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Are stocks good inflation hedges?

Theory and evidence from the Turkish stock market

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Abstract

We study the inflation hedging properties of stock investments. We first develop a simple mean-variance hedging strategy for inflation. We show that the inflation hedge portfolio can be found from a regression of inflation on a vector of asset returns, with adding-up restrictions on the slope coefficients. We then extend the model to multiple periods and a conditional setting where the investor hedges only the unexpected component of inflation. In the empirical part, we apply the various hedges to the case of Turkey, which has had high and volatile inflation over the last decades. We create quintile portfolios from stocks traded in the Turkish market, sorted on their inflation beta. A zero-net investment strategy based on these inflation-beta sorted portfolios, the stock market index and treasury bills provides a good inflation hedge, and hedges between 13% (monthly) and 43% (annually) of unexpected inflation. A portfolio designed to track inflation as closely as possible replicates between 46% (monthly) and 82% (annually) of the variation in inflation. Investments in foreign currency provide an additional inflation hedge, especially at the 12-month horizon.

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1 Introduction

Inflation has spiked up recently throughout the world, after a long period of moderate inflation. The outlook for inflation is highly uncertain. This begs the question of how investors can hedge this inflation risk.

Whether stocks provide a good inflation hedge has been debated in the literature. According to the hypothesis of Fisher (1930), stocks are a claim on real assets and therefore should provide an inflation hedge. In sharp contrast, most papers based on monthly or quarterly U.S. data find that the stock market has a negative contemporaneous correlation with inflation (see Fama and Schwert (1977) and many other papers reviewed in Section 2). In contrast, Boudoukh and Richardson (1993) find that U.S. stock returns over a longer time horizon (five years in their analysis) show positive correlations with realized and ex-ante inflation. Bekaert and Wang (2010) find that the relationship between stock returns and inflation is negative for developed economies but positive for developing economies. Ang, Brière and Signori (2012) sort all US stocks on their inflation beta and find that long-short portfolio of high minus low inflation beta stocks correlates positively with inflation.

The problem of optimal investment under inflation has been studied by Campbell and Viceira (2001) and Brennan and Xia (2002), building on the work of Merton (1973). These papers model (expected) inflation as a state variable; the optimal investment portfolio then contains a hedge term that provides the optimal exposure to changes in this state variable. A key insight from these papers is that (expected) inflation is a persistent process and shocks to inflation affect the long run more than the short run. Hence, inflation risk is relatively more important for investors with a long horizon. Long term bonds can be useful instruments for this hedge. On top of this, transitory unexpected inflation shocks must be hedged. However, nominal bonds are particularly poor instruments to hedge that risk, and either index linked bonds or other assets correlated with inflation must be used to hedge unexpected inflation risk.

This paper tries to bring these two approaches together. The optimal investment approach is useful as it provides investors with an advice on which assets to buy as inflation hedge. However, the results are strongly dependent on the model and results are very sensitive to model misspecification (see e.g. Balter, Coumans and de Jong, 2021).

The Fisher (1930) hypothesis literature on the other hand is model free, but does not give an immediate structure for the best investment portfolio. In this paper, we go back to simple principles of mean-variance optimal portfolios based on real returns, and build optimal portfolios under inflation risk for various investment horizons.

First, following Solnik (1978), we derive the mean-variance optimal portfolio of a one-period investor with a real wealth target. We study cases with and without a nominal risk-free asset. The results show that the optimal portfolio is composed of a speculative portfolio, an inflation hedge portfolio and the (nominal) minimum variance portfolio. An infinitely risk averse investor will only hold the latter two portfolios, which together form the best inflation replication portfolio of Bekaert and Wang (2010). The inflation hedge portfolio is a zero-net investment portfolio, closely related to the factor mimicking portfolios introduced by Breeden et al. (1989). We show that both the inflation hedge and inflation replication portfolio can be calculated from a regression of inflation on a vector of asset returns (including, if it exists, the one-period risk free asset), with the restriction that the coefficients of the regression add up to zero (for the hedge portfolio) or one (for the replication portfolio).

