Preferred and Non-Preferred Creditors*

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Abstract

International financial institutions (IFIs) generally enjoy preferred creditors treatment (PCT). Although PCT rarely appears in legal contracts, when sovereigns restructure bilateral or commercial debts they normally pay IFIs in full. This paper presents a model where a creditor, such as an IFI, that can commit to lend limited amounts at the risk-free rate and can refrain from lending into arrears is always repaid and adds value. The analysis suggests that IFIs and market lenders can both enhance welfare, even if banning commercial borrowing can sometimes be optimal. To maintain their status, preferred lenders should offer low cost financing in volumes that are consistent with countries' incentives to repay even in bad states. This suggests such lenders should not differentiate lending interest rates according to risk and should not participate in the restructuring of commercial debt.

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1 Introduction

The role of international financial institutions (IFIs) in the global financial architecture has been widely analyzed both by academics and by the international policy community.[1] Much has been discussed about their role as providers of emergency funding but a long-standing central puzzle remains. Namely, the International Monetary Fund (IMF) and the main multilateral development banks (MDBs) enjoy Preferred Creditor Treatment (PCT) in relation to their sovereign lending, meaning that they are expected to be repaid even if the borrower restructures private or bilateral debt. And yet, while PCT is critical for the operating model of IFIs, and the Paris Club Agreed Minutes exonerate IFIs from a "comparability of treatment" clause, their preferred standing is not strongly backed in international law.[2]

At times, the preferred treatment of IFIs has been called into question. For example, Greece fell into arrears with the IMF in July 2015.[3] On August 28, 2019, the Argentine economy minister announced the intention to reprofile domestic and external debt and appeared to include amortizations to the IMF.[4] Still, during the prolonged "trial of the century" following Argentina's 2002 default, the outstanding debts to the IMF were paid early and in full. Indeed, it is notable in that case that while there were many attempts from hold-outs to disrupt payments to creditors that accepted the 2005 restructuring and subsequent offers, there was virtually no mention of Argentina's preferred lenders, nor any serious attempt to crowd them in.[5] And in several recent bond restructurings, including Argentina's in 2020, private creditors suffered changes in contracts and present-value haircuts, but IFIs continued to be paid in full.[6]

The persistence of IFIs' preferred-standing is intriguing, especially given that it is a market practice, which is not backed by any contractual clause. The resilience of PCT, together with the lack of any strong legal foundation, suggests that it should be understood as an "equilibrium outcome." And yet we know of no economic model to date that shows this to be the case. Sovereign debt models that focus on "willingness to pay" do not consider seniority,[7] while those that focus on seniority[8] assume it, without explaining its origin. Despite the extensive literature on the international financial architecture and sovereign debt restructuring, and despite the critical nature of PCT to the operations of the main IFIs, to our knowledge there is no model that explains why sovereign borrowers treat such lenders as preferred. This paper attempts to fill this gap. Our contribution is to develop a model that endogenizes the repayment decision of both commercial and IFI creditors[9] to show how such decisions are interdependent, and to describe the potential advantages and

[1] We review relevant academic literature in the next section. As examples of the policy discussion, see Council of Foreign Relations (2018) and the section on the global financial safety net in G20 Eminent Persons Group on Global Financial Governance (2018).

[2] "The Paris Club Agreed Minutes 'comparability of treatment' clause aims to ensure balanced treatment of the debtor country's debt by all external creditors. In accordance with this clause, the debtor country undertakes to seek from non-multilateral creditors, in particular other official bilateral creditor countries that are not members of the Paris Club and private creditors (mainly banks, bondholders and suppliers), a treatment on comparable terms to those granted in the Agreed Minutes." See: http://www.clubdeparis.org/en/communications/page/what-does-comparability-of-treatment-mean On the other hand, see Martha (1990) and Schadler (2014) on the lack of strong legal standing for PCT in international law.

[3] See, for example, "Defaulting on the IMF: A stupid idea whose time has come," *Financial Times,* Alphaville, July 1st, 2015.

[4] See (in Spanish) "Hernán Lacunza: Propusimos al FMI dialogar para reperfilar los vencimientos de deuda" in La Nación newspaper August 28, and "Argentina seeks to restructure $101bn of debt" Financial Times August 29, 2019.

[5] For example, in Cruces and Samples (2016) account of Argentina's "trial of the century" there is virtually no mention of Argentina's senior creditors.

[6] Recent cases include Argentina, Barbados, and Ecuador. Previous cases include Belize, the Dominican Republic and Uruguay. A set of low income countries obtained debt relief under the Multilateral Debt Relief Initiative (MDRI). Broadly speaking, these countries did not have market access. Our focus is to understand why countries may treat IFIs as preferred in relation to market lending. Still, only two countries were in arrears with the IMF in 2019 and Oeking and Simlinski (2016) argue that persistent arrears to that organization may be a thing of the past. At the time of writing, Eritrea, Somalia, Sudan, the Syrian Arab Republic, and Zimbabwe were in arrears with the World Bank and Venezuela was in arrears with the Inter-American Development Bank.

[7] See, for instance, the classic papers by Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Kletzer and Wright (2000) or, for a more up-to-date discussion, Aguiar and Amador (2014).

[8] See, e.g., Bolton and Jeanne (2009), Boz (2011), Chaterterjee and Eyigungor (2015), Gonçalves and Guimaraes (2015), Hatcheondo et al (2017), and Corsetti et al (2018).

[9] Throughout the paper we use IFIs and multilateral lenders/debt interchangeably. We thus ignore the fact that some

trade-offs faced by countries that may borrow from both the market (commercial lenders) and IFIs.

In what follows, we develop a relatively simple model of emergency financing[10] that allows us to obtain a set of analytical results. This contrasts with much of the recent literature, which tends to rely on numerical simulations. Our model abstracts from several important real-world features such as liquidity and creditor coordination issues as well as reforms and conditionality. The aim is to understand the fundamental differences between IFIs and private lenders, focusing on the underlying incentives for a country to borrow and to repay each type of creditor. We show that IFIs are preferred because their bylaws allow them to commit to i) lend limited amounts at close to the risk-free rate under most circumstances, and ii) refrain from lending until any unpaid arrears are cleared. In contrast, atomistic private lenders are unable to coordinate and commit to a maximum amount of lending; they then face a type of dilution. This sets IFI lending aside from that of private lenders, and explains why, in many instances, the presence of IFIs may add value. However, we also find that preferred lending may be constrained by a country's willingness to honor the commitments made, with the constraint depending on the probability and the severity of future shocks, on repayment costs, and on the country's access to private lenders, which, in turn, depends on similar parameters.

In a similar vein to the common statement in corporate finance that if all financing is debt then none is (it turns into equity), we may quip that if all lending is preferred then no lending is. Preferred lending cannot be increased without limit, otherwise borrowers may cease to consider it preferred. Moreover, if emergency financial assistance is required frequently, or is extremely rare market solutions may be just as good. Finally, we find situations in which a country is better off if it cannot borrow from the market, providing a justification for restrictions on commercial lending under certain conditions.

In the next section, we review the literature on different aspects of PCT. In Section 3, we introduce the basic model and study the case in which the country borrows from an IFI. In Section 4, following the standard sovereign debt literature, we assume that the country relies only on private lenders. In Section 5, we allow for the simultaneous presence of multilateral and private lenders. Section 6 discusses extensions of the model analyzing cross-default clauses, conditionality, and the possibility of evergreening. Section 7 investigates how robust our results are to the relaxation of some critical assumption, while Section 8 provides an interpretation of the different parameters of the model and elaborates on whether our results would hold true in a more general set-up. Finally, Section 9 provides a set of policy implications and concludes.

2 On Preferred Creditor Treatment: A Brief Review

Our paper borrows from several strands of literature on IFIs and PCT. First, a number of papers have discussed potential explanations for why IFIs enjoy PCT, although none to our knowledge contains a model illustrating how it can be supported as an equilibrium outcome. Buiter and Fries (2002) suggest countries may confer PCT to IFIs in return for competitive lending rates. Levy Yeyati (2009) argues that PCT is related to insurance–namely the expectation that IFIs will extend credit during a crisis. Humphrey (2015) stresses IFIs' mutual ownership structure. Risk Control (2017) collects statistics related to three potential "drivers" of PCT (favorable rates, counter-cyclical lending and the cooperative nature of the institutions) and finds support for each. The paper also compares the degree of "mutuality" of each institution and discusses how that may affect preferential treatment.

Second, within the large empirical literature on sovereign defaults, a subset of papers considers the role of IFIs. Schlegl et al. (2015, 2019) analyze World Bank data on 127 countries from 1980 to 2006 and find a de facto hierarchy, with the IMF and MDBs as the most senior creditors. Bonds appear below MDBs in the pecking order, and then come bilateral lenders, banks and trade credit. Not all multilateral lenders are treated equally; for instance, during the European crisis the EFSF/ESM was not as preferred as other IFIs. Steinkamp and Westermann (2014) analyze the European crisis and present survey evidence on market participants' perception of seniority levels and claim the IMF was perceived as the most senior official

multilateral lenders such as the IFC (the private sector arm of the World Bank Group) do not generally claim preferred status.

[10] Focusing on emergency lending (and abstracting from lending for consumption-smoothing motives in normal times) provides a clean way to illustrate how a preferred lender could exist as an equilibrium outcome.

creditor. Finally, MDBs issue bonds on international markets and the IBRD and the four main regional MDBs (ADB, AfDB, EBRD and IDB) maintain AAA ratings. Moody's and Standard and Poor's both suggest these five organizations enjoy PCT, although methodologies vary regarding how much of a bonus this provides in formulating ratings.[11]

A set of papers shows that senior creditors may yield benefits but their status is assumed. Bolton and Jeanne (2009) argue that a seniority structure within creditors may ease the debt restructuring process without generating much inefficiency ex ante. In Gonçalves and Guimaraes (2015), a country can commit to a specific fiscal policy and borrow from a senior creditor–the IMF. Hatchondo, Martinez and Onder (2017) demonstrate that a country might be able to borrow more when there is a senior creditor. Our contribution, in contrast to these papers, is to show that seniority may be an equilibrium outcome consistent with the notion that preferred creditor status is a market norm, which, in general, is not written into legal contracts. Welfare improves through somewhat different mechanisms across these three papers and in ours. Our paper follows in the spirit of Grossman and van Huyck (1988). In their equilibrium, creditors default given a bad draw on income (an excusable default) but, otherwise, the long run value of the borrower's reputation exceeds the short-run benefits of default. In our case, we allow for two types of creditors and, as we discuss below, the preferred creditor may commit to a low interest rate and a low lending volume such that the long-run value of access outweighs the short-run benefits of default.

Figure 1 shows[12] the number of countries "in default" in each year from 1960 to 2016 classified by the type of creditor. These data stem from the Bank of Canada-Bank of England Sovereign Default Database—see Beers and Mavalwalla (2017) and Beers, Jones and Walsh (2020). As noted by these authors, there is no single accepted definition of default. For the purposes of the database "a default has occurred when debt service is not paid on the due date or within a specified grace period." For IFIs these "defaults" are arrears precisely because they expect at some point full repayment to be made. Each IFI has its own rules on provisioning against such events. We use the label "default" in the figure following the convention of the database. Private creditors here refers to foreign currency lending by commercial creditors including loan and bond financing. Note that very few countries are in arrears with the IMF or with the IBRD. The maximum number of countries in arrears with the IMF was 16 in 1989, and this figure falls to 2 in 2019. For the IBRD, there was a maximum of 9 countries in arrears in 1992, falling to just 1 in 2019. Arrears are much more frequent with the Paris Club group of bilateral lenders, with a peak of 46 countries in the year 2000, but falling to 12 in 2019. Regarding private creditors, there was a peak of 91 countries in 1994 and 1995, falling to 39 by 2019.

Emergency lending is commonly seen as a potential driver of PCT. At times of stress, private creditors perceiving higher default probabilities will demand higher interest rates. If IFIs act benevolently and the expectation is that they will be repaid, then they can lend at low interest rates (just above the riskless rate to support their operating costs) even during difficult times. For a country that may suffer future emergencies, this relationship may be very valuable. In our model, it is the value of this relationship that provides the incentives for repayment.

