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Antagonistic Cooperation: Factional Competition in the Shadow of Elections *

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Abstract

Intra-party competition is widespread and affects political parties' strength. This paper presents a model of elections in which intra-party factions can devote resources to campaigning for the party or undermining competing factions to obtain more power. The model shows that inter- and intra-party competition are substitutes: Internal competition increases when the electoral stakes are low—e.g., in consensus democracies granting power to the losing party—because the incentives to focus on the fight for internal power increase. Similarly, an increase in party polarization incentivizes factions to campaign to avoid a more costly electoral loss. Factions in the moderate party campaign more than those in the extreme party; conversely, when factions in the same party are ideologically divided, extreme factions campaign more. Finally, the model studies how internal rules affect intra-party competition, showing how parties design internal contests among factions to maximize campaigning.

Keywords: Party Organization, Electoral Competition, Party Factions, Portfolio Allocation.

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1. Introduction

Political parties are rarely monolithic organizations: they are often internally divided into separate factions. The presence of factions can benefit parties. Factions "might be instrumental in building integrated parties by facilitating their aggregate capacity" (**?**). Furthermore, factions can improve party deliberations and tie the views of extreme politicians to moderate faction leaders, which can enhance a party's welfare (**?**). Despite their potential benefits, factions can also be detrimental. In Federalist 10, Madison outlines the perils posed by factionalist interests against the public good (**?**). V. O. Key blamed factions for encouraging favoritism and graft (**?**). What brings opposing factions to compete against each other rather than working for the party's common good? This paper studies the tension between collective and factional interest, exploring under what conditions factions might enhance vs. hinder a party's welfare.

Parties that developed structured factions worldwide — for instance, the Italian Christian Democratic (DC) party during the First Republic, the Japanese LDP and the Indian National Congress — are characterized by chronic internal division and competition over power and resources. Such competition can hurt a party's electoral prospects if factions prioritize their preeminence over the party's common good. This happens, for example, when a faction invests in tangential activities meant to increase its power over other factions instead of supporting the party with campaigning activities aimed at enhancing party valence among voters.

To illustrate this competitive logic, consider Italy during the era of the DC's dominance (1946-1994). Factions in the DC were renowned for fighting over members, a metric of factions' internal power adopted by the party to distribute resources. Common ways used to inflate factions' members were to assign memberships to individuals who either were unaware, dead or had emigrated, or to impede subscriptions to competing factions with delays and procedural complications (**?**). Similar metrics of factions' power are commonly used in parties with formalized factions competing for electoral spoils. In Austria, for instance, the share of factions' membership is used to measure the strength of party subunits within the Social Democratic Party (SPÖ), the Freedom Party (FPÖ) and the Alliance for the Future of Austria (BZÖ) (**?**).

To show how factional conflict affects a party's welfare, I develop a model of electoral competition where parties are internally divided among factions competing in a contest for internal power. Each faction decides how much time to allocate to campaigning activities to support the party (e.g., constituency service that increases party valence), and to activities aimed at increasing its relative stance against the other (e.g., fighting over membership cards). In the model, I refer to the latter type of activity as "sabotage," as it constitutes a "deliberate act by one contestant to damage the performance of another in a contest" (**?**). While campaigning effort probabilistically raises the party's chances of winning the election, sabotage only helps a faction achieve internal power. Besides factions' efforts, the electoral outcome depends on party platforms, which are a function of factions' preferred platforms.

The central results of the paper show how features of the electoral environment — both institutional and non-institutional — influence the factions' trade-off between the two activities. Institutional features capture for example the degree of *inter-party power-sharing* (**??**). Previous work has found that in factionalized countries such as Italy and Japan, intra-party competition is substantially higher when the electoral system resembles a consensual democracy (**???**). Conversely, reforms that change the system towards a winner-takes-all reduce intra-party fights, as summarized in Table 1. The model explains these patterns, showing that a change from a more consensual to a majoritarian electoral system can intensify factional competition by incentivizing factions to focus on the internal fight for power. In other words, the model shows that inter- and intra-party competition are *substitutes*.

To illustrate the logic of this argument, consider the share of electoral spoils (e.g., cabinet portfolios or assignment to committees) obtained by a party and distributed among factions after the election. These spoils are determined by the amount of power-sharing of the electoral environment: majoritarian democracies concentrate power in the hands of the winning parties,

Table 1 – Institutional reforms and intra-party competition in Italy and Japan.

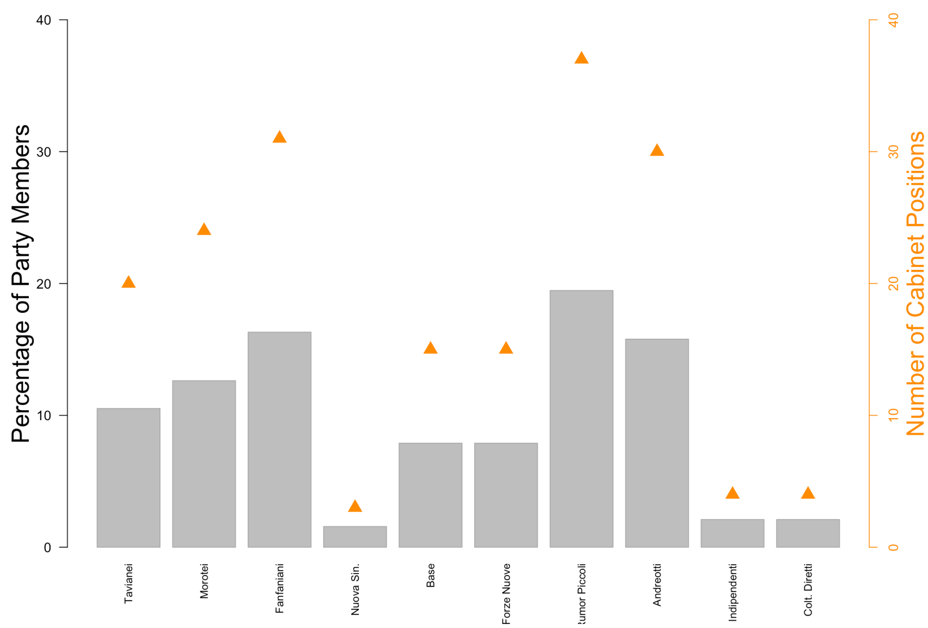
	Party competition	Factional competition	Evidence
Consensus Democracy	*low*	*high*	Italy's First Republic (PR) Japan's before 1994 reform (SNTV)
Winner Takes All	*high*	*low*	Italy's Second Republic (mixed system) Japan after 1994 (SMD)

while in consensus democracies resources are more evenly shared with minority parties. When the political system resembles a majoritarian democracy the stakes of the election are high, and factions support the party by investing in campaigning activities. Conversely, the more the system reflects a consensus democracy that grants power to losing parties, the more factions invest resources in the internal competition for power. Several constitutional design scholars warn against certain features of winner-takes-all electoral systems (**??**). This result underscores the overlooked element of intra-party incentives generated by institutions when comparing different democratic systems.

A similar logic applies to (party) ideological polarization. As the ideological distance between parties increases, factions invest more resources in campaigning to avoid a more costly electoral loss. Intuitively, by increasing the stakes of losing the election, an increase in polarization can motivate factions to campaign more for the party. Perhaps less intuitively, when one party is further away from the median voter's bliss point, factions in the more extreme party campaign less than those in the moderate party, as platform asymmetry dampens the incentives for factions in the extreme party to campaign as a result of a lower winning probability. This result differs from models of endogenous campaigning, where extreme parties typically campaign more than moderate ones (**??**).

The model then studies how *internal* incentives affect factional competition: as noted by **?**, "party organizational arrangements have a big role to play in the management of factionalism" (p. 475). Each faction's trade-off is affected by internal rules that determine how the electoral spoils (e.g., cabinet portfolios and committee chairs) controlled by the party are allocated among factions based on their relative strength. For instance, the strength of a faction within the Italian DC depended on its seat share in the national party council (**?**), itself a function of the number of factional members. The allocation method of the party followed an explicit formula according to which factions obtained cabinet positions in proportion to their total membership. Figure 1 provides an illustration of the method.[1]

Figure 1 – Empirical Realization of the Portfolio Allocation Method in the Italian DC (1973).



Notes: The bars represent the percentage of party members belonging to each faction. Based on this percentage, the total number of cabinet positions is divided proportionally among factions. The triangles show the number of cabinet positions obtained by factions in 1973.

To capture these internal rules, the model analyzes different incentive schemes that reward factions. Rewards depend on an intra-party contest representing various procedures used by parties to assess power among factions which may include factional affiliations among party members or geographical breakdowns in party support. In principle, the internal contest can be beneficial or detrimental to the party's electoral prospects. The internal contest is good for

[1] The method refers to the "Cencelli Manual," which has since become a common political idiom (**?**). The original table from the manual is available upon request.

the party when factions achieve power by prioritizing campaigning over sabotage. Conversely, competition damages the party when factions become more powerful by investing in sabotage, as this takes away resources that factions would otherwise invest in campaigning activities.

How do rewards affect factions' investment decision in equilibrium? The rewards span from low-powered (i.e., factions are equally rewarded, independently of how they fare in the internal contest) to high-powered (i.e., the pre-eminent faction obtains all spoils). When the internal contest is good for the party, high-powered incentives reward the faction that probabilistically invested more in campaigning activities, while low-powered incentives discourage campaigning. In this case, in equilibrium factions invest more in campaigning when incentives are high-powered. Conversely, when sabotage is more effective than campaigning activities to achieve internal power, high powered incentives encourage sabotage and factions invest more in campaigning when incentives are low-powered.

Finally, the model analyzes optimal rewards — i.e., the rewards that an office-motivated leadership would offer to factions to maximize the party's chances of winning — as a function of the internal contest (whether it is beneficial to the party or not). I assume that the party commits to a premium for the pre-eminent faction. Crucially, the party knows whether investment in campaigning or sabotage is more effective for winning the internal contest. When factions are tied to geographic strongholds, for example, the party can associate its electoral performance in a given area to a faction's campaigning. In this case, the internal contest is beneficial — in the sense that the pre-eminent faction is the one that probabilistically invested more in campaigning — and high-powered incentives motivate factions to campaign. The method of allocation of cabinet positions in the Austrian SPÖ is consistent with this prediction: the Social Democrats bias portfolio allocation towards their large *Land* factions, tied to the geographic strongholds of Vienna and Lower Austria (**?**). Conversely, when factions overlap geographically, the party must rely on poorer measures (e.g., party members). In this case, the internal contest is detrimental to the party and the optimal incentives reward factions equally.

Discouraging sabotage with zero incentives is no longer optimal if parties can offer policy concessions to factions. An extension of the model assumes that party platforms are weighted averages of factions' ideal points, where the weight is chosen by the party. Here, in addition to electoral spoils, the party can reward the higher-ranked faction by putting more weight on its preferred policy in the party manifesto. Results show that, when factions have different ideological preferences, the optimal incentives can reward the pre-eminent faction by favoring its policy *even when the internal contest is detrimental to the party*. I show that these equilibrium policy concessions are used to incentivize moderate factions to work for the party.

2. Literature

This paper provides a novel framework to study how factional competition shapes the life of a party, including both its internal institutions and its campaigning capacity. As such, it directly relates to the theoretical literature analyzing the role of factions within parties. In particular, this paper shares its focus on the rules of party organization that incentivize intra-party effort with the work of **?**. Methodologically, my paper builds upon theirs by modeling the party organization in a novel way and including ideological motives as a driver of factions' incentives. **?** studies a distinct form of intra-party competition (over ideology, not resources) in the form of public dissent against an ideological ally. Her model shows that by dissenting, or competing over ideology, factions can achieve a better policy outcome, complementing this paper's findings.

The model joins a substantial body of work on intra-party organization. Many existing models of party organization focus on nomination processes, specifically examining primary elections (e.g., **???**). A smaller theoretical literature studies party organizations outside of primaries (**???**). I contribute to this literature by formalizing a party's internal organization as the degree of *power-sharing* among its faction, and showing how this changes with features of the electoral environment such as polarization and electoral institutions.

Finally, this paper contributes to the formal literature on portfolio allocation by focusing on the distribution of the portfolio *within* parties rather than between parties forming coalitions. The main empirical pattern of portfolio allocation, known as Gamson's Law, shows that parties forming coalition governments obtain shares of portfolios proportional to the seats contributed (**??**). Since the pattern is at odds with non-cooperative bargaining models' predictions (**???**), several papers have developed models that theoretically justify Gamsonian allocations among coalition parties (**???**). In particular, Gamson's Law has been justified as the "optimal incentive scheme" to maximize campaigning effort of partners of a *pre-electoral* coalition under the assumption that parties can form binding agreements before elections (**?**).

My model builds on a similar intuition, analyzing how the optimal portfolio allocation changes when it is done within parties. This approach produces several contributions. I show that a Gamsonian division of spoils is not necessarily the optimal incentive scheme if the goal is to maximize *factional* effort rather than coalition partners' efforts, which brings a new interpretation to the mixed results of the empirical literature on portfolio allocation *within* parties (**????**). Furthermore, I show how party rules of portfolio allocation can affect the nature of campaigning investment (and not just the level) among different activities, as well as how the electoral environment affects factions' trade-offs given the party's equilibrium internal rules.

3. The Model

Consider a probabilistic model of electoral competition between two parties, *left* and *right*. There are three players in *left*: two factions denoted by L_1, L_2 and a party leader L (*she*). Players in *right* are denoted by R_1, R_2 and R. Denote *left*'s preferred platform by $x^L \in \mathbb{R}$, which is implemented when the party wins the election (the same holds for *right*). Both platforms (x^L, x^R) are common knowledge and fixed.[2] There exists a representative voter, denoted by V, who votes for *left* or *right*. The voter's ideal point is denoted by x^V. Without loss of generality, let

[2] The latter assumption is relaxed in Subsection 5.1, which analyzes endogenous platforms.

$x^V = 0$, and $x^L < x^V < x^R$. For exposition purposes, I will refer only to the *left* (the description of actors in *right* is analogous).

A faction is a unitary team of politicians who share the same ideological preferences: let x_i^L denote the policy preferred by faction L_i, where $i = 1, 2$. I start by assuming that factions in the same party share the same ideological preferences ($x_1^L = x_2^L = x^L$), and relax this assumption in Section 5. Each faction can use its resources to increase the party electoral chances by exerting campaigning effort, denoted by e_i^L, and/or to "sabotage" the other faction, a_i^L. Both actions are costly: $C(e_i^L) = (e_i^L)^2/2$ and $C(a_i^L) = (a_i^L)^2/2$.[3] The assumption of decreasing returns associated with each activity reflects that while initially it is easy to find slogans to convince voters and to invest in internal politicking, at the margin more resources are needed to have a substantial impact on the outcome of the contest.

The goal of the model is to understand which circumstances encourage campaigning effort and which encourage sabotage. In order to do so in a tractable way, the total amount of sabotage is assumed to be equal to $a_i^L = 1 - e_i^L$ — that is, all of the resources that are not used to promote the party are invested in damaging the rival faction (this assumption is relaxed in the Appendix). The binding budget constraint together with convex costs implies that a positive level of sabotage is cost efficient: the focus will be on how the equilibrium investment in effort and sabotage changes with parameters of interest.

Consider an office-motivated leadership that stands for the party organization and is above factions.[4] The leader incentivizes factions to invest in campaigning for the party by promising rewards contingent on the electoral outcome. Rewards depend on the total amount of electoral

[3] The assumption of separate, equally convex costs of campaigning and sabotage reflects the decreasing marginal return associated with each activity, while being neutral about the desirability of sabotage vs. campaigning effort, as it is not obvious that one action should be more time consuming or expensive resource-wise than the other. However, results would be unchanged if sabotage was costlier than campaigning effort (or vice-versa).