Second, we extend the model to multiple periods. The problem for a mean-variance buy-and-hold investor is almost identical to the one period problem, with a longer holding period of the asset. The portfolio weights can again be found from a regression of K-period inflation on a vector of K-period returns, with the restriction that the coefficients add up to zero or one. That regression can be estimated with overlapping data and the standard errors corrected using appropriate methods like Hansen and Hodrick (1980) or Newey and West (1983).

The problem for a multi-period investor who rebalances the portfolio every period is more complex. Here, we build on the work of Campbell and Viceira (2002) and derive the optimal portfolio from an explicit conditional utility maximization. The optimal portfolio weights can be found from a regressions of cumulative inflation up to the investment horizon, regressed on a vector of excess asset returns (returns in excess of the one period risk-free asset) and no restrictions on the regression coefficients. From the literature on testing predictability, it is known that such regressions are fraught with econometric difficulties. Following Wei and Wright (2013), we therefore also develop an approach

based on the reverse regressions of current inflation on long-horizon averages of past returns. This approach results in the regressions of inflation minus inflation expected K periods ago, on a K -period moving average of excess asset returns. For a one period conditional model, this simplifies to a regression of unexpected inflation on a vector of excess returns (which is different from the mean-variance approach, where inflation itself is regressed on the excess returns; the difference is the inclusion of expected inflation).

In the empirical section, we apply the methods to a sample of stock portfolios from the Turkish stock market. Turkey is an interesting case as over the past decades the country has consistently experienced high and volatile inflation, which is clearly visible in Figure 1. We show that stock returns in Turkey are positively correlated with inflation. Following the approach of Ang, Brière and Signori (2012), we create five quintile portfolios of firms in the BIST 100 index, sorted on their Bekaert and Wang (2010) inflation beta. The returns on these inflation-beta sorted portfolios, together with the return on the BIST 100 index and the one-month treasury bill, form the vector of asset returns.

The empirical results show that T-bill and the market return alone do not provide a good inflation hedge. The coefficient and the R-squared of regressions of (unexpected) inflation on the market excess return is close to zero, both for monthly returns as well as for annual returns. In contrast, adding the five inflation-beta sorted portfolios provides a good inflation hedge, with positive weights on the high inflation beta portfolio and negative weights on the low inflation beta portfolio. The R-squared of the hedge regressions ranges from 0.129 (for the one month unexpected inflation) to 0.487 for 12-month inflation. A portfolio designed to track inflation as closely as possible replicates between 46% (monthly) and 82% (annually) of the variation in inflation. The inflation replication portfolios are dominated by the T-bill return; this is no surprise as the T-bill is by far the least volatile asset and hence dominates the minimum variance portfolio part of the inflation replication portfolio. The results for the multi-period model with rebalancing are not so strong, and signs of the portfolio weights are erratic.

The paper is structured as follows. Section 2 reviews the empirical literature on inflation hedging with stocks. Sections 3 and 4 develop the theoretical framework. Section 5 presents the empirical analysis and Section 6 concludes the paper.

2 Literature Review

Inflation hedging capabilities of stocks have been an important point of discussion in the literature over the past few decades. The studies conducted cover both developed and developing economies and the findings document striking differences between developed and developing countries.