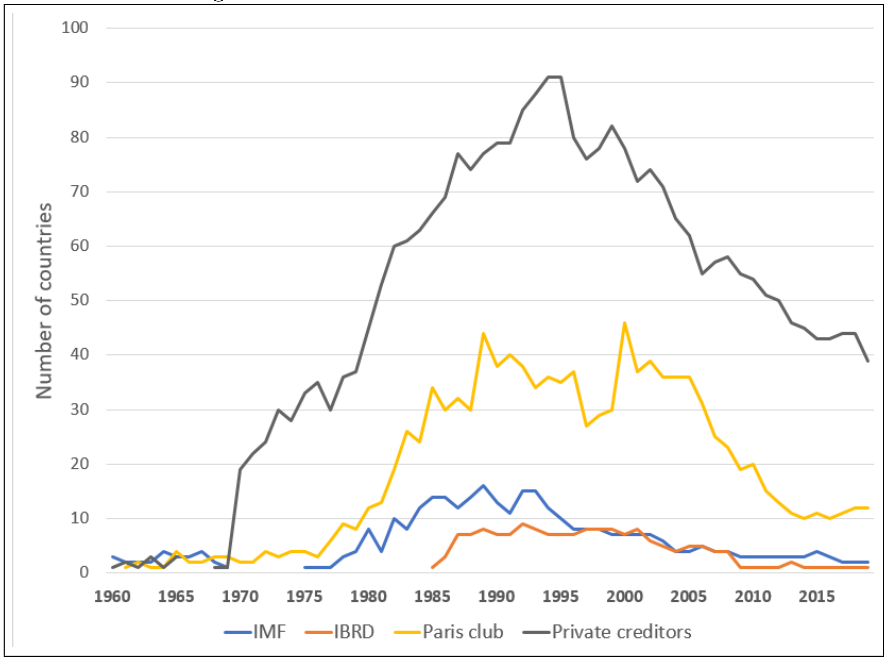
This implies that the amount that IFIs can lend to a country, and expecting to be repaid, will be related to the probability the country needs financial assistance in the future (e.g., because it is hit by a shock). Assuming that preferred lenders can credibly commit not to lend if a country has defaulted on its loans, this implies that a greater amount of preferred lending may be supported as the probability of shocks rises, suggesting a positive relationship between the probability of stress periods and the amount of preferred lending.[13]

[11] See Humphrey (2015), Perraudin et al (2016), and Risk Control (2017) for relevant commentary. Moody's (2017) and Standard and Poor's (2017) contain information on the ratings methodologies.

[12] The figure plots the number of countries "in default" for each group of creditors following the definition of default as employed in the Bank of Canada and Bank of England dataset on sovereign defaults. The IMF is the International Monetary Fund. IBRD is the International Bank of Reconstruction and Development (the non-concessional balance sheet for sovereign lending of the World Bank Group), Paris Club refers to those bilateral lenders that are members of the Paris Club (essentially OECD member countries) and Private Creditors here includes both bond and loan financing exclusively in foreign currency. Source: Bank of Canada–Bank of England Sovereign Default Database.

[13] In the working paper version of this paper, Cordella and Powell (2019), we provide evidence of a positive relation between

Figure 1: Number of Countries in "Default"



3 The Model

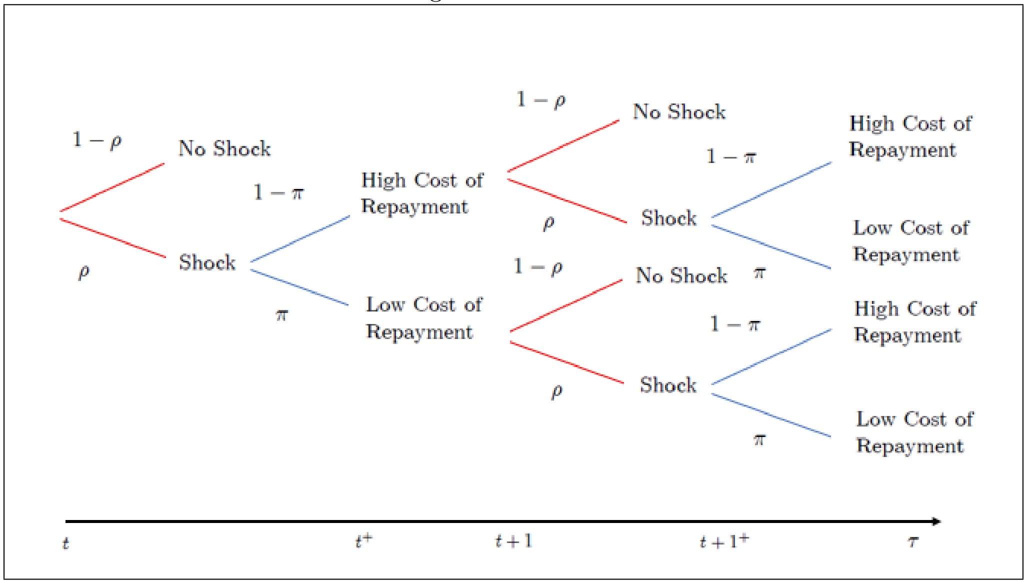
In this section, we present a stylized model where it is assumed that, with a certain probability, a country may require emergency financial assistance. In our set-up, time, τ, is discrete and runs from the initial period, t, to infinity. In the initial period, the country requires assistance with probability ρ; but if there is no such need in t, then we assume there will never be a need thereafter. One interpretation of this is that the country has then "graduated" from requiring emergency lending. If, instead, the country has not graduated, then with the same probability ρ that need reoccurs in $t + 1$, and so forth. These assumptions imply a simple Markovian structure and allow us to solve the model analytically rather than relying on numerical simulations. All borrowing in the model is short term: the full amount of the loan plus the interest should be repaid at the end of each period, which we refer to as τ^+. Figure 2 illustrates the timing of the model.

For the sake of simplicity, the country's discount factor and the (gross) risk-free interest rate are both set equal to 1, and we also normalize the utility in all states where no assistance is needed (the non-shock states) to zero; in those states where assistance is needed (the shock states), absent lending, we instead assume utility to be equal to to $-C$. However, by borrowing an amount L, the country can reduce the loss in utility by $aL - \frac{L^2}{2}$. This specification implies that the value of emergency lending rises with a, and exhibits decreasing marginal returns. Finally, there is some utility cost for the country to repay debt. This cost may vary depending on political, economic, or other considerations. To capture this, while keeping things simple, we assume that there are just two states, so this cost may be either high or low. We refer to these states as the high- and the low-repayment-cost states. We label the probability of the low-repayment-cost state as π, and we normalize the cost of repayment in that low-repayment-cost state to 1. We denote by k the cost of repayment in the high-repayment-cost state.

Our model employs five key parameters, namely the probability of requiring financial assistance (ρ), the utility value of that assistance (a), the probability of the realization of a state in which the cost of repayment of the debt is low (π), the cost of repayment in the high repayment-cost state (k), and the loss in

the probability of a country having IMF liabilities and the size of those liabilities, lending some support to this view.

Figure 2: Timeline



utility during an emergency period if there is no lending (C). The first parameter determines the value of maintaining a relationship with lenders and thus plays a critical role in the model. The more likely an event occurs where emergency lending is required, then the greater that value will be. The second determines how valuable lending is in that period. The third and fourth together determine the expected cost of repayment in utility terms and how this varies across states. Countries will tend to default when the utility cost of repayment is high. Finally, the loss of utility in an emergency period absent lending may be thought of as the cost of a crisis.

Having said that, we make four main assumptions. The first two simply ensure that the model is of interest. The last two are restrictions on the parameters to rule out certain outcomes that would complicate the main narrative of the paper. We explore the implications of relaxing these assumptions in Section 7 below. First, we require that C is such that the utility during a period when emergency lending is needed is never higher than the utility in non-emergency periods. Thus, we assume $C > \frac{a^2}{2}$, which assures that this is always the case. Second, for the model to be of interest we require that borrowing (from either the market or the IFI) creates value, that is, the marginal benefit of borrowing is higher than the marginal cost of repayment in the high-repayment-cost state; this is achieved when $a > k$. Third, there should be some situations in which the country would wish to borrow from the market and default in the high-repayment-cost state, rather than simply borrow from the IFI at all times–otherwise there would be no market borrowing at all. This implies that the cost of repayment in the high-repayment-cost state needs to be above a threshold, which may be expressed in terms of the other parameters of the model, namely $k > \frac{a-\pi}{a\pi-1}$. Finally, we assume that the cost of servicing the debt in the high-repayment-cost state is not so high that the country is tempted to default and then wait until a low-repayment-cost state materializes, clear the arrears and regain access to borrowing. This is ruled out by assuming that, $k < 1 + \frac{1}{\rho(2\pi-1)}$. We return to this condition when we discuss the robustness of the model below. It turns out that the last three restrictions can be stated as:

$$Min[a, 1 + \frac{1}{\rho(2\pi - 1)}] > k > \frac{a - \pi}{a\pi - 1}. \tag{A1}$$

In what follows, our main focus is on deriving a set of analytical results, but we also illustrate those results with numerical simulations. In our base case, we assume $a = 8$, $\pi = .6$, $k = 3.25$. This parameterization complies with (A1) for all $\rho \in [0, 1]$.

3.1 IFI Lending

We start our analysis by considering the case in which the country only has access to an IFI. IFIs are different from commercial lenders since, according to their internal rules, they normally lend at rates well below market, anticipating that they will receive preferential treatment and be repaid during periods of debt distress. However, as we investigate below, this means that, in equilibrium, countries should only borrow amounts that they are willing to repay under all possible circumstances. For this risk-free lending to be sustainable in equilibrium, we assume that, in contrast to the market, IFIs can commit to specific lending volumes. We also assume that, if a country defaults on an IFI, it is excluded from borrowing from any IFI until it fully repays the arrears. This reflects IFIs' actual rules and working practices.[14]

At time t, the value function for the country is, assuming that, if financial assistance is needed, it borrows L_{I_t} from an IFI at the risk-free interest rate, and it repays both in the high- and in the low-repayment-cost state, is given by:

$$V_{I_t} = \rho(-C + aL_I - \frac{L_{I_t}^2}{2} - L_{I_t}((1-\pi)k + \pi) + V_{I_{t+1}}), \tag{1}$$

where subscript I denotes IFI. Equating V_{I_t} with $V_{I_{t+1}}$, we can rewrite the value function as

$$V_I = \frac{\rho}{1-\rho}(-C + aL_I - \frac{L_I^2}{2} - L_I((1-\pi)k + \pi)). \tag{2}$$

For the loan to be risk free and support the specification of this value function, the country must be willing to service it in the high-repayment-cost state. If we rule out the possibility that the country may go into arrears, but then at some future date repay the IFI, then the following condition must be satisfied:

$$V_I - kL_I \geq -\frac{\rho C}{1-\rho}, \tag{3}$$

that is, the continuation value of the official lending relation V_I, net of the repayment cost kL_I, should be greater than the continuation value of defaulting, which is given by $-\sum_{\tau=t}^{\infty} \rho^{(t-\tau)} C = -\frac{\rho C}{1-\rho}$.

In writing equation (3), we indeed ruled out the possibility for the country to default on the official sector if the high-repayment-cost state materializes and then to clear its arrears in the next draw of a low-repayment-cost state. In the Appendix, we show that such a strategy is never in the interest of the country if (A1) holds. Were this not the case, the repayment constraint would be tighter. This would reduce the amount the IFI can lend risk free, but it would not affect our qualitative results, as we will show in the robustness section. We thus have that

Proposition 1 *The optimal amount of IFI lending is given by*

$$L_I^* = \begin{cases} 0, & \rho \leq \overline{\rho}_I \equiv \frac{k}{a+\pi(k-1)}; \\ 2(a - \pi - k(\frac{1}{\rho} - \pi)), & \overline{\rho}_I < \rho < \widehat{\rho}_I; \\ a - \pi - k(1-\pi), & \rho \geq \widehat{\rho}_I \equiv \frac{2k}{a+\pi(k-1)+k}; \end{cases} \tag{4}$$

and the associated utility by

$$V_I^* = \begin{cases} -\frac{\rho C}{1-\rho}, & \rho \leq \overline{\rho}_I; \\ -\frac{\rho C}{1-\rho} + \frac{2k(\rho(\alpha-\pi)-(1-\pi\rho)k)}{\rho}, & \overline{\rho}_I < \rho < \widehat{\rho}_I; \\ -\frac{\rho C}{1-\rho} + \frac{\rho(\alpha-\pi-(1-\pi)k)^2}{2(1-\rho)}, & \rho \geq \widehat{\rho}_I. \end{cases} \tag{5}$$

[14] For instance, the IMF's ability to lend to a country is (at least in principle) a multiple of the country's IMF quota. The IBRD lending envelope to any particular country is approved by the World Bank board every 4 years in the Country Partnership Strategy and is a function of each country's size, needs and ability to repay. As per the no-lending into arrears clause, there is indeed a mutual understanding between IFIs that arrears to different institutions have to be cleared simultaneously. For instance, according to IMF (2013), "The Fund maintains a policy of non-toleration of arrears to official creditors. Fund supported programs required the elimination of existing arrears and the non-accumulation of new arrears during the program period with respect to official creditors" (p. 48).

Proof: In Appendix.

