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# Leverage and Interest Rates<sup>\*</sup>

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## Abstract

We study the sensitivity of optimal leverage to the level of the risk-free interest rate. Our trade-off model implies a heterogeneous response depending on the presence of a sponsor backing company debt. A highly-leveraged, backed company optimally increases debt when interest rates fall, while a company without a sponsor reduces it despite having lower initial leverage. This heterogeneity implies divergent bankruptcy probability and recovery-upon-default, in the same interest rate scenarios, for the two company types. We also show that a lower risk-free rate reduces the sponsor's incentive to issue debt.

*Keywords:* capital structure, tax-bankruptcy trade-off, default, LBO, subsidiaries, securitization, restructurings, risk transfer.

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## 1. Introduction

Persistent reductions followed by sudden reversals in the level of interest rates make us wonder about the response of leveraged entities. Empirical studies show that this response is heterogeneous across firms. An opposite reaction appears in a comparison of Leveraged Buyout (LBO) targets to matched public companies (Axelson, Jenkinson, Stromberg and Weisbach, 2007), in that only LBO targets increase debt when interest rates fall. Such opposite reaction resurfaces across SMEs and large public companies (Caglio et al., 2021), as only the former increase borrowing during monetary expansions despite bearing a higher cost of debt. Yet, extant capital structure theories usually predict a one-sided association between corporate borrowing and the level of interest rates.

Against this background, this paper studies the sensitivity of optimal leverage to interest rates in a model that allows for a sponsor backing a company’s debt. In the real world, a sponsor appears in different contexts, such as a private equity fund behind a Leveraged Buyout target, or the originator of a Special Purpose Vehicle (SPV) in a securitization, or as a parent company in a multinational group. Such arrangements are characterized by high debt tax shields (see Kaplan (1989), Acharya et al. (2013), Renneboog et al. (2007) for private equity and Han et al. (2015) for securitization), and empirical studies support the trade-off theory for multinationals and business groups (see Section 3.3 in Hanlon and Heitzman (2022) and Brok (2022)). For this reason, we investigate the response of leverage to interest rates in a trade-off model. A “stand-alone” company will choose leverage trading off its own tax benefits of debt with its default costs, as in Leland (2007). Otherwise, the trade-off applies to the combination of a sponsor and its backed company, as in Nicodano and Regis (2019).

We uncover a marked difference in the interest rate sensitivity of optimal debt between the typical stand-alone company and the one backed by a sponsor protected by limited liability from the insolvency of the backed company. Our analysis, based on the parameters for BBB companies in Leland (2007), shows that a stand-alone company has no financial incentive to demand more debt when interest rates (that is, the level of the risk-free rate) fall, if its investment opportunities are unchanged. A lower risk-free rate leads to lower tax savings of debt and optimal leverage drops. This pattern reverses when a sponsor stands

ready to transfer bailout funds to its profitable but insolvent backed company. To understand why, consider that the backed company has comparatively higher leverage and higher credit spreads at the initial risk-free rate. When the latter falls, the tax shield contracts less in the backed than in the stand-alone company because the spread component of the tax shield is less sensitive to the risk-free rate. At the same time, the sponsor allows the backed company to repay debt over a larger state space, thereby containing the dead-weight costs of bankruptcy.

Some analytical results help us understand the underlying economic logic of these effects. For the stand-alone company we show that, at the initial face value of debt, not only taxes but also default costs increase due to the lower risk-free rate. The reason is that the after-tax profits available to repay debt fall due to the lower tax shield, increasing default probability. Therefore both tax savings and default costs provide the stand-alone an incentive to reduce the face value of debt, when interest rates fall. Clearly, the tax shield reduction is the driving force behind both effects. We also prove that the market value of the stand-alone company increases when the risk-free interest rate falls, at any fixed level of debt. The reason is that the future payoff of both its debt and its equity are discounted at a lower rate. This effect is especially strong for our case where debt is discount, only.

The numerical analysis of our model also sheds light on changes in both the optimal leverage of the sponsor and default probability triggered by interest rate changes. When the sponsor is optimally a zero-leverage entity, at the initial risk-free rate, it remains zero-leverage after the reduction in interest rates. This occurs when the backed company has trade-off parameters that are at least as favourable to debt as those of the sponsor. In such a case, the sponsor keeps zero-leverage to ensure maximum support to the highly leveraged company at the lower interest rate level. The zero-leverage sponsor reminds of both a Private Equity fund behind its LBO target, injecting equity to prevent insolvency in the face of cash-flow shocks (see Haque et al., 2022), and the originator of a securitization with its SPV. Our model thus suggests an explanation for the opposite responses to changes in interest rates by public companies and LBO targets, that are backed by a private equity fund (Axelson, Jenkinson, Stromberg and Weisbach, 2007). The model may also explain why falling interest rates in the first two decades of this century were accompanied by a disproportionate increase

in lending through securitization vehicles (Powell, 2019), giving rise to concerns regarding their defaults (Rosengren, 2019). In our model, a reduction in the risk-free rate implies both higher expected default costs and higher default probability for the company/SPV backed by a sponsor, due to its higher leverage. The implications of the model thus support such default concerns, with one important caveat. The joint default probability of the sponsor-backed company structure remains very low - and lower than that of two equivalent stand-alone companies - due to the sponsor's optimal zero leverage.

Another type of sponsor is a parent company that owns a subsidiary. A parent-subsidiary structure endogenously emerges when we allow for trade-off parameters that are more favourable in the parent than in the subsidiary. Consider, for instance, a positive tax rate differential when a parent incorporates a subsidiary in a tax haven, or a negative bankruptcy cost differential when a financial parent (such as a diversified holding company) has an industrial subsidiary with firm-specific fixed capital. In such cases, there is a trade-off between the incentive to lever up the parent due to more favourable trade-off parameters and the opposite incentive to deleverage the parent to increase subsidiary's support. The response of capital structure to reductions in the risk-free rate shows significant risk transfers from the parent to the subsidiary. Starting from a situation with debt outstanding in both the parent and its subsidiary, a reduction of the risk-free rate may lead to both an optimal deleveraging of the parent and higher optimal debt for the subsidiary. The parent may optimally become a zero-leverage sponsor, specializing in providing support, even if it has a higher tax rate than its subsidiary. This occurs because a lower interest rate shifts the balance between the incentive towards leverage versus the one of providing support towards the latter. We indeed show analytically that a lower interest rate reduces the sponsor's incentive to issue debt.

Such debt and default-risk transfer from the sponsor to the backed unit is stronger when the cash-flows of the two units are positively correlated, since the tax motive for pooling cash-flows is stronger than the diversification motive. The total optimal face value of debt of the two units will be higher at the lower level of the risk-free rate than at the initial one, provided that cash flow correlation, and hence the tax savings motive, is sufficiently high.

Our model therefore provides a financial rationale for the different response of borrowing by private and public companies in Caglio et al. (2021), if the share of private companies

with a sponsor (a parent company; a private equity fund as in Haque et al. (2022); an equivalent solution within a horizontal group) is sufficiently large in their sample. Under the same proviso, our results may also explain higher borrowing by firms with lower ratings in both the EU and the US (Darmouni and Papoutsis, 2021; Schularick, 2021) during the years of falling interest rates.

The paper unfolds as follows. Section 2 benchmarks closely related literature. Section 3 presents the model for stand-alone units and for companies backed by a sponsor. In Sections 4 and 5 we describe our numerical analysis, providing insights into leverage, default probability and lenders' losses-upon-default adjustments following a drop in interest rates. The former studies equal tax-bankruptcy parameters that give rise to a zero-leverage sponsor and a highly leveraged backed company. Section 5 shifts attention to a leveraged sponsor that may restructure in response to a reduction in the risk-free rate. Section 6 concludes.

## 2. Related Literature

Our set-up relies on the static trade-off theory of leverage and structured finance (Leland, 2007), where the interest spread is endogenous to the borrower's cash-flow diversification. Optimal leverage is jointly determined in units that are connected through guarantees (Luciano and Nicodano, 2014) and ownership links (Nicodano and Regis, 2019), that are themselves endogenous to the tax-bankruptcy parameters. By considering the extended structure of the company, these models deliver optimally zero-leverage units and fully-leveraged units, on top of the usual interior solutions. This paper relies on this set-up to pin down the effects of exogenous changes in the level of the risk-free rate. It shows a divergent response of the optimal face value of debt and default probability in companies with and without sponsor. This study confirms that the overall structure of the company contributes to a better understanding of leverage choices, as the response differs for a stand-alone and a backed company with the same characteristics.<sup>2</sup> Our results also suggest that bankruptcy prediction exercises may gain from differentiating between sponsor-backed and stand-alone firms when the interest rate scenario changes. While we know that information about the complex structure

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<sup>2</sup>This observation adds another strand for future research to the suggestions in Frank and Goyal (2022).

improves on the prediction (Beaver et al. (2019)), we show that the sign of the prediction flips depending on the presence of the sponsor.

The dynamic trade-off models of Fischer, Heinkel, and Zechner (1989), Leland (1994) and, more recently, the ones of Ju and Ou-Yang (2006) and Duarte, Öztekin, and Saporito (2022) with stochastic interest rates, imply optimal debt levels that are increasing in the level of interest rates, due to a rising tax benefit of debt accompanied by a less than proportional increase in default costs. Our numerical results for the static, stand-alone company are in line with theirs, although we are silent about the dynamics of adjustment. The ones for the sponsor-backed company depart instead from previous knowledge in corporate finance.

Some recent papers investigate the response of optimal leverage to a reduction in interest rates, without resorting to the trade-off theory. Observing that traditional corporate finance models cannot explain a leverage increase when interest rates fall, Fahri and Tirole (2009) argue that lower interest rates increase (aggregate) loan demand, when they generate the expectation of further accommodative monetary policy or bailouts. They also show that highly leveraged companies benefit most from such policies. In our corporate finance model, the risk-free rate is (expected to be) invariant for the relevant company horizon, once it has fallen. What leads to the leverage increase is the presence of sponsors backing them.

