Optimal Financial Policies for a Group *

Shiqi Chen

Cambridge Judge Business School

Bart M. Lambrecht

Cambridge Judge Business School and CEPR

June 9, 2022

Abstract

We model the financial policies of a private firm owned by a group of undiversified investors with heterogeneous capital contributions and risk preferences. The first-best expected life-time utility for each investor can be achieved by issuing financial claims resembling preferred stock with heterogeneous dividend caps and common stock, and by following procyclical investment and financing policies. These optimal financial policies and claims can be derived as the solution to a social planner problem that maximizes a weighted average of investors' life-time utility. Investors' utility weights are fixed at startup and determined by their participation constraints.

Keywords: investment, payout, capital structure, risk preferences, group policy

JEL: G32, G34, G35

*We thank the following conference discussants Anton Tsoy (2020 UBC Winter Finance Conference), Savitar Sundaresan (2020 EFA meetings), Ji Shen (Guanghua International Finance symposium), Li He (18th Corporate Finance Day), as well as Lorenzo Garlappi, Nadya Malenko, Lucio Sarno, and participants at the CERF Cavalcade, the 2019 SFA meetings, the 23rd Annual International Real Options Conference, 2021 ACBES conference, 2022 BAFA meetings, and seminar participants at the universities of Cambridge, Essex, Fudan, Lancaster, Leeds, Tilburg, and Hong Kong Baptist University, Korea University Business School, and UNC Charlotte for helpful comments and discussions. The authors thank the Cambridge Endowment for Research in Finance (CERF) for financial support. An earlier version of the paper was circulated under the title "Financial Policies and Internal Governance with Heterogeneous Risk Preferences". Comments can be sent to Shiqi Chen (sc884@jbs.cam.ac.uk) or Bart Lambrecht (b.lambrecht@jbs.cam.ac.uk).

1 Introduction

The corporate finance literature has traditionally studied optimal financial policies within the context of public corporations. A standard assumption is that all stockholders agree to maximize shareholder value, irrespective of their personal preferences. Investors who wish, for example, a higher payout than what is offered by the firm can sell some stock to smooth consumption. Likewise, investors can create homemade leverage if they prefer higher corporate gearing or risk exposure.

Once we leave the realm of publicly traded corporations with well diversified investors it is less clear what stockholders with heterogeneous risk preferences should optimize in the absence of a stock price. For example, small family firms are highly illiquid and typically owned and run by a few people with diverse preferences (e.g. due to age differences) whose entire livelihood depends on the income generated by the firm. Since each investor may want a different investment, financing and payout policy, one might jump to the conclusion that heterogeneous investors inevitably have to settle for a second-best compromise. Surprisingly, we find that it is possible for all investors to achieve their first-best expected lifetime utility. The first-best outcome can be achieved by issuing financial claims that are tailor-made to investors' risk preferences, and through an investment policy and net debt ratio that change dynamically over time in response to economic shocks. These claims and dynamic policies can be derived as the solution to a social planner problem that maximizes a weighted average of investors' life-time utility. The utility weights are set and fixed at the startup of the firm and determined by investors' participation constraints.

Why do investors want to work together in our model? Each investor has her own outside investment opportunity set which consists of a risky asset (project) and a line of credit (negative debt corresponds to cash savings). The projects are all subject to the same source of economic uncertainty but their Sharpe ratio differs across investors. Synergies can be generated by investors pooling their capital in the project of the lead investor or "entrepreneur" with the highest Sharpe ratio. We assume without loss of generality that all welfare gains from collaboration accrue to the entrepreneur, i.e. each co-investor supplies capital on a competitive basis and receives a utility weight that leaves her indifferent between joining the private firm or investing in her outside option as a sole proprietor.[1] In equilibrium, each co-investor's utility weight is increasing in her capital contribution and goes to zero as her capital contribution becomes vanishingly small. In contrast, the entrepreneur retains a strictly positive utility weight even in the absence of any monetary contribution. Her "excess weight" reflects the synergies (i.e. superior Sharpe ratio) she generates with her human capital. The entrepreneur's net worth stake in the firm is larger than the (monetary) capital she contributes. The difference represents a compensation for her human capital or "sweat equity". Conversely, co-investors' equity stake is smaller than the capital they contribute. The premium paid resembles an upfront "fee" for access to a project that generates superior expected returns.

We derive the group's optimal investment, payout and financing policies within a continuous-time open-horizon model where investors have heterogeneous risk preferences. We show that the optimal dynamic investment and financing policies for the group are a weighted average of each individual investor's optimal policy evaluated at the entrepreneur's superior Sharpe ratio. However, the weights (not to be confounded with the fixed utility weights) vary with the firm's net worth. As net worth increases (decreases) in good (bad) times, increasingly more weight is put on the least (most) risk averse investor, causing the group's coefficient of risk aversion – defined in terms of the group's indirect utility function – to converge to the lowest (highest) coefficient of risk aversion in the group. Consequently, leverage and investment are procyclical: the firm's

[1] In Section 4.4 we extend to model by distributing the synergies across all investors.

investment and its net debt ratio (NDR) rise when the firm is doing well, and fall in bad times. Businesses with little net worth optimally hold a net cash position (i.e. negative debt). As a firm and its net worth grow, it ultimately starts borrowing using a line of credit and builds up a net debt position. Firms dynamically delever (lever up) towards the NDR of the most (least) risk averse investor as net worth shrinks (grows). Only when all investors have the same coefficient of risk aversion does the firm maintain a constant NDR. By dynamically rebalancing assets and liabilities in response to income shocks, debt is kept safe at all times as investors wish to avoid default due to risk aversion and their reliance on the firm for future consumption. We believe these results to be original.

Our results may also cast some light on puzzling findings in the literature on group decisions. It is well known that groups behave differently from individuals. The literature offers two competing hypotheses for group decisions. The group shift hypothesis (e.g. Moscovici and Zavalloni (1969); Kerr (1992)) suggests that group decisions shift towards the one of the dominant person in a team. As that person typically holds very pronounced preferences, a team eventually gravitates towards extremes. Consequently, teams make more polarized decisions than individuals do. In contrast, an alternative hypothesis predicts that extreme preferences in a group are averaged out and teams eventually make less extreme decisions than individuals do. Although this kind of diversification hypothesis appears to enjoy strong empirical support (see Bär, Kempf and Ruenzi (2011)), it struggles to explain the puzzling observation that the average group is more (less) risk averse than the average individual in high (low) risk situations (Shupp and Williams (2008)). Our model reconciles both hypotheses as it shows that group choices are a time-varying weighted average of individuals' preferred decisions. Polarization is a special, limiting case in which (almost) all decision weight is put on one individual. As previously mentioned, polarization occurs in our model when the firm is performing either very well or very poorly.

Our model applies not only to private firms owned by a group of investors, but also to other groups that share the proceeds of their activities, such as fund management teams, partnerships, VC groups or syndicates. The key feature of our model is that each group member receives a tailor-made claim or contract that specifies her payout as a function of the group's net worth. Furthermore, an investor's payout and claim value depend not only on her own risk preferences, but also on the risk preferences of her co-investors. Investors that are relatively less risk averse opt for riskier claims with higher upside potential but more downside risk. We find that the payout and claim value of the least (most) risk averse investor is a convex (concave) function of the firm's net worth. For investors with intermediate levels of risk aversion, the payout and claim value are S-shaped in net worth (i.e. convex for low levels of net worth, and concave for high levels). Payouts and claim values converge to fixed caps as net worth increases, except for the least risk averse investor whose payout and claim become arbitrarily large as net worth goes to infinity. We show that the claim on the firm's assets of the least risk averse investor is similar to a common equity contract. The claims of investors with higher levels of risk aversion resemble preferred stock with more risk averse investors having a higher seniority but lower maximum dividend. Only when all investors have the same coefficient of risk aversion do we obtain payouts and claims that are linear in the firm's assets and net worth, reflecting equity contracts in which each investor gets a fixed proportion of the firm's total payouts and owns a constant fraction of the firm's net worth. Our model endogenously generates contracts resembling preferred stock for which investors' cash flows may not move in tandem with the control they exert. Gilson and Schizer (2003) argue that preferred stock facilitates the separation of control and cash flow rights.

Our paper relates to two literature strands: one strand of dynamic models for the behavior and decisions of groups, and a strand of continuous-time models studying the dynamics and interaction of corporate financial policies. Dynamic models of group decisions in corporate finance are very rare.[2] A notable exception is Garlappi et al. (2017) who study a dynamic corporate investment problem by a group of agents holding heterogeneous beliefs and adopting a utilitarian aggregation mechanism. They show that group decisions are dynamically inconsistent due to learning and that this may lead to underinvestment. Our policies are time-consistent because no learning takes place along the way, and investors' utility weights are fixed at startup. Garlappi et al. (2021) study a real option model where the decisions to invest in and abandon a project are made sequentially by a group of agents with heterogeneous beliefs who make decisions based on majority voting. Voting leads to inefficient underinvestment when group members' beliefs are polarized. Ebert et al. (2020) model a canonical real option investment problem under weighted discounting. Weighted discount functions may describe the discounting behavior of groups. They find that greater group diversity leads to delayed investment. Habib and Mella-Barral (2007) model the formation and duration of joint ventures when partners can acquire part of each other's knowhow. Antill and Grenadier (2019) develop a continuous-time dynamic bargaining model of corporate reorganization between equityholders and creditors. Some papers examine group decision within a principal-agent setting. For example, Grenadier et al. (2016) model a real options investment decision in an organization where an uninformed principal makes a timing decision interacting with an informed but biased agent. Rivera (2020) develops a dynamic model that integrates the risk-shifting problem between bondholders and shareholders with the moral-hazard problem between shareholders and the manager. These papers do not consider optimal capital structure, payout structure, and internal governance, and all agents have identical

[2]Most existing models on group decisions are static in nature and build on a seminal literature on partnerships and syndicates that studies how risk should be shared between partners (see e.g. Wilson (1968), Eliashberg and Winkler (1981), Pratt and Zeckhauser (1989)). More recently, Mazzocco (2004) examines the savings behavior of couples with homogenous HARA preferences and individual stochastic incomes. Hara et al. (2007) show within a static setting that heterogeneity in consumers' risk attitudes generates optimal sharing rules that are concave, convex, or initially convex and eventually concave, depending on investors' risk preferences.

(risk neutral) risk preferences.[3] Group members do not vote in our model, but achieve the efficient outcome by issuing tailor-made contracts and adopting dynamic financial policies.

Our paper also relates to a large literature in corporate finance that jointly models a firm's financial policies in a dynamic framework. Continuous-time papers in this strand include Gryglewicz (2011), Bolton et al. (2011), Décamps et al. (2017), Hugonnier and Morellec (2017), Lambrecht and Myers (2017), and Hartman-Glaser et al. (2020) among others. Some papers focus on entrepreneurial (rather than corporate) finance. Chen et al. (2010) study the effects of nondiversifiable risk on entrepreneurial finance by building on Leland (1994). Wang et al. (2012) develop an incomplete markets q-theoretic model to study entrepreneurship dynamics. Sorensen et al. (2014) analyze the portfolio-choice problem of a risk-averse Limited Partner (LP). Whereas existing papers focus on the decisions of a single agent, and take contracts as exogenously given, our paper considers a group of investors, and endogenizes their optimal claims.[4]

[3]A large body of dynamic asset pricing papers studies the effect of heterogeneous risk preferences on prices in a market equilibrium framework. For example, Dumas (1989), Wang (1996), Cvitanić et al. (2012), and Bhamra and Uppal (2014) consider a pure exchange economy where agents can trade their share in the exogenous, aggregate endowment of the consumption good and have access to a risk-free security. Wang (1996) shows that introducing heterogeneity in preferences can turn a flat term structure of interest rates into one that is time-varying as aggregate consumption changes. Biais et al. (2021) study risk sharing in a dynamic exchange economy where incentive problems make securities payoffs imperfectly pledgeable, limiting agents' ability to issue liabilities. In our setting the aggregate cash-flows are endogenously generated with group members exerting time-varying influence on the investment decision. Although group members do not trade their claims, non-linearities and heterogeneity in members' claims allow for time-variation in relative consumption entitlements. Our model has closed-form solutions with n different agents, whereas many asset pricing models are specialized to two classes of consumers.

[4]Heinkel and Zechner (1990) analyze the optimal mix of debt, common equity, and preferred equity. They provide a rationale for the use of preferred equity in the presence of asymmetric information and taxes.

