What is Missing in Asset-Pricing Factor Models?

Massimo Dello Preite
Imperial College London

Raman Uppal
Edhec Business School, CEPR

Paolo Zaffaroni
Imperial College London

Irina Zviadadze
HEC Paris, CEPR

Turin - Collegio Carlo Alberto

26 June 2023

Objective and Motivation

- Major challenge in finance is to price cross-section of stock returns.
  - That is, explain why do stocks differ in their expected returns?

- The first model proposed to address this challenge was the CAPM.

- When the CAPM failed, researchers then explored other candidate models with hundreds of systematic risk factors (factor zoo).

- However, there is still a sizable pricing error in returns, called alpha.

- In this paper we ask:
  What is missing in asset-pricing factor models?

Objective and Motivation

- Major challenge in finance is to price cross-section of stock returns.
    - That is, explain why do stocks differ in their expected returns?
- The first model proposed to address this challenge was the CAPM.
- When the CAPM failed, researchers then explored other candidate models with hundreds of systematic risk factors (factor zoo).
- However, there is still a sizable pricing error in returns, called alpha.
- In this paper we ask:
  What is missing in asset-pricing factor models?

Objective and Motivation

- Major challenge in finance is to price cross-section of stock returns.
  - That is, explain why do stocks differ in their expected returns?
- The first model proposed to address this challenge was the CAPM.
- When the CAPM failed, researchers then explored other candidate models with hundreds of systematic risk factors (factor zoo).
- However, there is still a sizable pricing error in returns, called alpha.
- In this paper we ask:
  What is missing in asset-pricing factor models?

Where we deviate from existing models

- Existing models typically assume that only systematic (common) risk is compensated in financial markets.

$$\mathbb{E}(R_{t+1} - R_{ft}1_N) - \beta\lambda = a = 0.$$

  - β is a vector of assets' exposures to systematic risk
  - λ is a vector of prices of unit of systematic risk

- Under the Arbitrage Pricing Theory (APT) setting of Ross (1976, 1977), we explore the possibility that unsystematic risk is also compensated.

$$\mathbb{E}(R_{t+1} - R_{ft}1_N) - \beta\lambda = a \neq 0.$$

Where we deviate from existing models

- Existing models typically assume that only systematic (common) risk is compensated in financial markets.

$$\boxed{\mathbb{E}(R_{t+1} - R_{ft}1_N) - \beta\lambda = a = 0.}$$

  - β is a vector of assets' exposures to systematic risk
  - λ is a vector of prices of unit of systematic risk

- Under the Arbitrage Pricing Theory (APT) setting of Ross (1976, 1977), we explore the possibility that unsystematic risk is also compensated.

$$\boxed{\mathbb{E}(R_{t+1} - R_{ft}1_N) - \beta\lambda = a \neq 0.}$$

Our line of attack ... use the SDF

- Each asset-pricing model implies a stochastic discount factor (SDF).

- The SDF adjusts cashflows for time and risk.

$$\text{price}_t^n = \mathbb{E}[M_{t+1} \times \text{cashflow}_{t+1}^n] \qquad \ldots \text{cashflows}$$

$$1 = \mathbb{E}[M_{t+1} \times R_{t+1}^n] \qquad \ldots \text{returns}$$

$$0 = \mathbb{E}\left[M_{t+1} \times (R_{t+1}^n - R_f)\right] \qquad \ldots \text{excess returns}$$

$$\mathbb{E}[\underbrace{R_{t+1}^n - R_f}_{\text{risk premium}}] = -\underbrace{\text{cov}(M_{t+1}, R_{t+1}^n)}_{\text{risk}} \times R_f \qquad \ldots \text{covariances}$$

- We examine misspecification in factor models through lens of SDF.

What we do: both Theory and Empirics

1. Under the APT setting
    - Identify the admissible SDF implied by the APT (in which $a \neq 0$);
    - Quantify the importance of unsystematic risk, a, by estimating the SDF.

2. Given some candidate factor model
    - Develop a methodology to identify what is missing in the candidate model;
    - Characterize what is missing in some popular models.

What we do: both Theory and Empirics

1. Under the APT setting
   - Identify the admissible SDF implied by the APT (in which $a \neq 0$);
   - Quantify the importance of unsystematic risk, a, by estimating the SDF.
2. Given some candidate factor model
   - Develop a methodology to identify what is missing in the candidate model;
   - Characterize what is missing in some popular models.

What we find: Empirical findings (first part)

- Unsystematic risk is priced in financial markets.
  - The unsystematic SDF component explains more than 72% of variation in the admissible SDF;
  - Several successful factors correlate with unsystematic SDF component, such as
    - Value (Fama and French, 2015),
    - Momentum (Jegadeesh and Titman, 1993).
- Systematic component of SDF is driven by Market factor.
  - The Market factor explains 95% of the variation in the systematic component of the SDF.
- What is missing in popular candidate models is, largely, compensation for unsystematic risk.

What we find: Empirical findings (first part)

- Unsystematic risk is priced in financial markets.
  - The unsystematic SDF component explains more than 72% of variation in the admissible SDF;
  - Several successful factors correlate with unsystematic SDF component, such as
    - Value (Fama and French, 2015),
    - Momentum (Jegadeesh and Titman, 1993).
- Systematic component of SDF is driven by Market factor.
  - The Market factor explains 95% of the variation in the systematic component of the SDF.
- What is missing in popular candidate models is, largely, compensation for unsystematic risk.

What we find: Empirical findings (first part)

- Unsystematic risk is priced in financial markets.
  - The unsystematic SDF component explains more than 72% of variation in the admissible SDF;
  - Several successful factors correlate with unsystematic SDF component, such as
    - Value (Fama and French, 2015),
    - Momentum (Jegadeesh and Titman, 1993).
- Systematic component of SDF is driven by Market factor.
  - The Market factor explains 95% of the variation in the systematic component of the SDF.
- What is missing in popular candidate models is, largely, compensation for unsystematic risk.

