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Abstract

This paper shows how green investing spurs companies to mitigate their carbon emissions by raising the cost of capital of the most carbon-intensive companies. Companies' emissions decrease when the wealth share of green investors and their sensitivity to climate externalities increase. We show that the impact of green investors primarily governs companies' long-run emissions. Companies are further incentivized to reduce their emissions when green investors anticipate tighter climate regulations and climate-related technological innovations. However, heightened uncertainty regarding future climate risks alleviates green investors' pressure on the cost of capital of companies and pushes them to increase their emissions. Calibrated on United States data, our model suggests that, albeit effective, the impact of green investors remains limited given their current wealth share and practices.

Keywords: Climate finance; socially responsible investing; ESG; impact investing.

JEL codes: G12, G11.

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1 Introduction

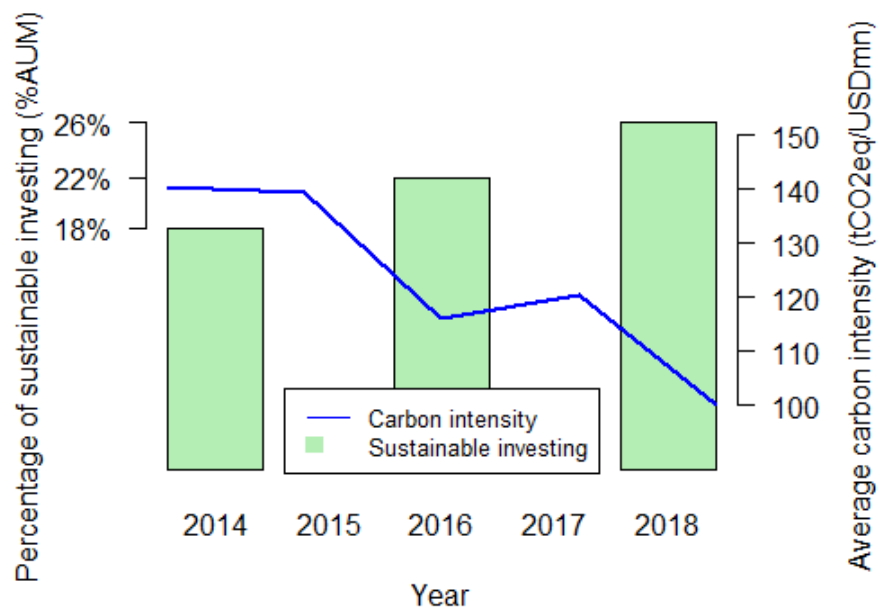


Figure 1. Percentage of sustainable investments and average carbon intensity of the AMEX, NASDAQ, and NYSE stocks. This figure presents the evolution of the proportion of sustainable investing relative to total managed assets over time, according to the Global Sustainable Investment Alliance (2018), as compared to the average carbon intensity of AMEX, NASDAQ and NYSE companies provided by S&P-Trucost between 2014 and 2018. The carbon intensity corresponds to the direct (scope 1 and 2) and indirect (upstream scope 3) greenhouse gas emissions of the companies, expressed in tCO₂e per million dollars of revenue generated.

From 2014 to 2018, sustainable investments grew from 18% to 26% of the total assets under management (AUM) in the United States (U.S.) (US SIF, 2018) while, over the same period, the average carbon intensity of the companies listed on the National Association of Securities Dealers Automated Quotations (NASDAQ), American Stock Exchange (AMEX), and New York Stock Exchange (NYSE) decreased from 140 tons of CO₂ equivalent per million dollars of revenue (tCO₂e/USDmn) to 100 tCO₂e/USDmn (Figure 1).¹ The downward trend in corporate greenhouse gas intensity may be driven by several factors, such as the reductions in the costs of green technologies, tighter climate regulations, consumer pressure for more sustainable practices, and pressure exerted by green investors.² The two main channels through which green investors can have an impact on companies' practices are portfolio screening and shareholder engagement. Through portfolio climate screening, by underweighting or excluding the most carbon-intensive³ companies from their investment scope, green investors increase these companies' cost of capital (Heinkel, Kraus,

¹The carbon intensity of a company is defined as its emission rate relative to its revenue over one year. This metric is expressed in terms of tons of equivalent carbon dioxide per million dollars.

²Green investing is a form of socially responsible investing aimed at contributing to environmental objectives, mostly reducing greenhouse gas emissions by internalizing climate externalities.

³We refer to *carbon-intensive* companies and companies with high *greenhouse gas emissions* interchangeably since carbon dioxide is the main gas contributing to global warming. In the United States (U.S.), it accounted for more than 80% of the total emissions in 2018: <https://www.epa.gov/ghgemissions/overview-greenhouse-gases>.

and Zechner, 2001; Pastor, Stambaugh, and Taylor, 2021b; Pedersen, Fitzgibbons, and Pomorski, 2021; Zerbib, 2021) and can push them to reform. We focus on the specific channel of climate screening (referred to as *green investing* hereinafter) and address the issue of impact investing by answering the following questions: does green investing push companies to reduce their greenhouse gas emissions? If so, what are the factors that lead companies to mitigate their emissions? And how do these factors affect the *dynamics* of companies' emissions?

We show that the development of green investing—both in terms of the proportion of AUM and the sensitivity to climate externalities of green investors—pushes companies to reduce their greenhouse gas emissions by raising their cost of capital. By internalizing the negative impact of green investors on their financial valuation, companies are incentivized to pay a price to mitigate their emissions by adopting less carbon-intensive technologies, thereby lowering their cost of capital. These incentives are further strengthened when investors anticipate tighter climate regulations, climate-related technological innovations, and when they account for the negative impact of other companies' emissions on the company under consideration. However, in a sufficiently large or diversified market, investors' uncertainty regarding future climate risks reduces the incentives for companies to mitigate their emissions.

We develop a dynamic equilibrium model populated by heterogeneous investors and companies. We model two different groups of investors with constant absolute risk aversion (CARA investors). Both groups determine their optimal allocation by maximizing their expected wealth at a given terminal date, but they differ in their climate beliefs. Of the two groups of investors, one is a group of *green investors* and the other one of *regular investors*. Because green investors are aware of environmental risks, their magnitude, and their timing, or better equipped to anticipate them, they internalize the expected financial impact of future climate externalities of the companies in which they invest, while regular investors do not. These externalities correspond, for example, to greenhouse gas emissions and reflect companies' exposure to various climate transition risks, such as the rise in carbon price (Jakob and Hilaire, 2015) or changes in consumer preferences. While climate externalities will have a negative financial impact for the most carbon-intensive companies, they may positively impact the greenest companies benefiting from their favorable position. In the first version of our model, green investors internalize *deterministic* climate externalities.

These investors invest in n companies, each with a different marginal cost of reducing greenhouse gas emissions (referred to as *marginal abatement cost* hereinafter). Corporate managers are stock-value optimizers, who balance the benefit of mitigating greenhouse emissions, thus attracting green investors, against the cost of implementing these reforms. To represent the fact that a company reforms its environmental practices over a long period of time, at the initial date, $t = 0$, each company chooses a deterministic greenhouse gas emission schedule up to a final date T to maximize its expected discounted future market value. We allow companies to have their own climate beliefs. In addition, each company accounts for the strategies adopted by all other companies, hence reducing the companies' problem to a nonzero-sum game. This framework notably differs from standard heterogeneous belief models because the choice of each company's emission schedule directly affects the parameter on which investors disagree—companies' climate externalities.

We obtain a tractable formula of the equilibrium asset prices and show that they include an *externality premium*. This premium increases with the future *financial impact of climate externalities* (for simplicity, referred to as *climate externalities* hereinafter) internalized by green investors, which can be either positive or negative, and scales in proportion to the relative wealth of green investors. Therefore, all else being equal, the asset price of a carbon-intensive company (also referred to as a *brown* company hereinafter) will be lower than that of a company with a low carbon footprint (also referred to as a *green* company hereinafter). Consequently, the expected returns increase when the climate externalities are negative and decrease when they are positive.

We characterize companies’ optimal emission schedules in a general setup and derive an explicit solution for the case when climate externalities are measured as a decreasing quadratic function of the company’s emissions. In equilibrium, emissions decrease with the proportion of assets managed by green investors, their sensitivity to climate externalities (also referred to as *climate sensitivity* hereinafter), as well as companies’ climate sensitivity, but increase with the marginal abatement cost. In particular, we show that investors’ climate sensitivity mainly drives long-term emissions whereas companies’ climate sensitivity predominantly drives short-term emissions. Therefore, when the average climate sensitivity of investors is lower than that of the companies, optimal emissions decrease over time. In addition, corporate emissions are a convex function of time, with the degree of convexity increasing in the rate of time preference. We calibrate the model on the AMEX, NASDAQ, and NYSE stocks between 2004 and 2018 using the carbon intensity of companies as a proxy for their emissions. We then simulate emissions’ mitigation in several scenarios by considering an electrical equipment manufacturing company. Consistent with the small effect of divestment on companies’ cost of capital estimated by Berk and van Binsbergen (2021), our simulations suggest that, albeit effective, green investors’ impact is still limited: an electrical equipment manufacturing company reduces its emissions by an average of 1% per year over a 20-year period when green investments account for 25% of the total AUM in the economy. When either green investments account for 50% of the AUM or when green investors’ climate sensitivity doubles, this company reduces its emissions by an average of 3% per year over the same period.

We also show that tighter climate regulations as well as climate-related technological innovations, when anticipated and internalized by green investors, increase the pressure on companies to further reduce their emissions. Using previous research, we recalibrate the marginal abatement cost and the climate sensitivities accounting for the effects of technological change and expected regulatory action, respectively. Our model estimates that emissions decline 2.2 times faster when green investors anticipate regulatory tightening. We also find that emissions decline 3.6 times faster when they anticipate climate-related technological change. Finally, when both changes are anticipated, declines are 4 times faster. In addition, we illustrate the effect of strategic interactions between companies by showing that when green investors internalize the negative financial impact of the economy’s average emissions, companies are further incentivized to curb their emission schedules.

We next extend our model to the case where green investors also internalize *uncertainty* about future climate externalities. Climate risks, such as a rise in carbon price or the occurrence of natural disasters, usually have non-Gaussian fat-tailed distributions (Weitzman, 2009; Barnett, Brock, and Hansen, 2020). Therefore, we model future climate risks internalized by green investors as a stochastic jump process (Poisson process). We give a tractable expression of optimal portfolio allocations, asset prices, expected returns and emission schedule in equilibrium. We show that, in a sufficiently large or diversified market, uncertainty about future climate risks leads green investors to lower the risk of their portfolios by reducing their tilt towards green assets, shifting portfolio allocations away from green assets and into brown assets. Consequently, climate uncertainty reduces the magnitude of the externality premium on expected returns. As a result, the incentive for companies to reform is also reduced, leading them to increase their optimal emission schedule in equilibrium.

The results of this paper are of interest to both investors and policymakers. For investors, we identify three major implications. First, the findings show that investors can increase their impact on companies by raising their environmental requirements, for example by restricting the range of companies in which they invest or by significantly underweighing the most carbon-intensive companies. However, the small simulated effect suggests that other impact strategies, such as shareholder engagement, might be more efficient at this stage, in line with the conclusions of Broccardo, Hart, and Zingales (2020). Indeed, as long as the wealth share of sustainable investors

is low, the assets they exclude will be purchased by regular investors whose wealth is large enough to dampen the effect on the cost of capital and thus the impact on corporate practices. Second, to increase the climate impact of their asset allocations, investors also have a key role to play as shareholders in pressing companies to increase transparency about future climate-related risks and raise their environmental standards. Third, impact investing is financially beneficial if investors favor companies that are on a pathway towards reducing their climate footprints. Investors can also benefit from financial gains by investing in green companies for which information on their climate footprints is still poorly available.

From the public authorities' viewpoint, the results of this paper have four implications. First, they highlight their role in supporting the development of green investments. In particular, they suggest policymakers should set rigorous standards for environmental impact assessments and disclosure to foster and increase impact investing. This is consistent with the recommendations of the European Union High Level Expert Group on Sustainable Finance (2018) and the European Commission (2018)'s Action Plan, which led to the recent development of a green taxonomy and an official standard for green bonds. Second, these results emphasize the importance of access to information regarding companies' climate footprints, which enables green investors to internalize climate externalities as accurately as possible, thereby maximizing their impact on companies. Third, these results clarify the effect of climate regulations and their predictability on the adjustment of investors' beliefs: a tighter climate regulation is amplified by the adjustment of green investors' expectations, which increases their pressure on companies' cost of capital, thereby forcing them to further reduce their emissions. Fourth, our analysis shows the importance of low-cost access to greener technological solutions as a lever for companies to mitigate their climate impact. Specifically, industries for which green alternatives are limited, such as cement or aircraft, face a structural barrier to which an increase in R&D is an essential response. In addition, enabling investors to better forecast and internalize the likelihood of future technological innovations increases their impact on corporate emissions.

Related literature. This paper contributes to the emerging literature on asset pricing and impact investing in sustainable finance. First, from an asset pricing perspective, we clarify the relationship between the development of sustainable investing⁴ and asset returns. Building on the seminal paper by Heinkel et al. (2001), three recent papers by Pastor et al. (2021b), Pedersen et al. (2021) and Zerbib (2021) study this relationship using a single-period model. These papers show that the stock returns of the brownest companies are increased by a positive premium. Bolton and Kacperczyk (2021), Ardia, Bluteau, Boudt, and Inghelbrecht (2021) and Pastor, Stalbaugh, and Taylor (2021a) empirically support this finding.⁵ Avramov, Lioui, Liu, and Tarelli (2021b)

⁴Sustainable investing can be motivated by pecuniary or non-pecuniary motives (Krüger, Sautner, and Starks, 2020). Riedl and Smeets (2017) and Hartzmark and Sussman (2020) highlight the positive effect of sustainable preferences on sustainable fund flows. Pro-social and pro-environmental preferences also impact asset returns since they induce an increase in the return on sin stocks (Hong and Kacperczyk, 2009), a decrease in the return on impact funds (Barber, Morse, and Yasuda, 2021) and a decrease in the return on bonds (Baker, Bergstresser, Serafeim, and Wurgler, 2018; Zerbib, 2019).

⁵It is worth noting that the global empirical literature on the effects of Environment, Social and Governance (ESG) integration on asset returns is mixed: some authors highlight the negative impact of ESG performance on asset returns, while others suggest a positive relationship or find no significant impact. For negative impacts, see Brammer, Brooks, and Pavelin (2006), Renneboog, Ter Horst, and Zhang (2008), Sharfman and Fernando (2008), ElGhoul, Guedhami, Kowk, and Mishra (2011), Chava (2014), Barber et al. (2021), and Hsu, Li, and Tsou (2019). For positive impacts, see Derwall, Guenster, Bauer, and Koedijk (2005), Statman and Glushkov (2009), Edmans (2011), Eccles, Ioannou, and Serafeim (2014), Krüger (2015) and Statman and Glushkov (2016). Finally, Bauer, Koedijk, and Otten (2005), Galema, Plantinga, and Scholtens (2008) and Trinks, Scholtens, Mulder, and Dam (2018) find no significant impact.

extend this modeling framework with dynamic preferences for sustainability. We contribute to this literature by characterizing the dynamic of asset prices and expected returns when green investors internalize non-Gaussian climate uncertainty. Consistent with Avramov, Cheng, Lioui, and Tarelli (2021a) who study the effect of uncertainty in a Gaussian setup, we show that, in the presence of non-Gaussian uncertainty about future climate risks, the expected return gap between brown and green assets narrows as green investors diversify their exposure to mitigate their risk.