2 The Model Setup

Consider n investors each endowed with an amount of wealth W_{i0} $(i = 1, ..., n)$ at startup $(t = 0)$. All n investors have a power utility function and therefore constant relative risk aversion. They can, however, have a different coefficient of risk aversion γ_i, i.e. investor i's utility function is given by $u_i(c_{it}) = \frac{c_{it}^{1-\gamma_i} - 1}{1-\gamma_i}$ where $\gamma_i > 0$ for $i = 1, ..., n$ and $\lim_{\gamma_i \to 1} u_i(c_{it}) = \ln(c_{it})$.[5] All investors are strictly risk averse $(\gamma_i > 0)$ and have a subjective discount rate ρ. Without loss of generality, we assume $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$.

We first consider the scenario where each investor operates a sole proprietorship, and then consider a firm owned by a group of investors.

2.1 Sole proprietorships

Building on the classic models of Fischer et al. (1989) and Leland (1994), we consider firms with assets that generate a continuous cashflow stream. Each investor can run a sole proprietorship on the basis of her own investment opportunity set, which is to invest in a risky asset (project), and borrow or save at the risk-free rate. The after-tax return on investor i's risky asset is given by the diffusion process:

$$\frac{dA_{it}}{A_{it}} = \mu_i'(1 - \tau_c)dt + \sigma_i'(1 - \tau_c)dB_t \equiv \mu_i dt + \sigma_i dB_t \tag{1}$$

where B_t is a Brownian motion representing shocks to the economy, and τ_c is the corporate tax rate. The drift and volatility of the process are μ_i and σ_i respectively.

Investor i's sole proprietorship finances its assets with a line of credit, D_{it}, and equity, W_{it}, i.e $A_{it} = W_{it} + D_{it}$. D_{it} is the amount of net debt held by investor i. If D_{it} is

[5]In the Internet Appendix C we present the results for investors with exponential utility $u_i(c_{it}) = -exp(-\eta_i c_{it})/\eta_i$.

negative, then the firm has a net cash position. The firm can continuously roll over its net debt position. We will show that the firm never defaults under the optimal policies and that debt is risk-free. The sole proprietorship can therefore borrow and save at the risk-free rate, and the firm's instantaneous after-tax cost (return) of debt (cash) is r, i.e.[6]

$$\frac{dD_{it}}{D_{it}} = r'(1-\tau_c)dt \;\equiv\; rdt \tag{2}$$

We define $\psi_i \equiv \frac{\mu_i - r}{\sigma_i}$ as the Sharpe ratio associated with the investor i's risky asset.

At each instant t the investor receives payout c_{it}^S $(i = 1, ..., n)$, and decides on the amount A_{it} to invest in risky assets, given the firm's net worth W_{it}. The superscript S refers to the sole proprietorship case. The net worth process is therefore:

$$dW_{it} = dA_{it} - dD_{it} - c_{it}^S dt = \left[(\mu_i - r)\, A_{it} + rW_{it} - c_{it}^S\right] dt + \sigma_i A_{it} dB_t \tag{3}$$

Define $\omega_{it}^S \equiv \frac{A_{it}}{W_{it}}$ as the fraction of the firm's net worth invested in risky assets. If $\omega_{it}^S > 1$ then the firm invests all its net worth W_{it} in risky assets and borrows an amount $(\omega_{it}^S - 1)W_{it}$ that is also invested in risky assets. The process for W_{it} is now

$$dW_{it} = \left[\left(\omega_{it}^S(\mu - r) + r\right) W_{it} - c_{it}^S\right] dt + \sigma_i\, \omega_{it}^S\, W_{it}\, dB_t \tag{4}$$

Each investor maximizes the expected life-time utility from her payouts with respect to the control variables ω_{it}^S and c_{it}^S. We assume the payout c_{it}^S is not subject to personal taxes, but relax this assumption in Internet Appendix D. The maximum life-time utility $I_i(W_{it}|\psi_i)$ investor i can generate from a sole proprietorship is given by the familiar

[6] Although corporate taxes increase optimal leverage in our model, we do not need taxes to obtain a positive debt level provided that investors' risk aversion is sufficiently low.

Merton (1969) solution:

$$I_i(W_{it}|\psi_i) = \max_{\{c^S_{it};\omega^S_{it}\}} E_t\left[\int_t^\infty e^{-\rho(s-t)}u_i(c^S_{is})ds\right] = a_i(\psi_i)^{\gamma_i}\frac{W_{it}^{1-\gamma_i}}{1-\gamma_i} - \frac{1}{\rho}\frac{1}{1-\gamma_i}, \quad (5)$$

$$\omega^S_{it}(\psi_i) = \frac{\psi_i}{\sigma_i\gamma_i} \quad \text{and} \quad c^S_{it}(\psi_i) = \frac{W_{it}}{a_i(\psi_i)} \quad \text{where} \quad (6)$$

$$a_i(\psi_i) = \frac{\gamma_i^2}{r\gamma_i^2 + (\rho + \frac{\psi_i^2}{2} - r)\gamma_i - \frac{\psi_i^2}{2}} \quad \text{for } i = 1, ..., n \quad (7)$$

$a_i(\psi_i)$ is the inverse of the entrepreneur's marginal propensity to consume.

Since $D_{it} = (\omega_i^S - 1)\, W_{it}$, it follows that the optimal debt policy for each investor is to maintain a constant debt to net worth ratio (i.e. a constant book leverage target). The sole proprietorship also adopts a constant payout to net worth ratio. The financing and payout policies both depend on the sole proprietor's coefficient of risk aversion γ_i.

2.2 Firm owned by a group of investors

Assume, without loss of generality, in what follows that $\psi_1 \equiv \xi \geq \psi_2 = \psi_3 = ... = \psi_n = \beta \geq 0$. The lead investor 1 is by definition the investor with the highest Sharpe ratio ξ. In our case, the lead investor also happens to have the lowest risk aversion, but that is not necessary. We refer hereafter to lead investor 1 for short as the "entrepreneur" and to investors 2 to n as the "co-investors". Co-investors have a lower (but non-negative) Sharpe ratio β, i.e. $0 \leq \beta \leq \xi$. A zero Sharpe ratio ($\beta = 0$) means that investors' personal opportunity set is restricted to a savings account giving the risk-free rate.

The positive wedge $\xi - \beta$ between the Sharpe ratios forms the basis for cooperation between the entrepreneur and the co-investors, and it determines the size of the synergies. The entrepreneur has skills or human capital that generate superior risk-adjusted returns ($\xi \geq \beta$), whereas co-investors supply additional capital. We assume that co-investors supply their capital competitively in that their participation constraints bind at startup.

Therefore, all synergies accrue to the entrepreneur. We extend the model to a general sharing rule for the synergies in Section 4.4. After joining the group, investors have to agree on common investment and payout policies. Before solving for the group's optimal policies, we first derive the first-best outcome for each individual investor.

2.3 First-best outcome for each individual group member

When joining the group, each investor i contributes an amount W_{i0} (for $i = 1, ..., n$) giving a total initial net worth for the firm equal to $W_0 = \sum_{i=1}^{n} W_{i0}$. In return for the capital contribution each investor acquires a net worth stake in the firm, which we denote by $\tilde{W}_{i0}$ with $\sum_{i=1}^{n} \tilde{W}_{i0} = W_0$. The first-best outcome for investor i with stake $\tilde{W}_{i0}$ is the expected life-time utility she can achieve by running the group according to her personal preferences. This would allow her to achieve a life-time utility equal to $I_i(\tilde{W}_{i0}|\xi)$ for a net worth stake $\tilde{W}_{i0}$. The net worth stake $\tilde{W}_{i0}$ that satisfies her participation constraint under her optimal investment and payout policies is the solution to:

$$I_i(\tilde{W}_{i0}|\xi) \;=\; I_i(W_{i0}|\beta) \qquad for\ i = 2, ..., n \tag{8}$$

Since $\xi \geq \beta$, it follows that $W_{i0} - \tilde{W}_{i0} \geq 0$ for $i = 2, ..., n$, which means that co-investors acquire their net worth stake in the firm at a premium. The lead investor's net worth after the startup is then:

$$\tilde{W}_{10} \;=\; \sum_{i=1}^{n} W_{i0} - \sum_{i=2}^{n} \tilde{W}_{i0} \;>\; W_{10} \tag{9}$$

It follows that the lead investor's participation constraint is automatically satisfied (since $I_1(\tilde{W}_{10}|\xi) \geq I_1(W_{10}|\xi)$). Using the Merton (1969) solution (see Equations (5), (6), and (7)), the first-best outcome can be summarized in the following proposition.

Proposition 1 *If investor i's net worth stake in the group is $\tilde{W}_{i0}$ then her first-best expected life-time utility is given by $I_i(\tilde{W}_{i0}|\xi)$ and her corresponding optimal investment and payout policies are:*

$$c^o_{it}(\tilde{W}_{it}|\xi) = c^S_{it}(\xi) \quad = \quad \frac{\tilde{W}_{it}}{a_i(\xi)} \tag{10}$$

$$\omega^o_{it}(\tilde{W}_{it}|\xi) = \omega^S_{it}(\xi) \quad = \quad \frac{\xi}{\sigma\gamma_i} \tag{11}$$

Investor i's net worth $\tilde{W}_{i0}$ in the group is defined by Equations (8) and (9). The evolution of her net worth under her optimal policy is given by:

$$\frac{d\tilde{W}_{it}}{\tilde{W}_{it}} = \left[\frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i(\xi)}\right] dt + \frac{\xi}{\gamma_i} dB_t \tag{12}$$

Note that investor i's policies are not optimal for her co-investors. Investor j's (for $j = 1,..,n$ and $j \neq i$) optimal investment and payout policies are given by $\frac{\xi}{\sigma\gamma_j}$ and $\tilde{W}_{jt}/a_j(\xi)$ (and not $\frac{\xi}{\sigma\gamma_i}$ and $\tilde{W}_{jt}/a_i(\xi)$). Therefore, finding a common policy for a group of investors with heterogeneous preferences is a non-trivial task.

2.4 The group's optimal policies

Previously, we considered a first-best scenario where each investor sets investment and payout according to her personal preferences. In practice, the investors have to set one single investment policy, ω_t, and jointly agree on how to share the firm's cash-flows among investors (i.e. c_{it} for $i = 1, ..., n$).

The n investors each contribute an amount W_{i0} ($i = 1, ..., n$) to the firm's net worth ($W_0 = \sum_{i=1}^{n} W_{i0}$) at startup. At each instant t the investors receive payout c_{it} ($i = 1, ..., n$), and set the amount A_t to invest in risky assets, given the firm's net worth W_t.

Define $\omega_t \equiv \frac{A_t}{W_t}$. To simplify the notation, define $\mu \equiv \mu_1$, $\sigma \equiv \sigma_1$, so that $\xi \equiv (\mu - r)/\sigma$. The firm's net worth process is then:

$$dW_t \;=\; \left[(\omega_t(\mu \,-\, r) \,+\, r)\, W_t \,-\, \sum_{i=1}^{n} c_{it} \right] dt \,+\, \sigma\, \omega_t\, W_t\, dB_t \tag{13}$$

We now state a conjecture that we verify later (see Proposition 5).

Conjecture 1 *Investors' first-best life-time utility level and the group's optimal policies ($c^*_{1t}, ..., c^*_{nt}$ and ω_t) can be obtained as the solution to the following social planner problem.*

$$J(W_t) = \max_{\{c_{1t},...,c_{nt};\ \omega_t\}} E_t \left[\sum_{i=1}^{n} \lambda_i \int_t^{\infty} e^{-\rho(s-t)} u_i(c_{is})\, ds \right] = \sum_{i=1}^{n} \lambda_i J_i(W_t) \tag{14}$$

subject to the transversality condition $\lim_{t\to\infty} [e^{-\rho t} J(W_t)] \,=\, 0$ *and the budget constraint (13). The utility weights λ_i are the solution to:*

$$\begin{aligned} J_i(W_0) \;&=\; I_i(W_{i0}|\beta) \quad \text{for} \quad i = 2, ..., n \quad \text{(participation constraints)} \\ \sum_{i=1}^{n} W_{i0} \;&=\; W_0 \quad \text{(total net worth constraint)} \end{aligned}$$

where $J_i(W_t)$ denotes investor i's life-time utility from being part of a firm with n investors and total net worth W_t.

Objective function (14) is known as the weighted-sum-of-utilities welfare function, a generalization of the classical utilitarian welfare function. Note that investors derive utility from dividends c_{it} only (i.e. they do not sell their claim to generate income). We will show that under the group's optimal policies, its members achieve their first-best expected life-time utility (i.e. $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi)$ for $i = 1, ..., n$). The first-best outcome is achieved without trading. The utility weights λ_i will be determined below by investors' participation constraint at startup. It is well known (see Duffie (2001), Chapter 10) that a necessary and sufficient condition for $(c_{1t}, ..., c_{nt})$ to be a Pareto efficient allocation is that there exist non-negative weights $(\lambda_1, \lambda_2, ..., \lambda_n)$ that solve the above optimization problem. We will verify that such positive weights indeed exist (see Proposition 4).