Johnson et al. (1971) analyze the relationship between stock returns and inflation in inflationary periods. They use the returns on the thirty stocks included in the Dow-Jones Industrial Average and conclude that none of these stocks proved to be good inflation hedgers on a consistent basis during these inflationary periods. Bodie (1976) addresses inflation hedging properties of NYSE stocks, for the period 1953-1972. He finds that real returns on stocks are negatively related to both expected and unexpected components of inflation. Nelson (1976) makes a similar conclusion. He investigates the post-war period, from 1953 to 1974, using the returns on the Scholes Index and the S&P 500 Index. He discovers a negative relationship between the returns on these indices and inflation. In their study covering almost the same period, 1953 to 1973, Fama and Schwert (1977) look at the inflation hedging capabilities of several asset classes. In line with the previous findings at that time, they also find an inverse relationship between expected inflation and returns on the NYSE stocks. To explain this result, Fama (1981) and Bekaert and Wang (2010) put forward the proxy hypothesis, which states that high inflation is a signal of poor expected economic growth. In line with these findings, the asset pricing literature (Boons et al., 2020) shows that stock portfolios that are strongly correlated with inflation have a positive risk premium, whereas one would expect a negative inflation risk premium if stocks are good hedges against inflation.¹

In contrast, Firth (1979) concludes that the British stock market provides some hedge against inflation, using the data for the period 1935-1976. In a study covering 26 countries and the data for the period 1947-1979, Gultekin (1983) does not find a positive relationship between stock returns and realized or expected inflation. He also notes that the inflation hedging capabilities of stocks are time-varying. Using a very long historical sample from 1914 to 1990, Boudoukh and Richardson (1993) find that annual stock

¹The relation is time-varying: in the periods before 2000, with high and variable inflation, the inflation risk premium is significantly positive, whereas after 2000 with low and stable inflation, the inflation risk premium is not significant.

returns in the U.S and the U.K. have almost no correlation with inflation, but five-year returns show positive and significant correlation with inflation. They also find that, in line with the Fisher (1930) hypothesis, long-horizon expected stock returns have an almost one-for-one exposure to ex-ante (expected) inflation. In a more recent study, Luintel and Paudyal (2006) analyze the issue in a cointegration framework using the returns on the British market index and seven other sector indices in the period 1955-2002. They find that these stock indices and inflation are cointegrated, meaning a positive relationship between the two variables in the long run. Bekaert and Wang (2010) analyze the data for 45 countries covering the period 1970-2010. They obtain the inflation betas for each country using the returns on the market index of the respective country. Then, they run pooled regressions to allow for different coefficients for regions, and for developed and emerging economies. They show that most of the inflation betas for individual countries are either negative or far from one. At a more aggregate level, they find that the relationship between stock returns and inflation is negative for developed economies but positive for developing economies.

As summarized above, most empirical evidence on the developed economies points to a negative relationship between stock returns and inflation. However, some studies focusing on developing markets provide a different picture. Choudhry (2001) looks into this relationship using the evidence from four high inflation countries: Argentina, Chile, Mexico, and Venezuela. He investigates the period 1981-1998 using the returns on the market indices of the respective countries. He finds a positive relationship between stock returns and inflation in Argentina and Chile. In a later study, Spyrou (2004) analyzes ten emerging economies using the data for the period 1989-2000. He discovers a positive relationship between stock returns and inflation for six countries (of which four are significant), and a negative relationship for the remaining four countries (of which only one is significant). In addition, the author tries to explain the discrepancy between the results for developed and developing economies by looking at the cointegration between consumer prices, money supply, and real output growth. For most countries in the sample, he finds that these variables move together. He then argues that in the sample period, the main determinant of inflation could be the money supply rather than real output growth, which in turn could explain the positive relationship between stock returns and

inflation in these markets. Maghyreh (2006) looks at potential non-linear relationships between stock returns and inflation in 18 developing economies in the long run.

Ang, Brière and Signori (2012) address the issue at the market, industry, and individual stock level using the data for U.S. stocks for the period 1989-2010. They conclude that the market index is a poor inflation hedge. In addition, they note that there is heterogeneity across individual stocks and that certain stocks have provided a hedge against inflation. A long-short portfolio of high-low beta stocks correlates positively with inflation. However, they argue that there is significant time variation in the inflation hedging capability and that it is difficult to predict the stocks that will provide a good hedge against inflation on an ex-ante basis using past data.

