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Risk Appetite Fluctuations in the Insurance Industry *

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Abstract

The risk appetite of insurance companies fluctuates over time in a quasi cyclical fashion. When their capitalization is high (low), companies choose portfolios with a high (small) share of risky assets. We show that this phenomenon may have the same source as the underwriting cycle, namely recapitalization costs. We build a simple dynamic model of the insurance sector where financial frictions prevent companies from maintaining a target leverage. Portfolio decisions of insurers fluctuate with their aggregate capitalization. The model rationalizes two apparently disjoint pieces of evidence: long-standing

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empirical evidence on underwriting cycles and more recent evidence on the fluctuations of insurance companies' risk appetite.

JEL classification numbers: G11, G12, G23

1 Introduction

Insurance companies are large investors, and the risk composition of their portfolios fluctuates over time in a substantial manner, as illustrated by the recurrent flight-to-quality episodes reported by Domanski, Shin and Sushko (2017) or Bijlsma and Vermeulen (2016), by reach-for-yield episodes such as the one detected by Becker and Ivashina (2015), or by the pre-COVID shift towards private investments, linked to the low-for-long scenario for interest rates.

Given the sizes of their portfolios, it is important to understand the reasons for these fluctuations in insurers' risk appetite. We suggest that it might be related to the well established evidence on the "underwriting cycle", i.e. sizable fluctuations in insurance supply (capacity) and premia. Up to our knowledge, there has been no attempt to link the fluctuations in insurers' assets and liabilities, and no theoretical attempt to link the cyclicality of premia and profits to the changes in risk tolerance of insurance companies.

This paper offers an explanation of the fluctuating risk aversion of insurance companies through a theoretical model of the insurance sector with financial frictions. In the model, asset allocation between safe and risky bonds is determined jointly with insurance capacity. The link between assets and liabilities is fully endogenized and its causality explained. Risk aversion, asset allocation and insurance capacity all depend on the aggregate capitalization of the insurance sector. This capitalization fluctuates because financial frictions prevent companies from constantly adjusting their capital to an optimal target, as would be implied by the trade off theory of leverage, as in Bolton et al. (2012).

Our model provides an explanation for the random duration of capacity-cycles, detected by Boyer et al. (2012). Cyclicality in insurance supply is indeed a well known phenomenon, documented since the past century, but fixed-length cycles do not fit the observed capacity levels. The theories provided so far to explain capacity cycles did not look at insurers' overall risk, but only to the risk of liabilities (see for instance Doherty and Kang (1988), Venezian (1985), Cummins and Outreville (1987), Winter (1988, 1991) and Gron (1989, 1994)). Within our model, the length of the fluctuations is stochastic, because it depends on the shocks to capital coming from assets as well as liabilities.

Our model also provides an explanation of underwriters' risk asset allocation. We explain the "flight-to quality" or "flight-to-liquidity" phenomenon, namely the tendency - which they share with other liability-driven institutional investors - to invest in less risky bonds instead of stocks, when they become financially constrained. This is again a widely observed phenomenon, difficult to explain with theories in which risk aversion of investors is exogenous, instead of being an endogenous function of capitalization, as here.

To obtain these results, we build a simple Markovian model of a competitive insurance market. In this model, the equity of insurance companies fluctuates between two bounds, which depend on recapitalization costs and on assets as well as liabilities risk premia. Insurance capacity and exposure to risky assets are increasing with the aggregate capitalization of the underwriters.

The outline of the paper is as follows: Section 2 reviews some background literature; Section 3 introduces our simple dynamic model of the insurance sector. Section 4 characterizes the competitive equilibrium of the model. Section 5 calibrates the model and shows the behavior of risk aversion, capacity, asset allocation and premia as functions of capital. Section 6 focuses on the fluctuations over time and the long-run behavior of the industry capital. It also endogenizes the risk premium. Section 7 uses the model to infer the risk aversion of the Italian insurance sector from their risky asset allocation over the decade 2011-2019. Section 8 concludes.

2 Background Literature

The background literature can be divided into several strands: the evidence on insurance asset allocation, the more recent empirical literature on fluctuating risk aversion of financial intermediaries, the large literature on the underwriting cycle, and the macro-finance literature with heterogeneous agents and endogenous market risk aversion.

The traditional view is that, as investors, insurance companies act as stabilizers of capital markets. They buy when asset prices are low and sell when they are high. This is because they are long term investors who tend to keep their assets in their balance sheets until they mature. Their tactical asset allocation is limited in scope and amount. However, recent empirical evidence goes against this conjecture. Fache Rousova and Giuzio (2019) find procyclical behavior with respect to risk premia. The study of Fache Rousova and Giuzio (2019) relies on an extensive dataset of 19 Euro area countries, and is corroborated by other studies on single countries. Using data from German insurers, for instance, Timmer (2018) finds that, over the period 2005 to 2014, insurance companies acted countercyclically. Using mainly UK data, instead, the Bank of England (2014) finds some evidence of procyclicality after the dot.com crash, and, to a smaller extent, during the Great Recession. Ellul et al. (2018) find procyclicality, at different points in time, in US insurance companies' asset allocation. The hedging activity of underwriters who issued guaranteed life insurance contracts indeed makes them pro-cyclical in market downturns, even if in normal times they act as "asset insulators".

Overall, the evidence described so far refers either to the EU before the introduction of the Solvency II regulation, or to the US, where regulation is much weaker than in the EU. The picture however should not be different after the introduction of the risk-based regulation in the EU, because the competent authorities introduced specific provisions in order to make the regulation as neutral as possible with respect to the financial cycle and asset price movements (see EIOPA 2018 a, b). Further revisions of the Solvency II regulation, which were due by 2020-21 but were delayed by the pandemic, are meant to reinforce this neutrality.

Our model relates fluctuations in asset allocation to insurers' capitalization, whose shocks drive the cycle. Asset allocation is procyclical in capital. When we endogenize risk premia in Section 6, risky asset allocation remains procyclical.

There is a recent evidence, provided by Ge and Weisbach (2019), who document that larger insurers choose more risky asset portfolios than smaller insurers. Larger insurers have higher ratings. Ge and Weisbach interpret higher ratings as a measure of financial flexibility, which corresponds to our high capitalization. Using a sample of 842 life insurers and 2084 P&C insurers over the period 2001-2015, and instrumenting for size and catastrophic exogenous losses, they show that less financially flexible companies have less risky bonds in their portfolios, and the relationship is causal. We rationalize their evidence, since our fluctuations depend on insurers' capital, not on market risk premia.

Our paper is also related to the large literature on the underwriting cycle. Most insurance markets are characterized by "soft" periods, in which premia are low and supply (capacity) is high, and "hard" periods, in which premia are higher, profits tend to raise and supply is comparatively low. Evidence of this underwriting cycle is well documented since Smith (1980), Venezian (1985), Doherty and Kang (1988). It exists both in the US and in other countries, as documented by Cummins and Outreville (1987). Recent evidence continues to show alternating soft and hard markets: see for instance Swiss Re (2019). Insurance Europe (2019), among others, documents the growth of life-insurance premia, including the unit-linked ones, over the period 2008-2017. A similar phenomenon occurred in the US. The most frequent explanation for these fluctuations, known as capacity theory, ascribes cycles to movements in companies' capitalization. Faced to losses, if those do not hit too much, companies' reduce capital, but still count on internal resources. To avoid default, though, they decrease their capacity and increase premia. According to capacity theory, this leads to higher profits, because internal capital is not very costly. Higher profits restore capitalization. When losses hit hard, insurance companies run out of internal capital and are forced to recapitalize, at a cost. After recapitalization, capacity increases, premia decrease. Profits decrease, because external capital is more costly than internal one. The inversion of the cycle takes place. Early models in this strand are Winter (1988, 1991) and Gron (1989, 1994). Doherty and Garven (1995), and later Gron (1994), find support for capacity theory. Harrington (2004) provides a survey. Boyer et al. (2012), using most of the datasets already investigated in the literature, do not exclude cycles, but reject any fixed length for them.

Also related to our paper are the recent macro-finance models where agents differ either in preferences, or beliefs, or access to markets. All of them produce countercyclical market prices of risk, exactly as our model does with respect to aggregate capital. Panageas (2020) builds a model which can incorporate both heterogeneity in beliefs, preferences or access to markets and shows that positive aggregate shocks increase the consumption share of less risk averse or more "optimistic" investors, which in turn deflates the risk premium. As a consequence, risk premia are countercyclical. This happens because with heterogeneity in risk aversion or beliefs ("optimistic" versus "pessimistic"), less risk-averse or more optimistic agents bear more aggregate risk than more risk averse or more pessimistic ones. Similar results hold with restricted participation.

In the same vein, in He and Krishnamurthy (2013) the economy is populated by two types of agents: short-lived households and long-lived intermediaries. Households can only invest in bonds and deposits, while intermediaries can invest in the equity market. He and Krishnamurthy have financial frictions through capital constraints for intermediaries. The combination of heterogeneity in the access to markets and capital constraints for intermediaries generates fluctuations in intermediaries' risk aversion, and countercyclical market risk premia. In Brunnermeier and Sannikov (2014) the economy is populated by households and skilled investors, or experts. The latter can borrow from households at the risk-free rate, have limited liability, and are forced to liquidate when they reach zero wealth. Similarly to He and Krishnamurthy (2013), Brunnermeier and Sannikov obtain state-dependent risk aversion, countercyclical risk premia and procyclical asset allocation by experts.

