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Bankruptcy Problems with Self-Serving Biased Reference Points

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Abstract

I study bankruptcy problems under the assumption that claimants display reference-dependent preferences and set self-serving biased reference points. I consider two possible specifications for the self-serving bias, additive and multiplicative, and investigate how they impact on claimants' perceived gains and losses, and thus ultimately on welfare. Focusing on the four most prominent allocation rules (Proportional, Constrained Equal Awards, Constrained Equal Losses, and Talmud), I show that all rules are welfare equivalent when the bias takes the additive form, whereas a clear ranking emerges, with the Constrained Equal Losses on top, when the bias is multiplicative.

Keywords: bankruptcy problems, self-serving bias, reference points, welfare.

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1. Introduction

A bankruptcy problem is the problem faced by an arbitrator who is called to decide on how to allocate a finite and perfectly divisible resource among several claimants whose claims sum to a greater amount than what is available (see Moulin, 2002, and Thomson 2003, 2015 for detailed surveys). Typical examples are the liquidation of a bankrupted firm among creditors or the division of an estate among heirs.

In this paper, I study bankruptcy problems when claimants display reference dependent preferences (RDPs) and set self-serving biased reference points. Both features seem to fit well within the context of bankruptcy problems. Think for instance about the above mentioned examples. Indeed, these are situations in which claimants are likely to form subjectively biased expectations about the allocation that the arbitrator will implement and then inevitably compare what they actually get with what they were expecting to get. RDPs capture the perceived gains and losses that stem out of this comparison, whereas the notion of self-serving bias (SSB) is a natural candidate for rationalizing claimants' distorted expectations. In fact, SSB leads people to overestimate their own merits and abilities, to favorably acquire and interpret information, and, very much in line with the the topic of the paper, to inflate their claims and contributions (see Babcock *et al.*, 1995, Babcock and Loewenstein, 1997, Farmer and Pecorino, 2002).

I thus argue that SSB affects claimants' reference points in a simple but systematic way. In line with Babcock and Loewenstein's (1997, p. 110) definition of the bias as a tendency "to conflate what is fair with what benefits oneself", I postulate that, everything else being equal, a self-serving biased agent sets a higher reference point with respect to the one that his hypothetical unbiased alter ego would have set. I then consider two possible formulations for the SSB: an additive one (the bias shifts the claimant's reference point up by a constant) and a multiplicative one (the bias magnifies the claimant's reference point). I study how the two formulations impact on claimants' perceived gains and losses, and thus ultimately on aggregate welfare. Taking the point of view of a benevolent arbitrator who aims at maximizing welfare, I thus evaluate how the four most prominent allocation rules (Proportional, Constrained Equal Awards, Constrained Equal Losses, and Talmud) perform. I show that when claimants' SSB takes the additive form, then the four rules are welfare equivalent. Instead, when claimants' bias takes the multiplicative form a clear ranking emerges, with the Constrained Equal Losses rule dominating the alternatives.

These results are coherent with those that appear in Gallice (2019), but with a twist. Gallice (2019) introduces and studies bankruptcy problems with reference-dependent preferences, but does not consider the role of SSB. He also finds that in some situations different allocation rules lead to different levels of welfare. The intuition is similar: when claimants have RDPs, and thus display diminishing marginal sensitivity to losses, the rules that better perform in terms of welfare are those that implement the more skewed distributions of perceived losses across claimants. In other words, from a welfare point of view, it is better to disappoint a lot a few individuals rather than disappoint a little all of them. However, Gallice (2019) shows that when claimants use as reference points the claims vector, the zero award vector or the minimal rights vector, it is in general the Constrained Equal Awards rule that achieves higher welfare. Here instead I show that when claimants' display self-serving biased reference points, the rule that better performs in terms of welfare is the Constrained Equal Losses rule.