Theory: Our methodology

Arbitrage Pricing Theory (APT) . . . our starting point

- Gross returns are described by a latent linear factor model

$$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta\big(f_{t+1} - \mathbb{E}(f_{t+1})\big) + e_{t+1},$$

- Expected excess returns are, for some a,

$$\mathbb{E}(R_{t+1} - R_f 1_N) = a + \beta\lambda,$$

- By asymptotic no-arbitrage, the vector a satisfies the no-arbitrage restriction

$$\forall N, \quad a' V_e^{-1} a \leq \delta_{\text{apt}} < \infty.$$

- f_{t+1} be the $K \times 1$ vector of common (latent) risk factors with risk premia λ and a $K \times K$ positive definite covariance matrix $V_f > 0$,
- $\beta = (\beta_1, \beta_2, \ldots, \beta_N)'$ is the $N \times K$ full-rank matrix of loadings,
- e_{t+1} is the vector of unsystematic shocks with zero mean and the $N \times N$ positive definite covariance matrix $V_e > 0$.

Arbitrage Pricing Theory (APT) ... our starting point

- Gross returns are described by a latent linear factor model

$$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta\big(f_{t+1} - \mathbb{E}(f_{t+1})\big) + e_{t+1},$$

- Expected excess returns are, for some a,

$$\mathbb{E}(R_{t+1} - R_f 1_N) = a + \beta\lambda,$$

- By asymptotic no-arbitrage, the vector a satisfies the no-arbitrage restriction

$$\forall N, \quad a' V_e^{-1} a \leq \delta_{\text{apt}} < \infty.$$

- f_{t+1} be the $K \times 1$ vector of common (latent) risk factors with risk premia λ and a $K \times K$ positive definite covariance matrix $V_f > 0$,
- $\beta = (\beta_1, \beta_2, \dots, \beta_N)'$ is the $N \times K$ full-rank matrix of loadings,
- e_{t+1} is the vector of unsystematic shocks with zero mean and the $N \times N$ positive definite covariance matrix $V_e > 0$.

Arbitrage Pricing Theory (APT) . . . our starting point

- Gross returns are described by a latent linear factor model

$$R_{t+1} - \mathbb{E}(R_{t+1}) = \beta\big(f_{t+1} - \mathbb{E}(f_{t+1})\big) + e_{t+1},$$

- Expected excess returns are, for some a,

$$\mathbb{E}(R_{t+1} - R_f 1_N) = a + \beta\lambda,$$

- By asymptotic no-arbitrage, the vector a satisfies the no-arbitrage restriction

$$\forall\, N, \quad a' V_e^{-1} a \leq \delta_{\text{apt}} < \infty.$$

- f_{t+1} be the $K \times 1$ vector of common (latent) risk factors with risk premia λ and a $K \times K$ positive definite covariance matrix $V_f > 0$,
- $\beta = (\beta_1, \beta_2, \ldots\ , \beta_N)'$ is the $N \times K$ full-rank matrix of loadings,
- e_{t+1} is the vector of unsystematic shocks with zero mean and the $N \times N$ positive definite covariance matrix $V_e > 0$.

Proposition 1: The SDF under the APT

- The SDF implied by the APT-model of asset returns is

$$M_{t+1} = M_{t+1}^{\beta} + M_{t+1}^{a}, \quad \text{where}$$

$$M_{t+1}^{\beta} = \frac{1}{R_f} - \frac{\lambda V_f^{-1}}{R_f}(f_{t+1} - \mathbb{E}(f_{t+1})) \quad \ldots \text{ systematic component}$$

$$M_{t+1}^{a} = -\frac{a' V_e^{-1}}{R_f} e_{t+1} \quad \ldots \text{ unsystematic component}$$

Implementation challenges: Non-negative and feasible SDF

- There are two challenges in implementing our SDF.
  1. The SDF may not always be strictly positive, thus possibly leading to negative asset prices.
  2. $M_{t+1}^{\beta mis}$ and M_{t+1}^{a} depend on unobservable quantities, such as, f_{t+1}^{mis} and e_{t+1}, respectively.
- We address both of them
  1. We specify the admissible SDF in exponential form.
  2. We use the exponential function of linear projections of the SDF on the set of assets.
- **Proposition 2**: our feasible SDF is asymptotically admissible.

Implementation challenges: Non-negative and feasible SDF

- There are two challenges in implementing our SDF.
  1. The SDF may not always be strictly positive, thus possibly leading to negative asset prices.
  2. $M_{t+1}^{\beta mis}$ and M_{t+1}^{a} depend on unobservable quantities, such as, f_{t+1}^{mis} and e_{t+1}, respectively.
- We address both of them
  1. We specify the admissible SDF in exponential form.
  2. We use the exponential function of linear projections of the SDF on the set of assets.
- **Proposition 2**: our feasible SDF is asymptotically admissible.

Implementation challenges: Non-negative and feasible SDF

- There are two challenges in implementing our SDF.
  1. The SDF may not always be strictly positive, thus possibly leading to negative asset prices.
  2. $M_{t+1}^{\beta mis}$ and M_{t+1}^{a} depend on unobservable quantities, such as, f_{t+1}^{mis} and e_{t+1}, respectively.
- We address both of them
  1. We specify the admissible SDF in exponential form.
  2. We use the exponential function of linear projections of the SDF on the set of assets.
- **Proposition 2**: our feasible SDF is asymptotically admissible.

Now, develop results for second question:

What is missing in asset-pricing factor models?

Correcting a candidate factor model

- Let's consider a candidate model with K^{can} observable risk factors f_{t+1}^{can}.

$$R_{t+1} - R_f 1_N = \alpha + \beta^{can}\lambda^{can} + \beta^{can}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \varepsilon_{t+1}$$

- The candidate model may omit
  1. Systematic risk factors f_{t+1}^{mis}
  2. Compensation for unsystematic risk a.

- We can rewrite α and ε_{t+1} as follows

$$\alpha = a + \beta^{mis}\lambda^{mis}$$

$$\varepsilon_{t+1} = \beta^{mis}(f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1}$$

$$V_\varepsilon = \mathrm{var}(\varepsilon_{t+1}) = \beta^{mis} V_{f^{mis}} \beta^{mis'} + V_e.$$

Correcting a candidate factor model

- Let's consider a candidate model with K^{can} observable risk factors f_{t+1}^{can}.

$$R_{t+1} - R_f 1_N = \alpha + \beta^{can}\lambda^{can} + \beta^{can}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \varepsilon_{t+1}$$

- The candidate model may omit
  1. Systematic risk factors f_{t+1}^{mis}
  2. Compensation for unsystematic risk a.