We share with this literature both endogeneity in risk aversion - or aggregate risk aversion - and countercyclical risk premia, while working in a different context. In most of the paper we indeed focus on insurance premia and show that they are countercyclical. When we endogenize risky asset premia in Section 6 though we replicate the counterciclicality result also for them.

Henriet et al. (2016) build a dynamic model with financial frictions that rationalizes the underwriting cycle. Our paper extends that paper by modelling simultaneously asset and liability decisions of insurance companies, thus providing a joint explanation for underwriting cycles and fluctuations in asset allocations.

In these macro-finance models, frictions are key to preventing homogeneity and solving a number of empirical puzzles. In Bolton et al. (2021) costly equity issuance lowers the target leverage of firms, makes it consistent with long standing empirical observations, and produces tolerance regions where firms' leverage is not immediately adjusted to its target level. In a similar vein, costly equity issuance in our model prevents insurance companies from adjusting to a target capacity or asset allocation. Both of them evolve stochastically as a function of equity, instead of being constant, despite iid returns on risky assets.

3 A Dynamic Model of the Insurance Market

Consider a continuous time economy with two types of agents: households, who are subject to wealth losses and competitive insurance companies. Household losses are modelled as $ldt - \sigma dZ_t$, where l and σ are positive numbers and $Z_t, t \geq 0$, is a one-dimensional Wiener process, common to all households[1]. A unit insurance contract compensates wealth losses $ldt - \sigma dZ_t$ in exchange for a loaded premium $(l + \pi)dt$ paid at date t. For convenience we refer to the loading factor π as the premium. It adds to the fair premium ldt, and compensates companies for underwriting aggregate risk σdZ_t. The underwriting profit on a unit contract is therefore

$$(\pi + l)dt - (ldt - \sigma dZ_t) = \pi dt + \sigma dZ_t.$$

Let y_t denote aggregate insurance capacity or supply, measured by the total number of unit contracts offered by all insurance companies at t. It is related to the insurance premium by the inverse demand function of households, which is denoted $\pi = \pi(y)$. Appendix A shows that, when maximum capacity is normalized to one and households are short-lived,

$$\pi(y) = \alpha\sigma^2 (1 - y), \tag{1}$$

where α is the risk aversion parameter of households.[2]

[1] Idiosyncratic shocks on households' wealth are not modelled.

[2] If households are not short-lived, the demand function in the text can be interpreted as the linear approximation to their demand function.

In the absence of financial frictions, the equilibrium premium would be identically zero and the aggregate volume of insurance would be constant and equal to $y^* = 1$. With financial frictions, aggregate supply will be systematically below $y^* = 1$.

There is a continuum of insurance companies indexed by i, where i is uniformly distributed on $(0, 1)$. Insurance companies can invest their capital W_t^i in a risky asset with stationary random returns

$$\mu dt + \varsigma dB_t,$$

where $\mu, \varsigma > 0$ and B is another Wiener process, orthogonal to Z. We work on the probability space $(\Omega, \mathcal{F}, P)$ where F is the complete, natural filtration generated by the couple of Wiener processes (Z, B) and P is the probability measure associated to it.

Insurance companies can also borrow or invest at the riskless rate $0 < r < \mu$. They must keep their capital non-negative: $W_t^i \geq 0$.

At each date t, company i chooses its supply y_t^i of insurance contracts and its investment x_t^i in risky assets, as well as its policy of dividend distribution/share repurchase or new equity issuance.[3] Let $dD_t^i \geq 0$ and $dI_t^i \geq 0$ denote stock repurchases and new equity issuance at date t. The processes $D_t^i \geq 0$ and $I_t^i \geq 0$ are non negative and non-decreasing ($dD_t^i \geq 0$ and $dI_t^i \geq 0$), defined on $(\Omega, \mathcal{F}, P)$, and will be determined below within that class. The equity issuance process will maintain capital non-negative.

Company i' s objective is to maximize its shareholder value:

$$\mathbb{E} \int_0^{+\infty} \exp(-\lambda t) \left[dD_t^i - (1+\gamma)\, dI_t^i \right],$$

[3]If the underwriter is in a net borrowing position, the liabilities on its balance sheet are equity capital W_t^i and debt $x_t^i - W_t^i$. If it is a net investor, its balance sheet has x_t^i risky assets and $W_t^i - x_t^i$ riskless assets on the asset side, capital W_t^i on the liability side.

which is the expected present value of future net dividends/stock repurchases dD_t^i net of the costs $(1+\gamma)dI_t^i$ of future recapitalizations, under the constraints $dD_t^i \geq 0$ and $dI_t^i \geq 0$. Financial frictions are captured by the marginal costs of new equity issuance $\gamma > 0$. The discount rate λ is greater than the riskless rate r, the difference $\lambda - r > 0$ representing the excess return on capital required by shareholders. The dynamics of the capital of firm i is thus given by

$$dW_t^i = \left[rW_t^i + (\mu - r)\, x_t^i\right] dt + x_t^i \varsigma dB_t + y_t^i(\pi_t dt + \sigma dZ_t) - dD_t^i + dI_t^i. \quad (2)$$

where the first term represents the expected return on financial assets, the second its diffusive term, the third represents the net profit from the underwriting activity, and the last two term represent depletions and increases in capital due respectively to dividend distribution and recapitalization.

The aggregate capitalization of the insurance sector W_t, which is the integral of W_t^i over i, will play an important part in the sequel. It satisfies an equation analogous to the individual one:

$$dW_t = [rW_t + (\mu - r)\, x_t]\, dt + x_t \varsigma dB_t + y_t(\pi_t dt + \sigma dZ_t) - dD_t + dI_t, \quad (3)$$

where the variables x, y, D, I are the integrals of the corresponding variables with respect to i.

The problem of insurer i boils down to finding the recapitalization, dividend, asset allocation and capacity strategies that solve the following control problem:

$$\mathcal{P}^i \begin{cases} J(W^i, W) \triangleq \max_{dI^i, dD^i, x^i, y^i} \mathbb{E} \int\limits_0^{+\infty} \exp(-\lambda t) \left[dD_t^i - (1+\gamma)\, dI_t^i\right] \\ \text{subject to (2), (3), with initial condition } W_0^i = W^i, W_0 = W. \end{cases}$$

The Bellman function $J(W^i, W)$ gives the market value of equity of company i, as a function of W_t^i, its book value of equity, and aggregate book equity W_t

of the insurance sector. Note that this optimization problem is homogeneous of degree one with respect to W^i. Indeed, by linearity of the objective function and the state equation in W_t^i , if the initial value of W_t^i is multiplied by a constant m, while everything else stays identical, then the optimal strategy (dI^i, dD^i, x^i, y^i) of firm i and the value function are also multiplied by the same constant. This means that J is linear with respect to W^i:

$$J(W^i, W) = W^i q(W).$$

We are looking for a Markov Competitive Equilibrium, where the price of insurance π_t and the aggregate variables x_t, y_t, D_t, I_t are deterministic functions of aggregate capitalization W_t.

4 The Competitive Equilibrium

This section shows that there is a unique Markov Competitive Equilibrium (MCE) and studies its properties. We start (subsection 4.1) by solving the Bellman equation that characterizes the optimal behavior of each insurance company. Then we aggregate these individual behaviors and show the existence and uniqueness of the MCE (subsection 4.2). Finally, we study the properties of this equilibrium and endogenize market risk aversion (subsection 4.3).

4.1 Optimal Behavior of Each Company

The value function of company i satisfies the Bellman equation:

$$\begin{aligned}
\lambda J &= \max_{dD^i \geq 0} dD^i\,(1 - J_1) + \max_{dI^i \geq 0} dI^i\,(J_1 - (1+\gamma)) &(4)\\
&+[rW + y\pi(y) + x(\mu - r)]J_2 + \frac{x^2\varsigma^2 + y^2\sigma^2}{2}J_{22} + rW^iJ_1 \\
&+\max_{x^i}[x^i(\mu - r)J_1 + \varsigma^2 x.x^i J_{12} + \frac{\varsigma^2(x^i)^2}{2}J_{11}] \\
&+\max_{y^i}[y^i\pi(y)J_1 + y.y^i\sigma^2 J_{12} + \frac{\sigma^2(y^i)^2}{2}J_{11}], &(5)
\end{aligned}$$

where lower indices denote partial derivatives. The problem is simplified by using the linearity of J in W^i, which we established at the end of Section 3:

$$J(W^i, W) = W^i q(W).$$

The function $J_1 = q$, which can be interpreted as the Market-to-Book Ratio of equity, is the same for all companies, and only depends on W. In the interior of the interval $(\underline{W}, \bar{W})$, we can substitute for the expression of J and its derivatives (see Appendix B) in the Bellman equation. After simplifying, we obtain the following equation for q:

$$\begin{aligned}
(\lambda - r)q(W)W^i &= \max_{dD^i \geq 0} dD^i\,(1 - q(W)) + \max_{dI^i \geq 0} dI^i\,(q(W) - (1+\gamma)) &(6)\\
&+[rW + y\pi(y) + x(\mu - r)]q'(W)W^i \\
&+\frac{\varsigma^2 x^2 + \sigma^2 y^2}{2}q''(W)W^i \\
&+\max_{x^i} x^i[(\mu - r)q(W) + \varsigma^2 x q'(W)] \\
&+\max_{y^i} y^i[\pi(y)q(W) + \sigma^2 y q'(W)].
\end{aligned}$$

Let us figure out the optimal controls. By doing that, we will also be able to simplify the equation and transform it into a second order ODE for q.