According to Proposition 1, if the probability of requiring emergency financial assistance is sufficiently low, $\rho \leq \overline{\rho}_I$, then risk-free IFI lending is not supported in equilibrium and the country cannot improve on$-C$ in that case. The reason is that, given the low probability of requiring assistance, the borrower will default in the bad state when repayment costs are high as the value of the relationship with the official lender is insufficient to outweigh such costs. On the other hand, when the probability of requiring assistance is sufficiently large, $\rho \geq \widehat{\rho}_I$, the lending relation is valuable enough to allow the IFI to lend the optimal unconstrained amount. For intermediate values of ρ ($\widehat{\rho}_I > \rho > \overline{\rho}_I$), IFI lending is supported in equilibrium, but the participation constraint (ensuring that the IFI is repaid in the bad state) binds. This constraint then determines the amount of IFI lending.

The optimal amount of IFI lending, as a function of ρ, is illustrated in Figure 3. In Proposition 1, we have expressed the three regimes as a function of ρ, the probability of requiring financial assistance. However, the critical values $\overline{\rho}_I$ and $\widehat{\rho}_I$ also depend on the other parameters. More precisely, $\overline{\rho}_I$ and $\widehat{\rho}_I$ both increase in a and in π, and decrease in k. As discussed above, a is a measure of the value of emergency lending and thus, the higher is a, the higher is the opportunity cost of defaulting. So higher values of a support larger amounts of IFI lending. Similarly, the higher is π and the lower is k, the lower is the expected cost associated with servicing the debt in the high-repayment-cost state, which may be written as $(1-\pi)k$, and thus the higher is the value of maintaining the relationship with the IFI and being able to borrow in the future. This, in turn, implies that higher values of π, or lower values of k, also support larger volumes of IFI lending.

4 Market Lending

Consider now the case in which the country only borrows from private lenders, which we refer to as the market. The key difference between market and IFI lenders is that atomistic private lenders are unable to coordinate and commit to a maximum amount of lending. As is standard in the literature, private lenders are competitive and risk neutral, and to break even they must charge an interest rate commensurate with the probability of default. Loans are short term as before. As mentioned above, we assume that if the country defaults it cannot regain access to credit from the market.[15]

To solve for the optimum amount of market lending, we start by assuming that the borrower always defaults in the high-repayment-cost state so that the gross interest rate is $1/\pi$. Again, we assume that, in each period τ, loans are short term and must be repaid at the end of the period. In the initial period t, the value function for the country—assuming it borrows L_{MD_t} from commercial lenders at the risk-adjusted interest rate and pays the total due amount $\frac{L_{MD_t}}{\pi}$ in the low-repayment-cost state, which occurs with probability π—is given by

$$V_{MD_t} = \rho(-C + aL_{MD_t} - \frac{L^2_{MD_t}}{2} + \pi(-\frac{L_{MD_t}}{\pi} + V_{MD_{t+1}}) + (1-\pi)\frac{-\rho C}{1-\rho}), \tag{6}$$

where superscript MD denotes market (M) lending when repayment occurs in the low-repayment-cost state and default (D) in high-cost-repayment state. Again, $-\frac{\rho C}{1-\rho}$ is the continuation value assuming default in the bad state and no lending. Equating V_{MD_t} with $V_{MD_{t+1}}$, the value function can be written as

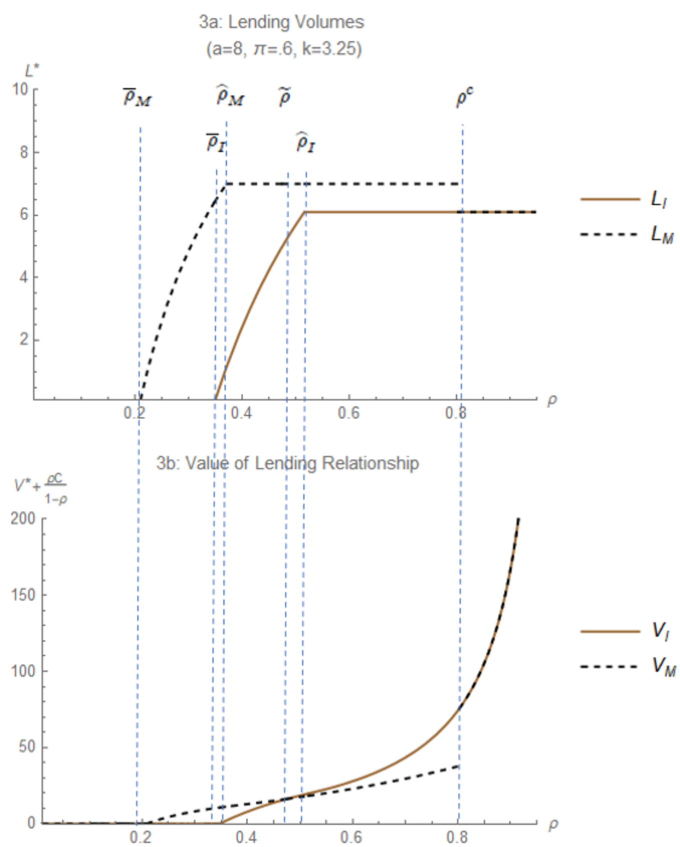
$$V_{MD} = \frac{\rho}{1-\pi\rho}(-C + aL_{MD} - \frac{L^2_{MD}}{2} - L_{MD} - (1-\pi)\frac{\rho C}{1-\rho}). \tag{7}$$

For the market to be willing to offer loans at this interest rate, the country must be willing to repay the debt in the low-repayment-cost state. This condition can be written as

$$V_{MD} - \frac{L_{MD}}{\pi} \geq -\frac{\rho C}{1-\rho}. \tag{8}$$

[15] Had we assumed that, after a default, the borrower could borrow again from the market with a certain probability in the subsequent period, the qualitative results of the paper would remain the same.

Figure 3: IFI and Market Lending: Volumes and Value (I)



The optimal amount of borrowing is then found by maximizing (7) subject to the constraint (8). The results are summarized in Lemma 1.

Lemma 1 *If the country defaults in the high-repayment-cost state, and it repays its debt in the low-repayment-cost state, the optimal amount of borrowing is given by*

$$L^*_{MD} = \begin{cases} 0, & \rho \leq \overline{\rho}_M \equiv \frac{1}{a\pi}; \\ \frac{2(a\rho\pi-1)}{\pi\rho}, & \overline{\rho}_M < \rho < \widehat{\rho}_M; \\ a-1, & \rho \geq \widehat{\rho}_M \equiv \frac{2}{\pi(1+a)}; \end{cases} \tag{9}$$

and the associated utility is given by

$$V^*_{MD} = \begin{cases} V^*_{MD1} = -\frac{\rho C}{1-\rho}, & \rho \leq \overline{\rho}_M; \\ V^*_{MD2} = -\frac{\rho}{1-\rho}C + \frac{2(a\pi\rho-1)}{\pi^2\rho}, & \overline{\rho}_M < \rho < \widehat{\rho}_M; \\ V^*_{MD3} = -\frac{\rho}{1-\rho}C + \frac{(a-1)^2\rho}{2(1-\rho\pi)}, & \rho > \widehat{\rho}_M. \end{cases} \tag{10}$$

Proof: In Appendix

Thus, also in the case in which the country borrows from the market (and defaults in the high-repayment-cost state), there are three different regimes. If the probability of requiring financial assistance is low, $\rho \leq \overline{\rho}_M$, the market cannot lend, as it will not be repaid: the value of the borrowing relationship is so low that the country always finds it in its interest to default (even in the low-repayment-cost state). On the other hand, if the probability of requiring emergency lending is high, $\rho \geq \widehat{\rho}_M$, then maintaining the relation with private lenders is very valuable, and the unconstrained optimal amount of market lending can be supported in equilibrium. In intermediate cases, $\overline{\rho}_M < \rho < \widehat{\rho}_M$, the private sector is willing to lend, but the constraint that the lender must be repaid in the low-repayment-cost state binds. This repayment constraint then determines the amount of market lending.

In the discussion above, we derived the optimal amount a country can borrow from the market when there is default in the high-repayment-cost state and when the country repays in the low-repayment-cost state. For this to be an equilibrium, it should indeed be the case that it is in the country's interest to default in the high-repayment-cost state, that is,

$$V_{ND} - k\frac{L^*_{MD}}{\pi} \leq -\frac{\rho C}{1-\rho}, \tag{11}$$

where V_{ND} is the value of borrowing from the market L^*_{MD} when the market charges the risk-adjusted rate, but the country never defaults. Of course, this condition is always verified if $\rho \leq \widehat{\rho}_M$: if the non-default constraint is binding in the low-repayment-cost state, it is *a fortiori* binding in the high-repayment-cost state. In the Appendix, we show that, when $\rho > \widehat{\rho}_M$ the country has no incentive to deviate from the MD strategy if

$$\rho \leq \frac{2k}{(a+2k-1)\pi} \equiv \rho^c, \tag{12}$$

with

$$\widehat{\rho}_I < \rho^c. \tag{13}$$

If, instead, $\rho > \rho^c$, then the country will not default in any state, and the market will be willing to offer the very same loan the IFI offers at the risk-free rate. This follows directly from the fact that if the country has no incentive to default on L^*_{MD} in the high-repayment-cost state, *a fortiori* it will have no incentives to default on the optimal amount of borrowing L^*_I. The argument is straightforward, and we omit the formal proof. Hence, it follows that

Proposition 2 *For sufficiently low values of* π, $\pi < \widehat{\pi} \equiv \frac{2k}{a+2k-1}$, $L_M^* = L_{MD}^*$ *is the equilibrium level of market borrowing, and the country defaults in the high-repayment-cost state; when, instead,* $\pi \geq \widehat{\pi}$, *if* $\overline{\rho}_M \leq \rho \leq \rho^c$, $L_M^* = L_{MD}^*$ *is the equilibrium level of market borrowing, and the country defaults in the high-repayment-cost state, while if* $\rho > \rho^c$, $L_M^* = L_I^*$ *is the equilibrium, the market charges the risk-free rate, and there is no default in either state.*

Proposition 2 confirms the intuition that, when the need for financial assistance is rare, the incentives of a country to repay are lower and market lending cannot be risk free. In contrast, if the need for financial assistance is more frequent, then the value of keeping the borrowing relation in good standing may be high enough that the market can mimic the official sector and also lend risk free.[16] In addition, if the probability of a low-repayment-cost state is low, then it is never the case that the market offers risk-free loans. The reason is that this negatively affects the value of the lending relationship, and thus reduces countries' incentives to repay lenders.

4.1 Comparison between Market and IFI Lending

Looking at Figure 3, which compares market and IFI lending, it is evident that, for low values of ρ, no lending, whether IFI or market, can be supported in equilibrium. When ρ is very small, the value of the lending relationship is low so that the country will have the incentive to default, no matter what the repayment cost is. At higher values of ρ, $\rho > \overline{\rho}_M$, market lending is supported, and the country will repay in the low-repayment-cost state and default in the high-repayment-cost state. At somewhat higher values of ρ, $\rho > \overline{\rho}_I$, the IFI can lend and, as the value of the relationship is higher, it can expect to be repaid even in the high-repayment-cost state. At still higher values of ρ, the market can replicate the IFI and also lend risk free. The comparison between market lending and IFI lending is most interesting for intermediate values or ρ, when ρ is high enough to support both types of lending but not so high as to eliminate the difference between the two. We can then show that

Proposition 3 *For low values of* ρ, $\rho < \widetilde{\rho}$, *the utility level associated with market lending is higher than that associated with IFI lending. For intermediate values of* ρ, $\rho \in [\widetilde{\rho}, Min\{\rho^c, 1\}]$, *the utility associated with IFI lending is higher than that associated with market lending. If* $\pi \geq \widehat{\pi}$, *for sufficiently high values of* ρ, $\rho \in [\rho^c, 1]$, *the utility value of market and IFI lending is the same.*

Proof: In Appendix.

Proposition 3 implies that, as shown in Figure 3, when the probability of requiring emergency financial assistance is low, market lending dominates IFI lending. The intuition is that, given the existence of a high-repayment-cost state, market lending *de facto* offers a state-contingent contract. Given the low probability that financial assistance will be needed again, the costs associated with losing market access are limited when compared with the gains associated with state-contingent repayments/defaults. Hence, for low values of ρ, the utility associated with market lending is higher than that associated with IFI lending. When the probability of requiring financial assistance in the future is large, the market and the IFI offer the very same volume of lending, at the same price, and thus they bring the same utility levels. For intermediate values of ρ, however, the utility associated with IFI lending is higher. It is more valuable for the country to obtain a lower rate of interest and to be assured continued access than to pay a higher rate, default and lose access. IFIs are able to offer this contract (and maintain preferred creditor status), as they can limit their lending to volumes that are consistent with repayment in all states. In our model, preferred creditors do not price risk, as by definition they lend risk free. Instead, they manage risk by restricting credit volumes, if and when it is necessary to do so.

