

What is Missing in Asset-Pricing Factor Models?

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Objective and Motivation

- Major challenge in finance is to price **cross-section of stock returns**.
 - That is, explain why do stocks **differ** in their expected returns?
- The first model proposed to address this challenge was the CAPM.
- When the CAPM failed, researchers then explored other **candidate** models with **hundreds** of systematic risk factors (**factor zoo**).
- However, there is still a sizable pricing error in returns, called **alpha**.
- In this paper we ask:
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Where we deviate from existing models

- Existing models typically assume that only **systematic** (common) risk is compensated in financial markets.

$$\mathbb{E}(R_{t+1} - R_{ft}1_N) - \beta\lambda = a = 0.$$

- β is a vector of assets' exposures to systematic risk
 - λ is a vector of prices of unit of systematic risk
- Under the **Arbitrage Pricing Theory** (APT) setting of Ross (1976, 1977), we explore the possibility that **unsystematic** risk is also compensated.

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Our line of attack . . . use the SDF

- Each asset-pricing model implies a **stochastic discount factor (SDF)**.
- The SDF adjusts cashflows for **time** and **risk**.

$$\text{price}_t^n = \mathbb{E}[M_{t+1} \times \text{cashflow}_{t+1}^n] \quad \dots \text{cashflows}$$

$$1 = \mathbb{E}[M_{t+1} \times R_{t+1}^n] \quad \dots \text{returns}$$

$$0 = \mathbb{E}[M_{t+1} \times (R_{t+1}^n - R_f)] \quad \dots \text{excess returns}$$

$$\underbrace{\mathbb{E}[R_{t+1}^n - R_f]}_{\text{risk premium}} = - \underbrace{\text{cov}(M_{t+1}, R_{t+1}^n)}_{\text{risk}} \times R_f \quad \dots \text{covariances}$$

- We examine misspecification in factor models through lens of SDF.

What we do: both Theory and Empirics

1. Under the APT setting

- Identify the admissible SDF implied by the APT (in which $a \neq 0$);
- Quantify the importance of unsystematic risk, a , by estimating the SDF.

2. Given some candidate factor model

- Develop a methodology to identify what is missing in the candidate model;
- Characterize what is missing in some popular models.

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What we find: Empirical findings (first part)

- **Unsystematic risk** is priced in financial markets.
 - The unsystematic SDF component explains more than **72% of variation** in the admissible SDF;
 - Several **successful factors** correlate with **unsystematic SDF component**, such as
 - **Value** (Fama and French, 2015),
 - **Momentum** (Jegadeesh and Titman, 1993).
- **Systematic component** of SDF is driven by **Market factor**.
 - The Market factor explains **95%** of the variation in the **systematic component** of the SDF.
- **What is missing** in popular candidate models is, largely, **compensation for unsystematic risk**.

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Theory: Our methodology



Arbitrage Pricing Theory (APT) . . . our starting point

- **Gross returns** are described by a **latent linear factor model**

$$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta(f_{t+1} - \mathbb{E}(f_{t+1})) + e_{t+1},$$

- **Expected excess returns** are, for some a ,

$$\mathbb{E}(R_{t+1} - R_f 1_N) = a + \beta\lambda,$$

- By **asymptotic no-arbitrage**, the vector a satisfies the **no-arbitrage restriction**

$$\forall N, \quad a' V_e^{-1} a \leq \delta_{\text{apt}} < \infty.$$

- f_{t+1} be the $K \times 1$ vector of common (latent) risk factors with risk premia λ and a $K \times K$ positive definite covariance matrix $V_f > 0$,
- $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$ is the $N \times K$ full-rank matrix of loadings,
- e_{t+1} is the vector of **unsystematic shocks** with zero mean and the $N \times N$ positive definite covariance matrix $V_e > 0$.

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Proposition 1: The SDF under the APT

- The SDF implied by the APT-model of asset returns is

$$M_{t+1} = M_{t+1}^{\beta} + M_{t+1}^a, \quad \text{where}$$

$$M_{t+1}^{\beta} = \frac{1}{R_f} - \frac{\lambda V_f^{-1}}{R_f} (f_{t+1} - \mathbb{E}(f_{t+1})) \quad \dots \text{systematic component}$$

$$M_{t+1}^a = -\frac{a' V_e^{-1}}{R_f} e_{t+1} \quad \dots \text{unsystematic component}$$

Implementation challenges: Non-negative and feasible SDF

- There are **two** challenges in implementing our SDF.
 1. The SDF **may not always be strictly positive**, thus possibly leading to negative asset prices.
 2. $M_{t+1}^{\beta mis}$ and M_{t+1}^a **depend on unobservable quantities**, such as, f_{t+1}^{mis} and e_{t+1} , respectively.
- We address both of them
 1. We specify the admissible SDF in **exponential** form.
 2. We use the exponential function of **linear projections** of the SDF on the set of assets.
- **Proposition 2**: our feasible SDF is asymptotically admissible.