We also contribute to the emerging literature on impact investing. In a seminal study, by constructing a single-period model in which green investors have the ability to exclude the most polluting companies, Heinkel et al. (2001) show that such companies are pushed to reform because exclusionary screening negatively impacts their valuations. Chowdhry, Davies, and Waters (2018) study the optimal contracting for a company that cannot commit to social objectives and show that impact investors must hold a large enough financial claim to incentivize the company to internalize social externalities. Oehmke and Opp (2019) show how sustainable investors can enable a scale increase of clean production by internalizing social costs in the presence of financing constraints. The authors emphasize the importance of sustainable investors unconditionally caring for sustainability issues (i.e., irrespective of whether they are investors in the firm) and coordinating among themselves. They also show that non-sustainable investors and sustainable investors can jointly achieve higher surplus than either investor type alone. When markets are subject to search friction, Landier and Lovo (2020) show that the presence of an ESG fund forces companies to partially internalize externalities. In contrast, by analyzing venture capital funds, Barber et al. (2021) suggest instead that the pressure exerted by sustainable investors on companies is tied to their willingness to pay to invest in impact funds. Finally, through an asset pricing model, Pastor et al. (2021b) show that green investors produce positive social impact by shifting investment towards green firms and making firms greener. We also contribute to the literature on impact investing from an asset pricing perspective along three different avenues. First, we characterize the *dynamics* of companies' optimal emissions and show that increases in either the share of green investors or their climate sensitivity pushes companies to cut their emissions by affecting their cost of capital. Notably, long-term emission dynamics are governed by green investors' beliefs, while short-term dynamics are driven by companies' beliefs. Second, when investors internalize future climate-related regulations and technological innovations, they increase their pressure on companies' cost of capital, thereby incentivizing them to further reduce their emissions. Finally, we show that green investors' uncertainty about future climate risks leads companies to increase their emissions.

The remainder of this paper is structured as follows. Section 2 introduces an economy populated by greenhouse gas emitting companies and investors with heterogeneous beliefs. Section 3 describes the equilibrium pricing equations and companies' emission schedules when green investors internalize deterministic climate externalities. Section 4 extends the model to non-Gaussian stochastic climate externalities. Section 5 concludes the paper. Proofs of the mathematical results are presented in detail in the Appendix.

2 A simple economy with greenhouse gas emitting companies and heterogeneous beliefs

We develop a simple model with heterogeneous beliefs in which climate externalities are internalized by green investors in a deterministic way. We introduce the dynamics of the assets available in the market and heterogeneous beliefs about climate externalities of three types of agents—a group of regular investors, a group of green investors and n companies. We then present the investors' and companies' optimization programs.

2.1 Securities market

In this section, we consider a financial market consisting of n risky stocks and a risk-free asset, which is assumed to be free of arbitrage and complete. The risk-free asset is in perfectly elastic supply and we assume that the risk-free rate is zero without loss of generality.⁶ Each stock $i \in \{1, \dots, n\}$ is in positive net supply of one unit and is a claim on a single liquidating dividend D_T^i at horizon T . The terminal dividend of the i -th asset, D_T^i , is broken down into three terms: (i) the cost of an environmental reform, decided by the company at time $t = 0$ and implemented between $t = 0$ and $t = T$, with an associated greenhouse gas emission schedule; (ii) the initial dividend forecast at time $t = 0$ net of the cost of the environmental reform; (iii) the entire cash flow news sequence between $t = 0$ and $t = T$. The values of the first two terms are public information at time $t = 0$ and their sum is also referred to as the initial dividend forecast. We isolate the cost of environmental reform from the initial dividend forecast because it is a parameter of interest in our model. This cost is known at the initial state because a company's decision to reduce its emissions is usually made over a long period of time. For example, the transformation of a generating fleet by an electric utility, or the development of a line of electric vehicles by a car manufacturer are the result of long-term decisions known to the public well in advance of their completion. Therefore, the vector of terminal dividends for all assets, $D_T \in \mathbb{R}^n$, reads

$$D_T = \underbrace{\int_0^T c_t(\psi_t - \psi_b)dt}_{\text{Cost of reform}} + \underbrace{d}_{\substack{\text{Initial dividend forecast} \\ \text{net of cost of reform}}} + \underbrace{\int_0^T \sigma_t dB_t}_{\text{Cash flow news}}. \quad (1)$$

Initial dividend forecast

In the first term of the above expression, representing the cost of an environmental reform, ψ_t is the vector of greenhouse gas emissions per unit of time of the companies at date t .⁷ We refer to greenhouse gas emissions for simplicity, but ψ can be seen as a measure of relative emissions compared to a level of production (e.g., carbon intensity). The constant $\psi_b \in \mathbb{R}_+^n$ is the vector of initial (or *baseline*) emissions, and c_t is a diagonal matrix with elements (c_t^i) , which correspond to the marginal abatement costs for each company. Thus, by reducing its emissions by x , the i -th company reduces its terminal dividend by $c_t^i x$ per unit of time t . The process $(\psi_t^i)_{t \in [0, T]}$, which is also referred to as the emission schedule of the i -th company, is determined at time $t = 0$: it is deterministic and does not depend on the future cash flow news so as to reflect the long-term nature of companies' reform strategies. The second term of the expression, d , is a constant vector corresponding to the initial dividend forecast under a reference probability measure net of the cost of the environmental reform. In the third term, $\sigma_t dB_t$ ($t \in [0, T]$) is the sequence of cash flow news, where $(B_s)_{s \in [0, T]}$ is a standard n -dimensional Brownian motion defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with a filtration $(\mathcal{F}_s)_{s \in [0, T]}$. We refer to \mathbb{P} as the *reference probability measure* and we will later introduce other probability measures for the companies and the investors, which reflect their different beliefs. For each $s \in [0, T]$, σ_s is a deterministic, $n \times n$, invertible matrix.

Denoting by $(p_t)_{t \in [0, T]}$ the equilibrium assets price process in \mathbb{R}^n , we assume $p_T = D_T$. We also denote the dividend forecast under the reference probability measure at $t \in [0, T]$ by

$$D_t = \mathbb{E}[D_T | \mathcal{F}_t] = \int_0^T c_t(\psi_t - \psi_b)dt + d + \int_0^t \sigma_s dB_s, \quad (2)$$

⁶As stressed by Atmaz and Basak (2018), the interest rate can be taken as exogenous since consumption occurs only at time T , i.e., there is no intermediate consumption.

⁷Formally, $\psi \in F([0, T], \mathbb{R}_+^n)$, where $F([0, T], \mathbb{R}_+^n)$ is the set of Borel-measurable functions of $[0, T]$ in \mathbb{R}_+^n .

and in particular,

$$D_0 = \mathbb{E}[D_T | \mathcal{F}_0] = \int_0^T c_t(\psi_t - \psi_b) dt + d.$$

This Gaussian continuous-time specification of the dividend dynamics is consistent with previous literature on models with heterogeneous beliefs that study investors' reaction to good and bad news (Veronesi, 1999), excess confidence (Scheinkman and Xiong, 2003) and extrapolation bias (Barberis, Greenwood, Jin, and Shleifer, 2015).⁸ We choose a setup with Gaussian dividends and prices because we are after explicit formulae for equilibrium prices, which companies use to endogenously determine their prospective greenhouse gas emissions.

2.2 Investors' and companies' beliefs

The market is populated by two types of investors, *regular* and *green*, who have different expectations regarding companies' future cash flow news. Regular investors only consider the information related to the flow of financial news, in addition to the initial dividend forecast. Therefore, under their probability measure \mathbb{P}^r , conditional on the information in t , they account for the past cash flow news, $\int_0^t \sigma_s dB_s$, which is known at time t , but the expectation of the future cash flow news, $\int_t^T \sigma_s dB_s$, is zero as B_s is a Brownian motion. Denoting by \mathbb{E}_t^r this conditional expectation,

$$\mathbb{E}_t^r(D_T) = D_t, \tag{3}$$

that is, the dividend forecast of the regular investors coincides with the dividend forecast under the reference probability measure. From the point of view of the properties of the cash flow news, $\sigma_s dB_s$, there is no difference between the measures \mathbb{P} and \mathbb{P}^r , and we can simply assume $\mathbb{P} = \mathbb{P}^r$. However, it should be noted that \mathbb{P} is a technical device and, as such, we make no assumptions about the realistic nature of this measure. The expectations of regular investors are thus not necessarily consistent with the realized events.

In contrast, compared to regular investors, green investors are more aware of environmental risks, their magnitude, and their timing, or better equipped to anticipate them. Therefore, they internalize the financial impact of the expected climate externalities of the companies in which they invest. These externalities correspond, for example, to greenhouse gas emissions and reflect companies' exposure to various climate transition risks, such as the rise in carbon price (Jakob and Hilaire, 2015) or changes in consumer preferences. While climate externalities will have a negative financial impact for the brownest companies, they may positively impact the greenest companies. In our first model specification (Sections 2 and 3), we assume that green investors have a perfect anticipation of the future climate externalities. As a result, in addition to accounting for the cash flow news and the initial dividend forecast as regular investors do, green investors internalize, under their probability measure, the financial impact of future climate externalities at date $t \in [0, T]$. The latter is expressed by

$$\int_t^T \theta_s(\psi_s) ds. \tag{4}$$

Here, $\theta_s(\psi_s) \in \mathbb{R}^n$ is the vector of the *financial impact of climate externalities* (for convenience, referred to as *climate externalities*) at date $s \in [0, T]$. Naturally, we assume that θ_s^i is a decreasing function of ψ_s^i so that higher emissions of the i -th company correspond to stronger negative financial

⁸Other articles on heterogeneous beliefs adopt this same setup in discrete time such as Hong and Stein (1999), Barberis and Shleifer (2003) and Barberis, Greenwood, Jin, and Shleifer (2018).

impact on the i -th asset. However, in a general case, θ_s^i depends on the emissions of all companies because more global emissions increases financial risks for each company. This is why θ_s^i is a function of the whole vector ψ_s . Moreover, θ_s^i is a function of time to reflect growing climate sensitivity of green investors, or their anticipation of stronger regulatory pressure. As a consequence, green investors internalize their climate beliefs regarding the i -th company by attributing a fundamental value to the i -th stock at time t that is higher (if $\int_t^T \theta_s^i(\psi_s) ds$ is positive) or lower (if $\int_t^T \theta_s^i(\psi_s) ds$ is negative) than the value of the dividend forecast (see Equation (2)). Denoting by \mathbb{E}_t^g the expectation of the green investors conditional on the information at time t , we have

$$\mathbb{E}_t^g(D_T) = D_t + \int_t^T \theta_s(\psi_s) ds, \quad (5)$$

that is, the dividend forecast of the green investors equals the forecast under the reference measure augmented by the financial impact of future climate externalities internalized by these investors. It is worth noticing that the variable D_t is constructed from the actual realization of the past cash flow news between 0 and t , which is a known quantity to investors at time t . However, from a probabilistic point of view, the stochastic process D_t under the probability \mathbb{P}^g includes an additional drift $\theta_s(\psi_s) ds$ associated with the beliefs of green investors.⁹

Alongside the two types of investors, we also introduce the productive sector by modeling the views of the companies about the n assets available on the market. As in Oehmke and Opp (2019), corporate managers (referred to as *companies* hereinafter) also have subjective beliefs about the financial impact of climate externalities on the dividend dynamics of each of the n companies. We denote by $\theta_s^c(\psi_s)$ the vector of the climate externalities internalized by all companies. Denoting by \mathbb{E}_t^c the expectation of the companies conditional on the information in t , we have

$$\mathbb{E}_t^c(D_T) = D_t + \int_t^T \theta_s^c(\psi_s) ds. \quad (6)$$

Similarly to Equation (5), the dividend forecast of the companies equals the forecast under the reference measure augmented by the financial impact of future climate externalities internalized by the companies. Here again, the variable D_t is constructed from the actual realization of the past cash flow news between 0 and t , which is known to market participants at time t . However, from a probabilistic point of view, the dynamic of the stochastic process D_t under the probability \mathbb{P}^c includes an additional drift $\theta_s^c(\psi_s)$ associated with the companies' beliefs.¹⁰

2.3 Investors' preferences and optimization

Regular and green investors are assumed to have CARA preferences. Subject to their budget constraints, investors maximize the expected exponential utility of their terminal wealth¹¹ W_T , which reads

$$\mathbb{E}^j(1 - e^{-\gamma^j W_T^j}), \quad \gamma^j > 0, \quad j \in \{r, g\},$$

⁹Formally, the probability measure of green investors is constructed through a change of measure. The Radon-Nikodym density that connects the two probability measures \mathbb{P}^g and \mathbb{P}^r is $Z_T = e^{\int_0^T \lambda_s^\top dB_s - \frac{1}{2} \int_0^T \|\lambda_s\|^2 ds}$, where $\lambda_t := \sigma_t^{-1} \theta_t(\psi_t)$. Under \mathbb{P}^g , the dynamics of the stochastic process D_t reads $D_t = \int_0^T c_s(\psi_s - \psi_b) ds + d + \int_0^t \sigma_s dB_s^g + \int_0^t \theta_s(\psi_s) ds$, where B^g is a brownian motion under \mathbb{P}^g .

¹⁰In the same way as presented in footnote 9, the companies' measure is constructed through a similar change of measure where $\theta_t(\psi_t)$ is replaced by $\theta_t^c(\psi_t)$.

¹¹As Atmaz and Basak (2018) point out, investors' preferences are based on their wealth at the terminal date rather than on intermediate dates, which would have led to endogenizing the interest rate in equilibrium.

where the superscripts r and g refer to the regular and green investors, respectively, and γ^j s are their absolute risk aversions. The wealth processes follow the dynamics

$$W_t^r = w^r + \int_0^t (N_s^r)^\top dp_s, \quad W_t^g = w^g + \int_0^t (N_s^g)^\top dp_s, \quad (7)$$

where N_t^r and N_t^g are quantities of assets held by the regular and green investors, respectively, at time t , and prices $(p_t)_{t \in [0, T]}$ are determined by the market clearing condition. The initial wealth levels of regular and green investors are denoted by w^r and w^g , respectively, and the symbol \top stands for the transposition operator.

In what follows, we denote by γ^* the global risk aversion, defined by $\frac{1}{\gamma^*} = \frac{1}{\gamma^r} + \frac{1}{\gamma^g}$, and set $\alpha = \frac{\gamma^r}{\gamma^r + \gamma^g}$ and $1 - \alpha = \frac{\gamma^g}{\gamma^r + \gamma^g}$. To simplify the interpretation of the impact of green and regular investors' wealth on the variables in equilibrium, and without losing generality, we assume that green and regular investors have equal relative risk aversions; that is, $\gamma^R = \gamma^g w^g = \gamma^r w^r$, where γ^R denotes the relative risk aversion. In this case, α is the proportion of the green investors' initial wealth at $t = 0$, and $1 - \alpha$ is that of the regular investors; that is, $\alpha = \frac{w^g}{w^g + w^r}$ and $1 - \alpha = \frac{w^r}{w^g + w^r}$.