Theory also points to investors' incentives to take on more risk when the level of interest rates falls. The increase in fund supply to low-rated firms is explained by the search for higher yield by investors (Martinez Miera and Repullo, 2017), which indeed appear to look for borrowers with higher losses-upon-default (Becker and Ivashina, 2015). Our model shifts the attention to the demand for funds, assuming an infinitely elastic supply at fair prices. This focus, while adding to our understanding of leverage changes, is in line with evidence in both Beck et al. (2022) and Caglio et al. (2021), which point to the absence of credit constraints and, more generally, stronger credit demand than supply effects. Absent moral hazard, riskier firms increase borrowing in the face of lower risk-free rates if backed by a sponsor.

Our findings explain the divergent response, by public companies and comparable LBO targets or SMEs, to lower interest rates which has been observed both before and after the Great Financial Crisis. Axelson, Jenkinson, Stromberg and Weisbach (2007) find that the

ratio of debt to EBITDA is higher in LBO targets but not in matched public companies when interest rates fall. The spreads they find depend on the type of debt. The median is equal to 262bp and 937bp for senior and junior bank loans respectively, reaching up to 916bp and 1048bp for senior and subordinated bonds respectively. The spread implied by our model, when 1% (5%) is the level of interest rates, is equal to 441bp (130bp) when cash-flow correlation between the fund and the LBO target is -0.8, reaching 1130bp (762bp) when cash-flow correlation is 0.2. Our numerical exercise also shows that these spreads allow to reduce the tax burden of the LBO target to up to one fifth of the taxes paid by a similar zero-leverage company.

Caglio et al. (2021) analyse data coming from the quarterly Capital Assessments and Stress Testing Report, covering approximately 70 percent of total corporate and industrial loans made to U.S. firms from 2012 to 2019. They conclude that the impact of monetary policy in the full sample is driven by private firms, which are mostly Small and Medium Enterprises (SMEs). They show that SMEs with higher leverage borrow more, at a higher cost, during monetary expansions. This result is driven by their higher demand for credit, while their lenders do not increase risk-taking. The higher cost of borrowing these leveraged SMEs pay relative to others is a result of their higher credit demand. On the contrary, highly leveraged public firms obtain less credit and pay higher spreads during monetary expansions. We can read this evidence through the lenses of our model, provided sponsors are more likely to be present in the sample of SMEs.

Rosengren (2019) raises concerns of potential financial instability observing the rise – from below 4 to above 5 - in the multiple of average total debt to EBITDA for leveraged transactions priced at or above LIBOR + 225bp. Powell (2019) also notes the increase in the debt-to-GDP ratios in the leveraged-loan market segment and in the overall business sector associated with low interest rates. Our results indicate that, when the leverage in structured finance increases in association with lower interest rates, so do both default probabilities and, to a lesser extent, lenders' losses upon default. However, our numerical exercise allows for neither increases in mean cash flows triggered by a lower risk-free rate nor for productivity increases relative to stand-alone companies that have been observed in some private equity research.



### 3. The Model

This section describes our set-up, that follows Leland (2007) in modeling Stand-Alone companies.

At time 0, a controlling entity owns two units,  $i = S, B$ . Each unit has a random exogenous operating cash flow  $X_i$  that is realized at time  $T$ . We denote with  $G(\cdot)$ , the cumulative distribution function and with  $f(\cdot)$  the density of  $X_i$ , identical for the two units;  $g(\cdot, \cdot)$  is the joint distribution of  $X_S$  and  $X_B$  and  $\rho$  their correlation. At time 0, the controlling entity selects how to finance the risky cash flows, either through a face value  $F_i$  zero-coupon debt with maturity  $T$  or equity. She does so to maximize the total arbitrage-free value ( $\nu_{SB}$ ) of equity,  $E_i$ , and debt,  $D_i$  in the two units:

$$\nu_{SB} = \max_{F_S, F_B} \sum_{i=S, B} (E_i + D_i). \quad (1)$$

Each unit pays a flat proportional income tax at an effective rate  $0 < \tau_i < 1$  and suffers proportional default costs  $0 < \alpha_i < 1$ .<sup>3</sup> Interest on debts are entirely deductible from taxable income. The tax advantage for debt generates a trade-off. On the one hand, increasing leverage generates tax benefits, while on the other it increases expected default costs because – everything else being equal – higher leverage increases default likelihood.

At time  $T$ , cash flows are realized and distributed to claim-holders. First, corporate income taxes are paid. Then, debt obligations are fulfilled, if possible. When a unit cannot meet its debt obligations, its income, net of taxes and the dead-weight costs of default, is distributed to the lenders. Once debt is fully repaid, equity-holders receive the net residual income.

Maximizing the value of debt and equity for the owner is equivalent to minimizing the expected cash flows not to be redistributed to claim-holders, namely expected taxes ( $T_i$ ) and default costs ( $C_i$ ):

$$\nu_{SB} = \min_{F_S, F_B} \sum_{i=S, B} T_i + C_i. \quad (2)$$

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<sup>3</sup>No tax credits or carry-forwards are permitted.

The expected tax burden of each unit is proportional to the expected operational cash flow  $X_i$ , net of the tax shield  $X_i^Z$ , defined as the interest deductions, which are equal to the difference between the nominal value of debt  $F_i$ , and its market value  $D_i$ :  $X_i^Z = F_i - D_i$ . Default costs are proportional to income.

Units can be owned as two separate unconnected units, or they can be connected through bailout and payoff transfers, conditional on some states of the world.

### 3.1. The Stand-Alone Companies

It is useful to start with the benchmark case of unconnected, stand-alone units. The expected tax burden in each stand-alone (SA) unit is equal to:

$$T_{SA}^i(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+], \quad (3)$$

where the expectation is computed under the risk neutral probability and  $\phi$  is the discount factor. The superscripts and subscripts,  $i$ , indicate whether the stand-alone unit is endowed with the sponsor ( $i = S$ ) or backed unit ( $i = B$ ) parameters.

Each stand-alone unit defaults when its realized net cash flow is lower than the face value of debt; in other words, default occurs when cash flows are lower than the default threshold,  $X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_i$ . Expected default costs, that are a dead-weight loss, are equal to:

$$C_{SA}^i(F_i) = \alpha_i \phi \mathbb{E} \left[ X_i 1_{\{0 < X_i < X_i^d\}} \right]. \quad (4)$$

They are proportional to the default cost parameter,  $\alpha_i$ , and they increase in realized cash flows, when the unit goes bankrupt. A rise in the nominal value of debt,  $F_i$ , increases the default threshold,  $X_i^d$ , thereby increasing the expected default costs.

When units are owned separately, the value of the objective function (2) is simply the sum of the values of the taxes and default costs in each unit. Notice also that the value of each unit can be written as:

$$V_i(F_i) = V_i(0) + TS^i(F_i) - C_{SA}^i(F_i),$$

where  $V_i(0)$  is the value of the unlevered unit, and  $TS^i(F_i) = T_{SA}^i(0) - T_{SA}^i(F_i)$  is the present value of the tax savings from leverage, equal to the difference between the taxes paid by an unlevered firm and a firm which issues debt  $F_i$ . It is possible to show that the tax shield of a stand-alone unit is a convex function of  $F_i$ . Increasing the nominal value of debt increases the tax shield, thereby reducing the tax burden because the market value of debt,  $D_i$ , increases with  $F_i$  at a decreasing rate (reflecting a higher risk). On the contrary, the default threshold  $X_i^d$  is concave in the face value of debt,  $F_i$ . Luciano and Nicodano (2014) prove that a stand-alone unit has positive optimal debt if the sum of tax burden and default costs is convex in the face value of debt. The Appendix shows that it raises positive debt even if the riskfree rate is zero, because of the endogenous spread.

We can now analyze the optimal response to a reduction in the risk-free rate, that increases the discount factor,  $\phi$ . At a given debt level  $F_i$ , the interest rate influences (3) and (4) through two channels. First, they are both discounted expected values, and depend on  $\phi$  directly. Second, they depend on the thresholds  $X_i^Z$  and  $X_i^d$ , which are influenced by the market value of debt  $D_i$ , which in turn depends on  $\phi$ . Hence, when the level of interest rate changes, expected taxes and default costs change. The following lemma describes how they change.

**Lemma 1.** *In a Stand-Alone company, the expected values of both taxes and default costs increase with the discount factor  $\phi$  (i.e. decrease with the interest rate), for a fixed face value of debt.*

**Proof.** See the Appendix. ■

The first part of the Lemma concerning taxes is straightforward, as the tax shield,  $X^Z$ , falls together with the risk-free interest rate. The result concerning the increase in default costs is instead not obvious. It stems from a reduction in net after-tax income available to repay debt, which owes to the reduction in the tax shield, which increases the probability of default. It turns out that, for reasonable values of the tax rate ( $\tau < \frac{1}{2}$ ), the tax shield decreases faster, as the risk-free rate falls, than the increase in the no-default threshold. The loss of benefits from leverage are therefore first order relative to the increase in default costs.

The Proposition below explores the associated effects on the market value of the Stand-

Alone company. Due to the increase in default probability, the present value of debt falls. However, the discount factor effect, which pushes debt value up as the interest rate falls, prevails. For fixed face value of debt, the market debt value increases as interest rates fall. Moreover, also the market value of equity increase in the discount factor, leading to the increase in value for any  $F_i$  and at the optimum, following a drop in interest rates:

**Proposition 1.** *In a Stand-Alone company, the market values of both debt and equity increase with the discount factor  $\phi$ , for fixed face value of debt,  $F_i$ . As a consequence, the market value of the firm increases with  $\phi$  (decreases with the interest rate) for any face value of debt (and, thus, at the optimum).*

**Proof.** See the Appendix. ■

### 3.2. The Unit Backed by a Sponsor

We allow the owner to set up units, which are connected through both bailout and payoff transfers, as in the structure of Nicodano and Regis (2019). For simplicity, we do not determine the optimal ownership shares and type of bailout transfers, referring respectively to Nicodano and Regis (2019) and Luciano and Nicodano (2014).

