What is Missing in Asset-Pricing Factor Models?

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Objective and Motivation

• Major challenge in finance is to price cross-section of stock returns.
  • That is, explain why do stocks differ in their expected returns?

• The first model proposed to address this challenge was the CAPM.

• When the CAPM failed, researchers then explored other candidate models with hundreds of systematic risk factors (factor zoo).

• However, there is still a sizable pricing error in returns, called alpha.

• In this paper we ask:
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  What is missing in asset-pricing factor models?
Where we deviate from existing models

- Existing models typically assume that only systematic (common) risk is compensated in financial markets.

\[ \mathbb{E}(R_{t+1} - R_{ft1_N}) - \beta \lambda = a = 0. \]

- \( \beta \) is a vector of assets’ exposures to systematic risk
- \( \lambda \) is a vector of prices of unit of systematic risk

- Under the Arbitrage Pricing Theory (APT) setting of Ross (1976, 1977), we explore the possibility that unsystematic risk is also compensated.

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\[ \mathbb{E}(R_{t+1} - R_{ft1_N}) - \beta \lambda = a \neq 0. \]
• Each asset-pricing model implies a stochastic discount factor (SDF).

• The SDF adjusts cashflows for time and risk.

\[
\text{price}_t^n = \mathbb{E}[M_{t+1} \times \text{cashflow}_{t+1}^n]
\]

\[
1 = \mathbb{E}[M_{t+1} \times R_{t+1}^n]
\]

\[
0 = \mathbb{E}[M_{t+1} \times (R_{t+1}^n - R_f)]
\]

\[
\mathbb{E}[R_{t+1}^n - R_f] = -\text{cov}(M_{t+1}, R_{t+1}^n) \times R_f
\]

• We examine misspecification in factor models through lens of SDF.
What we do: both Theory and Empirics

1. Under the APT setting
   - Identify the admissible SDF implied by the APT (in which $a \neq 0$);
   - Quantify the importance of unsystematic risk, $a$, by estimating the SDF.

2. Given some candidate factor model
   - Develop a methodology to identify what is missing in the candidate model;
   - Characterize what is missing in some popular models.
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What we find: Empirical findings (first part)

- **Unsystematic risk** is priced in financial markets.
  - The unsystematic SDF component explains more than 72% of variation in the admissible SDF;
  - Several **successful factors** correlate with **unsystematic SDF component**, such as
    - **Value** (Fama and French, 2015),
    - **Momentum** (Jegadeesh and Titman, 1993).

- **Systematic component** of SDF is driven by **Market factor**.
  - The Market factor explains 95% of the variation in the **systematic component** of the SDF.

- **What is missing** in popular candidate models is, largely, compensation for unsystematic risk.
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Theory: Our methodology
Arbitrage Pricing Theory (APT) . . . our starting point

- Gross returns are described by a latent linear factor model
  \[ R_{t+1} - \mathbb{E}(R_{t+1}) = \beta(f_{t+1} - \mathbb{E}(f_{t+1})) + e_{t+1}, \]

- Expected excess returns are, for some \( a \),
  \[ \mathbb{E}(R_{t+1} - R_f 1_N) = a + \beta \lambda, \]

- By asymptotic no-arbitrage, the vector \( a \) satisfies the no-arbitrage restriction
  \[ \forall N, \quad a' V_e^{-1} a \leq \delta_{\text{apt}} < \infty. \]

- \( f_{t+1} \) be the \( K \times 1 \) vector of common (latent) risk factors with risk premia \( \lambda \) and a \( K \times K \) positive definite covariance matrix \( V_f > 0 \),

- \( \beta = (\beta_1, \beta_2, \ldots, \beta_N)' \) is the \( N \times K \) full-rank matrix of loadings,

- \( e_{t+1} \) is the vector of unsystematic shocks with zero mean and the \( N \times N \) positive definite covariance matrix \( V_e > 0 \).
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Proposition 1: The SDF under the APT