2. The Model

There are $N = \{1, ..., n\}$ claimants with claims over an endowment of size $E \in \mathbb{R}_+$. Let the vector $c = (c_1, ..., c_n)$ with $c_i \in \mathbb{R}_+$ collect individual claims. A *bankruptcy problem* is then defined by the pair $(c, E) \in \mathbb{R}^N_+ \times \mathbb{R}_+$ with $\sum_i c_i \geq E$. I denote with $\mathbb{B}^N$ the class of all such problems. A *rule* R is a function that associates to any problem $(c, E) \in \mathbb{B}^N$ a unique awards vector $R(c, E) = (R_1(c, E), ..., R_n(c, E))$, with $0 \leq R_i(c, E) \leq c_i$ for any $i \in N$ and $\sum_i R_i(c, E) = E$. In what follows, I consider the four most prominent rules (Herrero and Villar, 2001):

- The *Proportional* (P) rule, which allocates the endowment proportional to claims:

$$P(c, E) = \lambda c \text{ with } \lambda = E/C. \tag{1}$$

- The *Constrained Equal Awards* (CEA) rule, which assigns equal awards to all claimants subject to the requirement that no one receives more than his claim:

$$CEA_i\left(c,E\right)=\min\left\{c_i,\lambda\right\} \text{ for all } i\in N \text{ with } \sum_i \min\left\{c_i,\lambda\right\}=E. \tag{2}$$

- The *Constrained Equal Losses* (CEL) rule, which assigns an equal amount of losses to all claimants subject to the requirement that no one receives a negative amount:

$$CEL_i\left(c,E\right)=\max\left\{0,c_i-\lambda\right\} \text{ for all } i\in N \text{ with } \sum_i \max\left\{0,c_i-\lambda\right\}=E. \tag{3}$$

- The *Talmud* (T) rule, which foresees two different solutions depending upon the relationship between the half-sum of the claims and the endowment:

$$\begin{aligned} T(c,E) &= CEA\left(\frac{1}{2}c,E\right) \text{ if } \frac{C}{2}\geq E,\\ T(c,E) &= \frac{1}{2}c+CEL\left(\frac{1}{2}c,E-\frac{C}{2}\right) \text{ if } \frac{C}{2}<E. \end{aligned} \tag{4}$$

Let $\mathcal{R}=\{P,CEA,CEL,T\}$ thus denote the set of rules and say that the arbitrator uses rule $R\in\mathcal{R}$. Following Gallice (2019), I endow claimants with reference dependent preferences à la Kőszegi and Rabin (2006). Generic claimant $i\in N$ then displays the following utility function:

$$u(R_i\left(c,E\right)\mid r_i)=R_i\left(c,E\right)+\mu(R_i\left(c,E\right)-r_i), \tag{5}$$

where $r_i\geq 0$ is the claimant's reference point.

The claimant thus derives direct utility from the possession / consumption of the actual award $R_i\left(c,E\right)$. However, his overall utility is also influenced by how the award compares with his reference point: the gain-loss function $\mu(\cdot)$ captures the (psychological) effects of these perceived gains and losses. In line with the original formulation of prospect theory (Kahneman and Tversky, 1979), the function $\mu(\cdot)$ is continuous, strictly increasing and such that $\mu(0)=0$. It is strictly convex in the domain of losses ($R_i\left(c,E\right)<r_i$) and strictly concave in the domain of gains ($R_i\left(c,E\right)>r_i$). Finally, losses loom larger than gains, i.e., $|\mu(-z)| > \mu(z)$ for any $z > 0$.

I am interested in studying how the rules perform in terms of *welfare* when claimants display self-serving biased reference points. As a measure of welfare, I rely on the notion of utilitarian welfare (see Moulin, 2003). By denoting with $r = (r_1, ..., r_n)$ the vector that collects claimants' reference points, the welfare generated by rule $R \in \mathcal{R}$ thus takes the following form:

$$\begin{aligned} W(R \mid r) &= \sum_i u(R_i(c,E) \mid r_i) \\ &= \sum_i (R_i(c,E) + \mu(R_i(c,E) - r_i)) \\ &= E + \sum_i \mu(R_i(c,E) - r_i). \end{aligned} \tag{6}$$

Finally, I formalize the notion of *self-serving biased reference point*. Let $r_i^{rat} = R_i(c,E)$ denote a claimant's rational reference point, which is thus given by what the claimant actually gets from the arbitrator.[1] I then postulate that a self-serving biased claimant sets a higher reference point with respect to the one that his hypothetical rational alter ego would set (see also Gallice, 2011). Thus, $r_i^{ssb} > r_i^{rat}$.