2.4 Companies' utility and optimization

A company's decision to implement reforms to mitigate its climate footprint is made over a sufficiently long time horizon. Therefore, at $t = 0$, the i -th company chooses its emission schedule, $(\psi_t^i)_{t \in [0, T]}$, up to the horizon T so as to maximize its future valuation. Consequently, in our setup, we endogenize companies' emissions through their market values: on the one hand, investors allocate their wealth according to companies' emission schedules,¹² thereby impacting companies' market values; on the other hand, companies take into account their market values to determine their emission schedules. We denote by ρ the rate of time preference and by ψ^{-i} the vector of emission schedules of companies other than the i -th company. The market value of the i -th company's asset at time t is denoted by $p_t^i(\psi^i, \psi^{-i})$ to reflect its dependence on the vector ψ of all companies' emission schedules. The companies have a linear utility and risk neutral preferences (Lambrecht and Myers, 2017; van Binsbergen and Opp, 2019). Therefore, at $t = 0$, the i -th company chooses $(\psi_t^i)_{t \in [0, T]}$ so as to maximize the following objective function:

$$\mathcal{J}^i(\psi^i, \psi^{-i}) = \mathbb{E}^c \left[\int_0^T e^{-\rho t} p_t^i(\psi^i, \psi^{-i}) dt \right]. \quad (8)$$

Maximizing the sum of the market values over the entire period is consistent with Pastor et al. (2021b) as well as recent studies on Chief Executive Officers' (CEO) compensation plans. Larcker and Tayan (2019), for example, report that "stock-based performance awards have replaced stock options as the most prevalent form of equity-based pay." In addition, CEOs are generally required to hold their companies' stocks. Managers are therefore directly interested in the valuation of their company's stock price at each date, which endogenizes the financial impact of the company's emission schedule. This optimization program is also in line with the approach of Heinkel et al. (2001) in the context of a multi-period model where the company's climate impact is endogenized.

The optimal emission schedule, ψ^* , corresponds to a Nash equilibrium in which each company $i \in \{1, \dots, n\}$ determines its own emissions, $\psi^{i,*}$, in $t = 0$, so that

$$\mathcal{J}^i(\psi^{*,i}, \psi^{*, -i}) \geq \mathcal{J}^i(\psi^i, \psi^{*, -i}), \quad \text{for all } \psi^i \in F([0, T], \mathbb{R}_+). \quad (9)$$

Table 1 summarizes the preferences and optimization programs of the different players and their interactions in the economy we model.

¹²Green investors internalize climate externalities and all investors account for the costs of environmental reforms.

Table 1 Summary of agents' actions. This table summarizes the optimization programs of each agent as well as their interactions between $t = 0$ and $t = T$.

Date	Agent	Choose	Given
At $t = 0$	Companies	Their deterministic emission schedule, ψ , from 0 to T	- Their expected market value between 0 and T - The cost of reducing their emissions between 0 and T
$\forall t \in [0, T]$	Regular investors	Their asset allocation, N^r	- The observed cash flow news between 0 and t , and the expected cash flow news between t and T - The cost of reducing companies' emissions between 0 and T
$\forall t \in [0, T]$	Green investors	Their asset allocation, N^g	- The observed cash flow news between 0 and t , and the expected cash flow news between t and T - The cost of reducing companies' emissions between 0 and T - The expected financial impact of companies' emissions between t and T

3 Equilibrium in the presence of greenhouse gas emitting companies and heterogeneous beliefs

This section presents equilibrium asset prices and returns in the model developed in Section 2. The optimal portfolio allocations of regular and green investors are found explicitly. We derive the optimal dynamics of companies' emissions, which we render tractable by assuming that climate externalities are quadratic. Finally, we analyze the effects of regulatory changes, technological changes, and companies' interactions on companies' optimal emission schedules.

3.1 Equilibrium stock prices and returns

In equilibrium, investors choose their allocations, which maximize their expected utility. Equilibrium prices are determined such that the market clears. Denoting $\Sigma_t = \sigma_t^\top \sigma_t$, and letting $\mathbf{1}$ be the vector of ones of length n , Proposition 1 gives the equilibrium prices and allocations.

Proposition 1. *Given an emission schedule $(\psi_t)_{t \in [0, T]}$, asset prices in equilibrium read*

$$p_t = D_t - \int_t^T \mu_s ds \quad \text{with} \quad \mu_t = \gamma^* \Sigma_t \mathbf{1} - \alpha \theta_t(\psi_t), \quad (10)$$

where $-\alpha \theta_t(\psi_t)$ is the externality premium. The optimal number of shares for the regular and green investors are

$$N_t^r = (1 - \alpha) \left(\mathbf{1} - \frac{1}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right) \quad \text{and} \quad N_t^g = \alpha \left(\mathbf{1} + \frac{1}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right), \quad (11)$$

respectively.

The different beliefs of green investors introduce an *externality premium*, which is an additional drift in the price dynamics. When future climate externalities are negative (i.e., the emissions are high), the price is adjusted downward proportionally to the fraction of the initial wealth held by the green investors, α . Conversely, when future externalities are positive (i.e., the emissions are low), green investors bid up the price, which is adjusted upwards. However, the cost of environmental reform arises in the dividend forecast, D_t (see Equation (2)), and impacts the price in the opposite direction: a reduction (increase) in emissions thus lowers (increases) the price. Therefore, the net effect of a change in emissions on the price depends on the intensity with which green investors internalize climate externalities (through θ_t^i) and the cost of reform (through c_t^i).

The effect of heterogeneous beliefs on climate externalities can also be analyzed in terms of expected dollar returns (referred to as *expected returns* hereinafter), $\mathbb{E}(dp_t) = \mu_t dt$. Since θ_t^i is a decreasing function of ψ_t^i , expected returns increase with companies' emissions. The externality premium on asset returns can be positive ($\theta_t^i(\psi) < 0$) or negative ($\theta_t^i(\psi) > 0$). This result is supported by extensive empirical evidence, including Renneboog et al. (2008), Sharfman and Fernando (2008), Chava (2014), Barber et al. (2021), Bolton and Kacperczyk (2021) and Hsu et al. (2019). It is also consistent with the theoretical works of Pastor et al. (2021b), Pedersen et al. (2021) and Zerbib (2021), who show, through a single-period model, that expected returns increase along with a company's climate impact as green investors require a higher cost of capital.

The number of shares purchased by investors is also adjusted by the climate externalities. Green investors overweigh assets with the higher positive externalities and underweigh or short assets with the higher negative externalities. Regular investors have a symmetrical allocation by providing liquidity to green investors. This result is consistent with optimal allocations in disagreement models where some investors have an optimistic market view and others a pessimistic one (Osambela, 2015; Atmaz and Basak, 2018): the risk is transferred from pessimists to optimists who increase their holding of the asset under consideration.

3.2 Equilibrium emission schedules

At the initial date, companies choose their optimal emission schedules by maximizing their expected market values between times 0 and T .

Proposition 2. *The i -th company's optimal emission schedule, ψ^i , given a vector ψ^{-i} of all other companies' emissions, is the one that maximizes for all $t \in [0, T]$*

$$\beta_t^c \theta_t^{c,i}(\psi_t) + \alpha \beta_t \theta_t^i(\psi_t) + c_t^i \psi_t^i, \quad (12)$$

where

$$\beta_t^c = \frac{e^{-\rho t} - e^{-\rho T}}{1 - e^{-\rho T}} \quad \text{and} \quad \beta_t = \frac{1 - e^{-\rho t}}{1 - e^{-\rho T}}.$$

At each time t , the i -th company maximizes the sum of three terms. The first and second terms measure the financial benefits associated to two climate externality premia: one endogenized by the company ($\theta_t^{c,i}(\psi_t)$) and the other endogenized by the green investors ($\alpha \theta_t^i(\psi_t)$), both adjusted by suitable time factors (β_t^c and β_t , respectively). The third one, $c_t^i \psi_t^i$, accounts for the financial benefits obtained by not reducing its emissions. The optimal emission schedule of a company is a trade-off between the positive effect of reducing its emissions—especially, the positive effect on its cost of capital through $\alpha \beta_t \theta_t^i(\psi_t)$ —and the cost of reform to achieve the target emission schedule.¹³

Research in environmental economics consensually suggests the use of a convex specification to model the economic damage associated with environmental risks (Dietz and Stern (2015); Burke,

¹³The companies' optimization problem in the reduced form of (12) is obtained by writing the equilibrium price

Hsiang, and Miguel (2015); Burke, Davis, and Diffenbaugh (2018)). Following Nordhaus (2014), who argues that “damages can be reasonably well approximated by a quadratic function of temperature change”, we use a quadratic climate damage function to model the economic impact associated with climate change. We assume that climate externalities are quadratic in carbon emissions and consider a first simple case where the climate externalities of a given company, θ_t^i , depend only on its own emissions, ψ_t^i .¹⁴ In this case, Proposition 2 yields a simple solution detailed in Corollary 3.

Corollary 3. *Assuming $\theta_t^i(x) = \kappa_{0,t} - \frac{\kappa_t}{2}x^2$ and $\theta_t^{c,i}(x) = \kappa_{0,t}^c - \frac{\kappa_t^c}{2}x^2$, for $x \geq 0$, where κ_t , κ_t^c , $\kappa_{0,t}$ and $\kappa_{0,t}^c$ are positive deterministic functions of time,¹⁵ the i -th company’s optimal emission schedule reads*

$$\psi_t^{*,i} = \frac{c_t^i}{\beta_t^c \kappa_t^c + \alpha \beta_t \kappa_t} \quad (13)$$

Emissions decrease with respect to the proportion of wealth held by green investors, α , and when green investors’ and companies’ sensitivities to climate externalities increase. Indeed, κ_t and κ_t^c measure the sensitivity with which green investors and companies, respectively, internalize climate externalities at time t (also referred to as *climate sensitivity*). Thus, green investors can increase their impact on companies by raising their climate sensitivity, for example by restricting the range of companies in which they invest or by significantly underweighing the most carbon-intensive companies. It should be noted that even if the company does not internalize climate externalities ($\kappa_t^c = 0$), green investors’ beliefs and the threat they pose to a company’s market value are sufficient to prompt a company to reduce its climate impact. In such a case, the optimal emission schedule simplifies to:

$$\psi_t^{*,i} = \frac{c_t^i}{\alpha \beta_t \kappa_t}. \quad (14)$$

As expected, the emissions of the i -th company decrease when the marginal abatement cost, c_t^i , decreases. In the special case where the marginal abatement cost is zero, the company cuts its emissions to zero. The marginal abatement cost is a company specific factor that plays an important role in the greening dynamics of the economy. R&D in industries where green alternatives are still limited (e.g., cement, aviation) is therefore a key tool to support and accelerate the ecological transition.

Irrespective of technological and regulatory changes, which will be analyzed in Section 3.3, the emission schedule is not necessarily constant over time. The effect of investors’ beliefs, $\alpha \kappa_t$, which changes with $\beta_t \simeq \frac{t}{T}$ (for small ρ), grows over time. In the long run, this effect becomes prominent over the effect of companies’ beliefs, κ_t^c , which changes with $\beta_t^c \simeq 1 - \frac{t}{T}$ (for small ρ) and fades over time. This effect is explained by the following reasoning: green investors make equilibrium

(10) under the companies’ probability measure \mathbb{P}^c :

$$p_t = D_t - \int_t^T \mu_s ds = \int_0^T c_t(\psi_t - \psi_b) dt + d + \int_0^t \sigma_s dB_s^c + \int_0^t \theta_s^c(\psi_s) ds - \gamma^* \int_t^T \Sigma_s \mathbf{1} ds + \alpha \int_t^T \theta_s(\psi_s) ds,$$

where B^c is a Brownian motion under the measure \mathbb{P}^c . Substituting this expression for the price into Equation (8) leads to the result of Proposition 2 after an integration by parts.

¹⁴The financial impact of climate externalities, θ_t^i , represents the opposite of a damage function (θ_t^i decreases as carbon emissions increase), so it is concave. Other types of climate externalities functions can be considered, such as an exponential function (Barnett et al., 2020). However, in the case of our model, the exponential specification does not lead to tractable solutions.

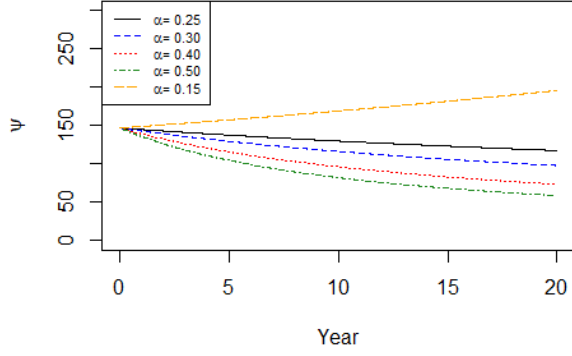
¹⁵For simplicity we assume that κ_t , κ_t^c , $\kappa_{0,t}$ and $\kappa_{0,t}^c$ are the same for all companies but the generalization to different constants is straightforward.

prices depend on their long-term beliefs (Equation (10)) by internalizing their perceived future climate risks in their investment decisions; therefore, since companies maximize their expected future valuations, they are pushed to adjust their long-term climate footprint according to green investors' beliefs. More precisely and from an analytical standpoint, this dynamic is driven by the fact that the asset prices at time t under the companies' probability measure depend on companies' climate beliefs between 0 and t ($\int_0^t \theta_s^c(\psi_s) ds$) and on green investors' climate beliefs between t and T through the externality premium ($\alpha \int_t^T \theta_s(\psi_s) ds$) as detailed in footnote 13. Therefore, by maximizing the expectation of the price's integral between 0 and T , companies give more weight to their beliefs at the beginning of the period and to green investors' beliefs at the end of the period. Consequently, when the climate sensitivity of companies is lower than the climate sensitivity of the average investor, $\kappa^c < \alpha\kappa$,¹⁶ they emit more at the beginning of the period and less at the end of the period because the pressure exerted by investors is more pronounced over the long run. Conversely, the emission schedule increases over time if companies have a higher climate sensitivity than that of the average investor. In the limiting case where companies' climate beliefs are equal to investors' average climate beliefs, $\kappa^c = \alpha\kappa$, the emission schedule is constant over time. In short, companies' climate sensitivity, κ^c , mainly drives short-term emissions, whereas green investors' proportion of wealth, α , and their climate sensitivity, κ , mainly drive long-term emissions.¹⁷ This result captures two main routes available to green investors to impact corporate decisions. First, green investors have the ability to reduce the long-term emission target of the companies in which they invest by internalizing climate externalities in their investment decisions. Second, they can also contribute to reducing companies' short-term emissions by pushing them to internalize climate externalities, for example through shareholder engagement (Dimson, Karakaş, and Li, 2015; Broccardo et al., 2020). Aligning corporate objectives with climate issues can be achieved via incentive mechanisms in managers' compensation schemes as suggested by Edmans, Gabaix, Sadzik, and Sannikov (2012), Varas (2018) and Aggarwal, Dizon-Ross, and Zucker (2020).

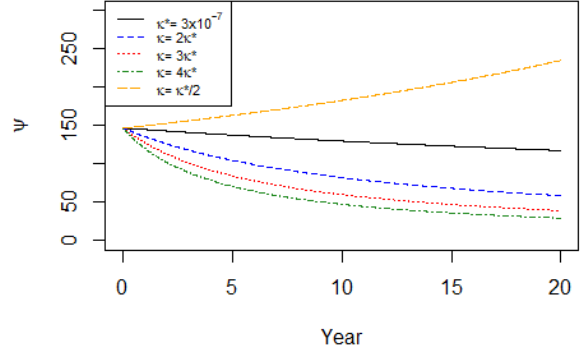
Figure 2 shows several optimal emission schedules for an electrical equipment company as a function of α , κ , κ^c , and ρ . The calibration is detailed in Appendix B. It is reasonable to assume a lag in a company's responses to pressure from investors. To represent this delay, we use as an example the case where the company internalizes climate externalities with slightly less sensitivity than the average investor, $\kappa^c < \alpha\kappa$, and thus where emissions decrease over time. With these parameter values, when 25% of total AUM are managed by green investors, the company reduces its emissions by only 1% per year on average. This magnitude is coherent with the small effect of divestment on companies' cost of capital estimated by Berk and van Binsbergen (2021). The emission reduction reaches 3% per year on average when green investments account for 50% of total

¹⁶ $\alpha\kappa$ is the climate sensitivity of the average investor because green investors' climate sensitivity, κ , is weighted by their proportion of wealth, α , and regular investors have zero climate sensitivity.