There is also some empirical evidence that focuses specifically on the Turkish stock market. Horasan (2008) conducts a study using the years 1990-2007 as the sample period and finds a positive relationship between the BIST 100 index return and inflation. In another relevant study, Aktürk (2016) investigates this issue at the market index, industry, and individual stock levels. He uses the data for the period 1986-2013 covering 26 industries and 170 individual stocks. He also separates inflation into its expected and unexpected components and uses the inflation expectation survey results conducted by the Turkish Central Bank as a proxy for expected inflation. He concludes that the Turkish stock market provides a hedge against expected inflation but fails to do so against realized inflation. Eyuboglu and Eyuboglu (2018) analyze this relationship for the period 2006-2016 using 15 sector indices. They find a negative relationship for 11 of these indices whereas a positive relationship for the remaining four indices.

As is evident from the above, empirical evidence on stocks inflation hedging capabilities is far from conclusive. Moreover, the evidence regarding the Turkish stock market is still limited.

3 Mean-Variance Inflation Hedge Portfolios

In this section, we derive the basic mean-variance optimal portfolio with inflation risk. The standard approach in the literature is to derive the optimal portfolio by minimizing the variance of the real return under adding-up constraints on the portfolio weights (Bekaert and Wang, 2010) and a constraint on the expected portfolio return (Solnik,

1978). We shall follow a slightly different approach by maximizing a mean-variance utility function. This approach is analytically simpler, and works both in the case with and the case without a risk-free return. The appendix shows that the optimal portfolios are equivalent to the ones derived by the existing literature.

Suppose we have k assets, whose return are collected in a $(k - 1) \times 1$ vector of returns R_{t+1} and a base asset return $R_{0,t+1}$. The base asset could be the T-bill return, but it doesn't have to be. The portfolio return thus is

$$R_{t+1}^p = w'R_{t+1} + (1 - \iota'w)R_{0,t+1} = w'(R_{t+1} - R_{0,t+1}) + R_{0,t+1} \quad (1)$$

where w is the vector of portfolio weights on the first $k - 1$ assets, and $1 - \iota'w$ is the portfolio weight on the base asset.² The investor has a one-period mean-variance utility function in real (after inflation) returns

$$U(w) = E [R_{t+1}^p - \pi_{t+1}] - \frac{\gamma}{2} Var (R_{t+1}^p - \pi_{t+1}) \quad (2)$$

Maximizing the utility with respect to w gives

$$w^* = \frac{1}{\gamma} Var(R_{t+1} - R_{0,t+1})^{-1} E [R_{t+1} - R_{0,t+1}] + Var(R_{t+1} - R_{0,t+1})^{-1} Cov(R_{t+1} - R_{0,t+1}, \pi_{t+1} - R_{0,t+1}) \quad (3)$$

and the optimal weight on the base asset is $w_0^* = 1 - \iota'w^*$.

We define the inflation *hedge* portfolio as the difference between this portfolio weight and the optimal weight in the case of constant inflation. Taking that difference gives

$$w^\pi = Var(R_{t+1} - R_{0,t+1})^{-1} Cov(R_{t+1} - R_{0,t+1}, \pi_{t+1}) \quad (4)$$

and $w_0^\pi = -\iota'w^\pi$. Hence, the weights in the inflation hedge portfolio add up to zero, and the inflation hedge portfolio is a zero-net-investment overlay on the strategy without the inflation hedge. These weights are easily calculated as the set of slope coefficients in a regression of inflation on the vector of asset return differences:

$$\pi_{t+1} = \alpha + (w^\pi)'(R_{t+1} - R_{0,t+1}) + \epsilon_{t+1} \quad (5)$$

²To simplify the notation, we write $R_{t+1} - \iota R_{0,t+1}$ as $R_{t+1} - R_{0,t+1}$.