In what follows, we solve the social planner problem backwards. We first solve for the firm's optimal investment and payout policies, and for investors' life-time utility taking the utility weights as given (Proposition 2). Next, we derive the utility weights that satisfy co-investors' participation constraints at startup (Proposition 4). We conclude by verifying the above conjecture that the solution to the social planner's problem is indeed first-best (Proposition 5).

3 Optimal Financial Policies

Solving for payout policies $(c_{1t}, ..., c_{nt})$ and investment policy ω_t that maximize the social planner's objective function (14) subject to the intertemporal budget constraint (13), and the transversality condition $\lim_{t\to\infty} [e^{-\rho t} J(W_t)] = 0$ gives the following proposition.

Proposition 2 *The group's optimal investment* (ω_t^*) *and payout* (c_{it}^*) *policies for any positive utility weights* λ_i *are:*

$$\omega_t^* = \left(\frac{A_t}{W_t}\right)^* = \frac{\xi}{\sigma}\frac{W'(z_t)}{W(z_t)} = \sum_{i=1}^{n} \phi_i(z_t)\frac{\xi}{\sigma\gamma_i} \tag{15}$$

$$c_{it}^* = \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} = \phi_i(z_t)\frac{W_t}{a_i(\xi)} \qquad \textit{for} \quad i = 1, ..., n \tag{16}$$

where the auxiliary state variable $z_t \equiv -ln(J'(W_t))$ *is the solution to*

$$W(z_t) = \sum_{i=1}^{n} a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} \equiv \sum_{i=1}^{n} \tilde{W}_{it} \quad \textit{and where} \tag{17}$$

$$\phi_i(z_t) \;=\; \frac{a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}}{\sum_{j=1}^n a_j(\xi)\lambda_j^{\frac{1}{\gamma_j}} e^{\frac{z_t}{\gamma_j}}} \quad \textit{and} \quad \sum_{i=1}^n \phi_i(z_t) \;=\; 1 \tag{18}$$

$a_i(\xi)$ is defined in Equation (7). The group's joint weighted life-time utility, $J(W_t)$, is given by:

$$J(W(z_t)) = \sum_{i=1}^n \lambda_i \, J_i(z_t) \;=\; \sum_{i=1}^n \lambda_i \left[\frac{a_i(\xi)}{1 \,-\, \gamma_i} (\lambda_i e^{z_t})^{\frac{1}{\gamma_i}-1} \,-\, \frac{1}{\rho} \frac{1}{1 \,-\, \gamma_i} \right] \tag{19}$$

The expressions for the endogenized utility weights λ_i are given in Proposition 4.

The investment and payout policies, and the group's weighted claim value are not explicitly expressed as a function of its total net worth W_t but as a function of an auxiliary variable z_t. Net worth is, however, a continuous, monotonically increasing function of z as expressed by Equation (17). z_t can be interpreted as some kind of "normalized" wealth variable that allows solutions to be expressed in closed form. As z_t ranges from $-\infty$ to $+\infty$, W_t varies from 0 to $+\infty$, i.e. $\lim_{z_t \to -\infty} W(z_t) = 0$ and $\lim_{z_t \to +\infty} W(z_t) = +\infty$. The firm never defaults because zero net worth is never reached.

3.1 Investment and net debt policies

We find that the optimal investment policy ω_t^* of a group of investors is a weighted average of the optimal investment policies $\omega_i^S(\xi)$ of the individual investors evaluated at the superior Sharpe ratio ξ. The weight $\phi_i(z_t)$ reflects member i's influence or "control" over the investment policy. We can rewrite $\phi_i(z_t)$ as

$$\phi_{it} \;\equiv\; \phi_i(z_t) \;=\; \frac{c_{it}^*}{W_t / a_i(\xi)} \;=\; \frac{c_{it}^*}{c_{it}^S(\xi)}$$

The optimal control weights ϕ_{it} (not to be confused with the "utility" weights λ_i) are given by the ratio $c_{it}^*/c_{it}^S(\xi)$ of the individuals' payout c_{it}^* and her optimal payout $c_{it}^S(\xi)$ if she ran the whole firm alone. Importantly, the control weights ϕ_{it} are time-varying as they depend on the firm's net worth W_t through the auxiliary variable z_t.

Corollary 1 *The firm's risky asset to net worth ratio, ω_t^*, increases in its net worth, i.e.* $\frac{\partial \omega_t^*}{\partial W_t} \geq 0$

The corollary implies that the firm invests more aggressively in risky assets in good times (when its net worth has grown), and reduces its weight in risky assets during bad times.[7] Consider the n-investor case where investor 1 (n) is least (most) risk averse (i.e. $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$). In that case,

$$\lim_{z_t \to +\infty} \omega^*(z_t) = \frac{\xi}{\sigma\gamma_1} \quad \text{and} \quad \lim_{z_t \to -\infty} \omega^*(z_t) = \frac{\xi}{\sigma\gamma_n} \tag{20}$$

As the firm accumulates (loses) more wealth, its investment policy converges towards the one of the least (most) risk averse investor. $\omega^*(z_t)$ is an S-shaped function situated between the asymptotes $\frac{\xi}{\sigma\gamma_1}$ and $\frac{\xi}{\sigma\gamma_n}$. $\omega^*(z_t)$ is constant for the knife-edge case of homogenous risk preferences ($\gamma_i = \gamma$ for all i), and given by the standard Merton (1969) ratio $\omega = \xi/(\sigma\gamma)$.

The private firm's optimal net debt level follows from the balance sheet identity $D_t = A_t - W_t = (\omega_t^* - 1)W_t$. If $D_t < 0$ then the firm's cash-holdings exceed its debt, so that the firm has a net cash surplus (or negative debt). The below corollary states the firm's net debt ratio under the optimal investment and payout policy. With negative debt, the traditional net debt ratio (NDR) is hard to interpret and can become unbounded. Lambrecht and Pawlina (2013) provide an economic rationale for using the below piecewise definition ($D_t \geq 0$ versus $D_t < 0$) which ensures that the NDR stays

[7]The proofs of the corollaries are in the Internet Appendix A.

within the interval $[-1,1]$. Corollary 1 and Equation (20) then lead to the following corollary.

Corollary 2 *Define the firm's net debt ratio as* $NDR_t \equiv \frac{D_t}{W_t+\Psi D_t}$ *where* $\Psi = 1$ *if* $D_t \geq 0$ *and* $\Psi = 0$ *if* $D_t < 0$*. The NDR is time-varying, procyclical, and given by*

$$-1 \leq \frac{\frac{\xi}{\sigma\gamma_n} - 1}{1 + (\frac{\xi}{\sigma\gamma_n} - 1)\Psi} < NDR_t = \frac{\omega_t^* - 1}{1 + (\omega_t^* - 1)\Psi} < \frac{\frac{\xi}{\sigma\gamma_1} - 1}{1 + (\frac{\xi}{\sigma\gamma_1} - 1)\Psi} \leq 1 \qquad (21)$$

The firm's leverage is procyclical and increases in W_t. For low levels of net worth (e.g. at startup) and assuming some investors are sufficiently risk averse, the firm initially keeps a net cash balance and only invests a fraction of its net worth in risky assets (i.e. $\omega_t^* < 1$). As the firm accumulates retained earnings, the optimal NDR rises and converges towards the optimal NDR of the least risk averse investor, which involves net borrowing if some investors have a sufficiently low coefficient of risk aversion. In bad times, as net worth shrinks, the firm rebalances by selling or shutting down risky assets, and using the proceeds to pay down its line of credit. As $W_t \to 0$ (or $z_t \to -\infty$), the NDR converges towards the most risk-averse investor's optimal NDR. Dynamic rebalancing in response to shocks allows the firm to keep debt safe all the time. In other words, safe debt arises endogenously from our model because of the firm's ability to continuously rebalance assets and liabilities.

As co-investor n becomes infinitely risk averse ($\gamma_n \to +\infty$), the lower bound for the NDR converges to -1, which represents an all equity-financed firm that invests in the safe asset (cash) only. As investor 1 (i.e. the entrepreneur) moves towards risk neutrality ($\gamma_1 \to 0$), the upper bound for the NDR converges to +1, which represents a near 100% debt-financed firm that invests in risky assets only. Therefore, if the most and least risk averse investors have highly divergent risk preferences, then the intertemporal variation in the firm's NDR across the business cycle could be large.

3.2 Payout policy

The payout (in dollars) to each investor is increasing but non-linear in the firm's net worth if investors have different coefficients of risk aversion. In contrast, the payout is linear in net worth for the sole proprietorship. Since $c_{it}^* = \phi_{it} W_t / a_i(\xi)$ (see Equation (16)), the non-linearity is driven by the time-varying control weight ϕ_{it}. For the n-investor case one can prove the following proposition:

Proposition 3 *Payout is a convex (concave) increasing function of the firm's total net worth W_t for the least (most) risk averse investor. For investors with an intermediate level of risk aversion, there exists a critical wealth level W_i^* $(i = 2, 3, ..., n-1)$ such that payout is convex in wealth for low levels of wealth (i.e. $W_t \leq W_i^*$) and concave in wealth for high levels of wealth (i.e. $W_t \geq W_i^*$). Furthermore, $\lim_{z_t \to -\infty} c_{it}^* = 0 \quad for \quad i = 1, ..., n,$ and*

$$\lim_{z_t \to +\infty} \frac{\partial c_{1t}^*}{\partial W_t} = \frac{1}{a_1(\xi)} \quad and \quad \lim_{z_t \to +\infty} \frac{\partial c_{jt}^*}{\partial W_t} = 0 \quad for \quad j = 2, 3, ..., n$$

Payout is linear in net worth if investors have homogeneous risk preferences:

$$\frac{\partial^2 c_{it}^*}{\partial W_t^2} = 0 \text{ and } c_{it}^* = \frac{\lambda_i^{\frac{1}{\gamma}}}{\sum_{j=1}^n \lambda_j^{\frac{1}{\gamma}}} \frac{W_t}{a(\xi)} \quad for \quad i=1,...,n \Leftrightarrow \gamma_1 = ... = \gamma_n = \gamma \tag{22}$$

It is Pareto optimal for the least (most) risk averse agent to have a convex (concave) payout function. Since all payouts are zero for $W_t = 0$, it follows that, among all investors, the most risk averse investor is paid the most for very low levels of net worth, whereas the least risk-averse investor gets paid the most for very high levels of net worth. The least risk averse investor (i.e. the entrepreneur) is the residual claimant who gets the upside potential in good times, but is heavily exposed in bad times. All other co-investors are less exposed in downturns, but their payouts are capped as the firm grows and accumulates net worth. The diversity in payouts reconciles group members' heterogeneous risk preferences, allows for efficient risk sharing, and means that investors do not have to trade their claims to smooth consumption. Each investor's payout is non-linear in total net worth, except if all investors have the same coefficient of risk aversion. These findings are also reflected in investors' relative cash flow rights:

$$\frac{c^*_{jt}}{c^*_{it}} = \frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z_t\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} \tag{23}$$

An investor's payout is strictly proportional to her utility weight λ_i if and only if all investors have logarithmic utility. For log utility the utility weights are equivalent to ownership shares that entitle investors to a constant share of the firm's total payout.

Consider next the firm's payout yield c^*_t/W_t. It follows from Proposition 2 that:

$$\frac{c^*_t}{W_t} = \sum_{i=1}^{n} \frac{c^*_{it}}{W_t} = \sum_{i=1}^{n} \frac{\phi_{it}}{a_i(\xi)} \tag{24}$$

The payout yield or average propensity to consume, c^*_t/W_t, is time-varying, except if all investors have the same coefficient of risk aversion γ, in which case

$$\frac{c^*_t}{W_t} = \frac{\sum_{i=1}^n \lambda_i^{\frac{1}{\gamma}} e^{\frac{z_t}{\gamma}}}{\sum_{i=1}^n a_i(\xi)\lambda_i^{\frac{1}{\gamma}} e^{\frac{z_t}{\gamma}}} = \frac{\rho}{\gamma} - \frac{r(1-\gamma)}{\gamma} - \frac{\xi^2(1-\gamma)}{2\gamma^2} \equiv \frac{1}{a(\xi)} \text{ if } \gamma_i = \gamma \text{ for all } i. \tag{25}$$

This is the marginal propensity to consume defined in Equation (7) and corresponds to the Merton (1969) solution (see Merton (1969, Equation (40)).