- We can rewrite α and ε_{t+1} as follows

$$\alpha = a + \beta^{mis}\lambda^{mis}$$

$$\varepsilon_{t+1} = \beta^{mis}(f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1}$$

$$V_\varepsilon = \text{var}(\varepsilon_{t+1}) = \beta^{mis} V_{f^{mis}} \beta^{mis'} + V_e.$$

Correcting a candidate factor model

- Let's consider a candidate model with K^{can} observable risk factors f_{t+1}^{can}.

$$R_{t+1} - R_f 1_N = \alpha + \beta^{can}\lambda^{can} + \beta^{can}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \varepsilon_{t+1}$$

- The candidate model may omit
    1. Systematic risk factors f_{t+1}^{mis}
    2. Compensation for unsystematic risk a.

- We can rewrite α and ε_{t+1} as follows

$$\begin{aligned} \alpha &= a + \beta^{mis}\lambda^{mis} \\ \varepsilon_{t+1} &= \beta^{mis}(f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1} \\ V_\varepsilon &= \text{var}(\varepsilon_{t+1}) = \beta^{mis} V_{f^{mis}} \beta^{mis'} + V_e. \end{aligned}$$

Proposition 4: Correcting the candidate SDF

- Under the APT assumptions, there exists an admissible SDF M_{t+1}

$$M_{t+1} = M_{t+1}^{\beta,can} + \underbrace{\left(M_{t+1}^{a} + M_{t+1}^{\beta,mis}\right)}_{=M_{t+1}^{\alpha}},$$

$$M_{t+1}^{\beta,can} = \frac{1}{R_f} - \frac{(\lambda^{can})' V_{f^{can}}^{-1}}{R_f}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}])$$

$$M_{t+1}^{\beta,mis} = -\frac{(\lambda^{mis})' V_{f^{mis}}^{-1}}{R_f}(f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}])$$

$$M_{t+1}^{a} = -\frac{a' V_{e}^{-1}}{R_f} e_{t+1}$$

Key insight of paper that follows from the proposition

- In the candidate model, α represents the pricing error:

$$\boxed{\alpha = \mathbb{E}[M_{t+1}^{\beta,can}(R_{t+1} - R_f 1_N)] \times R_f,}$$

- In the corrected model, α represents compensation for assets' exposures to the missing factors f_{t+1}^{mis} and unsystematic risk e_{t+1}:

$$\boxed{\begin{aligned}\alpha &= -\operatorname{cov}(M_{t+1}^{\alpha}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, \varepsilon_{t+1}) \times R_f.\end{aligned}}$$

- In particular, a is compensation for unsystematic risk e_{t+1}:

$$\boxed{\begin{aligned}a &= -\operatorname{cov}(M_{t+1}^{a}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, e_{t+1}) \times R_f.\end{aligned}}$$

Key insight of paper that follows from the proposition

- In the candidate model, α represents the pricing error:

$$\boxed{\alpha = \mathbb{E}[M_{t+1}^{\beta,can}(R_{t+1} - R_f 1_N)] \times R_f,}$$

- In the corrected model, α represents compensation for assets' exposures to the missing factors f_{t+1}^{mis} and unsystematic risk e_{t+1}:

$$\begin{aligned} \alpha &= -\operatorname{cov}(M_{t+1}^{\alpha}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, \varepsilon_{t+1}) \times R_f. \end{aligned}$$

- In particular, a is compensation for unsystematic risk e_{t+1}:

$$\begin{aligned} a &= -\operatorname{cov}(M_{t+1}^{a}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, e_{t+1}) \times R_f. \end{aligned}$$

Key insight of paper that follows from the proposition

- In the candidate model, α represents the pricing error:

$$\boxed{\alpha = \mathbb{E}[M_{t+1}^{\beta,can}(R_{t+1} - R_f 1_N)] \times R_f,}$$

- In the corrected model, α represents compensation for assets' exposures to the missing factors f_{t+1}^{mis} and unsystematic risk e_{t+1}:

$$\begin{aligned} \alpha &= -\operatorname{cov}(M_{t+1}^{\alpha}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, \varepsilon_{t+1}) \times R_f. \end{aligned}$$

- In particular, a is compensation for unsystematic risk e_{t+1}:

$$\begin{aligned} a &= -\operatorname{cov}(M_{t+1}^{a}, (R_{t+1} - R_f 1_N)) \times R_f \\ &= -\operatorname{cov}(M_{t+1}, e_{t+1}) \times R_f. \end{aligned}$$

Empirics: Apply methodology

Estimating the set of parameters

- Estimate the data-generating process of asset returns

$$R_{t+1} - R_f 1_N = a + \beta^{mis}\lambda^{mis} + \beta^{can}\lambda^{can} \\ + \beta^{can}(f_{t+1}^{can} - \mathbb{E}[f_{t+1}^{can}]) + \beta^{mis}(f_{t+1}^{mis} - \mathbb{E}[f_{t+1}^{mis}]) + e_{t+1}$$

- We use Gaussian maximum-likelihood estimator under the no-arbitrage restriction

$$\max_{\theta \in \Theta} \ \ell(\theta; K) \\ \text{s.t. } a' V_e^{-1} a \leq \delta_{apt}$$

- Determine the hyper-parameters (K, δ_{apt})
  - using cross-validation
  - with HJ-distance as selection metric.