Regarding bailouts, we let the sponsor transfer part of its net profits to an insolvent, but profitable, backed unit if such transfer is able to prevent the insolvency. Formally, the sponsor transfers an amount  $F_B - X_B^n$  to the backed unit, provided its net profits are large enough ( $X_S^n - F_S \geq F_B - X_B^n$ ), so that both units become solvent. Importantly, the sponsor enjoys limited liability relative to the debt of its backed company. The assets of the backed unit are instead subject to the claims of the sponsor's lenders, if there are any, should the sponsor default.

Regarding payoffs, we assume that the sponsor controls the backed unit and is entitled to receive its equity payoff at  $T$ . These consist of the backed unit's net profits, i.e. its cash flows after paying the tax authority and lenders,  $(X_B^n - F_B)^+$ , where  $X_B^n$  are the cash flows, net of corporate income taxes. The cash flow available to the sponsor, after receiving the internal payoff, increases to:

$$X_S^{n,\omega} = X_S^n + (X_B^n - F_B)^+. \quad (5)$$

It is clear from Equation (5) that the payoff transfer, when positive, generates an internal rescue mechanism. It provides the sponsor, if leveraged, with an extra buffer of cash that may help prevent the default that it would experience as a stand-alone company. Observe that payoff transfers are not conditional on the sponsor survival and are subject to the sponsor bankruptcy costs, should the sponsor be insolvent. On the contrary, bailouts are contingent on positive cash flows in the backed company.

The bailout and internal dividends modify the tax/bankruptcy trade-off, for fixed capital structure  $(F_S, F_B)$ , as follows. The bailout transfer never increases the default costs in the backed unit,  $C_B$ , and leaves both the default costs of the sponsor and the tax burden of the group unaffected. The value of the guarantee, i.e. the reduction in expected default costs  $(\Gamma)$ , is equal to:

$$\Gamma = \alpha_B \phi \mathbb{E} \left[ X_B 1_{\{0 < X_B < X_B^d, X_S \geq h(X_B)\}} \right] \geq 0, \quad (6)$$

where the indicator function  $1_{\{\cdot\}}$  defines the set of states of the world in which the rescue occurs, that is when the backed unit defaults without transfers (first term) and the sponsor cash flows are sufficient for rescue (second term). Notice that the rescue by the sponsor is likelier the smaller the sponsor debt,  $F_S$ . The function  $h(X_B)$  is defined in the Appendix.

Payoff transfers leave the backed unit trade-off unchanged, but affect both the default costs and the tax burden of the sponsor. They add to the cash flow in the sponsor – as in equation (5) – increasing the chances that it is solvent. They also increase the lenders' recovery rate in insolvency, should the sponsor go bankrupt anyway. This last feature differentiates internal payoff transfers from the bailout transfers described earlier, because only the former are subject to depletion due to bankruptcy costs in the sponsor's insolvency.

The value of internal payoff transfers, i.e. the default costs saved by sponsor,  $\Delta C$ , are equal to:

$$\Delta C = \alpha_S \phi \left[ \mathbb{E} \left[ X_S 1_{\{0 \leq X_S < X_S^d, X_B \geq k(X_S)\}} \right] - \mathbb{E} \left[ (X_B^n - F_B)^+ 1_{\{X_S < X_S^d, X_B < k(X_S)\}} \right] \right],$$

where  $k(X_S)$  is defined in the Appendix. The first term measures the default costs saved when the sponsor avoids insolvency thanks to the payoff transfer. The second term equals the default costs on the payoff transfer, when it is insufficient to repay the sponsor debt.

Higher debt in the backed unit decreases the payoff transfer (see Nicodano and Regis, 2019).

The simultaneous presence of bailout and payout transfers originates a trade-off between raising debt in the sponsor and in the backed unit. The higher the latter debt, the lower the payoff transfer, the higher the default costs in the leveraged sponsor. Similarly, increasing the sponsor debt reduces support to the backed unit through the bailout guarantee.<sup>4</sup>

We can prove the following proposition:

**Proposition 2.** *A lower interest rate amplifies the incentive for a zero-leverage sponsor.*

**Proof.** See Appendix. ■

#### 4. Optimal leverage and credit risk sensitivity to the risk-free rate

In this section, we numerically analyse the changes in the optimal capital structure following a drop in interest rates. We will compare stand-alone units with connected units displaying the same parameters. We track modifications in the endogenous default probability, spread and loss given default as we vary the level of the risk-free rate.

Table 1 displays the base-case calibration parameters, which we borrow from Leland (2007), that refer to a typical BBB company.

We set the tax rate and the proportional bankruptcy costs to  $\tau = 20\%$  and  $\alpha = 23\%$ , respectively, and then proceed to parametric changes. We fix the marginal distributions of cash flows at maturity (5 years) to a normal distribution with mean  $100 \cdot (1.05)^5$  and Standard Deviation  $\sigma = 22 \cdot \sqrt{5}$  and we maintain a joint normality assumption for connected units, letting correlation vary. We compare changes in the capital structure of the Stand-Alone and of the connected units, when the risk-free rate falls from 5% to 1%.

##### 4.1. The Stand-Alone Company

Our first observation is that, in the stand-alone case, the decrease in interest rates reduces the incentives towards leverage (see Table 2).

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<sup>4</sup>The trade-off would be sharpest if the payout transfer could be set to zero when the sponsor defaults; in other words, if the sponsor's lenders cannot file a revocatory action to recover their dividends. Savings in default costs become non-decreasing in internal ownership, for any density, and payoff transfers become the mirror image of bailouts.

<b>Table 1: Base-case parameters</b>		
<b>Symbol</b>	<b>Parameter</b>	<b>Value</b>
$\tau$	Tax Rate	20%
$\alpha$	Default Costs rate	23%
$r$	Interest rate	5%
$\phi$	Discount Factor	0.78
$X(0)$	Cash flow present value	100
$V_U$	Unleveraged firm value	80.05
$T$	Time Horizon	5
$\sigma$	Cash flow volatility	$22 * \sqrt{5}$
$\rho$	Cash flow correlation	0.2

Table 1: This table displays the base-case parameters, following Leland (2007).

Indeed, following the interest rate drop, the optimal face value of debt for a Stand-Alone company decreases by almost 40%, from 57.1 to 34.3. Also the total market value of debt drops almost 25%, from 42.2 to 31.8. While the value of a hypothetical zero-leverage company increases sharply (from 80.05 to 97.20) when the interest rate drops, due to the higher discount factor, the value of leverage, i.e. the difference between the optimally leveraged and the zero-leverage firm value, drops dramatically, from 1.42 to 0.25. The reduction in the value of leverage is explained by the lower relevance of the tax shield, which falls from 14.89 to 2.46. As a consequence, the tax savings from leverage fall, from 2.32 to 0.47. Symmetrically, taxes increase from 17.70 to 23.84.

Since the optimal debt is smaller, the default threshold shrinks, albeit less sharply than the tax shield, from 67.65 to 42.26. Default costs reduce accordingly, from 0.89 to 0.21, reflecting the lower risk for lenders.<sup>5</sup> Interestingly, default costs drop not only in absolute terms, but also relative to both the optimal value and the present value of expected cash flow. The lower riskiness of the optimal Stand-Alone company as interest rates fall is mirrored in a much smaller default probability. In the base-case, it is 11.14% at the 5-year horizon<sup>6</sup>. This is largely due to leverage, as the probability of default for the zero-leverage firm is

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<sup>5</sup>In a traditional trade-off model with exogenous bankruptcy probability, we would obtain an even sharper reduction in optimal leverage, with a falling tax benefit of debt together with fixed bankruptcy probability.

<sup>6</sup>The default probability is defined as the probability that the firm is not able to repay its debtholders after  $T = 5$  years, when the cash flows are realized, i.e.  $DP = \int_{-\infty}^{X^d} f(x)dx$ .

Table 2: Optimal stand-alone		
Parameter	Interest Rate	
	5%	1%
Principal ( $F^*$ )	57.1	34.3
Value ( $V^*$ )	81.47	97.45
Debt ( $D^*$ )	42.21	31.84
Equity ( $E^*$ )	39.26	65.61
Tax Shield ( $X_Z^*$ )	14.89	2.46
No-default threshold ( $X_d^*$ )	67.65	42.26
Unlevered Firm Value ( $V_U$ )	80.05	97.20
Value of Leverage ( $V^* - V_U$ )	1.42	0.25
Debt Yield( $y^*$ ) (spread, $s^*$ )	6.23% (1.23%)	1.50% (0.50%)
Taxes ( $T^*$ )	17.70	23.84
Tax Savings( $TS^*$ )	2.32	0.47
Default Costs ( $C^*$ )	0.89	0.22
5- Year Default Probability ( $DP^*$ )	11.14%	4.13%
Loss Given Default ( $LGD^*$ )	28.96	20.16

Table 2: This table displays the optimal figures of a Stand-Alone unit with parameters as in Table 1, for two levels of interest rates, 5% and 1%. The yield is computed as  $(F^*/D^*)^{\frac{1}{5}} - 1$ , the loss given default as  $\frac{F^* - D^* \frac{1}{\phi}}{DP^*}$ . Yields and spreads are annualized, while the default probability is over the 5-year horizon. Cash flow distribution is fixed:  $X \sim N(127.63, 49.19)$ .

0.47%, only. When interest rate drops to 1%, leverage decreases and the default probability decreases accordingly, down to 4.13%. The endogenous (annualized) spread<sup>7</sup> reflects such change, decreasing from 1.23% to 0.50%.

Lenders' losses upon default instead reduce in absolute terms as the interest rate decreases, from 28.96 to 20.16. However, this is due to the reduction in the optimal debt principal value. Indeed, as a percentage of the principal, the loss given default worsens, because only about 41% of the principal is recovered by lenders in default when the interest rate is 1% vs. 50% when it is 5%.<sup>8</sup> Summarizing, while defaults are less frequent, and this drives the drop in expected default costs, they have more severe consequences, as a proportion of outstanding debt. This happens because the default threshold is closer to zero, and defaults occur in most cases when after-tax profits are negative or very small, leading to zero

<sup>7</sup>The spread  $s$  is the difference between the yield, defined as  $y = (F/D)^{\frac{1}{5}}$  and the interest rate  $r$ .