Rewriting this regression gives

$$\pi_{t+1} = \alpha + (w^\pi)'R_{t+1} + (-l'w^\pi)R_{0,t+1} + \epsilon_{t+1} = \alpha + (\bar{w}^\pi)'\bar{R}_{t+1} + \epsilon_{t+1} \quad (6)$$

with

$$\bar{R} = \begin{bmatrix} R \\ R_0 \end{bmatrix}, \quad \bar{w}^\pi = \begin{bmatrix} w^\pi \\ -l'w^\pi \end{bmatrix} \quad (7)$$

This shows that the inflation hedge portfolio can be found by regressing inflation on the vector of all returns with the restriction that the slope coefficients add up to zero. This insight makes clear that the choice of base asset is innocuous and the same inflation hedge portfolio weights will result for a given set of assets. If the base asset is the T-bill return, the inflation hedge portfolio is the factor mimicking portfolio of Breeden et al. (1989).

In the literature, the inflation hedging properties of stocks are typically assessed by studying the so-called inflation beta of the stocks, see Bekaert and Wang (2010). These are found from regressing the stock returns on inflation

$$R_{i,t+1} = \alpha_i^\pi + \beta_i^\pi \pi_{t+1} + \varepsilon_{i,t+1} \quad (8)$$

From the properties of the OLS estimator, $\beta_i^\pi = Cov(R_{i,t+1}, \pi_{t+1})/Var(\pi_{t+1})$. The inflation hedge portfolio weights therefore are related to the inflation betas via

$$w^\pi = Var(\pi_{t+1})Var(R_{t+1} - R_{0,t+1})^{-1}(\beta^\pi - \beta_0^\pi) \quad (9)$$

The weights of the inflation hedge portfolio thus depend on the inflation beta's, but also on the mutual covariances of the asset returns. Only in the case of an excess return covariance matrix that is proportional to the identity matrix, the inflation hedge portfolio weights are proportional to the inflation betas minus the inflation beta of the base asset.

We also define the inflation *replication* portfolio as the portfolio that minimizes the variance of the difference between inflation and the portfolio return:³

$$\min_w Var(R_{t+1}^p - \pi_{t+1}) \quad (10)$$

³Bekaert and Wang (2010) call this the inflation 'mimicking' or 'tracking' portfolio.

where again $R_{t+1}^p = w'(R_{t+1} - R_{0,t+1}) + R_{0,t+1}$. The solution to this problem is

$$w^{rep} = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, \pi_{t+1} - R_{0,t+1}) \quad (11)$$

and $w_0^{rep} = 1 - \iota'w^{rep}$. Notice that the covariance term in the formula is different from the covariance term in the inflation hedge portfolio formula, and the portfolio weights add up to one. These weights can be found by regressing inflation minus the base asset return on the return differences

$$\pi_{t+1} - R_{0,t+1} = \alpha + (w^{rep})'(R_{t+1} - R_{0,t+1}) + \epsilon_{t+1} \quad (12)$$

and $w_0^{rep} = 1 - \iota'w^{rep}$. Rewriting this equation we find

$$\pi_{t+1} = \alpha + (w^{rep})'R_{t+1} + (1 - \iota'w^{rep})R_{0,t+1} + \epsilon_{t+1} = \alpha + (\bar{w}^{rep})'\bar{R}_{t+1} + \epsilon_{t+1} \quad (13)$$

which is a regression of inflation on the vector of all returns with the restriction that the slope coefficients add up to one, $\iota'\bar{w}^{rep} = 1$. The appendix shows that this regression-based method of calculating the inflation replication portfolio weights is equivalent to the methods of Solnik (1978) and Bekaert and Wang (2010) who solve the real return minimization problem by constrained optimization.