Data

- Basis assets: 202 characteristic-based portfolios with monthly returns: 1963:07 to 2019:08. list

- To interpret our results, we collected a comprehensive set of variables at monthly frequency potentially spanning the SDF
  - 457 traded strategies list
  - 103 non-traded variables list

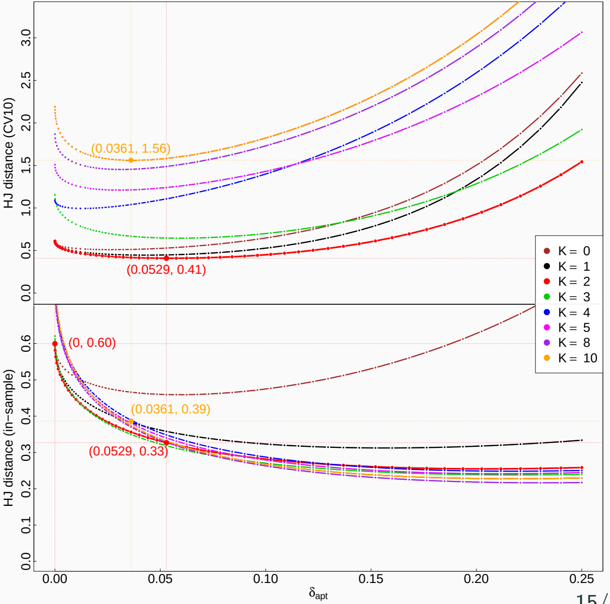
Data

- Basis assets: 202 characteristic-based portfolios with monthly returns: 1963:07 to 2019:08. list

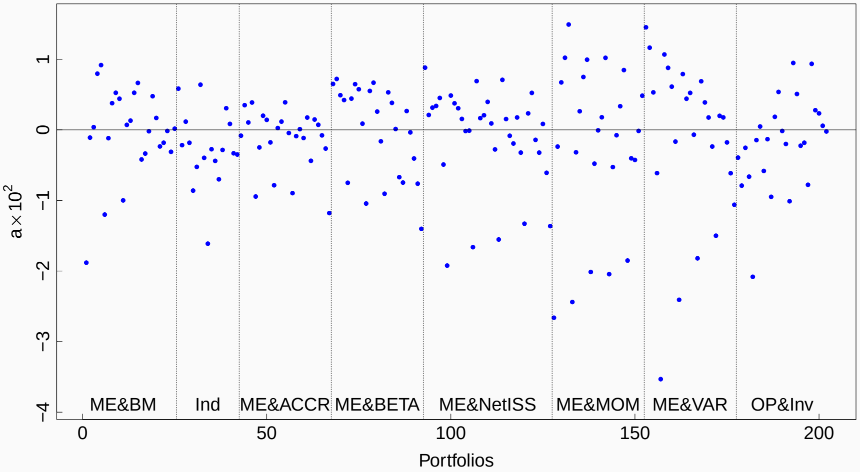
- To interpret our results, we collected a comprehensive set of variables at monthly frequency potentially spanning the SDF
  - 457 traded strategies list
  - 103 non-traded variables list

Model selection: determine hyper-parameters (K, δ_{apt})

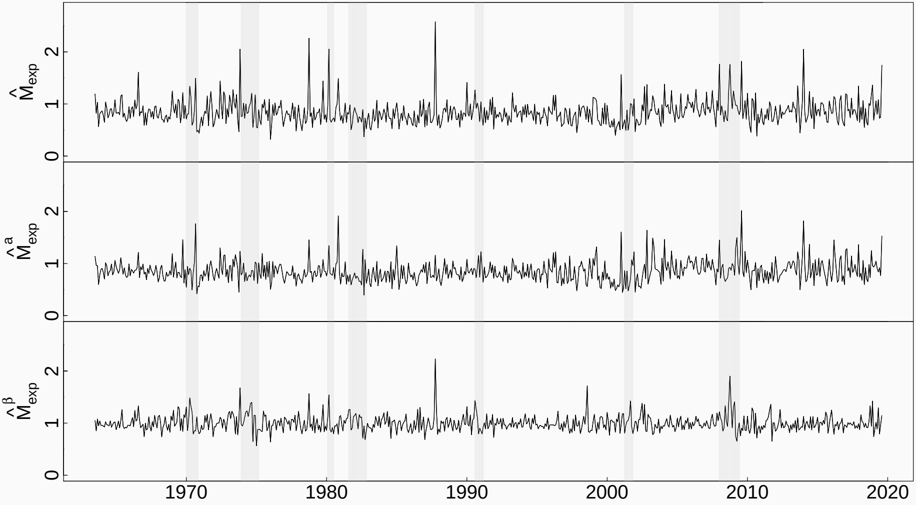
- $K = 2$
- $\delta_{apt} = 0.0529$
- $SR^a = 0.80$ p.a.
- Unsystematic risk is priced.



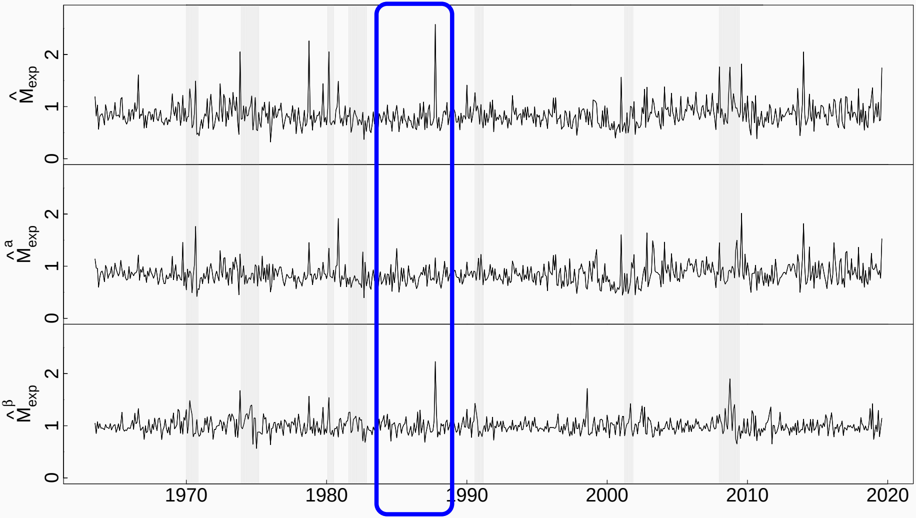
What is compensation for unsystematic risk