[16] Notice that the equilibrium characterized above is not necessarily the only market equilibrium in the interval $(\widetilde{\rho}, \rho^c)$–where $\widetilde{\rho}$ is the value of ρ, such that $\rho > \widetilde{\rho} \Longleftrightarrow V_I^* > V_{MD}^*$. There can be another equilibrium in this interval where the market believes that the country always repays creditors and thus charges the risk-free rate. However, we can show that Propositions (2), and (3) below hold true if we had selected the other equilibrium.

5 Market and IFI Lending: The Blended Case

In the previous section, we discussed IFI and market borrowing separately and independently of each other. However, in general, countries may borrow both from the market and from IFIs; in such a situation, the volume of market lending will affect optimal IFI lending and vice versa. In addition, even if the market does not lend, the very possibility that it can lend may affect the volume of lending extended by a preferred creditor. In this section, we thus allow the country to borrow both from a set of competitive private lenders and from an IFI. Once again we will investigate feasible and optimal lending allocations, imposing the restriction that the preferred lenders should be repaid in all states. We refer to this as the blended case, denoted by the subscript B.

When there are two types of lenders present, a critical assumption is what happens to the country in terms of access to further borrowing if it defaults. Our first approach is to say that if there is default on one type of lender, the borrower is excluded from further lending from that lender but there is no cross default. In other words, if there is default on the IFI, this curtails access to further borrowing from that source but not from the market. And if there is default on private lenders, this curtails access from that source but not from the IFI. Under this symmetric assumption of no cross-default, the space for the IFI to lend is constrained relative to a setting where default on the IFI also triggers a loss of access to the private market. The IFI would typically be able to lend more, and still expect to be repaid, under that arguably more realistic assumption . We start by exploring the stricter, symmetric, assumption as we wish to show that even under these unfavorable conditions IFI preferred lending may be supported in equilibrium. In the case where there is cross-default, preferred lending is supported for a wider set of parameter values, as we show in Section 6.

Assuming that the country always repays the IFIs, and that the country repays the market in the low-repayment-cost state and defaults in the high-repayment-cost state, the value function can be written as

$$V_{B_t} = V_{IB_t} + V_{MB_t}, \tag{14}$$

where

$$V_{IB_t} = \rho(-C + aL_{IB_t} - \frac{{L_{IB_t}}^2}{2} - L_{IB_t}(\pi + (1-\pi)k) + V_{IB_{t+1}}), \tag{15}$$

$$V_{MB_t} = \rho(aL_{MB_t} - \frac{(L_{IB_t} + L_{MB_t})^2 - L_{IB_t}^2}{2} - L_{MB_t} + \pi V_{MB_{t+1}}), \tag{16}$$

and where (15) denotes the value of the relation with the IFI, in a similar vein to (1), and (16) represents the additional utility associated with borrowing L_{MB} from the market at an interest rate of $1/\pi$, when the country has already borrowed L_{IB} from the IFIs.

As discussed previously, IFIs differ from the market in that they can commit to the amounts they lend, while commercial lenders are not able to make a similar commitment. It is therefore natural to solve the blended case assuming that the IFI moves first (as a Stackelberg leader) and decides how much to lend anticipating the volume of loans that the country would then choose to take from the market, with the interest rate charged by private lenders reflecting the default risk. When we solve for the optimal amount of market lending for a given level of IFI lending, we obtain

Lemma 2 *If the country defaults on the market in the high-repayment-cost state, and honors its debt in the low-repayment-cost state, for any given amount of risk-free IFI lending L_{IB}, the optimal amount of market borrowing is given by*

$$L^*_{MB}(L_{IB}) \begin{cases} a - 1 - L_{IB}, & \text{if} \quad 0 < L_{IB} < \widehat{L} \equiv a + 1 - \frac{2}{\pi\rho}; \\ \frac{2(\rho(a\pi - \pi L_{IB}) - 1)}{\pi\rho}, & \text{if} \quad \widehat{L} < L_{IB} < \overline{L} \equiv a - \frac{1}{\pi\rho}; \\ 0, & \text{if} \quad L_{IB} > \overline{L}; \end{cases} \tag{17}$$

and the associated utility by

$$V_B(L_{IB}) = \begin{cases} V_{B1} = -\frac{\rho C}{(1-\rho)} + \frac{L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)} + \frac{(1-a+L_{IB})^2\rho}{2(1-\pi\rho)}, & \textit{if} \quad 0 < L_{IB} < \widehat{L}; \\ V_{B2} = -\frac{\rho C}{(1-\rho)} + \frac{+L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)} + \frac{2((a-L_{IB})\pi\rho-1)}{\rho\pi^2}, & \textit{if} \quad \widehat{L} < L_{IB} < \overline{L}; \\ V_{B3} = -\frac{\rho C}{(1-\rho)} + \frac{L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)}, & \textit{if} \quad L_{IB} > \overline{L}. \end{cases} \tag{18}$$

Proof: In Appendix.

Once again, there are three different regimes and, in this case, we choose to differentiate them by the amount of IFI lending offered, so that (17) can be thought of as a reaction function—how market lending responds to the chosen volume of lending by the IFI.[17] The reaction function has a different specification in each regime. In the first regime, the volume of IFI lending is low and the optimal volume of market lending is unconstrained. This implies that, for each additional dollar of official lending, market lending is reduced one to one. In the second regime, the volume of IFI lending is higher, and the market-repayment constraint in the low-repayment-cost state is now binding. Here, for each additional dollar of IFI lending, market lending must thus fall more steeply. In the third regime, IFI lending is higher still and no market lending is supported.

The constraint that the private sector must be repaid in the low-repayment-cost state is embedded in these reaction functions but, so far, the fact that preferred creditors must be repaid in all states has been ignored. A necessary and sufficient condition for the IFI to always be repaid is that, in the high-repayment-cost state, the country is better off repaying, rather than defaulting and relying henceforth solely on the market.[18] Formally:

Definition 1 *L_I is risk free if*

$$V_B(L_{IB}) - kL_{IB} > V_M, \tag{19}$$

and the set $\mathbf{L}^*_{IB}$ where IFI lending is risk free is

$$\mathbf{L}^*_{IB} = \{L_{IB} \mid V_B(L_{IB}) - kL_{IB} > V_M\}. \tag{20}$$

Let us now start investigating under which conditions IFI lending is risk free and how the different variables affect the set $\mathbf{L}^*_{IB}$. We begin by proving that

Lemma 3 *If $k^a \equiv \frac{a-\pi}{1-\pi} > k > \frac{\pi(\pi(a-1)-2)}{4(1-\pi)} \equiv k^b$, for a sufficiently high probability of requiring financial assistance ($\rho > \rho^a > \overline{\rho}_I$) the set $\mathbf{L}^*_{IB}$ where official lending is risk free is non-empty.*

Proof: In Appendix.

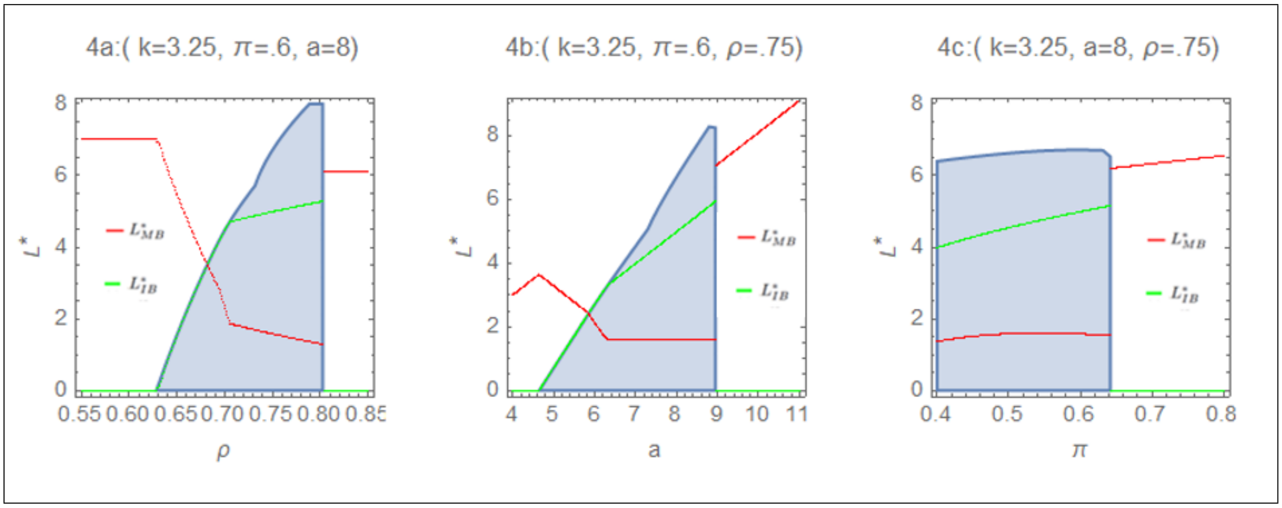
The blue/gray shaded area in Figure 4 depicts the set of the IFI's risk-free lending as a function of the different parameters of the model. At very low values of ρ (the probability of requiring financial assistance), neither preferred lending (the green line) nor market lending (the red line) is supported. As ρ rises, market lending becomes feasible and then, at still higher values of ρ, preferred lending also becomes feasible and the amount of IFI preferred lending increases as ρ increases. In cases where the value of financial assistance is large (high a) and when such assistance is required more frequently (high ρ), countries are able to borrow more as they have greater incentives to repay. Note that the volume of feasible IFI lending also increases

[17] Note that we do not allow IFIs to re-optimize should the country default on the private sector. A justification for this is that countries' lending envelopes with IFIs tend to be relatively fixed. Moreover, re-optimization would imply greater lending from IFIs, but this is generally frowned upon as it may be seen as rewarding a country that had defaulted on the market. Technically, the continuation value of only being able to borrow from IFIs is like a constant outside option and so this assumption has little bearing on the overall nature of most of the results.

[18] To simplify the analysis and to focus on the policy relevant cases, we rule out the possibility that the country would default on the IFI, then borrow from the market, and use the lending proceeds to repay its IFI arrears and resume IFI borrowing.

with π, that is, when the probability of the high-repayment-cost state declines. However, as ρ, a and π increase, at a certain point preferred lending becomes infeasible. This happens when the market becomes willing to offer loans on terms similar to those offered by IFIs, in which case there is very little cost[19] in defaulting on the IFI and thus no risk-free IFI lending can be supported in equilibrium.[20]

Figure 4: Optimal and safe lending (I)



5.1 Optimal Lending

Let us now switch our attention to the optimal amount of IFI lending. To compute the optimal lending levels, we not only take into account the reaction functions (17), computed assuming that the country defaults on the market in the high-repayment-cost state, but we also consider the possibility that the market can mimic IFIs, lending the same amount risk-free at the risk-free rate of interest. In the most interesting cases, from the standpoint of this paper, the country will choose to borrow both from the IFI, which offers the risk-free rate, and from the market, which offers a more expensive contract, anticipating default in the high-repayment-cost state.

Figure 4 also plots the optimal volume of IFI (preferred) and private (defaultable) lending as a function of ρ, α, and π. For the sake of brevity, we will focus our discussion on the first case, depicted in panel 4a. The optimal lending volume by the market is depicted by the red line and that of the preferred IFI lending by the green line. At very low values of ρ, lending is infeasible. As this region is not of interest we do not show it in the figure. At higher ρ (the probability of negative shocks), market lending is feasible and the optimum consists of solely borrowing from the market. As ρ rises, risk-free IFI lending becomes feasible but it is constrained and it rises withρ. As IFI lending rises, market lending falls in the blended optimum.

To understand fully the interaction between the market and IFI schedules (the red and the green lines) it is useful to note (comparing Figure 4 with Figure 3 or considering Figure 5 below) that when both the market

[19] To be precise, if $\rho > \rho^c$, for low values of k, $k < k^T \equiv \frac{1+\pi}{\pi}$, there are no gains for the country to borrow defaultable debt at the risk-adjusted rate and thus there is no cost in defaulting from the IFI. If $k > k^T$ it would instead be optimal for the country to borrow both from the IFI and from the market. However, such gains are not large enough to induce the country to repay the IFI.