The difference between the inflation replication portfolio and inflation hedge portfolio is the minimum variance portfolio:

$$w^{MV} = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, -R_{0,t+1}) \quad (14)$$

and $w_0^{MV} = 1 - \iota'w^{MV}$. The minimum variance portfolio can be calculated from a regression of minus the base asset return on the return differences

$$-R_{0,t+1} = \alpha + (w^{MV})'(R_{t+1} - R_{0,t+1}) + \epsilon_{t+1} \quad (15)$$

This is equivalent to the regression

$$0 = \alpha + (w^{MV})'(R_{t+1} + (1 - \iota'w^{MV})R_{0,t+1} + \epsilon_{t+1}) = \alpha + (\bar{w}^{MV})'\bar{R}_{t+1} + \epsilon_{t+1} \quad (16)$$

with the restriction $l'w^{MV} = 1$. Notice that even if there is a one-period T-bill, the weight on the T-bill in the minimum variance portfolio is not necessarily one, because the T-bill is not unconditionally risk free.

3.1 Multiple period inflation hedge and replication

The unconditional one-period model can easily be extended to a K period problem if the investor has a buy-and-hold strategy. Following the same logic as before, the K period inflation hedge portfolio is

$$w^{\pi,K} = Var \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k} \right)^{-1} Cov \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k}, \sum_{k=1}^K \pi_{t+k} \right) \quad (17)$$

and $w_0^{\pi,K} = -l'w^{\pi,K}$. This portfolio can be found by running the regression

$$\sum_{k=1}^K \pi_{t+k} = \alpha + (w^{\pi,K})' \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k} \right) + \epsilon_{t+K} \quad (18)$$

Similarly, the K -period inflation replication portfolio is

$$w^{rep,K} = Var \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k} \right)^{-1} Cov_t \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k}, \sum_{k=1}^K \pi_{t+k} - R_{0,t+k} \right) \quad (19)$$

which can be found from the regression

$$\sum_{k=1}^K \pi_{t+k} - R_{0,t+k} = \alpha + (w^{rep,K})' \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k} \right) + \epsilon_{t+K} \quad (20)$$

and $w_0^{rep,K} = 1 - l'w^{rep,K}$.

4 Conditional inflation hedge

In this section, we discuss how to construct inflation hedge portfolios in a conditional mean-variance framework. The investor has a one-period conditional mean-variance utility function

$$U(w_t) = E_t [R_{t+1}^p - \pi_{t+1}] - (\gamma/2)Var_t (R_{t+1}^p - \pi_{t+1}) \quad (21)$$

where again $R_{t+1}^p = w_t'(R_{t+1} - R_{0,t+1}) + R_{0,t+1}$. Maximizing the utility with respect to w_t gives

$$w_t^* = \frac{1}{\gamma} Var_t(R_{t+1} - R_{0,t+1})^{-1} E_t[R_{t+1} - R_{0,t+1}] + Var_t(R_{t+1} - R_{0,t+1})^{-1} Cov_t(R_{t+1} - R_{0,t+1}, \pi_{t+1} - R_{0,t+1}) \quad (22)$$

and the optimal weight on the base asset is $w_{0,t}^* = 1 - \iota' w_t^*$.

We define the conditional inflation hedge portfolio as the difference between this portfolio weight and the optimal weight in the case of constant inflation. Taking that difference gives

$$w_t^\pi = Var_t(R_{t+1} - R_{0,t+1})^{-1} Cov_t(R_{t+1} - R_{0,t+1}, \pi_{t+1}) \quad (23)$$

and $w_{0,t}^\pi = -\iota' w_t^\pi$. Hence, we have weights in the inflation hedge portfolio adding up to zero, and the inflation hedge portfolio is a zero-net-investment overlay on the strategy without the inflation hedge. The formula is based on conditional variances and covariances. Define the unexpected inflation $u_{t+1} = \pi_{t+1} - E_t[\pi_{t+1}]$. If we are willing to assume that the conditional variance-covariance matrix of the return differences is time-invariant, we can write the inflation hedge portfolio in terms of unconditional covariances as

$$w^u = Var(R_{t+1} - R_{0,t+1})^{-1} Cov(R_{t+1} - R_{0,t+1}, u_{t+1}) \quad (24)$$

