value of households' net worth re-allocated as banks' equity at the bailout stage (short-run costs) and the consequential improvement in financial conditions and investment productivity in later stages (long-run benefits).

1.1 Related literature

This paper belongs to the macro-finance (e.g. Brunnermeier and Sannikov, 2014; Cochrane, 2017; He and Krishnamurthy, 2019) and intermediary asset pricing literature (e.g. He and Krishnamurthy, 2011, 2013). More specifically, it relates to those studies in which agents act suboptimally at the individual level due to the presence of financial frictions, and their inefficiencies materialize in the aggregate as pecuniary externalities (Dávila and Korinek, 2018). Also, it connects to the theoretical stream of research on Capital Structure Models (CSM) (see for example Flannery, 1994; Hilscher and Raviv, 2014), especially to those papers deploying continuous-time models of endogenous default à la Leland (1994) to study banks' recapitalization decisions (e.g. Peura and Keppo, 2006; Berger et al., 2020).

Similar to Klimenko et al. (2016), we extend the partial equilibrium setting that is typical in CSM by including a structural dynamic model of bank capital à la Løkka and Zervos (2008) in a general equilibrium environment. Moreover, from the technical standpoint, we fundamentally build on the seminal work of Brunnermeier and Sannikov (2014) by introducing frictions in capital markets to characterize the relationship between banks' equity, investments dynamics, and financial stability.

By describing the externalities of banks' activities on the equilibrium allocation, this work is strictly related to the stream of research exploring the role of banks' recapitalization regimes (Caprio and Klingebiel, 1996) and macro-prudential regulation (Gersbach and Rochet, 2012) in addressing allocation inefficiencies that may arise from liquidity frictions (Gorton and Huang, 2004; Keister, 2016), herding (Acharya and Yorulmazer, 2007), and debt overhang problems (Philippon and Schnabl, 2013).[5] Different from the static models developed in these studies, our framework allows to highlight the trade-off, or dynamic complementarity, between the positive effect of bailouts in the long run and their short-run costs. In this respect, a closely related paper is Bianchi (2016), who studies the intertemporal role of bailouts on the corporate sector.

Several recent contributions exploring different aspects of banks' regulation and capitalization in a dynamic partial and general equilibrium environment are Nicolo et al. (2014) and Sandri and Valencia (2013); Phelan (2016); Hugonnier and Morellec (2017); Schroth (2020),

[5] On the relationship between debt overhang and Tobin's q, see also Hennessy (2004).

respectively. Sandri and Valencia (2013) build a financial accelerator DSGE model and show that bank recapitalization as a response to large losses may improve welfare by smoothing output fluctuations. Conversely, Phelan (2016) examines the role of leverage constraints in stabilizing the business cycle, but considers neither the possibility of inefficient (costly equity issuance) nor that of bailouts. In a complementary fashion, Hugonnier and Morellec (2017) investigate the effects of liquidity and leverage requirements on banks' insolvency. Differently, Schroth (2020) analyses the distributional effects of bailouts and show that, even when increasing aggregate welfare, they may have adverse redistributive effects from poor to wealthy households.[6]

Finally, this work relates to a recent paper by Mendicino et al. (2019) showing that capital requirements make banks safer by addressing long-run stability risks, but negatively impact aggregate demand by imposing short-run costs. In the same spirit, we describe the trade-off between the positive feedback loop (long-run benefit) that associates to the (short-run) cost of bailouts.

The remainder of the paper proceeds as follows. Section 2 describes the model setup, while Section 3 derives and numerically solves the competitive equilibrium in the baseline case with no bailouts. Section 4 extends the baseline framework by including bank bailouts and explores how they affect the macroeconomic dynamics and households' welfare in the long run. Section 5 concludes. All proofs are collected in the Appendix.

2 Model

Time is continuous and infinite. The economy features two goods: physical capital (such as trees, henceforth "capital") and perishable output good (such as apples), as well as four actors: productive firms, households, banks, and a government sector. The total amount of wealth within the economy at each time t consists of the aggregate capital stock, denoted K_t. All agents are price taker and exchange capital on frictionless markets at the competitive price q_t; output acts as numéraire.

[6] Another recent work exploring the redistributive effects of bailouts and their relationship to financial fragility is Mitkov (2020).

Productive firms are constituted at time zero via capital transfers from either households or banks. At each instant of time, they either receive additional transfers or pay back a fraction of their capital outstanding holdings to their shareholders. In other words, firms act as capital lessors. This gives rise to a moral hazard problem in capital markets as in Tirole (2010) that can be tackled by implementing costly monitoring. We assume that both households and banks do so by paying a fixed cost for each unit of capital deployed to constitute firms or transferred consequentially. Importantly, banks' monitoring costs are lower than households'.

Firms have two roles: first, they produce output by using capital; second, due to technological illiquidity, they invest a fraction of their production output to intertemporally generate new capital by means of a risky technology. They pay out the residual share of output to their shareholders as dividends. The stream of firms' dividends plus their capital gains can be interpreted as the return on risky claims written on their profits.

Households are infinitely lived. They are born at time zero with an initial endowment of capital, a fraction of which is exogenously allocated to constitute banks' equity. Households consume and allocate the remainder of their net worth in either firms issued risky claims or banks' (risk-free) Short-term Liabilities (StL).

Banks are constituted at time zero from a fraction of households' net worth. They issue StL and allocate their assets in firms issued risky claims. They choose their leverage, dividends payout, and equity issuance strategies to maximize their market value. For the sake of simplicity, we assume that households exclusively own and manage banks and thus, we abstract from any agency issues between them.[7]

The government sector has a re-distributive role: it collects state-contingent taxes from households' net worth to bail out banks. To summarize, Figure 1 graphically depicts the cross-relationships among all agents in the model. We now focus on each economic actor in greater detail.

[7] The model can be generalised to account for moral hazard by assuming that banks are owned by households but managed by financial intermediaries while imposing a suitable Incentive Compatibility (IC) constraint. However, this does not fundamentally affect the main results of the paper. All details can be found in Appendix A.1.

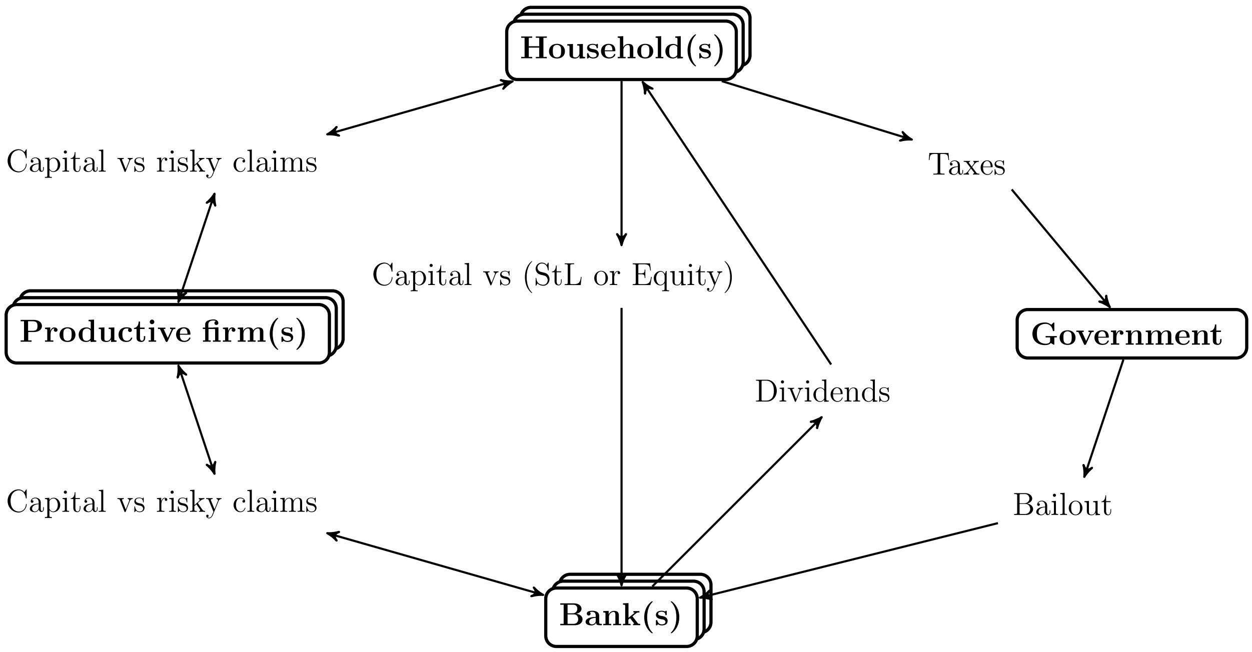

Figure 1: Cross relationships among productive firms, households, banks, and the government.

2.1 Firms, technologies, and costly monitoring

There exists a continuum of productive firms indexed $i \in \mathbb{I} := [0, 1)$. They operate a neoclassical AK technology producing perishable output good y_t^i by using capital as an input:

$$y_t^i = Ak_t^i, \tag{1}$$

in which A can be interpreted as the marginal capital productivity net of depreciation. Moreover, they are equipped with a stochastic technology that intertemporally generates new capital by investing a share ι_t^i of their output production. Let dW_t be a standard Brownian motion defined on the filtered probability space $(\Omega, \mathcal{H}, \mathbb{P})$, in which $\{\mathcal{H}_t, t > 0\}$ denotes the natural filtration over the measurable space $(\Omega, \mathcal{H})$. The capital stock k_t^i re-generated by firm i evolves with dynamics:

$$dk_t^i = k_t^i[\Phi(\iota_t^i)dt + \sigma dW_t], \tag{2}$$

in which σ is a positive constant. The function $\Phi(\bullet)$ is assumed to be concave in firms' investment rate ι_t^i; it is equivalent to a standard investment technology with convex adjustment costs (Bernanke et al., 1999). What is relevant to stress is that dW_t represents the unique source of systematic risk within the economy that is perfectly correlated across firms.

Other than their production technologies, firms have no initial endowment on their own. Accordingly, they are constituted at time zero by transfers of capital k_0^i that are either intermediated by banks or directly from households. In exchange for capital, they issue risky claims written on their profits with stochastic return dR_t^i. At each time $t \in [0, \infty)$ firms receive additional transfers ($dk_t^i > 0$) or pay back ($dk_t^i < 0$) a fraction of their outstanding capital holdings to their shareholders. As implied by their profit maximization (see Dindo et al., 2019), firms' risky claims are valued q_t for each unit of capital. Moreover, firm's i optimal investment rate ι_t^i relates to the price of capital by the so-called Tobin's q:

$$\partial_\iota \Phi(\iota_t^i) = \frac{1}{q_t} \Rightarrow \iota_t^i = \iota_t, \forall i \in \mathbb{I}. \tag{3}$$

Price dynamics and return on risky claims To characterize the return on firms issued risky claims, we can postulate a stochastic process for the dynamics of capital price q_t. As the only source of uncertainty in the economy is the systematic risk dW_t, we conjecture q_t to follow an Itô's process:

$$dq_t = q_t(\mu_t^q dt + \sigma_t^q dW_t), \tag{4}$$

whose drift μ_t^q and diffusion σ_t^q are endogenous $\mathcal{H}$-adapted stochastic processes that will be determined in equilibrium. By Itô's lemma, given the output production function (1), the investment technology (2), and the conjectured processes (4), the return on risky claims evolves with dynamics:[8]

$$dR_t^i := dR_t = \underbrace{\left[\frac{A-\iota_t}{q_t} + \Phi(\iota_t) + \mu_t^q + \sigma_t^q\sigma\right]}_{\mu_t} dt + \underbrace{(\sigma + \sigma_t^q)}_{\sigma_t} dW_t. \tag{5}$$

Costly monitoring Neither households nor banks directly hold capital stock, which is managed on their behalf by firms. By exerting costly effort, the latter can enhance their individual productivity. This gives rise to a moral hazard problem in capital markets that can be tackled either indirectly by writing suitable contracts between the firms and their shareholders, or directly by implementing costly monitoring of firms' effort decisions (Diamond, 1984; Tirole, 2010). We assume that direct monitoring is always more convenient than contracting and that banks have a cost advantage relative to households when doing so. Similar to Van Der Ghote (2020), we model households' and banks' monitoring costs so that they scale down the return on firms' risky claims by a fixed amount η^j, $j \in \{h, b\}$, for each unit of allocated capital, in which h and b denote households and banks, respectively. We define banks' "efficiency edge" as:

$$\eta := \eta^h - \eta^b \geq 0, \tag{6}$$

Banks' efficiency edge (6) and their supply of liquidity via StL are the only reasons that motivate their existence in the model. Without loss of generality we set $\eta^b = 0$ and thus $\eta = \eta^h$. Further details of the rationalization of banks' efficiency edge can be found in Appendix A.2.