[4] Subsection 5.1 discusses how the leader would tailor the party policy platform to factions' preferences if she belonged to a particular faction. In the baseline model, the leader can be thought of representing the outcome of a bargaining process maximizing the joint surplus of factions.

spoils, normalized to one: the winning party gets a share of spoils α, where $\alpha \in [1/2, 1]$, and the losing party gets $(1-\alpha)$. The parameter α refers to the degree of inter-party power-sharing: I use the short-hand "majoritarian democracies" to refer to systems with high α, and "consensus democracies" to refer to systems with low α.

The party leader does not directly observe factions' campaigning efforts. Instead, L observes the outcome of an internal contest, which is modeled as a "relative ranking" indicator that can take two values, $s^L \in \{1, 2\}$. When $s^L = 1$, faction L_1 is ranked higher than L_2; the opposite is true when $s^L = 2$. Both campaigning effort and sabotage help factions rank higher. Formally, the probability that faction L_1 obtains a higher rank than L_2 is:

$$\rho_1^L = \Pr\{s^L = 1\} = \frac{1}{2} + \frac{(e_1^L - e_2^L) + \gamma(a_1^L - a_2^L)}{\phi}. \tag{1}$$

The parameter $\gamma \in \mathbb{R}_+$, which is common knowledge, reflects the relative effectiveness of sabotage to rank higher: when $\gamma < 1$, the internal contest benefits the party (i.e., investing in campaigning effort is more effective than sabotage to rank higher), whereas when $\gamma > 1$, sabotage is more effective than campaigning effort to achieve a high internal ranking. The parameter ϕ is a normalization ensuring that the probability is bounded. The probability that faction L_2 obtains a higher rank is simply $\rho_2^L = 1 - \rho_1^L$.

The parameter γ represents features of the environment that make the internal contest among factions more detrimental to the party. Beside non-institutional features — characterizing for instance the extent to which factions overlap geographically — γ can represent *institutional* features of the environment such as the use of preference votes in PR systems. For example, a reform that changes the system from closed list PR (where parties select candidates and voters cannot indicate their preferences on the ballot) to open list PR (i.e., a decrease in γ) could make factional campaigning efforts more effective, as the party observes its candidates' preference votes and knows each candidate's faction.

Based on the indicator s^L, the leader distributes among factions the share of electoral spoils obtained contingent on the party winning (α) or losing the election $(1-\alpha)$ by assigning a *premium to the faction that ranks higher* within the party (e.g., faction L_1 if $s^L = 1$). Formally, L assigns π_v^L to L_1 if *left* wins and π_d^L if it loses (the subscript v stands for victory, while d represents defeat). Premia are assumed to be non–negative — that is, the leader can "punish" factions at most by offering zero incentives. Importantly, this assumption is motivated by real-world instances where party leaders are *de facto* constrained by the existing measures of factional performance and have no other choice but to abide by them.[5,6] Rewards for factions in *left* satisfy the following budget constraint:

$$\begin{aligned} \pi_v^L + 2b_v^L &= \alpha \\ \pi_d^L + 2b_d^L &= 1-\alpha, \end{aligned} \tag{2}$$

where b_v^L and b_d^L are baseline prizes offered to both factions in case of party victory and defeat, respectively. Given the leader's budget constraint assumption (2), the value of the baseline prizes is equal to $b_d^L = (1-\alpha-\pi_d^L)/2$, $b_v^L = (\alpha - \pi_v^L)/2$, and the only relevant choice for L is the vector of premia (π_d^L, π_v^L).[7] The leader chooses (π_d^L, π_v^L) to maximize the probability of winning the election, which is increasing in factions' campaigning effort and determined in equilibrium by the voter's choice. Note that the assumption that the leader cannot renege on the rewards promised to factions reflects dynamic considerations preventing the party leader from keeping

[5] In the Italian DC case, for instance, the leadership knew that a faction's relative higher ranking — corresponding to more membership cards brought to the party — was a symptom of its higher investment in sabotaging activities. Yet, leaders still could not punish factions for bringing more members to the party.

[6] The Appendix relaxes this assumption and shows that results are robust to a setup where leaders can punish the higher-ranked faction with a negative premium (i.e., rewarding the lower-ranked faction).

[7] Given the budget constraint assumption, L's incentive scheme can also be interpreted as the share of spoils offered to each faction under the event of party victory and defeat. I distinguish between baseline prizes and premia to reflect real instances of incentives used by parties. For instance, in the Japanese LDP *"the more numerous positions, notably in the cabinet, were allocated in close proportion to a faction's membership in the Diet, whereas scarcer positions [...] were balanced among the very largest factions"* (**?**).

resources.[8] It also closely reflects real instances of portfolio allocation in factionalized parties.[9]

The voter's payoff has two components. The first is a standard quadratic loss from the distance from party platforms, and the second depends on party valence, which refers to all attributes that are independent of ideology. Factions' campaigning activities such as constituency service increase the party's appeal to voters by increasing its valence. Formally, V's realized payoff if *left* wins is:[10]

$$u^V(e_1^L, e_2^L; x^L) = -\left(x^V - x^L\right)^2 + e_1^L + e_2^L. \tag{3}$$

Finally, before the election, an exogenous shock that favors party R affects the voter's payoff, where ξ is uniformly distributed in $\left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$. I assume that ψ is small enough to ensure a bounded probability of victory for both parties. The parameter ψ can be interpreted as the importance of the electoral campaign: as ψ increases, the support of the shock shrinks and factional campaigning activities (as well as party policy platforms) become more salient to the voter.

Let $\boldsymbol{\pi}^L = (\pi_d^L, \pi_v^L)$ and $\boldsymbol{e}^L = (e_1^L, e_2^L)$. The payoff of faction L_i can be expressed as:

$$u_i^L(\boldsymbol{e}^L, \boldsymbol{\pi}^L; x_i^L) = R_i^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L) - \left(x_i^L - x^*\right)^2 - C(e_i^L) - C(1 - e_i^L), \tag{4}$$

[8] In the Appendix I show that the equilibrium rewards are also an equilibrium of an infinitely repeated game where the leader can appropriate electoral spoils, provided that the leader's discount factor is high enough.

[9] For the Italian DC, all governments closely followed the Cencelli Manual in assigning office positions. If the proportions did not follow the manual's predictions, the penalized factions would jeopardize the government. Throughout the governing period of the DC (1946-1994) the threat of the vote of no confidence was sufficient to follow the division of spoils prescribed by the Manual. The case of the Japanese LDP is similar. Despite having the opportunity to revise the proposal of cabinet posts (by asking for a leadership change, leaving the party, or calling for a vote of no-confidence), no disagreement ever happened (**?**). Moreover, historically no cabinet formation required more than three days, which is a surprisingly short period given that the average length of government formation process in Western Europe is 28 days (**?**).

[10] Note that sabotage does not affect V's payoff directly. Assuming that the voter directly punishes sabotage does not qualitatively affect the model's results: that is, the marginal benefit of campaigning would be higher and in equilibrium factions would campaign more, but the main comparative statics of the model would still hold.

where $x^* \in \{x^L, x^R\}$ denotes the winning party's platform, and the reward $R_i^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)$ is a function of the incentive scheme $\boldsymbol{\pi}^L$.[11] The expected rewards from electoral victory and defeat are, respectively:

$$R_i^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)|_v = b_v^L + \rho_i^L(\boldsymbol{e}^{\boldsymbol{L}})\pi_v^L, \qquad (5)$$
$$R_i^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)|_d = b_d^L + \rho_i^L(\boldsymbol{e}^{\boldsymbol{L}})\pi_d^L.$$

That is, L_i is rewarded with π_v^L (π_d^L) only when it ranks higher than the other faction (which happens with probability ρ_i^L), and the size of the reward is determined by the institutional environment ($b_v^L + \pi_v^L > b_d^L + \pi_d^L$, as $\alpha > 1/2$).

The timing of the game is as follows. First, leaders announce an incentive scheme contingent on the electoral outcome. Second, factions decide how much of their resources to invest in campaigning and sabotage. Finally, elections are held, and prizes are distributed according to the contract. A strategy for L maps from the internal ranking s^L to an incentive scheme (π_d^L, π_v^L). For L_1 and L_2, a strategy is a mapping from the set of incentives to an allocation decision e_1^L, e_2^L (and analogously for R_1 and R_2). The voter votes for the party that gives her the higher payoff. The solution concept is Subgame Perfect Equilibrium.

4. Equilibrium Analysis

In what follows I compute the voter's decision, which determines the probability of each party winning the election. Given this winning probability, the expected payoff of each faction is derived as a function of the other factions' decision, as well as for each possible incentive scheme offered by the leader. Finally, I compute the incentive scheme chosen by the party leader and characterize the equilibrium of the game.

[11] Note that the reward function $R_i^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L)$ also depends on $\boldsymbol{\pi}^{\boldsymbol{R}}$ and $\boldsymbol{e}^{\boldsymbol{R}}$, which affect the incentive scheme $\boldsymbol{\pi}^L$. To be concise, this is omitted from the notation above.

The voter prefers the *left* party if:

$$u^V(e_1^L, e_2^L; x^L) \geq u^V(e_1^R, e_2^R; x^R) + \xi.$$

The probability that *left* wins the election is the probability that V prefers *left*. By the uniform assumption, this is simply a function of factions' efforts ($\boldsymbol{e}$):

$$p^L(\boldsymbol{e}) = \frac{1}{2} + \psi \left[u^V\left(e_1^L, e_2^L; x^L\right) - u^V\left(e_1^R, e_2^R; x^R\right) \right]. \tag{6}$$

Factions choose how many resources to allocate to campaigning and sabotage. Formally, L_1 solves:

$$\max_{e_1^L \in [0,1]} p^L(\boldsymbol{e}) \Big[b_v^L + \rho_1^L \pi_v^L \Big] + \left(1 - p^L(\boldsymbol{e})\right) \Big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2 \Big] - \frac{\left(e_1^L\right)^2}{2} - \frac{\left(1 - e_1^L\right)^2}{2},$$

where the first term (expected payoff from winning the election) does not include an ideological cost because $x_1^L = x^L$. Notice that e_1^L has two effects on factions' expected payoff. First, campaigning increases the party's electoral chances via higher $p^L(\boldsymbol{e})$. Second, e_1^L enters the probability of ranking higher within the party (ρ_1^L), with this component reflecting the strategic tension faced by factions.

Depending on the value of γ, the probability of ranking higher could be increasing or decreasing with campaigning. This follows from (1), and the assumption that $a_i^L = 1 - e_i^L$. When $\gamma < 1$, campaigning effort helps winning the election and improves the odds of being assigned a positive premium by the leader. The trade-off between campaigning effort and sabotage arises when $\gamma > 1$: while sabotage increases the odds of a high internal ranking (hence, a higher share of spoils), campaigning helps the party win the election. This trade-off is apparent in the faction's first-order condition, which can be written as:

$$\underbrace{\frac{\partial p^L(\boldsymbol{e})}{\partial e_1^L}\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\right]}_{\text{External Incentive}} + \underbrace{\frac{\partial \rho_1^L}{\partial e_1^L}\left[p^L \pi_v^L + (1-p^L)\pi_d^L\right]}_{\text{Internal Incentive}} + 1 - 2e_1^L = 0. \quad (7)$$

The first term represents the marginal return of effort on winning the election holding the competition inside the party fixed, while the second term corresponds to the marginal return of effort on ranking higher within the party holding the electoral incentives fixed. The first term is always positive. In other words, factions' campaigning always improves the party's electoral chances in the election. The sign of the internal incentive term depends on whether sabotage is more effective than campaigning. When $\gamma < 1$, the internal incentive term is positive (campaigning helps a faction achieve a high ranking), whereas it is negative when $\gamma > 1$ (sabotage is more effective than campaigning).

In the Appendix, I show that each faction's objective is concave, which implies that the first-order condition above identifies the solution to the faction's maximization problem. The solution of the system of first-order conditions, one for each faction in each party, determines factional effort as a best reply to the other factions' efforts for both incentives schemes (π_d^L, π_v^L) and (π_d^R, π_v^R). The following analysis refers to equilibrium effort as e_i^{L*}.

The leader chooses premia to maximize the probability of winning the election, which is increasing in campaigning effort. Substituting the value of the voter's realized payoffs into the probability of victory expression (6), we can formally express L's problem as:

$$\max_{\pi_d^L, \pi_v^L} \quad \frac{1}{2} + \psi\left[e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*} - (x^L)^2 + (x^R)^2\right]. \quad (8)$$

How can the leader give each faction a stake in the party's electoral success? Let campaigning effort be more effective than sabotage ($\gamma < 1$). The first result below shows that in this case the leader gives the highest feasible premium (all the electoral spoils) to the higher ranking faction, and the second faction is not rewarded (i.e., incentives are high-powered). By doing so, the leader increases factions' incentives to campaign. Conversely, when sabotage is more effective than campaigning effort ($\gamma > 1$) a positive premium to the higher ranking faction incentivizes sabotage. In this case, the leader's optimal strategy is to set zero premia — that is, both factions receive an equal share of spoils for all the election outcomes.

The first result derives the equilibrium when no party has an ex-ante electoral advantage over the other due to policy platforms. Since $x^L < 0 < x^R$, the assumption of no electoral advantage means that either both party platforms coincide with x^V (i.e., $x^L = x^R = 0$) or that their platforms are equidistant from it ($x^L = -x^R$). The assumption will be relaxed in the next section, which analyzes the equilibrium for any policy platforms such that $x^L < 0 < x^R$. While less general, this first result is valuable because it yields both a simple closed-form solution for the equilibrium effort and intuitive comparative statics.

Proposition 1. Equilibrium without ex-ante Electoral Advantage. *Suppose $x^L = -x^R$. Then, the optimal incentives offered by L in equilibrium (and, symmetrically, by R) are $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$ if $\gamma > 1$ and $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$ if $\gamma < 1$. The unique level of campaigning effort for both factions is:*

$$e^{L*} = \begin{cases} \dfrac{2 + \psi\big(2\alpha - 1 + 2\big(x^L - x^R\big)^2\big)}{4} & \text{if} \quad \gamma > 1 \\ \dfrac{5 - \gamma + 2\psi\big(2\alpha - 1 + 2\big(x^L - x^R\big)^2\big)}{8} & \text{if} \quad \gamma < 1 \end{cases} \tag{9}$$

Proof. Unless otherwise stated, all proofs are collected in the Appendix. □

When sabotage is more effective than campaigning effort in achieving primacy within the party ($\gamma > 1$), the leader's optimal strategy is not to reward it. Hence, incentives are low-powered, and the equilibrium incentive scheme has zero premia. Conversely, when $\gamma < 1$, the equilibrium incentive scheme features high-powered incentives.[12] Intuitively, in this case

[12] Note that when $\gamma = 1$ the probability of winning the internal contest is independent of factions' investment in campaigning effort and equal to one half because $a_i^L = 1 - e_i^L$. In this case there are clearly no optimal premia.

the incentives to exert campaigning effort arise from both the election and the internal ranking indicator, while sabotage only hurts factional welfare. Hence, the party leader designs the internal contest such that the higher ranking faction is rewarded with the highest premium feasible (i.e., all the electoral spoils) contingent on the electoral outcome, and the second faction is not rewarded. This result is in line with standard intuition from contest theory suggesting that the effort-maximizing incentive scheme is a winner-takes-all contest — i.e., high powered incentives are optimal in contests where the probability of winning is increasing in effort (**?**).[13]

The expression for the equilibrium campaigning effort e^{L*} in Proposition 1 allows us to inspect how factions' campaigning effort changes in equilibrium with (i) the importance of the electoral campaign (ψ), (ii) the amount of power sharing of the institutional setting $(1 - \alpha)$, and (iii) party polarization $(x^R - x^L)$, where the latter is defined as follows: Polarization increases if x^L decreases and x^R increases by the same amount, thus holding $x^L + x^R$ constant.[14] This ensures that any increase in polarization does not change the identity of the (ex-ante) advantaged party, and allows us to focus exclusively on the level of divergence between party platforms ($x^R - x^L$).