Linearity of the Bellman equation with respect to the dividend policy dD^i implies that the optimal policy is singular: retain earnings as long as the marginal utility of capital is greater than one ($J_1(W^i, W) = q(W) > 1$) and distribute dividends at the dividend boundary $\bar{W}$, defined implicitly by

$$J_1(W^i, \bar{W}) = q(\bar{W}) = 1.$$

Similarly, linearity of the Bellman equation with respect to the recapitalization policy dI^i implies that the optimal recapitalization policy is also singular: do not recapitalize as long as the marginal cost $1 + \gamma$ is greater than the marginal utility of capital $J_1(W^i, W) = q(W)$, and recapitalize only at the recapitalization boundary $\underline{W}$, defined implicitly by

$$J_1(W^i, \underline{W}) = q(\underline{W}) = 1 + \gamma.$$

The intervention boundaries are horizontal lines: $W = \underline{W}$ for the recapitalization boundary and $W = \bar{W}$ for the dividend boundary. In correspondence to them the following conditions must hold:

$$q(\underline{W}) = 1 + \gamma, \tag{7}$$

$$q(\bar{W}) = 1. \tag{8}$$

In the interior of the domain the q function satisfies:

$$\begin{aligned}(\lambda - r)q(W)W^i &= [rW + y\pi(y) + x(\mu - r)]q'(W)W^i \\ &+ \frac{\varsigma^2 x^2 + \sigma^2 y^2}{2} q''(W)W^i \\ &+ \max_{x^i} x^i[(\mu - r)q(W) + \varsigma^2 x q'(W)] \\ &+ \max_{y^i} y^i[\pi(y)q(W) + \sigma^2 y q'(W)].\end{aligned}$$

As for the other controls, risky asset allocation and capacity, we note that the equation for q is linear in x^i and y^i. The coefficients of these variables must vanish at any interior solution, so that the equation for q becomes

$$\begin{aligned}(\lambda - r)q(W) &= [rW + y\pi(y) + x(\mu - r)]q'(W) \\ &+ \frac{\varsigma^2 x^2 + \sigma^2 y^2}{2} q''(W)\end{aligned}$$

Nullification of the coefficients implies:

$$x(W) = \frac{\mu - r}{\varsigma^2}\left(\frac{-q(W)}{q'(W)}\right), \tag{9}$$

and

$$y(W) = \frac{\alpha\left(\frac{-q(W)}{q'(W)}\right)}{1 + \alpha\left(\frac{-q(W)}{q'(W)}\right)}. \tag{10}$$

These control values must be substituted in the equation for q, which then becomes a second-order ODE:

$$\begin{aligned}&(\lambda - r)q(W) \qquad (11)\\ = &\left[rW + \frac{\alpha\left(\frac{-q(W)}{q'(W)}\right)}{1 + \alpha\left(\frac{-q(W)}{q'(W)}\right)} \frac{1}{1 + \alpha\left(\frac{-q(W)}{q'(W)}\right)} \alpha\sigma^2 + \frac{(\mu - r)^2}{\varsigma^2}\left(\frac{-q(W)}{q'(W)}\right)\right] q'(W) \\ &+ \frac{\frac{(\mu-r)^2}{\varsigma^2}\left(\frac{-q(W)}{q'(W)}\right)^2 + \sigma^2\left(\frac{\alpha\left(\frac{-q(W)}{q'(W)}\right)}{1+\alpha\left(\frac{-q(W)}{q'(W)}\right)} \frac{1}{1+\alpha\left(\frac{-q(W)}{q'(W)}\right)}\right)^2}{2} q''(W)\end{aligned}$$

Conditions (7) and (8) act as its boundary conditions.

The classical controls for x and y deserve a comment. Even though shareholders are risk neutral (which manifests itself by the linearity of the Bellman function with respect to W^i), they behave as if they were risk averse. Indeed, equation (9) shows that the optimal share of risky asset in the portfolio of firm i is inversely proportional to the ratio $-\frac{q'(W)}{q(W)}$, which can be interpreted as the market (absolute) risk aversion index $R(W)$. Similarly, equation (10) defines the equilibrium insurance supply y as a function of this risk aversion $R(W)$:

$$x(W) = \frac{\mu - r}{\varsigma^2 R(W)}, \tag{12}$$

$$y(W) = \frac{\alpha}{\alpha + R(W)}, \tag{13}$$

These two conditions can be interpreted as optimal hedging conditions with respect to the two aggregate risks, namely asset risk and underwriting risk.

4.2 Equilibrium: Existence and Uniqueness

Homogeneity implies that insurance companies only differ by their initial size and behave in a synchronous fashion. They all recapitalize when aggregate capitalization reaches the lower boundary $\underline{W}$ and distribute dividends when it reaches the upper boundary $\bar{W}$. The growth rate of book equity is the same for all companies. By aggregating their individual behaviors, we can determine the values of the boundary points $\underline{W}$ and $\bar{W}$.

On top of the boundary conditions, equilibrium requires the respect of a *no-arbitrage* condition applied to the value of the aggregate capitalization of the insurance sector. No arbitrage implies that the marginal change in this aggregate capitalization after dividend distribution or recapitalization equals the flow of money injected in (or taken out) the industry by the shareholders. In other words $[Wq(W)]'$ equals $1 + \gamma$ in $\underline{W}$ and 1 in $\bar{W}$:

$$\begin{aligned} [\underline{W}q(\underline{W})]' &= q(\underline{W}) + (\underline{W})q'(\underline{W}) = 1 + \gamma, \\ [\bar{W}q(\bar{W})]' &= q(\bar{W}) + \bar{W}q'(\bar{W}) = 1 \end{aligned}$$

Using the boundary values of q determined above, the previous conditions are equivalent to $Wq'(W)$ being zero at both extremes of the capital interval:

$$\underline{W}q'(\underline{W}) = \bar{W}q'(\bar{W}) = 0$$

Assuming that, as will be shown below, q is decreasing in W, we prove in Appendix B that the only possibility for this no-arbitrage condition to hold is that

$$\underline{W} = 0, \tag{14}$$

$$q'(\bar{W}) = 0. \tag{15}$$

The first results says that, in the presence of purely proportional recapitalization costs, the recapitalization boundary is equal to zero. It is not optimal to recapitalize when capital is still positive, because this brings costs. The second result says that the upper boundary for capital, when dividends are distributed, is just the level where its marginal market-to-book value is zero. It will be determined while solving for the function q', using (15). Indeed, we do not solve straight away for q, but for the ratio q'/q, as follows.

Substitute for $R = -q'/q$ in the ODE for q, and use the fact that $-q''/q = R' - R^2$, since

$$R' = -\frac{q^{''}}{q} + \frac{\left(q^{'}\right)^2}{q^2} = -\frac{q^{''}}{q} + R^2$$

The second-order ODE becomes a first order ODE for R:

$$R'(W) = -R^2(W)\left[1 + 2\frac{\left(\lambda - r + rWR(W)\right)\left(\alpha + R(W)\right)^2}{\sigma^2\alpha^2R^2(W) + \left(\alpha + R(W)\right)^2\frac{(\mu-r)^2}{\varsigma^2}}\right]. \tag{16}$$

For any $\bar{W}$, the condition (15) on q' gives the only boundary condition for this ODE:

$$R(\bar{W}) = 0. \tag{17}$$

Note that (16) implies that $R(W) = 0$ for all $W > \bar{W}$ and R is decreasing ($R' < 0$ because the RHS of (16) is negative). So $R(W) > 0$ for $W < \bar{W}$, which means $q' < 0$ for $W < \bar{W}$, or q decreasing, as assumed above.

When reconstructing q from R however we must make sure that (8) and (7) are satisfied:

$$\begin{aligned} q(\underline{W}) &= q(0) = 1 + \gamma, && (18) \\ q(\bar{W}) &= 1. && (19) \end{aligned}$$

Compute $q(W)$ from $R(W)$ by using the fact that $q'/q = -R$ and $q(\bar{W}) = 1$:

$$q(W) = \exp\Big(\int_W^{\bar{W}} R(s)ds\Big).$$

By so doing, it remains to determine $\bar{W}$ so that

$$q(0) = \exp\Big(\int_0^{\bar{W}} R(s)ds\Big) = 1 + \gamma.$$

Numerically, we perform the calculation as follows. For any $z > 0$ we observe that there exists a unique solution of (16) that takes the value z for $W = 0$. Denote it by $R(W; z)$. Thus, by definition, $R(0; z) = z$. Since the solutions to the ODE do not cross,

$$z_1 < z_2 \text{ implies } R(s; z_1) < R(s; z_2) \text{ for all } s$$

Thus the RHS of q is increasing in z and for all γ there is a unique z that satisfies the boundary condition at 0, namely

$$q(0) = \exp\Big(\int_0^{\bar{W}(z)} R(s; z)ds\Big) = 1 + \gamma. \tag{20}$$

The next theorem summarizes our results so far.