[20] For this particular result to hold, the assumption that IFIs do not re-optimize the volume of lending when the country default on the market is critical. Were this not the case, we could end up in a situation where, notwithstanding the fact the risk-free lending is feasible, neither the market nor IFIs would be willing to lend because it would always be in the country's interest to default on one type of lender and borrow from the other thereafter.

and IFI lenders are present, preferred IFI lending is feasible only for values of ρ that are higher than those for which, absent market borrowing, IFI lending is feasible. The reason is that the very presence of a market alternative increases the incentives to default on IFI loans. Intuitively, when the V_I and the V_{MB} schedules cross, at $\rho = \widetilde{\rho}$, the value of market and IFI lending are the same. Hence, it would be in the country's interest to default on the IFIs (saving on debt service) and borrow from the market; this makes preferred lending infeasible. For higher values of ρ, $\rho > \rho^a \equiv \frac{2k}{2\pi k+(1-\pi)+\sqrt{(1-\pi)(4(a-1)k+(1-\pi)}}$, the advantages of IFI lending *vis-à-vis* market lending are greater and a positive volume of both IFI and market lending may be sustained in equilibrium. But then, at a higher value of ρ ($\rho = \rho^c$), the market is also able to offer risk-free loans, as we showed in Section 4, and this undermines IFIs' ability to lend risk free and be repaid in equilibrium. Formally, we can prove the following,

Proposition 4 *If $k \in [k^b, k^a]$ and $\rho \in (\rho^a, \rho^c)$, $L^*_{IB} > 0$ and the presence of IFIs strictly improves welfare.*

Proof: In Appendix

As IFI safe lending becomes feasible, it is at first constrained (the optimum is at the frontier of the feasible set), and as ρ rises, the optimal level of IFI lending increases and market lending falls. At higher values of ρ, IFI lending becomes unconstrained—this is where the optimum for IFI lending leaves the frontier of the feasible set. In this region, the IFI offers loans at lower interest rates, but those may become onerous to repay in the high-repayment-cost state; in contrast, the market offers loans at higher interest rates but will face default should the high-repayment-cost state materialize. Of course, the higher the probability of requiring emergency lending ρ, the higher is the appeal of relying on IFI *vis-à-vis* market lending so that, as ρ rises, optimal IFI lending also rises, while optimal market lending falls.

In order to better understand the welfare implications of the three different types of lending, in Figure 5, we plot the value of the different lending relation (5a), the lending volumes (5b), and the welfare associated with the blended case with that of only the market and only IFI lending—to assess the welfare contribution of IFI lending (5c and 5d).

As discussed, for low values of ρ ($\rho < \widetilde{\rho}$), relying solely on market lending is clearly preferred. Things become more interesting in the interval $[\widetilde{\rho}, \rho^a]$ where IFI lending would be optimal but it is not feasible in the blended case. This is because the very presence of market lenders would induce the country to default on IFIs. In this region, it would be in the country's self-interest to be barred from borrowing from private creditors and only borrow from IFIs. In the interval $[\rho^a, \rho^b]$ market lending does not completely crowd out IFIs' but, if private lending could be barred in this region, the country can borrow larger amounts from IFIs, and this would strictly improve welfare—see Figure 5d. Note that having the possibility of blended lending adds value (relative to just market) in the intermediate region for ρ—see Figure 5c. This is when the presence of IFIs strictly improves welfare.

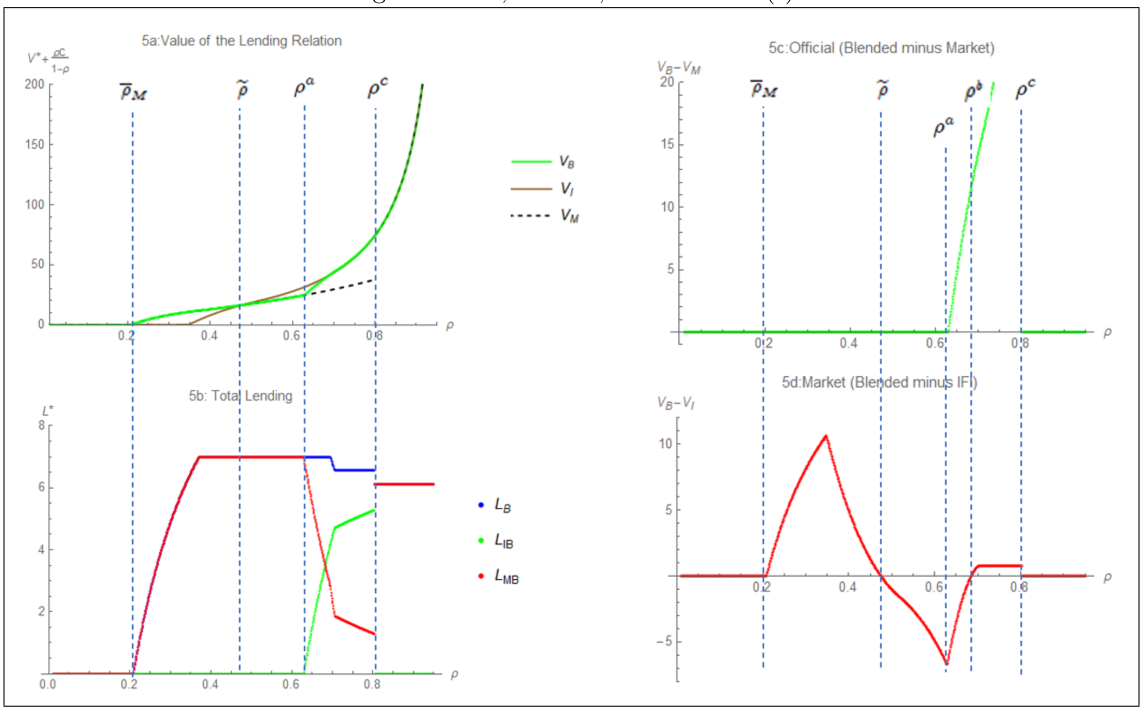
6 Extensions

6.1 Allowing for Cross-Default

In the previous analysis, we assumed that if a country defaults on one type of lender (the market or the IFI), this would not affect its ability to borrow from the other type of lender. Even under this strict assumption, we demonstrated that an equilibrium with value-added IFI lending could be sustained for a range of parameter values.

However, default on IFIs would no doubt affect a country's credit standing with private lenders and could indeed curtail the ability to borrow from the private market. Risk managers in pension funds, commercial banks and other institutions, exercising their due diligence, or fearful of a regulator's actions, may well prevent the purchase or continued holding of bonds issued by countries that had defaulted on a lender with a preferred status.

Figure 5: IFI, market, and blended (I)



In this section, we consider alternative assumptions. First, assume that if the country defaults on the preferred lender then it has no further access to both types of lending. In that case, V_M in (19) is replaced with a value function that assumes there is no subsequent access to borrowing, $V_{NA} = -\frac{-\rho C}{(1-\rho)}$. As $V_M > V_{NA}$, It follows immediately that the set $\widetilde{\mathbf{L}}^*$, where IFI lending is risk free, namely

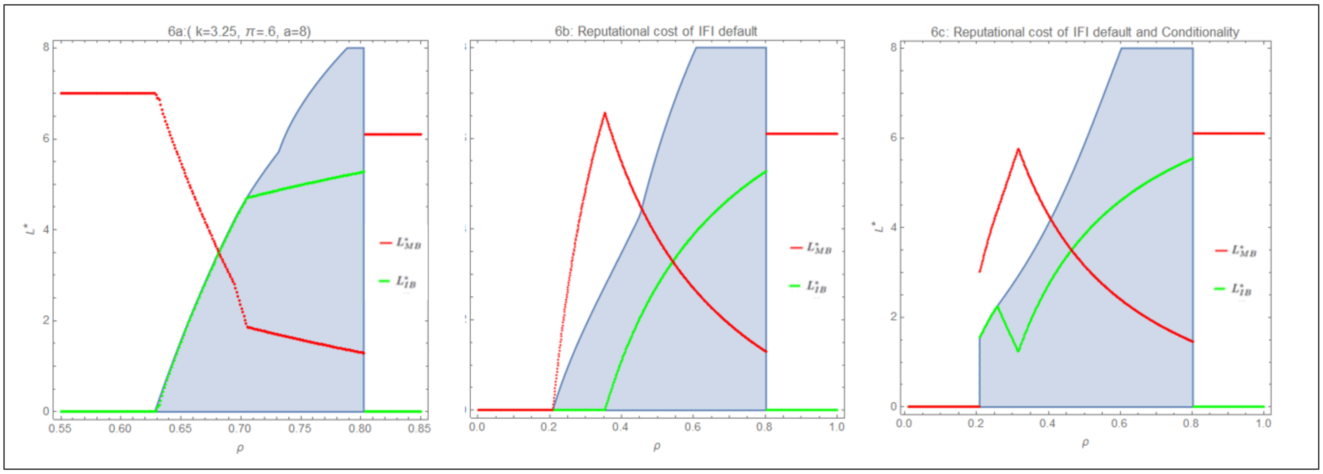
$$\widetilde{\mathbf{L}}^*_{IB} = \{L_{IB} \mid V_B(L_{IB}) - kL_{IB} > V_{NA}\}. \tag{21}$$

contains the set for risk-free lending, as previously defined in (20), where a default on the IFI does not affect the country's ability to borrow from the market. The main result in the paper, that preferred lending can be supported in equilibrium, is then reinforced by this change in assumptions. From the fact that the feasible set for lending contains the previous set, it immediately follows that welfare must be at least as high as in the previous case and it could be greater.

To illustrate, Figure 6 presents the results from a numerical simulation in similar vein to Figure 5. The impact of cross-default on the feasibility of greater IFI lending may be substantial. In the second panel, where cross-default is assumed, the feasible set is considerably larger and optimal IFI lending becomes positive at lower values of ρ. Note that as IFI lending increases, this crowds out private borrowing. As IFI lending was not feasible before, at these values of ρ, welfare is higher than in the case of no cross default. Indeed, whenever optimal IFI lending was constrained by the feasibility constraint in the case of no cross-default—that is, IFI lending was on the border of the feasibility set—welfare rises when there is cross-default and reputations are shared. This is clear from a simple inspection of Figure 7 where, for the same parameter values as in Figure 4a, we compare how the introduction of a reputational cost of IFI default (green line) increases welfare by relaxing the IFI constraint.

There is also the logical possibility of cross default in the opposite direction: that default on commercial lenders might exclude countries from borrowing from an IFI. This seems hard to square with the current thinking in global financial architecture debates, which calls for "private sector involvement" (meaning the

Figure 6: Optimal and safe lending (II)



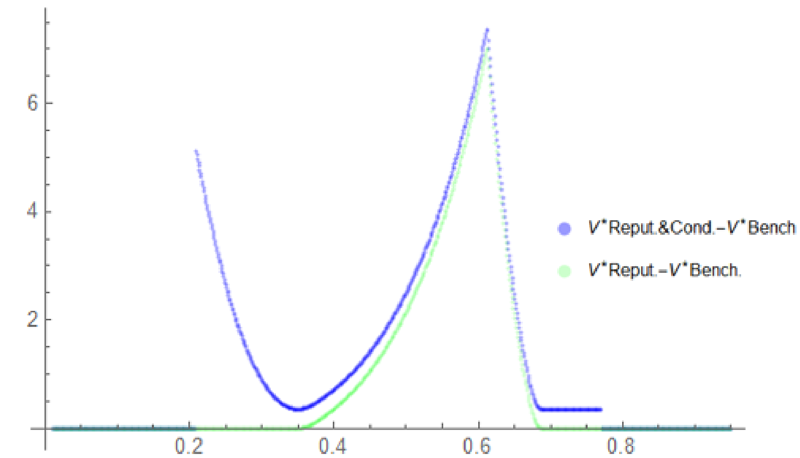
private sector taking a haircut) in the context of international support packages from preferred lenders. Moreover, in our model such an assumption would lead to the peculiar result that a country would default on the IFI in all situations in which it defaults on the market, since the relationship with the IFI becomes worthless after market default. In addition, if default on the IFI precludes market lending, then the only possible equilibrium is with only private lenders. Hence, shared reputation in this direction, if it does anything, reduces welfare.

6.2 Conditionality

An interesting further possibility is if commercial lending is made contingent on good standing with the IFI and, in addition, IFI lending is contingent on the repayment of market lenders in the low-repayment cost state. In other words, in this case we combine the idea that there would be no access to private lenders if there is default on the IFI with that of no IFI lending if the borrower defaults in the good state, which might be thought of as a non-excusable default. This could be considered a form of conditionality, where the condition is that countries should only default in bad states and not expect to borrow from the IFI if they do not repay private creditors in a good state of the world. This combination would be clearly welfare-improving for all situations in which market borrowing is constrained by the constraint that the country should repay the market in the low-repayment-cost state, as it would increase the amount that the country can borrow ex ante.

Again, a numerical simulation exercise is helpful to illustrate this point—see Figure 6c. The main effect of introducing such a form of conditionality is that, now, an increase in official lending can relax the low-repayment-cost state default constraint. This means that IFI and market lending may become complements rather than substitutes—the slope of the reaction function changes and may even change sign. In other words, an increase in IFI lending will increase the cost of defaulting on the market in the low-repayment-cost state, and this may relax the market borrowing constraint. This conditionality leads to a better mix of IFI and market borrowing, thus increasing welfare. Welfare is increased both with respect to the benchmark case and to the case where only reputation is shared so the country cannot borrow from the market if there is default on the IFI, see Figure 7, blue line.