- The SDF implied by the APT-model of asset returns is

\[ M_{t+1} = M_{t+1}^\beta + M_{t+1}^a, \quad \text{where} \]

\[ M_{t+1}^\beta = \frac{1}{R_f} - \frac{\lambda V_f^{-1}}{R_f} (f_{t+1} - E(f_{t+1})) \quad \text{... systematic component} \]

\[ M_{t+1}^a = -\frac{a' V_e^{-1}}{R_f} e_{t+1} \quad \text{... unsystematic component} \]
Implementation challenges: Non-negative and feasible SDF

There are two challenges in implementing our SDF.

1. The SDF may not always be strictly positive, thus possibly leading to negative asset prices.

2. $M_{t+1}^{\beta_{mis}}$ and $M_{t+1}^a$ depend on unobservable quantities, such as, $f_{t+1}^{mis}$ and $e_{t+1}$, respectively.

We address both of them

1. We specify the admissible SDF in exponential form.

2. We use the exponential function of linear projections of the SDF on the set of assets.

Proposition 2: our feasible SDF is asymptotically admissible.
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Now, develop results for second question: What is missing in asset-pricing factor models?
Correcting a candidate factor model

- Let’s consider a candidate model with $K^{can}$ observable risk factors $f^{can}_{t+1}$.

\[ R_{t+1} - R_f 1_N = \alpha + \beta^{can} \lambda^{can} + \beta^{can} (f^{can}_{t+1} - \mathbb{E}[f^{can}_{t+1}]) + \varepsilon_{t+1} \]

- The candidate model may omit
  1. Systematic risk factors $f^{mis}_{t+1}$
  2. Compensation for unsystematic risk $a$.

- We can rewrite $\alpha$ and $\varepsilon_{t+1}$ as follows

\[ \alpha = a + \beta^{mis} \lambda^{mis} \]

\[ \varepsilon_{t+1} = \beta^{mis} (f^{mis}_{t+1} - \mathbb{E}[f^{mis}_{t+1}]) + e_{t+1} \]

\[ \text{Var}(\varepsilon_{t+1}) = \beta^{mis} \text{Var}(f^{mis}_{t+1}) + \text{Var}(\epsilon) \]

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$$\mathbb{V}_\varepsilon = \text{var}(\varepsilon_{t+1}) = \beta^{mis} \mathbb{V}_{f^{mis}} \beta^{mis'} + \mathbb{V}_e.$$
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  \varepsilon_{t+1} = \beta^{mis} (f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1} \\
  V_{\varepsilon} = \text{var}(\varepsilon_{t+1}) = \beta^{mis} V_{f_{mis}} \beta^{mis'} + V_e.
  \]
Proposition 4: Correcting the candidate SDF

- Under the APT assumptions, there exists an admissible SDF $M_{t+1}$

$$
M_{t+1} = M_{t+1}^{\beta,can} + \underbrace{(M_{t+1}^a + M_{t+1}^{\beta,mis})}_{=M_{t+1}^\alpha},
$$

$$
M_{t+1}^{\beta,can} = \frac{1}{R_f} - \frac{(\lambda^{can})'V^{-1}_{f^{can}}}{R_f}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}])
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$$
M_{t+1}^a = -\frac{a'V_e^{-1}}{R_f}e_{t+1}
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Key insight of paper that follows from the proposition

- In the **candidate model**, \( \alpha \) represents the pricing error:

\[
\alpha = \mathbb{E}[M_{t+1}^{\beta,\text{can}}(R_{t+1} - R_f 1_N)] \times R_f,
\]

- In the **corrected model**, \( \alpha \) represents compensation for assets’ exposures to the missing factors \( f_{t+1}^{\text{mis}} \) and unsystematic risk \( e_{t+1} \):

\[
\alpha = -\text{cov}(M_{t+1}^{\alpha}, (R_{t+1} - R_f 1_N)) \times R_f \\
= -\text{cov}(M_{t+1}, e_{t+1}) \times R_f.
\]