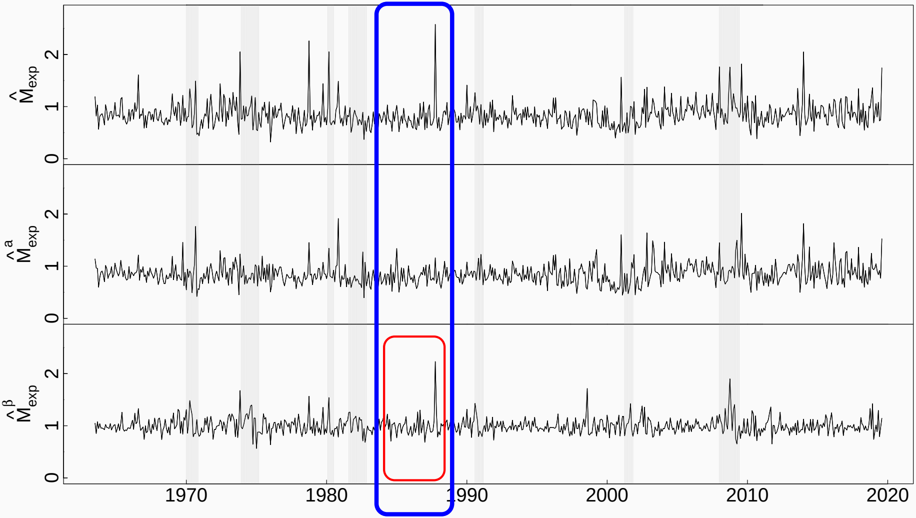
Time-series properties of SDF and its components



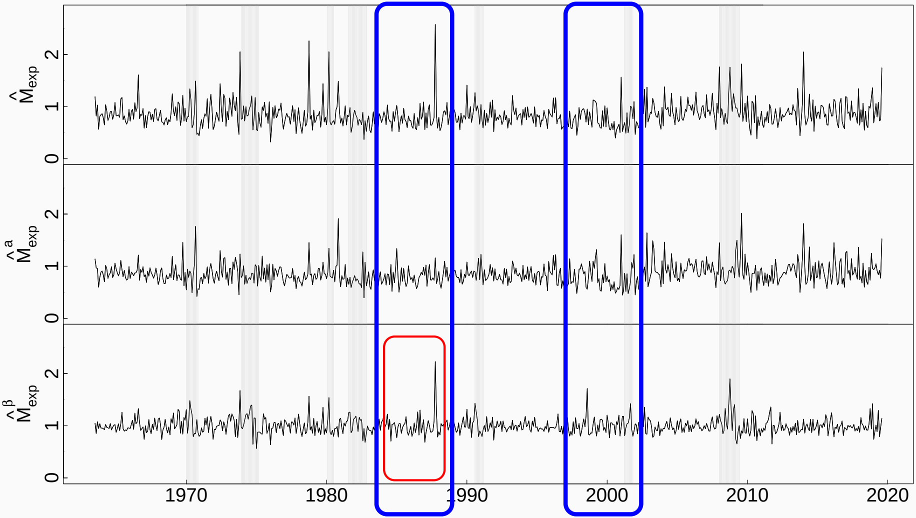
Time-series properties of SDF and its components



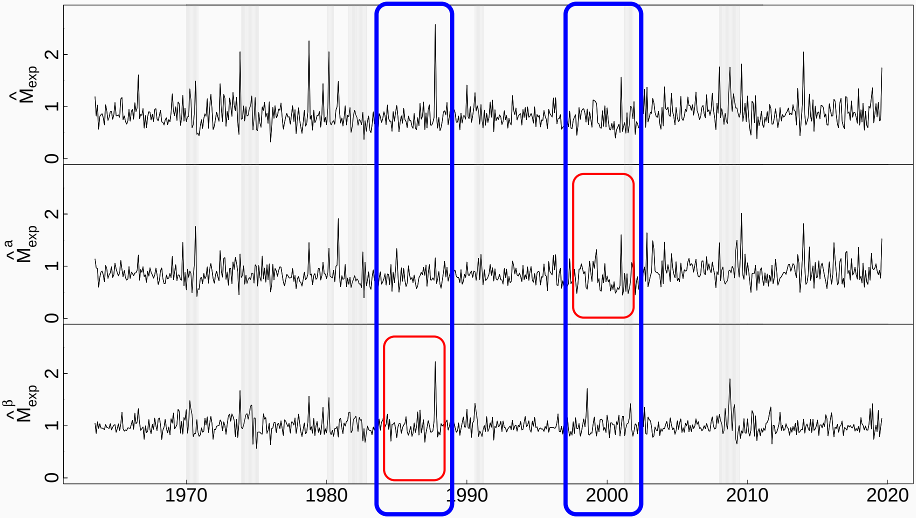
Time-series properties of SDF and its components



Time-series properties of SDF and its components



Time-series properties of SDF and its components



SDF: Systematic versus unsystematic risk

- Unsystematic risk accounts for 73% of variation in SDF
  - Sharpe ratio for the aggregate measure of unsystematic risk is 0.79
- Latent systematic factors explain only 27% of variation in SDF

	std dev	% var($\log(\hat{M}_{\text{exp},t+1})$)
$\log(\hat{M}_{\text{exp},t+1})$	0.89	100.00
$\log(\hat{M}^{a}_{\text{exp},t+1})$	0.79	72.60
$\log(\hat{M}^{\beta}_{\text{exp},t+1})$	0.51	27.40

Focus on the unsystematic SDF component

- Acyclical: no relation to the NBER recession indicator
- Idiosyncratic-volatility factor (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than 10% of variation of M^a_{t+1}.
- 307 out of 457 (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.
- Observed strategies do not fully span the unsystematic SDF component. graph
- The strategies with highest compensation for unsystematic risk, $\mathrm{RP}^a_{strategy}$, are attributed to frictions and behavioral biases in the literature.

$$\mathrm{RP}^a_{strategy} = -\mathrm{cov}(M^a_{t+1}, R_{strategy,t+1}) \times R_f.$$

Focus on the unsystematic SDF component

- Acyclical: no relation to the NBER recession indicator
- Idiosyncratic-volatility factor (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than 10% of variation of M^a_{t+1}.
- 307 out of 457 (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.
- Observed strategies do not fully span the unsystematic SDF component. graph
- The strategies with highest compensation for unsystematic risk, $\mathrm{RP}^a_{strategy}$, are attributed to frictions and behavioral biases in the literature.

$$\mathrm{RP}^a_{strategy} = -\mathrm{cov}(M^a_{t+1}, R_{strategy,t+1}) \times R_f .$$