Figure 7: Welfare under different cross-default rules



7 Robustness

The results outlined above rely on two critical assumptions raised in section 3 when we described the basics of the model. In this section we consider the implications of relaxing these assumptions.

First, we assumed that the cost of repayment in the high-repayment-cost state is large enough so that the country would wish to borrow from the market and default in the high-repayment-cost state, $k > \frac{a-\pi}{a\pi-1}$, but that it is not so large that the country is tempted to default on the IFI (in the high-repayment-cost state) and then wait until a low-repayment-cost state materializes, clear the arrears and regain access to borrowing, $k < 1 + \frac{1}{\rho(2\pi-1)}$.

If $k < \frac{a-\pi}{a\pi-1}$ then, for low values of ρ, no lending (from the IFI or from the market) can be supported in equilibrium. However, at higher values of ρ, $\rho > \overline{\rho}_I$, official lending is supported but market lending is not. It is only at somewhat higher values of ρ, $\rho > \overline{\rho}_M$, that the market can now lend as well. As before, at still higher values of ρ, the market may be able to replicate the IFI and also lend risk free. The utility associated with IFI lending is always higher than that associated with market lending, except, of course, in the cases in which the market can replicate the IFI and lend the same amounts, that is for $\rho > \rho^c$. Again, these general results can be illustrated via a numerical simulation—see Figure 8.

In the blended case, the picture is a more complicated one, see Figure 9. Now, for low values of ρ, official lending is the only possibility, but it is crowded out by market lending when the latter becomes an option. And it is only for much higher values of ρ the two can coexist in equilibrium.

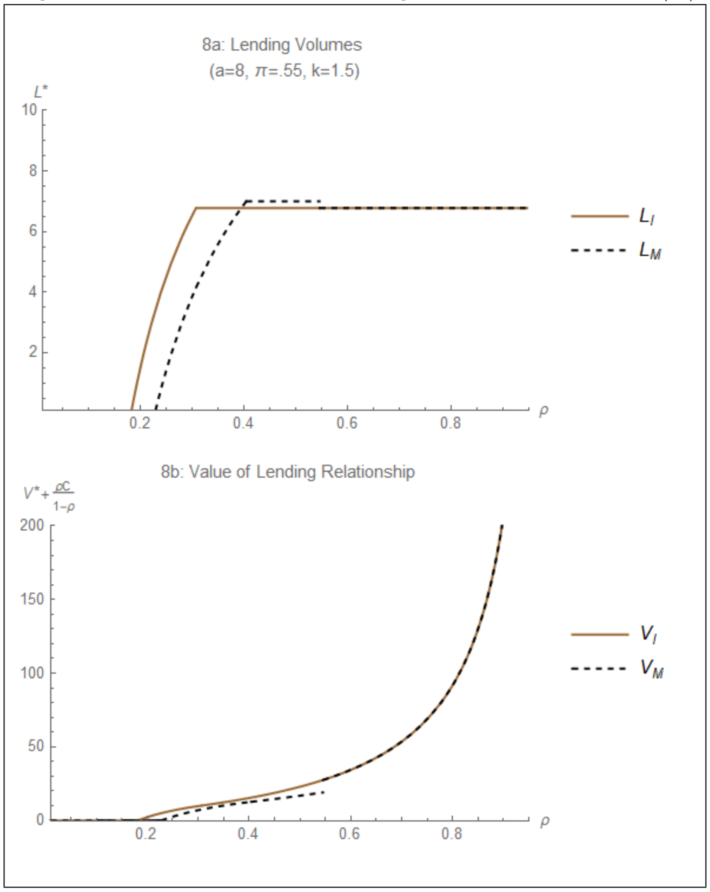
However, it is worth pointing out that in this situation market lending does not add value. This general observation is illustrated through a numerical simulation shown in Figure 10. Interestingly, this would imply that official lending should be contingent on no market borrowing in this case.

The second assumption is that $k < 1 + \frac{1}{\rho(2\pi-1)}$, which rules out the case that private creditors would default in a high-repayment-cost state but would repay if a low-repayment cost state materializes. This constraint puts an upper bound to the amount that the IFI can lend risk free, namely

$$L_I \leq 2(a - (k-1)\frac{1-(1-\pi)\rho}{\rho}).$$

If the constraint is not met ($k > 1 + \frac{1}{\rho(2\pi-1)}$), then the set L^*_{IB}, where official lending is risk free shrinks, reducing the amount the IFI can lend in equilibrium—see Appendix 1. This is a general result but there

Figure 8: IFI and Market Lending: Volumes and Value (II)



are different forces at play. To illustrate those forces a numerical simulation is again useful. In Figure 11 below, the yellow area denotes the reduction in the amount the IFI can lend risk-free in this case. Again, the red and the green lines denote, respectively, the optimal IFI and market lending in this constrained equilibrium—while the light blue and pink lines (labelled as "no evergreening" in Figure 11 illustrate the optimal amount of lending that would occur if the IFI could commit not to resume lending, if payment to the IFI is delayed but occurs in a future low-repayment-cost state. In other words, if the IFI is unable (or unwilling) to ban countries in default from borrowing again once arrears are cleared in a low repayment-cost state, then this reduces the amount that the IFI can lend risk-free. The associated welfare loss can be viewed as the cost of IFIs' inability to commit not to pursue this rather specific form of evergreening.

These arguments relate to the internal consistency of the model. In the real world, countries and IFIs may make decisions that are driven by forces beyond the logic of the model. For example, suppose an IFI, for whatever reason, lends more than it "should" if it wishes for its debt to remain default free. If this is the case, then we might speculate that the IFI may pursue a different type of evergreening, and it may also be in the interests of the country to agree to that. As this lies outside of the logic of our model, we simply suggest it as a possibility. To understand this fully would require a different approach and perhaps one that incorporates political as well as economic forces; we leave this as an interesting future line of research.

Figure 9: Optimal and safe lending (III)

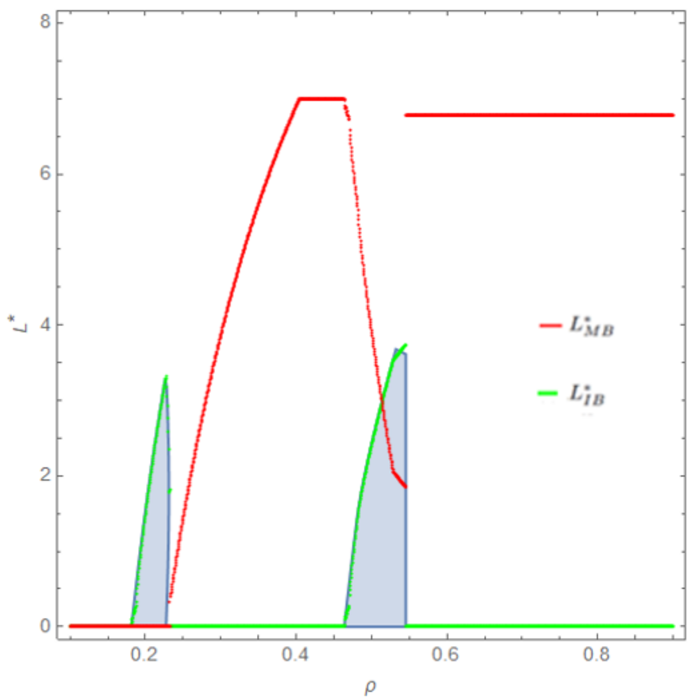
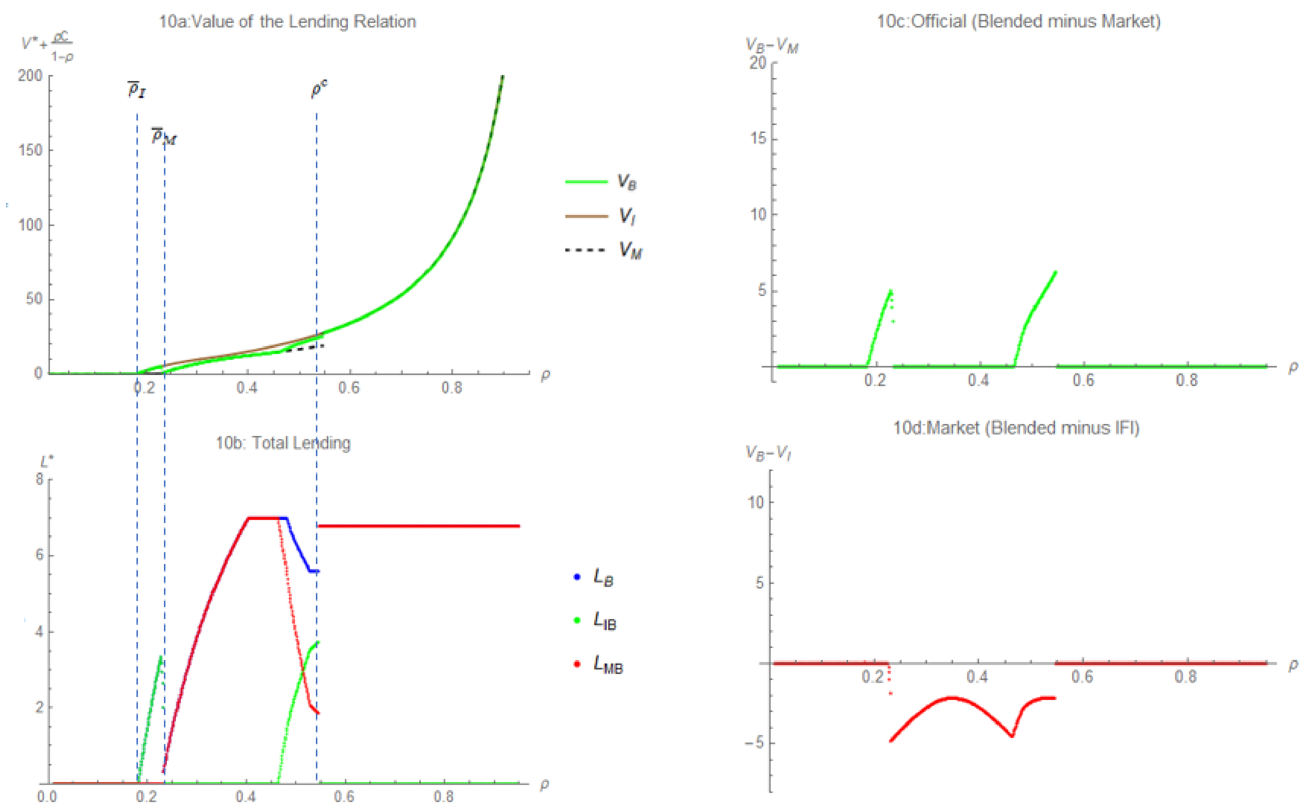


Figure 10: IFI, market, and blended (II)



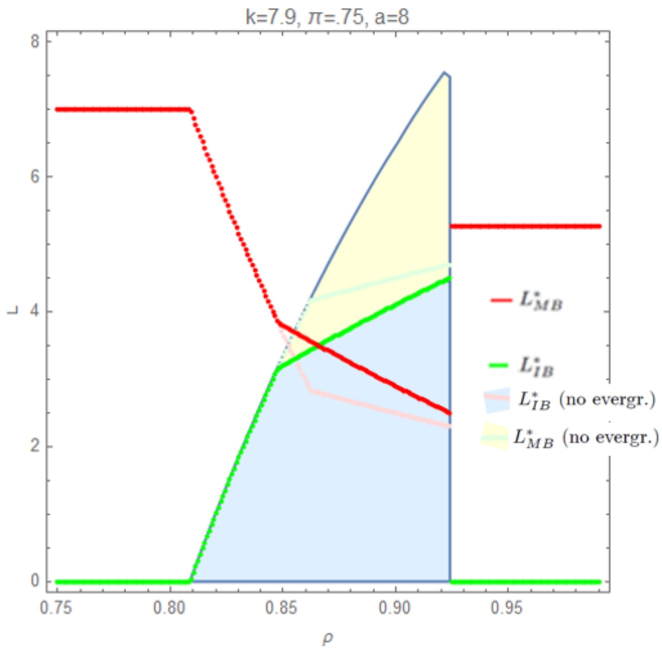
8 Interpretation of the Results

In our modeling strategy, we stripped the problem down to a set of core elements. This allowed us to obtain a set of closed-form, analytical results, something relatively rare in the current sovereign debt literature. However, questions may arise on how we should interpret the different parameters of the model and on whether our results would hold true in a more general set-up. In this section we address these questions.