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**Now, develop results for second question:
What is missing in asset-pricing factor models?**

Correcting a candidate factor model

- Let's consider a **candidate** model with K^{can} observable risk factors f_{t+1}^{can} .

$$R_{t+1} - R_f 1_N = \alpha + \beta^{can} \lambda^{can} + \beta^{can} (f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \varepsilon_{t+1}$$

- The candidate model may omit
 - Systematic risk factors f_{t+1}^{mis}
 - Compensation for unsystematic risk a .
- We can rewrite α and ε_{t+1} as follows

$$\alpha = a + \beta^{mis} \lambda^{mis}$$

$$\varepsilon_{t+1} = \beta^{mis} (f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1}$$

$$V_\varepsilon = \text{var}(\varepsilon_{t+1}) = \beta^{mis} V_{f^{mis}} \beta^{mis'} + V_{e_{t+1}}$$

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Proposition 4: Correcting the candidate SDF

- Under the APT assumptions, there exists an **admissible SDF** M_{t+1}

$$M_{t+1} = M_{t+1}^{\beta, can} + \underbrace{(M_{t+1}^a + M_{t+1}^{\beta, mis})}_{=M_{t+1}^{\alpha}}$$

$$M_{t+1}^{\beta, can} = \frac{1}{R_f} - \frac{(\lambda^{can})' V_{f^{can}}^{-1}}{R_f} (f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}])$$

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Key insight of paper that follows from the proposition

- In the **candidate model**, α represents the pricing error:

$$\alpha = \mathbb{E}[M_{t+1}^{\beta, \text{can}}(R_{t+1} - R_f 1_N)] \times R_f,$$

- In the **corrected model**, α represents compensation for assets' exposures to the missing factors f_{t+1}^{mis} and unsystematic risk e_{t+1} :

$$\begin{aligned}\alpha &= -\text{cov}(M_{t+1}^{\alpha}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\text{cov}(M_{t+1}, \varepsilon_{t+1}) \times R_f.\end{aligned}$$

- In particular, a is compensation for unsystematic risk e_{t+1} :

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Empirics: Apply methodology

Estimating the set of parameters

- Estimate the data-generating process of asset returns

$$R_{t+1} - R_f 1_N = a + \beta^{mis} \lambda^{mis} + \beta^{can} \lambda^{can} \\ + \beta^{can} (f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \beta^{mis} (f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1}$$

- We use Gaussian maximum-likelihood estimator under the no-arbitrage restriction

$$\max_{\theta \in \Theta} \ell(\theta; K) \\ \text{s.t. } a' V_e^{-1} a \leq \delta_{apt}$$

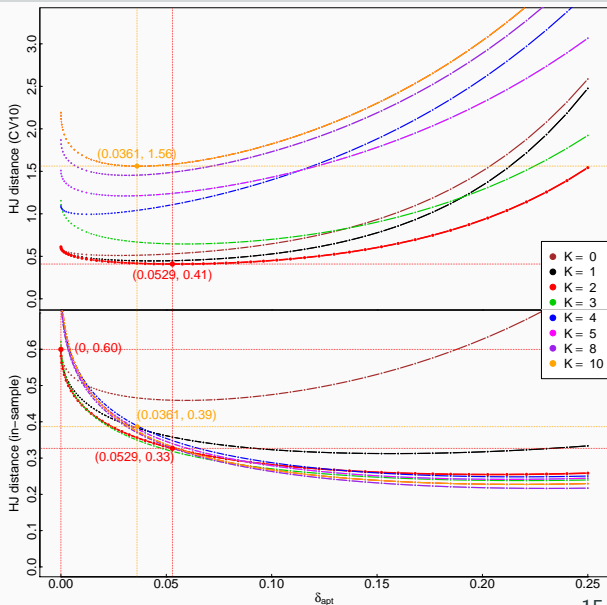
- Determine the hyper-parameters (K, δ_{apt})
 - using cross-validation
 - with HJ-distance as selection metric.