Corollary 1. Electoral Environment. *For all γ, factions' campaigning effort (e^{L*}):*

(i) increases with the importance of the electoral campaign (ψ),

(ii) decreases with the proportionality — or amount of power granted to minorities — of the institutional setting $(1 - \alpha)$,

(iii) increases with ideological polarization ($x^R - x^L$).

The proof follows by inspection of (9). Intuitively, when the support of the aggregate shock

[13] Note that both factional symmetry in the internal contest and the resource budget constraint are crucial for this result. In related work, I analyze optimal incentives when one faction could be ex-ante advantaged and when factions can keep resources for themselves. Under such conditions, a winner-takes-all incentive scheme is not always optimal (i.e., it does not always maximize factions' total campaigning).

[14] Note that this condition is trivially satisfied when no party has an ex-ante electoral advantage.

gets smaller (higher ψ), the electoral outcome depends less on the random component and more on factional campaigning effort. As a consequence, factional effort more effectively influences the voter's decision. Perhaps less intuitively, factional campaigning effort in equilibrium is strictly increasing in α (or, alternatively, factional sabotage is strictly decreasing in α). In other words, inter-party and intra-party competition are *substitutes*: In majoritarian democracies, intense competition among parties induces factions within parties to cooperate more. Conversely, in consensual democracies — featuring power sharing between parties — compromise among parties produces competition within parties.

The effect of α on campaigning suggests a simple yet neglected relationship between inter-party and intra-party competition. Constitutional design scholars typically focus on the incentives that institutions produce at the party level (**??**). In particular, majoritarian democracies are associated with adversarial fights for power, and consensual democracies with bargaining and compromise across parties. Proposition 1 suggests that an institutional change in the electoral stakes can also affect competition within parties. Note that a change in α might refer to a change in the electoral system (e.g., from winner-takes-all to proportional), or to an institutional change holding fixed the electoral system's proportionality (e.g., from executive dominance to legislative-executive balance). As such, the result applies to both two-party and multi-party systems.

The relationship of substitution between inter-party and intra-party competition similarly arises through an increase in party polarization ($x^R - x^L$): By increasing the electoral stakes, polarization motivates factions to campaign for the party. Conversely, as party platforms get closer, factions focus on the competition internal to the party. To see how polarization can affect factional behavior, consider factions' payoff from losing the election in the following two cases. First, when $x^L = x^R = 0$, the ex-ante probability of victory for each party is the same, and factions do not suffer any ideological cost from losing the election. When $x^L = -x^R$ and $x^L, x^R \neq 0$, parties' ex-ante winning probability does not change but factions now suffer a cost, increasing in $x^L - x^R$, from losing the election. This increasing cost in turn implies that, as ideological polarization increases, factions invest more resources in campaigning effort (refraining from sabotaging each other), in order to avoid a costly unfavorable electoral outcome.

4.1. Introducing Electoral Imbalance

One question that arises when parties are heterogeneous in their ex-ante winning probability is whether the leading or trailing party's factions campaign more. To answer this question, I relax the assumption that party platforms are equidistant from the voter's preferred platform, thus allowing for ex-ante imbalance in parties' electoral prospects.

The next result establishes the consequences of party ideological extremism on factional campaigning effort and parties' electoral prospects, for any x^L, x^R such that $x^L < 0 < x^R$. The first part of Proposition 2 generalizes the effect of polarization to the case of general platforms, showing that the comparative statics highlighted by Corollary 1 hold.

The second part of the result focuses on the difference between total campaigning effort in *left* and *right* ($2e^{L*} - 2e^{R*}$). When platforms are not equidistant from the median, an increase in platforms' extremism affects campaigning effort in equilibrium through an additional channel: the change in the odds of winning the election. The result shows that when $\gamma < 1$, an increase in a party's extremism — defined as the distance between the party platform and the voter's preferred platform — leads its factions to campaign less than the other party's factions. The next result assumes without loss of generality that the ex-ante winning probability of *left* is lower than that of *right*: $|x^L| > |x^R|$.[15]

Proposition 2. Party Polarization and Ideological Extremism.

(i) For all γ, factional campaigning effort in both parties (e^{L*}, e^{R*}) increases with polarization.

[15] In the Appendix, I derive the vector of equilibrium effort choices as the unique solution (in closed form) of the system of factions' first-order conditions. The closed form solution is omitted from the main text as it does not provide further intuition than that provided by (9). The Appendix also shows that the equilibrium incentive scheme is the same as the one derived in Proposition 1 when factions are equidistant from x^V.

(ii) When $\gamma < 1$, $\partial(2e^{L*} - 2e^{R*})/\partial|x^L| < 0$ for all x^R, and $\partial\big(2e^{R*} - 2e^{L*}\big)/\partial|x^R| < 0$ for all x^L. When $\gamma > 1$, factions in both parties campaign equally.

The intuition for the first part of Proposition 2 is analogous to that of Corollary 1: when polarization ($x^L - x^R$) increases, factions' expected payoff from losing decreases because of the ideological loss they suffer. This, in turn, increases the marginal return on campaigning effort to avoid electoral defeat. Note that this mechanism is analogous to an increase in α: that is, both institutions and polarization can increase the stakes of the election, thus affecting factions' decision. The second part of the Proposition shows that polarization affects campaigning effort through an additional channel, clarifying the distinction between the two mechanisms.

Proposition 2(ii) shows that when party platforms are asymmetric, factions in the moderate party campaign more than those in the extreme party. This happens because asymmetry dampens factions' internal incentives in the extreme party. When $\gamma < 1$, in equilibrium incentives are high-powered ($\pi_d^{L*} = \pi_d^{R*} = 1 - \alpha$, $\pi_v^{L*} = \pi_v^{R*} = \alpha$). Since $\alpha > 1/2$, the expected payoff from the election is lower for factions in *left* than for those in *right*. Put another way, the *internal incentive* term in the faction's first-order condition is lower for L_1, L_2 since in equilibrium $p^L(\boldsymbol{e}^*) < 1/2$. As x^L moves away from x^V, the *left*'s expected payoff from the election decreases (via a lower winning probability), which induces its factions to campaign less for the party. Conversely, factions in the more moderate party campaign more in equilibrium, and the difference in parties' total effort is increasing in the extremism of the trailing party's platform.

Proposition 2(ii) also suggests that factional incentives to sabotage increase as the party weakens. This prediction offers an unexplored explanation of observed empirical patterns of intra-party competition. **?** identify political fights within the Italian DC as one of the main causes of the corruption scandals involving the party's MPs. Plausibly, the increased political competition resulting from the steady rise of the left and the associated loss of spoils faced by DC factions contributed to increasing factional sabotage: as the stakes of the election decreased, the appeal of securing internal power became more important to factions.[16] This explanation is consistent with the high levels of intra-party competition and resulting corruption scandals that doomed the DC and contributed to the end of the First Republic.

One reasonable question is whether factions in *left* would indulge more in sabotage if platforms were endogenous instead of being fixed to parties' ideal points. In the Appendix, I study how the result in Proposition 2(ii) would change if parties could choose the platform location (in addition to premia) to maximize the probability of victory. The analysis reveals that factions' incentives to campaign are affected by a change in polarization via two mechanisms: On the one hand, as a party gets more extreme, its probability of winning the election is lower, which induces factions to campaign less. This is the force behind Proposition 2(ii). On the other hand, the marginal benefit of campaigning for the extreme factions increases as the platform chosen by the party becomes more extreme, because the net policy benefit becomes greater for the extreme party. The latter is the force behind results in the literature of endogenous campaigning showing that extreme parties campaign more (**??**). Which of the two mechanisms prevails depends on how extreme factions are: unless factions are too extreme, the former (negative) incentive to campaign prevails.

Finally, when $\gamma > 1$, in equilibrium premia are set to zero and $e^{L*} = e^{R*}$, regardless of the distance between policy platforms. With zero premia, the *internal incentive* term in the faction's first-order condition is equal to zero, and the incentive to campaign exclusively arises from the *external incentive*. The latter depends only on the difference between party platforms and is the same for factions in the *left* and *right*. Thus, when $\gamma > 1$ and $|x^L| > |x^R|$, both parties' factions exert the same amount of campaigning in equilibrium — even though the *left*'s ex-ante winning probability is lower than the *right*'s — and $\partial\big(e^{L*} - e^{R*}\big)/\partial|x^L| = 0$ in equilibrium.[17]

[16] In particular, the rise of the left was triggered by the Socialist Party becoming more moderate in the 1970s and 1980s.

[17] One extension in the Appendix shows that when allowing for negative premia, the result $\partial\big(e^{L*}-e^{R*}\big)/\partial|x^L| < 0$ generalizes for all values of γ. By allowing the party to punish the faction ranking higher (rewarding the faction

So far, I have assumed that factions in the same party (having the same ideological preferences) suffer the same ideological cost of election loss. The next section relaxes this assumption and shows how campaigning effort changes as a function of factions' ideological extremism.

5. Factions' Ideological Heterogeneity

Without loss of generality, let faction L_1 be more extreme than L_2 ($x_1^L < x_2^L < 0$). I assume that the policy platform implemented by a party corresponds to the simple average of its factions' ideological bliss points, $x^L = (x_1^L + x_2^L)/2$ (and symmetrically for x^R). This assumption is based on the empirical observation that comparing factions' ideal points with the overall party position, factions bound the party to its platform choice (**?**). The next extension considers party platforms as weighted averages of factions' bliss points, where the weights depend on factions' relative power.[18]

The next result illustrates the equilibrium when factions of the same party differ in ideology, showing how *within*-party polarization affects factional incentives to campaign for the party.

Proposition 3. Heterogeneous Factions. *In equilibrium, ideologically extreme factions campaign more than moderate ones, which devote more resources to sabotage. The equilibrium incentive scheme is as in the homogeneous case:* $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$ *if* $\gamma > 1$, *and* $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$ *if* $\gamma < 1$.

Proposition 3 demonstrates that when factions in the same party do not share the same ideological position, the extreme faction invests more in the electoral campaign than the moderate one. Intuitively, the moderate faction is ideologically closer than the extreme faction to *right*'s bliss point, thus suffering a lower cost from election loss. As a consequence, the moderate faction invests more resources into sabotage.

ranking lower) when $\gamma > 1$, negative premia create the same incentives to campaign as when $\gamma < 1$ and incentives are high-powered.

[18] In this extension, weights are decided by the party leader, who can reward the higher-ranking faction with "policy concessions" (granting more influence on the party platform).

This result underscores the different effect that ideological polarization produces *within* and *across* parties. Proposition 2 demonstrates that when parties are polarized, factions in the more extreme party sabotage more than factions in the leading, moderate party. In contrast, Proposition 3 shows that when *factions* within the same party are polarized, the more extreme faction invests more in campaigning than the moderate one. Intuitively, as a faction becomes more extreme, the expected payoff of both factions decreases (via a lower winning probability), leading to a reduction in campaigning by both factions. However, campaigning decreases asymmetrically: the extreme faction campaigns more because the stakes of the election are greater, and the faction will suffer a higher ideological loss if they lose the election.

This logic is consistent with the behavior of the extreme and moderate factions of the Labour Party during the 2017 UK electoral campaign. There is evidence that the Labour Left "largely relied on positive campaigning and mobilized grassroots activism to an extent rarely seen before, ensuring that it inspired new voters" (**?**). For example, the grassroots movement "Momentum" helped the Labour Party win 32 new seats in the 2017 election, with members of the movement even supporting moderate candidates to ensure a Labour Party win.[19] Conversely, moderate Labour MPs extensively engaged in sabotaging activities against Corbyn's campaign, as recently described in a Labour Party internal report.[20]

The reader might wonder why the leader would commit to using the ranking indicator at all when factions are ideologically diverse, given that she would be strictly better off rewarding the faction that exerted more effort (the more extreme faction, in equilibrium). That is, all else equal, L would want to design a non-anonymous contract that punishes the moderate faction more for its sabotage efforts. However, non-anonymous incentive schemes are typically not feasible since leaders are constrained by party legal rules and formal procedures, which

[19] See Lott, R. (2019) "Inside Momentum, Labour's Secret Weapon," Vice, 18 November.

[20] The report describes a "hyper-factional atmosphere" where more right-wing senior Labour staff actively seek to sabotage the work of those on the party's left, referring to them contemptuously as "Trots" (2020 Labour Antisemitism Report, Section 2.1.3.i).

are the same for all factions and are decided ex-ante. Yet, if the leader could decide over the party policy platform, she would be able to reward the moderate faction through a policy concession by setting the party platform closer to the moderate faction's preferred position. The next section analyzes this possibility, showing that leaders could tailor policies to factional preferences to increase the party's overall electoral chances.

5.1. Policy Concessions as Incentives for Factions

How can parties tailor policies to factional preferences in order to increase electoral chances? Typically, party manifestos weigh factions' preferred platforms based on their share of votes gained during congresses (**???**). This section endogenizes the weight of each faction, asking which policy weight the party adopts in equilibrium. Let the party leader L choose — in addition to premia — how much to weigh the preferred policy of the faction ranking higher, where the policy weight is denoted by $\lambda \in [1/2, 1]$. Formally, a strategy for L is defined by an incentive scheme (π_d^L, π_v^L) and a policy weight λ.

The timing of the game is unchanged. First, L announces a vector of incentives $(\pi_d^L, \pi_v^L, \lambda)$. Second, factions decide the level of resources to invest in campaigning and sabotage. Finally, elections are held, and premia (π_d^L, π_v^L) and policy concessions (determined by λ) are distributed. The voter compares the same party platforms — the simple averages of the factional bliss points — as in the baseline model. That is, I assume that the voter does not anticipate the post-electoral bargaining process taking place within parties when comparing party platforms.[21] Note that policy concessions are meted out once the internal ranking is revealed, which happens *after* the electoral outcome is known. These assumptions imply that λ affects factions' decisions only through their expected reward.[22]

[21] This assumption greatly simplifies the analysis. However, the main result of this extension would be qualitatively unchanged if the voter responded to policy concessions.

[22] The fact that policy concessions can only be post-electoral together with the voter's naïvete assumption imply that λ affects the probability of electoral victory only indirectly, through factions' campaigning efforts.

To see how λ affects factions' decisions, it is convenient to express faction L_1's expected reward *from electoral victory* as

$$R_1^L(\boldsymbol{\pi}^L, \boldsymbol{e}^L, \lambda)|_v = b_v^L + \rho_1^L \pi_v^L - \left(x_1^L - x^L(\lambda, \boldsymbol{e^L})\right)^2, \tag{10}$$

where

$$x^L(\lambda, \boldsymbol{e^L}) = \begin{cases} \lambda x_1^L + (1-\lambda) x_2^L & \text{if} \quad s^L(\boldsymbol{e^L}) = 1 \\ (1-\lambda) x_1^L + \lambda x_2^L & \text{if} \quad s^L(\boldsymbol{e^L}) = 2. \end{cases} \tag{11}$$

The policy incentive consists in a lower policy cost of ranking higher (and a higher cost of ranking lower), conditional on winning the election. Conversely, L_1's expected reward *from losing the election* does not depend on λ — and is equivalent to the baseline model (5) — because the implemented policy platform in the event of electoral defeat is chosen by the *right*.