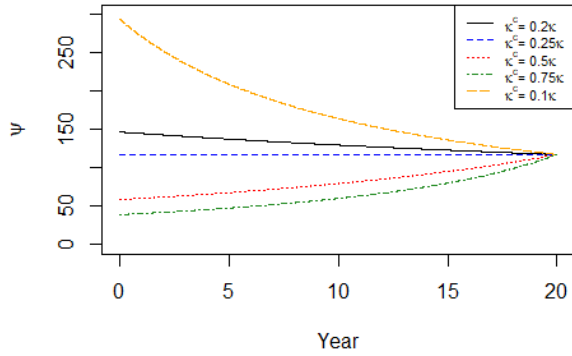
¹⁷Two special cases arise: when κ^c is zero (that is, companies do not internalize climate externalities), emissions tend to infinity close to time $t = 0$ (because β_t tends to 0); similarly, when $\alpha\kappa$ is zero (that is, there are no green investors), emissions tend to infinity close to time $t = T$ (because β_t^c tends to 0). This effect is due to the fact that, in order to obtain a tractable and interpretable solution for the emission schedule, we have opted for a constant marginal abatement cost for each date t , which is profitable to companies when their emissions are higher than the initial level, ψ_b (for example, because they are exempt from the cost of infrastructure maintenance). Therefore, companies benefit from letting their emissions grow when there are no incentives to reduce them (in $t = 0$ when κ^c is zero, and in $t = T$ when $\alpha\kappa$ is zero). In the paper, we focus on this simplified model because (i) the main objective is to analyze the impact of green investors-induced incentives on companies and (ii) the model allows us to obtain tractable formulae that explain the pressures exerted by investors and companies on greenhouse gas emissions. However, in the Internet Appendix, we study a less tractable version of the model in which companies have zero marginal gain from letting their emissions grow above the initial level, ψ_b . In that framework the emissions' dynamics is similar to the one observed here but for an important difference: companies never increase their emissions above the initial level, ψ_b , irrespective of whether α , κ or κ^c are zero.



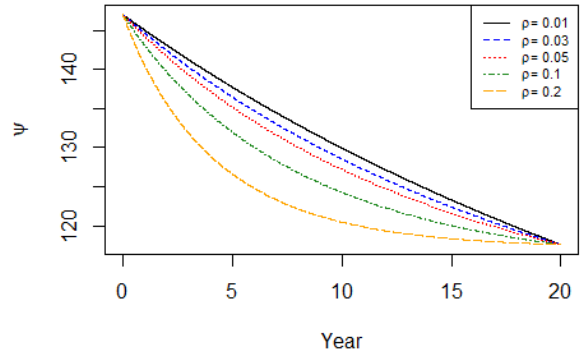
(a) ψ_t with different values for α



(b) ψ_t with different values for κ



(c) ψ_t with different values for κ^c



(d) ψ_t with different values for ρ

Figure 2. Emission schedules. This figure shows the optimal emission schedules, ψ_t , according to several values of the proportion of green investors (α , sub-figure (a)), the green investors' climate sensitivity (κ , sub-figure (b)), the companies' climate sensitivity (κ^c , sub-figure (c)), and the rate of time preference (ρ , sub-figure (d)). The parameters are calibrated according to the values estimated in Appendix B: $\psi_b = 147$, $\alpha = 0.25$, $\rho = 0.01$, $\kappa = 3 \times 10^{-7}$, $\kappa^c = 6 \times 10^{-8}$, $c^{\text{elec}} = 8 \times 10^{-6}$.

AUM, or when green investors’ climate sensitivity doubles. The decrease in emissions is convex in time because of the dynamics of the time factors β_t and β_t^c : when ρ is small, the emission schedule has a hyperbolic temporal dynamic of the form $1/t$. This convexity increases with the rate of time preference, ρ , which accelerates the substitution effect between the impact of the company’s beliefs (through β_t^c) and the impact of green investors’ beliefs (through β_t) on the optimal emission schedule.¹⁸ In the present example where the average investor is more climate sensitive than the company, $\kappa^c < \alpha\kappa$, green investors’ pressure on long-term emissions occurs earlier and accelerates the emissions decline, thereby increasing the convexity of the schedule. Therefore, in cases where executives have weak incentives to reduce their companies’ climate footprints, a strong preference for the present—for example, through short-term objectives in compensation schemes (Bolton, Scheinkman, and Xiong, 2006; Marinovic and Varas, 2019)—might mitigate their adverse impact on the companies’ optimal emission schedules. Conversely, a company with low climate requirements and a low preference for the present will emit more greenhouse gases at the optimum.¹⁹

As in Pastor et al. (2021b), this model extends the work of Heinkel et al. (2001) by (i) endogenizing the climate impacts of companies and (ii) allowing them to choose among a continuum of climate impacts, in contrast to Heinkel et al. (2001) where companies reform in a binary way (from *brown* to *green*). Compared to Pastor et al. (2021b), in this first approach where externalities are deterministic, we develop a dynamic model that allows us to characterize the *dynamics* of companies’ climate footprints (Section 3.2) as well as to study the dynamic impact on companies’ climate footprints of (i) technological changes (Section 3.3), (ii) regulatory changes (Section 3.3), and (iii) interaction effects between companies’ emissions (Section 3.4).

3.3 Technological and regulatory changes

The anticipation of technological changes and more demanding climate regulations by green investors can further push companies to reduce their climate footprints. We develop the analysis of this mechanism in this section.

Technological changes. Technological changes allowing companies to reduce their climate footprints can take three major forms: the use of new machines that improve energy efficiency, that is, the ratio of energy use over emissions; end-of-pipe innovations, such as carbon capture technologies for utilities, which reduce emissions without modifying the production process; and process innovations, which offer alternative production processes reducing the use of fossil fuels. Although the effect on the marginal abatement cost curve is not unequivocal (Amir, Germain, and Van Steenberghe, 2008; Bauman, Lee, and Seeley, 2008),²⁰ technological breakthroughs generally induce a decrease in the marginal abatement cost (Milliman and Prince, 1989; Palmer, Oates, and Portney, 1995; Jaffe, Newell, and Stavins, 2002).

Mekaroonreung and Johnson (2014) estimate the effect of technological change on nitrogen oxides (NO_x) marginal abatement costs of U.S. coal power plants in 2000–2008 by analyzing 325

¹⁸The Internet Appendix (Figure IA.1.) shows the dynamics of β_t^c and β_t for different rates of time preference, ρ .

¹⁹If companies optimize over an infinite horizon, the economic insights remain the same. Indeed, in equilibrium, the optimal allocations, prices and emission schedules converge to a well-defined limit when T tends to $+\infty$. Specifically, regarding optimal emissions, the direct consequence of an infinite horizon optimization is that the effect of green investors’ beliefs on climate externalities, which impact long-term emissions, substitutes more slowly for the effect of companies’ beliefs. For example, for a rate of time preference of 0.01, the two effects have the same weight after 69 years ($\beta_{69} = \beta_{69}^c = 0.5$).

²⁰The decrease in the marginal abatement cost is consensual for end-of-pipe innovations; for process innovations, the decrease in the marginal abatement cost is favored by a strong substitutability between the two factors of production (energy and capital).

boilers operating in 134 bituminous coal power plants. They find that technological change reduced the NO_x marginal cost by 28.3% in 2000–2004 and 26.5% in 2004–2008. Based on the order of magnitude of their result and using the parameters calibrated in Appendix B, we simulate the effect of technological changes that would reduce the marginal cost of carbon intensity abatement by 5% per year (Figure 3). Such technological changes, when anticipated by companies and investors, push companies to multiply the pace of emissions reduction by a factor of 3.6 (from 1% to 3.6% per year on average). Compared to the situation where no technological change is anticipated, the carbon intensity is reduced by 40% after 10 years and by 64% after 20 years.

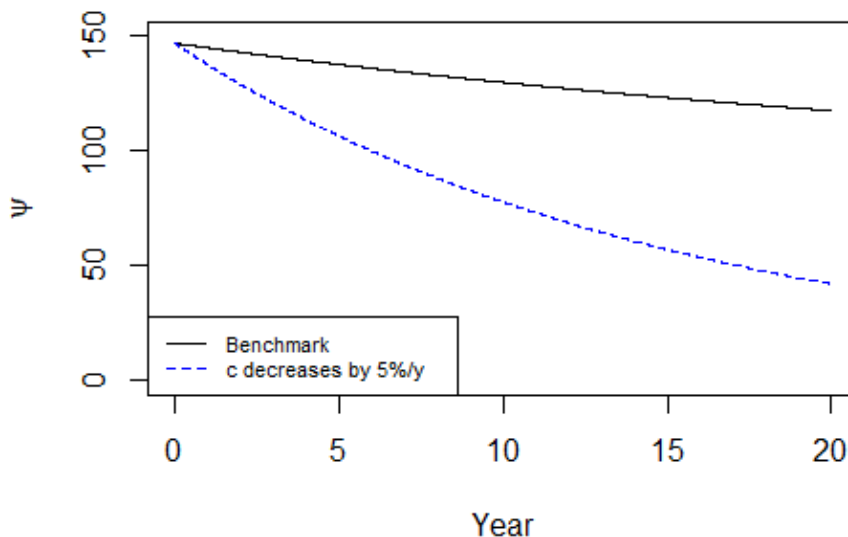


Figure 3. Emission schedule with technological change. This figure shows the optimal emission schedules, ψ_t , without technological change (Benchmark) and with technological change for which the marginal abatement cost decreases by 5% per year. The parameters are calibrated according to the values estimated in Appendix B: $\psi_b = 147$, $\alpha = 0.25$, $\rho = 0.01$, $\kappa = 3 \times 10^{-7}$, $\kappa^c = 6 \times 10^{-8}$, $c^{\text{elec}} = 8 \times 10^{-6}$ (Benchmark).

This result not only underscores the importance of R&D, particularly in sectors where marginal abatement costs are high, but also the need to enable agents to forecast and internalize a likely path of future technological change. Even if, by definition, the occurrence of climate-related innovations is unpredictable, it is possible to anticipate a vigorous dynamic of technological change when R&D is largely supported by public and private funding; the development of renewable energy infrastructures over the last 20 years as well as ongoing research on energy storage²¹ or carbon capture and storage²² are insightful examples.

Regulatory changes. Tightening climate regulations can take two main forms: the introduction of more demanding standards or an increase in the price of carbon, whether through taxes or

²¹https://ec.europa.eu/energy/sites/ener/files/documents/batstorm_d10_roadmap.pdf

²²<https://www.energy.gov/fe/science-innovation/carbon-capture-and-storage-research/carbon-capture-rd>

pollution permits. For green investors, such regulatory changes raise the future financial risks of the brownest companies, specifically the transition risks. Therefore, when investors anticipate regulatory tightening, given the same level of emissions, ψ_t , they adjust their expected financial impact of climate externalities, $\theta_t(\psi_t)$, by increasing their climate sensitivity, κ_t . However, tighter regulations can have an additional effect: as suggested by Porter and van der Linde (1995), they also encourage companies to innovate and may lower the marginal abatement cost. Porter’s hypothesis has been supported by empirical evidence, such as the introduction of sulfur emission standards in India (Sugathan, Bhangale, Kansal, and Hulke, 2018) and a carbon emissions permit trading system in China (Xian, Wang, Wei, and Huang, 2020).

To calibrate the effect of tighter regulation on the climate sensitivity of green investors, we use the carbon price trajectory. Although carbon prices vary from USD 1/tCO₂e to USD 119/tCO₂e in different jurisdictions, the High-Level Commission on Carbon Prices estimates that carbon prices of USD 40-80/tCO₂ by 2020 and USD 50-100/tCO₂ by 2030 are required to reduce emissions in line with the temperature goals of the Paris Agreement (World Bank, 2020). Consistent with this trajectory, we therefore consider an increase of κ_t by 2% per year.

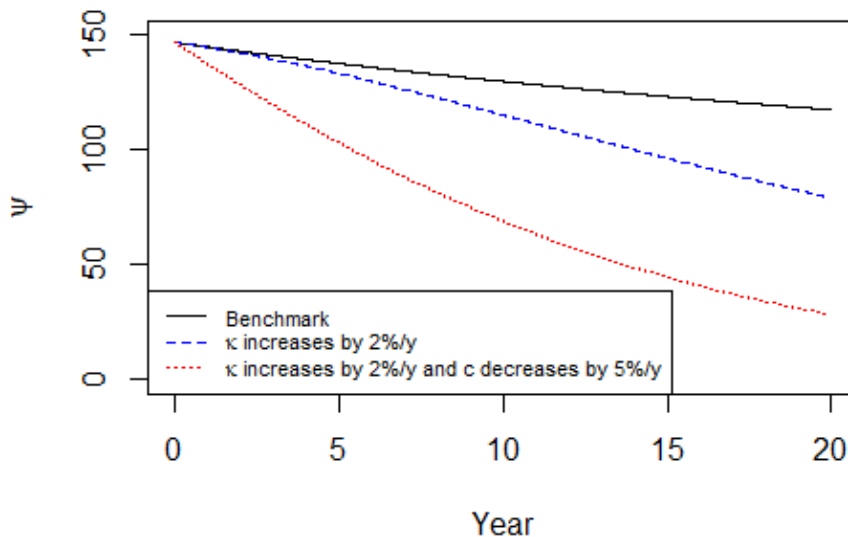


Figure 4. Emission schedule with regulatory change. This figure shows the optimal emission schedules, ψ_t , without regulatory change (Benchmark) and with regulatory change for which (i) green investors’ climate sensitivity, κ_t , increases by 2% per year, and (ii) green investors’ climate sensitivity, κ_t , increases by 2% per year and the marginal abatement cost, c^{elec} , decreases by 5% per year. The parameters are calibrated according to the values estimated in Appendix B: $\psi_b = 147$, $\alpha = 0.25$, $\rho = 0.01$, $\kappa = 3 \times 10^{-7}$ (Benchmark), $\kappa^c = 6 \times 10^{-8}$, $c^{\text{elec}} = 8 \times 10^{-6}$ (Benchmark).

Figure 4 shows the effects of tighter regulation on companies’ optimal emission schedule, using the parameters calibrated in Appendix B. In addition, we also show the effect of a simultaneous policy change and technological change by including a 5% annual drop in marginal abatement cost along with the 2% annual increase in climate sensitivity.

When investors anticipate regulatory tightening, they push companies to multiply the rate of

emissions reduction by a factor of 2.2 (2.2% per year on average). This rate is multiplied by four when investors and companies account for technological changes. Compared to the situation without regulatory changes, the carbon intensity decreases by 11% (47%) after 10 years and by 33% (76%) after 20 years without (with) technological change. Here again, the anticipation of future regulatory tightening encourages companies to launch projects that emit less greenhouse gases and to adopt a more ambitious emission schedule, partly as a result of the increased pressure exerted by investors on their cost of capital. Therefore, by announcing plans for more stringent climate standards or the future carbon price trajectory early enough, governments are sending a signal not only to companies but also to investors, which further strengthens the climate impact they have on companies.

3.4 Interaction effects

In Corollary 3 we have made the simplifying assumption that investors and companies internalize the financial impact of the i -th company's climate externalities as a function of its own emissions only, $\theta_t^i(\psi_t^i)$. However, a company can be financially impacted by the emissions of other companies in the economy. For example, the risk of tightening climate regulations is likely to be greater if the companies in the same geographical area have a large climate footprint. Proposition 4 shows that when agents internalize the negative impact of the economy's average emissions through an interaction effect, companies decrease their optimal emission schedules compared to the case with no such interaction.