Theorem 1 *There is a unique Markovian Competitive Equilibrium. In this equilibrium*

- *Market risk aversion $R(W)$ is a decreasing function of aggregate capital W.*

- *$R(W)$ is characterized by the ODE (16) and boundary condition $R(0) = z$, where z is defined implicitly by (20).*

- *Firms recapitalize when $W = 0$ and distribute dividends when $W = \bar{W}$, where $\bar{W}$ is such that (17) holds.*

4.3 Equilibrium Properties

Theorem 1 establishes that, as a consequence of the optimal recapitalization and dividend policies, aggregate capitalization, which follows the SDE (3), has 0 and $\bar{W}$ as reflecting barriers. It fluctuates between these two barriers. It is subject to shocks, which come from both the asset side (associated with dB_t) and from claims (associated with dZ_t).

With respect to total capitalization W, risk aversion is countercyclical, because, according to the same Theorem, it is decreasing in W.

A simple consequence of Theorem 1 is:

Corollary 2 *At the equilibrium of Theorem 1, insurance supply y and investment x in the risky asset are increasing functions of W, given respectively by (12) and (13).*

So, asset allocation and capacity are pro-cyclical. The main consequence is that risk premia π are countercyclical, or decreasing in capital, because they are decreasing in supply. This is consistent with the macro-finance results provided for the non-insurance assets in Panageas (2020) as well as He and Krishnamurthy (2013) or Brunnermeier and Sannikov (2014).[4]

Our theoretical result on asset allocation rationalizes the evidence in Ge and Weisbach (2019), who observe that higher capitalization is associated

[4]In Section 6 below we show that in our model also financial asset premia are countercyclical with respect to capital.

with higher exposure in risky assets. On the capacity side, Corollary 2 tells us that following negative shocks to capital, insurance companies reduce their supply and premia increase, in line with the capacity theory of the underwriting cycle. What is new is the joint determination of assets and liabilities. Asset allocation in the risky security is not only increasing in capital, but also decreasing in risk aversion, or increasing in risk tolerance. The same applies to capacity: following (13), it is increasing in capital and decreasing in risk aversion. So, assets and liabilities go hand in hand both as functions of capital and of endogenous risk aversion.

Also, the formula (12) for the investment x in the risky asset is similar to the one obtained for the portfolio problem of a risk-averse investor studied in Merton (1969). The difference is that the risk aversion R of an insurance company is endogenous and depends on capital, instead of being an exogenous parameter.

Corollary 2 implies that, following negative shocks to capital, insurance companies reduce their asset allocation in risky securities, as a consequence of an increase in risk aversion. The Corollary is therefore supportive of the "risk management" approach of insurance companies to negative shocks in their equity.[5] A policy consequence is that underwriters behave more prudently after negative shocks to capital even in the absence of a specific regulation.

Last, the corollary explains why insurance companies "fly to quality" in case of adverse movements in financial markets: with negative shocks to capital, their risk aversion increases and they reduce net asset allocation in the risky asset decreasing debt or increase their net exposure in riskless securities.

The Market-to-Book ratio, q, is characterized in the following:

Corollary 3 *The Market-to-Book ratio q is a decreasing, convex function of aggregate capitalization. Its upper bound is $q(0) = 1 + \gamma$, while its lower*

[5] If jumps (catastrophic losses) were introduced, risk shifting behaviour could appear as in Jensen-Meckling (1976).

bound is $q(\bar{W}) = 1$.

Proof. The fact that $q' < 0$ is an immediate consequence of the fact that $R = -q'/q > 0$ and $q > 0$. As for the sign of q'', recall that $R' = -\frac{q''}{q} + R^2$. Since R' is negative, it follows that $q'' > 0$. ■

5 Calibration

This section calibrates the model to Italian market data. It characterizes the relation between risk aversion, insurance supply and asset allocation on one side, market capitalization of the sector on the other.

We calibrate the standard deviation of losses, σ, to fit the 1998-2019 time-series of the claims in the income statements of Italian insurance companies, as from the Ania Statistical Appendix (Ania, 2020a). This is the longest time-series we can adopt, and serves to give a "long-run" value of the loss parameter. We interpret the actual claims as representing, year by year, the expected value of the losses, ly. Their standard deviation per unit of loss has to be used as σ. First, we compute the value of actual claims - in billion euros at year-end 2019 - using the consumer price index from the Italian Statistical Institute (ISTAT), i.e. we write all of them in nominal values of 2019. Then we compute the standard deviation of such a time series, and divide it by the series average. As a result, we obtain the standard deviation per unit of expected losses: $\sigma = 0.2844$.

The risky assets x showing up in the same financial statements include stocks and equity stakes, bonds, funds, and assets which back unit-linked contracts. We exclude assets backing unit-linked contracts, because their riskiness is borne by the clients. We focus on the asset mix of Italian insurers at year-end 2019 (see Ania 2020a), at book value.[6] At that date, 73.7% of

[6] The calibration is robust to market-based calibration of the asset mix. In order to obtain risky investments we exclude from Ania investments real estate, cash and guaranteed bonds.

the risky investments were in bonds, comprehensive of a large stake in Italian Government bonds, maturity close to ten years, and corporate bonds, mainly BBB financial sector, maturity 5 years. 13.4% of investments were in shares and equity stakes, with some more 12.9% in investment funds. Given that in the financial statements equity funds are not separated from fixed-income ones, we assume for simplicity that half of them is of the first type and half of the second. This gives us a stock percentage of risky assets equal to 20%.

Financial returns are taken from Campbell, Chan and Viceira (2003), so as to represent "long-run" expected values. Working on US data over the period 1952-1999, they obtain a risk free rate $r = 1.528\%$, a risk premium for equities equal to 6.40%, and for risky bonds equal to 0.9%. We compute the risk premium on the risky assets $\mu - r$ as the weighted average of the bond and equity premiums of Campbell, Chan and Viceira using the 80-20% weights explained above. We obtain $\mu - r = 2\%$.

As for the volatility of risky assets, we take the 2019 volatility of the 10-year Italian Government bonds and of the Eurostoxx Banks from Datastream, which are respectively 17% and 23%, and we compute the standard deviation of x assuming a perfect correlation between stocks and bonds returns.[7] It follows that $\zeta = 18\%$.

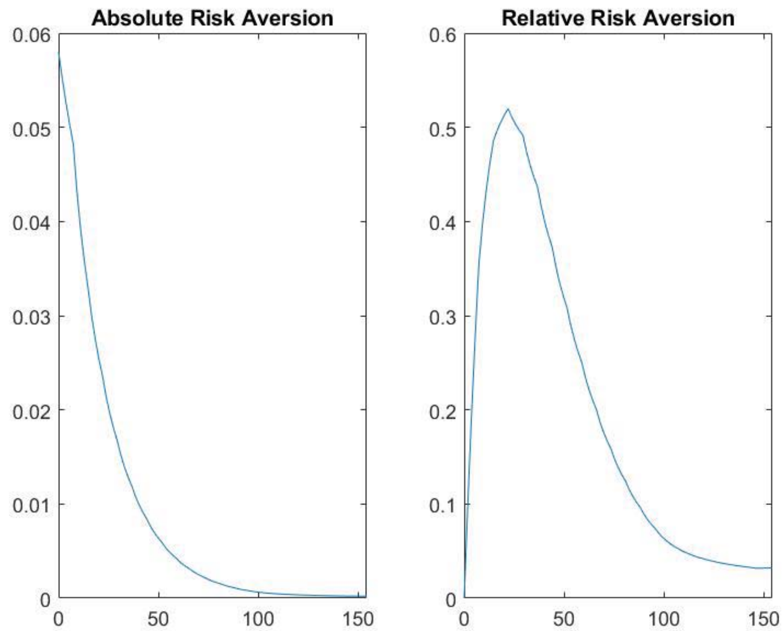
The household preference parameter is drawn from Kondor and Vayanos (2019): $\alpha = 2$. As in that paper, results are very robust to the choice of alternative values for α. We take the required return on equity (impatience parameter of insurers) λ equal to 20%. The recapitalization costs $\gamma = 0.2$ reflects empirical evidence on the maximum value of average Market-to-Book ratio of equity. We indeed know that the maximum ratio equates recapitalization costs in our model.

With the parameters above, we solve equation (16) under the boundary condition (17) for an initial value of z and repeat the procedure until (20)

[7] Different values of the correlation would have little impact on the results. The Eurostoxx Banks is chosen because the most stocks are financials.

is satisfied, for $\bar{W} = \bar{W}(z)$, and $R(\bar{W})$ tends to zero. As a result, we find $R(0) = 0.058, \bar{W} = 154.3$.

Absolute risk aversion R is decreasing, while relative risk aversion is hump shaped, as illustrated in Figure 1.