Recall that the ideologically extreme faction campaigns more than the moderate one in equilibrium, as the latter suffers a lower ideological cost from an electoral loss (Proposition 3). This difference is crucial for the next result, which shows that the leader might reward the preeminent faction by favoring its policy even when the internal contest is detrimental to the party.

Proposition 4. Policy Concessions. *Let* $|x_1^L - x_2^L| > 0$. *When* $\gamma > 1$, *in equilibrium the leader rewards the pre-eminent faction by setting* $\lambda^* = 1$, *and the premia are* $(\pi_d^{L*}, \pi_v^{L*}) = (0, 0)$. *When* $\gamma < 1$, *the equilibrium premia are* $(\pi_d^{L*}, \pi_v^{L*}) = (1-\alpha, \alpha)$, *and there exists* d' *such that if* $|x_1^L - x_2^L| < d'$, *then* $\lambda^* = 1$, *and if* $|x_1^L - x_2^L| \geq d'$, *then* $\lambda^* = 1/2$.

Proposition 4 states that, when the internal contest is detrimental to the party ($\gamma > 1$), L rewards the strongest faction by setting the party platform equal to the faction's preferred policy. When sabotage is more effective than campaigning to achieve a higher internal ranking, the higher ranking faction is the one that (probabilistically) sabotages more (Proposition 3). In this case, setting a positive premium corresponds to rewarding sabotage, and the equilibrium premia (π_d^{L*}, π_v^{L*}) are set to zero as in the baseline model. In equilibrium, the leader promises a policy concession *contingent on victory* to the faction for which the ranking is higher, by setting $\lambda^* = 1$. This motivates the moderate faction — which in equilibrium is more likely to obtain the policy concession — to campaign more for the party and to invest less in sabotaging the other faction.

To understand why this is the case, it is key to note that when the relative ranking indicator rewards sabotage, the more extreme faction campaigns more in equilibrium ($e_1^{L*} > e_2^{L*}$). This in turn implies that $\rho_2^L > \rho_1^L$. In other words, the internal contest among factions is not a coin flip anymore and the moderate faction L_2 has more chances to win the premium than L_1. In this case, a policy concession incentivizes L_2's to campaign to increase the party's chances of victory. Crucially, the extreme faction's effort (e_1^{L*}) is always greater than e_2^{L*} in equilibrium. This implies that even if e_2^{L*} increases under λ^*, $\rho_2^L > \rho_1^L$ and L_2 still ranks higher in equilibrium.

When $\gamma < 1$, the extreme faction (which campaigns more in equilibrium) is more likely to rank higher than the moderate one, as campaigning is more effective than sabotage. In this case, a high λ incentivizes factions to campaign in order to rank higher thus moving the party platform closer to their bliss point. This clearly helps the party win the election via more campaigning. Indeed, when the ideological distance between L_1 and L_2 is low enough, the leader sets $\lambda^* = 1$ to maximize total effort. Suppose now that $|x_1^L - x_2^L|$ is high enough. In this scenario, a high λ reduces the appeal of electoral victory to the moderate L_2 by shifting the party platform to the extreme x_1^L. When the distance between factions' bliss points is high enough, the loss from the moderate faction's sabotage outweighs the gain in campaigning by the extreme faction, and the leader chooses not to reward any faction with a policy concession.

Proposition 4 shows that moderate factions could be advantaged when the voter does not anticipate policy concessions. Note that this "moderate" bias would be even stronger with a forward-looking voter who takes into account the party equilibrium incentive schemes in her voting decision. Intuitively, when voters anticipate the post-electoral platform, policy rewards have two effects on the party's probability of winning: a *direct* effect via the platform evaluated by the voter, and an *indirect* effect via factions' investment decision (Proposition 4). The direct effect generates clear incentives for the party: The platform giving the party highest chances of victory is moderate. Therefore, when $\gamma > 1$, the direct effect goes in the same direction of the indirect one, reinforcing the party's incentive to reward the moderate faction with policy concessions.

Finally, the result suggests how the equilibrium incentive scheme would vary if the leader had ideological preferences. Suppose that L shared the ideological preferences of the extreme faction L_1, and suppose that $\gamma > 1$. In this case, the leader would trade off a lower ideological cost by decreasing λ — because in equilibrium the moderate L_2 would be more likely to rank higher — and a higher probability of victory by increasing λ, via higher total effort. By allowing leaders to share ideological preferences with factions, the forces highlighted in Proposition 4 would still be at work, but the leader would have to weigh the incentive to increase the party probability of victory with the ideological cost of using policy concessions to reward a platform distant from her own bliss point.

6. Conclusion

This paper studies the allocation of electoral spoils among factions competing within the same party, and embeds this framework in a model of elections. The baseline model shows that factions' competition over electoral spoils can be positive or destructive to the party depending on several features of the competitive environment. As the institutional power granted to minority parties increases, factions invest more resources into competing against each other and less in campaigning for the party. Conversely, when the stakes of the election increase — via polarization or institutional changes — factions invest more in campaigning. This finding improves our understanding of alternative democratic systems by highlighting the oft-neglected effect of different electoral institutions on intra-party competition. Furthermore, the model shows the asymmetric effect of party polarization on factional competition. As polarization increases, factions in both parties campaign more for the party to avoid a costly electoral defeat. However, in the presence of electoral imbalance, intra-party competition is greater among trailing parties.

Results also show how parties can design optimal internal rules to improve their electoral prospects. When the internal contest is beneficial to the party, the party encourages competition among factions through a winner-takes-all contest for electoral spoils. Conversely, when the factional contest is detrimental to the party, the optimal incentive scheme distributes electoral spoils in an egalitarian way to discourage destructive competition. This insight can be applied to several internal procedures used by parties to manage intra-party competition.

While nominations procedures such as primaries are outside the scope of the model, the paper suggests a mechanism by which primaries could have a positive effect on the management of intra-party competition. In the model, primary elections can be formalized as an egalitarian division of the spoils. That is, by bringing competition into the open, primaries can increase intra-party power sharing among factions. While there is evidence that primaries are deeply divisive and can hurt parties in general elections, the adoption of primaries in the United States has also had a positive effect on the management of intra-party competition (**?**). The model suggests that the adoption of primaries might be an effective tool to manage factional competition when this is detrimental to the party.

The framework advanced in this model is not limited to the application of electoral competition: the same logic could be extended to any public good provision context in organizations where members can either contribute to the good of the organization or compete against each other to influence the organization's decision maker (e.g., companies, departments and guerrillas). Suppose that a principal decides how to reward members based on how much they contribute to the organization's public good, contingent on the events of success and failure. The principal observes a noisy signal of public good provision which can be influenced by other members' activities. Members differ in their stakes of succeeding: some are more motivated then others, thus suffering a higher cost in the event of the organization's failure. All else equal, more motivated members are more likely to contribute to the public good. These simple ingredients arguably apply to several settings. Future research could exploit this framework to explore many relevant applications.

The model shows how incentives change when parties must respond to imperfect signals of effort in the presence of sabotaging activities. The same approach can be extended to compare the efficiency of different incentive schemes. The analysis of a proportional contest function — that is, a proportional allocation of electoral spoils relative to each faction's own performance rather than factions' relative performance — is a promising avenue for future research. This would shed light on the question of what the optimal party structure is to win elections.

A. Appendix: Preliminaries

A.1. The Faction's Problem

Faction L_1's problem is:

$$\max_{e_1^L \in [0,1]} \quad p^L \Big[b_v^L + \rho_1^L \pi_v^L \Big] + (1-p^L) \Big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2 \Big] - \frac{\left(e_1^L\right)^2}{2} - \frac{\left(1-e_1^L\right)^2}{2}, \tag{12}$$

where $\rho_1^L = \frac{1}{2} + (1-\gamma)(e_1^L - e_2^L)/2$ and $p^L = \frac{1}{2} + \psi[-(x^L)^2 + (x^R)^2 + e_1^L + e_2^L - e_1^R - e_2^R]$. All other factions solve their respective problem. The first-order condition associated to L_1 is

$$2e^L = 1 + \left(\frac{1-\gamma}{2}\right) \Big[p^L \pi_v^L + (1-p^L)\pi_d^L \Big] + \psi \Big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + \left(x^L - x^R\right)^2 \Big] \tag{13}$$

and likewise for L_2, R_1 and R_2. Notice that the factions' objective function is concave. The second-order condition can be expressed as

$$\psi(1-\gamma)(\pi_v^L - \pi_d^L) - 2,$$

which is negative for ψ sufficiently small. Hence, the first-order conditions of the factions' problem identify a maximum.

Lemma A.1. *There exists an equilibrium, i.e., a solution to the system of first-order conditions of each faction. The equilibrium is unique.*

Proof. Solving the system of four first-order conditions yields a unique closed-form solution for the effort exerted by factions in both parties. The solution is symmetric for factions in the same party (i.e., $e_1^{L*} = e_2^{L*} = e^{L*}$) and equal to

$$e^{L*} = \frac{\big[(\gamma-1)(\pi_d^R - \pi_v^R)\psi - 2\big]\big[2 + (2\alpha-1)\psi + 2\psi(x^L - x^R)^2\big] + (\gamma-1)(\pi_v^L + \pi_d^L) - \theta(\pi_v^R)\pi_v^L + \theta(\pi_d^R)\pi_d^L}{4\big[-2 + \psi(\gamma-1)(\pi_d^R + \pi_d^L - \pi_v^R - \pi_v^L)\big]},$$

$$e^{R*} = \frac{\big[(\gamma-1)(\pi_d^L - \pi_v^L)\psi - 2\big]\big[2 + (2\alpha-1)\psi + 2\psi(x^L - x^R)^2\big] + (\gamma-1)(\pi_v^R + \pi_d^R) - \theta(\pi_v^L)\pi_v^R + \theta(\pi_d^L)\pi_d^R}{4\big[-2 + \psi(\gamma-1)(\pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R)\big]},$$

where $\theta(\pi)$, for $\pi = \{\pi_d^L, \pi_v^L, \pi_d^R, \pi_v^R\}$, equals

$$\theta(\pi) = \psi\big[\pi(1-\gamma) + 2 + 2[(x^L)^2 - (x^R)^2] + \psi(1-2\alpha) + 2\psi(x^L - x^R)^2\big],$$

and has the following properties: If $\gamma < 1$ ($\gamma > 1$), $\theta(\pi)$ is increasing (decreasing) in π. If $(x^L)^2 > (x^R)^2$ (i.e., if party L is electorally disadvantaged) and $\gamma < 1$, $\theta(\pi)$ is positive. □

Claim 1 (Interior Effort.). *The following condition ensures that* $e^{L*} \in (0,1)$*:*

$$\alpha + (x^L - x^R)^2 < \frac{1}{2\psi} \tag{14}$$

Proof. (I) Condition for $e^{L*} < 1$. Using the expression for the faction's first-order condition (13), we can evaluate when the first-order condition evaluated at $e^{L*} = 1$ is negative:

$$\psi\Big[(b_v^L - b_d^L) + \frac{1 + (1-\gamma)(1-e_2^L)}{2}(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\Big] + \frac{1-\gamma}{4}(\pi_v^L + \pi_d^L)$$
$$+ \psi\Big(\frac{1-\gamma}{2}\Big)\Big[-(x^L)^2 + (x^R)^2 + 1 + e_2^L - e_1^R - e_2^R\Big] < 1,$$

for which a sufficient condition is

$$\alpha + (x^L - x^R)^2 < \frac{1}{2\psi}. \tag{15}$$

(II) Condition for $e^{L*} > 0$. Using the expression for the faction's first-order condition (13) we can evaluate when the first-order condition evaluated at $e^{L*} = 0$ is positive:

$$\psi\Big[(b_v^L - b_d^L) + \frac{1 + (1-\gamma)(-e_2^L)}{2}(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\Big] + \frac{1-\gamma}{4}(\pi_v^L + \pi_d^L)$$
$$+ \psi\Big(\frac{1-\gamma}{2}\Big)\Big[-(x^L)^2 + (x^R)^2 + e_2^L - e_1^R - e_2^R\Big] > 0,$$

where the LHS is greater than

$$\psi\Big(\frac{\alpha-1}{2}\Big)+1.$$

Hence, the following is a sufficient condition for campaigning effort to be positive in equilibrium:

$$(1-\alpha)<\frac{2}{\psi}, \tag{16}$$

which is again satisfied for ψ small enough. Since effort is continuous, the conditions identified in (15) and (16) ensure that the unique level of effort exerted by factions in equilibrium must be interior. □

A.2. The Party Leader's Problem

Each party leader maximizes the probability of winning the election with respect to the party's two premia. By Lemma A.1, we know that when L_1, L_2 face a symmetric problem, in equilibrium $e_1^{L*}=e_2^{L*}=e^{L*}$. This allows us to re-write L's objective as:

$$\max_{\pi_d^L,\pi_v^L} \quad u^L(\pi_d^L,\pi_v^L)=\frac{1}{2}+\psi\big[2e^{L*}\big(\pi_d^L,\pi_v^L\big)-2e^{R*}\big(\pi_d^L,\pi_v^L\big)-(x^L)^2+(x^R)^2\big]. \tag{17}$$

Since party L's premia affect the probability of winning only through the effort level of party L's factions and party R's factions, we can also express the leader's objective (53) as the maximization of the following transformed utility:

$$\max_{\pi_d^L,\pi_v^L} \quad \tilde{u}^L(\pi_d^L,\pi_v^L)=2e^{L*}\big(\pi_d^L,\pi_v^L\big)-2e^{R*}\big(\pi_d^L,\pi_v^L\big).$$

The game between the two party leaders is zero-sum, as each party wins what the other loses:

$$\tilde{u}^L(\pi^L,\pi^R)+\tilde{u}^R(\pi^L,\pi^R)=2e^{L*}-2e^{R*}+\big(2e^{R*}-2e^{L*}\big)=0. \tag{18}$$

Given Equation 18, we can define the payoff function of this zero-sum game as $\tilde{u}^L(\pi^L, \pi^R) = u$, with $\tilde{u}^R(\pi^L, \pi^R) = -u$. Party L's leader maxmin value is given by

$$\underline{v}^L = \max_{\{\pi_d^L, \pi_v^L\}} \quad \min_{\{\pi_d^R, \pi_v^R\}} \tilde{u}^L(\pi_d^L, \pi_v^L, \pi_d^R, \pi_v^R) \tag{19}$$

and party R's leader maxmin value is given by

$$\underline{v}^R = \max_{\{\pi_d^R, \pi_v^R\}} \quad \min_{\{\pi_d^L, \pi_v^L\}} -\tilde{u}^L(\pi_d^L, \pi_v^L, \pi_d^R, \pi_v^R) = \min_{\{\pi_d^R, \pi_v^R\}} \quad \max_{\{\pi_d^L, \pi_v^L\}} \tilde{u}^L(\pi_d^L, \pi_v^L, \pi_d^R, \pi_v^R).$$

Substituting the values of equilibrium efforts into (18), we can express the payoff function of the game in closed form as follows:

$$u = \frac{(\gamma - 1)\Big[\pi_d^L - \pi_d^R + \pi_v^L - \pi_v^R + 2\psi\big(\pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R\big)\big(x_L^2 - x_R^2\big)\Big]}{-4 + 2(\gamma - 1)\psi\big(\pi_d^L + \pi_d^R - \pi_v^L - \pi_v^R\big)}, \tag{20}$$

where $\tilde{u}^L(\pi^L, \pi^R) = u$ and $\tilde{u}^R(\pi^L, \pi^R) = -u$. Hence, L (R) maximizes (minimizes) the payoff function (20) with respect to $\boldsymbol{\pi^L}$ ($\boldsymbol{\pi^R}$).