Proposition 4. *Let $\psi_t^{*,i} = c_t^i(\beta_t^c \kappa_t^c + \alpha \beta_t \kappa_t)^{-1}$ be the i -th company's optimal emission schedule without interaction effect (Corollary 3), $\bar{\psi}_t^* = \frac{1}{n} \sum_{j=1}^n \psi_t^{*,j}$ the average optimal emissions of the n companies without interaction effect, and $\varepsilon \geq 0$ an elasticity parameter such that*

$$\theta_t^i(\psi) = \kappa_{0,t} - \frac{\kappa_t}{2} \left[(\psi_t^i)^2 + \varepsilon \psi_t^i \bar{\psi}_t \right] \quad \text{and} \quad \theta_t^{c,i}(\psi) = \kappa_{0,t}^c - \frac{\kappa_t^c}{2} \left[(\psi_t^i)^2 + \varepsilon \psi_t^i \bar{\psi}_t \right],$$

where $\bar{\psi}_t = \frac{1}{n} \sum_{j=1}^n \psi_t^j$, and $\kappa_{0,t}$, $\kappa_{0,t}^c$, κ_t and κ_t^c are deterministic functions of time. Then, we have the following results:

- (i) *Companies lower their optimal emissions compared to the case without interaction effect (that is, when $\varepsilon = 0$), and the i -th company's optimal emission schedule reads*

$$\psi_t^{*,\varepsilon,i} = \psi_t^{*,i} - \frac{\varepsilon}{2n + \varepsilon} \left(\psi_t^{*,i} + \frac{2n^2}{2n + (n+1)\varepsilon} \bar{\psi}_t^* \right). \quad (15)$$

- (ii) *When the number of companies is large and much larger than the elasticity parameter, i.e., $n \gg \varepsilon \geq 0$, the optimal emission schedule is approximated by*

$$\psi_t^{*,\varepsilon,i} \simeq \psi_t^{*,i} - \frac{\varepsilon}{\varepsilon + 2} \bar{\psi}_t^*. \quad (16)$$

In the above proposition, we show that when investors and companies internalize the negative financial impact of the economy's average emissions, companies are further incentivized to curb their optimal emission schedules. Let us take a simple example where a company has an optimal carbon intensity of 150 tCO₂e per million dollars of revenue generated (tCO₂e/USDmn), while the average optimal carbon intensity of the $n = 1000$ companies in the economy is 100 tCO₂e/USDmn in the absence of interaction effect. If the agents internalize the impact of the economy's average emissions with an elasticity of $\varepsilon = 0.5$, the optimal emissions decrease to 130 tCO₂e/USDmn, resulting in a

13% reduction. The downward adjustment of a company’s emission schedule in an economy with a sufficiently large number of companies is proportional to $\frac{\varepsilon}{\varepsilon+2}$ times the average emissions of the economy. Therefore, in the particular case where companies give equal weight to their emissions and to the average emissions of the economy ($\varepsilon = 1$), they reduce their optimal emission schedules by one third of the average emissions of the economy. As expected, in the absence of interaction effect ($\varepsilon = 0$), the optimal emission schedule is maximal and it equals $\psi_t^{*,i} = c_t^i(\beta_t^c \kappa_t^c + \alpha \beta_t \kappa_t)^{-1}$.

4 Equilibrium with climate uncertainty

We extend the model presented in Section 2 to the case where the climate externalities are internalized by green investors as a non-Gaussian stochastic process. We characterize the optimal emission schedule under the new setup, and we show that uncertainty about future climate externalities reduces the incentive for companies to lower their emissions.

4.1 Climate uncertainty

The internalization of deterministic climate externalities is an imperfect approach. Barnett et al. (2020) note that “given historical evidence alone it is likely to be challenging to extrapolate climate impacts on a world scale to ranges which many economies have yet to experience. Both richer dynamics and alternative nonlinearities may well be essential features of the damages that we experience in the future due to global warming.” Indeed, climate risks are characterized by fat tails (Weitzman, 2009, 2011) and abrupt changes beyond tipping points (Alley, Marotzke, Nordhaus, Overpeck, Peteet, Pielke Jr., Pierrehumbert, Rhines, Stocker, and Wallace, 2003; Lontzek, Cai, Judd, and Lenton, 2015; Cai, Judd, Lenton, Lontzek, and Narita, 2015) that will severely impact the world economy (Dietz, 2011).

We therefore extend our model to the case where green investors internalize uncertainty about climate-related financial risks. As climate-related financial risks are not Gaussian and occur in jerks and turns, we describe the arrival of these risks by a Poisson process. On the same filtered probability space, $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$, we define a time-homogeneous Poisson process $\mathcal{N} := (\mathcal{N}_t)_{t \in [0, T]}$ (counter of the shocks), whose intensity is denoted by λ . To ensure comparability with the deterministic case, we make the average effect of jumps in climate externalities equal to the average effect of deterministic linear externalities over a given period: we denote by $\mathcal{N}_t^\lambda := \lambda^{-1} \mathcal{N}_t$ the normalized version of \mathcal{N} , so that $\mathbb{E}[\mathcal{N}_t^\lambda] = t$ for any λ . Therefore, the only factor governing uncertainty is the intensity parameter, λ , which modulates the strength of climate uncertainty. When λ is small, climate uncertainty is strong: the shocks are rare but large; when λ is large, climate uncertainty is low: the shocks are frequent but small.

4.2 Investors’ and companies’ beliefs

As before, we assume that regular investors do not internalize the financial impact of climate externalities. Therefore, according to regular investors beliefs, the vectors of terminal dividends, D_T , and dividend forecast, $\mathbb{E}_t^g(D_T)$, are still given by Equations (1) and (3), respectively. However, in contrast to Sections 2 and 3, green investors internalize climate externalities by taking into account their uncertainty. Although the expression of the expected terminal dividends, $\mathbb{E}_t^g(D_T)$, given in Equation (6) continues to hold, it offers an average picture of green investors’ beliefs and, therefore, does not allow us to specify the dynamics of jumps introduced in this section. To account properly for the jump process, we need to slightly modify the definition of the terminal dividends as perceived by the green investors. Specifically, green investors assume that the vector of terminal

dividends contains an additional factor that depends on the climate externalities of the companies, $\theta^s(\psi^s)$, and the Poisson process, \mathcal{N}^λ :

$$D_T = \int_0^T c_t(\psi_t - \psi_b)dt + d + \int_0^T \sigma_t dB_t + \int_0^T \theta_s(\psi_s) d\mathcal{N}_s^\lambda, \quad (17)$$

where now B is a standard Brownian motion under all probability measures that we consider (that is, under the reference measure $\mathbb{P} = \mathbb{P}^r$ and under the measures of the green investors \mathbb{P}^g and of the companies \mathbb{P}^c).

This framework generalizes the first model where climate externalities were deterministic. Indeed, when the intensity, λ , increases, the number of shocks increases and their size decreases; in the limiting case where λ tends to $+\infty$, the uncertainty disappears, and we recover the setting of Section 2:

$$\lim_{\lambda \rightarrow \infty} \int_0^T \theta_s(\psi_s) d\mathcal{N}_s^\lambda = \int_0^T \theta_s(\psi_s) ds.$$

Like in the case where the externalities were deterministic, the variable D_t is constructed from the actual realization of the past cash flow news between 0 and t , which is known at time t . However, from a probabilistic point of view, the dynamic of the stochastic process D_t under green investors' beliefs (probability measure \mathbb{P}^g) includes a jump process corresponding to the internalization of the uncertainty about future climate externalities:

$$D_t = \int_0^t c_t(\psi_t - \psi_b)dt + d + \int_0^t \sigma_t dB_t + \int_0^t \theta_s(\psi_s) d\mathcal{N}_s^\lambda.$$

Similarly, under companies' beliefs (probability measure \mathbb{P}^c), we define the dynamics of the terminal dividend using $\theta_s^c(\psi_s)$ as

$$D_T = \int_0^T c_t(\psi_t - \psi_b)dt + d + \int_0^T \sigma_t dB_t + \int_0^T \theta_s^c(\psi_s) d\mathcal{N}_s^\lambda,$$

and the dynamic of the stochastic process D_t as

$$D_t = \int_0^t c_t(\psi_t - \psi_b)dt + d + \int_0^t \sigma_t dB_t + \int_0^t \theta_s^c(\psi_s) d\mathcal{N}_s^\lambda.$$

4.3 Equilibrium stock prices and returns

The optimization framework and notation remain similar to those of the first model. The *equilibrium price process* is denoted by $(p_t)_{t \in [0, T]}$, and it is assumed that $p_T = D_T$. In equilibrium, investors choose their allocation to maximize the expected exponential utility of their terminal wealth (Equations (7)), and equilibrium prices are determined by the market clearing condition. Proposition 5 gives the equilibrium price and allocations.

Proposition 5. *Given an emission schedule $(\psi_t)_{t \in [0, T]}$, the asset price in equilibrium reads*

$$p_t = D_t - \int_t^T \mu_s ds \quad \text{with} \quad \mu_t = \gamma^* \Sigma_t \mathbf{1} - \alpha y_\lambda \theta_t(\psi_t), \quad (18)$$

and the optimal number of shares for the regular and green investors are

$$N_t^{r, \lambda} = (1 - \alpha) \left(\mathbf{1} - \frac{y_\lambda}{\gamma^g} \Sigma_t^{-1} \theta_t(\psi_t) \right) \quad \text{and} \quad N_t^{g, \lambda} = \alpha \left(\mathbf{1} + \frac{y_\lambda}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right), \quad (19)$$

respectively, where $y_\lambda > 0$ is obtained by solving the one-dimensional equation

$$y_\lambda = \exp \left\{ -\frac{\alpha\gamma^g}{\lambda} \left(\mathbf{1}^\top \theta_t(\psi_t) + \frac{y_\lambda}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t) \right) \right\}. \quad (20)$$

When green investors internalize uncertainty about climate externalities, an additional factor, y_λ , arises in the equilibrium allocations, $N_t^{r,\lambda}$ and $N_t^{g,\lambda}$, the equilibrium price, p_t , and in particular the expected return, $\mu_t dt$. Notice that y_λ is determined at each time t by Equation (20). Thus, it should be denoted by $y_\lambda(t)$. We choose the simpler notation y_λ as no confusion shall arise. The equation for y_λ admits a unique solution.²³ Proposition 6 explains the effect of y_λ on the optimal allocations and expected returns depending on the properties of green investors' optimal portfolio *without* uncertainty, N_t^g .

Proposition 6. *Fix an emission schedule ψ_t . Recall that $N_t^g = \alpha \left(\mathbf{1} + \frac{1}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right)$ is the optimal portfolio of green investors with deterministic climate externalities, and $N_t^{g,\lambda} = \alpha \left(\mathbf{1} + \frac{y_\lambda}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right)$ is the optimal portfolio of green investors when the uncertainty level of climate externalities equals λ . Then,*

- (i) *When the level of climate uncertainty increases, green investors decrease the market risk of their portfolio, i.e., $\lambda \mapsto (N_t^{g,\lambda})^\top \Sigma_t N_t^{g,\lambda}$ is increasing in λ .*
- (ii) *If the green investors' portfolio in the absence of climate uncertainty has positive climate externalities, meaning that $\theta_t(\psi_t)^\top N_t^g > 0$, then, the function $\lambda \mapsto y_\lambda$ is positive, monotonically increasing and satisfies $\lim_{\lambda \rightarrow \infty} y_\lambda = 1$. Consequently, for a given level of emission schedule, ψ_t , the increase in climate uncertainty (λ decreases)

 - pushes green investors to reallocate their wealth towards brown assets;
 - decreases (increases) the expected returns, $\mu_t dt$, of the brown (green) companies.*
- (iii) *If the green investors' portfolio in the absence of climate uncertainty has negative climate externalities, meaning that $\theta_t(\psi_t)^\top N_t^g < 0$, then, the function $\lambda \mapsto y_\lambda$ is positive, monotonically decreasing and satisfies $\lim_{\lambda \rightarrow \infty} y_\lambda = 1$. Consequently, for a given level of emission schedule, ψ_t , the increase in climate uncertainty (λ decreases)

 - pushes green investors to reallocate their wealth towards green assets;
 - increases (decreases) the expected returns, $\mu_t dt$, of the brown (green) companies.*
- (iv) *If the green investors' portfolio in the absence of climate uncertainty has neutral climate externalities, meaning that $\theta_t(\psi_t)^\top N_t^g = 0$, then $y_\lambda = 1$ for all $\lambda > 0$. Consequently, green investors' portfolio and companies' expected returns do not depend on the level of climate uncertainty.*

Uncertainty about future climate externalities is an additional source of risk for green investors who, as a result, reduce the overall risk of their portfolio. However, the adjustment of their asset allocation depends on the climate externalities of their optimal portfolio without uncertainty, $\theta_t(\psi_t)^\top N_t^g$. Lemma 1 further elaborates on the case in which the green investors' optimal portfolio without climate uncertainty is green, that is, $\theta_t(\psi_t)^\top N_t^g > 0$.

Lemma 1. *Fix an emission schedule ψ_t . Assume that the sum of the companies' climate externalities is not too negative, such that $\mathbf{1}^\top \theta_t(\psi_t) > -\frac{1}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t)$. Then, the green investors' portfolio in the absence of climate uncertainty has positive climate externalities, that is, $\theta_t(\psi_t)^\top N_t^g > 0$.*

²³Existence and uniqueness of y_λ are given as part of the proof of the next proposition.

The situation described by Lemma 1 is the most common.²⁴ It requires that the sum of the climate externalities of the companies in the market is greater than $-\frac{1}{\gamma^r}\theta_t(\psi_t)^\top \Sigma_t^{-1}\theta_t(\psi_t)$, which is always negative. Typically, this occurs when the market is not excessively composed of brown assets, for example, when it is sufficiently diversified in terms of companies' climate externalities, that is, when the market is formed by companies with positive and negative $\theta_t^i(\psi_t)$. Moreover, the condition is more easily verified for large markets because the quadratic correction, $\frac{1}{\gamma^r}\theta_t(\psi_t)^\top \Sigma_t^{-1}\theta_t(\psi_t)$, becomes larger. Pastor et al. (2021b) assume that the ESG externalities of the market portfolio are zero—here $\mathbf{1}^\top \theta_t(\psi_t) = 0$ —which is a sufficient condition for Lemma 1 to be valid. Furthermore, Zerbib (2021) validates the Pastor et al. (2021b) hypothesis by estimating the climate externalities of the market portfolio in the U.S. between 2007 and 2019 at a value close to zero.

In the main case described by Lemma 1 where the market is not excessively brown, green investors are able to build a green portfolio in the absence of uncertainty. Therefore, $y_\lambda \in [0, 1[$, which means that uncertainty about future climate externalities pushes green investors to lower the risk of their portfolio by reducing their exposure towards green assets. The size of this effect scales with the degree of uncertainty: the larger the uncertainty, the more green investors decrease their allocation in green assets and increase their allocation in brown assets. Consequently, for a given level of emission schedule, ψ_t , climate uncertainty reduces the effect of the externality premium, $-\alpha y_\lambda \theta_t(\psi_t)$, on expected returns, which lowers the cost of capital of brown companies and increases the cost of capital of green companies. This result is consistent with that of Avramov et al. (2021a) who show that gaussian uncertainty weakens the negative relationship between ESG and financial performances.

In less common cases, green investors fail to construct a green optimal portfolio in the absence of uncertainty. This situation is possible, for example, when all or much of the economy is brown. Therefore, $y_\lambda > 1$, which indicates that the increase in climate uncertainty pushes green investors to invest more in green assets and divest from brown assets to diversify their allocation and mitigate their risk. Consequently, for a given level of emission schedule, ψ_t , climate uncertainty amplifies the effect of the externality premium, $-\alpha y_\lambda \theta_t(\psi_t)$, on expected returns, which reduces the cost of capital of green companies and increases that of brown companies.