- **Basis assets: 202 characteristic-based portfolios** with monthly returns: 1963:07 to 2019:08. [list](#)
- To interpret our results, we collected a comprehensive set of variables at monthly frequency potentially spanning the SDF
 - **457 traded strategies** [list](#)
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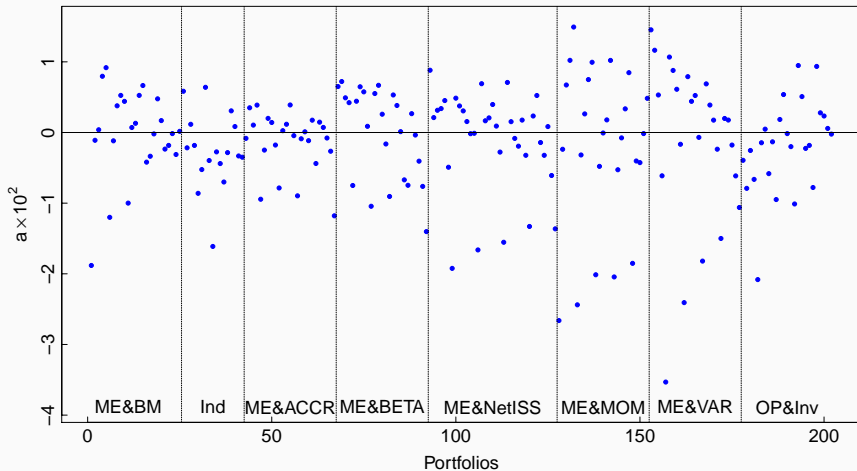
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Model selection: determine hyper-parameters (K, δ_{apt})

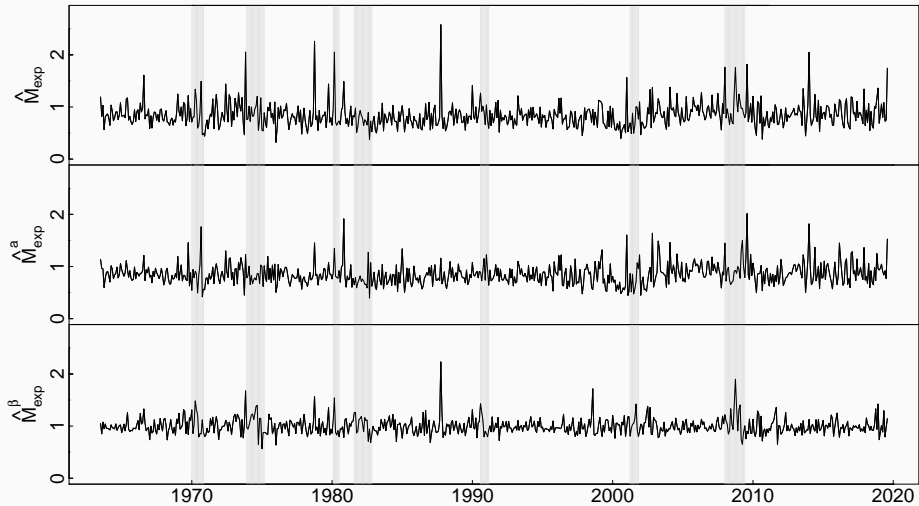
- $K = 2$
- $\delta_{apt} = 0.0529$
- $SR^a = 0.80$ p.a.
- Unsystematic risk is priced.



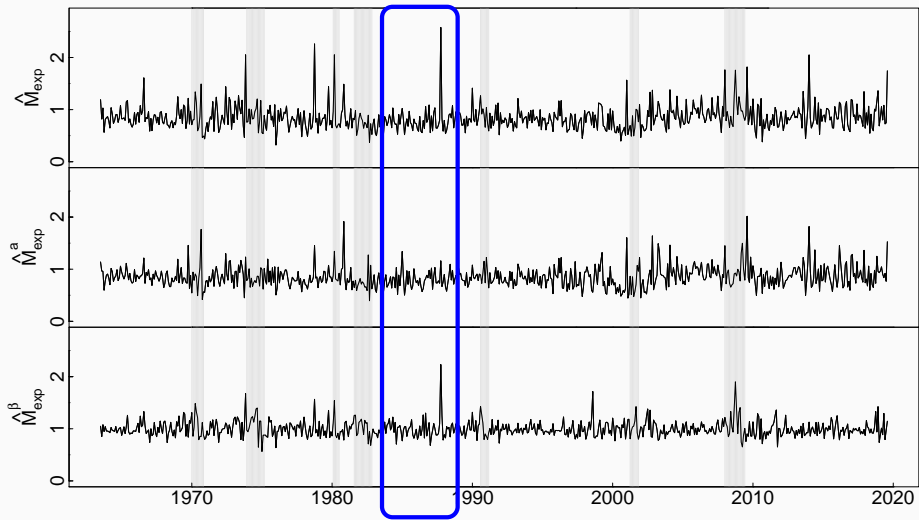
What is compensation for unsystematic risk