Lemma A.2. *The payoff function of the game u is*

- *increasing in π_v^L for $\gamma < 1$*
- *decreasing in π_v^L for $\gamma > 1$*

Proof. Recall that the payoff function of the zero-sum game among party leaders is defined by $u = 2e^{L*} - 2e^{R*}$. By symmetry across factions in the same party, we can rewrite the first-order condition of L_1, L_2 as

$$2e = 1 + \psi\big(b_v^L + \rho_1^L \pi_v^L\big) + p^L\Big(\frac{1-\gamma}{2}\Big)\pi_v^L - \psi\big(b_d^L + \rho_1^L \pi_d^L\big) + (1 - p^L)\Big(\frac{1-\gamma}{2}\Big)\pi_d^L \tag{21}$$

Taking the partial derivative of the faction's equilibrium effort (21) with respect to π_v^L yields

$$\frac{\partial 2e^{L*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[p^L + \psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_v^L} - \frac{\partial 2e^{R*}}{\partial \pi_v^L}\Big)\big(\pi_v^L - \pi_d^L\big)\Big] + \psi\Big[-\frac{1}{2} + \rho_1^L\Big], \tag{22}$$

which follows from the budget constraint assumption on b_v^L, the fact that $\partial p^L/\partial e^{L*} = 2\psi$ and the fact that $\partial \rho_1^L/\partial \pi_v^L = 0$ in equilibrium. Define $\mathcal{X}_1$ as

$$\mathcal{X}_1 := 2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big),$$

and notice that in equilibrium $\rho_1^L = 1/2$. Then, the partial derivative of effort with respect to π_v^L (22) can be simplified as

$$\frac{\partial 2e^{L*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[p^L + \psi\mathcal{X}_1\big(\pi_v^L - \pi_d^L\big)\Big], \tag{23}$$

where $2e^{L*}$ is the first component of L's payoff function. Next, we need to characterize $\partial 2e^{R*}/\partial \pi_v^L$. To do so, observe that

$$\frac{\partial 2e^{L*}}{\partial \pi_v^R} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_v^R} - \frac{\partial 2e^{R*}}{\partial \pi_v^R}\Big)\big(\pi_v^L - \pi_d^L\big)\Big].$$

Hence, by symmetry

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{R*}}{\partial \pi_v^L} - \frac{\partial 2e^{L*}}{\partial \pi_v^L}\Big)\big(\pi_v^R - \pi_d^R\big)\Big],$$

which is equivalent to

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\psi(-\mathcal{X}_1)\big(\pi_v^R - \pi_d^R\big). \tag{24}$$

Using (23) and (24), we can re-express $2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big)$ as

$$\mathcal{X}_1 = \Big(\frac{1-\gamma}{2}\Big)p^L + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_1\big(\pi_v^L - \pi_d^L\big) + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_1\big(\pi_v^R - \pi_d^R\big),$$

which yields

$$\mathcal{X}_1\Big[1 - \frac{1-\gamma}{2}\psi\big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)\Big] = \Big(\frac{1-\gamma}{2}\Big)p^L.$$

Substituting $\mathcal{X}_1 = 2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big)$ allows us to evaluate how the payoff function changes with π_1:

$$2\Big(\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L}\Big) = \frac{\Big(\frac{1-\gamma}{2}\Big)p^L}{2-(1-\gamma)\psi\big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)}.$$

Notice that $\big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big) \in [0, 2\alpha] \subset [0, 2]$. Hence, the denominator is positive either when $\gamma > 1$, or when $\gamma < 1$ and ψ is small enough. When this is the case, the sign of $\mathcal{X}_1$ is driven by the numerator, which is positive when $\gamma < 1$ and negative when $\gamma > 1$, which completes the proof. □

Lemma A.3. *The payoff function of the game u is*

- *increasing in π_d^L for $\gamma < 1$*
- *decreasing in π_d^L for $\gamma > 1$*

Proof. Taking the partial derivative of the faction's equilibrium effort (21) with respect to π_d^L yields

$$\frac{\partial 2e^{L*}}{\partial \pi_d^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_v^L} - \frac{\partial 2e^{R*}}{\partial \pi_v^L}\Big)\big(\pi_v^L - \pi_d^L\big) + (1-p^L)\Big] + \psi\Big[-\frac{1}{2} + \rho_1^L\Big]. \tag{25}$$

Let $\mathcal{X}_0 := 2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big)$. Then, (25) simplifies to

$$\frac{\partial 2e^{L*}}{\partial \pi_d^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[2\psi\mathcal{X}_0\big(\pi_v^L - \pi_d^L\big) + 1 - p^L\Big] - \psi, \tag{26}$$

which is the first component of the payoff function.

Next, we need to characterize $\frac{\partial 2e^{R*}}{\partial \pi_d^L}$. To do so, observe that

$$\frac{\partial 2e^{L*}}{\partial \pi_d^R} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{L*}}{\partial \pi_d^R} - \frac{\partial 2e^{R*}}{\partial \pi_d^R}\Big)\big(\pi_v^L - \pi_d^L\big)\Big].$$

Hence, by symmetry

$$\frac{\partial 2e^{R*}}{\partial \pi_d^L} = \Big(\frac{1-\gamma}{2}\Big)\Big[\psi\Big(\frac{\partial 2e^{R*}}{\partial \pi_d^L} - \frac{\partial 2e^{L*}}{\partial \pi_d^L}\Big)\big(\pi_v^R - \pi_d^R\big)\Big],$$

which is equivalent to

$$\frac{\partial 2e^{R*}}{\partial \pi_v^L} = \Big(\frac{1-\gamma}{2}\Big)\psi(-\mathcal{X}_0)\big(\pi_v^R - \pi_d^R\big). \tag{27}$$

Using (26) and (27), we can express $2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big)$ as

$$\mathcal{X}_0 = \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_0\big(\pi_v^L - \pi_d^L\big) + \Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi + \Big(\frac{1-\gamma}{2}\Big)\psi\mathcal{X}_0\big(\pi_v^R - \pi_d^R\big),$$

which yields

$$\mathcal{X}_0\Big[1 - \frac{1-\gamma}{2}\psi\big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)\Big] = \Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi.$$

Substituting $\mathcal{X}_0 = 2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big)$ allows us to evaluate how the payoff function changes with π_d:

$$2\Big(\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L}\Big) = \frac{\Big(\frac{1-\gamma}{2}\Big)(1-p^L) - \psi}{2 - (1-\gamma)\psi\big(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R\big)}.$$

As in the previous case, the denominator is positive either when $\gamma > 1$, or when $\gamma < 1$ and ψ is small enough. When this is the case, the sign of $\mathcal{X}_0$ is driven by the numerator, which is positive when $\gamma < 1$ (for ψ small enough) and negative when $\gamma > 1$, which completes the proof. □

B. Main Results

B.1. Proof of Proposition 1

Proof. To derive the equilibrium incentive scheme, note that by Lemma A.2 and Lemma A.3 the following is true when $\gamma < 1$:

$$\frac{\partial e^{L*}}{\partial \pi_d^L} - \frac{\partial e^{R*}}{\partial \pi_d^L} > 0$$
$$\frac{\partial e^{L*}}{\partial \pi_v^L} - \frac{\partial e^{R*}}{\partial \pi_v^L} > 0.$$

Thus, in equilibrium L sets $\pi_d^{L*} = (1-\alpha)$, $\pi_v^{L*} = \alpha$. When $\gamma > 1$, the inequality is reversed. Therefore, the optimal premia are $\pi_d^{L*} = \pi_v^{L*} = 0$. Given the payoff of the game (20), the *right* party faces a problem that is symmetric to L's problem: $\max_{\{\pi_d^R, \pi_v^R\}}\{-u\}$. Therefore, R sets in equilibrium $\pi_d^R = (1-\alpha)$, $\pi_v^R = \alpha$ when $\gamma < 1$, and $\pi_d^R = \pi_v^R = 0$ when $\gamma > 1$.

When no party has an ex-ante electoral advantage over the other, faction L_1 solves the following problem:

$$\max_{e_1^L \in [0,1]} \left\{ \begin{array}{c} \left[\frac{1}{2} + \psi(e_1^L + e_2^L - e_1^R - e_2^R)\right]\left[b_v^L + \left(\frac{1}{2} + \frac{(1-\gamma)(e_1^L - e_2^L)}{2}\right)\pi_v^L\right] + \\ \left[\frac{1}{2} - \psi(e_1^L + e_2^L - e_1^R - e_2^R)\right]\left[b_d^L + \left(\frac{1}{2} + \frac{(1-\gamma)(e_1^L - e_2^L)}{2}\right)\pi_d^L - \left(x^L - x^R\right)^2\right] \\ -\frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2} \end{array} \right\}.$$

The first-order condition associated to faction L_1 is:

$$\psi\left[b_v^L + \left(\frac{1}{2} + \frac{(1-\gamma)(e_1^L - e_2^L)}{2}\right)\pi_v^L\right] - \psi\left[b_d^L + \left(\frac{1}{2} + \frac{(1-\gamma)(e_1^L - e_2^L)}{2}\right)\pi_d^L - (x^L - x^R)^2\right] +$$
$$\frac{1-\gamma}{2}\pi_v^L\left[\frac{1}{2} + \psi(e_1^L + e_2^L - e_1^R - e_2^R)\right] + \frac{1-\gamma}{2}\pi_d^L\left[\frac{1}{2} - \psi(e_1^L + e_2^L - e_1^R - e_2^R)\right] + 1 - 2e_1^L = 0,$$

and likewise for faction L_2 in party L and factions R_1 and R_2 in party R. Solving the system of four first order conditions, one for each faction, yields a unique solution for e_1^L,e_2^L,e_1^R,e_2^R:

$$e^{L*} = \frac{\begin{array}{c}[(\gamma-1)(\pi_d^R-\pi_v^R)\psi-2][2+(2\alpha-1+2(x_L-x_R)^2)\psi]+(\gamma-1)\pi_v^L(1+\psi)(-2+(\gamma-1)\pi_v^R \\ +(2\alpha-1-2(x_L-x_R)^2)\psi)+(\gamma-1)\pi_d^L(1+\psi(2+\pi_d^R-\gamma\pi_d^R+[2\alpha-1+2(x_L-x_R)^2)\psi)]\end{array}}{4\psi(\gamma-1)(\pi_d^L+\pi_d^R-\pi_v^L-\pi_v^R)-8} \tag{28}$$

and analogously for factions in *right*. Substituting the equilibrium premia yields

$$e^{L*} = \begin{cases} \dfrac{2+\psi\big(2\alpha-1+2\big(x^L-x^R\big)^2\big)}{4} & \text{if} \quad \gamma > 1 \\ \dfrac{5-\gamma+2\psi\big(2\alpha-1+2\big(x^L-x^R\big)^2\big)}{8} & \text{if} \quad \gamma < 1, \end{cases}$$

which completes the proof. □

B.2. Proof of Proposition 2

Proof. (i) The proof simply follows by inspection of the first-order conditions and by observing that an increase in polarization $\big(x^L - x^R\big)$ only increases the external incentive term.

(ii) The following first-order condition identifies the equilibrium effort of factions in *left*:

$$\psi\big(b_v^L+\rho_1^L\pi_v^L\big)+p^L\Big(\frac{1-\gamma}{2}\Big)\pi_v^L-\psi\big[b_d^L+\rho_1^L\pi_d^L-\big(x^L-x^R\big)^2\big]+(1-p^L)\Big(\frac{1-\gamma}{2}\Big)\pi_d^L+1-2e^L=0,$$

and can also be expressed as

$$2e^L = 1+\Big(\frac{1-\gamma}{2}\Big)\Big[p^L\pi_v^L+(1-p^L)\pi_d^L\Big]+\psi\Big[b_v^L-b_d^L+\rho_1^L(\pi_v^L-\pi_d^L)+\big(x^L-x^R\big)^2\Big]. \tag{29}$$

Differentiating with respect to $|x^L|$ yields

$$\begin{aligned}\frac{\partial 2e^L}{\partial |x^L|} &= \Big(\frac{1-\gamma}{2}\Big)\frac{\partial p^L}{\partial |x^L|}(\pi_v^L-\pi_d^L)+2\psi\big(x^L-x^R\big) \\ &= -(1-\gamma)\psi|x^L|(\pi_v^L-\pi_d^L)+2\psi\big(x^L-x^R\big),\end{aligned} \tag{30}$$

where the second equality follows from $\partial p^L/\partial|x^L| = -2\psi|x^L|$.

Differentiating with respect to $|x^R|$ yields

$$\frac{\partial 2e^L}{\partial|x^R|} = \Big(\frac{1-\gamma}{2}\Big)\frac{\partial p^L}{\partial x^R}(\pi_v^L - \pi_d^L) - 2\psi\big(x^L - x^R\big)$$
$$= (1-\gamma)\psi x^R(\pi_v^L - \pi_d^L) - 2\psi\big(x^L - x^R\big). \tag{31}$$

By symmetry,

$$\frac{\partial 2e^R}{\partial|x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^R - \pi_d^R) + 2\psi\big(x^L - x^R\big). \tag{32}$$

Differentiating $(2e^L - 2e^R)$ with respect to $|x^L|$ yields

$$\frac{\partial(2e^L - 2e^R)}{\partial|x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^L + \pi_v^R - \pi_d^L - \pi_d^R). \tag{33}$$

In equilibrium, when $\gamma < 1$ the optimal contract offered by both leaders is $\pi_v^{L*} = \pi_v^{R*} = \alpha$, and $\pi_d^{L*} = \pi_d^{R*} = 1-\alpha$. Substituting the optimal contract yields

$$\frac{\partial(2e^{L*} - 2e^{R*})}{\partial|x^L|} = -2(1-\gamma)\psi|x^L|(2\alpha - 1), \tag{34}$$

which is always negative. That is, the difference in equilibrium efforts $(2e^{L*} - 2e^{R*})$ decreases in L's ideological extremism. The proof of $\frac{\partial(2e^{R*}-2e^{L*})}{\partial|x^R|} < 0$ is analogous and, therefore, omitted.

Finally, when $\gamma > 1$, the optimal contract offered by both leaders is $\pi_v^{L*} = \pi_v^{R*} = \pi_d^{L*} = \pi_d^{R*} = 0$, which substituted into (33) yields zero for every x^L. □

Endogenous Platforms. In order to evaluate the statement in Proposition 2 when parties' platforms are endogenously chosen, suppose that factions have the following ideal points: $\hat{x}^L < 0 = x^V = \hat{x}^R$. That is, parties are asymmetric and *right* is aligned with the voter's bliss point. Let x^L, x^R be the platforms chosen by *left* and *right*, respectively. As in the baseline model, parties choose the reward scheme and then factions choose the campaigning level.