The following section presents the companies' optimal emission schedules when they account for the climate uncertainty internalized by green investors.

4.4 Equilibrium emission schedule

Companies fix their emission schedules at the initial date by maximizing their future market values as in the deterministic case (Equation (8)). The situation here is more complex, however, because of the appearance of the new parameter y_λ in the price vector (see Proposition 5). Indeed, for a fixed value of y_λ , the optimal emissions schedule of the i -th company is the one that maximizes for all $t \in [0, T]$ the expression:

$$\beta_t^c \theta_t^{c,i}(\psi_t) + \alpha \beta_t y_\lambda \theta_t^i(\psi_t) + c_t^i \psi_t^i, \quad (21)$$

by the same arguments as the ones used in Proposition 2 for deterministic externalities. However, this shows that the equilibrium emissions schedule depends on the choice of y_λ , which will affect the investors' portfolio allocations and asset prices in Proposition 5, creating a feedback effect. In order to maintain tractability in our model we make an assumption of a large market that allows us to partially decouple the optimization problems for investors and companies. Then we derive

²⁴The proof of the lemma is an immediate consequence of the expression for N_t^g (see Equation (11)).

equilibrium emission schedules in the next proposition.²⁵

Proposition 7. *Let the functions of climate externalities, θ_t^i and $\theta_t^{c,i}$, be defined as in Corollary 3. Assuming that the market is large enough so that the aggregated climate externalities of the market are not strongly affected by a change in the climate externalities of each company, the optimal emission schedule for the i -th company is*

$$\psi_t^{i,*}(y_\lambda^*) = \frac{c_t^i}{\alpha\beta_t\kappa_t y_\lambda^* + \beta_t^c \kappa_t^c}, \quad (22)$$

where y_λ^* is a solution of the fixed-point equation

$$y = \exp \left\{ -\frac{\alpha\gamma^g}{\lambda} \left[\mathbf{1}^\top \theta_t(\psi_t(y)) + \frac{\alpha y}{\gamma^r} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)) \right] \right\}. \quad (23)$$

As climate uncertainty adds a multiplicative factor y_λ to the externality premium, it also adds a multiplicative factor y_λ^* to the company's optimization program (Equation (12)) on the factor driven by green investors' beliefs, $\alpha\beta_t y_\lambda^* \theta_t^i(\psi_t)$. As a result, the optimal emission schedule in the presence of uncertainty (Equation (22)) is adjusted by y_λ^* in its denominator. Proposition 8 clarifies the behavior of y_λ^* depending on λ .

Proposition 8. *Under the same assumptions as in Proposition 7, the following holds true:*

- (i) *Existence: For all $\lambda \in (0, \infty)$, Equation (23) admits at least one positive solution.*
- (ii) *Uniqueness: If, for all $i \in \{1, \dots, n\}$, and all $t \in [0, T]$,*

$$\inf_{y>0} \sum_{j=1}^n (\Sigma_t^{-1})_{ij} y \theta_t^j(\psi_t^j(y)) \geq -\frac{\gamma^r}{\alpha} \quad (24)$$

the solution of Equation (23) is unique. In particular, in the asymptotic regime where uncertainty is low ($\lambda \rightarrow \infty$), the solution is unique.

- (iii) *Assume that the condition (ii) is satisfied and denote by y_λ^* the unique solution of Equation (23). Let $\psi_t^{*,0,i} = c_t^i(\beta_t^c \kappa_t^c + \alpha\beta_t \kappa_t)^{-1}$ be the equilibrium emission schedule without climate uncertainty (given by (13)), and $N_t^g = \alpha \left(1 + \frac{1}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t^{*,0}) \right)$ be the green investors' corresponding optimal allocation without climate uncertainty (given by (11)).*

- *If the green investors' portfolio in the absence of climate uncertainty has positive climate externalities, meaning that $\theta_t(\psi_t^{*,0})^\top N_t^g > 0$, then, the function $\lambda \mapsto y_\lambda^*$ is positive, monotonically increasing and satisfies $\lim_{\lambda \rightarrow \infty} y_\lambda^* = 1$. Consequently, an increase in climate uncertainty (λ decreases) increases the companies' optimal emission schedule, ψ_t^* , given by (22).*
- *If the green investors' portfolio in the absence of climate uncertainty has negative climate externalities, meaning that $\theta_t(\psi_t^{*,0})^\top N_t^g < 0$, then, the function $\lambda \mapsto y_\lambda^*$ is positive, monotonically decreasing and satisfies $\lim_{\lambda \rightarrow \infty} y_\lambda^* = 1$. Consequently, an increase in climate uncertainty (λ decreases) decreases the companies' optimal emission schedule, ψ_t^* , given by (22).*

²⁵If the market is large enough, we can assume that the aggregate quantities are not strongly affected by each individual company, and therefore, the i -th company may maximize its optimization functional by assuming that y_λ is constant. This assumption is standard, for example, in mean-field game optimization problems. At the mathematical level, our assumption says that each company solves (21) for a fixed value of y_λ , thus obtaining $\psi_t^i(y_\lambda)$. Plugging this expression back into (20), we arrive at a fixed point equation for y_λ . This equation is still one-dimensional but different from Equation (20) due to the additional dependence of ψ_t on y_λ .

- If the green investors’ portfolio in the absence of climate uncertainty has neutral climate externalities, meaning that $\theta_t(\psi_t^{*,0})^\top N_t^g = 0$, then, the function $\lambda \mapsto y_\lambda^*$ is constant and equal to 1. Consequently, climate uncertainty does not affect the companies’ optimal emission schedule, ψ_t^* , given by (22).

In the most common case described by Lemma 1 where green investors have a green optimal portfolio in the absence of climate uncertainty, $\theta_t(\psi_t^{*,0})^\top N_t^g > 0$, increasing uncertainty (λ decreases) decreases y_λ below 1. This effect dampens the contribution of green investors’ beliefs, $\alpha\beta_t y_\lambda \theta_t^i(\psi_t)$, in the companies’ optimization program (Equation (12)). Indeed, by mitigating the externality premium on expected returns (Proposition 6), green investors reduce the incentive for companies to decrease their emissions at unchanged abatement cost, $c_t^i \psi_t^i$. Therefore, all companies increase their optimal emission schedules in the presence of climate uncertainty compared to the situation without uncertainty.

In the case where green investors have a brown optimal portfolio in the absence of climate uncertainty, $\theta_t(\psi_t^{*,0})^\top N_t^g < 0$, increasing climate uncertainty (λ decreases) increases y_λ above 1 and thus amplifies the contribution of green investors’ beliefs, $\alpha\beta_t y_\lambda \theta_t^i(\psi_t)$, in the companies’ optimization program. By strengthening the incentive for companies to reduce their emissions, green investors push companies to reduce their emissions compared to the situation without uncertainty.

Figure 5 shows optimal emission schedules according to different levels of uncertainty: the cases where information on climate risks (or the materialization of climate risks) becomes available on average annually, every five years, every ten years, and every twenty years are displayed. When green investors internalize these levels of uncertainty, companies increase their emissions by 0.4%, 2%, 4%, and 8.5%, respectively, over a 20-year horizon compared to the case where climate risks are perfectly known.

This result underscores the value of increasing the transparency of companies’ climate impacts as well as improving the forecasting of climate-related financial risks. It also emphasizes the importance of predictability of public policies in favor of climate transition, notably, the carbon price upward trajectory. Transparency and predictability are key pillars for a better integration of climate-related financial risks in green investors’ asset allocation, which provides incentives for companies to better internalize their climate externalities and thus reduce their climate footprints more rapidly.

5 Conclusion

In this paper we show how green investing impacts companies’ practices by increasing their cost of capital. Companies are pushed to internalize their climate externalities and thereby reduce their greenhouse gas emissions. Green investors’ impact is further strengthened when they anticipate tighter climate regulations, technological advances, and when they account for the negative financial impact of the economy’s average emissions. However, uncertainty about climate risks pushes green investors to diversify their asset allocation, thereby reducing the incentive for companies to mitigate their climate footprints.

The results of this paper suggest that investors can increase their impact on companies by raising their environmental requirements as well as by pressing companies to increase transparency and their environmental standards. In addition, impact investing is financially beneficial if investors favor companies that are on a pathway towards reducing their climate footprints or green companies for which information on their climate footprints is still poorly available. From the viewpoint of public authorities, this study emphasizes the importance of developing a regulatory framework that supports the development of green investing and encourages the transparency of information on

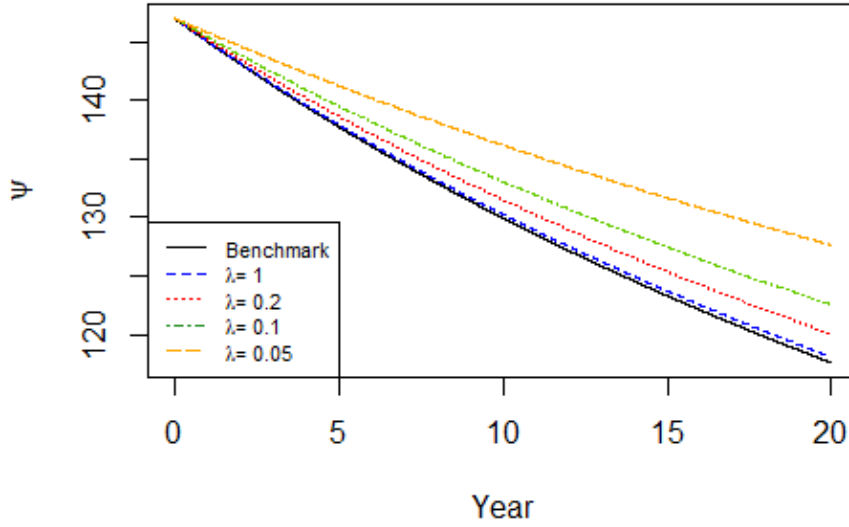


Figure 5. Emission schedule with uncertainty. This figure shows the optimal emission schedules with uncertainty of an electrical equipment company with an initial carbon intensity of $\psi_b^{\text{elec}} = 147$ tCO₂e/USDmn. This company operates in a market with two companies: the second company is a coal company with an initial carbon intensity of $\psi_b^{\text{coal}} = 555$ tCO₂e/USDmn. The correlation between the assets of these two companies is 50%. We consider different levels of uncertainty through λ . The parameters are calibrated according to the values estimated in Appendix B: $\alpha = 0.25$, $\rho = 0.01$, $\kappa = 3 \times 10^{-7}$, $\kappa_0 = 0.047$, $\kappa^c = 6 \times 10^{-8}$, $c^{\text{elec}} = 8 \times 10^{-6}$, $\gamma_r = 0.1$.

companies' climate footprints. These actions are naturally compatible with the strengthening of climate regulation and support for climate-related technological innovation, which, when anticipated by green investors, enhance the pressure the latter exert on companies to cut their emissions.

A natural avenue for future research would be to estimate the impact of green investing based on the equilibrium equations of this model. However, we acknowledge that rigorous identification is challenging for at least two major reasons. First, the proportion of green investors, their sensitivities to climate risks, and their perception of climate uncertainty must be approximated. Using green fund holdings is an ambitious approach because it is necessary to identify funds that truly have green practices (Raghunandan and Rajgopal, 2021; Yang, 2021) and to use a dynamic fund list that is not subject to survivor bias. Second, it is crucial to control for the other effects that may impact companies' practices, such as shareholder engagement, global environmental policy stringency, and climate-related technological innovation.

Besides, impact investing may go beyond climate screening, for example, by favoring brown companies that are inclined to green up quickly or small green companies that would benefit from financial support to grow (Green and Roth, 2020; Heeb and Kölbel, 2020). Therefore, future research could also analyze the impact of these new forms of investment on corporate practices, including their ability to further reduce the aggregate emissions of an economy. In addition, the impact of climate screening could be empirically compared to that of shareholder engagement, which Broccardo et al. (2020) find more effective in reducing the environmental footprint of companies. Finally, a line of theoretical research could introduce the ability for companies to reform dynamically in response to the stochastic dynamics of cash flow news and investors' portfolio allocations.

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Appendix A: Proofs

In this appendix we collect proofs and some supporting mathematical materials, needed to justify rigorously our claims.

Proof of Proposition 1

Since the market is assumed to be free of arbitrage and complete, there exists a unique *state price density* ξ_T , i.e., a positive \mathcal{F}_T -measurable integrable random variable such that the market price at time t of every contingent claim with terminal value X_T , satisfying $\mathbb{E}[\xi_T|X_T] < \infty$, is given by

$$\xi_t^{-1} \mathbb{E}[\xi_T X_T | \mathcal{F}_t], \quad (25)$$

where $\xi_t := \mathbb{E}[\xi_T | \mathcal{F}_t] = \mathbb{E}_t[\xi_T]$. In particular, since the interest rate is zero, $\mathbb{E}[\xi_T] = 1$. It is worth recalling that $\mathbb{P} = \mathbb{P}^r$ and that $(B_t)_{t \in [0, T]}$ is a Brownian motion under this measure.

The optimization problems of the two investors read:

$$\min_{W_T^r \in \mathcal{A}_T} \mathbb{E}^r [e^{-\gamma^r W_T^r}], \quad \min_{W_T^g \in \mathcal{A}_T} \mathbb{E}^g [Z_T e^{-\gamma^g W_T^g}], \quad (26)$$

subject to the budget constraints

$$\mathbb{E}[\xi_T W_T^r] = w^r, \quad \mathbb{E}[\xi_T W_T^g] = w^g, \quad (27)$$

where $w^r > 0$ and $w^g > 0$ are the initial wealth of the regular and green investor, respectively. Both investors use the real-world probability measure for pricing but every investor uses her subjective measure for computing the utility function. Here we consider admissible controls from the class

$$\mathcal{A}_T := \{X \in \mathcal{F}_T : \mathbb{E}^r[\xi_T | X] < \infty\}$$

and denote by Z_T the Radon-Nikodym density that connects the two probability measures \mathbb{P}^g and \mathbb{P}^r . More precisely, recalling (3) and (5), we have

$$Z_T = e^{\int_0^T \lambda_s^\top dB_s - \frac{1}{2} \int_0^T \|\lambda_s\|^2 ds}, \quad (28)$$

where we set $\lambda_t := \sigma_t^{-1} \theta(\psi_t)$, to simplify the notation, and $\|\cdot\|$ is the Euclidean norm in \mathbb{R}^n .

The optimization problem is over the set of all admissible contingent claims, but we shall see later that the optimal claims will be attainable. Moreover, we assume that

$$\mathbb{E}^r[\xi_T \log \xi_T] < \infty \quad \text{and} \quad \mathbb{E}^r[\xi_T \log Z_T]. \quad (29)$$

This assumption will be checked a posteriori for the equilibrium state price density.

By the standard Lagrange multiplier argument, the solutions to problems (26)-(27) are given by

$$W_T^r = w^r - \frac{1}{\gamma^r} \log \xi_T + \frac{1}{\gamma^r} \mathbb{E}^r[\xi_T \log \xi_T], \quad W_T^g = w^g - \frac{1}{\gamma^g} \log \frac{\xi_T}{Z_T} + \frac{1}{\gamma^g} \mathbb{E}^r \left[\xi_T \log \frac{\xi_T}{Z_T} \right]. \quad (30)$$

The equilibrium state price density ξ_T is found from the market clearing condition

$$W_T^r + W_T^g = \mathbf{1}^\top D_T + K,$$

where K is a constant that allows the market to clear since the bond supply is endogenous. Recall that the interest rate and the initial wealth are exogenous.