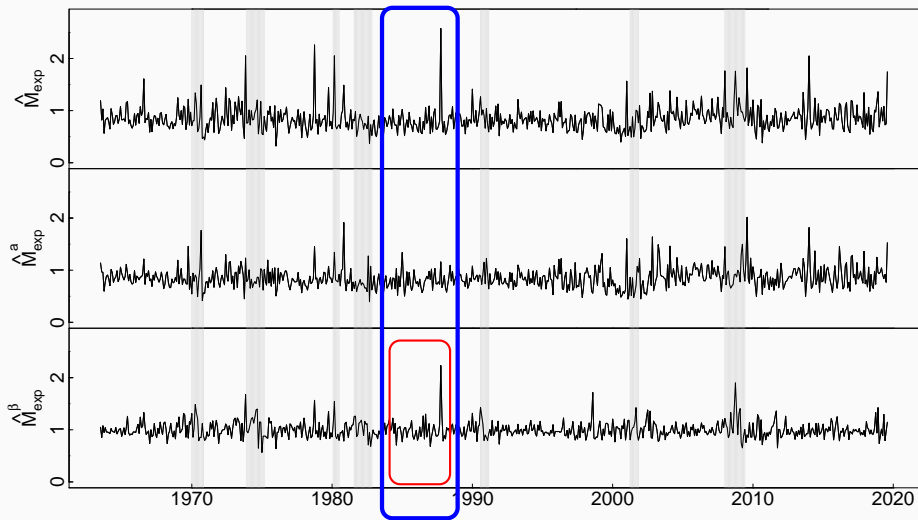
Time-series properties of SDF and its components



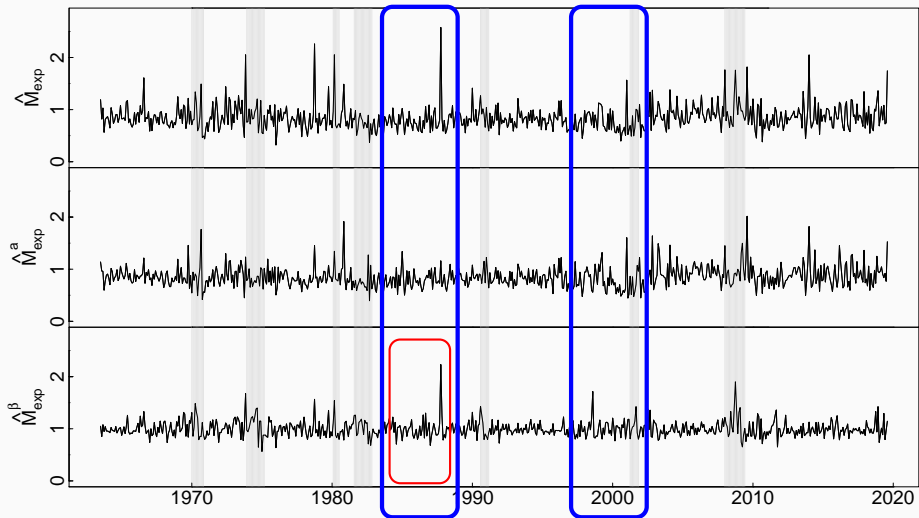
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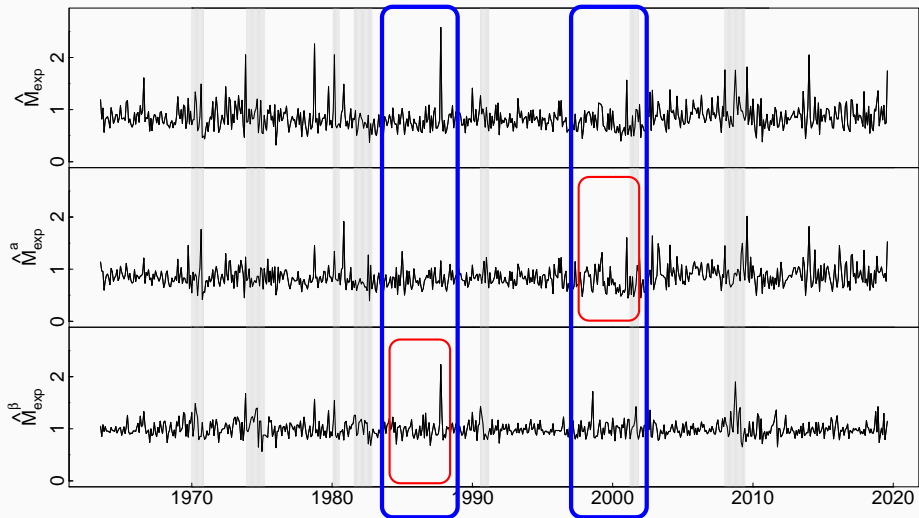
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SDF: Systematic versus unsystematic risk