Given this setup, the following first-order condition identifies the equilibrium effort of factions in *left*:

$$2e^L = 1 + \left(\frac{1-\gamma}{2}\right)\left[p^L\pi_v^L + (1-p^L)\pi_d^L\right] + \psi\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) - \left(\hat{x}^L - x^L\right)^2 + \left(\hat{x}^L - x^R\right)^2\right]. \quad (35)$$

which differs from (29) because the external incentive term includes $-\left(\hat{x}^L - x^L\right)^2$. Differentiating with respect to $|x^L|$ yields:

$$\frac{\partial 2e^L}{\partial |x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^L - \pi_d^L) + 2\psi\left(\hat{x}^L - x^L\right), \quad (36)$$

which shows that the marginal benefit of campaigning of *left*'s factions is strictly increasing in x^L. It is straightforward to derive $\frac{\partial(2e^L - 2e^R)}{\partial|x^L|}$, which produces:

$$\frac{\partial(2e^L - 2e^R)}{\partial|x^L|} = -(1-\gamma)\psi|x^L|(\pi_v^L + \pi_v^R - \pi_d^L - \pi_d^R) + 2\psi\hat{x}^L. \quad (37)$$

Equation (37) shows that an increase in ideological extremism does not have an obvious effect on the difference in factions' campaigning. In particular, factions in *left* campaign more if and only if

$$\hat{x}^L < |x^L|(1-\gamma)(\pi_v^L + \pi_v^R - \pi_d^L - \pi_d^R). \quad (38)$$

In other words, the positive incentive to campaign prevails on the negative one when factions' ideological preferences are sufficiently extreme.

B.3. Proof of Remark 1

Proof. Differentiating $\mathcal{W}^L(x^L)$ with respect to $|x^L|$ yields

$$\frac{\partial \mathcal{W}^L(x^L)}{\partial |x^L|} = \frac{\partial p^L}{\partial |x^L|}\left[2\alpha - 1 + 2\left(x^L - x^R\right)^2\right] - 4|x^L - x^R|\left(1 - p^L\right),$$

where $\frac{\partial p^L(x^L)}{\partial|x^L|} = -2\psi|x^L|$ is always negative. □

B.4. Proof of Proposition 3

Proof. When factions are heterogeneous, we can express L_1's first-order condition as

$$e_1^{L*} = \frac{1}{2} + \left(\frac{1-\gamma}{4}\right)\left[p^L\pi_v^L + (1-p^L)\pi_d^L\right] + \frac{\psi}{2}\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) - \left(x_1^L - x^L\right)^2 + \left(x_1^L - x^R\right)^2\right], \quad (39)$$

and L_2's first-order condition as

$$e_2^{L*} = \frac{1}{2} + \left(\frac{1-\gamma}{4}\right)\left[p^L\pi_v^L + (1-p^L)\pi_d^L\right] + \frac{\psi}{2}\left[b_v^L - b_d^L + \left(1-\rho_1^L\right)(\pi_v^L - \pi_d^L) - \left(x_2^L - x^L\right)^2 + \left(x_2^L - x^R\right)^2\right]. \quad (40)$$

From (39) and (40) it is clear that $e_1^{L*} > e_2^{L*}$ for $|x_1^L| > |x_2^L|$. To derive the equilibrium incentive scheme, first take the sum $e_1^{L*} + e_2^{L*}$:

$$e_1^{L*}+e_2^{L*} = 1+\left(\frac{1-\gamma}{2}\right)\left[\pi_d^L+2p^L(\pi_v^L-\pi_d^L)\right]+\frac{\psi}{2}\left[2\alpha-1-\left(x_1^L-x^L\right)^2+\left(x_1^L-x^R\right)^2-\left(x_2^L-x^L\right)^2+\left(x_2^L-x^R\right)^2\right].$$

Differentiating $e_1^{L*} + e_2^{L*}$ with respect to π_v^L yields

$$\frac{\partial(e_1^{L*} + e_2^{L*})}{\partial\pi_v^L} = \frac{1-\gamma}{2}\left[p^L + \frac{\partial p^L}{\partial\pi_v^L}(\pi_v^L - \pi_d^L)\right].$$

Analogously,

$$e_1^{R*}+e_2^{R*} = 1+\left(\frac{1-\gamma}{2}\right)\left[\pi_v^R+2(1-p^L)(\pi_d^R-\pi_v^R)\right]+\frac{\psi}{2}\left[2\alpha-1-\left(x_1^R-x^R\right)^2+\left(x_1^R-x^L\right)^2-\left(x_2^R-x^R\right)^2+\left(x_2^R-x^L\right)^2\right],$$

and

$$\frac{\partial(e_1^{R*} + e_2^{R*})}{\partial\pi_v^L} = \frac{(1-\gamma)}{2}\frac{\partial p^L}{\partial\pi_v^L}(\pi_d^R - \pi_v^R).$$

Define χ as $\frac{\partial(e_1^{L*}+e_2^{L*}-e_1^{R*}-e_2^{R*})}{\partial\pi_v^L}$. We have:

$$\chi = \frac{1-\gamma}{2}\left[p^L + \frac{\partial p^L}{\partial\pi_v^L}(\pi_v^L - \pi_d^L - \pi_d^R + \pi_v^R)\right]. \quad (41)$$

Since

$$\frac{\partial p^L}{\partial \pi_v^L} = \psi \frac{\partial(e_1^{L*} + e_j^{L*} - e_1^{R*} - e_2^{R*})}{\partial \pi_v^L} = \psi\chi,$$

the expression for χ (41) can be re-written as

$$\chi = \frac{\partial(e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*})}{\partial \pi_v^L} = \frac{p^L}{\frac{2}{1-\gamma} - \psi(\pi_v^L - \pi_d^L - \pi_d^R + \pi_v^R)},$$

which is positive for $\gamma < 1$ and ψ small enough. This implies $\pi_v^{L*} = \alpha$. The sign of the derivative is negative for $\gamma > 1$, which implies $\pi_v^{L*} = 0$. The proof for π_d^{L*} is analogous and, therefore, omitted. □

B.5. Proof of Proposition 4

Proof. Faction L_1's realized payoff from winning the election and having a high party ranking is

$$\begin{aligned} u_1^L(\pi_v^L, s^L = 1) &= b_v^L + \pi_v^L - \big(x_1^L - \big(\lambda x_1^L + (1-\lambda)x_2^L\big)\big)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2} \\ &= b_v^L + \pi_v^L - (1-\lambda)^2(x_1^L - x_2^L)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2}. \end{aligned} \tag{42}$$

Similarly, L_1's realized payoff from winning the election and having a low ranking is

$$u_1^L(\pi_v^L, s^L = 2) = b_v^L - \lambda^2(x_2^L - x_1^L)^2 - \frac{(e_1^L)^2}{2} - \frac{(1-e_1^L)^2}{2}. \tag{43}$$

Given these expressions, we can write L_1's first-order condition as

$$\begin{aligned} e_1^{L*} =& \frac{1}{2} + \frac{\psi}{2}\Big[b_v^L - b_d^L + \rho_1^L\big(\pi_v^L - \pi_d^L - (1-\lambda)^2(x_1^L - x_2^L)^2\big) - \big(1-\rho_1^L\big)\lambda^2(x_2^L - x_1^L)^2 + \big(x_i^L - x^R\big)^2\Big] \\ &+ \Big(\frac{1-\gamma}{2}\Big)\Big[\pi_d^L + p^L(\pi_v^L - \pi_d^L) - (1-\lambda)^2(x_1^L - x_2^L)^2 + \lambda^2(x_2^L - x_1^L)^2\Big]. \end{aligned} \tag{44}$$

Similarly, we can write L_2's first-order condition as

$$e_2^{L*} = \frac{1}{2} + \frac{\psi}{2}\left[b_v^L - b_d^L + \left(1-\rho_1^L\right)\left(\pi_v^L - \pi_d^L - (1-\lambda)^2(x_2^L - x_1^L)^2\right) - \rho_1^L\lambda^2(x_1^L - x_2^L)^2 + \left(x_i^L - x^R\right)^2\right]$$
$$+ \left(\frac{1-\gamma}{2}\right)\left[\pi_d^L + p^L(\pi_v^L - \pi_d^L) - (1-\lambda)^2(x_2^L - x_1^L)^2 + \lambda^2(x_1^L - x_2^L)^2\right]. \tag{45}$$

Summing the two factions' equilibrium efforts after substituting the budget constraint and simplifying yields

$$e_1^{L*} + e_2^{L*} = 1 + \frac{\psi}{2}\left[2\alpha - 1 - (\lambda^2 + (1-\lambda)^2)(x_2^L - x_1^L)^2 + \left(x_1^L - x^R\right)^2 + \left(x_2^L - x^R\right)^2\right]$$
$$+ \left(1-\gamma\right)\left[\pi_d^L + p^L(\pi_v^L - \pi_d^L) + p^L(x_2^L - x_1^L)^2(2\lambda - 1)\right],$$

from which we can differentiate with respect to λ to obtain

$$\frac{\partial(e_1^{L*} + e_2^{L*})}{\partial\lambda} = -\left(2\lambda - 1\right)\psi\left(x_2^L - x_1^L\right)^2 + \left(1-\gamma\right)\left[\frac{\partial p^L}{\partial\lambda}\left(\pi_v^L - \pi_d^L + (x_2^L - x_1^L)^2(2\lambda-1)\right) + 2p^L\left(x_2^L - x_1^L\right)^2\right].$$

Similarly, summing the first-order conditions of factions in *right* yields

$$e_1^{R*} + e_2^{R*} = 1 + \frac{\psi}{2}\left[2\alpha - 1 - (x_1^R - x^R)^2 + \left(x_2^R - x^R\right)^2 + \left(x_1^R - x^L\right)^2 + \left(x_2^R - x^L\right)^2\right] + \left(1-\gamma\right)\left[\pi_v^R + p^L(\pi_d^R - \pi_v^R)\right],$$

from which we can differentiate with respect to λ to obtain

$$\frac{\partial(e_1^{R*} + e_2^{R*})}{\partial\lambda} = \left(1-\gamma\right)\frac{\partial p^L}{\partial\lambda}\left(\pi_d^R - \pi_v^R\right).$$

Define χ_λ as

$$\chi_\lambda \equiv \frac{\partial(e_1^{L*} + e_2^{L*} - e_1^{R*} - e_2^{R*})}{\partial\lambda}.$$

We have:

$$\chi_\lambda = \left(1-2\lambda\right)\psi\left(x_2^L - x_1^L\right)^2 + \left(1-\gamma\right)\psi\chi_\lambda\left(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\right) + 2p^L\left(x_2^L - x_1^L\right)^2,$$

which simplified yields

$$\chi_\lambda = \frac{(x_2^L - x_1^L)^2 \Big[2p^L + \psi(2\lambda - 1)\Big]}{1 - \big(1-\gamma\big)\psi\left(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\right)}. \tag{46}$$

The numerator of (46) is always positive, as $\lambda \in [1/2, 1]$. The denominator is positive when

$$(1-\gamma)\big[\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R + (x_2^L - x_1^L)^2(2\lambda - 1)\big] < \frac{1}{\psi}. \tag{47}$$

When $\gamma > 1$, the LHS in (47) is always negative, which implies that in equilibrium $\lambda^* = 1$. That is, when sabotage is more effective than campaigning effort it is always optimal to set a policy concession for the higher ranking faction. Since premia are constrained to be nonnegative, in this case there is no trade-off between eliciting campaigning and setting a winning platform.

When $\gamma < 1$, the sign of LHS in (47) depends on the value of $(x_2^L - x_1^L)^2$. A sufficient condition for the denominator of χ_λ (46) to be positive is

$$2\alpha + (x_2^L - x_1^L)^2(2\lambda - 1) < \frac{1}{\psi(1-\gamma)}, \tag{48}$$

which requires the distance between factions' ideological bliss points to be low enough. A sufficient condition for the denominator to be negative is

$$(x_2^L - x_1^L)^2(2\lambda - 1) > \frac{1}{\psi(1-\gamma)} + 2(1-\alpha), \tag{49}$$

which requires the distance between factions' ideological bliss points to be high enough. That is, when the ideological distance is high enough and $\gamma < 1$, the optimal incentive scheme is to set low-powered incentives. When the distance is low enough, the optimal incentive scheme features high-powered incentives.

Lastly, we need to derive the optimal premia. To do so, notice that

$$\frac{\partial(e_1^{L*}+e_2^{L*})}{\partial\pi_v^L}=(1-\gamma)\left[\frac{\partial p^L}{\partial\pi_v^L}\big(\pi_v^L-\pi_d^L+(x_2^L-x_1^L)^2(2\lambda-1)\big)+p^L\right], \tag{50}$$

and

$$\frac{\partial(e_1^{R*}+e_2^{R*})}{\partial\pi_v^L}=(1-\gamma)\frac{\partial p^L}{\partial\pi_v^L}\Big(\pi_d^R-\pi_v^R\Big), \tag{51}$$

which yields

$$\frac{\partial(e_1^{L*}+e_2^{L*}-e_1^{R*}-e_2^{R*})}{\partial\pi_v^L}=\frac{p^L}{\frac{1}{1-\gamma}-\psi\Big[\pi_v^L-\pi_d^L-\pi_d^R+\pi_v^R+(x_2^L-x_1^L)^2(2\lambda-1)\Big)\Big]}. \tag{52}$$

By inspection, $\partial p^L/\partial\pi_v^L<0$ for $\gamma>1$, and $\partial p^L/\partial\pi_v^L>0$ for $\gamma<1$ and ψ small enough, which completes the proof. □

C. Extensions

C.1. Endogenous Platforms

This section extends the baseline model to allow the the party leader to choose the location of the party platform (in addition to the premia). Suppose that factions have the following ideal points: $\hat{x}^L<0=x^V<\hat{x}^R$. Let x^L,x^R be the platforms chosen by *left* and *right*, respectively. As in the baseline model, parties choose the reward scheme and then factions choose the campaigning level. Formally, L's problem is:

$$\max_{\pi_d^L,\pi_v^L,x^L}\quad p^L(\pi_d^L,\pi_v^L,x^L)=\frac{1}{2}+\psi\big[2e^{L*}\big(\pi_d^L,\pi_v^L,x^L\big)-2e^{R*}\big(\pi_d^L,\pi_v^L,x^L\big)-(x^L)^2+(x^R)^2\big]. \tag{53}$$

It is easy to prove that the optimal premia (π_d^L,π_v^L) are identical to those derived in Lemma A.2-A.3: differentiating factions' equilibrium effort with respect to π_v^L yields the same expression as (22), and the same holds for π_d^L. This is intuitive, since premia only affect the party's probability of victory via factions' campaigning effort. We now derive the optimal policy x^{L*}.