Following Farmer and Pecorino (2002), I consider two explicit analytic formulations for the self-serving bias, namely an additive bias (index $ssb+$) and a multiplicative bias (index $ssb*$). With an additive bias, a self-serving biased reference point is given by

$$r_i^{ssb+} = r_i^{rat} + k, \tag{7}$$

where $k > 0$ captures the bias. Instead, with a multiplicative bias, a self-serving biased reference point takes the form

$$r_i^{ssb*} = (1+\beta)\, r_i^{rat}, \tag{8}$$

where the parameter $\beta > 0$ captures the proportion by which the bias inflates the agent's reference point with respect to the rational benchmark.

[1]Thus, in a RDPs framework, a rational claimant would experience no perceived gains or losses, $u(R_i(c,E) \mid r_i) = R_i(c,E)$, see equation (5).

3. Welfare Analysis

Trivially, when claimants have rational reference points welfare amounts to $W(R \mid r^{rat}) = E$ for any $R \in \mathcal{R}$. Instead, when claimants display self-serving biased reference points, and independently from the multiplicative or additive formulation of the bias, welfare gets dampened: $W(R \mid r^{ssb}) < E$ for any $R \in \mathcal{R}$. This is a direct consequence of the bias being detrimental at the individual level: a claimant who sets a biased reference point experiences larger perceived losses or smaller perceived gains with respect to having set a rational reference point.

More interestingly, for any given profile of self-serving biased reference points, *different* rules may lead to *different* allocations of perceived gains and losses across claimants. Because of the properties of the gain-loss function $\mu(\cdot)$, this asymmetry may affect welfare. Proposition 1 investigates the issue for the case of the additive bias and shows that in such a case welfare is actually constant across rules. Proposition 2 instead refers to the case of the multiplicative bias where indeed different rules lead to different levels of welfare. These differences are systematic so that a clear ranking of the rules emerges.

Proposition 1. *When claimants have a self-serving biased reference point $r_i^{ssb+} = r_i^{rat} + k$ with $k > 0$, all rules are welfare equivalent.*

Proof 1. *By substituting (7) into (6) one obtains:*

$$\begin{aligned} W\left(R \mid r^{ssb+}\right) &= \sum_i u\left(R_i\left(c, E\right) \mid r_i^{ssb+}\right) \\ &= E + \sum_i \mu\left(R_i\left(c, E\right) - r_i^{ssb+}\right) \\ &= E + n\mu(-k), \end{aligned}$$

so that welfare is independent of the specific rule that the arbitrator uses: $W(R \mid r^{ssb+}) = \{W(Q \mid r^{ssb+}))$ *for any* $R, Q \in \{P, CEA, CEL, T\}$. □

When instead the bias is multiplicative, the distribution of perceived losses across claimants is not constant across rules. Because of the diminishing sensitivity to losses displayed by the $\mu(\cdot)$ function, different rules then generate different levels of welfare. The ranking is then determined by how (un)evenly the rules allocate the endowment across claimants. As a measure of inequality, I use the standard Lorenz dominance criterion (see Sen, 1973), which I briefly review.

Consider two rules, R and Q, that lead to the award vectors $R(c,E)$ and $Q(c,E)$. Define the new vectors $\hat{R}(c,E)$ and $\hat{Q}(c,E)$ that display the coordinates of the original vectors in increasing order, i.e., $\hat{R}_i(c,E) \leq \hat{R}_{i+1}(c,E)$ and $\hat{Q}_i(c,E) \leq \hat{Q}_{i+1}(c,E)$ for any $i \in \{1,...,n-1\}$. I say that $R(c,E)$ Lorenz dominates $Q(c,E)$, and I denote such a relation by writing $R(c,E) \succ_L Q(c,E)$, if:

$$\begin{aligned}
&\hat{R}_1(c,E) \geq \hat{Q}_1(c,E), \\
&\hat{R}_1(c,E) + \hat{R}_2(c,E) \geq \hat{Q}_1(c,E) + \hat{Q}_2(c,E), \\
&... \\
&\hat{R}_1(c,E) + ... + \hat{R}_{n-1}(c,E) \geq \hat{Q}_1(c,E) + ... + \hat{Q}_{n-1}(c,E).
\end{aligned} \tag{9}$$