- In particular, \( a \) is compensation for unsystematic risk \( e_{t+1} \):

\[
a = -\text{cov}(M_{t+1}^{a}, (R_{t+1} - R_f 1_N)) \times R_f \\
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Empirics: Apply methodology
Estimating the set of parameters

- Estimate the data-generating process of asset returns

\[ R_{t+1} - R_f 1_N = a + \beta^{mis} \lambda^{mis} + \beta^{can} \lambda^{can} \]
\[ + \beta^{can} (f^{can}_{t+1} - \mathbb{E}[f^{can}_{t+1}]) + \beta^{mis} (f^{mis}_{t+1} - \mathbb{E}[f^{mis}_{t+1}]) + e_{t+1} \]

- We use Gaussian maximum-likelihood estimator under the no-arbitrage restriction

\[
\max_{\theta \in \Theta} \ell(\theta; K) \\
\text{s.t. } a' V_e^{-1} a \leq \delta_{apt}
\]

- Determine the hyper-parameters \((K, \delta_{apt})\)
  - using cross-validation
  - with HJ-distance as selection metric.
Data

- **Basis assets:** 202 characteristic-based portfolios with monthly returns: 1963:07 to 2019:08.

- To interpret our results, we collected a comprehensive set of variables at monthly frequency potentially spanning the SDF:
  - 457 traded strategies
  - 103 non-traded variables
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Model selection: determine hyper-parameters \((K, \delta_{apt})\)

- \(K = 2\)
- \(\delta_{apt} = 0.0529\)
- \(SR^a = 0.80\) p.a.
- Unsystematic risk is priced.
What is compensation for unsystematic risk

![Graph showing compensation for unsystematic risk](image-url)
Time-series properties of SDF and its components
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\begin{figure}
\centering
\includegraphics[width=\textwidth]{sdf_properties}
\end{figure}
SDF: Systematic versus unsystematic risk

- Unsystematic risk accounts for 73% of variation in SDF
  - Sharpe ratio for the aggregate measure of unsystematic risk is 0.79
- Latent systematic factors explain only 27% of variation in SDF

<table>
<thead>
<tr>
<th></th>
<th>std dev</th>
<th>% var(log(\hat{M}_{\text{exp},t+1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(\hat{M}_{\text{exp},t+1})</td>
<td>0.89</td>
<td>100.00</td>
</tr>
<tr>
<td>log(\hat{M}^a_{\text{exp},t+1})</td>
<td>0.79</td>
<td>72.60</td>
</tr>
<tr>
<td>log(\hat{M}^\beta_{\text{exp},t+1})</td>
<td>0.51</td>
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</table>
Focus on the **unsystematic** SDF component

- **Acyclical**: no relation to the NBER recession indicator

- Idiosyncratic-volatility factor (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than 10% of variation of $M_{t+1}^a$.

- 307 out of 457 (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.

- Observed strategies **do not fully span** the unsystematic SDF component.

- The strategies with highest compensation for unsystematic risk, $R_{strategy}^a$, are attributed to frictions and behavioral biases in the literature.

\[
R_{strategy}^a = -\text{cov}(M_{t+1}^a, R_{strategy,t+1}) \times R_f.
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- To explain 99% of variation in the systematic SDF-component, need to **add** to the Market factor:
  - Sales-to-market,
  - Dollar trading volume, (... highly correlated with the Size factor)
  - Bid-ask spread,
  - Days with zero trades.

- To explain 99.5% of variation in the systematic SDF-component, we need to use 17 observable tradable factors.
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Empirical results for second question:
What is missing in candidate models?
What is missing in CAPM, C-CAPM, and FF3?

- All three models omit systemic risk factors and compensation for unsystematic risk.
- The main source of misspecification (by far) is compensation for unsystematic risk.

<table>
<thead>
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<th>Model</th>
<th>Std Dev or Sharpe ratio (p.a.)</th>
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What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
  - systematic risk factors and compensation for unsystematic risk.
  - The main source of misspecification (by far) is compensation for unsystematic risk.