- Unsystematic risk accounts for **73%** of variation in SDF
 - Sharpe ratio for the aggregate measure of unsystematic risk is **0.79**
- Latent systematic factors explain only **27%** of variation in SDF

	std dev	% var($\log(\hat{M}_{\text{exp},t+1})$)
$\log(\hat{M}_{\text{exp},t+1})$	0.89	100.00
$\log(\hat{M}_{\text{exp},t+1}^a)$	0.79	72.60
$\log(\hat{M}_{\text{exp},t+1}^\beta)$	0.51	27.40

Focus on the **unsystematic** SDF component

- **Acyclical**: no relation to the NBER recession indicator
- **Idiosyncratic-volatility factor** (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than **10%** of variation of M_{t+1}^a .
- **307 out of 457** (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.
- Observed strategies **do not fully span** the unsystematic SDF component. [graph](#)
- The strategies with highest compensation for unsystematic risk, $RP_{strategy}^a$, are attributed to **frictions** and **behavioral biases** in the literature.

$$RP_{strategy}^a = -\text{cov}(M_{t+1}^a, R_{strategy,t+1}) \times R_f.$$

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- **Cyclical**: related to the NBER recession indicator
- **Market factor explains 95.22%** of variation in systematic SDF component.
- To explain **99%** of variation in the systematic SDF-component, need to **add** to the Market factor:
 - Sales-to-market,
 - Dollar trading volume, (. . . highly correlated with the **Size** factor)
 - Bid-ask spread,
 - Days with zero trades.
- To explain **99.5%** of variation in the systematic SDF-component, we need to use 17 observable tradable factors. [graph](#)

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Empirical results for **second** question:
What is **missing** in candidate models?

What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
 - systematic risk factors and compensation for unsystematic risk.
 - The main source of misspecification (by far) is compensation for unsystematic risk.

Model	Std Dev or Sharpe ratio (p.a.)				Variance decomp. (%)		
	log of				log of		
	$\hat{M}_{\text{exp},t+1}$	\hat{M}_{t+1}^a	$\hat{M}_{t+1}^{\beta,\text{can}}$	$\hat{M}_{t+1}^{\beta,\text{mis}}$	\hat{M}_{t+1}^a	$\hat{M}_{t+1}^{\beta,\text{can}}$	$\hat{M}_{t+1}^{\beta,\text{mis}}$
APT	0.89	0.79	0.51		72.60	27.40	
CAPM	0.89	0.80	0.42	0.27	74.14	18.48	7.38
C-CAPM	0.92	0.79	0.36	0.42	66.05	15.92	18.03
FF3	0.99	0.80	0.67	0.27	55.49	38.30	6.21

What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
 - systematic risk factors and compensation for unsystematic risk.
 - The main source of misspecification (by far) is compensation for unsystematic risk.

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Consistent correction of candidate factor models

- After correction, SDFs implied by these models are **almost perfectly correlated to each other**;

Correlations

	$\log(\hat{M}_{\text{exp},t+1})$				$\log(\hat{M}_{\text{exp},t+1}^a)$			
	Corrected				Corrected			
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

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Microfoundations for pricing of unsystematic risk

Micro-foundations for priced asset-specific risk

- Merton (1987) develops an equilibrium model in which
 - only a proportion q_i of investors are informed about asset i ;
 - Returns, the SDF and its components have the same functional form as what we have specified in our APT-based model.
- Proposition 5: When $N \rightarrow \infty$
 - Equilibrium asset returns are

$$R_i - R_f = a_i + \beta_i(R_m - R_f) + e_i, \quad \text{where } a_i = \gamma \sigma_i^2 \left(\frac{1}{q_i} - 1 \right) \frac{V_i}{V_m};$$