<sup>8</sup>Such increase in losses upon default, which are defined as  $LGD = \frac{F - D \frac{1}{\phi}}{DP}$ , derives from the increase in the discount factor in the calculation of expected discounted losses (at the numerator) and by the decrease in the default probability (at the denominator).



or little recovery for debt-holders.

The decrease in optimal leverage when interest rate decreases is a consistent pattern across parametric changes. It occurs when cash flow volatility is higher (44%) or lower (15%), when the tax rate is higher (24%) or lower (16%) and when the proportional default cost parameter is higher (26%) or lower (20%), consistent with the insight deriving from Proposition 1. The decrease in leverage due to a fall in interest rates is milder the higher the incentive toward leverage, i.e. the higher the volatility and the tax rate and the lower the default cost rate.

We can summarize these results as follows, assuming that everything else is unchanged including the distribution of future cash flows:

**Observation 1.** *In a Stand-Alone company, the optimal leverage falls when the risk-free interest rate decreases. Both expected default costs and default probability decrease, while losses upon default increase.*

#### *4.2. A Zero-Leverage Sponsor: the Private Equity Case*

In this section we turn to the case where a sponsor supports the service of debt of a backed company. We start by addressing the case where both are endowed with the base-case parameters presented in Table 1. Table 3 reports the optimal capital structure and relevant figures for different levels of correlation between cash flows between the connected units. However, we will focus our discussion on the case of a weak (0.2) positive correlation between unit cash flows. This is also the correlation maintained in Figure 1 and Figure 2, that display the implied changes in connected units vs. two equivalent stand-alone units when the risk-free rate varies in the interval [1%,5%].

When the risk-free rate is 5%, the sponsor has optimal zero leverage as in Luciano and Nicodano (2014). On the contrary, the bankruptcy-remote backed company has an optimal face value of debt which is almost four times that of the Stand-Alone (220 versus 57.1). Such polarized capital structure is typical of both sponsor/SPV arrangements (Gorton and Souleles, 2006) and private equity fund-LBO target firm (Cohn et al., 2014). Thanks to zero leverage, the sponsor maximizes the bailout support provided to the highly leveraged company, which is in turn able to maximally exploit its tax shield. The tax shield indeed reaches

103 in the subsidiary unit, up from 14.9 in the stand-alone company. As a consequence, tax savings from leverage in the backed unit are far higher than in the stand-alone (14.62 vs. 2.32). Expected default costs increase as well, and are almost ten times larger than in a stand-alone, reaching 8.13 (vs. 0.89). Such "extreme" exploitation of the tax-bankruptcy trade-off is allowed for by the conditional guarantee provided by the sponsor, that limits the rise in default costs relative to tax savings. Indeed, the sponsor/backed unit organization is more valuable than two equivalent stand-alone (166.59 vs. 162.94). As evident also from Figure 2, despite the sponsor, the backed unit is optimally a very risky entity. Indeed, its default threshold (i.e. the cash flow level below which default occurs) grows to a startling 242, up from 67.7 of the stand-alone case. Its default probability is much higher and the losses upon default are far larger than those of a stand-alone unit (47% vs. 11% and 148.98 vs. 28.96, respectively). Lenders' losses upon default are larger because the sponsor supports the backed company when the latter has positive cash-flows (and the sponsor has enough funds). As a consequence, the endogenous spread, which reflects the credit risk compensation demanded by the lenders rises to 8.45%, up from 1.23% in the stand-alone. It is the large spread that leads to the high tax savings we just illustrated.

Let us now turn to changes in response to a drop in the risk-free rate. In the supported unit the face value of debt increases as the interest rate decreases, reaching 247 when the risk-free rate is 1%. For any interest rate level, it remains optimal for the sponsor to have zero leverage. Indeed, while the incentive towards leverage in the stand-alone company decreases due to the lower interest rate, in the sponsor/backed unit organization the drop in interest rates results in a more extreme exploitation of the tax-bankruptcy trade-off. We now study how this result derives from endogenous debt pricing and costly default.

When debt increases, several things happen. First, tax savings increase. Second, the default probability increases, driving up the spread. As a consequence, the dead-weight costs of default increase as well. However, in the sponsor/backed unit arrangement, they are mitigated by the bailout transfer occurring when the backed unit has positive cash flows. More precisely, the tax savings increase as the interest rate falls, from 14.6 to 19.1 and the expected default costs (concentrated in the backed unit) almost double, rising from 8.1 to 14.5. The ratio of expected default costs to total group value rises from 4.88% to 7.29%,

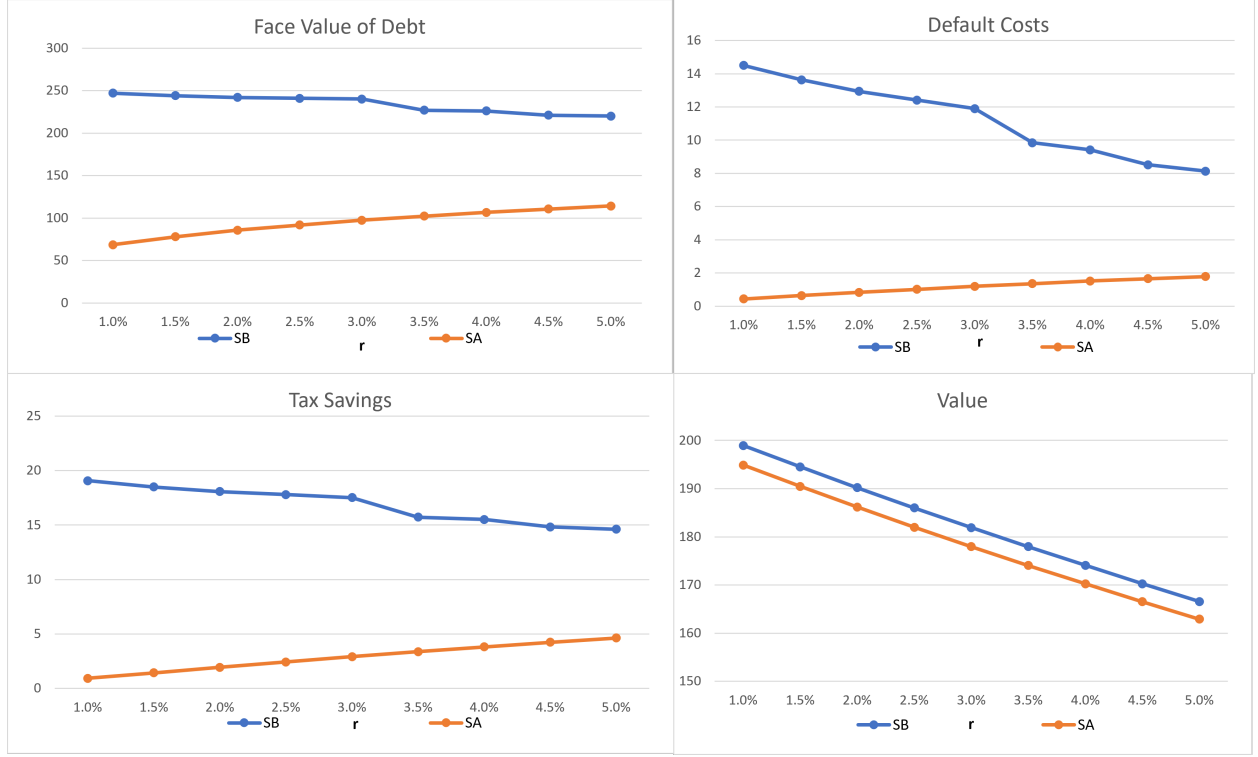


Figure 1: This figure contrasts the optimal face value of debt, default costs, taxes and total value of a company with its bankruptcy-remote sponsor (in blue) and the equivalent stand-alone arrangements (in orange) for different interest rate levels, ranging from 1% to 5%. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

outpacing the growth of expected tax savings to group value, which instead rises from 8.78% to 9.59%. While the value of leverage, i.e. the difference between the optimal and the zero-leverage value, falls from 6.49 to 4.54, the sponsor/backed unit remain 2.3% more valuable than the stand-alone (198.96 vs. 194.42). While debt market value – concentrated in the backed company – increases by 14%, from 117.06 to 133.43, the equity value (concentrated in the sponsor) increases much more, by almost 33%. This is why the market value of leverage does not increase. Such increase in equity value, due to the discounting effect of lower interest rates, provides the capital buffer needed to enhance the support provision to the backed unit.

