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**CCA**

Allievi Program, Master in Economics, and Ph.D. in Economics

# MEASURE THEORY

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Instructor: Bertrand Lods

## Contact Information

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## Course Description

The course introduces the theory of Lebesgue integration. After defining appropriate structures of sets such as algebras, semi-algebras and sigma-algebras, we provide the notion of Lebesgue-Stieltjes measures, and their construction from semi-algebras to sigma-algebras. We then constructively define integrals with respect to a Lebesgue-Stieltjes measure and state their most relevant properties. The relationship with classical Riemann integration and the connection with probability theory are also examined.

## Exam

Written exam at the end of the course.

## Course Outline

- Classes of subsets: algebras, semi-algebras, sigma-algebras, monotone classes
- Measures: definition and properties; finite-additivity and sigma-additivity; construction of measures on sigma-algebras; completions of measures; Lebesgue-Stieltjes measures
- Measurable functions
- Lebesgue-Stieltjes integrals: construction and properties
- Convergence theorems
- Null-measure sets and properties holding almost everywhere
- Comparison with Riemann integration
- Radon-Nikodym's theorem

## Textbooks

Detailed lecture notes will be provided. Some reference books are:

- BILLINGSLEY, P. (1986) — *Probability and measure*. Wiley.
- DUDLEY, R.M. (2004) — *Real analysis and probability*. CUP.
- FOLLAND, G.B. (1999) — *Real analysis*. Wiley.
- ROYDEN, H.L. (1988) — *Real analysis*. Prentice-Hall.
- RUDIN, W. (1976) — *Principles of mathematical analysis*. McGraw-Hill.