2.2 Government, taxes, and bailout

The economy features a government sector which collects instantaneous lump sum taxes from households' net worth, denoted dT_t^π. The taxes finance aggregate wealth transfers from

[8] As in Brunnermeier and Sannikov (2014), the left-hand side term of μ_t can be interpreted as firms' dividend yield. It is proportional to the marginal productivity of capital A net of firms' instantaneous investments ι_t, which summarizes the stock of output that is deployed to generate new capital. Complementarily, the right-hand side term of μ_t captures the capital gain, that is, the instantaneous stock plus value change in firms' managed capital associated to their investments.

households' to banks' net worth (equity) and can thus be interpreted as a bank bailout.

The government carries out the bailout contingent on an exogenous recapitalization threshold. It does so in households' best interest, that is, looking forward to maximizing their welfare in the long run.[9] The recapitalization threshold can be thought of as the minimal required capital buffer that is regarded as necessary for the banking sector to work.

Formally, bailouts take place as follows: first, taxes are collected; second, the aggregate tax revenues are evenly rebated across banks net of a fixed and proportional administrative cost $\lambda^G \in \mathbb{R}^+$. In other words, the bailout recapitalization flow dT_t^π absorbs $dT_t^\pi(1+\lambda^G)$ units of households' net worth.

2.3 Households and banks

Households There exists a continuum of households indexed $h \in \mathbb{H} := [0,1)$. They are risk-neutral, infinitely lived, and discount the future at a constant rate ρ. Each household is born at time zero with an initial endowment e_0, a fraction of which exogenously constitutes bank b's initial equity e_0^b, valued $\nu_0 e_0^b$.

Household h is manager and shareholder of bank b. As such, she receives bank b's dividends $d\Delta_t^b$ and pays for her individual equity issuance $d\Pi_t^b$. Similarly to Løkka and Zervos (2008), we assume that issuing one unit of bank equity entails the additional payment $\lambda \leq \lambda^G \in \mathbb{R}^+$. Accordingly, the instantaneous flow of resources spent to refinance bank b equals $d\Pi_t^b(1+\lambda)$, in which the parameter λ captures all administrative and organizational costs.[10]

Households gain utility from consumption and allocate their residual net worth (that is not already designated as bank equity) between banks' StL d_t^h and firms issued risky claims k_t^h, valued $q_t k_t^h$. Direct investments in firms yield a stochastic return (5), but require the payment of a monitoring cost $\eta^h k_t^h$. Conversely, banks' StL are remunerated at the risk-free rate r_t. Households value the latter due to their liquidity services, that is, they enjoy a utility

[9] As we shall see in Section 4.1, the long-run welfare is defined as not conditional on the current state of the economy (or "ex-ante").

[10] This friction can be also thought of as a reduced form that captures the market illiquidity banks face when they issue securities in a moment of distress. Empirical evidence of the negative relationship between stock liquidity and its issuance costs is in Butler et al. (2005).

flow $\mathit{\Gamma}(d_t^h)dt$, with $\mathit{\Gamma}(\bullet)$ being a non decreasing and concave function of d_t^h. In summary, households' net worth at time t equals:

$$e_t = \nu_t e_t^b + \underbrace{d_t^h + q_t k_t^h}_{e_t^h}, \tag{7}$$

and their problem reads as follows:

$$H_0^h \coloneqq \sup_{\left\{c_t^h, d_t^h, k_t^h\right\} \in G_t^h} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[c_t^h + \mathit{\Gamma}(d_t^h)\right] dt, \tag{8}$$

subject to the dynamic budget constraint

$$G_t^h : de_t = r_t d_t^h dt + q_t k_t^h dR_t - \eta k_t^h dt - c_t^h dt + d\left(\nu_t e_t^b\right) - dT_t^\pi (1 + \lambda^G). \tag{9}$$

Banks There exists a continuum of banks indexed $b \in \mathbb{B} \in [1, 2)$. They are owned and managed by households who choose their dividends $d\Delta_t^b$, recapitalization $d\Pi_t^b$, and leverage strategies to maximize their market value. Banks collect additional resources other than their outstanding equity endowment e_t^b either by borrowing (issuing StL d_t^b) at the endogenous rate r_t or by issuing new equity at a fixed cost $\lambda \in \mathbb{R}^+$. They allocate their assets in firms issued risky claims k_t^b, valued $q_t k_t^b$, with stochastic return (5). Formally, banks' problem reads as follows:

$$J_0^b \coloneqq \sup_{\left\{d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b\right\} \in B_t^b} \mathbb{E}_0 \lim_{T \to \infty} \sup_{\tau \in [0, T)} \int_0^{\tau \wedge T} e^{-\rho t} \left[d\Delta_t^b - (1 + \lambda) d\Pi_t^b\right], \tag{10}$$

subject to the dynamic budget constraint:

$$B_t^b : de_t^b = d_t^b r_t dt + q_t k_t^b dR_t - d\Delta_t^b + d\Pi_t^b + dT_t^\pi, \tag{11}$$

in which $d\Delta_t^b \coloneqq d\delta_t^b e_t^b$ and $d\Pi_t^b \coloneqq d\pi_t^b e_t^b$ represent the dividends and recapitalization (equity issuance) flows, respectively, dT_t^π denotes the counter-value of government bailout, and $\tau \coloneqq \inf\left\{t \in [0, \infty) : e_t^b \leq 0\right\}$ denotes the equity issuance ($\mathcal{F}_t$ - stopping) time.

To pin down the optimal stopping time τ that maximises (10), one must set a suitable boundary condition to banks' value $J^b(e_t^b)$ (see Stokey, 2009, Chapter 6). Intuitively, that can be done by noticing that, due to the continuity of dW_t in (2), bank b never expires her equity over the interval $[t, t+dt)$ and thus, due to the time value of money, it is always optimal to delay recapitalization as long as $e_t^b > 0$. It follows that there exists some arbitrarily small but positive ϵ such that it is optimal to issue equity before (11) reaches $(-\infty, 0)$ (Løkka and Zervos, 2008). Given the recursive structure of banks' problem, the optimal strategy that defines τ either lets the equity process (11) hit the boundary $(-\infty, 0]$ (absorbing barrier) or it prevents the process from reaching $(-\infty, 0]$ (reflecting barrier). Accordingly, Problem (10) shall be complemented with the following (*smooth pasting*) condition:[11]

$$\max\left\{-J^{b*}(0), \partial_e J^{b*}(0) - (1+\lambda)\right\} = 0, \tag{12}$$

in which

$$J^{b*}(e_t^b) = \sup_{\left\{d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b\right\}} J_t^b(d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b, e_t^b). \tag{13}$$

Appendix A.3 provides a heuristic proof that as long as λ is small enough, then $\tau = \infty$ and $\partial_e J^{b*}(0) = 1+\lambda$. In other words, it is always optimal to finance bank b's equity issuance. Under this condition, banks' optimal strategies $\left\{d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b\right\}$ satisfy the Hamilton-Jacobi-Bellman Equation (HJBE) :

$$\rho J_t^b = \sup_{\left\{d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b\right\} \in B_t^b} \left\{ d\Delta_t^b - (1+\lambda) d\Pi_t^b + \frac{1}{dt}\mathbb{E}_t dJ_t^b \right\}, \tag{14}$$

with transversality condition $\lim_{t\to\infty} \mathbb{E}_0 e^{-\rho t} J_t^b = 0$.

[11] A formal statement of the more general case when the drift of banks' equity is positive jointly and a proof of uniqueness is in Løkka and Zervos (2008). In this respect, they show that bankruptcy happens at the time when banks' equity (due to homogeneity, here also in the aggregate) hits the boundary $(-\infty, 0)$ instead of $(-\infty, 0]$ because, otherwise, there does not exist an optimal recapitalization strategy $d\Pi_t^b$. A model in which a positive capital buffer before banks' recapitalization exists by introducing discontinuities in the noise process, such as Poisson jumps, rather than postulating delays when banks collect new capital on the markets as, for example, in Peura and Keppo (2006).

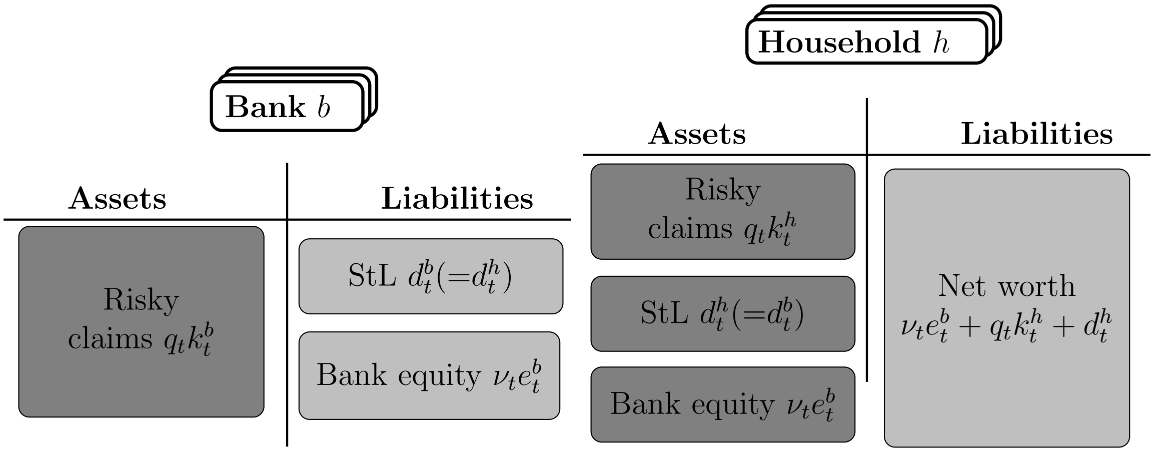

Figure 2: Banks' and households' balance sheet at time t.

3 Competitive equilibrium and numerical solution

This section characterizes the economy's competitive equilibrium, provides its numerical solution, and discusses the properties of the baseline model with no bailouts.

3.1 Equilibrium characterization

Informally, the competitive equilibrium of this economy is a map from histories of systematic shocks $\{dW_t\}$ to prices $\{q_t, \nu_t\}$, returns on firms issued risky claims $\{dR_t\}$, risk-free rates $\{r_t\}$, production $\{K_t, \iota_t\}$ and consumption choices $\left\{c_t^h\right\}$, capital allocations $\left\{k_t^{h \wedge b}, d_t^{h \wedge b}\right\}$ as well as dividends payout and equity issuance strategies $\left\{d\Delta_t^b, d\Pi_t^b\right\}$ so that: firms maximise their profits, households maximise their utility, banks maximise their value, and all markets clear. The formal statement of the equilibrium is in Appendix A.4.

Before we characterize households' and banks' optimal strategies in equilibrium, Figure 2 provides a summary snapshot of the relationship between their balance sheets at each point in time.

Proposition 1. ***Households' strategies***
Households' strategies $\left\{c_t^h, d_t^h, k_t^h\right\}$ *that are optimal controls to Problem (8) satisfy:*

1. *Households' consumption flows* $\left\{c_t^h\right\}$ *are always positive. In equilibrium, their level is*

determined by firms' investment decisions and banks' leverage capacity:

$$c_t^h = (A - \iota_t)\left(k_t^h + k_t^b\right) - \eta k_t^h.$$

2. *Households' portfolio choices* $\left\{d_t^h, k_t^h\right\}$ *satisfy the following asset pricing (in)equalities:*

$$r_t = \rho - \partial_d \mathit{\Gamma}(d_t^h), \tag{15}$$

$$\mu_t - \frac{\eta}{q_t} \leq \rho. \tag{16}$$

Proof. See Appendix A.5. □

Concerning households' portfolio choices, they are always willing to hold banks' StL at a (risk-free) rate r_t that equals their intertemporal discount rate ρ minus the marginal utility of liquidity (15). Conversely, they are not willing to directly invest in firms when the net return on their risky claims shrinks below ρ, and (16) holds slack ($k_t^h = 0$ and $e_t^h = d_t^h$). In that case, they utterly mandate investments in firms to banks. Conversely, households are indifferent in their portfolio choices when (16) holds with equality ($k_t^h, d_t^h > 0$). In equilibrium, when that happens, they directly invest in firms consistently with banks' leverage capacity.