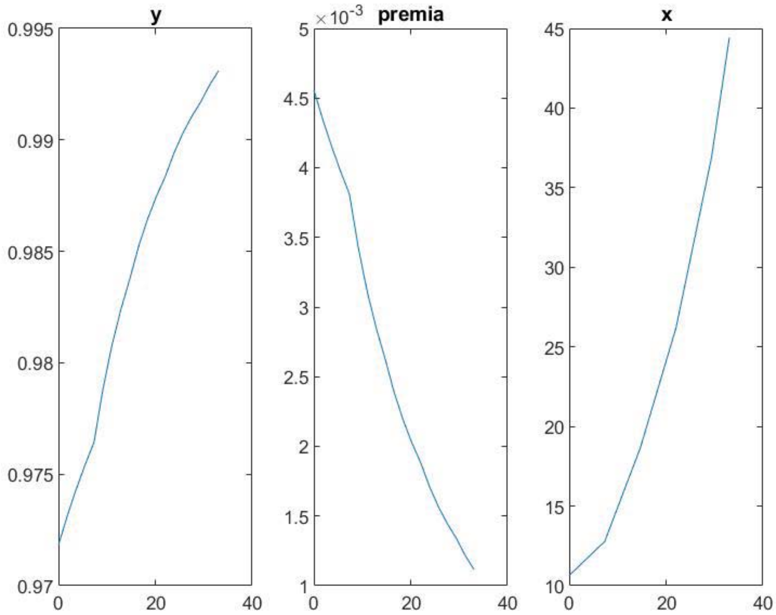


Left plot: absolute risk aversion R as a function of capital, up to the dividend-distribution level $\bar{W}$. Right plot: relative risk aversion $R \times W$ as a function of capital, up to the dividend-distribution level $\bar{W}$.

Figure 2 shows insurance supply, premia and risky asset allocation, again as a function of capital.[8] As expected, insurance supply is increasing and prices are decreasing with respect to W. Note that premia are "small",

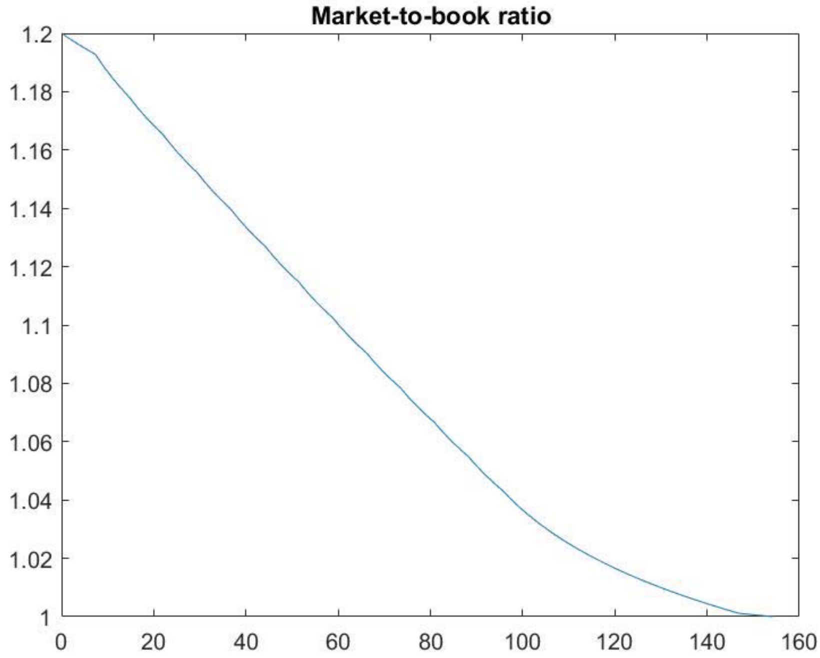
[8]The plot is up to the 75-th percentile of wealth, which is 33.7 billions of euros. We focus on the values of capital up to its long-run 75-th percentile, because the stationary density of wealth is highly concentrated at low values (see below, Section 6).

since they are in excess of fair premia, and do not cover underwriters' costs other than the recap ones, because we have neglected them.



The figure plots insurance supply y (left graph), unit premia π (center graph) and risky assets x (right graph) as a function of capital. The horizontal axis is truncated at the 75th percentile of capital.

Last, with the above parameters, we plot the Market-to-Book ratio as a function of book capital in the figure below.



Market-to-Book ratio as a function of capital. The maximum value of capital is the dividend distribution boundary $\bar{W}$.

Observe that, as established in Corollary 3, the Market-to-Book ratio has a maximum, greater than one, corresponding to the cost of raising external capital, and flats out to one when the dividend triggering capital is reached. The actual, empirically observed q is most of the time higher than one, and consistent with quite high recapitalization costs in our model. In the Italian case, q in the year 2019 was 1.003 for Generali, 1.10 for Unipol SaI, and 1.35 for Poste Italiane. While the business of Generali covers both life and non-life, with life premia being approximately the double of non-life ones, Unipol SaI is mainly a PC insurer, and Poste is mostly life insurance. So, we consider a value of q above 1 representative of both sectors.

6 Insurers' Capital: Long-run Behavior and Patterns

We have established the existence and uniqueness of a Markovian Competitive Equilibrium of the Insurance sector, where the risk aversion of the companies is a decreasing function R of aggregate capitalization W.

The long-run behavior of the model depends on the existence and properties of a long run distribution of capital. The capital process is reflected at the boundaries 0 and $\bar{W}$. In the interior of its domain, it follows from (3) that capital has drift

$$\mu_W = rW + (\mu - r)\, x(W) + y(W)\pi(y(W))$$

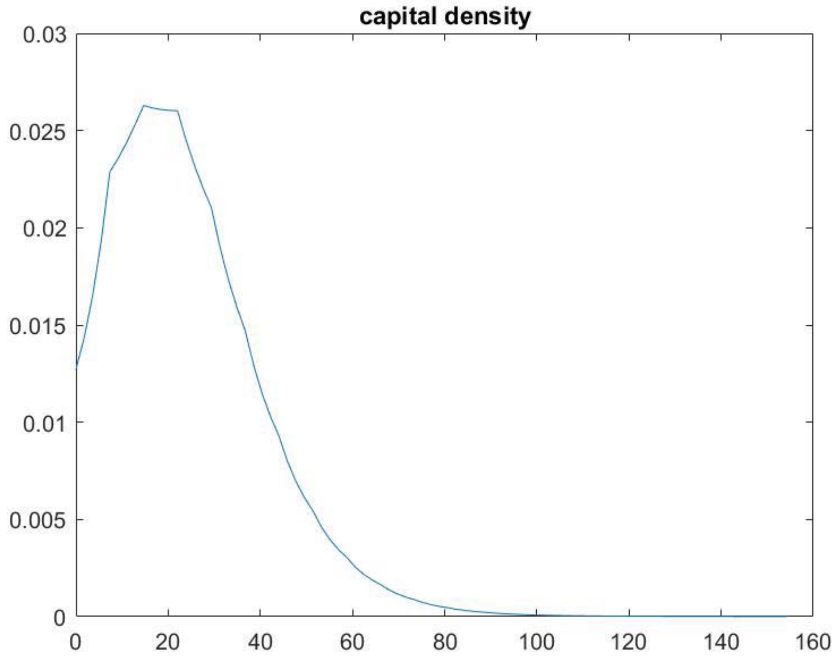
and instantaneous variance

$$\sigma_W^2 = x^2(W)\varsigma^2 + \ y^2(W)\sigma^2$$

Once the optimal values of x, y, π have been substituted in, these coefficients enter the Fokker-Planck equation for a stationary distribution. Following that procedure, and solving the Fokker-Planck equation, Appendix C shows that

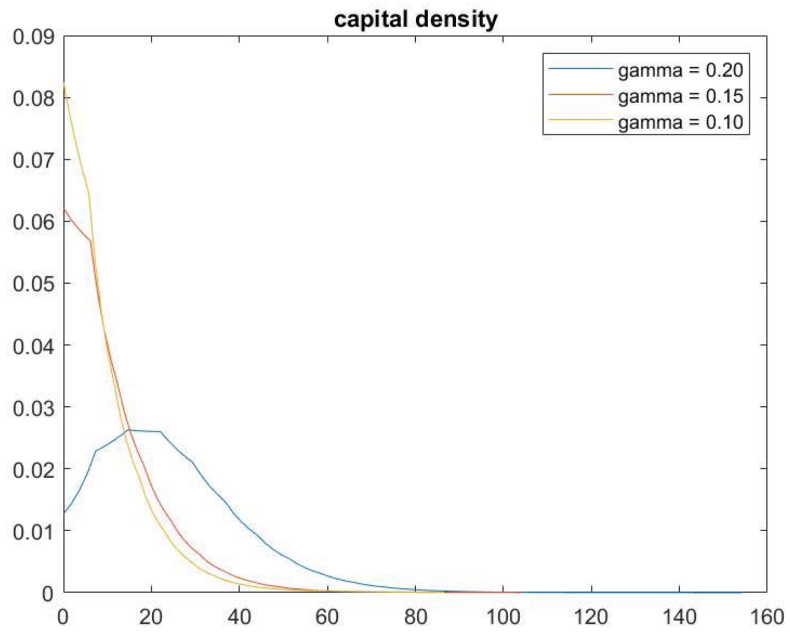
Proposition 4 *The capital process is ergodic, and its stationary density is positive at zero and null at $\bar{W}$.*

With the parameters of the calibration, the stationary density is hump-shaped, as the following picture illustrates:



Stationary density of capital W, over its entire support.