Substituting the formulas for W_T^r and W_T^g , yields

$$\xi_T = c \exp \left(-\gamma^* \mathbf{1}^\top D_T + \frac{\gamma^*}{\gamma^g} \log Z_T \right)$$

for some constant c , where we recall $\frac{1}{\gamma^*} = \frac{1}{\gamma^r} + \frac{1}{\gamma^g}$. Note that since D_T and $\log Z_T$ are Gaussian, our a priori assumptions (29) are satisfied.

We can now use the fact that $\mathbb{E}^r[\xi_T] = 1$ to conclude that:

$$\xi_T = \frac{\exp \left(-\gamma^* \mathbf{1}^\top D_T + \frac{\gamma^*}{\gamma^g} \log Z_T \right)}{\mathbb{E}^r \left[\exp \left(-\gamma^* \mathbf{1}^\top D_T + \frac{\gamma^*}{\gamma^g} \log Z_T \right) \right]}.$$

Substituting the explicit formulae for D_T and Z_T (see (1) and (28)) and using that

$$\int_0^T \left(-\gamma^* \mathbf{1}^\top \sigma_t + \frac{\gamma^*}{\gamma^g} \lambda_t^\top \right) dB_t$$

is normally distributed with zero mean and variance

$$\int_0^T \left\| -\gamma^* \mathbf{1}^\top \sigma_t + \frac{\gamma^*}{\gamma^g} \lambda_t^\top \right\|^2 dt,$$

because $(\sigma_t)_{t \in [0, T]}$ and $(\lambda_t)_{t \in [0, T]}$ are deterministic, we have:

$$\xi_T = \mathcal{E} \left(\int_0^T \left\{ -\gamma^* \mathbf{1}^\top \sigma_t + \frac{\gamma^*}{\gamma^g} \lambda_t^\top \right\} dB_t \right). \quad (31)$$

Here \mathcal{E} denotes the stochastic exponential, i.e., for any adapted square integrable process $X \in \mathbb{R}^n$,

$$\mathcal{E} \left(\int_0^t X_s dB_s \right) = \exp \left(\int_0^t X_s dB_s - \frac{1}{2} \int_0^t \|X_s\|^2 ds \right).$$

From (31) and (28) we can easily verify that (29) holds, since (σ_t) and (λ_t) are deterministic.

Using the no-arbitrage pricing rule (25), the vector of equilibrium prices is then given by

$$p_t = \xi_t^{-1} \mathbb{E}_t^r [\xi_T D_T] = D_0 + \int_0^t \sigma_s dB_s + \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^T \sigma_s dB_s \right],$$

where \mathbb{Q} is the risk-neutral measure defined by

$$\frac{d\mathbb{Q}}{d\mathbb{P}^r} \Big|_{\mathcal{F}_T} = \xi_T.$$

Under \mathbb{Q} , the process

$$\tilde{B}_t = B_t - \int_0^t \left\{ -\gamma^* \sigma_s^\top \mathbf{1} + \frac{\gamma^*}{\gamma^g} \lambda_s \right\} ds$$

is a standard Brownian motion. Hence, the equilibrium prices are computed as follows.

$$\begin{aligned} p_t &= \xi_t^{-1} \mathbb{E}_t^r [\xi_T D_T] \\ &= D_0 + \int_0^t \sigma_s dB_s + \int_t^T \sigma_s \left\{ -\gamma^* \sigma_s^\top \mathbf{1} + \frac{\gamma^*}{\gamma^g} \lambda_s \right\} ds \\ &= D_t + \int_t^T \{ -\gamma^* \Sigma_s \mathbf{1} + \alpha \theta_s(\psi_s) \} ds, \end{aligned} \quad (32)$$

with

$$D_t = D_0 + \int_0^t \sigma_s dB_s, \quad \Sigma_t = \sigma_t \sigma_t^\top, \quad \theta_t(\psi_t) = \sigma_t \lambda_t, \quad \text{and} \quad \alpha = \frac{\gamma^r}{\gamma^r + \gamma^g}.$$

This completes the proof of (10).

Next we determine the number of shares that each investor holds in her portfolio. The values of the investors' portfolios are determined through the no-arbitrage pricing rule (25). In particular, we have

$$\begin{aligned} W_t^r &= \xi_t^{-1} \mathbb{E}_t^r [\xi_T W_T^r] \\ &= w^r - \frac{1}{\gamma^r} \mathbb{E}_t^r \left[\frac{\xi_T}{\xi_t} \left(\log \frac{\xi_T}{\xi_t} + \log \xi_t \right) \right] + \frac{1}{\gamma^r} \mathbb{E}^r \left[\xi_t \left(\frac{\xi_T}{\xi_t} \log \frac{\xi_T}{\xi_t} \right) + \left(\frac{\xi_T}{\xi_t} \right) \xi_t \log \xi_t \right], \end{aligned}$$

by simple algebraic manipulations. Then, using that ξ_T/ξ_t is independent of \mathcal{F}_t (hence of ξ_t) and that $\mathbb{E}^r[\xi_t] = \mathbb{E}^r[\xi_T] = \mathbb{E}_t^r[\xi_T/\xi_t] = 1$ we obtain the wealth at time t of the regular investor

$$W_t^r = w^r - \frac{1}{\gamma^r} \log \xi_t + \frac{1}{\gamma^r} \mathbb{E}^r [\xi_t \log \xi_t]. \quad (33)$$

By construction $W_t^r = \mathbb{E}^{\mathbb{Q}}[W_T^r | \mathcal{F}_t]$, hence it is a \mathbb{Q} -martingale. Moreover, by (33) we see that the only stochastic term in the dynamics of (W_t^r) is $-1/\gamma^r \log \xi_t$. Then, using

$$\xi_t = \mathcal{E} \left(\int_0^t \left\{ -\gamma^* \mathbf{1}^\top \sigma_s + \frac{\gamma^*}{\gamma^g} \lambda_s^\top \right\} dB_s \right)_t,$$

we can conclude that, under the measure \mathbb{Q} , the process (W_t^r) has martingale dynamics

$$W_t^r = w^r + (1 - \alpha) \int_0^t \left\{ \mathbf{1}^\top \sigma_s - \frac{1}{\gamma^g} \lambda_s^\top \right\} d\tilde{B}_s.$$

The price derived in (32), on the other hand, has martingale dynamics under the measure \mathbb{Q} given by

$$p_t = p_0 + \int_0^t \sigma_s d\tilde{B}_s,$$

where

$$p_0 = D_0 + \int_0^T (-\gamma^* \Sigma_s \mathbf{1} + \alpha \theta_s(\psi_s)) ds.$$

It follows that the optimal claim for the investor is replicable by a self-financing portfolio whose value can be written as follows:

$$\begin{aligned} W_t^r &= w^r + (1 - \alpha) \int_0^t \left\{ \mathbf{1}^\top \sigma_s - \frac{1}{\gamma^g} \lambda_s^\top \right\} \sigma_s^{-1} dp_s \\ &= w^r + (1 - \alpha) \int_0^t \left\{ \mathbf{1}^\top - \frac{1}{\gamma^g} \theta_s(\psi_s)^\top \Sigma_s^{-1} \right\} dp_s. \end{aligned}$$

We conclude that the vector of quantities of shares held by the regular investor at time t is given by

$$N_t^r = (1 - \alpha) \left\{ \mathbf{1} - \frac{1}{\gamma^g} \Sigma_t^{-1} \theta_t(\psi_t) \right\},$$

while that of the green investor is given by

$$N_t^g = \alpha \left\{ \mathbf{1} + \frac{1}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right\}.$$

The latter can be obtained by the former and the market clearing condition. Alternatively, the risk-neutral pricing principle and calculations analogous to the ones above allow us to deduce that

$$W_t^g = \xi_t^{-1} \mathbb{E}_t^r[\xi_T W_T^g] = w^g + \alpha \int_0^t \left\{ \mathbf{1}^\top + \frac{1}{\gamma^r} \theta_t(\psi_s)^\top \Sigma_s^{-1} \right\} dp_s$$

from the formula in (30). Hence, the expression of N_t^g follows.

Proof of Proposition 2

Recalling (6), the measure \mathbb{P}^c has density with respect to the measure \mathbb{P}^r given by

$$Z_T^c = e^{\int_0^T (\lambda_s^c)^\top dW_s - \frac{1}{2} \int_0^T \|\lambda_s^c\|^2 ds},$$

where $\lambda_t^c := \sigma_t^{-1} \theta^c(\psi_t)$.

Using (32) and Girsanov theorem, the vector of expected equilibrium prices under the measure \mathbb{P}^c reads

$$\mathbb{E}^c(p_t) = d + \int_0^T c_t(\psi_t - \psi_b)dt + \int_0^t \theta_t^c(\psi_s)ds + \alpha \int_t^T \theta_t(\psi_s) - \gamma^* \int_t^T \Sigma_s \mathbf{1} ds.$$

Then, the profit function of the i -th company reads

$$\mathcal{J}^i(\psi^i, \psi^{-i}) = \int_0^T e^{-\rho t} \left(d + \int_0^T c_s^i(\psi_s^i - \psi_b^i)ds + \int_0^t \theta_s^{c,i}(\psi_s)ds + \alpha \int_t^T \theta_s^i(\psi_s)ds - \gamma^* \int_t^T [\Sigma_s \mathbf{1}]_i ds \right) dt,$$

where $[\Sigma_s \mathbf{1}]_i$ is the i -th coordinate of the vector $\Sigma_s \mathbf{1}$.

Maximizing $\mathcal{J}^i(\psi^i, \psi^{-i})$ over ψ^i is equivalent to maximizing

$$\tilde{\mathcal{J}}^i(\psi^i, \psi^{-i}) = \int_0^T e^{-\rho t} \left(\int_0^T c_s^i \psi_s^i ds + \int_0^t \theta_s^{c,i}(\psi_s)ds + \alpha \int_t^T \theta_s^i(\psi_s)ds \right) dt.$$

Applying integration by parts to the integral with respect to ‘ dt ’ we have

$$\tilde{\mathcal{J}}^i(\psi^i, \psi^{-i}) = \int_0^T \left(\frac{e^{-\rho t} - e^{-\rho T}}{\rho} \theta_t^{c,i}(\psi_t) + \alpha \frac{1 - e^{-\rho t}}{\rho} \theta_t^i(\psi_t) + c_t^i \frac{1 - e^{-\rho T}}{\rho} \psi_t^i \right) dt. \quad (34)$$

The problem reduces to maximizing the integrand above along the entire trajectory of $(\psi_t^i)_{t \in [0, T]}$. That is

$$\max_{\psi_t^i} \left(\frac{e^{-\rho t} - e^{-\rho T}}{\rho} \theta_t^{c,i}(\psi_t) + \alpha \frac{1 - e^{-\rho t}}{\rho} \theta_t^i(\psi_t) + c_t^i \frac{1 - e^{-\rho T}}{\rho} \psi_t^i \right),$$

and the claim follows (see (12)).

Proof of Proposition 4

Recall the optimization problem (12). Let us denote by F_i the function that the i -th company needs to maximize:

$$F_i(\psi_t^i, \psi_t^{-i}) := \beta^c(t) \theta_t^{c,i}(\psi_t) + \alpha \beta(t) \theta_t^i(\psi_t) + c_t^i \psi_t^i$$

Since $\psi \mapsto F_i(\psi, \varphi)$ is concave for each φ , it is enough to impose first order conditions:

$$\beta^c(t) \partial_{\psi^i} \theta_t^{c,i}(\psi_t) + \alpha \beta(t) \partial_{\psi^i} \theta_t^i(\psi_t) + c_t^i = 0$$

for all $i = 1, 2, \dots, n$. This leads to

$$\psi_t^{*,\varepsilon,i} = \psi_t^{*,i} - \frac{\varepsilon}{2n} \left(\sum_{j=1}^n \psi_t^{*,\varepsilon,j} + \psi_t^{*,\varepsilon,i} \right), \quad (35)$$

where $\psi_t^{*,i} := c_t^i [\beta^c(t) \kappa^c + \alpha \beta(t) \kappa]^{-1}$ is the solution for $\varepsilon = 0$ (i.e., without interaction).

Taking sums over $i = 1, 2, \dots, n$ on both sides of the equation, we have

$$\sum_{i=1}^n \psi_t^{*,\varepsilon,i} = \sum_{i=1}^n \psi_t^{*,i} - \frac{\varepsilon}{2} \sum_{i=1}^n \psi_t^{*,\varepsilon,i} - \frac{\varepsilon}{2n} \sum_{i=1}^n \psi_t^{*,\varepsilon,i},$$

which gives,

$$\sum_{i=1}^n \psi_t^{*,\varepsilon,i} = \frac{2n}{2n + (n+1)\varepsilon} \sum_{i=1}^n \psi_t^{*,i}.$$

Substituting $\sum_{i=1}^n \psi_t^{*,\varepsilon,i}$ back into Equation (35), we get

$$\psi_t^{*,\varepsilon,i} = \psi_t^{*,i} - \frac{\varepsilon}{2n + \varepsilon} \left(\psi_t^{*,i} + \frac{2n^2}{2n + (n+1)\varepsilon} \bar{\psi}_t^* \right),$$

where $\bar{\psi}_t^* = \frac{1}{n} \sum_{j=1}^n \psi_t^{*,j}$. The term in brackets in the expression above is positive, hence proving (i).

In addition, it is clear that

$$\frac{\varepsilon}{2n + \varepsilon} \left(\psi_t^{*,i} + \frac{2n^2}{2n + (n+1)\varepsilon} \bar{\psi}_t^* \right) \underset{n \rightarrow +\infty}{\sim} \frac{\varepsilon}{\varepsilon + 2} \bar{\psi}_t^*, \quad (36)$$

which proves (ii).

Proof of Proposition 5 and 6

The standard approach to the problem, via dynamic programming, requires us to introduce the value processes for the two agents:

$$V_t^r = \min_{N \in \mathcal{A}_{t,T}^r} \mathbb{E}_t^r [\exp(-\gamma^r W_T^r)], \quad V_t^g = \min_{N \in \mathcal{A}_{t,T}^g} \mathbb{E}_t^g [\exp(-\gamma^g W_T^g)],$$

where, for $t \leq T$ and $j \in \{r, g\}$, we define

$$\mathcal{A}_{t,T}^j := \{(N_s^\lambda)_{t \leq s \leq T} : N^\lambda \text{ is } \mathbb{R}^n\text{-valued, } (\mathcal{F}_s)_{t \leq s \leq T}\text{-adapted and } \mathbb{P}^j\text{-square integrable}\}$$

and \mathbb{P}^j -square integrable means

$$\mathbb{E}^j \left[\int_0^T |N_t^\lambda|^2 dt \right] < +\infty.$$

Moreover, we assume that the equilibrium price has the following dynamics.

$$p_t = p_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s + \int_0^t \theta_s(\psi_s) dN_s^\lambda \quad (37)$$

under the probability \mathbb{P}^g of the green investors and

$$p_t = p_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s \quad (38)$$

under the probability \mathbb{P}^r of the regular investors, where μ is deterministic and must be found in equilibrium. We shall show a posteriori that an equilibrium price process of this form can indeed be found.