3.3 Group risk aversion and financial policies

Corollary 3 *The group's level of relative risk aversion (RRA_G) is endogenous and can be defined in terms of the value function as:*

$$RRA_G \equiv -W_t \frac{J_{WW}}{J_W} = \frac{W(z_t)}{W'(z_t)} = \frac{\xi}{\sigma\,\omega_t^*} \tag{26}$$

Using our comparative statics for ω_t^* (see Section 3.1), it follows that:

$$\frac{\partial RRA_G}{\partial W_t} \leq 0 \quad \text{and} \quad \lim_{z_t \to +\infty} RRA_G = \gamma_1 \quad \text{and} \quad \lim_{z_t \to -\infty} RRA_G = \gamma_n \tag{27}$$

Although each individual investor has a constant level of relative risk aversion, the group's level of relative risk aversion decreases in its net worth, and converges to the level of the least (most) risk averse investor as net worth goes to infinity (zero). This explains the earlier described behavior of the firm's investment and financing policy. More generally, it may also help explain why groups are very bullish during booms and extremely risk averse during recessions (see Shupp and Williams (2008)). Finally, $RRA_G = \gamma$ if all investors have the same coefficient of risk aversion γ.

4 Utility weights and investors' claims

4.1 Utility weights

Investors' utility weights λ_i are fixed at startup ($t = 0$) and determine investors' future cashflows from the firm. Co-investors receive utility weights such that their expected life-time utility from joining the firm with Sharpe ratio ξ equals the expected life-time utility they can achieve as a sole proprietor with Sharpe ratio β, i.e. $J_i(W_0) = I_i(W_{i0}|\beta)$ for $i =$

$2, ..., n$. Solving the participation constraints and the net worth constraint $\sum_{i=1}^{n} W_{i0} = W_0$ for the utility weights gives the following proposition.

Proposition 4 *Investors' utility weights are given by:*

$$\lambda_1 = \left[W_{10} + \sum_{i=2}^{n} \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \right) W_{i0} \right]^{\gamma_1} a_1(\xi)^{-\gamma_1} \, e^{-z_0} \tag{28}$$

$$\lambda_i = W_{i0}^{\gamma_i} \left[\frac{a_i(\beta)^{\gamma_i}}{a_i(\xi)} \right]^{\frac{\gamma_i}{1-\gamma_i}} e^{-z_0} \qquad for \; i = 2, ..., n \tag{29}$$

The factor $e^{-z_0} (= J'(W_0))$ implies that Proposition 4 uniquely defines the weights in relative terms, not in absolute terms. More importantly, investors' utility weights are not proportional to the amounts of capital they invest at startup. The exception is when all investors have log utility.

Corollary 4 *Investors' utility weights are proportional to the equity capital they invest if and only if all investors have logarithmic utility (i.e. $\forall i : \gamma_i = 1$), in which case:*

$$\frac{\lambda_i}{\lambda_j} = \frac{W_{i0}}{W_{j0}} \quad for \; i, j = 1, ..., n. \tag{30}$$

If all investors have log utility then the utility weights are equivalent to traditional ownership shares that are strictly proportional to the equity capital contributed by each investor. In that case, the objective function (14) has a straightforward interpretation in that utilities are weighted according to investors' ownership share in the firm. Any utility function other than log utility generates skewed utility weights (see Section 4.5 for a numerical example). Strikingly, the entrepreneur receives a positive utility weight λ_1 even if she contributes zero capital (i.e. $W_{10} = 0$). The entrepreneur's "excess" weight is a compensation for the superior technology, human capital or "sweat equity" she contributes, as reflected in the wedge $\xi - \beta$.

4.2 Investors' claims

Let us consider next $\tilde{W}_{it}$, investors' claim in the private firm's net worth, as defined by Equation (17) in Proposition 2:

$$\tilde{W}_{it} \;=\; a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} \;=\; a_i(\xi) c^*_{it} \;=\; \phi_{it}\, W_t \;\; \text{with} \;\; \sum_{i=1}^{n} \tilde{W}_{it} = W_t \tag{31}$$

The following corollary describes investors' equity claim value as a function of the firm's net worth.[8] Since this value is a constant multiple of payout (i.e. $\tilde{W}_{it} = a_i(\xi)c^*_{it}$) the corollary mirrors the results we obtained in Proposition 3 for investors' payout policies c^*_{it}.

Corollary 5 *The claim value of the most (least) risk averse investor is a concave (convex) function of the firm's total net worth W_t. The claim of investors with intermediate levels of risk aversion is S-shaped in net worth. Moreover,* $\lim_{z_t\to-\infty} \tilde{W}_{it} \;=\; 0$ *for* $i = 1, ..., n$*, and*

$$\lim_{z_t\to+\infty} \frac{\partial \tilde{W}_{1t}}{\partial W_t} \;=\; 1 \quad \mathit{and} \quad \lim_{z_t\to+\infty} \frac{\partial \tilde{W}_{it}}{\partial W_t} \;=\; 0 \quad \mathit{for} \quad i = 2, ..., n \tag{32}$$

$$\lim_{z_t\to-\infty} \frac{\partial \tilde{W}_{nt}}{\partial W_t} \;=\; 1 \quad \mathit{and} \quad \lim_{z_t\to-\infty} \frac{\partial \tilde{W}_{it}}{\partial W_t} \;=\; 0 \quad \mathit{for} \quad i = 1, ..., n-1 \tag{33}$$

Finally, $\exists\, W^*_i \in \mathbb{R}^+$ *such that* $\frac{\partial^2 \tilde{W}_{it}}{\partial W_t^2} \;\geq (\leq)\; 0 \;\;\Leftrightarrow\;\; W_t \;\leq (\geq)\, W^*_i$ *for* $i = 2, ..., n-1$.

As the firm becomes very wealthy, an additional dollar of net worth is almost entirely to the benefit of the least risk averse investor (i.e $\lim_{z_t\to+\infty} \frac{\partial \tilde{W}_{1t}}{\partial W_t} \;=\; 1$), whereas the other investors' claims converge to a fixed value. Conversely, for a firm with little or no wealth, the marginal dollar accrues almost entirely to the most risk averse investor (i.e $\lim_{z_t\to-\infty} \frac{\partial \tilde{W}_{nt}}{\partial W_t} = 1$).

[8] The corollary can be reformulated to describe claim values as a function of the firm's holding in risky assets A_t. The results are very similar.

The strictly convex claim of the least risk averse investor ($i = 1$), the entrepreneur, is unbounded and resembles a common stock contract. The claims of investors 2 to $n-1$ are convex (concave) for low (high) values of net worth, with each claim converging towards a maximum value and resembling a preferred stock contract.[9] In simple terms, one could view preferred stock as a debt instrument when net worth is high, and as an equity instrument when net worth is low. This explains why preferred stock values can be convex and concave for, respectively, low and high values of net worth W_t.

The claim of the most risk averse investor ($i = n$) is strictly concave in W_t, and corresponds to the most senior preferred stock contract. For the lowest levels of net worth, an extra dollar for the firm accrues almost entirely to the most risk averse investor. As the firm accumulates more net worth, more junior preferred stock holders start sharing in the payouts. The claims of these other investors resemble preferred stock contracts for which the level of seniority increases in the degree of risk aversion, and for which the face value of the contract (achieved as $W_t \to +\infty$) decreases in the degree of risk aversion. The maximum value of each preferred stock contract is determined by the maximum payout rate c_{it} as $W_t \to +\infty$. The variation in the shapes of the claim values is entirely driven by differences in investors' coefficient of risk aversion, as illustrated by the following corollary:

Corollary 6 *If all investors have the same coefficient of risk aversion ($\gamma_1 = \gamma_2 = ... = \gamma_n = \gamma$), then the investors' claim values are linear in the firm's total net worth, i.e.*

$$\tilde{W}_{it} = \frac{\lambda_i^{\frac{1}{\gamma}}}{\sum_{i=1}^{n} \lambda_i^{\frac{1}{\gamma}}} W_t \quad for \quad i = 1, 2, ..., n \tag{34}$$

[9]In practice, preferred stock contracts are senior to common stock in that dividends may not be paid to common stock, unless the dividend is paid on all preferred stock. The dividend rate on preferred stock is usually fixed at the time of issue. The preferred stock is junior to the debt claim D_t. Unlike debt contracts, the firm is not in default when it suspends dividends to preferred stockholders.

If all investors have the same coefficient of risk aversion, then they share profits according to fixed proportions. This gives rise to common stock contracts that are linear in net worth. If all investors have log utility ($\gamma_i = 1$ for $i = 1, ..., n$) and the utility weights sum to one, then each investor's equity claim value is proportional to her utility weight λ_i (i.e. $\tilde{W}_{it} = \lambda_i W_t$) and utility weights can be interpreted as ownership shares.

Corollary 7 *The entrepreneur gets her equity claim in the firm ($\tilde{W}_{10}$) at a discount, whereas co-investors buy their equity claim ($\tilde{W}_{i0}$, $i = 2, ..., n$) at a premium:*

$$W_{10} - \tilde{W}_{10} = W_{10} + \sum_{i=2}^{n} \tilde{W}_{i0} - W_0 = \sum_{i=2}^{n} \left[\left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} - 1 \right] W_{i0} \leq 0 \quad (35)$$

$$W_{i0} - \tilde{W}_{i0} = \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \right) W_{i0} \geq 0 \quad for\ i = 2, ..., n \quad (36)$$

At startup, co-investors' ($i = 2, ..., n$) net worth stake in the firm ($\tilde{W}_{i0}$) is less than the equity capital (W_{i0}) they each contribute to the firm. This leads to an instantaneous drop in co-investors' payout compared to what they would get as sole proprietors. However, in return for paying a premium $W_{i0} - \tilde{W}_{i0}$, co-investors acquire assets that have a higher Sharpe ratio ($\xi \geq \beta$) and that generate a payout stream with a higher expected growth and higher volatility (see Internet Appendix B for further details). Conversely, the entrepreneur acquires her stake in the firm at a discount since $W_{10} \leq \tilde{W}_{10}$. The discount is an upfront compensation for sharing her superior technology or human capital with co-investors and leads to an instantaneous rise in her payout level. Since the entrepreneur's Sharpe ratio is unaffected by the capital infusion, neither is the rate of growth and volatility of her payout. The entrepreneur's discount and the co-investors' premium goes to zero as β converges to ξ (see Equations (35) and (36)).

The premium paid by co-investors effectively "locks" them into the private firm as it is not in their interest to pull out their net worth stake $\tilde{W}_{i0}$ once invested. The co-investors' life-time utility in the firm exceeds what they can achieve outside the firm by reinvesting their net worth stake $\tilde{W}_{i0}$ since $J_i(W_0) = I_i(W_{i0}|\beta) = I_i(\tilde{W}_{i0}|\xi) \geq I_i(\tilde{W}_{i0}|\beta)$ for $i = 2, ..., n$. The premium is similar to a joining fee for access to superior risk-adjusted expected returns. Once the co-investors have paid this "fee" it no longer makes sense for them to withdraw their net worth stake from the firm, even if they were able or allowed to do so.[10] The entrepreneur neither has an incentive to abandon the firm as she achieves the same life-time utility from the private firm as from a sole proprietorship with net worth $\tilde{W}_{10}$ and Sharpe ratio ξ. Furthermore, the entrepreneur benefits from co-investors joining the firm, i.e. $J_1(W_0) = I_1(\tilde{W}_{10}|\xi) \geq I_1(W_{10}|\xi)$. We can then prove the following proposition, which verifies conjecture 1.

Proposition 5 *Investors' life-time utility from the group is identical to the first-best outcome presented in Proposition 1, i.e.* $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi)$*. Also investors' payouts are at all time the same, i.e.* $c^*_{it}(W_t) = c^o_{it}(\tilde{W}_{it}|\xi)$ *for all* t*. The evolution of investor* i*'s net worth stake* $\tilde{W}_{it}$ *in the group is characterized by Equation (12) and is the same as if investor* i *ran a sole proprietorship with Sharpe ratio* ξ *and net worth* $\tilde{W}_{it}$*.*

Despite the fact that heterogeneous investors have to agree on a common investment policy ω_t for the group, each individual investor i achieves exactly the same life-time utility from the group as a sole proprietor with Sharpe ratio ξ and net worth stake $\tilde{W}_{it}$. The group achieves this first-best outcome by adopting dynamic investment and payout policies that are non-linear in the firm's net worth.

[10] Partnerships or private firms usually do not allow investors unilaterally to withdraw their capital from the firm, or only under very strict conditions.