The riskiness of the backed unit increases, as the risk-free rate decreases. Indeed, as portrayed in Figure 2, the default probability increases as the rate decreases, hitting 63.3% when the risk-free interest rate is 1%, up from 47.38% when  $r = 5\%$ . The spread of the backed unit increases accordingly, topping 12.11%, as does the loss given default, from 148.98

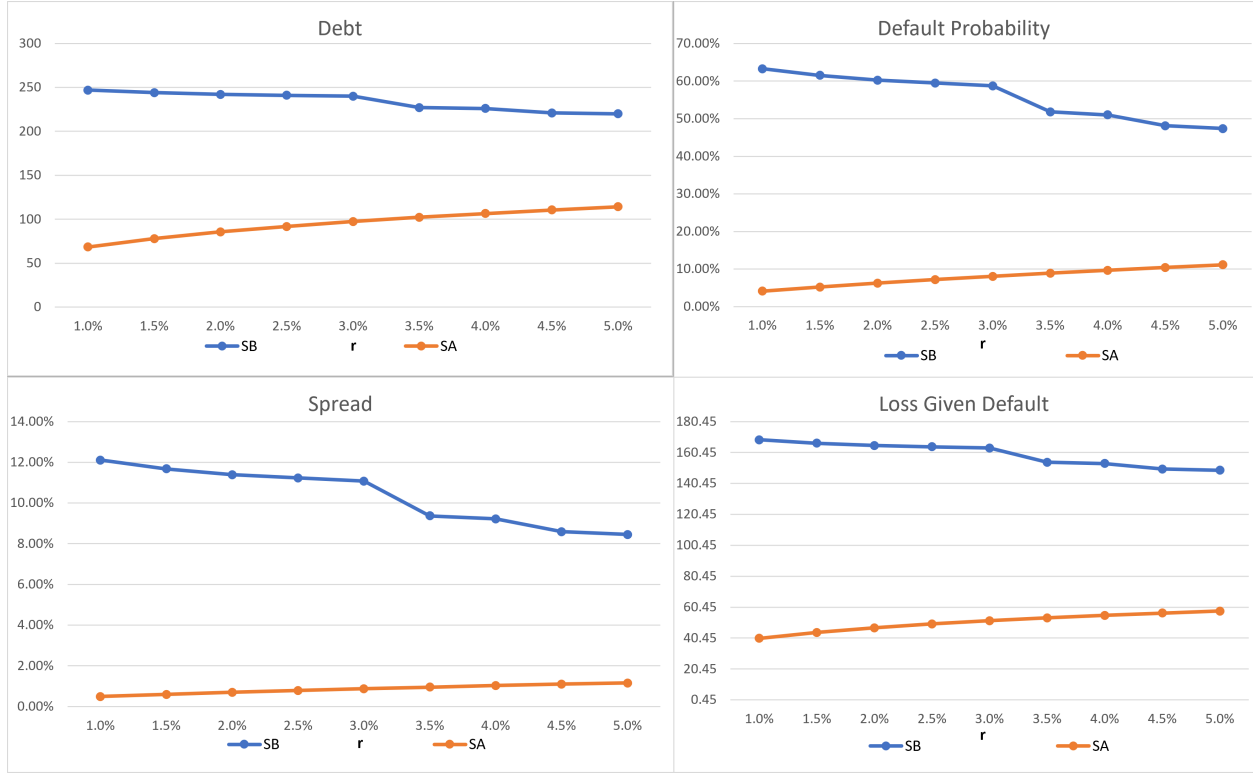


Figure 2: This figure contrasts the optimal face value of debt, 5-year default probability, annualized credit spread and loss given default of a company backed by a bankruptcy-remote sponsor (in blue) and the equivalent stand-alone (in orange) for different interest rate levels, ranging from 1% to 5%. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

to 168.68 (or from 67.72% to 68.29% in percentage terms). Again, the backed unit behaves very differently relative to the stand-alone, whose default probability and spread decrease when the interest rate decreases. On the contrary, the zero-leverage sponsor is insolvent very rarely in all interest rate scenarios. This happens only when both its cash flow realization is negative and the backed unit payout is not large enough to cover such losses. This is why the joint default probability is unaltered when the interest rate changes.

Figure 1 and Figure 2 show that the patterns we just discussed hold uniformly as the level of the risk free rate falls from 5% to 1%. Furthermore, the above results qualitatively hold true for different correlation levels, as reported in Table 3.

In particular, higher cash flow correlation makes support more valuable because it allows for higher tax savings. To obtain such tax savings, the sponsor has to be able to provide funds when the supported unit has positive cash flows. Both the optimal face value of debt

Table 3: Optimal Value and Debt: Sponsor/Backed Unit						
Parameter	Correlation					
	-0.8		0.2		0.8	
	Interest Rate		Interest Rate		Interest Rate	
	5%	1%	5%	1%	5%	1%
Face Value of Debt	183 (0;183)	201 (0;201)	220 (0;220)	247 (0;247)	227 (0;227)	257 (0;257)
Market Debt Value	133.58 (0;133.58)	153.38 (0;153.38)	117.06 (0;117.06)	133.43 (0;133.43)	115.53 (0;115.53)	133.55 (0;133.55)
Equity Value	32.74 (32.74;0)	42.88 (42.88;0)	49.52 (49.52;0)	65.53 (65.53;0)	51.84 (51.84;0)	66.70 (66.70;0)
Total Value	166.32 (32.74;133.58)	196.26 (42.88;153.38)	166.59 (49.52;117.06)	198.96 (65.53;133.43)	167.36 (51.83; 115.53)	200.24 (66.70;133.54)
Value of Leverage	6.23	1.86	6.50	4.56	7.27	5.86
Tax Savings	7.57 (0; 7.57)	8.87 (0; 8.87)	14.62 (0; 14.62)	19.07 (0; 19.07)	15.50 (0; 15.50)	20.16 (0; 20.16)
Taxes	32.45 (20.01;12.44)	39.73 (24.30;15.43)	25.40 (20.01;5.39)	29.53 (24.30;5.23)	24.52 (20.01;4.51)	28.44 (24.30;4.14)
Default Costs	1.91 (0;1.91)	7.29 (0;7.29)	8.13 (0;8.13)	14.50 (0;14.50)	8.24 (0;8.24)	14.29 (0;14.29)
Yield Sponsor (Spread)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)	N/A (N/A)
Yield Backed Unit (Spread)	6.50% (1.50%)	5.56% (4.56%)	13.45% (8.45%)	13.11% (12.11%)	14.46% (9.46%)	13.99% (12.99%)
Default Probability Sponsor	0%	0%	0%	0%	0%	0%
Default Probability Backed Unit	11.05%	30.76%	47.38%	63.30%	50.43%	64.73%
Joint Default Probability	0.01%	0.22%	0.47%	0.47%	0.47%	0.47%
Loss Given Default Sponsor	0	0	0	0	0	0
Loss Given Default Backed Unit	113.28	129.38	148.98	168.68	157.77	180.21

Table 3: This table displays the optimal figures of a backed unit with its bankruptcy remote sponsor when both units displaying the parameters in Table 1, for two levels of interest rates, 5% and 1%. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8. Yields and spreads are annualized, the default probabilities are the probabilities that debtholders are not repaid in full when cash flows are realized at  $T = 5$  years. Sponsor and backed Unit figures are reported in brackets, respectively.

of the backed unit and its riskiness, as captured by the spread, increase with correlation and top 257 and 12.99%, respectively, when cash flow correlation is equal to 0.8 and the interest rate is 1%.

The statement below summarizes these patterns, assuming that only the risk-free rate varies:

**Observation 2.** *In a company backed by a zero-leverage sponsor, the optimal debt increases when the risk-free interest rate decreases. Both the spread and tax savings, along with default probabilities and losses upon default increase. These changes are larger the higher is the cash-flow correlation between the sponsor and its backed unit.*

Summarizing, this section shows that a Stand-Alone company does not increase its demand for debt when the risk-free rate falls. With a tax-bankruptcy trade-off, marginal tax savings decrease more than marginal default costs when the risk-free rate falls. In sponsor/backed unit constructions, instead, marginal default costs are kept lower than marginal tax savings by the internal support mechanisms. As a consequence, optimal debt increases and becomes concentrated in one unit.

These findings offer a rationale for the observations reported in the introduction. The trend of falling interest rates in the first two decades of this century has been accompanied

by a disproportionate increase in lending through securitization vehicles such as CLOs in the US (Powell, 2019) and in borrowing by firms with lower ratings in both the EU and the US (Darmouni and Papoutsis, 2021; Schularick, 2021).

## 5. A Leveraged Sponsor: the Parent-Subsidiary Case

The previous section shows that the optimal debt of a backed company may increase in response to a decrease in interest rates, in contrast to the case of a stand-alone company. This pattern holds when the sponsor is optimally zero-leverage before the interest rate reduction and remains optimally zero-leverage afterwards. In turn, this occurs when the tax/bankruptcy-cost ratio is equal across the sponsor and backed company, or when it is lower for the sponsor than for the backed company.

A change in the level of the risk-free interest rate may however prompt a restructuring of the sponsor’s capital structure, as well. This section analyzes one such case for robustness purposes. The sponsor initially displays positive optimal leverage at the base-case interest rate level because of a higher tax-bankruptcy-cost ratio than its backed company. This case is empirically relevant since we often see a leveraged parent in multinationals and other corporate groups, which also acts as sponsor by backing its subsidiaries’ debt (Bianco and Nicodano, 2006; Brok, 2022; Anantavasilp et al., 2020). This case is theoretically interesting because it involves a trade-off between leveraging the backed subsidiary, in order to exploit the sponsor’s limited liability, and leveraging the parent, which enjoys higher marginal tax rates and/or a lower proportional bankruptcy cost parameter.

A numerical example shows conditions such that the optimal response to a reduction in interest rates is the creation of a LBO-like structure with debt concentrated in the backed company and a zero-leverage sponsor. In other words, when interest rates drop, a parent may find it profitable to sell its subsidiary to a zero-leverage private equity fund that will lever up the LBO target, rather than entertaining itself this financial restructuring.

Let us consider the case when the tax rate of the sponsor company ( $\tau_S = 24\%$ ) exceeds the one of the backed unit ( $\tau_B = 16\%$ ). Since the incentive to raise debt also in the sponsor is in general stronger the higher is cash flow volatility, we set  $\sigma_S = \sigma_B = \sigma = 44 * \sqrt{5}$ , similarly to Nicodano and Regis (2019). We first focus on the base-case correlation ( $\rho = 0.2$ ).

Figure 3 reports the optimal debt, market leverage, tax savings and default costs of the sponsor/backed unit organization and the equivalent two stand-alone units for interest rate levels ranging between 1% and 5%. Figure 4 displays the optimal debt, default probability, spread and loss given default of the sponsor and the backed unit, comparing them with their equivalent stand-alone values. Table 4 reports the numerical values of the optimal characteristics.