In the characterization of the analytical solutions to our model, we have mostly focused on the parameter ρ, the probability that a country requires financial assistance. A main finding is that IFI lending is more valuable for intermediate values of ρ. The intuition is that if ρ is low, no lending would be supported because the country will be tempted to default and, if ρ is very large, the value of the lending relationship is so high that the country will always repay and can thus borrow risk free from the market. An important assumption behind these results is that the probability of requiring such assistance and its value (or intensity governed by the parameter a) are orthogonal, which in reality may not be the case. Had we assumed that frequency and intensity were correlated, perhaps negatively, both components would affect the value of the lending relationship.[21] In addition, in the case of a high probability of requiring assistance but where such assistance had a relatively low value, we then find that the market could provide lending with a value similar to that of IFIs. This could be considered akin to normal business-cycle lending, perhaps more common in some advanced economies.

[21] It is worth noting that in a more general model the relative weight of the two might also depend on risk aversion.

Figure 11: Optimal and safe lending (IV)



In our model, the parameter a can be thought of as the marginal value of obtaining the first dollar of financing, and hence as a measure of the severity of the emergency in which financial assistance is required. In a standard sovereign debt model this would be analogous to the marginal utility of consumption. Our results indicate that as a rises, the value of the lending relationship with the preferred lender is increased, supporting higher lending volumes from the IFI. At high levels of a, however, the value of the lending relationship might become so great that the country will choose not to default on commercial borrowers. This implies that the country could then default on the IFI and borrow from the market; borrowing from the IFI would be rendered infeasible.

Another key parameter in our analysis is k, the cost of repayment in the high-repayment-cost state. The higher is k, the more valuable is market lending because of the associated "state contingency"—in the bad (or high repayment-cost) state, the country will default on the market. A higher cost of repayment may reflect lower fiscal revenues at the time repayment is required or the political dynamics in the country concerned. In some cases, repaying debts can become a political issue, because the same resources could be used to satisfy urgent needs of the local population. It can also be thought of as the inverse of the additional cost of default in a standard debt model. The higher is k, the greater are the net benefits from defaulting in the high-repayment-cost state. Notice also that, as we normalized the cost of repayment in the low-repayment-cost state to one, k can be thought of as a measure of the volatility of the cost of servicing the debt. Hence, it may reflect the volatility of fiscal revenues or the volatility of political tensions regarding debt repayment.

With the aim of simplifying the analysis, we assumed that k could only assume two values, which we can think of as high or low. In such a world, to offer loans at the risk-free interest rate, IFIs need to limit their lending to be repaid in both states. The market may then offer contracts that will be paid only in the low-repayment-cost state. Notice, however, that if we had a continuum of values of k, IFIs may not wish to be constrained to only lend expecting to be repaid in absolutely all states. Consider, for example, the aftermath of a major financial crisis, a conflict, or a large natural catastrophe. Probably, in such states, the country will either enter into arrears with IFIs, or its obligations will be "evergreened." Strictly speaking, totally risk-free lending may not exist. Hence, in a more general set-up, it may be optimal for IFIs to offer loans that are repaid in full in almost all states, while the market will offer loans that are repaid less often. Our qualitative results would carry through.

We solved for the optimal amount of IFI and market lending, and we illustrated the parameter region where IFIs can lend and are expected to be preferred—the preferred lender feasibility set. It is interesting to note that, as ρ rises and preferred lending becomes feasible, the optimum amount of preferred lending is constrained. Optimal IFI lending is on the frontier of the preferred lender feasibility set. In this region, it might be argued that IFIs should be cautious. An error in the sense of the IFI lending a bit more than the optimum would imply straying from the feasibility set with the consequent danger that the country would not treat the IFI as preferred. In our analysis, it is assumed that all parameter values are known with certainty, but in the real world this may not be the case. On the other hand, for higher values of ρ, the IFI becomes unconstrained as the optimum is inside the feasibility set for preferred lending. At these higher values of ρ, straying slightly from the optimal amount would not endanger being considered as preferred.

9 Conclusion

One of the most persistent puzzles regarding the global financial architecture has been in relation to the preferred creditor status (PCT) of the main IFIs. While PCT is critical to the operations of the IMF and the major MDBs, it is not generally stated in legal contracts, and it is normally described as a market custom. This suggests that it should be understood as an endogenous outcome of the relation of a country with its creditors rather than something that is imposed. And yet we know of no paper to date that provides an explanation of why this may be the case. Our contribution is to provide a relatively simple model that illustrates how IFIs can lend at close to the riskless interest rate and expect to be repaid in all states of nature (as it is always in the country's interest to repay) and how the existence of preferred creditors, in addition to private lenders (which may suffer default), may be valuable. Existing papers that consider sovereigns' willingness to pay do not include seniority, and those models that include seniority do not appear to consider willingness to pay. We believe that this is the first paper that attempts to provide a theoretical justification for both the existence of preferred lenders and a rationale for how they may improve welfare.

What is special about IFIs that they can offer this contract? Their critical characteristic is that they can commit to lend at the risk-free interest rate and they can credibly limit the amount they lend; this is what makes lending at the risk-free interest rate sustainable. Atomistic private lenders are not able to coordinate to replicate this contract. We do not focus here on potential multiple equilibria among private lenders, rather we focus on the standard framework where the country determines the lending volume and private lenders demand an interest rate commensurate with the resulting probability of default.

Naturally, if there is more than one preferred lender there would need to be coordination between them to ensure that the equilibrium is sustainable. In practice, IFIs do coordinate closely, especially when it comes to emergency lending. In general, the IMF takes a leading role, and it is common practice for any IMF program to include programmed assistance from the World Bank and the main regional development banks, depending on the location of the country. These programming commitments are normally agreed upon in consultations between the IFIs and are sometimes referred to as "burden-sharing."

The potentially binding constraint for the preferred lender is that it must be repaid in bad states of nature, when the cost of repayment is high, while the potentially binding constraint for private lenders is that they must be repaid in the good state when that cost is low. Hence, while both lenders offer loans with the same fair ex ante returns, ex post outcomes may differ, and this has implications. Indeed, if markets are incomplete (repayments cannot be made state contingent), having two lenders that offer different conditions is a way of "completing the market." The repayments of market lenders are "state contingent," but they come at the cost of market exclusion after the realization of a high-repayment-cost state; IFI repayments are not state contingent but ensure access even if there is default on the market. As there are two different types of lenders, to some degree they are "substitutes," and this reduces the cost of defaulting on either of them, assuming there is no reputation-sharing. However, increasing the "leverage" of the IFI by assuming that any borrower that defaults on the IFI is excluded from market lending allows for greater IFI lending in equilibrium and increases welfare. This enhanced leverage increases the value of having the two types of lenders, allowing for greater market completeness. Instead, making IFI lending contingent on market repayment is a bad idea, as it rules out the possibility of having two types of lenders. It reduces market completeness, even compared to the case of no reputation-sharing.

Our paper also sheds light on some broader questions about the role of IFIs. In our model the market and IFIs play different roles, as they offer different contracts, one which in the end is state contingent, although subject to the penalty of exclusion in the case of default, and the other where there is always repayment. Given these different roles, there is no reason to think that IFIs should mirror the behavior of private lenders. Indeed, IFIs add value precisely because they behave differently. If IFIs priced loans to risk, as private lenders do, then this would be tantamount to admitting that their lending is risky and that borrowers may then default. Taking this logic to the limit, IFIs would then become just one more (defaultable) lender among many, and may even lose their preferred creditor treatment.

It is worth noting that, in addition to other caveats discussed, we follow much of the sovereign debt literature in assuming that the market and IFIs cannot offer state-contingent contracts, where repayment depends on the realization of the state of nature. Were this the case, countries would be able to borrow more and perhaps avoid costly defaults altogether. There are various reasons why such contracts do not exist. Some argue that a finance minister is generally not blamed if there is a slump in the global market for an important commodity export, but that if a costly hedge is purchased and not needed then there may be an ex post inquiry as to why resources were "wasted." A further intriguing reason, more related to our model, is that, if a state-contingent contract takes the form of requiring lower payments in bad states but higher repayments in good states, then the willingness to repay in the good state may be threatened. In the comparison of the continuing value of the lending relationship versus the cost of repayment, this may actually restrict lending.[22]

A further interesting result is that, for some parameter values, it would be in a country's own best interest to not be able to borrow from private markets. This is an important issue that is discussed at length in IFIs and even merits an acronym, NCBP, which stands for non-concessional borrowing policy, and applies to countries that received debt relief under the Multilateral Debt Relief Initiative (MDRI). More precisely, IFIs sought to restrict the amount of commercial debt such countries could contract, both to avoid the repeat of the build-up of unsustainable levels of debt and in order to preserve PCT. IFIs have tried to implement NCBPs with the threat that future concessional resources might be curtailed if countries did not abide. But it is not easy to enforce such a rule. Countries have many ways to contract debt. For example, public companies may provide an indirect way of borrowing that is hard to monitor.

To conclude, international financial institutions that expect to be paid in all states of nature are very different animals from commercial lenders. While commercial lenders price loans according to risk and should therefore expect restructuring if unfavorable states materialize, IFIs that expect loans to be risk-free play by different rules. But in order to play by those rules their behavior must be consistent. In our analysis, the preferred creditor standing of IFIs can indeed be thought of as a market outcome and, due to the different contracts that are offered and sustained in equilibrium, preferred IFIs may add value.

[22] Anderson, Gilbert and Powell (1989) suggest that the World Bank may wish to guarantee such state contingent contracts. The argument is that markets can manage price risks while multilaterals may have a comparative advantage in controlling "performance risks." This argument depends on the multilateral having some other power of persuasion on the country perhaps coming from its governance structure.

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10 Appendix

Proof of Proposition 1:

Substituting (2) into (3), and solving for L_I, the maximum amount of official lending that will be repaid in both states is given by

$$\overline{L}_I = 2(a - \pi + (\pi - \frac{1}{\rho})k). \tag{22}$$

so that

$$\overline{L}_I > 0 \Longleftrightarrow \rho > \frac{k}{a + \pi(k-1)} \equiv \overline{\rho}_I. \tag{23}$$

Thus, if $\rho \leq \overline{\rho}_I$, no official lending occurs. Consider now the case $\rho > \overline{\rho}_I$. Absent default constraints, in the case of a shock, in period t, the country would choose

$$\widehat{L}_I \equiv \arg\max_{L_I} - C + \ aL - \frac{L_I^2}{2} - ((1-\pi)k + \pi)L_I, \tag{24}$$

that is,

$$\widehat{L}_I = a - \pi - (1-\pi)k. \tag{25}$$

Hence, the solution will be constrained, if $\widehat{L}_I - \overline{L}_I = \frac{2k - \rho(a+k-\pi(k-1)}{\rho} > 0 \Longleftrightarrow \rho < \frac{2k}{a+\pi(k-1)-\pi} \equiv \widehat{\rho}_I$. So the optimum lending will be the constrained solution, $L_I^* = \overline{L}_I$ if $\rho < \widehat{\rho}_I$, and the unconstrained one, $L_I^* = \widehat{L}_I$ if $\rho \geq \widehat{\rho}_I$. Substituting L_I^* into (2) we can then solve for the optimal value of the value function V_I^* for each case.

In writing equation (3), we ruled out the possibility for the country to default on the official sector if the high-repayment-cost state materializes and then to clear its arrears in the next draw of a low-repayment-cost state. Actually, such a strategy is never in the interest of the country if:

$$(k-1)L_I \leq \frac{\rho}{1-(1-\pi)\rho}(aL_I - \frac{1}{2}L_I^2), \tag{26}$$

where $(k-1)L_I$ is the amount the country "saves" by repaying the creditors in the low-repayment-cost states, and $\frac{\rho}{1-(1-\pi)\rho}(aL_I - \frac{1}{2}L_I^2) = (\rho + (1-\pi)\rho^2 + (1-\pi)^2\rho^3 + ...)(aL_I - \frac{1}{2}L_I^2)$ is the expected cost of the lack of access to credit until repayment occurs. Condition (2) can be rewritten as

$$L_I \leq 2(a - (k-1)\frac{1-(1-\pi)\rho}{\rho}) \tag{27}$$

It is easy to verify that if (A1) holds, (23) implies (26).□

Proof of Lemma 1:

Substituting (7) into (8), the maximum amount of lending that will be repaid in the high-repayment-cost state is given by

$$\overline{L}_{MD} = \frac{2(\rho a\pi - 1)}{\pi\rho}, \tag{28}$$

and

$$\overline{L}_{MD} > 0 \Longleftrightarrow \rho > \frac{1}{a\pi} \equiv \overline{\rho}_M. \tag{29}$$

Hence, if $\rho \leq \overline{\rho}_M$, no market lending occurs. Consider the case $\rho > \overline{\rho}_M$. Absent default constraints, in the case of a shock, in period t, the country would choose

$$\widehat{L}_{MD} \equiv \arg\max_{L} - C + \alpha L_{MD} - \frac{L_{MD}^2}{2} - L_{MD}, \tag{30}$$

that is,

$$\widehat{L}_{MD} = a - 1. \tag{31}$$

Hence, the solution will be constrained, if $\widehat{L}_{MD} - \overline{L}_{MD} = \frac{2-\pi\rho(a+1)}{\pi\rho} > 0 \iff \rho < \frac{2}{\pi(a+1)} \equiv \widehat{\rho}_M$. So the optimum lending will be the constrained solution, $L^*_{MD} = \overline{L}_{MD}$ if $\rho < \widehat{\rho}_M$, and the unconstrained solution, $L^*_{MD} = \widehat{L}_{MD}$, if $\rho \geq \widehat{\rho}_M$. Substituting L^*_{MD} into (7), we can then solve for the optimal value of the value function V^*_{MD} for each case.□

Derivation of Condition 12:

Assuming that if emergency lending is required, the country borrows $\widehat{L}_{MD}$ at the risk-free interest rate, and the country repays both in the high and in the low repayment cost state, then the value function is given by:

$$V_{ND} = \rho(-C + a\widehat{L}_{MD} - \frac{\widehat{L}^2_{MD}}{2} - \frac{\widehat{L}_{MD}}{\pi}((1-\pi)k + \pi) + V_{ND}). \tag{32}$$

Solving for V_{ND}, and using (31), we have that

$$V_{ND} = \frac{((2K - (a-1)^2)\pi - 2(a-1)(1-\pi)k)\rho}{2\pi(1-\rho)}, \tag{33}$$

and substituting this value into that condition (11), and using (31), we obtain

$$\rho \leq \frac{2k}{(a+2k-1)\pi} \equiv \rho^c \iff V_{ND} - k\frac{L^*_{MD}}{\pi} \leq -\frac{\rho C}{1-\rho}.$$

Proof of Proposition 3:

To prove Proposition 3, it is useful to start by proving two Lemmas.