Differentiating p^L with respect to x^L yields:

$$\frac{\partial p^L}{\partial x^L} = \psi\left(\frac{\partial 2e^{L*}\big(\pi_d^L, \pi_v^L, x^L\big) - 2e^{R*}\big(\pi_d^L, \pi_v^L, x^L\big)}{\partial x^L} - 2x^L\right), \tag{54}$$

from which it is clear that an increase in x^L (i.e., *left*'s platform moving closer to $x^V = 0$) always has a *direct*, positive effect on *left*'s probability of victory. The total effect of a change in x^L also depends on the *indirect* effect on factions' campaigning decision. The following first-order condition identifies the equilibrium effort of factions in *left*:

$$2e^L = 1+\left(\frac{1-\gamma}{2}\right)\left[p^L\pi_v^L+(1-p^L)\pi_d^L\right]+\psi\left[b_v^L-b_d^L+\rho_1^L(\pi_v^L-\pi_d^L)-\big(\hat{x}^L-x^L\big)^2+\big(\hat{x}^L-x^R\big)^2\right], \tag{55}$$

which differs from (29) because the external incentive term includes $-\big(\hat{x}^L - x^L\big)^2$. Differentiating with respect to x^L yields:

$$\frac{\partial 2e^L}{\partial x^L} = 2\psi\big(\hat{x}^L - x^L\big) + \frac{(1-\gamma)}{2}\psi\left(\frac{\partial 2e^L\big(x^L\big)}{\partial x^L} - \frac{\partial 2e^R\big(x^L\big)}{\partial x^L} - 2x^L\right)(\pi_v^L - \pi_d^L). \tag{56}$$

Define $\mathcal{X}_{x^L} := \frac{\partial 2e^L\left(x^L\right)}{\partial x^L} - \frac{\partial 2e^R\left(x^L\right)}{\partial x^L}$. To derive $\frac{\partial 2e^R\left(x^L\right)}{\partial x^L}$, observe that:

$$\frac{\partial 2e^L}{\partial x^R} = -2\psi\big(\hat{x}^L - x^R\big) + \frac{(1-\gamma)}{2}\psi\left(\frac{\partial 2e^L\big(x^R\big)}{\partial x^R} - \frac{\partial 2e^R\big(x^R\big)}{\partial x^R} + 2x^R\right)(\pi_v^L - \pi_d^L), \tag{57}$$

and, by symmetry:

$$\frac{\partial 2e^R}{\partial x^L} = -2\psi\big(\hat{x}^R - x^L\big) - \frac{(1-\gamma)}{2}\psi\left(\mathcal{X}_{x^L} - 2x^L\right)(\pi_v^R - \pi_d^R). \tag{58}$$

Straightforward derivation then yields:

$$\frac{\partial \left(2e^{L}\left(x^{L}\right)-2e^{R}\left(x^{L}\right)\right)}{\partial x^{L}}=\frac{2\psi\left(\hat{x}^{L}-x^{R}\right)-\psi\frac{(1-\gamma)}{2}x^{L}\left(\pi_{v}^{L}-\pi_{d}^{L}+\pi_{v}^{R}-\pi_{d}^{R}\right)}{1-\frac{(1-\gamma)}{2}\psi\left(\pi_{v}^{L}-\pi_{d}^{L}+\pi_{v}^{R}-\pi_{d}^{R}\right)}, \tag{59}$$

where the expression (59) captures the indirect effect of a change in *left*'s platform location on the probability of electoral victory via factions' effort in *left*.

Claim 2. *The sign of the indirect effect (59):*

- *is always negative for $\gamma > 1$;*
- *depends on factions' ideological extremism for $\gamma < 1$. In particular, it is positive (negative) when factions in left are sufficiently moderate (extreme).*

Proof. Let $\gamma > 1$. Then, the denominator in (59) is always positive, since $\left(\pi_{v}^{L}-\pi_{d}^{L}+\pi_{v}^{R}-\pi_{d}^{R}\right) \subset [0, 2\alpha] \subset [0, 2]$. The sign is driven by the numerator, which is clearly negative: As x^{L} moves towards the voter's bliss point, factions in L (who are more extreme) campaign less and focus on the internal contest.

Let $\gamma < 1$. Then, the denominator in (59) is positive for ψ small enough, and the sign is driven by the numerator. The first term of the numerator is always negative by assumption, while the second term is negative for $\hat{x}^{L} < x^{L} < 0$. The expression is negative when:

$$2\psi\left(\hat{x}^{L}-x^{R}\right)<\psi\frac{(1-\gamma)}{2}x^{L}\left(\pi_{v}^{L}-\pi_{d}^{L}+\pi_{v}^{R}-\pi_{d}^{R}\right), \tag{60}$$

which means that when factions are sufficiently moderate, a more moderate platform incentivizes more campaigning effort. □

It is straightforward to derive the optimal x^{L*} when $\gamma > 1$. In this case, all premia are set to zero in equilibrium, and the platform that maximizes p^{L} is $x^{L*} = 2\psi^{2}\left(\hat{x}^{L}-x^{R}\right)$. When $\gamma < 1$, the optimal premia are $\pi_v^L = \alpha$, $\pi_d^L = 1 - \alpha$. Substituting this value into the leader's first-order condition produces:

$$\psi \left(\frac{2\psi \left(\hat{x}^L - \hat{x}^R\right) - \psi \frac{(1-\gamma)}{2} x^L (4\alpha - 2)}{1 - \frac{(1-\gamma)}{2}\psi(4\alpha - 2)} - 2x^L \right) = 0, \tag{61}$$

which yields the following optimal platform:

$$x^{L*} = \frac{2\psi \left(\hat{x}^L - \hat{x}^R\right)}{3\left(1 - \psi \frac{(1-\gamma)}{2} x^L (4\alpha - 2)\right)}. \tag{62}$$

C.2. Reward Scheme in a Repeated Game

The baseline model assumes that the party leader cannot appropriate any of the electoral spoils obtained by the party. Formally, rewards need to satisfy the budget constraint: L distributes α $(1 - \alpha)$ when *left* wins (loses) the election. In what follows, I relax this assumption and analyze whether the equilibrium reward scheme of the baseline game is robust to an infinitely repeated game between a long-lived leader and two long-lived factions, where the leader can appropriate resources for herself.

Without loss of generality, I focus on the case of $\gamma < 1$ (i.e., campaigning effort is more effective than sabotage for ranking higher within the party). In this case, the reward scheme that maximizes total factional effort is $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$. For simplicity, I will focus on how premia affect the probability of electoral victory only through effort in *left*, shutting down their effect on factional effort in *right*.

We are interested in the following grim-trigger equilibrium candidate of the repeated game: $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$, and e^{L*} (as defined in Proposition 1, Equation 9 in the main text) conditional on obtaining rewards (π_d^{L*}, π_v^{L*}), and minimal effort for every period following a deviation from (π_d^{L*}, π_v^{L*}). Intuitively, the only interesting player in the repeated game is the party leader: If L gives (π_d^{L*}, π_v^{L*}), then the game continues as in the baseline model, if not the relationship collapses to the reversion equilibrium where factions split their resources half in sabotage half in effort, which is the faction-optimal allocation from a cost-saving standpoint. The party leader evaluates the future according to a discount factor $\delta \in (0, 1)$. We are interesting in proving the following Claim.

Claim 3. *When $\gamma < 1$, there exists an equilibrium of the infinitely repeated game where $(\pi_d^{L*}, \pi_v^{L*}) = (1-\alpha, \alpha)$, and factional effort is defined by e^{L*} as in Proposition 1.*

Proof. I begin with the stage game. Let $B \in \mathbb{R}_+$ denote the benefit the leader L obtains when its party wins the election (in the baseline model, $B = 1$ since the leader's only objective is to win the election). The expected stage game payoff of L given a vector of rewards (π_d^L, π_v^L) can be expressed as:

$$U_l(\pi_d^L, \pi_v^L) = p^L \left(B + \alpha - \pi_v^L\right) + (1 - p^L)\left(1 - \alpha - \pi_d^L\right), \tag{63}$$

which differs from the baseline model in that L can keep $\alpha - \pi_v^L$ $(1 - \alpha - \pi_d^L)$ conditional on party victory (defeat). Since the expected stage game payoff is decreasing in both premia, in a one-shot game L optimally sets $\pi_d^L = \pi_v^L = 0$ for any $\alpha \in (1/2, 1)$. Absent any incentives from the internal contest, factions will choose the effort and sabotage allocation that minimizes their costs: formally, $e_i^{L*} = e_i^{L*} = 1/2$ (from the binding resource constraint of factions). Thus, in a one-shot game, or in the "punishment" phase of a repeated game, $\pi_d^{L*} = \pi_v^{L*} = 0$. It is also clear that L always chooses $\pi_d^{L*} = \pi_v^{L*} = 0$ once in the punishment phase.

In order for the equilibrium in Claim 3 to be sustainable in the infinitely repeated game, the following incentive compatibility condition must hold:

$$(1 - \delta)B + \delta U_l(\pi_d^{L*}, \pi_v^{L*}) \geq (1 - \delta)(B + \alpha) + \delta U_l(0, 0), \tag{64}$$

where the left-hand side represents the leader's value of abiding by the equilibrium incentive scheme (π_d^{L*}, π_v^{L*}) in the repeated game under the event of electoral victory, and the right-hand side represents the value of defecting, i.e., keeping all the electoral spoils and giving zero premia. Note that under the incentive scheme $(\pi_d^{L*}, \pi_v^{L*}) = (1-\alpha, \alpha)$ the expected stage game payoff simplifies to:

$$U_l(\pi_d^{L*}, \pi_v^{L*}) = p^L\left(2e^{L*}(\pi_d^{L*}, \pi_v^{L*})\right)B, \tag{65}$$

whereas under $(\pi_d^L, \pi_v^L) = (0,0)$ we have:

$$U_l(0,0) = p^L\left(2e^{L*}(0,0)\right)(B+\alpha) + \left(1 - p^L\left(2e^{L*}(0,0)\right)\right)(1-\alpha). \tag{66}$$

By inspection of (65-66), it is clear that the leader's expected stage game payoff is higher under the candidate equilibrium (π_d^{L*}, π_v^{L*}) whenever the benefit from winning (B) is high enough, since this reward scheme increases the chances of a favorable electoral outcome with respect to appropriating all resources.

Rearranging Equation 64 yields the following condition that sustains an equilibrium with premia (π_d^{L*}, π_v^{L*}):

$$\delta \geq \frac{\alpha}{(U_l(\pi_d^{L*}, \pi_v^{L*}) - U_l(0,0) + \alpha)}. \tag{67}$$

Thus, for values of the discount factor above the identified threshold, the strategy profile of the baseline game is also an equilibrium of the repeated game, which completes the proof. □

C.3. Negative Premia

In the baseline model, the leader is constrained to choosing an incentive scheme that rewards the faction that ranks higher according to the internal monitoring device with non-negative premia. However, from a theoretical standpoint one might argue that the party should recognize and avoid such inefficient metrics and be able to punish factions that campaign less. That is, when $\gamma > 1$, negative premia should be strictly better than positive ones: by promising all electoral spoils to the lower-ranking faction, leaders can ensure higher campaigning effort.

In terms of information inferred from the ranking indicator, if negative premia are allowed, then having the ranking indicator increasing in mobilization effort is equivalent to it increasing in sabotage. The main results of the baseline model are robust to a specification that allows leaders to punish high-ranking factions.

Corollary 2 (Equilibrium with negative premia). *When premia can be negative, the optimal premia offered by L in equilibrium (and, symmetrically, by R) are $(\pi_d^{L*}, \pi_v^{L*}) = (\alpha - 1, -\alpha)$ if $\gamma > 1$, and $(\pi_d^{L*}, \pi_v^{L*}) = (1 - \alpha, \alpha)$ if $\gamma < 1$.*

Proof. The proof directly follows from Lemma A.2 and A.3, which show that the leader's expected payoff is strictly increasing (decreasing) in both premia when $\gamma < 1$ ($\gamma > 1$). □

Intuitively, the highest incentive to invest in campaigning coincides with a punishment for ranking higher when $\gamma > 1$. When the factions' incentives are aligned to those of the leader ($\gamma < 1$), high-powered incentives are optimal, as in the baseline model.

How does allowing for negative premia change the equilibrium investment decision of factions? The baseline model shows that factions in extreme parties campaign less than factions in moderate parties when $\gamma < 1$. Extending the analysis to negative premia shows that this is true for every value of γ, i.e.,

$$\frac{\partial\big(e^{L*} - e^{R*}\big)}{\partial |x^L|} < 0.$$

Since it is equivalent to have the internal ranking determined by sabotage or campaigning, the effect of polarization on campaigning effort is the same whether $\gamma > 1$ or $\gamma < 1$. The baseline model also shows that when factions are heterogeneous, rewarding sabotage with a positive premium contingent on electoral victory increases the moderate faction's campaigning. This result is robust to a specification that allows for negative premia.

C.4. Non-binding Resource Constraint

The baseline model assumes $a_i^L + e_i^L = 1$ — that is, effort (e_i^L) and sabotage (a_i^L) exhaust the faction's unitary budget of resources. In what follows I analyze the general case $a_i^L + e_i^L \leq 1$. I show that in equilibrium (i) effort must be positive, (ii) sabotage is either positive or zero. That is, both $a_i^{L*} > 0,\ e_i^{L*} > 0$ and $a_i^{L*} = 0,\ e_i^{L*} > 0$ are possible in equilibrium.

Consider the decision of faction L_1 in party L. Faction L_1's maximization problem is

$$\max_{e_1^L, a_1^L} \quad p^L\big[b_v^L + \rho_1^L \pi_v^L\big] + (1-p^L)\big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2\big] - \frac{(e_1^L)^2}{2} - \frac{(a_1^L)^2}{2}$$
$$\text{s.t.} \quad e_1^L + a_1^L \leq 1,$$
$$e_1^L, a_1^L \geq 0.$$

where $p^L = \frac{1}{2} + \psi\big[-(x^L)^2 + (x^R)^2 + e_1^L + e_2^L - e_1^R - e_2^R\big]$ and $\rho_1^L = \frac{1}{2} + \frac{(e_1^L - e_2^L) + \gamma(a_1^L - a_2^L)}{\phi}$. For simplicity, but without loss of generality, let $x^L = -x^R$ (no party has an ex-ante electoral advantage).

The Lagrangean associated with L_1's problem can be expressed as

$$\mathcal{L}(e_1^L, a_1^L) = p^L\big[b_v^L + \rho_1^L \pi_v^L\big] + (1-p^L)\big[b_d^L + \rho_1^L \pi_d^L - (x^L - x^R)^2\big] - \frac{(e_1^L)^2}{2} - \frac{(a_1^L)^2}{2} \tag{68}$$
$$- \lambda_1(a_1^L + e_1^L - 1) + \lambda_2(e_1^L) + \lambda_3(a_1^L) \tag{69}$$

The optimization problem satisfies the constraint qualifications. Hence, the solution of the faction's maximization is the solution of the following Karush-Kuhn-Tucker conditions:

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L \pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L - \lambda_1 + \lambda_2 = 0 \tag{70}$$

$$(2) \quad \frac{\gamma}{\phi}\Big[p^L \pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L - \lambda_1 + \lambda_3 = 0 \tag{71}$$

$$(3) \quad a_1^L + e_1^L - 1 \leq 0 \qquad \wedge \qquad \lambda_1\big[a_1^L + e_1^L - 1\big] = 0 \tag{72}$$

$$(4) \quad e_1^L \geq 0 \qquad \wedge \qquad \lambda_2\big(e_1^L\big) = 0 \tag{73}$$

$$(5) \quad a_1^L \geq 0 \qquad \wedge \quad \lambda_3\big(a_1^L\big) = 0 \tag{74}$$

$$(6) \quad \lambda_1 \geq 0,\ \lambda_2 \geq 0,\ \lambda_3 \geq 0, \tag{75}$$

where conditions (1) and (2) are the first order conditions with respect to e_1^L and a_1^L. Before proceeding with the cases for evaluation, notice that the following holds in equilibrium: (i) $e_1^{L*} = e_2^{L*}$ and $a_1^{L*} = a_2^{L*}$, which implies $\rho_1^L = 1/2$, (ii) $p^L(e_1^{L*}, e_2^{L*}, e_1^{R*}, e_2^{R*}) = 1/2$, since $x^L = -x^R$.

Given the inequality constraint, there are four cases to consider.