4.3 Financial policies and claim values: a numerical example

Figure 1 numerically evaluates the firm's optimal payout and financing policies. The firm has 4 investors with coefficients of relative risk aversion $\gamma_1 = 0.7, \gamma_2 = 2, \gamma_3 = 3, \gamma_4 = 9$, and with initial endowments $W_{10} = W_{20} = W_{30} = W_{40} = 25$.[11] We compare the group's policies with the investors' individual optimal policies (i.e. the policies each investor would adopt if she could run the firm with Sharpe ratio ξ according to her personal preference).

Panel A of Figure 1 plots investors' payouts as a function of the firm's net worth. For very low (high) net worth levels payout is increasing (decreasing) in investors' coefficient of risk aversion. In bad times, the more risk averse investors enjoy a higher payout level at the expense of less risk averse investors who take the hit. In good times, the least risk averse investor enjoys the upside, whereas all other agents' payout is capped. The cap decreases with investors' coefficient of risk aversion. This means that the payout function for each investor, except for the least risk averse one (represented by the dotted line), converges to a cap as wealth becomes large. The payout function for the least (most) risk averse investors is strictly convex (concave). For the other investors, the payout function is initially convex up to some inflection point, and concave thereafter.

Panel B plots investors' claim values as a function of total net worth. The claim values mirror the behavior of the payout functions in Panel A. The claim is strictly concave (convex) for the most (least) risk averse investor, and S-shaped for all other investors.

[11] Risk preferences are known to vary widely across the population (see for instance Barsky et al. (1997)). The most commonly accepted measures of the coefficient of relative risk aversion lie between 1 and 3, but there is a wide range of estimates in the literature, from as low as 0.2 to 10 and higher (See Chetty (2006), Campo et al. (2011), among others). Our parameter values for γ_i fall within this range. We set $r = 0.03$ and $\tau_c = 0.21$ in line with current practice in the US. We set the subjective discount rate $\rho = 0.1$, which is within the range commonly used in the corporate finance literature (for example, Bolton et al. (2011) and Lambrecht and Myers (2017)). We set $\mu = 0.6$ and $\sigma = 0.89$, in line with the estimates of the annualized volatility of venture capital investment returns, which reflects investment returns on startups (see Cochrane (2005), Korteweg and Sorensen (2010), Gornall and Strebulaev (2020)). The corresponding Sharpe ratio of the entrepreneur's investment opportunity, ξ, is 0.64.

Except for the claim held by the least risk averse investor, all other claims converge to a bounded value as $W_t \to +\infty$. The bounded claims resemble preferred stock claims.[12]

Panel C plots the payout yields as a function of net worth. The optimal payout yield is constant when investors operate on their own (or all have the same coefficient of risk aversion) as depicted by the four constant, horizontal lines. The optimal payout yields are an inverted U-shaped function of the coefficient of RRA, a standard feature of the Merton (1969) solution. In our example, investor 2 ($\gamma_2 = 2$) receives the highest payout yield equal to 11.3%. The payout yield of the group c_t^*/W_t (given by the thick solid line) initially increases in net worth, reaches a maximum of about 7.41% and thereafter, as W_t goes to infinity, converges to 0.71%, the optimal yield of investor 1 ($\gamma_1 = 0.7$).

Panel D plots the firm's NDR as a function of net worth (thick solid line). The firm holds a net cash position (i.e. negative debt) most of the time given the high riskiness of the firm's assets ($\sigma = 0.89$), co-investors' high risk aversion and the relatively low corporate tax rate ($\tau_c = 21\%$). The firm's NDR is procyclical and converges towards a maximum of 0.027 as $W_t \to +\infty$. The firm delevers in economic downturns, and reduces its holdings in risky assets. The NDR approaches -0.92 as $W_t \to 0$. A negative NDR of -0.92 means that the firm's net worth is invested for 92% in cash and the remainder in risky assets. In line with Proposition 2 the firm's NDR is bounded below and above by the optimal NDR for the most and least risk averse investor, respectively. Therefore, our model generates a large amount of intertemporal variation in leverage if investors' risk preferences are widely dispersed. For a single investor with coefficient of risk aversion equal to 0.7, 2, 3 or 9, the sole proprietorship adopts a constant net debt ratio of, respectively, 0.027, -0.64, -0.76 and -0.92.

[12]For example, the preferred stock valuation model by Emanuel (1983) produces claim values for cumulative preferred stock and non-cumulative preferred stock that are graphically very similar to ours (see Figures 3 and 4 on pages 1148 - 1149 in Emanuel (1983)), even though his valuation formulas are very different.

Panel E plots investors' control weights ϕ_{it}. As net worth rises, less risk averse investors' influence over the firm's investment and financing policy increases. This relation can, however, be non-monotonic as illustrated by the control weights of investors 2 and 3, which first increase and then decrease in the firm's net worth. This highlights the complicated nature of group behavior.

Panel F plots the group's level of relative risk aversion, RRA_G. Group risk aversion is fairly low (around 0.71) for most levels of net worth, but spikes up dramatically once net worth drops below 100, converging to 9 as W_t goes to zero. During busts the group's risk aversion and financial policies are determined (almost) entirely by the most risk averse investor.

4.4 Group synergies

The following corollary compares the aggregate utility of the group $J_G(W)$ with the aggregate utility $I(W_1, ..., W_n)$ of the n sole proprietorships.

Corollary 8 *The welfare gain created by combining the $n-1$ co-investors with the entrepreneur in one firm is:*

$$J_G(W_0) - I(W_{10}, ..., W_{n0}) = \sum_{i=1}^{n} J_i(z_0) - \left[I_1(W_{10}|\xi) + \sum_{i=2}^{n} I_i(W_{i0}|\beta) \right] = J_1(W_0) - I_1(W_{10}|\xi)$$

As co-investors' Sharpe ratio β approaches the entrepreneur's Sharpe ratio ξ, the welfare gain goes to zero.

The synergies and resulting welfare gains arise from the entrepreneur generating a higher Sharpe ratio ξ than what co-investors can achieve on their own. Since capital is supplied competitively and co-investors do not add any expertise that might increase the firm's Sharpe ratio, all welfare gains accrue to the entrepreneur.

Our model can be generalized to allow synergies to be distributed across all investors. We know from Equations (35) and (36) that the total net worth transfer from the co-investors to the entrepreneur equals:

$$\Delta \equiv \sum_{i=2}^{n} \left[1 - \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \right] W_{i0} \tag{37}$$

The minimum net worth stakes $\underline{\tilde{W}}_{i0}$ in the group that satisfy investors' participation constraint are:

$$\underline{\tilde{W}}_{10} = W_{10} \quad \text{and} \quad \underline{\tilde{W}}_{i0} = W_{i0} \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \quad \text{for} \quad i = 2, ..., n \tag{38}$$

If $(s_1, ..., s_n)$ is a sharing rule that distributes the utility gains across all investors, then the corresponding net worth stakes and utility weights are given by:

$$\tilde{W}_{i0} = \underline{\tilde{W}}_{i0} + s_i \Delta \;\; \text{with} \;\; \sum_{i=1}^{n} s_i = 1 \;\; \text{for} \;\; i = 1, ..., n \tag{39}$$

$$\lambda_i = \left(\frac{\tilde{W}_{i0}}{a_i(\xi)} \right)^{\gamma_i} e^{-z_0} \;\; \text{for} \;\; i = 1, ..., n \tag{40}$$

For example, $s_1 = 1$ and $s_i = 0$ (for $i = 1, .., n$) give the entrepreneur all synergy surplus, and correspond to the case we previously developed.

4.5 Utility weights and synergies: A numerical example

We conclude this section by providing a numerical example that calculates investors' utility weights, net worth claim value, and life-time utility. Table 1 considers the same firm (as in Figure 1), which has 4 investors with different coefficients of risk aversion ranging from $\gamma_1 = 0.7$ to $\gamma_4 = 9$. All investors make an initial capital contribution equal to 25. The parameter values are the same as for Figure 1 (see footnote 11).

Table 1: Investors' utility weights, net worth claims, and life-time utilities

Investors			A. $\beta = 0$				B. $\beta = 0.5\xi$				C. $\beta = \xi$			
ID	γ_i	W_{i0}	λ_i	$\tilde{W}_{i0}$	$I_i(\beta)$	J_i	λ_i	$\tilde{W}_{i0}$	$I_i(\beta)$	J_i	λ_i	$\tilde{W}_{i0}$	$I_i(\beta)$	J_i
1	0.7	25	0.29	68.75	245.18	343.93	0.13	59.32	245.18	327.60	0.01	25	245.18	245.18
2	2	25	0.34	7.47	-0.46	-0.46	0.36	10.89	2.83	2.83	0.24	25	6.87	6.87
3	3	25	0.33	9.34	-1.74	-1.74	0.42	12.77	1.39	1.39	0.40	25	4.06	4.06
4	9	25	0.04	14.43	-20.90	-20.90	0.09	17.01	-4.69	-4.69	0.35	25	0.98	0.98
sum		100	1	100	222.07	320.83	1	100	244.70	327.12	1	100	257.08	257.08

The table presents investors' utility weights (λ_i), net worth claims ($\tilde{W}_{i0}$), life-time utility from running a sole proprietorship with Sharpe ratio β ($I_i(\beta)$), and life-time utility (J_i) from the private firm for different values of β. β represents the Sharpe ratio of the co-investors' alternative investment opportunity. Panels A, B and C correspond to $\beta = 0$, 0.5ξ and ξ, respectively. Each investor has the same initial capital contribution (W_{i0}) but a different coefficient of relative risk aversion (γ_i). The parameter values are: $r = 0.03$, $\tau_c = 0.21, \rho = 0.1$, $\mu = 0.6$, $\sigma = 0.89$ and $\xi = 0.64$.

The table considers three different values for the Sharpe ratio of co-investors' outside investment opportunity: $\beta = 0$ (Panel A), $\beta = 0.5\xi$ (Panel B), and $\beta = \xi$ (Panel C). The following patterns leap out from the table. First, as one would expect, co-investors' life-time utility is decreasing in the coefficient of risk aversion. Second, when $\beta \leq \xi$, co-investors' net worth claim in the private firm, $\tilde{W}_{i0}$, is *increasing* in risk aversion, with less risk averse co-investors paying a higher premium to the entrepreneur for the net worth they acquire in the private firm. Less risk averse investors have a stronger preference for risky assets or projects than their more risk averse peers, and are therefore willing to pay a higher premium. The premium for the least risk averse co-investor (investor 2) can be large. In return for a capital contribution of 25, co-investor 2 gets a net worth claim in the firm equal to $\tilde{W}_{20} = 7.47$, $\tilde{W}_{20} = 10.89$ and $\tilde{W}_{20} = 25$, for $\beta = 0$, $\beta = 0.5\xi$ and $\beta = \xi$, respectively. In contrast, the corresponding equity claim values for co-investor 4 are 14.43, 17.01 and 25. The premiums paid by co-investors translate into a substantial discount for the entrepreneur who, for the three different levels of co-investors' Sharpe ratio, acquires an equity claim equal to 68.75, 59.32 and 25, in return for a capital contribution of 25.

A low Sharpe ratio for co-investors' alternative investment opportunity dramatically skews the utility weights towards the entrepreneur with $\lambda_1 = 0.29$ and $\lambda_1 = 0.13$ for $\beta = 0$ and $\beta = 0.5$, respectively, compared to $\lambda_1 = 0.01$ for $\beta = \xi$. In this example, co-investors' utility weights are a decreasing or inverted U-shaped function of the coefficient of risk aversion with the largest co-investor weight being $\lambda_2 = 0.34$, $\lambda_3 = 0.42$ and $\lambda_3 = 0.40$ for $\beta = 0$, $\beta = 0.5\xi$ and $\beta = \xi$, respectively. The inverted U-shape arises for medium levels of initial capital contribution by investors. One can numerically show that, all else equal, for low (high) levels of initial capital contribution, co-investors' utility weights decline (increase) in investors' coefficient of risk aversion. The results in the table illustrate that heterogeneity in risk preferences can significantly skew the utility weights and therefore investors' cashflows.

5 Extensions

5.1 Equity issues

Suppose that m new investors join the firm at time τ and each contributes $W_{i\tau}$ in capital (for $i = n+1, ..., n+m$), bringing the firm's total net worth after the equity issue, W'_τ, to $W'_\tau = W_\tau + \sum_{j=n+1}^{n+m} W_{j\tau}$. In what follows z' and λ'_i, respectively, denote the auxiliary variable and utility weights after the equity issue. Our analysis is similar to the one for group synergies in Section 4.4.