Similarly to households, due to the assumptions of risk neutral preferences and homogeneous controls, banks' absolute value is a linear function of their individual net worth e_t^b. However, their marginal value must differ from one due to the presence of equity issuance costs. Accordingly, the suitable ansatz for the value function J_t^b satisfies, under the optimal strategy $\left\{d_t^b, k_t^b, d\Delta_t^b, d\Pi_t^b\right\}$:

$$J_t^{b*} = \mathbb{E}_t \int_t^{\infty} e^{-\rho s}\left[d\Delta_s^b - (1+\lambda) d\Pi_s^b\right] \coloneqq \nu_t e_t^b. \tag{17}$$

The term $\nu_t \geq 1$, which is endogenously determined, represents the marginal value of a bank endowed with net worth e_t^b at time t, that is, the market price of its equity (on this point, see Brunnermeier and Sannikov, 2014).[12]

[12]In other words, $\nu_t e_t^b$ is the maximal expected net present value that the bank may attain conditional on having book value e_t^b. The term ν_t is a proportionality coefficient that summarizes the way market

To further characterize banks' behaviour, we can postulate a process that describes the motion of ν_t. In this way, it is possible to derive analytically the stochastic differential $d\left(\nu_t e_t^b\right)$ that appears in banks' HJBE (14), and thus obtain intuitive closed form solutions describing their optimal strategies. Similarly to the dynamics of capital price (18), let ν_t follow an Itô's process:

$$d\nu_t = \nu_t(\mu_t^\nu dt - \sigma_t^\nu dW_t), \tag{18}$$

whose drift μ_t^ν and diffusion σ_t^ν are endogenous $\mathcal{H}$-adapted stochastic processes. Their values, jointly with the associated optimal banks' strategies, are pinned down in the following.

Proposition 2. ***Bank's strategies - dynamic capital structure***
The banks' strategies $\left\{d_t^b, k_t^b, d\delta_t^b, d\pi_t^b\right\}$ *that are optimal controls to Problem (13) are so that:*

1. *The dynamics of banks' equity marginal value (market price) (18) has drift:*

$$\mu_t^\nu = \partial_d \mathit{\Gamma}(d_t^b). \tag{19}$$

   Its law of motion is bounded between an upper and a lower "reflecting barrier":

$$1 \leq \nu_t \leq 1 + \lambda, \quad \forall t. \tag{20}$$

   The associated "singular controls" $\left\{d\delta_t^b, d\pi_t^b\right\}$, *which regulate banks' dividends payout and recapitalization strategies, satisfy:*

   (a) $d\delta_t^b > 0 \Longleftrightarrow \nu_t = 1; \quad d\delta_t^b = 0 \quad$ *else.*

   (b) $d\pi_t^b > 0 \Longleftrightarrow \nu_t = 1 + \lambda; \quad d\pi_t^b = 0 \quad$ *else.*

2. *Banks' portfolio choices* $\left\{d_t^b, k_t^b\right\}$ *satisfy the following asset pricing inequality:*

$$\mu_t - r_t \leq -\frac{1}{dt}\mathbb{C}ov_t\left(\frac{d\nu_t}{\nu_t}, dR_t\right). \tag{21}$$

conditions (other than the banks' own equity endowment) affect their market value. According to Phelan (2016); Klimenko et al. (2016) ν_t can be interpreted as the *market-to-book value* of bank b so that e_t^b represents the book value of bank b equity. However, this definition may be misleading because, within the framework of the model, banks' equity is valued mark-to-market rather than at book values.

When (21) holds with equality, then the diffusion of banks' equity marginal value (market price) (18) equals their Sharpe ratio:

$$\sigma_t^\nu = \frac{\mu_t - r_t}{\sigma_t} := SR_t^b. \tag{22}$$

Proof. See Appendix A.5. □

The first implication of Proposition 2 concerns the relationship between the market price of banks' equity ν_t and their dynamic capital structure. Such a relationship is characterized as a map from the boundary values (or "reflecting barriers") of the guessed process (18) to banks' dividends payout and recapitalization strategies (or "singular controls"). Banks are willing to pay dividends at a rate $d\delta_t^b > 0$ when the marginal value of their equity ν_t equals that of their capital payouts. The state $\nu_t = 1$ classifies the upper reflecting barrier because banks might always pay out the full value of their equity instantaneously, guaranteeing a net value of at least e_t^b. Symmetrically, banks issue equity at a rate $d\pi_t^b > 0$ when their market price equals the cost of their recapitalization. The state $\nu_t = 1 + \lambda$ classifies the lower reflecting barrier. In all "interior" states, instead, in which ν_t takes values between 1 and $1 + \lambda$, banks neither pay dividends nor issue equity. Conversely, they are willing to gather additional resources other than their outstanding equity endowment either by retaining dividends or by issuing StL. Coherently, the drift of their equity market price dynamics μ_t^ν equals the marginal utility of their liquidity supply d_t^b (19).

What is relevant to stress is that banks' recapitalization takes place at $e_t^b \to 0$, when their outstanding equity buffers are not sufficient to remunerate their StL (on this point, see also Section 2.3). When that happens, banks' shareholders "withdraw" the counter-value $d\Pi_t^b$ from their risky stakes in firms and instantaneously re-finance banks' equity to keep them solvent. This mechanism guarantees the absolute safety of banks' StL (as for example in Stein, 2012).

The second important result summarized in Proposition 2 concerns the relationship between banks' portfolio choices (leverage), their precautionary motif, and asset prices. When allocating their assets, banks price risk by looking at the covariance between the marginal value of their equity and the return on their risky stakes in firms. When (21) holds with equality, banks are indifferent between holding risky claims and issuing StL. Else, (21) holds slack and banks are unwilling to hold risky stakes in firms ($k_t^b = 0$ and $d_t^b = e_t^b$). However, this case never takes place in equilibrium. It follows that banks' leverage is always positive ($k_t^b > 0$ and $d_t^b < 0$) and the diffusion term of their equity market price's dynamics σ_t^ν equals their Sharpe ratio (22).[13] In this sense, risk neutral banks act *as if* they were risk averse and the risk premiums relate to their precautionary motif through the relative size of their equity buffers (on this point, see Brunnermeier and Sannikov, 2014).

As we shall see in our numerical results, when banks' equity buffers are large enough, then households utterly mandate their risky investments to them; else, they directly allocate a fraction of their net worth in firms and pay the associated monitoring cost. In equilibrium, this fundamentally affects the marginal productivity of capital (its price) and, by a pecuniary externality, firms' investment decisions through the Tobin's q relationship (3).

The state variable We now have all ingredients to formally define the state variable of the economy, that is, the banking sector's relative capitalization (henceforth, "relative size"):

$$\psi := \frac{E_t^b}{E_t^b + E_t^h}, \tag{23}$$

in which $E_t^b = \int_{\mathbb{B}} e_t^b db$ and $E_t^h = \int_{\mathbb{H}} e_t^h dh$ denote bank' and households' aggregate net worths. From now on, we restrict our search to the class of equilibria that are Markov in ψ, and consistently drop all time subscripts t. All equilibrium aggregates can be expressed as functions of ψ, which evolves as a regulated Itô's process. The result is summarized in the following.

Proposition 3. ***State variable dynamics***

1. *The state variable (23) has low of motion:*

$$d\psi = \psi(\mu^\psi(\psi)dt + \sigma^\psi(\psi)dW + d\Xi), \tag{24}$$

[13] In the spirit of He and Krishnamurthy (2013), this is an intermediary asset pricing model because bank capital has a central role in determining the price of real and financial assets. Empirical evidence that the marginal value of the financial sector wealth provides relevant information for asset pricing is in Adrian et al. (2014).

whose dynamics is regulated within the interval $\Psi := (0, \bar{\psi} \leq 1]$ *by the singular control* $d\Xi$ *(24). Moreover,*

$$\mu^{\psi}(\psi) = \frac{1}{\theta}\left(\frac{1+\theta A}{q(\psi)} - 1\right) + [\sigma + \sigma^{q}(\psi)]\left\{\left[\omega^{b}(\psi) - 1\right]\sigma^{\nu}(\psi) + \sigma + \sigma^{q}(\psi)\right\}; \qquad (25)$$

$$\sigma^{\psi}(\psi) = \left[\omega^{b}(\psi) - 1\right][\sigma + \sigma^{q}(\psi)]; \qquad (26)$$

$$\omega^{b}(\psi) := \frac{q(\psi)K^{b}}{E^{b}};$$

$$d\Xi = \int_{\mathbb{B}} \left[d\delta^{b} - d\pi^{b}\right] db. \qquad (27)$$

2. *The upper and lower boundaries of* Ψ *(reflecting barriers) are characterized by the couple:*

$$\lim_{\psi \to 0} \nu(\psi) = 1 + \lambda; \quad \nu(\bar{\psi}) = 1 \ .$$

Proof. See Appendix A.6 □

As long as the banking sector's relative size ψ fluctuates in the interior of Ψ, then ψ evolves as an Itô's process with state-dependent drift $\psi\mu^{\psi}(\psi)$ and diffusion $\psi\sigma^{\psi}(\psi)$. In those states, neither dividends nor equity issuances take place, the control term $d\Xi$ equals zero (see also Proposition 2, point 1), and banks build capital buffers by retaining dividends. Conversely, when ψ reaches either of its reflecting barriers, $d\Xi$ acts as an impulse that adjusts to motion of (24) by creating a regulated diffusion. The adjustment takes place either when banks pay out dividends or when they issue equity, but not contemporaneously (27).[14] The boundaries of Ψ, where the adjustments take place, relate to those of $\nu(\psi)$ and are uniquely determined by the cost of new equity issuance λ.

Note that the choice of (23) as a state variable is appropriate because there is no idiosyncratic risk affecting individual banks' net worth e^{b}, of which banks' strategies are linear functions. However, due to their homogeneous exposure to systematic risk, banks' dividends and recapitalization decisions are always synchronous. As a result, the market price of their

[14]As long as $\lambda \geq 0$, banks never pay dividends and issue equity at the same time. In the limit case where $\lambda = 0$ there is no friction over banks' capital flows, that is, the FOCs for $d\delta^{b}$ and $d\pi^{b}$ are such that $d\Delta > 0, d\Pi > 0 \iff \nu_t = 1, \forall t$. In such a case, banks pay dividends and issue equity to keep ψ at the level in which the marginal value holds equal to 1. We briefly discuss this benchmark case in Appendix A.7.

equity $\nu(\psi)$ is a non-linear function of the *aggregate* banking sector's relative size ψ. As we will see next, the fact that banks do not internalize the effect of their individual strategies on the dynamics of ψ leads to socially inefficient recapitalization decisions and, consequently, capital allocations.

3.2 Numerical solution and discussion

This section discusses the model's numerical solution in the baseline case featuring no bailouts ($dT^{\pi} = 0$).[15] In doing so, it highlights the pecuniary externalities of the banking sector's relative size and their effect on the macroeconomic dynamics.

For sake of analytical tractability, we assume the following functional forms for firms' investment technology (2) and households' liquidity demand (8):

$$\Phi(\iota) = \ln(1 + \theta\iota)^{\frac{1}{\theta}};\ \Gamma(d) = \Gamma d,$$

in which θ parametrizes the degree of capital's technological illiquidity, and Γ parametrizes households' marginal utility for liquidity. From now on, unless specified otherwise, we consider the following parametric values: $A = 0.4$, $\eta = 0.1$, $\sigma = 0.25$, $\theta = 2$, $\lambda = 0.15$, $\rho = 0.05$, $\delta = 0.25$, and $\Gamma = 0.015$, which imply a dividends payout threshold $\bar{\psi} \approx 0.29$.[16]

Pecuniary externalities: banks' leverage capacity and firms' investments The top panels of Figure 3 report the equilibrium price of capital, firms' investment rate, and the market price of banks' equity as functions of the banking sector's relative size ψ. The bottom panels of the same figure display households' and banks' Sharpe ratios, and banks' leverage. The dotted red lines highlight the upper dividends payout threshold $\bar{\psi}$; the green

[15] The solution method is similar to Brunnermeier and Sannikov (2014); all details are in Appendix A.8.