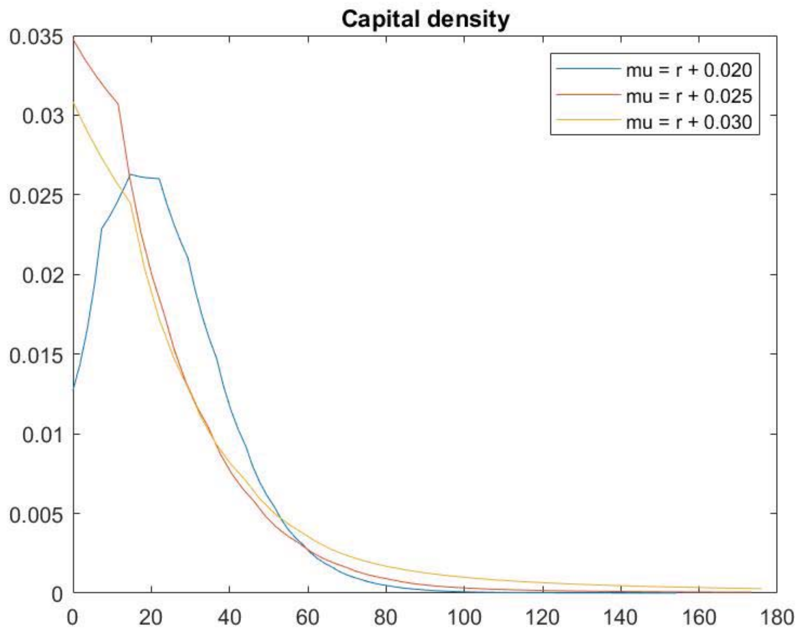
With lower levels of recapitalization costs, the density may become decreasing in its domain, as the following picture illustrates:



Stationary density of capital in correspondence to three levels of recapitalization costs: $\gamma = 20\%$ (the base calibration case, blue line), $\gamma = 15\%$ (red line), $\gamma = 10\%$ (yellow line). The dividend distribution level of capital changes from 154.3 to 104 to 86.

The behavior follows our intuition. We indeed know from standard ruin theory that default of insurers occurs with probability one if premia are fair. In our model premia have a loading, π, which covers recapitalization cost, so that the density is not a Delta-Dirac function centered at zero. As a result of insurer's optimization, the density is positive at 0, and capital tends to assume more frequently negligible values, when costs are low ($\gamma = 10\%$, yellow line in the figure). In the limit, with zero costs and fair premia, we would get the Dirac behavior of frictionless models. With high costs reflected on premia, the peak of the density at zero is lower ($\gamma = 10\%$, red line). Capital tends to stay far from the zero recapitalization threshold, with an hump-shaped density, when costs are even higher ($\gamma = 20\%$, blue line, the base calibration case).

A decreasing density occurs also when risky assets are more profitable than in the base case, namely when the risky premium is higher than in the calibration, as the following picture illustrates.



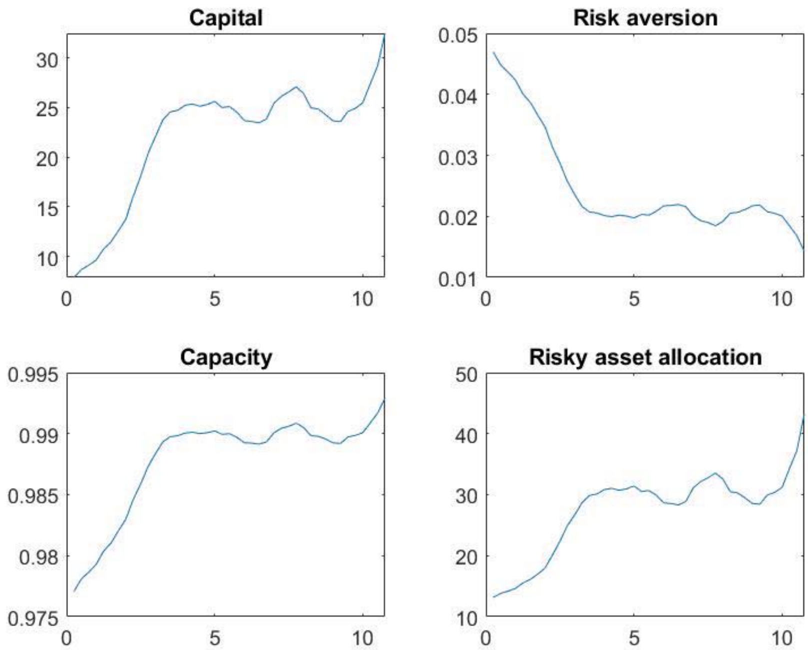
Stationary density of capital in correspondence to three levels of risk premium: $\mu - r = 2\%$ (the base calibration case, blue line), $\mu - r = 2.5\%$ (yellow line), $\mu - r = 3\%$ (red line). The dividend distribution level of capital changes from 154.3 to 173 to 176.

With higher premia the density is maximal at 0, showing that low levels of capital are more plausible. This happens because risk aversion is decreasing at 0, but overall risk taking increases, and as a result the likelihood of capital close to recapitalization goes up.

What Proposition 4 states is that capital also tends to stay away from the dividend distribution threshold, because the density is low in its neighbourhood, and actually zero at the threshold. This happens, as the last figures illustrates, both for low and high recapitalization costs and risk premia.

When the capital density is hump-shaped, the densities of risk aversion, supply, risky asset allocation, market-to-book ratio and prices may be non-monotonic, while it will be monotonic when the density of capital is, namely for low costs and high risk premia.

The existence of a stationary density affects the actual fluctuations of capital and of all the other variables. In order to illustrate the fluctuations, the following pictures display a simulated path of capital, risk aversion, capacity and risky asset allocation using the calibrated parameters, over a time interval of 10 years. Negative shocks to capital increase risk aversion and insurance premia, decrease risky investments and insurance supply, as due.



Simulated capital, absolute risk aversion, insurance supply and risky asset allocation, over a ten year interval. The simulated paths are constructed using the parameter values of the calibration.

From the upper left plot we observe that capital stays not far from its mean (24.7) and median (22.2). This happens as a result of using as initial distribution for the simulation the stationary distribution of capital, which we know to be hump-shaped, with a peak at 14.7, where the simulated values fall. So, fluctuations do encompass the whole range of capital (0, 154.3); in single simulations, they may encompass a smaller range of values, if, as here, they are obtained starting from the stationary distribution. As a consequence of capital staying close to its mean, also risk aversion, supply and asset allocation do not cover the whole range of possible values, but only a smaller range. Absolute risk aversion moves anti-cyclically below 5%, as a result of capital being low at the beginning of the simulation. Risky exposures on the liability and asset side, y and x, move pro-cyclically, with assets reacting more abruptly to capital changes, as intuition suggests. When capital goes from its peak to its trough, the other variables follow in a big swing. Using a stationary distribution peaked at 0, namely with lower costs or higher risk premia, we would obtain simulations closer to the minimal value of capital and consequently to the maximal value of risk aversion and minimal value of capacity and risky asset allocation.

While the previous pictures are important to give the sense of the fluctuations, the stationary distribution used to initialize them is then important to make capital spend more or less time closer to its lower boundary. In any case the upper boundary occurs less frequently, and dividends would be rarely seen in the simulations, given that the density is decreasing when getting close to the dividend boundary.

6.1 Endogenous financial risk premia

All of the previous results hold when the supply of risky assets is infinitely elastic, and the risk premium associated to them, $\mu - r$, is constant with respect to capital. We can fully endogenize the risk premium assuming, as in Kondor and Vayanos (2019), that the risky asset pays a dividend with the following dynamics:[9]

$$dD = \delta dt + \zeta dB$$

where δ is a constant. We should also assume, as in Kondor and Vayanos, that there exists a third category of agents. Let these agents, who form a continuum of measure 1 and whom we call financial investors, be short lived, as the households. The agent living from t to $t+dt$ receives an endowment u in risky assets at time t and consumes all of its wealth w at time $t+dt$; between t and $t+dt$ he can invest $-x_t$ in risky assets, which costs P per unit of time,

[9] We thank Peter Kondor for having discussed with us the risk premia issue.

for a total of $-x_t P dt$, and produces a final value $-x_t dD$. The return on risky assets is therefore $dD - Pdt = (\delta - P)\, dt + \zeta dB$. The notation $-x_t$ for their position already takes into account the fact that in equilibrium their position, summed to the underwriters' one, must sum up to zero: $-x_t + x_t = 0$. The rest of their wealth is invested in riskless assets. As a result of their investment policy, the financial investors' wealth is:

$$dw_t = rw_t dt - x_t(dD - Pdt) + udD$$

Since they are short lived, assuming risk aversion a, their utility function is

$$\frac{\mathbb{E}(dw_t) - \frac{a}{2}\mathrm{var}(dw_t)}{dt}, \tag{21}$$

It follows that their demand for risky assets is the usual tangent plus hedging portfolio