with at least one strict inequality. I instead write $R(c,E) \succeq_L Q(c,E)$ if none of the above listed inequalities holds strictly. I can then say that rule R Lorenz dominates rule Q, a relation that I denote as $R \succ_L Q$, whenever $R(c,E) \succeq_L Q(c,E)$ for any possible vector of claims and $R(c,E) \succ_L Q(c,E)$ for at least one vector of claims. In other words, the relation $R \succ_L Q$ indicates that rule R always implements an allocation that is (weakly) less skewed with respect to the allocation that rule Q implements. Focusing on the set $\mathcal{R} = \{P, CEA, CEL, T\}$, Bosmans and Lauwers (2011) show that the Constrained Equal Awards rule Lorenz dominates all other rules, whereas the Constrained Equal Losses rule is Lorenz dominated by all other rules. There is not a Lorenz dominance relation between the Proportional and the Talmud rules. More formally, $CEA \succ_L \{P, T\} \succ_L CEL$. Such a result plays a key role in the proof of the following proposition:

Proposition 2. *When claimants have a self-serving biased reference point $r_i^{ssb*} = (1+\beta)\, r_i^{rat}$ with $\beta > 0$, different rules lead to different levels of welfare. In particular, the following ranking holds: $W(CEL \mid r^{ssb*}) \geq \{W(P \mid r^{ssb*}), W(T \mid r^{ssb*})\} \geq W(CEA \mid r^{ssb*})$.*

PROOF 2. *By substituting (8) into (6) one obtains:*

$$\begin{aligned} W\left(R \mid r^{ssb*}\right) &= \sum_i u\left(R_i\left(c,E\right) \mid r_i^{ssb*}\right) \\ &= E + \sum_i \mu\left(R_i\left(c,E\right) - r_i^{ssb*}\right) \\ &= E + \sum_i \mu(-\beta R_i(c,E)), \end{aligned}$$

so that the distribution of losses across claimants is proportional to the distribution of the awards vector. The strict convexity of the μ function in the domain of losses implies that $\mu(a) + \mu(b) < \mu(a-\epsilon) + \mu(b+\epsilon) < 0$ for any $a \leq b < 0$ and $\epsilon \in (0, |b|)$. Therefore, the rule that implements the more skewed distribution of $R_i(c,E)$ is the rule that achieves highest welfare $W\left(R \mid r^{ssb*}\right)$. Given that $CEA \succ_L \{P, T\} \succ_L CEL$ (Bosmans and Lauwers, 2011) and $\beta > 0$, it follows that $W(CEL \mid r^{ssb*}) \geq \{W(P \mid r^{ssb*}), W(T \mid r^{ssb*})\} \geq W(CEA \mid r^{ssb*})$. □

Notice that, when the SSB takes the multiplicative form, the four rules are actually welfare equivalent only when all agents have identical claims, $c_i = c_j$ for all $i, j \in N$.[2] This implies that, whenever claimants are asymmetric ($c_i \neq c_j$ for some $i, j \in N$), the ranking defined in Proposition 2 holds strictly.

The following example illustrates the results of Propositions 1 and 2 within a specific bankruptcy problem.

EXAMPLE 1. *Consider the bankruptcy problem (c, E) with $c = (30, 50, 80)$, $E = 100$ and let claimant $i \in \{1, 2, 3\}$ have utility function:*

$$u(R_i\left(c,E\right) \mid r_i) = \begin{cases} R_i\left(c,E\right) + \sqrt{R_i\left(c,E\right) - r_i} & \text{if } R_i\left(c,E\right) \geq r_i \\ R_i\left(c,E\right) - 3\sqrt{|R_i\left(c,E\right) - r_i|} & \text{if } R_i\left(c,E\right) < r_i \end{cases}$$

The four rules select the awards vectors $P(c,E) = (18.75, 31.25, 50)$, $CEA(c,E) = (30, 35, 35)$, $CEL(c,E) = (10, 30, 60)$, and $T(c,E) = (15, 27.5, 57.5)$.

*(a) The case with an **additive bias**. Let $k = 9$ so that the vectors of self-serving biased reference*

[2] Indeed, when this is the case, all rules select the egalitarian allocation $R(c,E) = (E/n, ..., E/n)$, so that $W\left(R \mid r^{ssb*}\right) = E + n\mu\left(-\frac{\beta E}{n}\right)$ for any $R \in \{P, CEA, CEL, T\}$.

points are given by $\left(r^{ssb+} \mid P\left(c,E\right)\right) = (27.75, 40.25, 59)$, $\left(r^{ssb+} \mid CEA\left(c,E\right)\right) = (39, 44, 44)$, $\left(r^{ssb+} \mid CEL\left(c,E\right)\right) = (19, 39, 69)$, $\left(r^{ssb+} \mid T\left(c,E\right)\right) = (24, 36.5, 66.5)$ *and welfare, in line with Proposition 1, is* $W\left(R \mid r^{ssb+}\right) = 100 - 3\left(3\sqrt{9}\right) = 73$ *for any* $R \in \{P, CEA, CEL, T\}$.