[16] While these choices are not the result of calibration, they produce reasonable qualitative results. From the analysis of the equilibrium's outcomes it is straightforward that the model dynamics strictly relates to the marginal value of banks' equity, and so on the level of the financial friction that determines the cost of their equity issuance at the lower threshold. As such, further questions may arise: how does the upper threshold change with respect to the main parametric values? How does that relate to ν? To answer these questions, a comparative statics analysis can be found in Appendix A.9. Note that the lower threshold at which banks issue new equity always equals zero as long as there is no delay in collecting capital from the households and the risk sources within the economy are of the diffusion type, i.e. shocks take place continuously. Thus, the only degree of freedom to define the equilibrium state-space is summarized by $\bar{\psi}$.

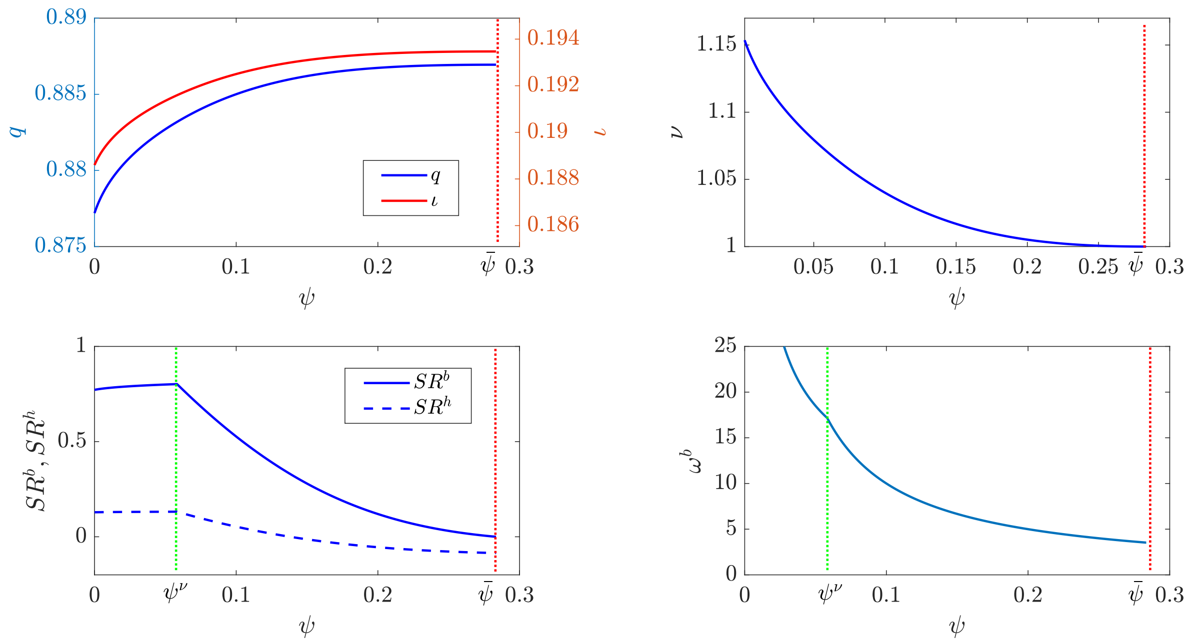

Figure 3: Top: capital price (blue, left), investment rate (red, left), and banks' equity market price (blue, right) as functions of the banking sector's relative size ψ. Bottom left: banks' (solid) and households' (dashed, left) Sharpe ratios. Bottom right: banks' leverage. The dotted lines depict banks' dividends payout $\bar{\psi}$ (red) and capital allocation ψ^ν (green) thresholds, respectively.

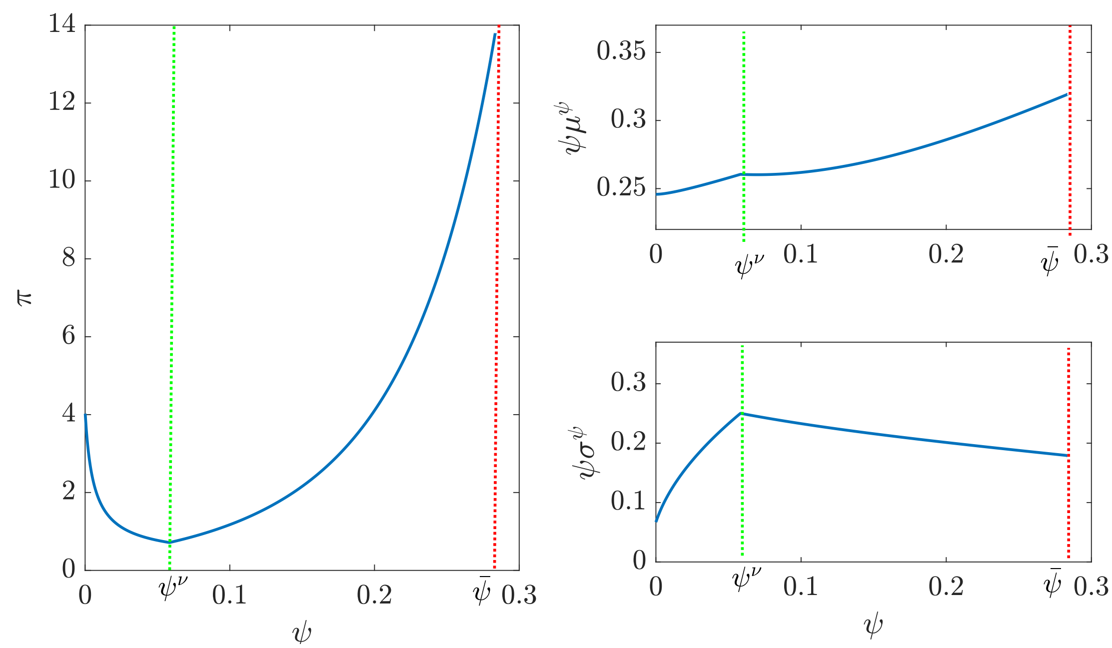

Figure 4: Stationary density (left), state drift (top, right), and diffusion (bottom, right) as functions of the banking sector's relative size ψ. The dotted lines represent the banks' dividends payout $\bar{\psi}$ (red) and capital allocation ψ^ν (green) thresholds, respectively.

dotted lines pin down the "capital allocation threshold" ψ^{ν} above which investment in firms is utterly bank-intermediated.

The first pecuniary externality in the model occurs through the market price of banks' equity $\nu(\psi)$ that, in equilibrium, is a decreasing and convex function of ψ. This happens because, due to the presence of recapitalization costs, risk neutral banks act *as if* they were risk averse (Brunnermeier and Sannikov, 2014). As a consequence, they constitute precautionary capital buffers against negative systematic shocks. By doing so, they moderate the magnitude of the co-variations between the market price of their equity and the return on their risky stakes in firms (see Proposition 2). The externality takes place because banks' equity buffers depend on the level and motion of $\nu(\psi)$, but they do not internalize the effect of their individual (but synchronous) recapitalization strategies on the dynamics of ψ and, in turn, on how that affects their own leverage capacity through $\nu(\psi)$.

The dynamics of the mechanism behind this externality can be better understood by looking at the bottom panels of Figure 3 in which, due to banks' precautionary motif, their leverage and Sharpe ratio feature a kink at the capital allocation threshold ψ^{ν}. The kink separates the constrained from the unconstrained regions of the Markov equilibrium. To the right-hand side of the kink, banks' equity is relatively cheap and they hold sufficiently high buffers to withstand households' liquidity demand. In this region, banks issue StL inelastically (they are unconstrained) and households utterly mandate to banks the risky investments in firms ($k^h = 0$ and $\omega^b = 1/\psi$). To the left-hand side of the kink, instead, banks' equity is relatively costly and their leverage capacity is bounded by their precautionary motif. Due to the pecuniary externality of their synchronous recapitalization strategies, banks do not internalize how their individual choices influence the economy's transition dynamics from the constrained to the unconstrained region, and vice versa.

The connection between banks' equity buffers and the allocation of capital between households and banks helps explain the second pecuniary externality in the model. The externality occurs through the effect of the capital price level $q(\psi)$ on firms' investment rate $\iota(q(\psi))$ that, in equilibrium, are both increasing and concave functions of ψ (Figure 3, top left panel). The underlying mechanism reads as follows.

When banks are constrained to the right-hand side of the kink, households are compelled to directly allocate a fraction of their net worth in firms' risky claims ($k^h > 0$ and $k^b > 0$ but $\omega^b < 1/\psi$). By doing so, they also pay the associated monitoring cost (ηk^h), therefore reducing the overall capital productivity and thus its equilibrium price. In turn, this negatively affects the investment rate $\iota(q(\psi))$ through the Tobin's q relationship (3).[17] Conversely, when banks reach the unconstrained region to the right-hand side of the kink, they are able to utterly intermediate investments in firms' risky claims on households' behalf. By doing so, due to their cost-efficient monitoring technology, they increase the overall capital productivity (price) and, in turn, firms' investment rate $\iota(q(\psi))$. The pecuniary externality takes place because banks do not internalize how their individual recapitalization strategies affect the aggregate allocation of capital between households and banks through the dynamics of ψ.

In summary, the level of the banking sector's relative size ψ regulates, through the market price of bank equity $\nu(\psi)$, individual banks' leverage capacity. In turn, banks' leverage determines the allocation of capital between more and less productive agents, namely, the fraction of firms' capital that is directly provided by households or intermediated by banks. Consecutively, the allocation of capital drives the motion of ψ while affecting the price level $q(\psi)$, and therefore firms' investment decisions.

When considered jointly, the two externalities suggest that there may exist a trade-off between the dis-utility of decreasing $\nu(\psi)$ versus the benefits of increasing $q(\psi)$ (and therefore $\iota(q(\psi))$). Accordingly, there may be room for the government to improve households' welfare by implementing a reallocation policy (bailout) that recapitalizes the banking sector in bad states, in which its leverage capacity is bounded and capital is allocated inefficiently due to banks' precautionary motif.

Stationary density Another feature of the equilibrium that is relevant to discuss relates to long-run transition dynamics of the state variable ψ. To do so, Figure 4 plots the stationary density $\pi(\psi)$ jointly with the drift and diffusion of the banking sector's relative size as functions of the state ψ (details of the derivation of the stationary density of an Itô's process can be found in Risken, 1996).

The state dynamics always drifts towards the upper boundary of Ψ, at which the banks'

[17] Consistently, the banks' SR is higher for lower ψ, but decreasing in banking sector relative size. Moreover, banks' SR is always higher than that of households $SR^b > SR^h$ (see Figure 3, dashed line).

dividend payouts are positive and their leverage capacity is unconstrained. Accordingly, the economy spends most of the time in states where the banks' equity buffers are sufficiently high and thus capital is allocated efficiently. However, the state's stationary density exhibits the so-called "stationary instability" described in Brunnermeier and Sannikov (2014). In other words, when ψ fluctuates in the neighbourhood of the capital allocation threshold ψ^{ν}, then the economy may shift abruptly to persistently "bad" regimes featuring low investments and high risk premiums (as well as limited liquidity supply).[18] In bad states, the allocation of capital is suboptimal due to banks' precautionary motif, and banks are endogenously hindered on their path towards recovery. For this reason, a stream of adverse shocks can dampen the system in the neighbourhood of $\psi \to 0$, in which frequent recapitalization may be required, for arbitrarily long periods. In this respect, a bailout recapitalization regime may shorten the transition through bad states, foster firms' investment, and thus benefit households' welfare in the long run.

4 Welfare and bailout

So far we have exclusively considered the baseline case in which banks' recapitalization is given by their individual strategies. In this respect, further questions arise spontaneously: are individual recapitalization policies also optimal from the social standpoint? And if not, is there any scope for bailouts to be beneficial?

We answer these questions by considering the possibility of a tax-financed recapitalization regime that complements banks' individual strategies (bailout). This regime is enforced by the government, who imposes households with lump sum taxes and evenly redistributes its revenues across banks net of administrative costs. In this respect, the government recapitalization can be interpreted as a systematic bailout across banks. The aim of bailouts is to maximize long-run (or "ex-ante") households' welfare.