Focus on the unsystematic SDF component

- Acyclical: no relation to the NBER recession indicator
- Idiosyncratic-volatility factor (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than 10% of variation of M^a_{t+1}.
- 307 out of 457 (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.
- Observed strategies do not fully span the unsystematic SDF component. graph
- The strategies with highest compensation for unsystematic risk, $\mathrm{RP}^a_{strategy}$, are attributed to frictions and behavioral biases in the literature.

$$\mathrm{RP}^a_{strategy} = -\mathrm{cov}(M^a_{t+1}, R_{strategy,t+1}) \times R_f.$$

Focus on the unsystematic SDF component

- Acyclical: no relation to the NBER recession indicator
- Idiosyncratic-volatility factor (Ang, Hodrick, Xing, and Zhang, 2006) explains no more than 10% of variation of M^a_{t+1}.
- 307 out of 457 (about 70%) of trading-strategy returns have significant correlations with the unsystematic SDF component.
- Observed strategies do not fully span the unsystematic SDF component. graph
- The strategies with highest compensation for unsystematic risk, $\text{RP}^a_{strategy}$, are attributed to frictions and behavioral biases in the literature.

$$\text{RP}^a_{strategy} = -\text{cov}(M^a_{t+1}, R_{strategy,t+1}) \times R_f.$$

Focus on the systematic SDF component

- Cyclical: related to the NBER recession indicator
- Market factor explains 95.22% of variation in systematic SDF component.
- To explain 99% of variation in the systematic SDF-component, need to add to the Market factor:
  - Sales-to-market,
  - Dollar trading volume, (. . . highly correlated with the Size factor)
  - Bid-ask spread,
  - Days with zero trades.
- To explain 99.5% of variation in the systematic SDF-component, we need to use 17 observable tradable factors. graph

Focus on the systematic SDF component

- Cyclical: related to the NBER recession indicator
- Market factor explains 95.22% of variation in systematic SDF component.
- To explain 99% of variation in the systematic SDF-component, need to add to the Market factor:
  - Sales-to-market,
  - Dollar trading volume, (... highly correlated with the Size factor)
  - Bid-ask spread,
  - Days with zero trades.
- To explain 99.5% of variation in the systematic SDF-component, we need to use 17 observable tradable factors. graph

Focus on the systematic SDF component

- Cyclical: related to the NBER recession indicator
- Market factor explains 95.22% of variation in systematic SDF component.
- To explain 99% of variation in the systematic SDF-component, need to add to the Market factor:
  - Sales-to-market,
  - Dollar trading volume, (...highly correlated with the Size factor)
  - Bid-ask spread,
  - Days with zero trades.
- To explain 99.5% of variation in the systematic SDF-component, we need to use 17 observable tradable factors. graph

Focus on the systematic SDF component

- Cyclical: related to the NBER recession indicator
- Market factor explains 95.22% of variation in systematic SDF component.
- To explain 99% of variation in the systematic SDF-component, need to add to the Market factor:
    - Sales-to-market,
    - Dollar trading volume, (...highly correlated with the Size factor)
    - Bid-ask spread,
    - Days with zero trades.
- To explain 99.5% of variation in the systematic SDF-component, we need to use 17 observable tradable factors. graph

Empirical results for second question:

What is missing in candidate models?

What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
  - systematic risk factors and compensation for unsystematic risk.
  - The main source of misspecification (by far) is compensation for unsystematic risk.

	Std Dev or Sharpe ratio (p.a.)				Variance decomp. (%)		
	log of				log of		
Model	$\hat{M}_{\exp,t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$
APT	0.89	0.79	0.51		72.60	27.40	
CAPM	0.89	0.80	0.42	0.27	74.14	18.48	7.38
C-CAPM	0.92	0.79	0.36	0.42	66.05	15.92	18.03
FF3	0.99	0.80	0.67	0.27	55.49	38.30	6.21

What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
  - systematic risk factors and compensation for unsystematic risk.
  - The main source of misspecification (by far) is compensation for unsystematic risk.

	Std Dev or Sharpe ratio (p.a.)				Variance decomp. (%)		
	log of				log of		
Model	$\hat{M}_{\exp,t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$
APT	0.89	0.79	0.51		72.60	27.40	
CAPM	0.89	0.80	0.42	0.27	74.14	18.48	7.38
C-CAPM	0.92	0.79	0.36	0.42	66.05	15.92	18.03
FF3	0.99	0.80	0.67	0.27	55.49	38.30	6.21

What is missing in CAPM, C-CAPM, and FF3?

- All three models omit
  - systematic risk factors and compensation for unsystematic risk.
  - The main source of misspecification (by far) is compensation for unsystematic risk.

	Std Dev or Sharpe ratio (p.a.)				Variance decomp. (%)		
		log of			log of		
Model	$\hat{M}_{\exp,t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$	$\hat{M}^{a}_{t+1}$	$\hat{M}^{\beta,can}_{t+1}$	$\hat{M}^{\beta,mis}_{t+1}$
APT	0.89	0.79	0.51		72.60	27.40	
CAPM	0.89	0.80	0.42	0.27	74.14	18.48	7.38
C-CAPM	0.92	0.79	0.36	0.42	66.05	15.92	18.03
FF3	0.99	0.80	0.67	0.27	55.49	38.30	6.21

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Consistent correction of candidate factor models

- After correction, SDFs implied by these models are almost perfectly correlated to each other;

Correlations

	$\log(\hat{M}_{\exp,t+1})$				$\log(\hat{M}^a_{\exp,t+1})$			
		Corrected				Corrected		
	APT	CAPM	C-CAPM	FF3	APT	CAPM	C-CAPM	FF3
APT	1.00	0.99	0.97	0.98	1.00	0.97	1.00	0.94
CAPM	0.99	1.00	0.96	0.97	0.97	1.00	0.97	0.93
C-CAPM	0.97	0.96	1.00	0.94	1.00	0.97	1.00	0.93
FF3	0.98	0.97	0.94	1.00	0.94	0.93	0.93	1.00

- The pricing performances are aligned across the corrected models.

Microfoundations
for pricing of unsystematic risk

Micro-foundations for priced asset-specific risk

- Merton (1987) develops an equilibrium model in which
  - only a proportion q_i of investors are informed about asset i;
  - Returns, the SDF and its components have the same functional form as what we have specified in our APT-based model.