Lemma 4 *If $\rho \in [\overline{\rho}_I, 1)$, $V_I - V_{MD}$ increases with ρ.*

Proof: Under Assumption (A1), we have that either $\overline{\rho}_M < \overline{\rho}_I < \widehat{\rho}_M < \widehat{\rho}_I$, or $\overline{\rho}_M < \widehat{\rho}_M < \overline{\rho}_I < \widehat{\rho}_I$. We show that $\frac{\partial(V_I - V_{MD})}{\partial\rho}$ is positive in all the different sub-intervals of ρ. Let first consider the case $\overline{\rho}_M < \overline{\rho}_I < \widehat{\rho}_M < \widehat{\rho}_I$.

(i) In the interval $\rho \in (\overline{\rho}_I < \widehat{\rho}_M]$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{2(k^2\pi^2-1)}{\pi^2\rho^2} > 0$, because of (A1).

(ii) In the interval $\rho \in (\widehat{\rho}_M, \widehat{\rho}_I]$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{2k^2}{\rho^2} - \frac{(a-1)^2}{2(1-\pi\rho)^2}$. We further have that $\underset{\rho\to\widehat{\rho}_M^+}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(a+1)^2(k\pi^2-1)}{2} > 0$, and $\underset{\rho\to\widehat{\rho}_I^-}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(k+1)(1-\pi)(2a-1+k(1-\pi)-\pi)(a+k+(k-1)\pi)^2}{2(a+k-(1+k)\pi)^2} > 0$. Notice further that $\frac{\partial(V_I - V_{MD})}{\partial\rho} = 0$ iff $k = \pm\frac{\rho}{2}\frac{a-1}{1-\pi\rho}$. This means that there is at most one value of $k > 0$ in the interval for which the expression can change sign. Hence the expression is positive in the whole interval.

(iii) In the interval $[\widehat{\rho}_I, 1)$, $\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{1}{2}(\frac{(a-k-\pi+k\pi)^2}{(1-\rho)^2} - \frac{(a-1)^2}{(1-\pi\rho)^2})$. We further have that $\underset{\rho\to\widehat{\rho}_I}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} = \frac{(k+1)(1-\pi)(2a-1+k(1-\pi)-\pi)(a+k+(k-1)\pi)^2}{2(a+k-(1+k)\pi)^2} > 0$ and that $\underset{\rho\to 1}{Lim}\frac{\partial(V_I - V_{MD})}{\partial\rho} > 0$. Notice further that $\frac{\partial(V_I - V_{MD})}{\partial\rho} = 0$ iff $\rho = \frac{k-1}{a-1+(k-1)\pi} \equiv \rho^d$ or $\rho = \frac{2a-1-k+\pi(k-1)}{a(1+\pi)-1-\pi(k-\pi(k-1)} \equiv \rho^e$. Since $k < a \iff \rho^f > 1$, then there is at most one value of $\rho \in (\widehat{\rho}_I, 1]$ for which the expression can change sign. Hence $\frac{\partial(V_I - V_{MD})}{\partial\rho}$ is positive in the interval.

As per the case $\overline{\rho}_M < \widehat{\rho}_M < \overline{\rho}_I < \widehat{\rho}_I$, we should just consider the interval $\rho \in (\overline{\rho}_I < \widehat{\rho}_I]$, where V_I increases in ρ and V_M not. This, together with the fact that $V_I - V_{MD}$ is a continuous function proves the Lemma.□

Lemma 5 *There is a $\widetilde{\rho} \in [\overline{\rho}_I, 1)$, such that for $\rho > \widetilde{\rho} \iff V_I > V_{MD}$.*

Proof: In the interval $[\overline{\rho}_M, \overline{\rho}_I]$, $V_M > V_I$ follows directly for the fact that $L_M > L_I = 0$. We further have that in the interval $\rho \in (\widehat{\rho}_I, 1)$, $V_I - V_{MD} = \frac{\rho}{2}\left(\frac{(\alpha-\pi-(1-\pi\rho)k)^2}{(1-\rho)} - \frac{(a-1)^2}{(1-\rho\pi)}\right)$, so that $\underset{\rho\to1}{Lim}(V_I - V_{MD}) > 0$. This together with the fact that $V_M > V_I$ at $\rho = \overline{\rho}_I$, and that $V_I - V_M$ is increasing in ρ in the interval $\rho \in [\overline{\rho}_I, 1)$, because of Lemma 4, proves the Lemma.□

Now, to prove Proposition 3, first, notice that, if $\rho < \widetilde{\rho}$, $V_{MD} > V_I$, and hence $L_M^* = L_I^*$ cannot be an equilibrium, so that $L_M^* = L_{MD}^*$. For sufficiently high values of ρ, $\rho > \rho^c$, we have that $L_M^* = L_I^*$. Thus if $\rho^c < 1$, that is when $\pi < \widehat{\pi}$, we have that there is a non-empty interval $\rho \in [\rho^c, 1]$ for which $V_M^* = V_I^*$. It remains to prove that that the interval $[\widetilde{\rho}, Min\{\rho^c; 1\}]$ is non-empty so that the interval in which $V_I^* > V_M^*$ is also non-empty. Since, $\rho^c > \widehat{\rho}_I > \widehat{\rho}_M$, where the right inequality follows from A1, it is enough to show that

$$\rho^c > \rho : V_{MD}^* \left|_{\rho > \widehat{\rho}_M} = V_I^* \right|_{\rho > \widehat{\rho}_I} \equiv \rho^f.$$

Using (5) and (10), we have that

$$\rho^f = \frac{(a-1)^2 - (a - k(1-\pi) - \pi)^2}{(a-1)^2 - \pi(a - k(1-\pi) - \pi)^2},$$

with $\rho^f < 1$ and (after some algebra) $a > 1 \Longrightarrow \rho^c > \rho^f$. This proves the Proposition.□

Proof of Lemma 2:

For a given L_{IB}, the additional utility associated with borrowing L_{MB} at an interest rate $1/\pi$ from the market V_{MB_t} is given by (16). Equating V_{MB_t} with V_{MB_t+1} the value of market borrowing (on top of official borrowing) can be written as:

$$V_{MB} = \frac{L_{MB}(2(a-1) - L_{MB} - 2L_{IB})\rho}{2(1-\pi\rho)}. \tag{34}$$

For the market to be willing to offer risky loans, the country must be willing to service this debt in the low-repayment-cost state. This condition can be written as

$$V_{MB} \geq \frac{L_{MB}}{\pi}. \tag{35}$$

Substituting (34) into (35) at equality, the maximum amount of lending that will be repaid in the low-repayment-cost state is given by:

$$\overline{L}_{MB} = \frac{2(\rho\pi(a - L_{IB}) - 1)}{\pi\rho}. \tag{36}$$

We further have that

$$\overline{L}_{MB} > 0 \Longleftrightarrow L_I < a - \frac{1}{\pi\rho} \equiv \overline{L}.$$

Absent default constraints, in period t, the country would choose

$$\widehat{L}_{MB} \equiv \underset{L_{MB}}{\arg\max} V_{MB_t} = a - 1 - L_{IB}. \tag{37}$$

Hence, the solution will be constrained, if $\widehat{L}_{MB} - \overline{L}_{MB} = a + 1 - L_I - \frac{2}{\pi\rho} > 0 \Longleftrightarrow L_I < a + 1 - \frac{2}{\pi\rho} \equiv \widehat{L}$. We thus have that

$$L_{MB}^*(L_{IB}) = \begin{cases} a - 1 - L_{IB}, & \text{if} \quad 0 < L_{IB} < \widehat{L}; \\ \frac{2(\pi\rho(a-L_{IB})-1)}{\pi\rho}, & \text{if} \quad \widehat{L} < L_{IB} < \overline{L}; \\ 0, & \text{if} \quad L_{IB} > \overline{L}. \end{cases} \tag{38}$$

Substituting these values into (34)

$$V_{MB}\begin{cases}\frac{(1-a+L_{IB})^2\rho}{2(1-\pi\rho)}, & \text{if } \ 0<L_{IB}<\widehat{L};\\ \frac{2(\pi\rho(a-L_{IB})-1)}{\rho\pi^2}, & \text{if } \ \widehat{L}<L_{IB}<\overline{L};\\ 0, & \text{if } \ L_{IB}>\overline{L}.\end{cases} \tag{39}$$

We further have that the value of the relation with official lenders in the blended case, V_{IB_t}, is given by

$$V_{IB_t}=\rho(-C+aL_{IB_t}-\frac{{L_{IB_t}}^2}{2}-(1-\pi)kL_{IB_t}-\pi L_{IBt}+V_{IB_{t+1}}), \tag{40}$$

and equalizing V_{IB_t} and V_{IB_t+1} we have that

$$V_{IB}(L_{IB})=\frac{-2\rho C+L_{IB}\rho(2a-L_{IB}-2((1-\pi)k+\pi)}{2(1-\rho)}. \tag{41}$$

Finally, the overall value function (official and market lenders) is then given by $V_B=V_{IB}(L_{IB})+V_{MB}(L_{IB})$. Using (39), and (41), and assuming that the country is always willing to repay any $L_{IB}\in[0,\overline{L}]$, we obtain (18). This proves the Lemma.□

Proof of Lemma 3:

First notice that if $\rho\le\overline{\rho}_I$ there is no risk-free lending in the IFI lending scenario and thus, a fortiori, there can be no risk-free IFI lending in the blended case. Consider now the interval $\rho>\widehat{\rho}>\overline{\rho}_I$. A sufficient condition for the existence of risk-free IFI lending if the market does not offer the risk-free loan is

$$k<\underset{L_{IB}\to 0}{Lim}\frac{\partial(V_{B1}-V^*_{MD3})}{\partial L_{IB}}=\underset{L_{IB}\to 0}{Lim}\frac{\partial V_{B1}}{\partial L_{IB}}\Longleftrightarrow\rho>\rho^a\equiv\frac{2k}{2\pi k+(1-\pi)+\sqrt{(1-\pi)(4(a-1)k+(1-\pi)}}, \tag{42}$$

and

$$\rho^a<1\Longleftrightarrow k<\frac{a-\pi}{1-\pi}\equiv k^a.$$

It remains to verify that the market does not offer the risk-free loan; this is the case if

$$\rho^c-\rho^a=\frac{2k\left(\sqrt{(1-p)(4k(a-1)+1-p)}+1-ap\right)}{p(a+2k-1)\left(\sqrt{(p-1)(-(4ak-4k-p+1))}+2kp+1-p\right)}>0$$

and

$$\rho^a>\rho^c\Longleftrightarrow k>\frac{\pi(\pi(a+1)-2)}{1-\pi}\equiv k^b.$$

The fact that $k^a>k^b$ completes the proof.□

Proof of Proposition 4:

From Lemma 3, we know that official lending is feasible if $\rho>\rho^a$ and that $\underset{L_{IB}\to 0}{Lim}\frac{\partial V_{B1}}{\partial L_{IB}}>0$. Hence, in the interval $k\in[k^a,k^b]$ there is a level of official lending that strictly improves welfare.□