The remainder of this section is organized as follows. First, we define households' short- and long-run welfare. Then, we describe the bailout regime, outline the assumption involved

[18] It is relevant to highlight that the instability in the dynamics of ψ is generated by the highly non-linear behaviour of the diffusion term $\psi\sigma^{\psi}$ that is maximal approaching the region where banks' precautionary motif constraints are binding.

in its implementation, and discuss why this policy can be socially optimal in the model. By doing so, we characterize the trade-off that is entailed in bailout recapitalization decisions as a dynamic complementarity between investing in banks' equity at the moment of distress (short-run costs) and its positive externality in terms of firms' investments, banks' valuation, and the economic transitional dynamics in subsequent states (long-run benefit).

4.1 Short- and long-run Welfare

Definition 1. ***Households' short-run welfare***

Households' short-run (or "ex-post") welfare is defined as households' value (8) conditional on being in state ψ_0. Formally, it equals to:

$$H(\psi_0, K_0) = K_0 h(\psi_0, 1) = \mathbb{E}_0 \int_0^\infty e^{-\rho t} K_t \Theta(\psi_t) dt, \tag{28}$$

in which

$$\Theta(\psi) := \underbrace{[A - \iota(\psi)]}_{\text{Consumption, } \Theta^c} - \underbrace{\left[\omega^b(\psi)\psi - 1\right]\eta}_{\text{Monitoring, } \Theta^m} + \underbrace{\mathit{\Gamma}\psi q(\psi)\left[\omega^b(\psi) - 1\right]}_{\text{Liquidity, } \Theta^l}; \tag{29}$$

$$h(\psi, 1) = q(\psi)\left[(1-\psi) + \nu(\psi)\psi\right], \tag{30}$$

and K_t and ψ_t evolve with dynamics (2) and (24), respectively. The details of the analytical derivation can be found in Appendix A.10.

Equation (28) can be interpreted as households' welfare "ex-post", that is, conditional on the realization of state ψ_0. Intuitively, it can be decomposed into three components: direct consumption, monitoring, and liquidity values. Details of each component can be found in Appendix A.10.

To evaluate the long-run benefit of bank bailouts, as well as the associated dynamic trade-off, we shall not exclusively rely on the "raw" short-run measure (28). This is because ex-post welfare is intrinsically static, and does not fully capture the transition across states that is embodied in the dynamics of ψ. To by-pass this problem, we can adopt a measure of households' welfare that is "ex-ante", i.e., non contingent on the state ψ_0. This is suitable because ex-ante welfare captures the full macroeconomic dynamics when $t \to \infty$, and ψ

visits every state in Ψ with density $\pi(\psi)$. Formally, households' welfare in the long run is defined in the following.[19]

Definition 2. ***Households' long-run welfare***
Households' long-run welfare W is defined as the short-run welfare (28) integrated over Ψ and weighted by the stationary density $\pi(\psi)$:

$$W \propto \int_{\Psi} h(\psi)\pi(\psi)d\psi. \tag{31}$$

4.2 Bailout

How do bailouts affect short- and long-run welfare? To answer this question, we include in the baseline model discussed in Section 3.2 the bailout policy introduced in Section 2.2.

The government implements (and commits to) the bailout policy by taxing and redistributing resources dT^{π} from households' to banks' net worth. It does so conditional on the banking sector's relative capitalization ψ reaching the exogenous recapitalization threshold ψ^{Ω}. For sake of analytical tractability, we approximate the instantaneous process dT^{π}_t with a linear and deterministic function of households' holding in banks' StL and risky stakes in firms E^h (7):

$$dT^{\pi} \coloneqq T^{\pi}E^{h}dt, \tag{32}$$

in which T^{π} denotes the instantaneous tax rate on households' net worth. The government chooses the level of T^{π} to maximize long-run households' welfare (31). Its optimal policy is summarized in the following.

Definition 3. ***Socially optimal tax rate - bailout***
Given the approximation (32), conditional on an exogenous recapitalization threshold ψ^{Ω}, the government implements a bailout policy by raising taxes on households' net worth. It does so by choosing the tax rate $T^{\pi,*}$ that maximizes households' long-run welfare (31):

$$T^{\pi,*} = \arg\max_{T^{\pi}} W. \tag{33}$$

[19]Note that households' value function is homogeneous of degree one in the aggregate capital stock K. For this reason, we henceforth normalize it for $K = 1$.

Once the possibility of government bailout is accounted for, it is possible to show that the dynamics of banks' relative capitalization, denoted $d\psi^{\Omega}$, evolves as:

$$\frac{d\psi^{\Omega}}{\psi^{\Omega}} = \frac{d\psi}{\psi} + \underbrace{\mathbb{I}_{\psi \leq \psi^{\Omega}} T^{\pi} (1-\psi) \left(\frac{1+\psi\lambda^{G}}{\psi} \right)}_{\text{Bailout term}} dt, \tag{34}$$

in which $\mathbb{I}$ denotes the indicator function that takes value one when ψ is at or below the recapitalization threshold ψ^{Ω}, zero otherwise. The details of the analytical derivation can be found in Appendix A.11).

To evaluate the effects of bailouts on the macroeconomic dynamics and households' welfare, we now solve for the economy's competitive equilibrium by considering the state dynamics given by (34) in place of (24). For our numerical example, we consider the following parametric values: $\psi^{\Omega} = 0.01$, $T^{\pi} = 0.1$.

Bailout, banks' equity, and capital price The top panels of Figure 5 plot the market prices of capital $q(\psi)$ and banks' equity $\nu(\psi)$ before (blue, solid) and after (green, dashed) considering the bailout policy. The bottom panels of the same figure display households' short-run welfare $h(\psi)$ and the stationary density $\pi(\psi)$. To complete the picture, Figure 6 shows banks' leverage and Sharpe ratio. All the red dotted lines highlight the dividends payout threshold $\bar{\psi}$.

What stands out is that, on the one hand, government bailouts increase the price of capital $q(\psi)$ for all $\psi \in \Psi$. This is because, when banks internalize the possibility of bailouts, they respond by increasing their leverage. They do so to extract value, in expectation, from the tax-financed injection of new equity at the moment of bailout. When that happens, capital prices react positively because a higher share of capital is bank intermediated and lower monitoring costs are paid. A higher price of capital fosters firms' investment rate and thus channels higher economic growth (see Equation 2).[20] On the other hand, however, government bailouts reduce the marginal value of banks' equity $\nu(\psi)$ for all $\psi \in \Psi$. They do so especially in those states in which the banking sector's aggregate size $\psi \to 0$. The

[20]Remember that bank intermediated investments are more productive due to their cost advantage at monitoring firms, while the re-investment rate ι relates to capital prices by the Tobin's q relationship (3).

reason is that households take into account that the government may impose on them to recapitalize the banking sector, and that a share of the tax revenues would be depleted in administrative costs. Moreover, lower levels of ψ associate to higher bank leverage and thus risk, conditional on being in bad states, which the market prices consistently with banks' precautionary motif by setting a higher Sharpe ratio.

What is relevant to stress is that the negative relationship between bailouts and the marginal value of banks' equity does not trivially apply to their overall market valuation. In fact, while decreasing the price of banks' equity, bailouts also increase the "book" value of their equity (or net worth) e^b through their pecuniary externality on $q(\psi)$. Moreover, bailouts also have a "dynamic" effect by increasing the average banking sector's relative size in the long run.

The "static" component of the trade-off between the variation of $q(\psi)$ and that of $\nu(\psi)$, that is, conditional on the level of ψ, can be evaluated by looking at households' short-run welfare reported in the bottom panel of Figure 5 (on this point, see also Equation 30). By doing so we find that, all in all, bailouts generate a short-run loss because the negative effect $\nu(\psi)$ overtakes the benefit in terms of $q(\psi)$. However, if we look at their effects over the stationary density, that is, in the long run, we notice that bailouts fundamentally increase the likelihood of being in "good" states while reducing that of transitioning through "bad" ones. In this perspective, there exists a "dynamic" trade-off between the short-run costs of recapitalization and the long-run benefits of a more efficient capital allocation.

The trade-off as a dynamic complementarity The dynamic trade-off that associates to the bailout recapitalization boils down to that between the cost of its pecuniary externalities in the short run and their beneficial effects on the long-run macroeconomic dynamics. The mechanism through which the trade-off relates to households' welfare can be understood by re-writing (31) as the sum of the (expected) banking sector's market value plus the expected value of households' "bail-out-able" net worth:

$$W \propto \underbrace{\mathbb{E}^{\pi}\left[q(\psi)\nu(\psi)\psi\right]}_{\text{Bank' value}} + \underbrace{\mathbb{E}^{\pi}\left[q(\psi)(1-\psi)\right]}_{\text{``Bail-out-able'' net worth}}, \tag{35}$$

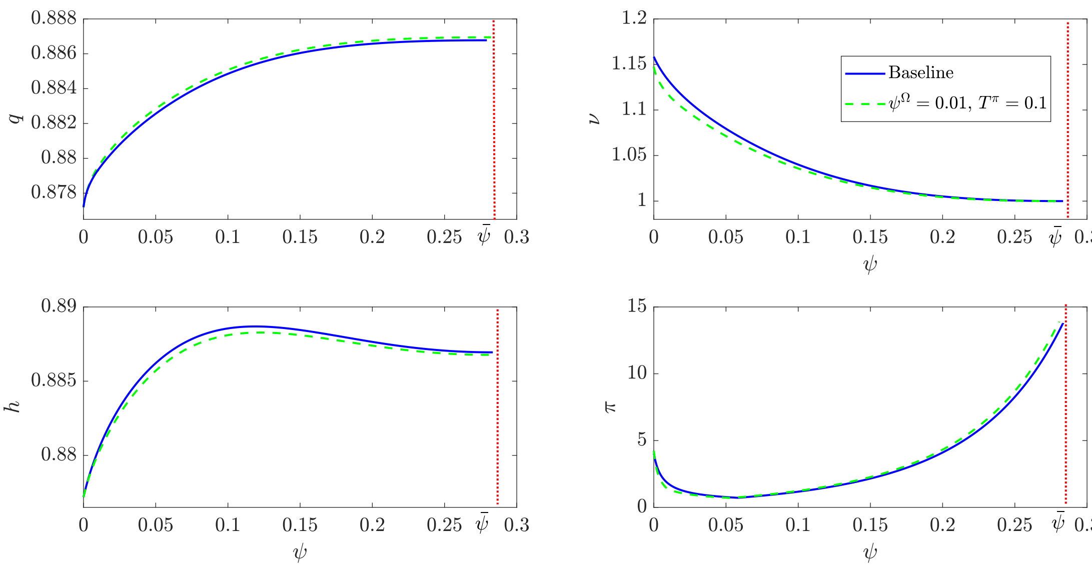

Figure 5: Top: capital price (left) and banks' equity market price (right) before (blue, solid) and after (green, dashed) considering the bailout policy. Bottom: households' short-run welfare (left) and equilibrium stationary density $\pi(\psi)$ before (blue, solid) and after (green, dashed) the bailout implementation. The red dotted lines in all panels represent banks' dividends payout threshold $\bar{\psi}$.

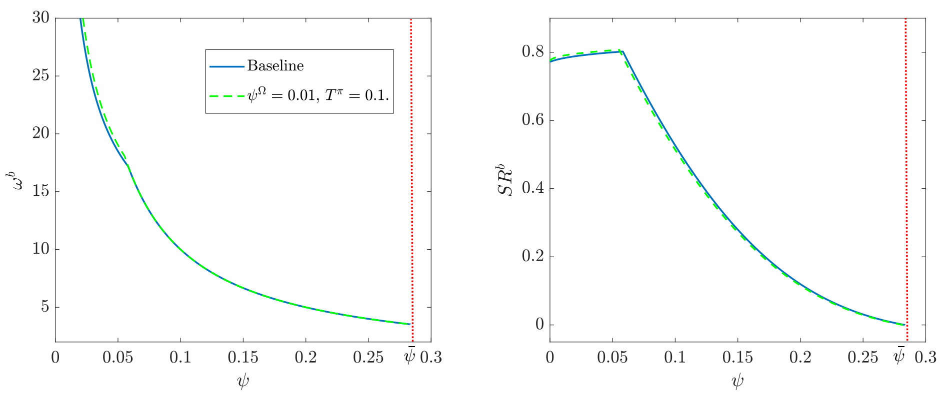

Figure 6: Banks' leverage (left) and Sharpe ratio (right) before (blue, solid) and after (green, dashed) the bailout implementation. The red dotted lines in all panels represent the dividends payout threshold $\bar{\psi}$.