(b) The case with a ***multiplicative bias****. Let* $\beta = 0.2$ *so that the vectors of self-serving biased reference points are given by* $\left(r^{ssb*} \mid P\left(c,E\right)\right) = (22.5, 37.5, 60)$, $\left(r^{ssb*} \mid CEA\left(c,E\right)\right) = (36, 42, 42)$, $\left(r^{ssb*} \mid CEL\left(c,E\right)\right) = (12, 36, 72)$, $\left(r^{ssb*} \mid T\left(c,E\right)\right) = (18, 33, 69)$ *and welfare is* $W\left(P \mid r^{ssb*}\right) = 77.2$, $W\left(CEA \mid r^{ssb*}\right) = 76.8$, $W\left(CEL \mid r^{ssb*}\right) = 78$, $W\left(T \mid r^{ssb*}\right) = 77.6$. *Therefore, and in line with Proposition 2,* $W(CEL \mid r^{ssb*}) > W(T \mid r^{ssb*}) > W(P \mid r^{ssb*}) > W(CEA \mid r^{ssb*})$.

4. Conclusions

I introduced the notion of self-serving biased reference points in the context of bankruptcy problems and showed that when the bias takes the multiplicative form the four most prominent rules can be ranked on the basis of the level of welfare that they generate. Aside from bankruptcy problems, self-serving biased reference points are likely to play a role also in other contexts such as disputes and litigations, auctions, and bargaining problems. The proposed framework can help to analytically investigate the consequences of this pervasive bias also in these situations.

References

[1] Babcock, L., Loewenstein, G., 1997. Explaining bargaining impasse: the role of self-serving biases. *Journal of Economic Perspectives* 11 (1), 109-126.

[2] Babcock, L., Loewenstein, G., Issacharoff, S., Camerer, C., 1995. Biased judgments of fairness in bargaining. *American Economic Review* 85 (5), 1337-1343.

[3] Bosmans, K., Lauwers, L., 2011. Lorenz comparisons of nine rules for the adjudication of conflicting claims, *International Journal of Game Theory*, 40, 791-807.

[4] Farmer, A., Pecorino, P., 2002. Pretrial bargaining with self-serving bias and asymmetric information. *Journal of Economic Behavior and Organization*, 48 (2), 163-176.

[5] Gallice, A., 2011. Self-serving biased reference points, *Carlo Alberto Notebooks*, No. 223.

[6] Gallice, A., 2019. Bankruptcy problems with reference-dependent preferences, *International Journal of Game Theory* 48 (1), 311-336.

[7] Gallice, A., 2020. Self-serving biased reference points in bankruptcy problems, *Working paper 067, Dipartimento di Scienze Economico-Sociali e Matematico-Statistiche*.

[8] Herrero, C., Villar, A., 2001. The three musketeers: Four classical solutions to bankruptcy problems, *Mathematical Social Sciences*, 39, 307-328.

[9] Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decision under risk, *Econometrica* 47, 263-291.

[10] Kőszegi, B., Rabin, M., 2006. A model of reference-dependent preferences, *Quarterly Journal of Economics* 121, 1133-1165.

[11] Moulin, H., 2002. Axiomatic cost and surplus-sharing. In: Arrow, K., Sen, A., Suzumura, K. (Eds.): *The Handbook of Social Choice and Welfare*, Vol. 1. Amsterdam: Elsevier.

[12] Moulin, H., 2003. *Fair Division and Collective Welfare*, MIT Press: Cambridge, MA, USA.

[13] Sen, A., 1973. *On Economic Inequality*, Clarendon Press: Oxford, UK.

[14] Thomson, W., 2003. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: A survey, *Mathematical Social Sciences*, 45, 249-297.

[15] Thomson, W., 2015. Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update, *Mathematical Social Sciences*, 74, 41-59.