If new investors supply capital competitively then their participation constraint pins down their utility weights, i.e. for investors $i = n+1, ..., n+m$ we get:

$$I_i(W_{i\tau}|\beta) = J_i(W'_\tau|n+m\, investors) \Longleftrightarrow \lambda'_i = W_{i\tau}^{\gamma_i} \left(\frac{a_i(\beta)^{\gamma_i}}{a_i(\xi)}\right)^{\frac{\gamma_i}{1-\gamma_i}} e^{-z'_\tau} \quad (41)$$

The net worth transfer from the new investors to the existing investors is:

$$\Delta' \equiv \sum_{i=n+1}^{n+m} \left(W_{i\tau} - \tilde{W}_{i\tau} \right) = \sum_{i=n+1}^{n+m} \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \right) W_{i\tau} > 0 \tag{42}$$

If $(s'_1, ..., s'_n)$ is a sharing rule that distributes the surplus across all existing investors, then the corresponding net worth stakes and utility weights after the equity issue are:

$$\tilde{W}'_{i\tau} \equiv \tilde{W}_{i\tau} + s'_i \Delta' \quad with \quad \sum_{i=1}^{n} s'_i = 1 \quad for \quad i = 1, ..., n \tag{43}$$

$$\lambda'_i = \left(\frac{\tilde{W}'_{i\tau}}{a_i(\xi)} \right)^{\gamma_i} e^{-z'_\tau} \quad for \ i = 1, ..., n \tag{44}$$

Finally, note that the firm's financial policies after equity issue are as described in Proposition 2 but for the enlarged set of $n + m$ investors and for the new utility weights λ'_i.

5.2 Personal and corporate taxes

We now briefly discuss whether and how personal and corporate taxes affect investors. Recall from Equation (1) that corporate taxes reduce the drift and volatility of the risky project's return process (i.e. $\mu_i \equiv \mu'_i(1 - \tau_c)$ and $\sigma_i \equiv \sigma'_i(1 - \tau_c)$). Personal taxes reduce the net payout investors receive. Assuming the personal tax rate is τ_p for all investors, then investor i's utility from payout is $u_i((1 - \tau_p)c_{it})$. Introducing personal taxes into our model is therefore straightforward, and a detailed analysis is given in Internet Appendix D. We summarize the key results here.

The effect on investors of a change in the personal or corporate tax rate depends on whether the change is announced before or after investors' utility weights are fixed at startup (i.e. whether the tax change is anticipated or unanticipated). In the former case changes in τ_p do not affect investors' pre-tax payout, nor their net worth stake at startup. The reason is that the utility weights are endogenous and internalize the new personal tax rate causing payout and net worth stakes at startup to be independent of τ_p. In contrast, higher corporate taxes increase (decrease) payouts of investors with a coefficient of relative risk aversion below (above) one.

If the tax change is announced after the utility weights are fixed, then this unanticipated change in the personal or corporate tax rate has a redistributing effect on investors' payout and net worth stake. Higher personal or corporate taxes increase (decrease) the net worth stake of the most (least) risk averse investor. For investors with intermediate levels of risk aversion, tax rises reduce (increase) their net worth if the firm's total net worth is below (above) some threshold.

5.3 Investors with exponential utility

As exponential utility has often been the workhorse utility function in the literature, we briefly revisit our model for investors with heterogeneous constant absolute risk aversion (CARA) preferences, i.e. $u_i(c_{it}) = -\frac{1}{\eta_i}\exp(-\eta_i c_{it})$, where $\eta_i > 0$ for all $i = 1, ..., n$. The aggregation theorem applies in a straightforward fashion for investors with exponential utility. As is well known, the problem for the group can be reformulated as one of a single representative investor with CARA utility and coefficient of risk aversion $\left(\sum_{i=1}^{n}\frac{1}{\eta_i}\right)^{-1}$.

Optimal payout is linear in net worth with CARA utility. The absolute dollar amount invested in risky assets (A_t) remains constant with CARA. Therefore, as net worth increases the proportion of net worth invested in risky assets falls, and leverage declines. In contrast, the ratio of risky assets to net worth increases (is constant) in W_t for investors with heterogeneous (homogenous) CRRA preferences. Explicit expressions and derivations of the optimal financial policies are available in Internet Appendix C.

5.4 Investor selection and matching

Which investors should the firm select from a pool of investors with heterogeneous risk preferences? We know that an investor's expected life-time utility is declining in her coefficient of risk aversion. Hence, if one wants to maximize the aggregate life-time utility of the firm's investors then one should select investors in order of increasing risk aversion, i.e. starting with the least risk averse investor.

The same ranking is true if an entrepreneur with all bargaining power (i.e. co-investors supply capital competitively) makes the selection. Ex post (i.e. after the startup) investors' life-time utility is not affected by the risk aversion level of the other group members because investors receive their first-best expected life-time utility based on the net worth stake they acquire in the firm, irrespective of the risk aversion of the other group members. However, ex ante (i.e. before the startup) investors' preferences do matter because the premium an investor is willing to pay to join the firm decreases in her risk aversion. It is therefore in the entrepreneurs interest to select in the first instance those investors with the lowest level of risk aversion.

Finally, consider the scenario where each investor has to select one co-investor to form a pair. Assuming that investors' bargaining power is independent of who they partner with (e.g. synergies are always shared equally), then each investor wants to partner with the investor who has the lowest risk aversion because this is the investor who is willing to pay the highest premium for achieving a higher Sharpe ratio from joining the team. Likewise, the investor with the lowest coefficient of risk aversion wants to team up with the least risk averse investor that is available. Consequently, it is in the interest of the two least risk averse investors to pair up by mutual consent. The two least risk averse investors from the remaining pool of investors will team up for the same reasons, and so on till no more profitable pairs can be formed. The matching process results in homophilous teams where investors who are most alike in their level of risk aversion connect with each other.

6 Empirical Implications and Conclusions

Our paper has empirical implications for several areas in financial economics.

6.1 Firm leverage

Our model predicts that startups are initially all-equity financed holding a net cash balance. A line of credit is gradually introduced to finance growth as retained earnings and net worth accumulate. To our knowledge, we are the first to show that heterogeneity in investors' risk preferences can lead to procyclical leverage, and be a significant source of intertemporal variation in a firm's debt ratio.[13]

The model generates a wide range of time-varying net debt ratios between -1 and +1 without relying on taxes, bankruptcy costs, asymmetric information or other frictions traditionally invoked by standard capital structure theories. Our findings may apply not only to private firms but also to public corporations that prohibit managers from trading the stock they own in the firm. Chava and Purnanandam (2010) show that CEOs' risk preferences affect leverage and cash-holding policies, whereas CFOs' risk preferences are relatively more important in explaining debt maturity structure and accrual decisions. They conclude that "closer attention should be paid to the risk preferences and attitudes

[13]The empirical evidence of whether optimal leverage ratios evolve pro-cyclically or counter-cyclically over the business cycle is mixed and primarily focused on public corporations. Korajczyk and Levy (2003) find that target leverage of financially unconstrained firms varies counter-cyclically with macroeconomic conditions while that of constrained firms shows pro-cyclical dynamics. Korteweg and Strebulaev (2015) report that target leverage evolves pro-cyclically over the business cycle. Halling et al. (2016) show that, on average, target leverage ratios evolve counter-cyclically, with a small minority of firms displaying pro-cyclical dynamics.

of managers to better understand the corporate financial decision making".

6.2 Capital structure, preferred stock, and ownership

Our model generates an optimal capital structure that includes debt, preferred stock and equity. The model predicts that the firm's claims are tailored to investors' risk preferences, with less (more) risk averse investors having claims that tend to be relatively more convex (concave) in the firm's net worth. Importantly, an investor's optimal contract not only depends on her own level of risk aversion, but also on the risk preferences of the other investors. An investor may opt for a contract that is relatively performance sensitive when matched with more risk averse co-investors, and choose a contract that is relatively performance insensitive when matched with less risk averse investors. These findings generate new empirical hypotheses regarding the capital structure of small businesses and private firms more generally. The results may also be relevant for the compensation structure of other group entities, such as fund management teams.

Our model generates an optimal capital structure that contains a variety of claims resembling preferred stock. We have shown that these contracts allow first-best efficient risk sharing among a group of investors with heterogeneous risk preferences without any trading among investors taking place. The contracts we derive cap the payout of more risk averse investors at lower levels in good times, but give them priority in terms of payout in bad times. This novel rationale for using preferred stock may help explain the huge variety of preferred stock contracts that we observe in practice (see e.g. Kaplan and Strömberg (2003)).

Our model shows that co-investors acquire net worth in the private firm at a premium. The premium is similar to a "fee" co-investors pay to the entrepreneur for access to assets or projects with a superior Sharpe ratio. As a result, the entrepreneur whose human capital generates the higher returns acquires her net worth at a discount as a compensation for the "sweat equity" she contributes. A premium (discount) translates into a lower (higher) utility weight for the investor. Our model provides one possible rationale why some investors acquire equity stakes at a discount, and other investors pay a premium.

An investor's utility weight fixed at startup, depends not only on the (monetary or human) capital she contributes but also on her risk preferences and outside investment option. We show that increasing heterogeneity in investors' risk preferences leads, all else equal, to higher utility weight concentration. As a result, investors are not treated equally in terms of cashflow and control rights even if they contribute the same amount of capital. Utility weights, cash flow rights and control are proportional to the amount of capital invested only if all investors have logarithmic utility, in which case the utility weights coincide with standard ownership shares.

6.3 Risk aversion and the dynamics of group behavior

Existing literature offers two competing hypotheses for group behavior: the group shift hypothesis (e.g. Moscovici and Zavalloni (1969); Kerr (1992)) and the diversification of opinions hypothesis (e.g. Sah and Stiglitz (1986, 1988)). Although the latter hypothesis appears to enjoy strong empirical support (see Bär, Kempf and Ruenzi (2011)), it struggles to explain the puzzling observation that the average group is more (less) risk averse than the average individual in high (low) risk situations. Our model provides a possible explanation by showing that group behavior is not merely a weighted average of individuals' preferred decisions but, crucially, the weights vary endogenously over time. In our model, more weight shifts towards the least risk averse agent as the firm grows during booms, whereas in busts increased weight shifts towards the most risk averse agent. The group's risk aversion is determined primarily by the most risk averse investor when net worth falls to critically low levels. During "normal" times the member with the median level of risk aversion has most influence (consistent with the findings of the experimental study by Ambrus et al. (2015)). As such our model reconciles the diversification of opinions hypothesis with the group shift hypothesis. Although each investor has a constant coefficient of RRA, the group's level of RRA is a time-varying weighted average of investors' coefficient of risk aversion. This causes the group's risk aversion to be decreasing in the firm's net worth. Our model may help explain the behavior of other group entities such as fund management teams, syndicates or partnerships.

Our paper highlights the dynamics of optimal group policies and generates a rich diversity of corporate liabilities in the firm's optimal capital structure. These features efficiently reconcile the differences in investors' preferences without trading. The optimal financial policies require continuous rebalancing, which ensures the debt remains risk-free. A static policy or discrete-time rebalancing reduce aggregate welfare, and the firm's debt may no longer be risk-free if net worth drops too much before rebalancing takes place. Despite these complexities, we managed to obtain tractable, closed-form solutions for the dynamic model.