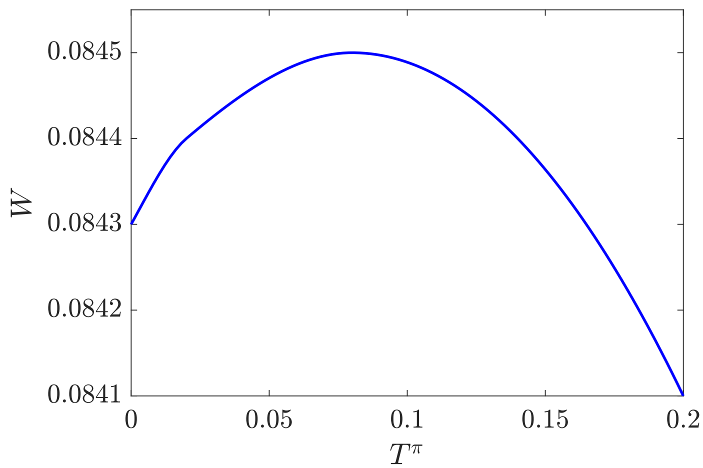

Figure 7: Households' long-run welfare as a function of the tax rate T^π.

in which $\mathbb{E}^\pi$ denotes the expected value taken with respect to the stationary density $\pi(\psi)$. By taking the partial derivative of (35) with respect to the tax rate T^π, the following first order optimality condition emerges:

$$\partial_{T^\pi} W = 0 \Rightarrow \underbrace{\partial_{T^\pi} \mathbb{E}^\pi \left[q(\psi)\psi\nu(\psi) \right]}_{\text{Expected bank value gain}} = \underbrace{-\partial_{T^\pi} \mathbb{E}^\pi \left[q(\psi)(1-\psi) \right]}_{\text{Expected ``Bail-out-able'' net worth loss}} . \tag{36}$$

The meaning of Equation (36) is that, at the optimum, the expected marginal loss in households' "bail-out-able" net worth due to the bailout must be compensated by the expected marginal gain in terms of the banking sector's market valuation. In other words, the (expected) cost of the bailouts shall be compensated by an increment in the (expected) discounted value of banks' future dividends. In this respect, the intertemporal trade-off between bailout costs and its benefits can be characterized by the dynamic complementarity that exists between "investing" capital in bank bailouts in bad states and the consequential increments in firms' investments productivity, and therefore banks' valuation, at consequent stages. Accordingly, there is room for welfare-improving bailouts as long as the "value effect" (Cordella and Levy-Yeyati, 2003) of reallocating resources to the banking sector dominates.

In conclusion, to visually evaluate the dynamic trade-off, Figure 7 plots households' long-run welfare as a function of the tax rate T^π for our baseline parametric specification. Overall, our numerical results suggest that, conditional to a relatively low default threshold

ψ^{Ω}, there exists a tax rate $T^{\pi,*} \geq 0 (\approx 0.08)$ that maximizes households' long-run welfare. Note that, due to the extremely stylized nature of our model, its quantitative implications are not to be taken by the book. However, going beyond the figures, our findings provide a few pertinent intuitions concerning the mechanism that interlinks bank resolution regimes to their aggregate outcomes in a dynamic environment.

5 Conclusions

Are individual banks' recapitalization decisions socially optimal? If not, can bank bailouts be beneficial? How do they relate to the macroeconomic dynamics and social welfare in the long run? We have investigated these questions by means of a suitable dynamic model of a productive economy with a banking sector and financial frictions.

Our findings suggest that systematic bank bailouts can be welfare-enhancing in the long run by improving the allocation of capital in bad states, in which the overall banking sector's leverage capacity is restricted by banks' precautionary motif, and the economy is persistently constrained in a low-growth regime. This happens because, in equilibrium, perfectly competitive agents do not internalize the pecuniary externalities of their individual (but synchronous) recapitalization strategies for the aggregate banking sector's leverage capacity, firms' investment decisions and, in turn, economic growth.

Our analysis complements the theoretical banking literature by showing that implementing a bailout regime entails an intertemporal trade-off between its short-run costs (taxes) and its long-run benefits in accelerating (and stabilizing) the transition from bad to good states. Moreover, we characterize the trade-off through the dynamic complementarity that takes place between "investing" in banks' equity (bailing them out) in distressed times and the increment in their investments' productivity (and thus long-run valuation) at subsequent stages.

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A Appendix - Supplementary material

A.1 Banks, financial intermediaries, and IC constraints

This Appendix outlines a generalization of the baseline model described in Section 2 that accounts for the Limited Enforcement Problem (LCP) that may arise between households and banks when the latter are managed by some intermediaries.

Let us assume that banks are owned by households but managed by financial intermediaries. The latter may finance banks assets either via their equity or by issuing risk-free short-term liabilities. Similar to the limited enforcement problem outlined in Bernanke et al. (1999) (see also Van Der Ghote, 2020, for a similar setting in continuous-time), immediately after raising StL intermediaries can choose to liquidate banks total assets net of an exogenous haircut $\chi \in (0,1)$ that constitutes an additional financial friction in the economy.

Provided that the remainder of the liquidated assets $(1-\chi)$ is not sufficient to redeem their principal in full, intermediaries would have the incentive to default on their short-term liabilities. In order to prevent them for exploiting the diversion strategy, it must hold that the value of the bank under their management holds greater than (or equal to) the haircut value of the liquidated assets. Note that, for the limited enforcement friction to be meaningful, one must assume that each intermediary is owned by a single household and that she borrows only from households other than their direct owner (on this point, see also Maggiori, 2017).

Formally, let J_t^b denote the value of a bank with equity book value e_t^b at time t and ω_t^b be her leverage. Under the optimal policy, bank's total assets a_t^b can be expressed as $a_t^b = \omega_t^b e_t^b$, and the IC constraint holds as:

$$a_t^b(1-\chi) \leq J_t^b, \tag{37}$$

for each $t \in [0, \infty)$. Note that, when banks' value J_t^b can be expressed as a linear function of the book value of her equity, that is, $J_t^b = \nu_t^b e_t^b$ (as we shall see, this is going to be the case of this paper, where ν_t represents the marginal value of banks' equity), then the IC constraint in Equation (37) implies an endogenous constraint over banks' leverage:

$$\omega_t^b \leq \frac{\nu_t}{1-\chi},$$

that is, it must hold smaller than or equal to a multiple $\frac{1}{1-\chi}$ of the marginal value of their equity.

A.2 Banks' efficiency edge

Similar to Van Der Ghote (2020), this Appendix provides a rationalization of banks' cost advantage at monitoring firms that generates the efficiency edge $\eta = \eta^h - \eta^b$.

Let the productivity rate across firms be idiosyncratic and stochastic. In particular, it may take high and low values A^h and A^l with probabilities p and $1-p$, respectively. By exerting costly effort ε per unit of rent capital, firms can increase the probability p to $p^\varepsilon > p > 0$. Then, as long as

$$A^h p^\epsilon + A^l\left(1-p^\epsilon\right) - \varepsilon \geq A^h p + A^l\left(1-p\right)$$

there exists a moral hazard problem between productive firms and their shareholders (households or banks). The problem can be tackled either directly, via costly monitoring, or indirectly implementing an optimal contract.

(Direct) Costly monitoring Costly monitoring is so that households and banks pay a fixed cost (η^h and η^b, respectively) for each unit of capital supplied, that is, out of firms' dividends. The monitoring activity is assumed to be efficient and prevents firms from not exerting effort.

(Indirect) Optimal contract Alternatively, households and bank may decide to write an incentive compatible contract conditional on the productivity realization. The contract is so that the shareholders commit to pay a premium x to the producers in order to remunerate their effort. Without loss of generality, by setting $A^l = 0$, the premium x must be so that

$$xp^\varepsilon - \varepsilon = xp \Rightarrow x = \frac{\varepsilon}{p^\varepsilon - p}.$$

We assume that $\eta^h < \frac{\varepsilon}{p^\varepsilon - p}$, so that households (and banks, because $\eta^h > \eta^b$) always decide to pay the monitoring cost rather than adopting the incentive compatible contract.

A.3 Bank recapitalization - optimal stopping time

Let us fix an initial condition on bank b's book value $e^b = e_0$. Henceforth, we omit redundant sup and subscripts for sake of clear notation. Also, let τ be the first time that the bank's equity reaches $(-\infty, 0)$, that is, the bank decides not to issue equity and goes bankrupt. Moreover, let J be the solution of (10) with complementary condition (12).

By definition, the bank's continuation value satisfies

$$J(e_0) = \int_0^{\tau \wedge t} e^{-\rho t} \left[(d\Delta_t - (1+\lambda) d\Pi_t \right] + \underbrace{e^{-\rho(t\wedge\tau)} J(e_{t\wedge\tau})}_{\text{Continuation}}. \tag{38}$$

By taking the differential $d(J(e)e^{-\rho t})$, applying Itô's lemma, and integrating over $(0, t\wedge\tau)$, the following relationship between the dynamics of bank's equity (book value) and continuation value holds:

$$e^{-\rho(t\wedge\tau)} J(e_{t\wedge\tau}) = J(e_0) + \int_0^{t\wedge\tau} e^{-\rho s} \left[-\rho J(e_s) + \mu^e e \partial_e J(e_s) + \frac{1}{2} (e\sigma^e)^2 \partial^2_{ee} J(e_s) \right] ds + \\ + \int_0^{t\wedge\tau} e^{-\rho s} e\sigma^e \partial_e J(e_s) dW_t - \int_0^{t\wedge\tau} \partial_e J(e_s) e^{-\rho s} d\Delta_s + \int_0^{t\wedge\tau} \partial_e J(e_s) e^{-\rho s} d\Pi_s, \tag{39}$$

in which μ^e and σ^e denote the drift and diffusion of the stochastic process that describes the dynamics of bank's b equity $d(e)/e$.

By matching (38) and (39) we obtain the following expression that relates bank's market value J to the dynamics of her book value e:

$$\int_0^{\tau\wedge t} e^{-\rho t}\left[(d\Delta_t-(1+\lambda)d\Pi_t\right]dt \leq$$

$$-e^{-\rho(t\wedge\tau)}J(e_{t\wedge\tau})+\int_0^{\tau\wedge t} e^{-\rho t}\left[\underbrace{1-\partial_e J(e_s)}_{\leq 0}\right]d\Delta_t+$$

$$+\int_0^{\tau\wedge t} e^{-\rho t}\left[\underbrace{\partial_e J(e_s)-(1+\lambda)}_{\leq 0}\right]d\Pi_t+J(e_0)+$$

$$+\int_0^{t\wedge\tau} e^{-\rho s}\left[\underbrace{-\rho J(e_s)+\mu^e e\partial_e J(e_s)+\frac{1}{2}\left(e\sigma^e\right)^2\partial_{ee}^2 J(e_s)}_{=0}\right]ds+$$

$$+\int_0^{t\wedge\tau} e^{-\rho s}e\sigma^e\partial_e J(e_s)dW_t. \quad (40)$$

The third term of the right-hand side of (40) is the bank's HJBE and, by standard optimal control theory, always holds with equality over the support $[0,e^{\max}]$. Conversely, the second and third term always equal zero when the bank does neither pay dividends nor issue equity ($d\Delta = 0$ and/or $d\Pi = 0$), it is negative otherwise (remind that bank's marginal value is decreasing in her capitalization $\partial_e V(e^{max}) \leq \partial_e J(e) \leq \partial_e V(0)$).