When  $r = 5\%$ , our choice of parameters leads to optimal positive leverage in both units. The total face value of optimal debt exceeds the one in two equivalent stand-alone units (191 vs. 169), and the sponsor raises only slightly more debt than its backed unit (98 vs. 93). This may seem counter-intuitive, because the parent has a much higher tax rate, and the debt tax shield is therefore more valuable in that unit. However, to preserve its ability to provide support, the sponsor raises less debt than the equivalent stand-alone (98 vs. 103), while the backed unit raises more (93 vs. 68). The sponsor bears much lower default costs than the stand-alone peer as it receives the payout from the backed unit, a mechanism that emerges in Anantavasilp et al. (2020). In turn, to endow the sponsor with a positive payoff, the subsidiary is not as leveraged as in the private equity case depicted in the previous Section 4.2, but still raises higher debt than the stand-alone. Overall, tax savings decrease relative to the stand-alone case (9.92 vs. 10.32), but default costs are mitigated (5.08 vs. 6.09), leading to higher total value (170.48 vs. 169.88). The sponsor and the backed unit appear to be similarly risky, with a (5-year ahead) default probability of around 31%, but they display respectively a lower (4.82% vs. 7.25%) and higher (annualized) spread (5.49% vs. 4.74%) than their stand-alone equivalent.

As the interest rate decreases to 1%, we highlight two main effects. First, the total face value of debt raised by the organization tends to increase, from 191 to 223. As Figure 4 captures, this is in sharp contrast with what happens to the two equivalent stand-alone units, whose combined optimal debt falls monotonically from 170 to 121 as interest rates drop. Second, debt is entirely raised in the backed unit only when the interest rate is small enough. Indeed, when the interest rate falls below a certain level, the sponsor optimally specializes in providing support as in the private equity-like structure described in Section 4.2. This occurs because a lower interest rate shifts the balance between two opposing incentives, increasing

the sponsor tax shield versus providing additional support to the backed unit, toward the latter. The drop in interest rates reduces the incentive to leverage up the sponsor, since it should bear higher default costs to reach the same tax savings level. The combination of connected units is however able to exploit the tax shield, while shielding the sponsor from bankruptcy, by leveraging up the backed unit. While the sponsor becomes zero leverage, the backed unit maximally exploits the tax shield, allowing the organization to become more valuable at the cost of increasing its riskiness.

The default costs of the organization increase sharply as the interest rate falls. In particular, those of the backed unit are more than 11 times the default costs of an equivalent stand-alone unit (9.08 vs. 0.79) and the (5-year) default probability reaches 54% when the interest rate is 1%. Losses upon default deteriorate as well, rising to 168.63 from 67.32 (75.62% vs. 72.38% in percentage terms) relative to the 5% interest rate case. These changes affect the endogenous (yearly) spread, which rises from 5.49% to 10.32%. The default probabilities of the two units move in two opposite directions. In the sponsor, as debt decreases the default probability drops when the interest rate moves from 5% to 1%. Below 3%, when the sponsor becomes a zero-leverage entity, it defaults only when its realized cash flows are negative (9.68% probability), bearing no losses due to its limited liability.<sup>9</sup>

For our selected parameters, the transformation to a zero-leverage structure composed of a sponsor and a backed unit occurs for every correlation level, leading us to the following observation, under the usual *coeteris paribus* assumption:

**Observation 3.** *When the risk-free rate falls, a parent-subsidiary structure with balanced debt may transform into a zero-leverage sponsor and a highly leveraged company, even if the tax rate (proportional bankruptcy cost) of the parent exceeds (is lower than) the subsidiary's.*

In Table 4 we let cash flow correlation vary for two risk-free interest rate value: 5% and 1%. First, we observe that, at the 5% interest rate level, the sponsor raises more debt than its backed unit when cash flow correlation is positive. Indeed, when cash flow correlation is

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<sup>9</sup>As the risk-free rate approaches 3% from above, lenders' losses upon default get closer to 100% since the default threshold approaches zero, implying that there is hardly any recovery due to negligible or negative after-tax profits.



negative there are diversification benefits, since payouts from the backed unit help support an insolvent sponsor, but little tax savings from debt. This is because the sponsor tends to have positive cash flows when the backed unit has negative cash flows, resulting in zero support being provided. On the contrary, when cash flow correlation is positive, tax savings are maximized thanks to the support mechanism. Indeed, support to the backed unit is more effective in saving default costs the higher the correlation. With negative cash flow, the sponsor would be able to provide support when the backed company would not suffer from default, or viceversa the sponsor would not be able to provide support when the backed company faces positive default costs. Under both interest rate scenarios, as a consequence, the total face value of debt (with the only small exception of moving from 0 to 0.2 correlation in the 5% interest rate case) increases with correlation. Total default costs, accordingly, follow the same pattern.

When moving from a 5% to a 1% level of the risk-free rate, we find that total debt of the connected units always increases. This occurs for high enough correlation ( $|\rho| > -0.2$ ). In those cases, the support mechanism is valuable enough to mitigate the increased marginal default cost associated with higher leverage. Default costs, instead, increase in absolute value for all correlation levels (and as a percentage of group value, when correlation is not extreme ( $|\rho| < 0.8$ )).

An important difference appears when looking at the magnitude of default costs relative to stand-alone units. When the interest rate is 5% default costs in the connected units are always smaller than the costs in the two equivalent stand-alone units (7.52), as in the case presented in Figure 3, unless correlation is very high (0.8). On the contrary, when the interest rate is 1%, this happens only for the lowest correlation level (-0.8), because cash flow diversification limits joint default. Thus, transformation to a sponsor/backed unit following a rate drop is privately optimal, but the organization is not welfare-optimal, as stand-alone units display lower default costs. In the 5% interest rate scenario, instead, the privately optimal and socially optimal firm combination is the same for almost all correlation levels.

Finally, default probabilities and spreads increase in the subsidiary for all correlation levels following a drop in interest rates (with the only exception of the default probability when  $\rho = -0.8$ , which only slightly decreases). The 5-year-ahead default probability tops an

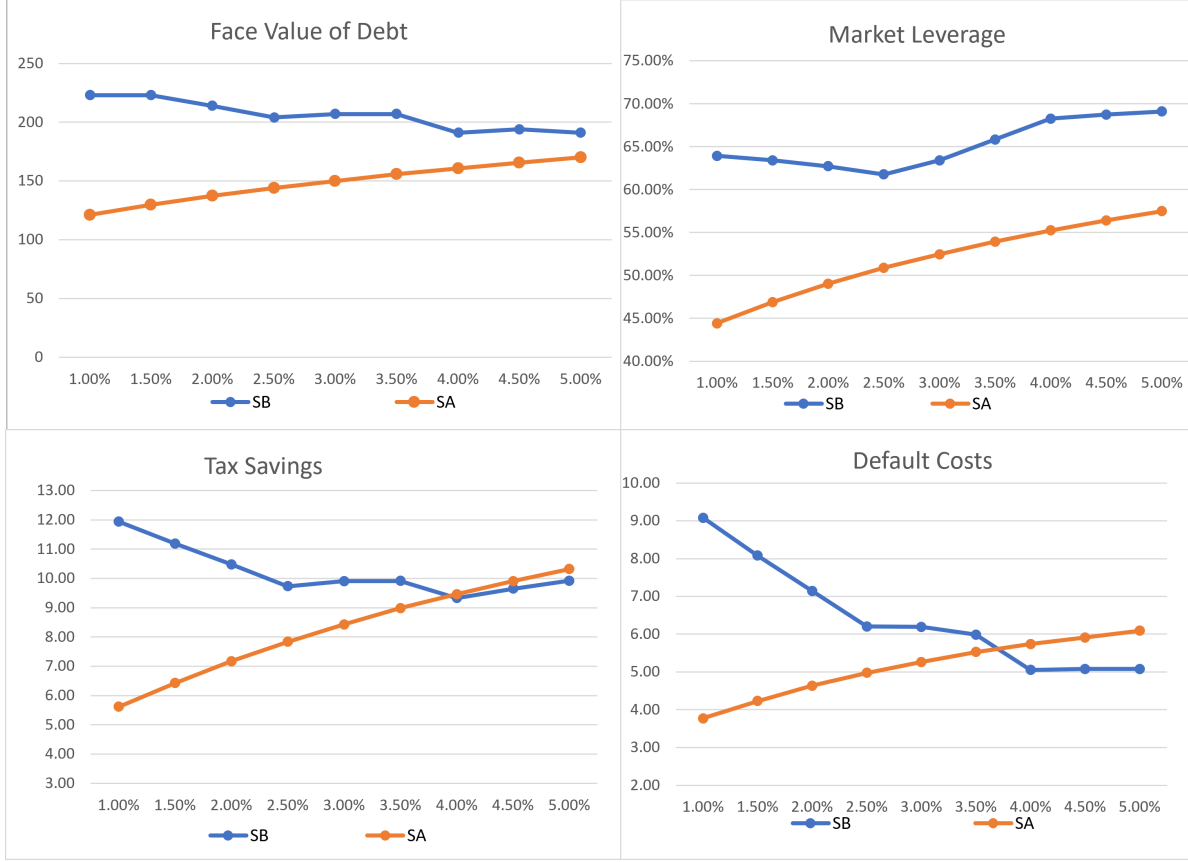


Figure 3: This figure portrays the optimal debt (face value and market value), default costs and tax savings of the sponsor/backed unit arrangement (in blue) when interest rate ranges from 1% to 5% and compares the figures with those of equivalent stand-alone units (orange). In the upper left panel, the red line depicts the optimal debt of the sponsor.

impressive 57.44% when  $\rho = 0.8$ , and the annualized spread consequently reaches 12.85%.

Two further effects are worth noticing. Firstly, the probability of a joint default decreases for all analyzed correlation levels when interest rate drops to 1%. This happens because of the limited liability of the sponsor, which, being zero-leverage, defaults only when its cash flows are negative. On the contrary, total default costs always increase, because debt is concentrated in one very risky unit, which is very likely to default. Secondly, not only the default probability (with the exception of very low correlation levels), but also losses upon default in the subsidiary increase.