- **Proposition 5**: When $N \to \infty$
  - Equilibrium asset returns are

$$R_i - R_f = a_i + \beta_i(R_m - R_f) + e_i, \quad \text{where} \ \ a_i = \gamma\sigma_i^2\left(\frac{1}{q_i} - 1\right)\frac{V_i}{V_m};$$

  - Equilibrium SDF is

$$M = \underbrace{-\frac{a'V_e}{R_f}e}_{M^a} + \underbrace{\frac{1}{R_f} - \frac{\mathbb{E}(R_m - R_f)}{R_f \times \text{var}(R_m)}(R_m - \mathbb{E}(R_m))}_{M^\beta}$$

Micro-foundations for priced asset-specific risk

- Merton (1987) develops an equilibrium model in which
  - only a proportion q_i of investors are informed about asset i;
  - Returns, the SDF and its components have the same functional form as what we have specified in our APT-based model.

- **Proposition 5**: When $N \to \infty$
  - Equilibrium asset returns are

$$R_i - R_f = a_i + \beta_i(R_m - R_f) + e_i, \quad \text{where } \; a_i = \gamma\sigma_i^2 \left(\frac{1}{q_i} - 1\right) \frac{V_i}{V_m};$$

  - Equilibrium SDF is

$$M = \underbrace{-\frac{a' V_e}{R_f} e}_{M^a} + \underbrace{\frac{1}{R_f} - \frac{\mathbb{E}(R_m - R_f)}{R_f \times \text{var}(R_m)}(R_m - \mathbb{E}(R_m))}_{M^\beta}$$

Conclusion

Conclusion

- Develop a methodology
  - to identify what is missing in factor models;
  - use this to examine potential significance of unsystematic risk.
- Key insight: quantitative importance of unsystematic risk
  - for theorists: vital for developing microfounded models;
  - for empiricists: essential for resolving the factor zoo;
  - for corporate finance: crucial for estimating cost of capital.

Thank you!

Please send comments to

m.dello-preite18@imperial.ac.uk
raman.uppal@edhec.edu
p.zaffaroni@imperial.ac.uk
zviadadze@hec.fr

References

References i

Almeida, C., and R. Garcia, 2012, "Assessing Misspecified Asset Pricing Models With Empirical Likelihood Estimators," *Journal of Econometrics*, 170, 519–537.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang, 2006, "Cross-Section of Volatility and Expected Returns," *Journal of Finance*, 61.

Baker, S. R., N. Bloom, and S. J. Davis, 2016, "Measuring Economic Policy Uncertainty," *Quarterly Journal of Economics*, 131, 1593–1636.

Breeden, D. T., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 265–296.

Bryzgalova, S., J. Huang, and C. Julliard, 2023, "Bayesian Solutions for the Factor Zoo: We Just Ran Two Quadrillion Models," *Journal of Finance*, 78, 487–557.

Chen, A. Y., and T. Zimmermann, 2022, "Open Source Cross-Sectional Asset Pricing," *Critical Finance Review*, 11, 207–264.

Cochrane, J. H., 2011, "Presidential Address: Discount Rates," *Journal of Finance*, 66, 1047–1108.

Daniel, K., and S. Titman, 1997, "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns," *Journal of Finance*, 52, 1–33.

References ii

Fama, E. F., and K. R. French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33, 3–56.

Fama, E. F., and K. R. French, 2015, "A Five-Factor Asset Pricing Model," *Journal of Financial Economics*, 116.

Feng, G., S. Giglio, and D. Xiu, 2020, "Taming the Factor Zoo: A Test of New Factors," *Journal of Finance*, 75, 1327–1370.

Freyberger, J., A. Neuhierl, and M. Weber, 2020, "Dissecting Characteristics Nonparametrically," *Review of Financial Studies*, 33, 2326–2377.

Gabaix, X., 2011, "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 79, 733–772.

Ghosh, A., C. Julliard, and A. P. Taylor, 2017, "What Is the Consumption-CAPM Missing? An Information-Theoretic Framework for the Analysis of Asset Pricing Models," *Review of Financial Studies*, 30, 442–504.

Giglio, S., Y. Liao, and D. Xiu, 2021, "Thousands of Alpha Tests," *Review of Financial Studies*, 34, 3456–3496.

Giglio, S., and D. Xiu, 2021, "Asset Pricing with Omitted Factors," *Journal of Political Economy*, 129, 1947–1990.

Goyal, A., and P. Santa-Clara, 2003, "Idiosyncratic risk matters!," *Journal of Finance*, 58, 975–1007.

Hansen, L. P., and R. Jagannathan, 1997, "Assessing Specification Errors in Stochastic Discount Factor Models," *Journal of Finance*, 52, 557–590.

Harvey, C. R., Y. Liu, and H. Zhu, 2015, "...and the Cross-Section of Expected Returns," *Review of Financial Studies*, 29, 5–68.

Herskovic, B., B. Kelly, H. Lustig, and S. Van Nieuwerburgh, 2016, "The Common Factor in iIdiosyncratic Volatility: Quantitative Asset Pricing Implications," *Journal of Financial Economics*, 119, 249–283.

Hou, K., H. Mo, C. Xue, and L. Zhang, 2021, "An Augmented Q-Factor Model with Expected Growth," *Review of Finance*, 25, 1–41.

Hou, K., C. Xue, and L. Zhang, 2015, "Digesting Anomalies: An Investment Approach," *Review of Financial Studies*, 28, 650–705.

References iv

Huang, D., J. Li, and L. Wang, 2021, "Are Disagreements Agreeable? Evidence from Information Aggregation," *Journal of Financial Economics*, 141, 83–101.

Jegadeesh, N., and S. Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, 48, 65–91.

Jensen, T. I., B. T. Kelly, and L. H. Pedersen, 2021, "Is There a Replication Crisis in Finance?," Working Paper.

Jurado, K., S. C. Ludvigson, and S. Ng, 2015, "Measuring Uncertainty," *American Economic Review*, 105, 1177–1216.

Korsaye, S., A. Quaini, and F. Trojani, 2019, "Smart SDFs," University of Geneva, Working Paper.