It follows that, when $\lambda < \infty$, then Inequality (40) is well defined, and thus it is convenient for the bank to issue new equity. Her value J is maximal when:

$$\begin{cases} d\Delta(e) > 0 \Longleftrightarrow \partial_e V(e) = 1, & e = e^{max}; \\ d\Pi(e) > 0 \Longleftrightarrow \partial_e V(0) = 1+\lambda, & e = 0, \end{cases}$$

and (40) holds with equality $\forall e \in [0,e^{\max}]$. In such a case, the process de is reflected at 0 and $e^{\max}$, and does not assume values in $(-\infty,0)\cup(e^{max},\infty)$. Accordingly, $\tau=\infty$ and (40) reduces to:

$$\lim_{t\to\infty}\int_0^{\tau\wedge t} e^{-\rho t}\left[(d\Delta_t-(1+\lambda)d\Pi_t\right]dt = -\lim_{t\to\infty} e^{-\rho(t\wedge\tau)}J(e_{t\wedge\tau})+$$

$$+\lim_{t\to\infty}\int_0^{t\wedge\tau} e^{-\rho s}e\sigma^e\partial_e J(e_s)dW_t+V(e_0).$$

By taking expected value at time $t = 0$ and considering the transversality condition

$$\lim_{t\to\infty} \mathbb{E}_0 e^{-\rho(t\wedge\tau)} J(e_{t\wedge\tau}) = 0,$$

it holds that

$$J(e_0) = \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[(d\Delta_t - (1+\lambda)d\Pi_t\right] dt.$$

A.4 Equilibrium

The formal statement of the competitive equilibrium reads as follows:

Definition 1. ***Competitive equilibrium***

Conditional on an initial allocation of capital between banks' equity and households ψ_0, an equilibrium is an adapted stochastic process that maps histories of exogenous systematic shocks $\{dW_t\}$ to prices $\{q_t, \nu_t\}$, return on risky claims $\{dR_t\}$, risk-free interest rate on short-term bank liabilities $\{r_t\}$, production choices $\{K_t, \iota_t\}$, consumption choices $\left\{c_t^h : h \in \mathbb{H}\right\}$, allocations $\{d_t^i, k_t^i : i \in \{\mathbb{H}, \mathbb{B}\}\}$, as well as dividend and recapitalization strategies $\left\{d\delta_t^b, d\pi_t^b : b \in \mathbb{B}\right\}$ so that:

1. *The firms maximise their profits:*

$$\{K_t, \iota_t\} = \arg \max_{\{K_t,\iota_t\}\in T} \left\{\mathbb{E}_t \left[V_s e^{-\int_t^s r_u du}\right] - K_t q_t\right\}, \tag{41}$$

   where V_s are the firms revenues at between t and $s = t + dt$ at time s.

2. *The household $h \in \mathbb{H}$ maximise their utility:*

$$\left\{c_t^h, d_t^h, k_t^h\right\} \in \arg \sup_{\left\{c_t^h, d_t^h, k_t^h\right\}\in G_t^h} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[c_t^h + \mathit{\Gamma}(d_t^h)\right] dt. \tag{42}$$

3. *Banks $b \in \mathbb{B}$ maximise their market value:*

$$\left\{d_t^b, k_t^b, d\delta_t^b, d\pi_t^b\right\} \in \arg \sup_{\left\{d_t^b, k_t^b, d\delta_t^b, d\pi_t^b\right\}\in B_t^b} \mathbb{E}_0 \lim_{t\to\infty} \sup_\tau \int_0^{\tau\wedge t} e^{-\rho t} \left[d\delta_t^b - (1+\lambda)d\pi_t^b\right] dt. \tag{43}$$

4. All markets clear:

(a) Short-term liabilities:

$$\int_{\mathbb{H}} d^h dh + \int_{\mathbb{B}} d^b db = 0; \tag{44}$$

(b) Consumption:

$$\int_{\mathbb{H}} (A - \iota_t - \eta)\, k_t^h dh + \int_{\mathbb{B}} (A - \iota_t)\, k_t^b db = C_t^h; \tag{45}$$

(c) Capital:

$$\int_{\mathbb{H}} q_t k_t^h dh + \int_{\mathbb{B}} q_t k_t^b db = q_t K_t. \tag{46}$$

A.5 Households' and banks' problem

For sake of clear notation, we omit all time subscripts.

Households By standard continuous-time stochastic control methods, households' strategies satisfy the following:

$$0 = \sup_{d^h} \left\{ \mathit{\Gamma}(d^h) + (r - \rho)\, d^h \right\} + \sup_{k^h} \left\{ k^h \left(\frac{1}{dt}\mathbb{E}dR - \rho \right) \right\} + \mathbb{E}\left[dB^b \right] + dT,$$

where $dB_t^b := d\Delta_t^b - (1+\lambda)d\Pi_t^b$ denotes the dynamics of bank b value, dR denotes the return on risky claims in firms, and dT denotes the government tax. The FOCs for capital holdings and banks' short-term liabilities satisfy:

$$\frac{1}{dt}\mathbb{E}dR \leq \mu - \frac{\eta}{q} = \rho, \tag{47}$$

with equality when $k^h > 0$, and:

$$r = \rho - \partial_d \mathit{\Gamma}(d^h), \tag{48}$$

The FOC for consumption c^h is indeterminate; it is pinned down by the market clearing conditions.

Banks By standard continuous-time stochastic control methods, the banks HJBE satisfies the following:

$$\rho J^b := \sup_{\left\{d^b,k^b,d\Delta^b,d\Pi^b\right\}\in B^b} \left\{d\Delta^b - (1+\lambda)d\Pi^b + \frac{1}{dt}\mathbb{E}dJ^b\right\}. \tag{49}$$

By following Brunnermeier and Sannikov (2014), we can characterize (49) by postulating the following form for the value $J^b := \nu(\psi)e^b$. Given the guess:

$$d\nu = \nu(\mu^\nu dt + \sigma^\nu dW),$$

by Itô's Lemma, (49) holds as:

$$\rho - r = \sup_{\left\{d\delta_t^b\right\}} \left\{\frac{d\delta^b}{\nu} - d\delta^b\right\} + \sup_{\left\{d\pi_t^b\right\}} \left\{d\pi^b - (1+\lambda)\frac{d\pi^b}{\nu}\right\} + \sup_{\left\{\omega^b\right\}} \left\{\omega^b\left[\left(\mu^b - r\right) - \sigma^\nu\sigma\right]\right\} + \mu^\nu,$$

in which $\omega^b = \frac{qk^b}{e^b}$. Note that we look for the equilibrium at its stationary limit, so that $\frac{\partial J}{\partial t} = 0$. By taking the FOCs, the asset pricing, dividends, and equity issuances policies follow straightforward. Moreover, under the optimal strategy $\left\{d^b, k^b, d\Delta^b, d\Pi^b\right\}$, it hold that, in equilibrium:

$$\mu^\nu = \rho - r = \partial_d\Gamma(d).$$

A.6 The state variable

We omit the time subscript for sake of clear notation. Consider the state $\psi = \frac{E^b}{Kq}$. By Itô's Lemma,

$$\frac{d\psi}{\psi} = \frac{Kq}{E^b}\frac{\partial\psi}{\partial E^b}dE^b + \frac{Kq}{E^b}\frac{\partial\psi}{\partial Kq}d\left(Kq\right) + \frac{1}{2}\frac{Kq}{E^b}\frac{\partial^2\psi}{\partial\left(Kq\right)^2}d\left(Kq\right)^2,$$

which, given:

$$\frac{dKq}{Kq} = \left(\Phi(\iota) + \mu^q + \sigma^q\sigma\right)dt + \left(\sigma + \sigma_t^q\right)dW_t,$$

and

$$\frac{dE^b}{E^b} = rdt + \omega^b\left(dR - rdt\right) - \int_{\mathbb{B}}\left[d\delta^b + d\pi^b\right]db,$$

can be written as:

$$\frac{d\psi}{\psi} = \underbrace{\left\{\omega^b \frac{A-\iota}{q} + \left(\omega^b - 1\right)\left[\Phi(\iota) + \mu^q + \sigma^q\sigma - r\right] dt + (\sigma + \sigma^q)^2\right\}}_{\mu^\psi} dt+$$

$$+ \underbrace{\left(\omega^b - 1\right)(\sigma + \sigma^q)}_{\sigma^\psi} dW + \underbrace{\int_{\mathbb{B}} \left[d\pi^b - d\delta^b\right] db}_{d\Xi}.$$

A.7 The economy with no banks

The natural benchmark case of our analysis is the economy with no banks, where $\psi = \mu^\psi = \sigma^\psi = 0$, and no bank recapitalization takes place. In this economy the price q and the investment rate ι are constant and equal:

$$\iota = \frac{q-1}{\theta}, \tag{50}$$

$$q = \max_\iota \frac{A - \eta - \iota}{\rho + \delta - \Phi(\iota)}. \tag{51}$$

The aggregate consumption holds as $C^h = K\left[A - \iota - \eta\right]$, and all aggregates evolve as a Geometric Brownian Motion (GBM) with dynamics:

$$\frac{dC^h}{C^h} = \frac{dK}{K} = \Phi(\iota)dt + \sigma dW_t. \tag{52}$$

A.8 Equilibrium dynamics & numerical solution

The banking sector's relative size is the only relevant variable that jointly determines the dynamics of equilibrium prices $q(\psi)$ and $\nu(\psi)$. Their is pinned down by Itô's Lemma. The result is summarized in the following lemma.

Lemma 1. ***Equilibrium dynamics***
Given the law of motion of the state ψ in (24), the dynamics of the competitive equilibrium is fully represented by the following system of SDEs:

$$\frac{dq(\psi)}{q(\psi)} = \mu^q(\psi)dt + \sigma^q(\psi)dW,$$

$$\frac{d\nu(\psi)}{\nu(\psi)} = (\rho - r)\, dt - \sigma^{\nu}(\psi) dW,$$

whose drifts and diffusions solve the following system of ODEs

$$\begin{cases} \mu^q(\psi) = \frac{\partial_\psi q(\psi)}{q(\psi)} \psi \mu^\psi(\psi) + \frac{1}{2} \frac{\partial_{\psi\psi} q(\psi)}{q(\psi)} \left[\psi \sigma^\psi(\psi)\right]^2, \\ \mu^\nu(\psi) = \frac{\partial_\psi \nu(\psi)}{\nu(\psi)} \psi \mu^\psi(\psi) + \frac{1}{2} \frac{\partial_{\psi\psi} \nu(\psi)}{\nu(\psi)} \left[\psi \sigma^\psi(\psi)\right]^2, \\ \sigma^\nu(\psi) = -\frac{\partial_\psi \nu(\psi) \psi}{\nu(\psi)} \sigma^\psi(\psi), \\ \sigma^q(\psi) = \frac{\partial_\psi q(\psi) \psi}{q(\psi)} \sigma^\psi(\psi), \end{cases} \tag{53}$$

with mixed boundary conditions $\lim_{\psi \to 0} q(\psi) = \frac{A - \eta - \iota(0)}{r - \rho - \Phi(\iota(0))}$, $\partial_\psi q(\bar{\psi}) = 0$, *and* $\lim_{\psi \to 0} \nu(\psi) = 1 + \lambda$, $\nu(\bar{\psi}) = 1$, $\partial_\psi \nu(\bar{\psi}) = 0$.

Note that the extra boundary condition for ν is required to determine $\bar{\psi}$ by smooth pasting at the upper bound $\partial J_e = \bar{\psi} \partial \nu(\bar{\psi}) + \nu(\bar{\psi}) = 1 \Rightarrow \partial \nu(\bar{\psi}) = 0$.

Numerical solution method System (53) can be solved numerically by the following procedure similar to the one proposed by Brunnermeier and Sannikov (2014). Let us consider the difference between banks' (21) and households' (16) pricing equations:

$$\frac{\eta}{q(\psi)} + \rho - r \leq \sigma^\nu(\psi) \left(\sigma + \sigma^q(\psi)\right). \tag{54}$$

Moreover, let us define the following auxiliary variable $\kappa(\psi) = \psi \omega^b(\psi)$.