$$-x = \frac{\delta - P}{a\zeta^2} - u$$

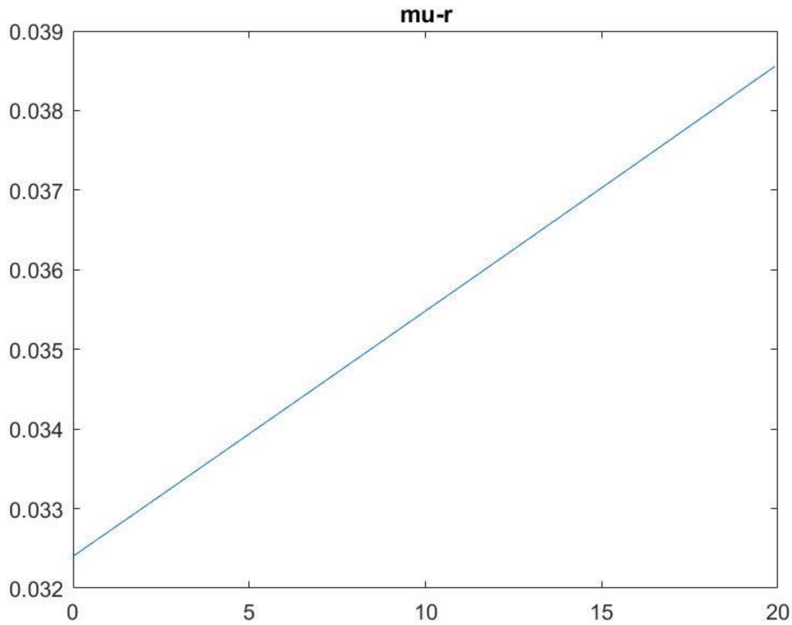
It must sum up to zero with the demand of underwriters, which is still $x = \frac{(\mu - r)}{\varsigma^2 R(W)} =$, with $\mu = \delta - P$. To guarantee market clearing, prices solves the equation

$$P(W) = \delta - \frac{u\zeta^2 a}{1 + a/R(W)}$$

which means that the equilibrium risk premium, $\mu - r = D - P(W) - r$ is

$$\mu(W) - r = \frac{a\zeta^2 u}{1 + a/R(W)} - r$$

Substituting for $\mu(W)$ in the problem of underwriters we would obtain an equation for R similar to the one solved above, and we would still have a unique solution and a unique CME. Solving for $u =!/100$ and $a = 1$, for instance, we would obtain the following $\mu - r$ levels:



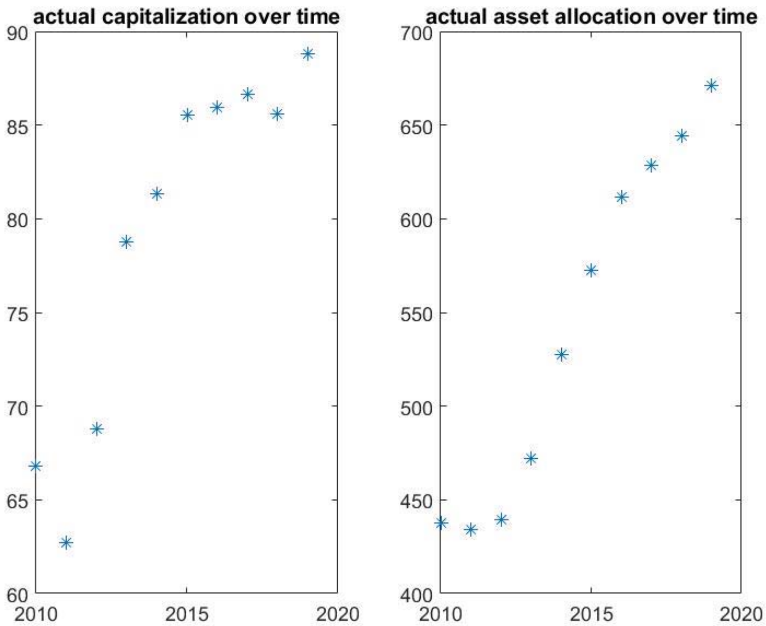
Endogenous risk premium as a function of capital. The dividend distribution level is 20.

The behavior of the model would not be much affected, since the behavior of risk aversion and of the other endogenous functions of capital is similar to the one with exogenous, numerically similar risk premium and market price of risk.

7 Risk aversion from market data

In the current section, we use the observed capitalization and asset allocation of Italian insurance companies over the period 2010-2019, to infer their implicit risk aversion. We show that risk aversion is countercyclical with respect to their capitalization, as predicted by our theoretical model.

The actual year-by year capital and asset allocation in the risky asset classes, in billions of euros, is depicted in the next Figure.



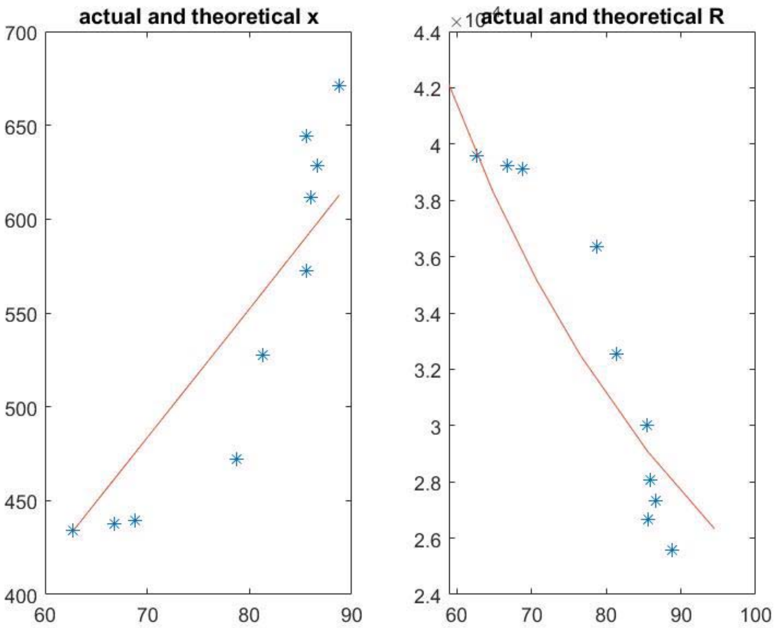
Left picture: Aggregate capitalization of the Italian life and non-life insurance companies, in billions of euros, year-end 2010-2019. Source: Ania (2020a). Data are in euros of 2019, and are obtained from book values using the ISTAT consumer price index. Right picture: Book value of the aggregate, actual asset allocation in risky bonds, funds and stocks in the time-period 2010-2019 (source: Ania (2020a)). The aggregation is over Italian life and non-life companies. Data are in nominal 2019 values, billions of euros.

The two time series are from Ania (2020a) and reveal that the aggregate capitalization of the sector is non-monotonic over the period 2010-2019. It fluctuates, consistently with our model. From the same data source we obtain the 2010-2019 time series of asset allocation in risky assets, which is non-monotonic and fluctuating too. Starting from these two time series, we compute the theoretical risky asset allocation of Italian insurance companies, as well as the risk aversion which corresponds to it. Last, we compare that risk aversion (named theoretical) with the risk aversion corresponding to the actual risky asset allocation. The theoretical and actual magnitudes will be very close, and will indicate a good descriptive power of the model.

To compute the theoretical risk aversion implicit in the portfolio choices of the undertakings under exam, we fit the function $R(W)$ to the actual asset allocation, when the capitalization is the actual one. The best fit obtains by adapting the volatility of the risky assets (unobserved). The weights of bonds and stocks on the total risky assets in the decade 2010-2019 were 83 and 17%, and the Datastream volatility of bonds and stocks is 16 and 31%.

We also compute the actual risk aversion using (9), together with the actual risky asset allocation: the actual risk aversion is the one which obtains by inserting in (9) the actual asset allocation and inverting with respect to R. The next figure compares actual and theoretical risk aversion (right plot), both represented as a function of capital.

Last, from the theoretical risk aversion we compute the theoretical risky asset allocation using the FOC (9), The next figure compares also the theoretical and the actual risky asset allocation (left plot), as a function of capital. The reader can appreciate the fact that in both the risk aversion and capital case the fitted and actual values are very close one to the other.



Left plot: actual and theoretical asset allocation in risky assets as a function of actual capital of the aggregate insurance sector in 2010-2019, Italian companies. Both entries are in billions. Stars correspond to the actual allocation, the solid line is the theoretical one. Right plot: actual Risk Aversion (stars) versus theoretical one (solid line), in billions, 2010 to 2019, Italian insurance companies.

8 Conclusion

We build a structural model with financial frictions that endogenizes insurers' risk aversion and implies a covariation of their insurance supply and asset allocation. The model implies that insurers' risk aversion is a decreasing function of their aggregate capitalization. This capitalization fluctuates over time, following underwriting losses and stochastic asset returns. Recapitalization costs - the only friction in our model - prevent companies from perfectly adjusting their level of equity to an optimal target. Risk aversion makes them manage their risk, and adopt a prudential approach to supply and asset allocation even in the absence of regulation.

The model fits well aggregate data from the Italian Insurance sector over the decade 2010-2019. These data show that risk aversion fluctuations have a substantial impact on insurers' asset allocations.

A natural extension would be to introduce prudential regulation, in the form of a minimum capital requirement depending both on assets and liabilities, as in Solvency II, but excluding the neutrality of Solvency with respect to the cycle, which has been practically enforced with lots of care. We expect that it would reinforce our results, as companies would be reluctant to fall into a regime where they would be constrained by regulatory capital requirements. This is likely to increase their risk aversion, especially when aggregate capitalization is low.

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10 Appendix A

Assume that short-lived, competitive and risk-averse households come as non-overlapping generations. The generation who lives from t to $t + dt$ suffers at

$t + dt$ a random shock in wealth equal to

$$dw_t = -ldt + \sigma dZ_t,$$

where $l, \sigma \in \mathbb{R}^{++}$ and $Z_t, t \geq 0$, is a *one*-dimensional Wiener process. We work on the probability space $(\Omega, \mathcal{F}, P)$ defined in the text.