The hedge portfolio weight vector w^u is thus easily calculated as the set of slope coefficients in a regression of unexpected inflation on the vector of return differences:

$$u_{t+1} = \alpha + (w^u)'(R_{t+1} - R_{0,t+1}) + \epsilon_{t+1} \quad (25)$$

and $w_0^u = -\iota' w^u$.

The conditional equivalent of the Bekaert and Wang (2010) inflation betas are found from regressing the stock returns on unexpected inflation

$$R_{i,t+1} = \alpha_i^u + \beta_i^u u_{t+1} + \varepsilon_{t+1} \quad (26)$$

The unexpected-inflation hedge portfolio weights therefore are related to the unexpected-

inflation betas via

$$w^u = Var(u_{t+1})Var(R_{t+1} - R_{0,t+1})^{-1}(\beta^u - \beta_0^u) \quad (27)$$

The unexpected-inflation hedge portfolio formula can be extended to multiple periods for a buy-and-hold investor. The unexpected-inflation hedge portfolio then is

$$w^{u,K} = Var\left(\sum_{k=1}^K R_{t+k} - R_{0,t+k}\right)^{-1} Cov\left(\sum_{k=1}^K R_{t+k} - R_{0,t+k}, \sum_{k=1}^K \pi_{t+k} - E_t\left[\sum_{k=1}^K \pi_{t+k}\right]\right) \quad (28)$$

Again, this can be estimated from a regression of unexpected inflation on return differences

$$\sum_{k=1}^K \pi_{t+k} - E_t\left[\sum_{k=1}^K \pi_{t+k}\right] = \alpha + (w^{u,K})' \left(\sum_{k=1}^K R_{t+k} - R_{0,t+k}\right) + \epsilon_{t+K} \quad (29)$$

and $w_0^{u,K} = -l'w^{u,K}$.

As a practical matter, these regressions require a measure of unexpected inflation. There are several ways to construct unexpected inflation. The traditional way (used e.g. by Boons et al., 2020) is to fit a time series model to inflation, and use the innovations of that model as the unexpected inflation. Unexpected inflation can also be constructed from analyst expectations.

4.1 Interpretation of conditional vs. unconditional hedges

The unconditional inflation hedge portfolio is

$$w^\pi = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, \pi_{t+1}) \quad (30)$$

Now write realized inflation as the sum of one-period ahead expected inflation and unexpected inflation $\pi_{t+1} = \pi_t^e + u_{t+1}$ with $\pi_t^e = E_t[\pi_{t+1}]$. The unconditional inflation hedge then can be written as

$$w^\pi = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, \pi_t^e + u_{t+1}) \quad (31)$$

and hence is the sum of a hedge against one-period ahead expected inflation and the hedge against unexpected inflation. The unconditional replication portfolio is

$$w^{rep} = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, \pi_{t+1} - R_{0,t+1}) \quad (32)$$

which, as we showed before, tracks one-period inflation as closely as possible. Writing again $\pi_{t+1} = \pi_t^e + u_{t+1}$, this hedge portfolio can be decomposed as

$$w^{rep} = Var(R_{t+1} - R_{0,t+1})^{-1}Cov(R_{t+1} - R_{0,t+1}, u_{t+1} - (R_{0,t+1} - \pi_t^e)) \quad (33)$$

Now the term $R_{0,t+1} - \pi_t^e$ does not generally have a nice interpretation. However, we can follow the models of Campbell and Viceira (2001) and Brennan and Xia (2002) and assume the short term nominal interest rate $R_{f,t}$ is the sum of expected inflation and the real interest rate and choose the base asset to be the Treasury bill. In that case, $R_{0,t+1} = R_{f,t} = \pi_t^e + r_t$ and the inflation replication portfolio can be written as

$$w^{rep} = Var(R_{t+1} - R_{f,t})^{-1}Cov(R_{t+1} - R_{f,t}, u_{t+1} - r_t) \quad (34)$$

So, the replication portfolio contains an unexpected inflation hedge and a real rate hedge, both for one period ahead. Of course, we can make the unit period as long as we want to get a longer horizon hedge, assuming a buy-and-hold strategy over the unit period.