Following a well-known ansatz we expect

$$V_t^r = \exp(-\gamma^r W_t^r + Q_t^r), \quad V_t^g = \exp(-\gamma^g W_t^g + Q_t^g),$$

where Q^r and Q^g are absolutely continuous deterministic processes with

$$dQ_t^r = q_t^r dt \quad \text{and} \quad dQ_t^g = q_t^g dt.$$

Applying the Itô's formula for jump processes to V^g under the green investor measure yields

$$\begin{aligned} dV_t^g &= V_{t-}^g \left(-\gamma^g dW_t^g + q_t^g dt + \frac{(\gamma^g)^2}{2} d[W^g]_t^c + (e^{-\gamma \Delta W_t^g} - 1 + \gamma \Delta W_t^g) \right) \\ &= V_{t-}^g \left(-\gamma^g (N_t^{g,\lambda})^\top dp_t + q_t^g dt + \frac{(\gamma^g)^2}{2} (N_t^{g,\lambda})^\top d[p]_t^c N_t^{g,\lambda} + (e^{-\gamma (N_{t-}^{g,\lambda})^\top \Delta p_t} - 1 + \gamma^g (N_{t-}^{g,\lambda})^\top \Delta p_t) \right) \\ &= V_{t-}^g \left(-\gamma^g (N_t^{g,\lambda})^\top \mu_t + q_t^g + \frac{(\gamma^g)^2}{2} (N_t^{g,\lambda})^\top \sigma_t \sigma_t^\top N_t^{g,\lambda} + \lambda (e^{-\frac{\gamma^g}{\lambda} (N_t^{g,\lambda})^\top \theta_t(\psi_t)} - 1) \right) dt + M_t, \end{aligned}$$

where (M_t) is a \mathbb{P}^g -martingale on $[0, T]$ and $[W^g]^c$ is the continuous part of the quadratic variation of the process W^g . Since V^g must be a martingale along the trajectory of the optimal process $(N_t^{g,\lambda})$ and a submartingale along every trajectory, we conclude that the drift term in ' dV_t^g ' must be non-negative and

$$\min_{N_t^\lambda} \left(-\gamma^g (N_t^\lambda)^\top \mu_t + q_t^g + \frac{(\gamma^g)^2}{2} (N_t^\lambda)^\top \Sigma_t N_t^\lambda + \lambda (e^{-\frac{\gamma^g}{\lambda} (N_t^\lambda)^\top \theta_t(\psi_t)} - 1) \right) = 0 \quad (39)$$

for each $t \in [0, T]$. Since Σ is nondegenerate, the function to be minimized is strictly convex and coercive (i.e., it tends to $+\infty$ as $\|N_t^\lambda\| \rightarrow \infty$), thus the unique minimum is always attained. With a slight abuse of notation, we denote the minimizer of (39) (which does not depend on q_t) by $N_t^{g,\lambda}$, as this will be the number of assets held by the green investors. By imposing first order conditions we have that $N_t^{g,\lambda}$ must be the unique solution of

$$\mu_t - \gamma^g \Sigma_t N_t^{g,\lambda} + e^{-\frac{\gamma^g}{\lambda} (N_t^{g,\lambda})^\top \theta_t(\psi_t)} \theta_t(\psi_t) = 0.$$

By the same logic, the regular investors use the measure \mathbb{P}^r to compute the dynamic ' dV_t^r ' and find the optimal quantity of assets. In particular, the optimal quantity $N_t^{r,\lambda}$ is the minimizer of

$$\min_{N_t^\lambda} \left(-\gamma^r (N_t^\lambda)^\top \mu_t + q_t^r + \frac{(\gamma^r)^2}{2} (N_t^\lambda)^\top \Sigma_t N_t^\lambda \right) = 0.$$

Since the regular investors do not take into account the climate uncertainty, there is no jump term in this formula, and the minimizer is given explicitly by

$$N_t^{r,\lambda} = \frac{1}{\gamma^r} \Sigma_t^{-1} \mu_t.$$

The market clearing condition therefore allows to compute $(\mu, N^{r,\lambda}, N^{g,\lambda})$ by solving the following system of equations:

$$\begin{aligned} N_t^{r,\lambda} &= \frac{1}{\gamma^r} \Sigma_t^{-1} \mu_t; \\ \mu_t - \gamma^g \Sigma_t N_t^{g,\lambda} + e^{-\frac{\gamma^g}{\lambda} (N_t^{g,\lambda})^\top \theta_t(\psi_t)} \theta_t(\psi_t) &= 0; \\ N_t^{r,\lambda} + N_t^{g,\lambda} &= \mathbf{1}. \end{aligned} \quad (40)$$

Substituting μ_t from the second equation into the first one, allows to eliminate it, obtaining the following equation:

$$\gamma^r \Sigma_t (\mathbf{1} - N_t^{g,\lambda}) - \gamma^g \Sigma_t N_t^{g,\lambda} + e^{-\frac{\gamma^g}{\lambda} (N_t^{g,\lambda})^\top \theta_t(\psi_t)} \theta_t(\psi_t) = 0. \quad (41)$$

The left-hand side of this equation coincides with the gradient of the strictly convex, differentiable and coercive function

$$f(N) := -\gamma^r \mathbf{1}^\top \Sigma_t N + \frac{\gamma^r + \gamma^g}{2} N^\top \Sigma_t N + \frac{\lambda}{\gamma^g} e^{-\frac{\gamma^g}{\lambda} N^\top \theta_t(\psi_t)},$$

which proves existence and uniqueness of the solution of (41). Let us write $y_\lambda = e^{-\frac{\gamma^g}{\lambda} (N_t^{g,\lambda})^\top \theta_t(\psi_t)}$ in (41) and solve for $N_t^{g,\lambda}$ to obtain the explicit expression

$$N_t^{g,\lambda} = \alpha \left(\mathbf{1} + \frac{y_\lambda}{\gamma^r} \Sigma_t^{-1} \theta_t(\psi_t) \right).$$

Plugging this back into (41) we find y_λ by solving the one-dimensional equation

$$y_\lambda = e^{-\frac{\alpha \gamma^g}{\lambda} \left\{ \mathbf{1}^\top \theta_t(\psi_t) + \frac{y_\lambda}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t) \right\}}.$$

Finally the expression for p_0 is obtained from the condition $p_T = D_T$.

First we show existence and uniqueness of the solution y_λ to Eq. (20). Writing

$$g(y) := \alpha (\mathbf{1}^\top \theta_t(\psi_t) + \frac{y}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t)),$$

Equation (20) becomes

$$-\lambda \log y = \gamma^g g(y), \quad (42)$$

where it is also worth noticing that $g(1) = \theta_t(\psi_t)^\top N_t^g$. Since g is increasing and linear and the function $y \mapsto -\lambda \log y$ is strictly decreasing, continuous and maps $(0, \infty)$ onto \mathbb{R} , there exists one and only one positive solution of (20).

Next we show the monotonicity of the map $\lambda \mapsto y_\lambda$ stated in (ii)–(iv). If $g(1) > 0$, the solution belongs to the interval $(0, 1)$, and it is easy to see that it is monotonically increasing in λ . If $g(1) = 1$, the solution is constant and equal to 1. If $g(1) < 0$, the solution satisfies $y_\lambda > 1$ and it is easy to see that it is decreasing in λ .

To study the asymptotic behavior of y_λ as $\lambda \rightarrow \infty$, we expand the expression on the right hand side of (20) using Taylor up to the first order in λ^{-1} . That gives

$$y_\lambda = 1 - \frac{\alpha \gamma^g}{\lambda} \left(\mathbf{1}^\top \theta_t(\psi_t) + \frac{y_\lambda}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t) \right) + O(\lambda^{-2}).$$

Then we substitute $y_\lambda = y_0 + y_1 \lambda^{-1}$ on both sides of the expression above and, equating terms of the same order in λ^{-1} , we find $y_0 = 1$ and

$$y_1 = -\alpha \gamma^g \left(\mathbf{1}^\top \theta_t(\psi_t) + \frac{1}{\gamma^r} \theta_t(\psi_t)^\top \Sigma_t^{-1} \theta_t(\psi_t) \right).$$

Since $y_\lambda = y_0 + y_1 \lambda^{-1}$ it is now immediate to obtain $\lim_{\lambda \rightarrow \infty} y_\lambda = 1$ in all cases.

It remains to prove (i). Since ψ_t is fixed, for simplicity we omit it from some of the formulae below. A straightforward computation yields the following expression for $(N_t^{g,\lambda})^\top \Sigma_t N_t^{g,\lambda}$:

$$(N_t^{g,\lambda})^\top \Sigma_t N_t^{g,\lambda} = \|\Sigma_t^{1/2} N_t^{g,\lambda}\|^2 = \alpha^2 \|N_t^g\|^2 + \frac{2\alpha^2}{\gamma^r} (y_\lambda - 1) \theta_t^\top \mathbf{1} + \frac{\alpha^2}{(\gamma^r)^2} (y_\lambda^2 - 1) \theta_t^\top \Sigma_t^{-1} \theta_t.$$

Let

$$f(y) := \frac{2}{\gamma^r} (y - 1) \theta_t^\top \mathbf{1} + \frac{1}{(\gamma^r)^2} (y^2 - 1) \theta_t^\top \Sigma_t^{-1} \theta_t$$

and notice that f is a quadratic function with $f'(y) = 2/(\gamma^r \alpha) g(y)$. From the arguments above, if $g(1) > 0$ we have $\lambda \mapsto y_\lambda \in (0, 1)$ and increasing so that by Equation (42), $g(y^\lambda) > 0$, and therefore $\lambda \mapsto f(y^\lambda)$ is increasing as well. If, on the other hand $g(1) < 0$, then $\lambda \mapsto y_\lambda \in (1, \infty)$ and decreasing so that by Equation (42), $g(y^\lambda) < 0$, and therefore $\lambda \mapsto f(y^\lambda)$ is increasing once again. The claim holds trivially in the remaining case $g(1) = 0$.

Proof of Propositions 7 and 8

Proof of Proposition 7. For each $y > 0$ the derivation of Eq. (22) follows immediately from (21), as in Corollary 3. Next we prove solvability of Eq. (23) and some properties of its solution.

Proof of Proposition 8. Writing

$$g^*(y) := \mathbf{1}^\top \theta_t(\psi_t(y)) + \frac{\alpha y}{\gamma^r} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)),$$

Equation (23) becomes

$$-\lambda \log y = \alpha \gamma^g g^*(y). \quad (43)$$

Unlike $g(y)$, used in the proof of Proposition 6, $g^*(y)$ is a nonlinear and possibly non-monotonic function of y , which is more difficult to study. Nevertheless, it is clear that $g^*(0) < \infty$ and $g^*(y) \sim \frac{\alpha \kappa_0^2}{\gamma^r} \mathbf{1}^\top \Sigma_t^{-1} \mathbf{1} y$ as $y \rightarrow \infty$, which implies that Equation (23) admits a solution on $(0, \infty)$. Then (i) holds.

To prove uniqueness (that is (ii)) it is sufficient to show that (24) implies that g^* is monotonic increasing. It is immediate to check

$$\frac{d}{dy} \mathbf{1}^\top \theta_t(\psi_t(y)) = \sum_{i=1}^n \left(\frac{d}{d\psi} \theta_t^i(\psi_t^i(y)) \right) \left(\frac{d}{dy} \psi_t^i(y) \right),$$

and

$$\begin{aligned} \frac{d}{dy} \left(\frac{\alpha y}{\gamma^r} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)) \right) &= \frac{\alpha}{\gamma^r} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)) + \frac{\alpha y}{\gamma^r} \frac{d}{dy} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)) \\ &\geq \frac{\alpha y}{\gamma^r} \frac{d}{dy} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)). \end{aligned}$$

The latter expression can be expanded as

$$\frac{d}{dy} \theta_t(\psi_t(y))^\top \Sigma_t^{-1} \theta_t(\psi_t(y)) = \sum_{i,j=1}^n (\Sigma_t^{-1})_{ij} \theta_t^j(\psi_t^j(y)) \left(\frac{d}{d\psi} \theta_t^i(\psi_t^i(y)) \right) \left(\frac{d}{dy} \psi_t^i(y) \right).$$

Combining the calculations above we obtain

$$\frac{d}{dy}g^*(y) \geq \sum_{i=1}^n \left(1 + \frac{\alpha}{\gamma^r} y \sum_{j=1}^n (\Sigma_t^{-1})_{ij} \theta_t^j(\psi_t^j(y)) \right) \left(\frac{d}{d\psi} \theta_t^i(\psi_t^i(y)) \right) \left(\frac{d}{dy} \psi_t^i(y) \right).$$

Recalling the expressions for $\psi_t(y)$ and $\theta_t(\psi)$ (see (22) and Corollary 3, respectively), we immediately see that

$$\left(\frac{d}{d\psi} \theta_t^i(\psi_t^i(y)) \right) \left(\frac{d}{dy} \psi_t^i(y) \right) \geq 0, \quad \text{for all indexes } i.$$

Then the condition (24) is sufficient to guarantee that g^* is monotonic increasing and the solution to (43) is unique.

All the statements in (iii) are shown using the same arguments of proof as in Proposition 6, using that g^* is increasing and that $\psi_t^*(1) = \psi_t^{*,0}(1)$.

Appendix B: Calibration

We choose the rate of time preference, ρ , equal to 0.01 (Gollier, 2002; Gollier and Weitzman, 2010). We estimate the share of assets managed taking into account climate criteria, α , at 25% (US SIF, 2018).

We estimate κ and κ_0 by using the estimates in Zerbib (2021) of the externality premium of the electrical equipment (-1.11%) and the coal (+0.12%) industries in the U.S. between 2013 and 2019: knowing that this premium is equal to $-\alpha\theta(\psi)$ in the present paper, with $\theta(\psi) = \kappa_0 - \frac{\kappa}{2}\psi^2$, and that the average carbon intensity of the electrical equipment and coal industries are $\psi^{\text{elec}} = 147$ tCO₂e/USDmn and $\psi^{\text{coal}} = 555$ tCO₂e/USDmn, respectively, we get $\kappa = 3 \times 10^{-7}$ and $\kappa_0 = 0.047$. Considering companies' responsiveness to internalize climate externalities following market pressure, we choose the companies' climate sensitivity slightly lower than that of the average investor sensitivity ($\alpha\kappa = 0.25\kappa$): $\kappa_c = 0.2\kappa = 5.5 \times 10^{-8}$.

We calibrate the marginal abatement costs for an electrical equipment company, which has an average carbon intensity of 147 tCO₂/USDmn, and a coal company, which has an average carbon intensity of 555 tCO₂/USDmn. To do so, we use the equilibrium equation of the emission schedule without climate uncertainty (Equation (13)) and we assume that the initial emissions of the companies ($\psi_b^{\text{elec}} = 147$ tCO₂e/USDmn and $\psi_b^{\text{coal}} = 555$ tCO₂e/USDmn) are adjusted to the optimal level: $\psi_b = \psi_0^*$. Therefore, $c^{\text{elec}} = 8 \times 10^{-6}$ and $c^{\text{coal}} = 3 \times 10^{-5}$.

Finally, to calibrate the parameters needed for the simulations of the model with climate uncertainty, we assume that the correlation between the two assets is 50% and we take regular investors' absolute risk aversion, γ_r , equal to 0.1 (Barberis et al. (2015)). Thus, as $\alpha = \frac{\gamma_r}{(\gamma_r + \gamma_g)} = 0.25$, $\gamma_g = 0.3$. In the simulations, we consider different levels of intensities, λ , and find y_λ^* as the solution of the fixed point Equation (23).

Table 2 summarizes the calibrated parameters.

Table 2 Calibrated parameters. This table gives the values of the parameters calibrated based on the estimates in this section and used for the simulations presented in Figures 2, 3, 4, and 5.

Parameter	Value
α	0.25
ρ	0.01
κ	3×10^{-7}
κ_0	0.047
κ^c	5.5×10^{-8}
ψ_b^{elec}	147
ψ_b^{coal}	555
c^{elec}	8×10^{-6}
c^{coal}	3×10^{-5}
$Cor(\text{elec}, \text{coal})$	0.5
γ_r	0.1
γ_g	0.3