Without a counterpart, namely insurers, willing to trade with them, the households change in wealth would be $dv_t = dw_t$. Since households are risk-averse, they have an incentive to get rid of the negative shocks, $-ldt + \sigma dZ_t$ when $dZ_t < ldt/\sigma$, even though they give away the positive shocks, $-ldt + \sigma dZ_t$ when $dZ_t > ldt/\sigma$, and pay a premium for that. Suppose that at time t insurance companies are willing to write a contract and absorb the shocks $-ldt + \sigma dZ_t$, for a loaded premium $(l + \pi_t)\, dt$, or $l + \pi_t$ per unit of time.

Denote with y_t the number of such contracts, or aggregate insurance supply. With insurance in place, the change dv_t in the households' wealth in $(t, t + dt)$ is[10]

$$\begin{aligned} dv_t &= dw_t + y_t\,(ldt - \sigma dZ_t) + y_t\,(-\,(l + \pi_t)\,dt) &\quad (22)\\ &= -ldt - y_t \pi_t dt + (1 - y_t)\sigma dZ_t. &\quad (23)\end{aligned}$$

If households born at t have VNM utility, because their horizon is dt, they can, without loss of generality, be represented as mean-variance investors. They maximize

$$V \triangleq \frac{\mathbb{E}(dv_t) - \frac{\alpha}{2}\mathrm{var}(dv_t)}{dt}, \qquad (24)$$

where the risk aversion parameter is $\alpha \in \mathbb{R}^{++}$. The usual symbols for expectation $\mathbb{E}$ and variance var under P are used.

Mean variance households with the objective function (24) appear also in Kondor and Vayanos (2019). They approximate any investor who lives an

[10] Consistently with reality, at each renewal - here for each generation of households - prices are updated.

infinitesimal period, consumes dv and has a VNM utility function with risk aversion α.

Given the expression above for dv_t, V is equal to

$$V(y_t) = -l - y_t\pi_t - \frac{\alpha}{2}(1-y_t)^2\sigma^2.$$

If we maximize V with respect to y_t, we get the optimal demand for insurance[11] as a function of its loading at t

$$-y_t = \frac{\pi_t}{\alpha\sigma^2} - 1, \tag{25}$$

or, equivalently, the loading as a function of the optimal demand

$$\pi_t = \alpha\sigma^2\left(1-y_t\right). \tag{26}$$

The loading becomes zero at $y^* = 1$.

11 Appendix B

This Appendix transforms the PDE for the value function J (4) into an ODE for the function q. It deals also with their boundary conditions.

Because the value function is linear in individual capital:

$$J(W^i, W) = W^i q(W),$$

[11] The demand for insurance is not modified if we assume that households receive at birth a non-random wealth $v_t > 0$, which returns the riskless rate $r > 0$.

its derivatives are

$$
\begin{aligned}
J_1(W^i, W) &= q(W), \\
J_{11}(W^i, W) &= 0, \\
J_2(W^i, W) &= W^i q'(W), \\
J_{22}(W^i, W) &= W^i q''(W), \\
J_{12}(W^i, W) &= q'(W).
\end{aligned}
$$

Substituting for the value function and its derivatives in the Bellman equation, after elicitation of the optimal controls, singular (D and I) and non-singular (x and y), we get a second-order ODE for q.

The boundary condition for J at $W = \bar{W}$ becomes a boundary condition for q :

$$J_1(W^i, \bar{W}) = q(\bar{W}) = 1.$$

The boundary condition at $W = \underline{W}$ is:

$$J_1(W^i, \underline{W}) = q(\underline{W}) = 1 + \gamma.$$

With q decreasing the no-arbitrage argument works as follows:

$$[Wq]' = q + Wq',$$

equates 1 at $\bar{W}$

$$q\left(\bar{W}\right) + \bar{W}q'\left(\bar{W}\right) = 1,$$

iff $q'\left(\bar{W}\right) = 0$, since $q\left(\bar{W}\right) = 1$ and $\bar{W} > 0$. It equates $1 + \gamma$ at $\underline{W}$

$$q\left(\underline{W}\right) + \underline{W}q'\left(\underline{W}\right) = 1 + \gamma,$$

iff $\underline{W} = 0$, since $q\left(\underline{W}\right) = 1 + \gamma$ while $q'\left(\underline{W}\right) > 0$, thanks to the monotonicity of q.

12 Appendix C

The density of the ergodic distribution of capital $h(W)$ exists if there is a non-negative solution of the following Fokker Planck equation (see Karlin and Taylor (1981), page 221)

$$-\mu_W h + \frac{1}{2}\left(\sigma_W^2 h\right)' = 0$$

Let us define the function

$$D(W) = h(W)\sigma_W^2(W).$$

From the ODE for h it follows that D satisfies the equation

$$d\ln D(W) = 2\frac{\mu_W}{\sigma_W^2}dW.$$

Solving for D we get

$$D(W) = D(0)\exp\left(2\int_0^W \frac{\mu_W(u)}{\sigma_W^2(u)}\right)du$$

which implies

$$h(W) = \frac{K}{\sigma_W^2(W)}\exp\left(2\int_0^W \frac{\mu_W(u)}{\sigma_W^2(u)}\right)du$$

where the normalization constant K must be such that

$$\int_0^{\bar{W}} h(u)du = 1.$$

In order to prove existence of a stationary distribution it is sufficient to observe that h is non-negative and to prove that K is finite. Substituting for the moments of capital $\mu_W(u)$ and $\sigma_W^2(u)$ we can compute the density $h(W)$

as

$$
\begin{aligned}
h(W) &= \\
&= \frac{K}{x^2(W)\varsigma^2 + \ y^2(W)\sigma^2} \exp\left(\int_0^W 2\frac{ru + (\mu - r)\,x(u) + y(u)\pi(y(u))}{x^2(u)\varsigma^2 + \ y^2(u)\sigma^2}du\right),
\end{aligned}
$$

Substituting for the optimal x and y in h we get

$$
\begin{aligned}
h(W) &= \\
&= \frac{K}{\frac{(\mu-r)^2}{\varsigma^2 R^2(W)} + \frac{\alpha^2\sigma^2}{(\alpha+R(W))^2}} \\
&\quad \times \exp 2 \left(\begin{array}{c} \int_0^W \frac{ru}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} du \\ + \int_0^W \frac{(\mu-r)\frac{(\mu-r)}{\varsigma^2 R(u)} + \frac{\alpha}{\alpha+R(u)}\pi(\frac{\alpha}{\alpha+R(u)})}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} R(u)du \end{array} \right).
\end{aligned}
$$

and, simplifying,

$$
\begin{aligned}
h(W) &= \\
&= \frac{K}{\frac{(\mu-r)^2}{\varsigma^2 R^2(W)} + \frac{\alpha^2\sigma^2}{(\alpha+R(W))^2}} \\
&\quad \times \exp\left(2\int_0^W \left(\frac{ru}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} + R(u)\right)du\right).
\end{aligned}
$$

We now prove that the integration constant K is finite:

$$
K = \frac{1}{\int_0^{\bar{W}} h_u(u)du} < \infty
$$

where $h_u = h/K$ is the un-normalized version of the density.

Let us study whether the integrand in the integration constant, namely $h_u(W)$, is bounded over the (finite) integration interval $[0, \bar{W}]$. We study separately the point $W = 0$, the open interval $(0, \bar{W})$ and the point $W = \bar{W}$.

At 0 we have:

$$h_u(0) = \frac{1}{\frac{(\mu-r)^2}{\varsigma^2 R^2(0)} + \frac{\alpha^2\sigma^2}{(\alpha+R(0))^2}},$$

Since risk aversion is bounded at 0, $R(0) < \infty$, then $h_u(0) < \infty$.

Under the same assumption, h is bounded in $(0, \bar{W})$, because R remains positive and finite when $W < \bar{W}$.

Consider the upper bound $\bar{W}$.

$$\begin{aligned} h_u(\bar{W}) &= \lim_{W\to\bar{W}} \frac{1}{\frac{(\mu-r)^2}{\varsigma^2 R^2(W)} + \frac{\alpha^2\sigma^2}{(\alpha+R(W))^2}} \\ &\quad \times \exp\left(2\int_0^W \left(\frac{ru}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} + R(u)\right) du\right). \end{aligned}$$

Since $R(\bar{W}) = 0$,

$$\begin{aligned} \frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2} &\to \infty, \\ \frac{1}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} &\to 0 \end{aligned}$$

Since

$$\frac{ru}{\frac{(\mu-r)^2}{\varsigma^2 R^2(u)} + \frac{\alpha^2\sigma^2}{(\alpha+R(u))^2}} + R(u)$$

is bounded in the whole interval $(0, \bar{W})$, the integral that appears in the exponential is finite and the exponential is. Therefore, $h_u(\bar{W}) = 0$. It follows that the integral in K is the integral over a finite interval of a bounded function, namely that K is finite. The stationary distribution of capital is well defined.

The existence of the stationary distribution of R, y, x follows from the existence of a stationary distribution of capital and the boundedness of the derivatives of the functions R, y, x with respect to capital.