Again, some model-based insights are broadly consistent with observation. The model predicts increased LBO activity when interest rates fall, and a consequent increase in default risk. Lower interest rates have indeed accompanied higher LBO activity, although there may

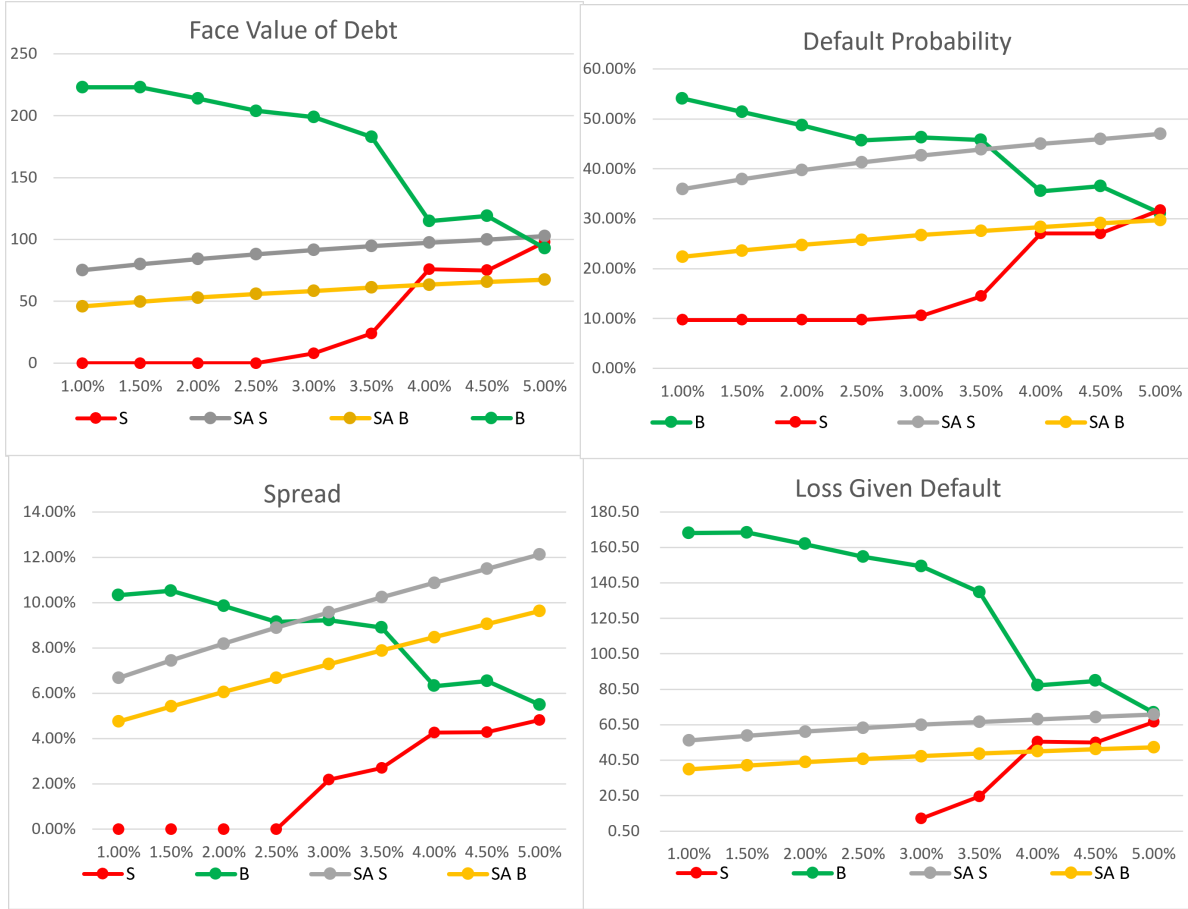


Figure 4: This figure portrays the debt (face value), default probabilities, spreads and loss upon defaults of the optimal sponsor/backed unit arrangement (blue), the sponsor alone (red), the backed unit (green) and compares their figures with those of equivalent stand-alone units (in grey and yellow, respectively). Spreads are annualized, the default probabilities are the probabilities that debtholders are not repaid in full when the cash flows are realized at  $T = 5$  years.

be other factors behind such association (Ivashina, 2022) beyond the demand-side factors we stress. As for the default risk, the one of Ba issuers doubles after being acquired by a private equity firm while that of B issuers rises by 75% (Moody's, 2006). When adding other controls in a regression analysis, the ratio of sponsored to non-sponsored default, 3 years after the acquisition, is about 1.22 to 1.30 but depends on the industries the firms belong to.

Table 4: Optimal Value and Debt: Transformation of a Parent Subsidiary Structure

Parameter	Correlation											
	-0.8				-0.2				0			
	Interest Rate		1%		Interest Rate		1%		Interest Rate		1%	
	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
Face Value of Debt	167 (30.117)	156 (0.156)	191 (66.125)	189 (0.189)	103 (72.121)	214 (0.214)	191 (98.93)	232 (0.232)	237 (175.62)	251 (0.251)	237 (175.62)	251 (0.251)
Market Value of Debt	115.46 (38.1977.26)	124.63 (0.124.63)	118.79 (41.9373.86)	125.45 (0.125.45)	118.31 (47.47; 70.83)	129.11 (0.129.11)	117.83 (61.3656.47)	130.46 (0.130.46)	129.29 (90.36; 38.89)	131.22 (0.131.22)	129.29 (90.36; 38.89)	131.22 (0.131.22)
Equity Value	55.04 (55.04.0)	79.03 (79.03.0)	51.48 (51.48.0)	77.94 (77.94.0)	52.06 (52.06.0)	74.64 (74.64.0)	52.65 (52.65.0)	73.60 (73.60.0)	43.37 (43.37.0)	74.53 (74.53.0)	43.37 (43.37.0)	74.53 (74.53.0)
Total Value	170.50 (93.24; 77.26)	203.66 (79.03; 124.63)	170.26 (96.40; 73.86)	203.40 (77.94; 125.46)	170.37 (99.54; 70.83)	203.75 (74.64; 129.11)	170.48 (114.02; 56.47)	204.06 (73.60; 130.46)	172.62 (133.73; 38.89)	205.75 (74.53; 131.22)	172.62 (133.73; 38.89)	205.75 (74.53; 131.22)
Value of Leverage	4.85	2.51	4.62	2.26	4.73	2.62	4.86	2.94	7.02	4.66	7.02	4.66
Taxes	35.14 (22.87; 12.27)	46.13 (30.17; 15.95)	32.48 (21.35; 11.13)	42.25 (30.17; 12.08)	32.02 (20.79; 11.23)	39.96 (30.17; 9.79)	31.48 (18.88; 12.60)	38.35 (30.17; 8.17)	26.14 (12.12; 14.01)	36.77 (30.17; 6.59)	26.14 (12.12; 14.01)	36.77 (30.17; 6.59)
Tax Savings	6.27 (1.08; 4.29)	4.16 (0.4; 1.6)	8.93 (3.50; 5.43)	8.03 (0.8; 0.3)	9.39 (4.05; 5.34)	10.33 (0.10; 3.3)	9.92 (5.96; 3.96)	11.94 (0.11; 9.4)	15.28 (12.73; 2.55)	13.52 (0.13; 5.2)	15.28 (12.73; 2.55)	13.52 (0.13; 5.2)
Default Costs	1.44 (0.31; 1.13)	1.68 (0.1; 0.68)	4.28 (1.29; 2.99)	5.82 (0.5; 8.2)	4.65 (1.61; 3.04)	7.89 (0.7; 8.9)	5.08 (3.02; 2.06)	9.08 (0.9; 0.8)	8.92 (7.68; 1.24)	8.97 (0.8; 9.7)	8.92 (7.68; 1.24)	8.97 (0.8; 9.7)
Yield Sponsor (Spread)	5.53% (0.53%)	N/A (N/A)	8.00% (3.00%)	N/A (N/A)	8.69% (3.69%)	N/A (N/A)	9.82% (4.82%)	N/A (N/A)	14.13% (9.13%)	N/A (N/A)	14.13% (9.13%)	N/A (N/A)
Yield Backed Unit (Spread)	8.65% (3.65%)	4.59% (3.59%)	11.10% (6.10%)	8.54% (7.54%)	11.30% (6.30%)	10.63% (9.63%)	10.49% (5.49%)	11.32% (10.32%)	9.78% (4.78%)	13.85% (12.85%)	9.78% (4.78%)	13.85% (12.85%)
DP Sponsor	4.70%	9.68%	20.18%	9.68%	24.03%	9.68%	31.74%	9.68%	52.32%	9.68%	52.32%	9.68%
DP Backed Unit	19.48%	19.32%	34.00%	40.16%	35.16%	48.88%	31.10%	54.09%	27.63%	57.44%	27.63%	57.44%
Joint Default Probability	2.08%	0.91%	14.35%	6.80%	17.93%	8.47%	21.63%	9.30%	27.55%	9.74%	27.55%	9.74%
LGD Sponsor	26.69	0	42.29	0	47.47	0	62.01	0	114.07	0	114.07	0
LGD Backed Unit	94.39	129.44	90.40	142.27	87.03	160.19	67.32	168.63	44.74	196.88	44.74	196.88

Table 4: This table displays the optimal figures of a Parent/Subsidiary structure when the parameters for the two units are those in Table 1, apart from  $\tau_P = 24\%$ ,  $\tau_S = 16\%$ ,  $\sigma_P = \sigma_S = \sigma = 44 = \%$  for two levels of interest rates, 5% and 1%. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8. Sponsor and Backed Unit figures are reported in brackets, respectively.

## 6. Concluding Remarks

This paper studies the heterogeneous sensitivity of optimal corporate debt to interest rates, in a tax-bankruptcy trade-off model, highlighting the role of sponsored debt.

In our model, companies optimally decrease leverage when interest rates drop, because the tax shield of debt become less valuable relative to expected default costs. However, the presence of a sponsor mitigates the increase in default costs in the backed company - arising from the reduction of the tax shield. This way optimal debt in the backed company increases when the level of interest rates fall. This is the first corporate finance theory, to our knowledge, that explains increased demand for debt in response to a lower level of interest rates, complementing existing explanations that stress supply considerations - such as investors' search for yield and time-inconsistent monetary policy.