Appendix

Proof of Proposition 1

For a given $\tilde{W}_{i0}$, investor i's first-best outcome is achieved by running the whole firm (with Sharpe ratio ξ) according to her preference. Investor i's preferred policies are given by Equations (10) and (11), which follow directly from Equation (6). The evolution of investor i's net worth stake (Equation (12)) is obtained by substituting her optimal policies into Equation (4). ■

Proof of Proposition 2

For any positive utility weights λ_i, the group's indirect value function $J(W_t)$ satisfies the following Hamilton–Jacobi–Bellman (HJB) equation

$$\rho J(W_t) = \max_{\{c_{it},\omega_t\}} \sum_{i=1}^{n} \lambda_i u_i(c_{it}) + \left[(\omega_t(\mu - r) + r)\, W_t - \sum_{i=1}^{n} c_{it}\right] J'(W_t) + \frac{1}{2}\sigma^2\omega_t^2 W_t^2 J''(W_t) \tag{45}$$

The first-order conditions are $\omega_t^* = -\frac{\xi}{\sigma}\frac{J'(W_t)}{W_t J''(W_t)}$ and $c_{it}^* = \left(\frac{J'(W_t)}{\lambda_i}\right)^{-\frac{1}{\gamma_i}}$. Substituting the first-order conditions into the HJB yields

$$\sum_{i=1}^{n}\left[\frac{\gamma_i}{1-\gamma_i}\lambda_i^{\frac{1}{\gamma_i}} J'(W_t)^{1-\frac{1}{\gamma_i}} - \frac{\lambda_i}{1-\gamma_i}\right] + rW_t J'(W_t) - \frac{1}{2}\xi^2\frac{J'(W_t)^2}{J''(W_t)} - \rho J(W_t) = 0 \tag{46}$$

This non-linear ordinary differential equation (ODE) cannot be solved analytically in W_t. We introduce an auxiliary state variable, $z_t \equiv -\ln(J'(W_t))$, to obtain closed-form solution. Substituting $J'(W_t) = e^{-z_t}$ and $J''(W_t) = -\frac{e^{-z_t}}{W'(z_t)}$ into Equation (46) gives:

$$\sum_{i=1}^{n}\left[\frac{\gamma_i}{1-\gamma_i}\lambda_i^{\frac{1}{\gamma_i}}\left(e^{-z_t}\right)^{1-\frac{1}{\gamma_i}} - \frac{\lambda_i}{1-\gamma_i}\right] + rW(z_t)e^{-z_t} + \frac{1}{2}\xi^2 W'(z_t)e^{-z_t} - \rho J(W(z_t)) = 0 \tag{47}$$

We further differentiate this transformed ODE with respect to z_t, and then divide both sides by e^{-z_t}, through which we obtain the following second-order linear ODE in z_t:

$$\sum_{i=1}^{n}\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} + \left(r - \frac{1}{2}\xi^2 - \rho\right) W'(z_t) + \frac{1}{2}\xi^2 W''(z_t) - rW(z_t) = 0 \tag{48}$$

The solution to the linear ODE can be written as $W_t = W_{pt} + W_{ct}$, where W_{pt} is the particular solution and W_{ct} is the complementary solution. One can conjecture and verify that $W_p(z_t) = \sum_{i=1}^{n} a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}$, where $a_i(\xi)$ is defined in Equation (7). The complementary solution is of the form $W_c(z_t) = B_1 e^{b_+ z_t} + B_2 e^{b_- z_t}$, where B_1 and B_2 are constants to be determined and $b_\pm = \frac{\left(\rho + \frac{1}{2}\xi^2 - r\right) \pm \sqrt{(\rho + \frac{1}{2}\xi^2 - r)^2 + 2\xi^2 r}}{\xi^2}$. It can be shown that $b_+ > 0$

and $b_- < -1$. The particular solution captures the expected lifetime utility under a particular payout and investment policy. The complementary solution captures the value from growth options and abandonment options, which the firm and investors do not have. Therefore, the complementary part is zero, i.e. $B_1 = B_2 = 0$, as otherwise, an exploding bubble component would be added when $z \to \pm\infty$. Thus, $W_t = W_p(z_t)$. Substituting W_t into Equation (47) and the first-order conditions gives the group's indirect utility function (19), the optimal investment (Equation (15)) and the payouts (Equation (16)), respectively. The solution also needs to satisfy the feasibility condition and the transversality condition, which are given by

$$r\gamma_i^2 + \left(\rho + \frac{\xi^2}{2} - r\right)\gamma_i - \frac{\xi^2}{2} > 0 \quad \text{and} \quad \rho > (1-\gamma_i)\left(\frac{\xi^2}{2}\frac{1+\gamma_i}{\gamma_i} + 2\right) \tag{49}$$

The two conditions together implies that for $i = 1, ..., n$,

$$\gamma_i > \gamma^* \equiv \frac{-(\rho - r) + \sqrt{(\rho - r)^2 + 2\xi^2(r + \frac{\xi^2}{2})}}{2(r + \frac{\xi^2}{2})} \in (0, 1) \tag{50}$$

To verify the second-order conditions, denote the right-hand side of (45) as $g(W_t)$. We show that with c_{it}^* and ω_t^*, $g_{\omega\omega} = -\frac{\sigma^2 W(z_t)^2}{W'(z_t)} e^{-z_t} < 0$, $g_{c_i c_i} = -\lambda_i \gamma_i (c_i^*)^{-1-\gamma_i} < 0$ and $g_{\omega c_i} = g_{c_i c_j} = 0$ for $i \neq j$. The determinants of the leading principle minors of the Hessian matrix are: $\det(H_1) = g_{\omega\omega}$ and $\det(H_k) = g_{\omega\omega}\prod_{i=1}^{k-1} g_{c_i c_i}$ for $k > 1$ and alternate in sign. Hence, the sufficient conditions for optimality are satisfied. ■

Proof of Proposition 3

For all i, one can show that $\lim_{z_t \to -\infty} c_{it}^* = \lim_{z_t \to -\infty} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} = 0$, and

$$\frac{\partial c_{it}^*}{\partial W_t} = \left[a_i(\xi) + \sum_{j=1}^{i-1} a_j(\xi)\frac{\gamma_i}{\gamma_j}\frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z_t\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} + \sum_{j=i+1}^{n} a_j(\xi)\frac{\gamma_i}{\gamma_j}\frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z_t\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)}\right]^{-1} \tag{51}$$

As $\gamma_1 \leq ... \leq \gamma_n$, Equation (51) implies $\lim_{z_t \to +\infty} \frac{\partial c^*_{1t}}{\partial W_t} = \frac{1}{a_1(\xi)}$ and $\lim_{z_t \to +\infty} \frac{\partial c^*_{it}}{\partial W_t} = 0$ for $i = 2, 3, ..., n$. The second derivative of the payout policy with respect to W_t is

$$\frac{\partial^2 c^*_{it}}{\partial W_t^2} = \frac{(\lambda_i e^{z_t})^{\frac{1}{\gamma_i}}}{\gamma_i W'(z_t)^3} \sum_{j=1}^{n} \frac{a_j(\xi)}{\gamma_j} (\lambda_j e^{z_t})^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j} \right) \geq (\leq)\, 0$$
$$\Leftrightarrow \sum_{j=1}^{i-1} \frac{a_j(\xi)}{\gamma_j} (\lambda_j e^{z_t})^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i} \right) \leq (\geq) \sum_{j=i+1}^{n} \frac{a_j(\xi)}{\gamma_j} (\lambda_j e^{z_t})^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j} \right) \qquad (52)$$

We can easily see that $\frac{\partial^2 c^*_{1t}}{\partial W_t^2} \geq 0$ and $\frac{\partial^2 c^*_{nt}}{\partial W_t^2} \leq 0$. Note that both sides of (52) are positive and convex monotonically increasing functions of z_t. Moreover, because $\frac{1}{\gamma_n} \leq ... \leq \frac{1}{\gamma_1}$, we can see that when $z_t \to -\infty$, the left-hand side of (52) is dominated by the right-hand side, while $z_t \to +\infty$, the reverse holds. Thus, the single-crossing property of two convex monotonically increasing functions implies that for $i = 2, ..., n-1$, there exist a $z^*_i \in \mathbb{R}$ (i.e. $W^*_i \equiv W(z^*_i) \in \mathbb{R}^+$) such that (52) is binding, and $\frac{\partial^2 c^*_{it}}{\partial W_t^2} \geq (\leq)\, 0$ for $W_t \leq (\geq)\, W(z^*_i)$. In addition, $W(z^*_2) \geq ... \geq W(z^*_{n-1})$. Lastly, setting $\gamma_1 = ... = \gamma_n = \gamma$ in (52), one can see that c^*_{it} becomes linear in W_t, i.e. $\frac{\partial^2 c^*_{it}}{\partial W^2} = 0$. ■

Proof of Proposition 4

The binding participation constraints for co-investors $i = 2, ..., n$ imply

$$J_i(W_0) = \frac{a_i(\xi)}{1-\gamma_i} (\lambda_i e^{z_0})^{\frac{1}{\gamma_i} - 1} - \frac{1}{\rho} \frac{1}{1-\gamma_i} = a_i(\beta)^{\gamma_i} \frac{W_{i0}^{1-\gamma_i}}{1-\gamma_i} - \frac{1}{\rho} \frac{1}{1-\gamma_i} = I_i(W_{i0}|\beta)$$

Solving for λ_i gives Equation (29). λ_1 (Equation (28)), is pinned down by substituting other co-investors' weights into the total net worth constraint:

$$W_0 = \sum_{i=1}^{n} a_i(\xi) \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_0}{\gamma_i}} = W_{10} + \sum_{i=2}^{n} W_{i0} = a_1(\xi) \lambda_1^{\frac{1}{\gamma_1}} e^{\frac{z_0}{\gamma_1}} + \sum_{i=2}^{n} a_i(\xi) W_{i0} \Big[\frac{a_i(\xi)}{a_i(\beta)^{\gamma_i}} \Big]^{-\frac{1}{1-\gamma_i}}$$

■

Proof of Proposition 5

We prove this proposition by first showing that the evolution of investor i's net worth stake $\tilde{W}_{it}$ in the group is the same as the corresponding one under the first-best outcome given by the stochastic differential equation (12). By substituting the optimal investment and payout policies (Equations (15) and (16)) into Equation (13), one can get

$$dW_t = \left[\xi^2 W'(z_t) + rW(z_t) - \sum_{i=1}^{n} (\lambda_i e^{z_t})^{\frac{1}{\gamma_i}}\right] dt + \xi W'(z_t) dB_t \tag{53}$$

Applying Itô's Lemma to $z(W_t)$ gives

$$dz_t = \left[\frac{\xi^2 W'(z_t) + rW(z_t) - \sum_{i=1}^{n} c_{it}^*}{W'(z_t)} - \frac{1}{2}\xi^2 \frac{W''(z_t)}{W'(z_t)}\right] dt + \xi dB_t \equiv \mu_z dt + \sigma_z dB_t \tag{54}$$

Next, we apply Itô's lemma to the net worth stake $\tilde{W}_{it} = a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}$ and show that

$$\frac{d\tilde{W}_{it}}{\tilde{W}_{it}} = \left[\frac{\mu_z}{\gamma_i} + \frac{1}{2}\frac{\sigma_z^2}{\gamma_i^2}\right] dt + \frac{\sigma_z}{\gamma_i} dB \equiv \alpha_{\tilde{W}_i} dt + \sigma_{\tilde{W}_i} dB_t \tag{55}$$

We then compare the process (55) with (12). Denote the drift term in (12) as $\alpha_i(\xi)$. We can indeed show that $\alpha_{\tilde{W}_i} = \alpha_i(\xi)$ and $\sigma_{\tilde{W}_i} = \frac{\xi}{\gamma_i}$. The second equality is satisfied as $\sigma_z = \xi$. The first equality also holds as

$$\alpha_{\tilde{W}_i} - \alpha_i(\xi) = \frac{\xi^2}{\gamma_i} + \frac{rW(z_t) - \sum_{i=1}^{n} c_{it}^*}{\gamma_i W'(z_t)} - \frac{1}{2}\xi^2 \frac{W''(z_t)}{\gamma_i W'(z_t)} + \frac{1}{2}\frac{\xi^2}{\gamma_i^2} - \left(\frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i(\xi)}\right) = 0$$
$$\Leftrightarrow \quad \sum_{i=1}^{n} \left(r - \frac{\xi^2 - 2(\rho + \frac{1}{2}\xi^2 - r)\gamma_i - 2r\gamma_i^2}{-2\gamma_i^2} + \frac{\rho + \frac{1}{2}\xi^2 - r}{\gamma_i} - \frac{\xi^2}{2\gamma_i^2}\right) \tilde{W}_i(z_t) = 0$$

We can also show that the initial net worth stakes that satisfy the participation and total net worth constraints in the first-best scenario (Equations (8) and (9)) are the same as the ones described in Equations (35) and (36). This means that the two processes have the same initial value. Therefore, we can conclude that investor i's net worth stake $\tilde{W}_{it}$ evolves in the same manner as the net worth stake she can achieve under the first-best outcome (i.e. the two processes (12) and (55) are identical). This further implies that the investors' life-time utility from the group replicates the first-best outcome (i.e. $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi)$), and $c^*_{it}(W_t) = c^o_{it}(\tilde{W}_{it}|\xi)$ for all t. We obtain $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi)$ by substituting λ_i (Equations (28) and (29)) into J_i defined in Equation (19). ■

References

Ambrus, A., B. Greiner, and P. A. Pathak (2015): "How individual preferences are aggregated in groups: An experimental study," *Journal of Public Economics*, 129, 1–13.

Antill, S., and S. R. Grenadier (2019): "Optimal capital structure and bankruptcy choice: Dynamic bargaining versus liquidation," *Journal of Financial Economics*, 133(1), 198–224.

Bär, M., A. Kempf, and S. Ruenzi (2011): "Is a team different from the sum of its parts? Evidence from mutual fund managers," *Review of Finance*, 15(2), 359–396.

Barsky, R. B., F. T. Juster, M. S. Kimball, and M. D. Shapiro (1997): "Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement study," *Quarterly Journal of Economics*, 112(2), 537–579.