(I) $a_1^L > 0,\ e_1^L > 0.$ Then, $\lambda_2 = \lambda_3 = 0$ from conditions (4) and (5). We can find the value of λ_1 from condition (1) and (2):

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L = \lambda_1$$

$$(2) \quad \frac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L = \lambda_1.$$

Substituting (1) into (2) yields

$$e_1^L - a_1^L = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big].$$

Recall that rewards are simply: $b_v^L = \frac{\alpha - \pi_v^L}{2}$ and $b_d^L = \frac{1-\alpha-\pi_d^L}{2}$. Hence we can re-write $e_1^L - a_1^L$ as

$$e_1^L - a_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big].$$

In equilibrium, effort is increasing (decreasing) in both premia when $\gamma < 1$ ($\gamma > 1$). Hence, premia are set to ($\pi_d^{L*} = 1-\alpha$, $\pi_v^{L*} = \alpha$) when $\gamma < 1$, and to ($\pi_d^{L*} = \pi_v^{L*} = 0$) when $\gamma > 1$. Substituting in (π_d^{L*}, π_v^{L*}) and $p^L = 1/2$ yields

$$a_i^{L*} = e_i^{L*} - \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] - \mathbb{1}\{\gamma < 1\}\frac{1-\gamma}{\phi}, \tag{76}$$

which substituted into condition (1) yields

$$\lambda_1 = \mathbb{1}\{\gamma < 1\}\frac{1}{\phi} + \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] - e_i^{L*},$$

where the first two terms are nonnegative because $\phi > 0$, $\psi > 0$, $\alpha \geq 1/2$. We can then find a sufficient condition for $\lambda_1 \geq 0$, which is

$$\psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] \geq 1 - \mathbb{1}\{\gamma < 1\}\frac{1}{\phi}.$$

$\boxed{\text{(II)}\ a_1^L > 0,\ e_1^L = 0.}$ Then, $\lambda_3 = 0$ from (5). We can find the value of λ_1 from condition (2):

$$(2) \quad \frac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - a_1^L = \lambda_1.$$

But then, substituting the value found for λ_1 into (1) we get

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1-\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + a_1^L + \lambda_2 = 0$$

which clearly contradicts $\lambda_2 \geq 0$.

$\boxed{\text{(III)}\ a_1^L = 0,\ e_1^L > 0.}$ Then, $\lambda_2 = 0$ from (4). We can find the value of λ_1 from condition (1):

$$(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - e_1^L = \lambda_1,$$

which substituted into (2) yields

$$(2) \quad \frac{\gamma-1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] - \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + e_1^L + \lambda_3 = 0.$$

In equilibrium,

$$\lambda_3 = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1-\gamma}{\phi} - e_1^{L*}. \tag{77}$$

We can then find a sufficient condition for $\lambda_3 \geq 0$, which is

$$\psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] \geq 1 - \frac{1-\gamma}{\phi},$$

which becomes harder to satisfy as γ increases.

(IV) $a_1^L = e_1^L = 0.$ Then, $\lambda_1 = 0$ from condition (3), and

$$\begin{aligned}
&(1) \quad \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \lambda_2 \leq 0 \\
&(2) \quad \frac{\gamma}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big] + \lambda_3 \leq 0,
\end{aligned}$$

which again contradicts $\lambda_2 \geq 0$ and $\lambda_3 \geq 0$.

C.5. Two Separate Actions: Different Cost Functions

This section extends the analysis to consider two separate actions, $e_1^L \in [0,1]$ and $a_1^L \in [0,1]$, and different convex cost functions.

Separate Quadratic Costs. Let faction L_1's maximization problem be

$$\max_{e_1^L, a_1^L} \quad p^L\big[b_v^L + \rho_1^L\pi_v^L\big] + (1-p^L)\big[b_d^L + \rho_1^L\pi_d^L - (x^L - x^R)^2\big] - (e_1^L)^2 + (a_1^L)^2,$$

where $p^L = 1/2 + \psi(-x_L^2 + x_R^2 + e_1^L + e_2^L - e_1^R - e_2^R)$ and $\rho_1^L = 1/2 + \phi^{-1}[e_1^L - e_2^L + \gamma(a_1^L - a_2^L)]$. We start the analysis with the following observations.

Remark 1. *Factions do not invest in sabotage ($a_i^{L*} = 0$) if and only if both premia are set to zero. When $\pi_v^L > 0$ and/or $\pi_d^L > 0$, in equilibrium factions exert some positive level of sabotage.*

Proof. To show this, observe that

$$\frac{\partial U_1^L}{\partial a_1^L} = \frac{\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2a_1^L, \tag{78}$$

where $\partial U_1^L / \partial a_1^L \leq 0$ for every a_1^L if and only if $\pi_v^L = \pi_d^L = 0$. Hence, when premia are set to zero, $a_i^{L*} = 0$. To show the second part of the claim, notice that when either premia is positive $\partial U_1^L / \partial a_1^L \geq 0$ at $a_1^L = 0$, implying $a_i^{L*} > 0$. □

Remark 2. *Factions always exert positive effort in equilibrium.*

Proof. The proof simply follows by inspection of

$$\frac{\partial U_1^L}{\partial e_1^L} = \psi\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\right] + \frac{1}{\phi}\left[p^L \pi_v^L + (1 - p^L)\pi_d^L\right] - 2e_1^L, \tag{79}$$

which is positive at $e_1^L = 0$ for every value of the premia. □

It follows that factions invest in sabotage in equilibrium for any incentive scheme with non-negative premia. The following result derives the equilibrium incentive scheme, which always features positive premia. Given this result, it is always true that factions exert positive sabotage in equilibrium with separate quadratic cost of sabotage and effort.

Lemma C.1. *L's objective function is always increasing in both premia, implying $\pi_v^{L*} = \alpha$, $\pi_d^{L*} = 1-\alpha$.*

Proof. The two first-order conditions of the faction's problem are:

$$2e_1^L = \psi\left[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L)\right] + \frac{1}{\phi}\left[p^L \pi_v^L + (1 - p^L)\pi_d^L\right] \tag{80}$$

$$2a_1^L = \frac{\gamma}{\phi}\left[p^L \pi_v^L + (1 - p^L)\pi_d^L\right] \tag{81}$$

Differentiating effort with respect to π_v^L yields:

$$\frac{\partial e_1^L}{\partial \pi_v^L} = \frac{\psi}{\phi}\Big[\frac{\partial(e_1^L - e_2^L)}{\partial \pi_v^L} + \gamma\frac{\partial(a_1^L - a_2^L)}{\partial \pi_v^L}\Big](\pi_v^L - \pi_d^L) + \psi\Big[\frac{1}{2} + \frac{e_1^L - e_2^L}{2} + \gamma\Big(\frac{a_1^L - a_2^L}{2}\Big)\Big] + \frac{1}{\phi}\Big[p^L + \frac{\partial p^L}{\partial \pi_v^L}(\pi_v^L - \pi_d^L)\Big]$$

When factions are symmetric (same ideological preferences) $a_1^{L*} = a_2^{L*}$ and $e_1^{L*} = e_2^{L*}$, which implies

$$\frac{\partial e^L}{\partial \pi_v^L} = \frac{\psi}{2} + \frac{1}{\phi}\Big[p^L + \psi\frac{\partial 2(e^L - e^R)}{\partial \pi_v^L}(\pi_v^L - \pi_d^L)\Big]$$

and

$$\frac{\partial e^L}{\partial \pi_v^R} = \frac{1}{\phi}\Big[\psi\frac{\partial 2(e^L - e^R)}{\partial \pi_v^R}(\pi_v^L - \pi_d^L)\Big].$$

By symmetry,

$$\frac{\partial e^R}{\partial \pi_v^L} = \frac{1}{\phi}\Big[\psi\frac{\partial 2(e^R - e^L)}{\partial \pi_v^L}(\pi_v^R - \pi_d^R)\Big],$$

which allows us to express L's objective function as

$$\frac{\partial(e^L - e^R)}{\partial \pi_v^L} = \frac{\psi}{2} + \frac{1}{\phi}p^L + \frac{2\psi}{\phi}\frac{\partial(e^L - e^R)}{\partial \pi_v^L}(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)$$

Rearranging, we can easily see that

$$\frac{\partial(e^L - e^R)}{\partial \pi_v^L} = \frac{\frac{\psi}{2} + \frac{1}{\phi}p^L}{1 - \frac{2\psi}{\phi}(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)},$$

which is positive for ϕ large enough. This implies $\pi_v^{L*} = \alpha$. The proof for $\pi_d^{L*} = 1 - \alpha$ is analogous and, therefore, omitted. □

Hence with this functional form assumption, the optimal premia do not depend on γ and are always set at the maximum. Premia are independent of γ because L does not internalize the cost of sabotage: with separate actions and separate costs for both actions, sabotage does not imply a lower investment in campaigning activities. The next cost function restores this property of the model while keeping the two actions separate.

Campaigning and Sabotage as Substitutes. Consider the following cost function, which preserves the crucial property of decreasing return of both activities and the fact that more investment in one activity increases the marginal cost of the other:

$$C(e_1^L, a_1^L) = (e_1^L + a_1^L)^2. \tag{82}$$

The two first-order conditions of the faction's problem can now be expressed as:

$$\begin{aligned} 2e_1^L &= \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L) + (x^L - x^R)^2\big] + \frac{1}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2a_1^L, \\ 2a_1^L &= \frac{\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big] - 2e_1^L, \end{aligned} \tag{83}$$

from which we can conclude that $e_1^{L*} = a_1^{L*} = 0$ is never a solution. This simply follows by inspection of $\partial U_1^L / \partial e_1^L$, which is always positive at $e_i^{L*} = a_i^{L*} = 0$. Similarly, when $\gamma < 1$, effort must be positive as the next result shows.

Claim 4. *If $\gamma < 1$, $e_1^{L*} = 0$, $a_1^{L*} > 0$ is not a solution.*

Proof. We can replace a_1^{L*} into the first-order condition with respect to e_1^{L*} and obtain

$$\frac{\partial U_1}{\partial e_1^L} = \psi\big[b_v^L - b_d^L + \rho_1^L(\pi_v^L - \pi_d^L)\big] + \frac{1-\gamma}{\phi}\big[p^L\pi_v^L + (1-p^L)\pi_d^L\big],$$

which is clearly positive. Hence, $e_1^{L*} > 0$. □

On the other hand, it is possible to have $e_1^{L*} > 0$, $a_1^{L*} = 0$. This happens in equilibrium when sabotage is less effective than effort, as the next result shows.

Claim 5. *If $\gamma < 1$, $a_1^{L*} = 0$.*

Proof. Suppose $a_1^L = 0$. Because of the budget constraint assumption and the fact that in equilibrium $e_1^{L*} = e_2^{L*}$, $a_1^{L*} = a_2^{L*}$, we can re-write the first-order condition with respect to e_1^L as

$$2e_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\Big[p^L\pi_v^L + (1-p^L)\pi_d^L\Big],$$

which when substituted into $\partial U_1^L/\partial a_1^L|_{a_1^L=0}$ yields the following condition:

$$\frac{\gamma-1}{\phi}\Big[p^L\pi_v^L+(1-p^L)\pi_d^L\Big]-\psi\Big[\alpha-\frac{1}{2}+(x^L-x^R)^2\Big]<0. \tag{84}$$

When the inequality holds, $a_1^{L*}=0$. By inspection of (84), a sufficient condition for $\partial U_1^L/\partial a_1^L|_{a_1^L=0}<0$ to hold is $\gamma<1$, but it is not necessary: the condition also holds when both premia are zero or when the first term of the LHS is sufficiently low. □

The case left to establish is whether $e_1^{L*}>0,\ a_1^{L*}>0$ can be true in equilibrium. In order to prove it, it is first necessary to show what the optimal incentive scheme is. This is not straightforward. On the one hand, high-powered incentives always elicit campaigning effort. On the other, when sabotage is highly effective, high-powered incentives might reduce campaigning via an increase in the marginal cost of effort. Unfortunately, finding the optimal incentive scheme when $e_1^L>0$, $a_1^L>0$ is not analytically tractable. However, the next result shows that factions exert positive effort in equilibrium when $\pi_v^{L*}=\alpha$ and γ is large enough. The proof is organized as follows. I begin by assuming that $e_1^L>0$, and $a_1^L=0$. The proof shows that the equilibrium incentive conditional on an electoral victory is always $\pi_v^{L*}=\alpha$. Then, the optimal incentive is substituted in $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0}$ to prove that, when γ is high enough, the sign of the derivative is positive for every π_d^L, which implies that $a_1^{L*}>0$. Intuitively, when γ is large, the return from sabotage is high and factions sabotage in equilibrium when incentives are high powered.

Claim 6. *When $\pi_v^{L*}=\alpha$ and γ is large enough, $e_1^{L*}>0,\ a_1^{L*}>0$.*

Proof. Consider the case $e_1^L>0$, $a_1^L=0$. Since $e_1^{L*}=e_2^{L*}$ and $a_1^{L*}=a_2^{L*}$, and by the budget constraint assumption $b_v^L+\pi_v^L/2=\alpha/2$ and $b_d^L+\pi_d^L/2=(1-\alpha)/2$, the first-order condition of L_1 with respect to e_1^L simplifies to

$$2e_1^L=\psi\Big[\alpha-\frac{1}{2}+(x^L-x^R)^2\Big]+\frac{1}{\phi}\big[p^L\pi_v^L+(1-p^L)\pi_d^L\big] \tag{85}$$

To find the optimal value of π_v^L, plug in the expression for p^L, and consider the symmetric first-order condition of faction R_1:

$$2e_1^L = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\Big[\pi_d^L + \Big(\frac{1}{2} + \psi(-x_L^2 + x_R^2 + 2e_1^L - 2e_1^R)\Big)(\pi_v^L - \pi_d^L)\Big],$$
$$2e_1^R = \psi\Big[\alpha - \frac{1}{2} + (x^R - x^L)^2\Big] + \frac{1}{\phi}\Big[\pi_d^R + \Big(\frac{1}{2} - \psi(-x_L^2 + x_R^2 + 2e_1^L - 2e_1^R)\Big)(\pi_v^R - \pi_d^R)\Big].$$

Subtracting $2(e_1^L - e_1^R)$ yields

$$2(e_1^L - e_1^R) = \frac{1}{\phi}\big[\pi_d^L - \pi_d^R + \Big(\frac{1}{2} + \psi(-x_L^2 + x_R^2)\Big)(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R) + \psi(2e_1^L - 2e_1^R)(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)\big],$$

which can be re-expressed as

$$2(e_1^L - e_1^R) = \frac{(\frac{1}{2} + \psi(-x_L^2 + x_R^2))(\pi_v^L + \pi_v^R) + (\frac{1}{2} - \psi(-x_L^2 + x_R^2))(\pi_d^L + \pi_d^R)}{\phi - \psi(\pi_v^L - \pi_d^L + \pi_v^R - \pi_d^R)},$$

which is increasing in π_v^L. In the numerator, π_v^L is pre-multiplied by the probability of victory of the left, and as π_v^L increases, the negative term in the denominator becomes smaller. Intuitively, increasing the power of the incentives increases equilibrium campaigning, which is what L wants to maximize. Hence in equilibrium $\pi_v^{L*} = \alpha$. Substituting π_v^{L*} into $\partial U_1^L / \partial e_1^L = 0$ yields

$$2e_1^{L*} = \psi\Big[\alpha - \frac{1}{2} + (x^L - x^R)^2\Big] + \frac{1}{\phi}\big[p^L \alpha + (1 - p^L)\pi_d^{L*}\big],$$

where the optimal level of π_d^{L*} is unknown. Recall that $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0} = \frac{\gamma}{\phi}\big[p^L \pi_v^L + (1 - p^L)\pi_d^L\big] - 2e_1^L$. We want to show that $\frac{\partial U_1^L}{\partial a_1^L}|_{a_1^L=0}$ can be positive. To find a sufficient condition, consider $\pi_d^L = 0$. Substituting the equilibrium values expressions for e_1^{L*} and π_v^{L*} yields

$$\frac{\gamma - 1}{\phi} p^L \alpha - \psi\Big(\alpha - \frac{1}{2}\Big),$$

which is clearly positive for γ high enough. Hence it must be that $a_1^{L*} > 0$ for $\gamma > \phi\psi/2$. □