Kozak, S., S. Nagel, and S. Santosh, 2020, "Shrinking the Cross-section," *Journal of Financial Economics*, 135, 271–292.

Lettau, M., and M. Pelger, 2020, "Factors that Fit the Time-series and Cross-section of Stock Returns," *Review of Financial Studies*, 33, 2274–2325.

McCracken, M. W., and S. Ng, 2015, "FRED-MD: A Monthly Database for Macroeconomic Research," Working Paper.

Mehra, R., S. Wahal, and D. Xie, 2021, "Is Idiosyncratic Risk Conditionally Priced?," *Quantitative Economics*, 12, 625–646.

Merton, R. C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42, 483–510.

Novy-Marx, R., 2013, "The Other Side of Value: The Gross Profitability Premium," *Journal of Financial Economics*, 108, 1–28.

Pasquariello, P., 2014, "Financial Market Dislocations," *Review of Financial Studies*, 27, 1868–1914.

Raponi, V., R. Uppal, and P. Zaffaroni, 2022, "Robust Portfolio Choice," Working paper, Imperial College London.

Ross, S., 1976, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory*, 13, 341–360.

Ross, S. A., 1977, "Return, Risk, and Arbitrage," in Irwin Friend, and J.L. Bicksler (ed.), *Risk and Return in Finance*, Ballinger, Cambridge, MA.

Sharpe, W., 1964, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance*, 1919, 425–442.

Basis assets

- Basis assets: 202 characteristic-based portfolios with monthly returns: 1963:07 to 2019:08:
  - 25 size and book-to-market portfolios,
  - 17 industry portfolios,
  - 25 investment profitability and investment,
  - 25 size and variance portfolios,
  - 35 size and net issuance portfolios,
  - 25 size and accruals portfolios,
  - 25 size and beta portfolios,
  - 25 size and momentum portfolios

back

Data on 457 tradable factors potentially spanning the SDF

- Factors used in Chen and Zimmermann (2022), Jensen, Kelly, and Pedersen (2021), and Kozak, Nagel, and Santosh (2020).
- Industry-adjusted value, momentum, and profitability factors (Novy-Marx, 2013).
- Intra-industry value, momentum, and profitability factors, and basic profitable-minus-unprofitable factor.
- Expected growth factor of Hou, Mo, Xue, and Zhang (2021) and the momentum Up minus Down (UMD) factor.
- Factors from Bryzgalova, Huang, and Julliard (2023).

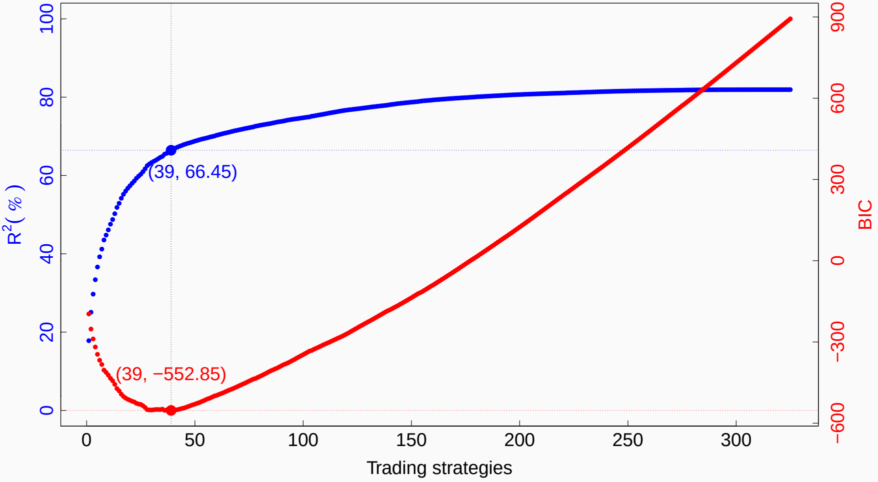
back

Data on 103 macro factors potentially spanning the SDF

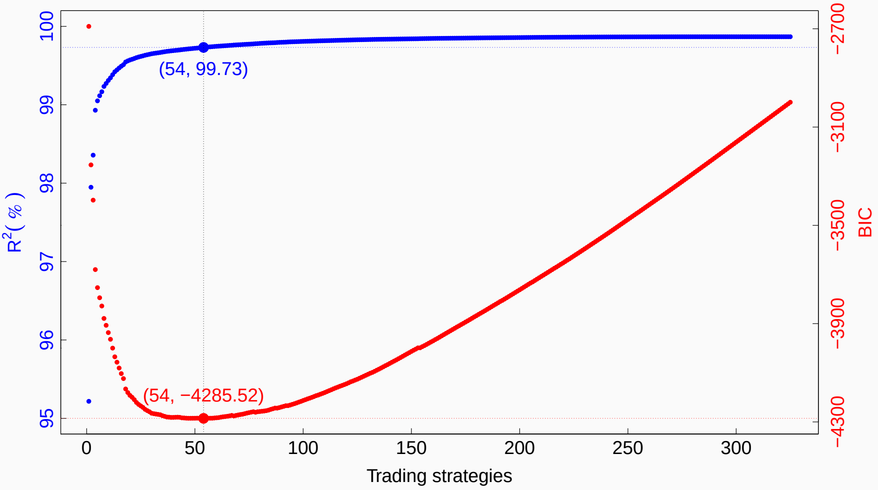
- Macroeconomic and business-cycle variables
  - 3 principal components and their VAR residuals for 279 macro variables (Jurado, Ludvigson, and Ng, 2015).
  - 8 principal components and their VAR residuals for 128 macro variables (McCracken and Ng, 2015).
- Consumption and inflation variables.
- Sentiment and confidence indexes.
- Volatility and uncertainty measures
  - Market-dislocations index (Pasquariello, 2014)
  - Disagreement index (Huang, Li, and Wang, 2021)
  - Chicago Board Options Exchange volatility index (VIX, from CBOE)
  - US econ. policy uncertainty index EPU (Baker, Bloom, and Davis, 2016)
  - Equity-mkt vol. (EMV) tracker (Baker, Bloom, and Davis, 2016).

back

Spanning M^a_{t+1} with observed factors



Spanning $M_{t+1}^{\beta,mis}$ with observed factors



back