Bhamra, H. S., and R. Uppal (2014): "Asset prices with heterogeneity in preferences and beliefs," *Review of Financial Studies*, 27(2), 519–580.

Biais, B., J. Hombert, and P.-O. Weill (2021): "Incentive constrained risk sharing, segmentation, and asset pricing," *American Economic Review*, 111(11), 3575–3610.

Bolton, P., H. Chen, and N. Wang (2011): "A unified theory of Tobin's q, corporate investment, financing, and risk management," *Journal of Finance*, 66(5), 1545–1578.

Campo, S., E. Guerre, I. Perrigne, and Q. Vuong (2011): "Semiparametric estimation of first-price auctions with risk-averse bidders," *Review of Economic Studies*, 78(1), 112–147.

Chava, S., and A. Purnanandam (2010): "CEOs versus CFOs: Incentives and corporate policies," *Journal of Financial Economics*, 97(2), 263–278.

Chen, H., J. Miao, and N. Wang (2010): "Entrepreneurial finance and nondiversifiable risk," *Review of Financial Studies*, 23(12), 4348–4388.

Chetty, R. (2006): "A new method of estimating risk aversion," *American Economic Review*, 96(5), 1821–1834.

Cochrane, J. H. (2005): "The risk and return of venture capital," *Journal of Financial Economics*, 75(1), 3–52.

Cvitanić, J., E. Jouini, S. Malamud, and C. Napp (2012): "Financial markets equilibrium with heterogeneous agents," *Review of Finance*, 16(1), 285–321.

Décamps, J.-P., S. Gryglewicz, E. Morellec, and S. Villeneuve (2017): "Corporate policies with permanent and transitory shocks," *Review of Financial Studies*, 30(1), 162–210.

Duffie, D. (2001): *Dynamic asset pricing theory*. Princeton University Press, 3rd edn.

Dumas, B. (1989): "Two-person dynamic equilibrium in the capital market," *Review of Financial Studies*, 2(2), 157–188.

Ebert, S., W. Wei, and X. Y. Zhou (2020): "Weighted discounting – On group diversity, time-inconsistency, and consequences for investment," *Journal of Economic Theory*, 189, 105089.

Eliashberg, J., and R. L. Winkler (1981): "Risk sharing and group decision making," *Management Science*, 27(11), 1221–1235.

Emanuel, D. (1983): "A theoretical model for valuing preferred stock," *Journal of Finance*, 38(4), 1133–1155.

Fischer, E. O., R. L. Heinkel, and J. Zechner (1989): "Dynamic capital structure choice: Theory and tests," *Journal of Finance*, 44(1), 19–40.

Garlappi, L., R. Giammarino, and A. Lazrak (2017): "Ambiguity and the corporation: Group disagreement and underinvestment," *Journal of Financial Economics*, 125(3), 417–433.

——— (2021): "Group-managed real options," *Review of Financial Studies*, hhab100.

Gilson, R. J., and D. M. Schizer (2003): "Understanding venture capital structure: a tax explanation for convertible preferred stock," *Harvard Law Review*, 116(3), 874–916.

Gornall, W., and I. A. Strebulaev (2020): "Squaring venture capital valuations with reality," *Journal of Financial Economics*, 135(1), 120–143.

Grenadier, S., A. Malenko, and N. Malenko (2016): "Timing decisions in organizations: Communication and authority in a dynamic environment," *American Economic Review*, 106(9), 2552–2581.

Gryglewicz, S. (2011): "A theory of corporate financial decisions with liquidity and solvency concerns," *Journal of Financial Economics*, 99(2), 365–384.

Habib, M. A., and P. Mella-Barral (2007): "The role of knowhow acquisition in the formation and duration of joint ventures," *Review of Financial Studies*, 20(1), 189–233.

Halling, M., J. Yu, and J. Zechner (2016): "Leverage dynamics over the business cycle," *Journal of Financial Economics*, 122(1), 21–41.

Hara, C., J. Huang, and C. Kuzmics (2007): "Representative consumer's risk aversion and efficient risk-sharing rules," *Journal of Economic Theory*, 137(1), 652–672.

Hartman-Glaser, B., S. Mayer, and K. Milbradt (2020): "Corporate liquidity management under moral hazard," Working paper.

Heinkel, R., and J. Zechner (1990): "The role of debt and prerferred stock as a solution to adverse investment incentives," *Journal of Financial and Quantitative Analysis*, 25(1), 1–24.

Hugonnier, J., and E. Morellec (2017): "Bank capital, liquid reserves, and insolvency risk," *Journal of Financial Economics*, 125(2), 266–285.

Kaplan, S. N., and P. Strömberg (2003): "Financial contracting theory meets the real world: An empirical analysis of venture capital contracts," *Review of Economic Studies*, 70(2), 281–315.

Kerr, N. L. (1992): "Group decision making at a multialternative task: Extremity, interfaction distance, pluralities, and issue importance," *Organizational Behavior and Human Decision Processes*, 52(1), 64–95.

Korajczyk, R. A., and A. Levy (2003): "Capital structure choice: macroeconomic conditions and financial constraints," *Journal of Financial Economics*, 68(1), 75–109.

Korteweg, A., and M. Sorensen (2010): "Risk and return characteristics of venture capital-backed entrepreneurial companies," *Review of Financial Studies*, 23(10), 3738–3772.

Korteweg, A., and I. Strebulaev (2015): "An empirical target zone model of dynamic capital structure," Working paper.

Lambrecht, B. M., and S. C. Myers (2017): "The dynamics of investment, payout and debt," *Review of Financial Studies*, 30(11), 3759–3800.

Lambrecht, B. M., and G. Pawlina (2013): "A theory of net debt and transferable human capital," *Review of Finance*, 17(1), 321–368.

Leland, H. E. (1994): "Corporate debt value, bond covenants, and optimal capital structure," *Journal of Finance*, 49(4), 1213–1252.

Mazzocco, M. (2004): "Saving, risk sharing, and preferences for risk," *American Economic Review*, 94(4), 1169–1182.

Merton, R. C. (1969): "Lifetime portfolio selection under uncertainty: The continuous-time case," *The Review of Economics and Statistics*, 51(3), 247–257.

Moscovici, S., and M. Zavalloni (1969): "The group as a polarizer of attitudes," *Journal of Personality and Social Psychology*, 12(2), 125–135.

Pratt, J. W., and R. J. Zeckhauser (1989): "The impact of risk sharing on efficient decision," *Journal of Risk and Uncertainty*, 2(3), 219–234.

Rivera, A. (2020): "Dynamic moral hazard and risk-shifting incentives in a leveraged firm," *Journal of Financial and Quantitative Analysis*, 55(4), 1333–1367.

Sah, R. K., and J. E. Stiglitz (1986): "The architecture of economic systems: Hierarchies and polyarchies," *The American Economic Review*, 76(4), 716–727.

——— (1988): "Committees, hierarchies and polyarchies," *The Economic Journal*, 98(391), 451–470.

Shupp, R. S., and A. W. Williams (2008): "Risk preference differentials of small groups and individuals," *The Economic Journal*, 118(525), 258–283.

Sorensen, M., N. Wang, and J. Yang (2014): "Valuing private equity," *Review of Financial Studies*, 27(7), 1977–2021.

Wang, C., N. Wang, and J. Yang (2012): "A unified model of entrepreneurship dynamics," *Journal of Financial Economics*, 106(1), 1–23.

Wang, J. (1996): "The term structure of interest rates in a pure exchange economy with heterogeneous investors," *Journal of Financial Economics*, 41(1), 75–110.

Wilson, R. (1968): "The theory of syndicates," *Econometrica*, 36(1), 119–132.

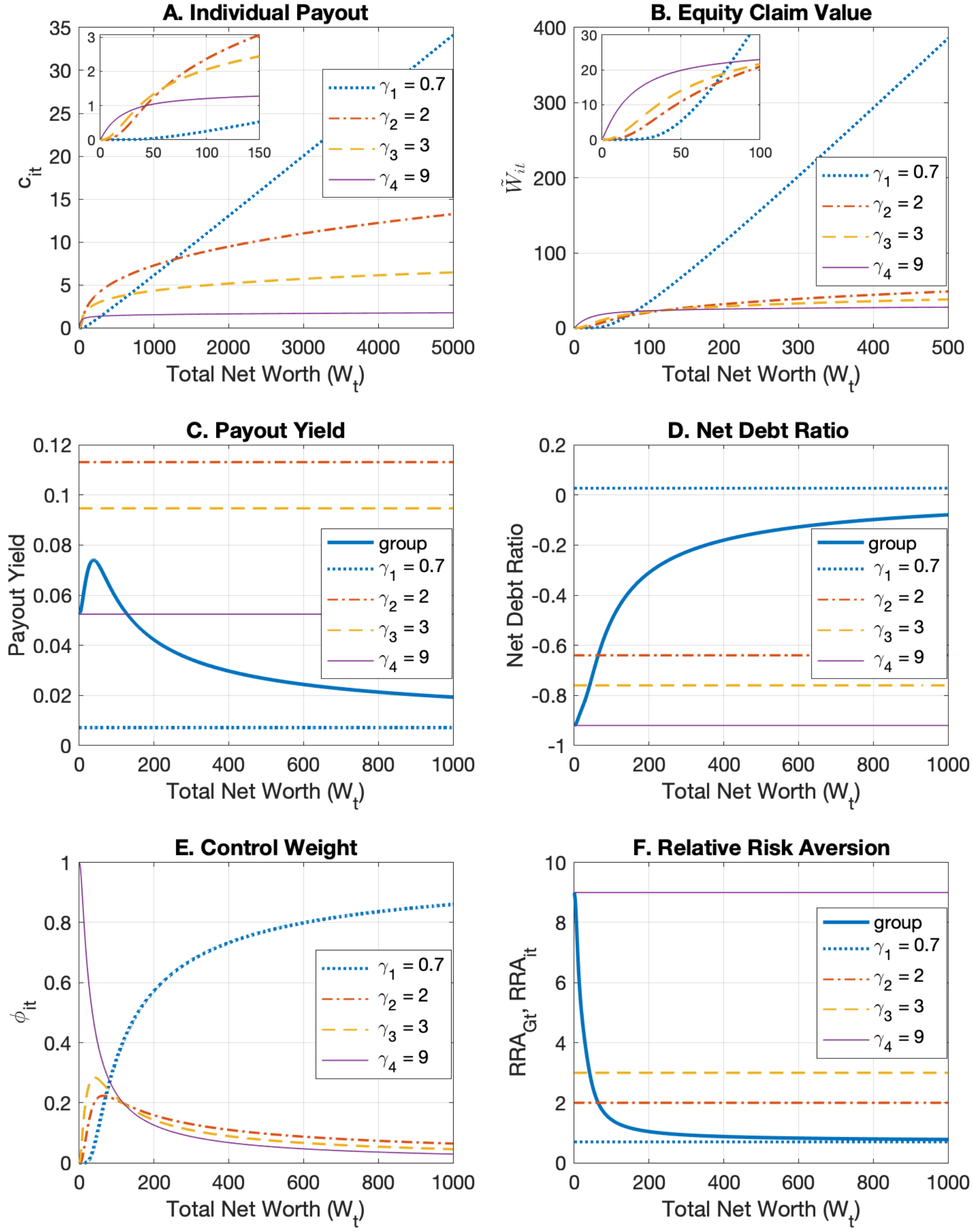

Figure 1: Financial policies and claim values

In this figure, we consider a group of 4 investors with heterogeneous risk aversion ($\gamma_1 = 0.7, \gamma_2 = 2, \gamma_3 = 3$, and $\gamma_4 = 9$) and initial endowments ($W_{10} = W_{20} = W_{30} = W_{40} = 25$). The parameter values for the entrepreneur's (investor 1's) investment project are: drift (μ) = 0.6, volatility (σ) = 0.89 and the corresponding Sharpe ratio (ξ) = 0.64. The Sharpe ratio of co-investors' outside investment opportunity is $\beta = 0.9\xi$. The other parameter values are: risk-free rate (r) = 0.03, subjective discount rate (ρ) = 0.1, corporate tax rate (τ_c) = 0.21. Panels A, B, C, D, E and F plot, respectively, payouts from the group c^*_{it}, investors' equity claims $\tilde{W}_{it}$, the group's payout yield, the firm's net debt ratio NDR_t, investors' control weights ϕ_{it} and the group's level of relative risk aversion RRA_G as functions of the firm's net worth W_t. In Panels C and D, the thick solid lines represent the payout yield and NDR of the private firm, whereas the other lines represent the optimal policies of the corresponding sole proprietorships (evaluated at ξ). The small windows inside Panels A and B zoom in on the payouts and equity claim values, respectively, when the firm's total net worth W_t is relatively small.