The spread of the backed company in our model exceeds the one of its stand-alone equivalent, due to its higher leverage and default probability. The model implications are thus broadly consistent with the observed disproportionate increase in borrowing by firms with higher spreads and lower credit ratings, both in the EU and in the US.

Our model also suggests that the share of private equity funds (parent companies) providing support should decrease (increase) in the level of the interest rates. In other words, the model predicts that more assets will be sold out to private equity funds when interest rates fall beyond a certain threshold, which depends on the tax-bankruptcy parameters.

The implications of our model concerning optimal leverage and spreads are broadly consistent with the observed concentration of leverage increases among high-risk companies in the years of falling interest rates. The model also indicates that bankruptcy rates prediction, in different interest rates scenario, flip sign depending on the presence and the characteristics of a bankruptcy-remote sponsor.

Our model supports the financial stability concerns arising from the increasing leverage of riskier entities, that appeared in association with lower interest rates. In fact, backed companies in our model display higher default probabilities and default costs in comparison to stand-alone counterparts. This conclusion is however conditional on the companies having the same cash-flow distribution and the same horizon, as in our set up, while it is often the case that highly leveraged ones have shorter horizons and operate in defensive industries.

Furthermore, bankruptcy-remote sponsors hardly ever default contrary to their stand-alone counterparts. Last but not least, the default probability of sponsored entities increases in those interest rates scenarios when the one of stand-alone activities falls. This suggests that heterogeneous company types smooth variation across interest rate scenarios of aggregate defaults. A thorough assessment of financial stability in alternative interest rate scenarios therefore deserves a much closer scrutiny.

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## 7. Appendix

### 7.1. Definition of the $h(\cdot)$ and $k(\cdot)$ functions

The function  $h(X_B)$  defines the set of states of the world in which the sponsor has enough funds to intervene and save its affiliate from default while at the same time remaining solvent. The rescue happens if the cash flows of the sponsor  $X_S$  are enough to cover both its own debt obligations and the remaining part of those of the subsidiary. The function  $h(X_B)$ , which defines the level of parent cash flows above which the rescue occurs, is defined as:

$$h(X_B) = \begin{cases} X_S^d + \frac{F_B}{1-\tau_B} - \frac{X_B}{1-\tau_B} & X_B < X_B^Z, \\ X_S^d + X_B^d - X_B & X_B \geq X_B^Z. \end{cases}$$

Similarly, the function  $k(X_S)$  describes the level of dividends required to rescue the sponsor from default. It is defined as

$$k(X_S) = \begin{cases} X_B^d + \frac{F_S - X_S}{(1-\tau_B)} & X_S < X_S^Z, \\ X_B^d + \frac{F_S - \tau_S X_S^Z - (1-\tau_S)X_S}{(1-\tau_B)} & X_S \geq X_S^Z. \end{cases}$$

When  $X_B < X_B^Z$  ( $X_S < X_S^Z$ ) the cash flow  $X_B$  ( $X_S$ ) of the subsidiary does not give rise to any tax payment, as it is below the tax shield generated in that unit.

### 7.2. Proof of Lemma 1

The derivatives of the expected discounted values of taxes and default costs are respectively (we suppress dependence on  $F_i$  and the subscript  $i$  for notational convenience):

$$\begin{aligned} \frac{\partial T}{d\phi} &= \frac{T}{\phi} - \frac{\partial X^Z}{d\phi} (1 - F(X^Z)) \phi \tau \\ \frac{\partial C}{d\phi} &= \alpha \phi \frac{\partial X^d}{d\phi} X^d f(X^d) \phi \alpha + \frac{C}{\phi}. \end{aligned}$$

Recalling that  $X_Z = F - D$ ,  $X^d = F + \frac{\tau}{1-\tau}D$ , indeed we have:  $\frac{\partial X^Z}{\partial \phi} = -\frac{\partial D}{\partial \phi}$  and  $\frac{\partial X^d}{\partial \phi} = \frac{\tau}{1-\tau} \frac{\partial D}{\partial \phi}$ .

Hence, we need to focus on the derivative of market debt value with respect to  $\phi$ , for fixed  $F$  (dependence of  $D$  on  $F$  is suppressed for notational convenience):

$$\begin{aligned}\frac{\partial D}{\partial \phi} &= \frac{D}{\phi} + \phi \left[ (1-\alpha) \frac{dX^Z}{d\phi} X^Z f(X^Z) + (1-\alpha-\tau) \left[ \frac{dX^d}{d\phi} X^d f(X^d) - \frac{dX^Z}{d\phi} X^Z f(X^Z) \right] + \right. \\ &\quad \left. - F \frac{dX^d}{d\phi} f(X^d) \right]. \\ \frac{D}{\phi} &= \frac{\partial D}{\partial \phi} \left[ 1 + \phi(1-\alpha) X^Z f(X^Z) - \phi(1-\alpha-\tau) \frac{\tau}{1-\tau} X^d f(X^d) + \right. \\ &\quad \left. - \phi(1-\alpha-\tau) X^Z f(X^Z) + \phi F \frac{\tau}{1-\tau} f(X^d) \right].\end{aligned}$$

The derivative of  $D$  is positive if the term multiplying it is positive. Rearranging it:

$$\begin{aligned}1 &+ \phi(1-\alpha) X^Z f(X^Z) - \phi(1-\alpha-\tau) \frac{\tau}{1-\tau} X^d f(X^d) - \phi(1-\alpha-\tau) X^Z f(X^Z) + \phi F \frac{\tau}{1-\tau} f(X^d) > 0 \\ 1 &+ \phi \tau X^Z f(X^Z) + \phi F \frac{\tau}{1-\tau} f(X^d) - \phi(1-\alpha-\tau) \frac{\tau}{1-\tau} (F + \frac{\tau}{1-\tau} D) f(X^d) > 0. \\ 1 &+ \phi \tau X^Z f(X^Z) - \phi \frac{\tau}{1-\tau} F f(X^d) (-\alpha-\tau) - \phi \left( \frac{\tau}{1-\tau} \right)^2 D f(X^d) (1-\alpha-\tau).\end{aligned}$$

The first two terms are positive. We need to look at whether the (algebraic) sum of the last two terms is non-negative:

$$\phi(\alpha+\tau) F \frac{\tau}{1-\tau} f(X^d) - \phi(1-\alpha-\tau) \left( \frac{\tau}{1-\tau} \right)^2 D f(X^d) \geq 0.$$

Since  $F \geq D$  as soon as  $\phi \leq 1$ , it follows that the above inequality is satisfied if

$$\begin{aligned}\alpha + \tau &\geq (1-\alpha-\tau) \frac{\tau}{1-\tau} \\ \alpha + \tau - \alpha\tau - \tau^2 &\geq \tau - \alpha\tau - \tau^2 \\ \alpha + \tau &\geq \tau,\end{aligned}$$

which is always true. Hence the market value of debt is increasing in  $\phi$  for fixed  $F$ . As a consequence, we have that  $\frac{\partial T}{\partial \phi} > 0$  and  $\frac{\partial C}{\partial \phi} > 0$ , which proves the lemma.

### 7.3. Proof of Proposition 1

When proving Lemma 1, we proved that  $\frac{\partial D}{\partial \phi} > 0$  for fixed  $F$ . We want to prove now that the equity value is increasing in  $\phi$  as well, for fixed  $F$ :

$$\begin{aligned} \frac{\partial E}{\partial \phi} &= \frac{E}{\phi} + \phi \left[ -(1-\tau) \frac{\partial X^d}{\partial \phi} X^d f(X^d) + F \frac{\partial X^d}{\partial \phi} X^d f(X^d) \right] = \\ &= \frac{E}{\phi} + \phi \frac{\partial X^d}{\partial \phi} f(X^d) \left[ \tau X^d - \frac{\tau}{1-\tau} D \right] = \frac{E}{\phi} + \phi \frac{\partial X^d}{\partial \phi} f(X^d) \left[ \tau F + \tau \frac{\tau}{1-\tau} D - \frac{\tau}{1-\tau} D \right] = \\ &= \frac{E}{\phi} + \phi \frac{\partial X^d}{\partial \phi} f(X^d) [\tau F - \tau D] > 0. \end{aligned}$$

This implies that the value of the firm, which is the sum of  $D$  and  $E$ , is increasing in  $\phi$  (decreasing in the interest rate) for any  $F$  and, a fortiori, at the optimum.

Finally, notice that, since  $\frac{\partial X^Z}{\partial \phi} = -\frac{\partial D}{\partial \phi}$  and  $\frac{\partial X^d}{\partial \phi} = \frac{\tau}{1-\tau} \frac{\partial D}{\partial \phi}$ , it follows that

$$\begin{aligned} \left| \frac{\partial X^Z}{\partial \phi} \right| > \left| \frac{\partial X^d}{\partial \phi} \right| &\implies 1 > \frac{\tau}{1-\tau} \\ &\text{i.e. } \tau < \frac{1}{2}. \end{aligned}$$

### 7.4. Proof of Proposition 2

Following Nicodano and Regis (2019), a sufficient condition for the sponsor to be zero-leverage is:

$$\frac{\tau_S(1-\tau_S)G(0)(1-G(0))}{\alpha_B[1-\tau_S G(0)]} \leq \int_0^{X_{SA}^{Z,B*}} xg\left(x, \frac{F_{SA}^{B*}}{1-\tau_B} - \frac{x}{1-\tau_B}\right) dx + \int_{X_{SA}^{Z,B*}}^{X_{SA}^{d,*}} xg\left(x, X_{SA}^{d,B*} - x\right) dx$$

The right hand side of the above inequality is increasing in  $\phi$ , because its derivative relative to  $\phi$  is

$$\frac{\partial X_{SA}^d(F_{SA}^*)}{\partial \phi} X_{SA}^d g(X_{SA}^d) > 0$$

Hence, for fixed  $\tau_S$ ,  $\alpha_B$  and  $G(0)$  the condition is more likely to be satisfied the higher is  $\phi$ , i.e. the lower the interest rate.