The algorithm can be summarized in the following steps:

1. Guess $\partial_\psi \nu \in (0, -\infty)$ and $\kappa \in \left[\psi, \psi + \frac{q(\psi)}{\partial_\psi q(\psi)}\right]$ so that (54) holds with equality, where $\sigma^q(\psi)$ and $\sigma^\nu(\psi)$ are pinned down by system (53), and $\sigma^\psi(\psi)$ and $\mu^\psi(\psi)$ by (25) and (26), respectively;

2. If $\kappa(\psi) \geq 1$, set $\kappa(\psi) = 1$ and recompute (54);

3. Solve numerically system (53) where, by considering (48),

$$\mu^\nu = \Gamma,$$

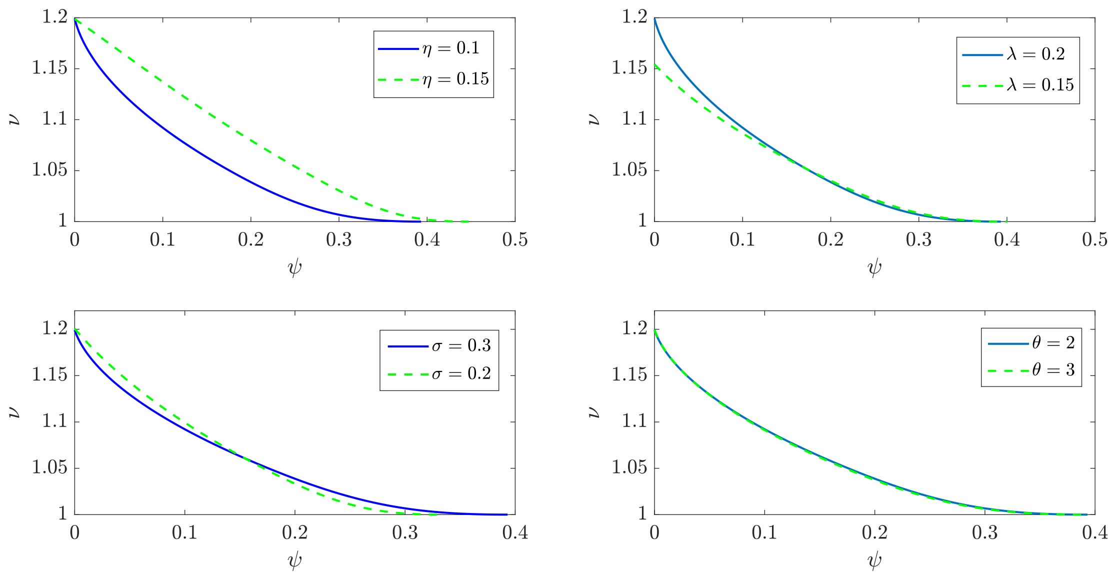

Figure 8: Comparative statics of banks' equity marginal value as a function of the key parameters of the model: the banking efficiency edge η (top, left), recapitalization cost λ (top, right), exogenous systematic volatility σ (bottom, left), and technological illiquidity θ (bottom, right).

while, by (47),

$$\mu^q(\psi) = \rho + \frac{\eta}{q(\psi)} - \sigma^q(\psi)\sigma - \Phi(\iota(\psi)),$$

and stop when either $\partial_\psi \nu(\bar{\psi})$ or $\partial_\psi q(\bar{\psi})$ (or both) equal zero;[21]

4. Rescale $\nu(\psi)$ so that $\nu(\bar{\psi}) = 1$ and compute $\omega^b = \frac{\kappa(\psi)}{\psi}$;

5. Check whether the initial boundary condition $\lim_{\psi \to 0} \nu(\psi) = 1 + \lambda$ is met. If yes, stop. Else, update the initial guess for $\partial_\psi \nu$ and repeat from 1.

A.9 Comparative statics

Figure 8 plots the equilibrium marginal value of banks' equity ν with respect to low (blue) and high (green) parametric values for the banks' efficiency edge η (top, left), the recapitalization cost λ (top, right), the exogenous volatility component σ (bottom, left), and the technological illiquidity θ (bottom, right) over the interval $\psi \in \Psi$.

First, a higher efficiency edge η fundamentally increases the marginal value of the banks'

[21] We implement this step via Matlab *ODE45*.

equity. Accordingly, it shifts forward the threshold $\bar{\psi}$ at which dividends are paid out. As the households extract more value from the banks having a greater capitalization, they are willing to wait longer before receiving dividends. This is because a higher premium corresponds to "more profitable" monitoring services.

Second, increasing the exogenous volatility parameter σ had an ambiguous effect on the banks' value depending on the level of ψ. It increases it for lower levels of ψ, it decreases it for higher ones. All in all, higher systematic volatility increases the equilibrium capital buffer held by the banking sector.

Third, similarly as for σ, higher recapitalization costs λ have an ambiguous effects on ν. Higher λ increases the marginal value of banks for lower values of ψ, while it increases it for higher ones. Overall, according to our numerical results the level of λ does not change fundamentally the dividend threshold $\bar{\psi}$.

Fourth, an increase in the technological illiquidity parameter θ does not fundamentally change banks' equity marginal value.

A.10 Welfare

Under the optimal strategy $\left\{C^h, D^h\right\}$, households' welfare H satisfies (we omit time subscript for sake of clear notation)

$$E = H(\psi, E^h) = \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} \left[C^h + \mathit{\Gamma}(D^h)\right] dt.$$

By considering the market clearing condition for consumption, it holds that:

$$C^h = K\left[(A-\iota) - \left(1-\psi\omega^b\right)\eta\right],$$

while the aggregate banks' short-term liabilities satisfy

$$\mathit{\Gamma}(D^h) = \Gamma D^h = \omega^h \frac{E^h}{Kq} Kq = \psi\left(\omega^b - 1\right) Kq.$$

It follows that:

$$H(\psi, K) = \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} K \left\{ [A - \iota(q(\psi))] - \left[1 - \omega^b(\psi)\psi\right] \eta + \Gamma \psi \left[\omega^b(\psi) - 1\right] q(\psi) \right\} dt. \quad (55)$$

By Feynman-Kac formula, the function H as represented in (55) must solve the following PDE:

$$\rho H(\psi, K) = K\Theta(\psi) + \partial_{\psi} H \psi \mu^{\psi} + \frac{1}{2}\partial^2_{\psi\psi} H \left(\psi \sigma^{\psi}\right)^2 + \partial_K H K \left[\Phi(\iota(q(\psi)) - \delta\right] + \\ + \frac{1}{2}\partial^2_{KK} \left(K\sigma\right)^2 + \partial^2_{K\psi} \psi \sigma^{\psi} \sigma K, \quad (56)$$

Finally, since the model is scale invariant in aggregate capital stock K, we postulate that $H(\psi, K) = Kh(\psi)$. By substituting in (56) and rearranging:

$$\left\{\rho - \left[\Phi(\iota(q(\psi)) - \delta\right]\right\} h(\psi) = \Theta(\psi) + \partial_{\psi} h(\psi) \psi \left[\mu^{\psi} + \sigma^{\psi}\sigma\right] + \frac{1}{2}\partial^2_{\psi\psi} h \left(\psi\sigma^{\psi}\right)^2,$$

with boundary conditions $h(0) = \frac{A-\eta-\iota(0)}{\rho+\delta-\Phi(\iota(0)}$ and $\partial_{\psi} h(0) = 0$, in which

$$\Theta(\psi) \coloneqq [A - \iota(\psi)] - \left[\omega^b(\psi)\psi - 1\right] \eta + \Gamma \psi q(\psi) \left[\omega^b(\psi) - 1\right].$$

Accordingly, households' (short-run) welfare can be written as:

$$Kh(\psi, 1) = \mathbb{E} \int_0^{\infty} e^{-\rho t} K \Theta(\psi) dt,$$

and

$$h(\psi, 1) = q(\psi) \left[1 + \psi \left(\nu(\psi) - 1\right)\right].$$

Households' short-run welfare and its components To better understand the elements that contribute at determining households' welfare, the three panels of Figure 9 depicts the components of function Θ in equation (29) as functions of ψ.

Households' welfare is characterized by the flow of consumption Θ^c (left panel) minus the monitoring expenditure Θ^m (middle) plus the utility flow that stems from the liquidity

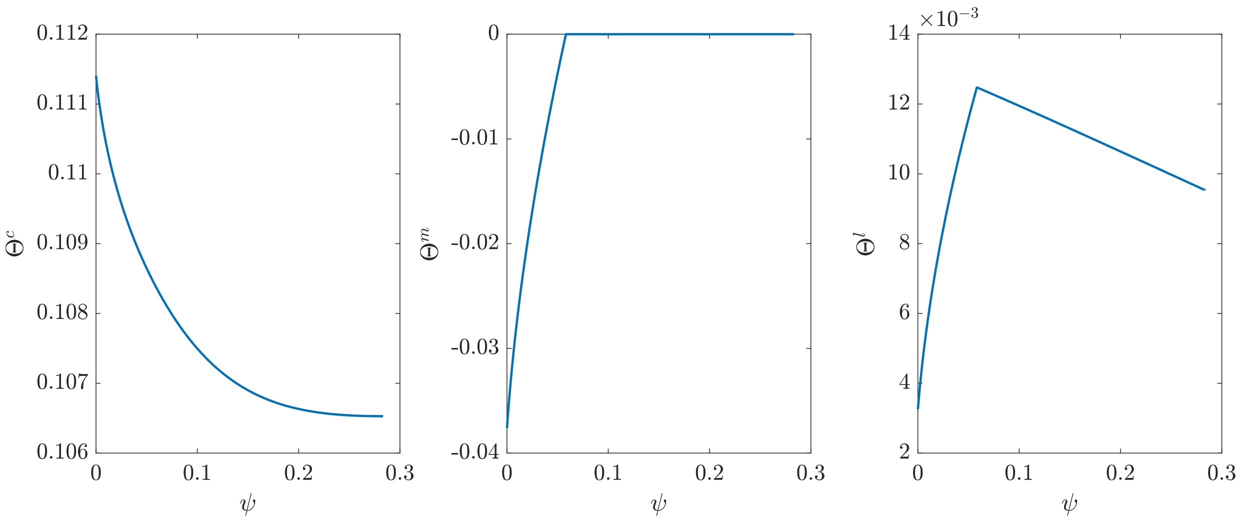

Figure 9: The three components of households' short-run welfare: consumption flow (left panel), monitoring expenditure (middle panel), and liquidity (right panel).

of their holdings in short-term bank liabilities Θ^l (right).

The fist component, the consumption flow Θ^c, is decreasing in ψ because investments $\iota(\psi)$ are increasing in banks' capitalization. In a dynamic perspective, this positively affects the growth rate of capital as well as that of banks' relative size. The second component, the monitoring term θ^m, is also increasing in ψ. This is because the higher banks' capitalization, the higher the stock of capital they can deploy to finance firms directly, therefore reducing the resources depleted after monitoring costs. The third and last component, the liquidity term Θ^l, is strictly increasing up to the point when banks have sufficient capitalization to absorb households' spare capital by issuing short-term liabilities. Then, it slightly decreases as the dis-utility of lower relative size $(1-\psi)$ overtakes the benefit of higher prices $q(\psi)$.[22]

Trivially, when either η or Γ equal zero, then households do not benefit from banks' capitalization neither through their liquidity services nor through the pecuniary externality of their cost efficient monitoring.

[22]Note that, when banks have enough capital to absorb households' capital by their liabilities, then $\omega^b = \frac{1}{\psi}$. Thus, $\Theta^l(\psi) \propto q(\psi)\,(1-\psi)$. In fact, when ψ approaches $\bar{\psi}$, $q(\psi)$ progressively slopes towards zero (see also Figure 9, top left panel).

A.11 State dynamics with bailout - sketch

Let $\psi^{\Omega} \geq 0$ be the exogenous bailout thresholds - banking sector minimum require capital buffer - at/below which the government may implement the bailout recapitalization. Let $\psi^{\Omega} := \frac{E^b}{(Kq)^{\Omega}}$ be the banking sector's relative size under the bailout policy $T^{\pi} \geq 0$. Then, the dynamics of the value of aggregate capital stock, henceforth denoted $d(Kq)^{\Omega}$, evolves as (we omit time subscripts for sake of clear notation):

$$\frac{d\left(Kq\right)^{\Omega}}{\left(Kq\right)^{\Omega}} = \frac{d(Kq)}{Kq} - \mathbb{I}_{\psi \leq \psi^{\Omega}} \lambda^G \frac{T^{\pi} E^h}{(Kq)^{\Omega}},$$

because $dT^{\pi}\lambda^G$ resources are lost after bailout administrative costs. Likewise, aggregate banks' equity evolves as (see also Equation 11):

$$\frac{dE^b}{E^b} = \frac{dE^b}{E^b} + \mathbb{I}_{\psi \leq \psi^{\Omega}} \frac{T^{\pi} E^h}{E^{b,\Omega}}.$$

By Itô's Lemma (see also Appendix A.6), the dynamics of relative banking sector's capitalization with bailout can be rearranged as:

$$\frac{d\psi^{\Omega}}{\psi^{\Omega}} = \frac{d\psi}{\psi} + \mathbb{I}_{\psi \leq \psi^{\Omega}} T^{\pi} \left(\frac{E^h}{E^b} + \lambda^G \frac{E^h}{Kq} \right) dt.$$