- Equilibrium SDF is

$$M = \underbrace{-\frac{a' V_e}{R_f} e}_{M^a} + \underbrace{\frac{1}{R_f} - \frac{\mathbb{E}(R_m - R_f)}{R_f \times \text{var}(R_m)}}_{M^b} (R_m - \mathbb{E}(R_m))$$

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Conclusion



Conclusion

- **Develop a methodology**
 - to identify what is missing in factor models;
 - use this to examine potential significance of unsystematic risk.
- **Key insight: quantitative importance of unsystematic risk**
 - for **theorists**: vital for developing **microfounded models**;
 - for **empiricists**: essential for **resolving the factor zoo**;
 - for **corporate finance**: crucial for estimating **cost of capital**.

Thank you!

Please send comments to

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Basis assets

- **Basis assets: 202 characteristic-based portfolios** with monthly returns: 1963:07 to 2019:08:
 - 25 size and book-to-market portfolios,
 - 17 industry portfolios,
 - 25 investment profitability and investment,
 - 25 size and variance portfolios,
 - 35 size and net issuance portfolios,
 - 25 size and accruals portfolios,
 - 25 size and beta portfolios,
 - 25 size and momentum portfolios

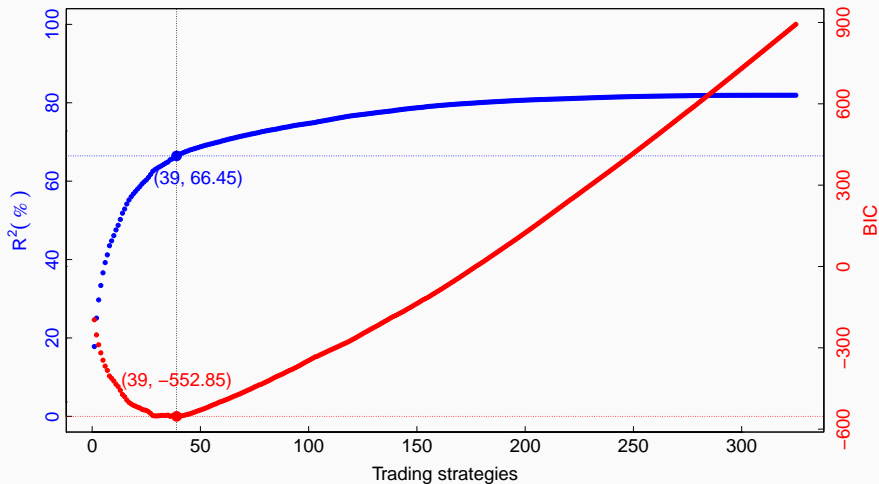
Data on 457 tradable factors potentially spanning the SDF

- Factors used in [Chen and Zimmermann \(2022\)](#), [Jensen, Kelly, and Pedersen \(2021\)](#), and [Kozak, Nagel, and Santosh \(2020\)](#).
- Industry-adjusted value, momentum, and profitability factors ([Novy-Marx, 2013](#)).
- Intra-industry value, momentum, and profitability factors, and basic profitable-minus-unprofitable factor.
- Expected growth factor of [Hou, Mo, Xue, and Zhang \(2021\)](#) and the momentum Up minus Down (UMD) factor.
- Factors from [Bryzgalova, Huang, and Julliard \(2023\)](#).

Data on 103 macro factors potentially spanning the SDF

- **Macroeconomic and business-cycle variables**
 - **3 principal components** and their VAR residuals for **279 macro variables** (Jurado, Ludvigson, and Ng, 2015).
 - **8 principal components** and their VAR residuals for **128 macro variables** (McCracken and Ng, 2015).
- **Consumption and inflation variables.**
- **Sentiment and confidence indexes.**
- **Volatility and uncertainty measures**
 - Market-dislocations index (Pasquariello, 2014)
 - Disagreement index (Huang, Li, and Wang, 2021)
 - Chicago Board Options Exchange volatility index (VIX, from CBOE)
 - US econ. policy uncertainty index EPU (Baker, Bloom, and Davis, 2016)
 - Equity-mkt vol. (EMV) tracker (Baker, Bloom, and Davis, 2016).

Spanning M_{t+1}^a with observed factors



Spanning $M_{t+1}^{\beta, mis}$ with observed factors