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Consistent correction of candidate factor models

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Microfoundations for pricing of unsystematic risk
Micro-foundations for priced asset-specific risk

- Merton (1987) develops an equilibrium model in which
  - only a proportion \( q_i \) of investors are informed about asset \( i \);
  - Returns, the SDF and its components have the same functional form as what we have specified in our APT-based model.

- **Proposition 5:** When \( N \rightarrow \infty \)
  - Equilibrium asset returns are
    \[
    R_i - R_f = a_i + \beta_i (R_m - R_f) + e_i, \quad \text{where} \quad a_i = \gamma \sigma_i^2 \left( \frac{1}{q_i} - 1 \right) \frac{V_i}{V_m};
    \]
  - Equilibrium SDF is
    \[
    M = -\frac{a'Ve}{R_f} e + \frac{1}{R_f} - \frac{\mathbb{E}(R_m - R_f)}{R_f \times \text{var}(R_m)} (R_m - \mathbb{E}(R_m)) \]
      \[
      \begin{aligned}
        &M^a \\
        &M^\beta
      \end{aligned}
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  where
  - $M^a = -\frac{a'Ve}{R_f}e$
  - $M^b = \frac{1}{R_f}$
  - $M^c = \frac{\mathbb{E}(R_m - R_f)}{R_f \times \text{var}(R_m)} (R_m - \mathbb{E}(R_m))$
Conclusion
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- Develop a methodology
  - to identify what is missing in factor models;
  - use this to examine potential significance of unsystematic risk.

- Key insight: quantitative importance of unsystematic risk
  - for theorists: vital for developing microfounded models;
  - for empiricists: essential for resolving the factor zoo;
  - for corporate finance: crucial for estimating cost of capital.
Thank you!

Please send comments to

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raman.uppal@edhec.edu
p.zaffaroni@imperial.ac.uk
zviadadze@hec.fr
References
References


Basis assets

- **Basis assets**: 202 characteristic-based portfolios with monthly returns: 1963:07 to 2019:08:
  - 25 size and book-to-market portfolios,
  - 17 industry portfolios,
  - 25 investment profitability and investment,
  - 25 size and variance portfolios,
  - 35 size and net issuance portfolios,
  - 25 size and accruals portfolios,
  - 25 size and beta portfolios,
  - 25 size and momentum portfolios
Data on 457 tradable factors potentially spanning the SDF

- Factors used in Chen and Zimmermann (2022), Jensen, Kelly, and Pedersen (2021), and Kozak, Nagel, and Santosh (2020).

- Industry-adjusted value, momentum, and profitability factors (Novy-Marx, 2013).

- Intra-industry value, momentum, and profitability factors, and basic profitable-minus-unprofitable factor.

- Expected growth factor of Hou, Mo, Xue, and Zhang (2021) and the momentum Up minus Down (UMD) factor.

- Factors from Bryzgalova, Huang, and Julliard (2023).
Data on 103 macro factors potentially spanning the SDF

- Macroeconomic and business-cycle variables
  - 3 principal components and their VAR residuals for 279 macro variables (Jurado, Ludvigson, and Ng, 2015).
  - 8 principal components and their VAR residuals for 128 macro variables (McCracken and Ng, 2015).

- Consumption and inflation variables.

- Sentiment and confidence indexes.

- Volatility and uncertainty measures
  - Market-dislocations index (Pasquariello, 2014)
  - Disagreement index (Huang, Li, and Wang, 2021)
  - Chicago Board Options Exchange volatility index (VIX, from CBOE)
  - US econ. policy uncertainty index EPU (Baker, Bloom, and Davis, 2016)
  - Equity-mkt vol. (EMV) tracker (Baker, Bloom, and Davis, 2016).
Spanning $M_t^{a}$ with observed factors
Spanning $M_{t+1}^{\beta, mis}$ with observed factors