4.2 Multiple period inflation hedging with rebalancing

Extending the conditional model to multiple periods with portfolio rebalancing is more involved. We follow Campbell and Viceira (2001) and model a dynamic investment strategy with horizon K . The portfolio return in period $t + 1$ is $R_{t+1}^p = x_t' R_{t+1}^e + R_{0,t+1}$ where the portfolio weights x_t are chosen every period. The problem is to maximize expected utility of real terminal wealth $W_{t+K} = (1 + r_{t:t+K}^p)W_t$ with the K -period real cumulative portfolio return given by

$$r_{t:t+K}^p = \sum_{k=1}^K (x_{t+k-1}' (R_{t+k} - R_{0,t+k}) + R_{0,t+k} - \pi_{t+k}) \quad (35)$$

This K -period portfolio return formula leaves out compounding effects and is therefore an approximation, see Viceira and Campbell (2001). We make one important assumption, that excess returns are IID.

Maximizing a mean-variance utility function with respect to x_t gives the first-order condition

$$E[R_{t+1} - R_{0,t+1}] - \gamma x'_{t+1} \Sigma_t - \gamma Cov_t \left(R_{t+1} - R_{0,t+1}, \sum_{k=1}^K R_{0,t+k} - \pi_{t+k} \right) = 0 \quad (36)$$

with $\Sigma_t = Var_t(R_{t+1} - R_{0,t+1})$ and solution

$$x_t = \frac{1}{\gamma} \Sigma_t^{-1} E[R_{t+1} - R_{0,t+1}] - \Sigma_t^{-1} Cov_t \left(R_{t+1} - R_{0,t+1}, \sum_{k=1}^K R_{0,t+k} - \pi_{t+k} \right) \quad (37)$$

The inflation hedge component of this portfolio weight then is

$$x_t^{\pi,K} = \Sigma_t^{-1} Cov_t \left(R_{t+1} - R_{0,t+1}, \sum_{k=1}^K \pi_{t+k} \right) \quad (38)$$

To get an unconditional expression, we subtract the conditional expectations from the inflation and assume that variances and covariances are time-invariant. This gives the (time-invariant) conditional inflation hedge portfolio weights

$$x^{\pi,K} = Var(R_{t+1} - R_{0,t+1})^{-1} Cov \left(R_{t+1} - R_{0,t+1}, \sum_{k=1}^K \pi_{t+k} - E_t \left[\sum_{k=1}^K \pi_{t+k} \right] \right) \quad (39)$$

Suppose we have a model for the inflation expectations, then we can find the hedge portfolio weights from the long-horizon regression

$$\sum_{k=1}^K \pi_{t+k} - E_t \left[\sum_{k=1}^K \pi_{t+k} \right] = \alpha + (x^{\pi,K})' (R_{t+1} - R_{0,t+1}) + \epsilon_{t+K} \quad (40)$$

and $x_0^{\pi,K} = -\iota' x^{\pi,K}$.

Reverse regressions

The long horizon regressions are notoriously prone to small sample biases and inference problems. Wei and Wright (2013) propose an alternative approach via reverse regres-

sions. Re-write the covariance term in equation (39) as (it is convenient now to write the portfolio return in terms of excess returns $R_{t+1}^e = R_{t+1} - R_{0,t+1}$)

$$\sum_{k=1}^K Cov(R_{t+1}^e, \pi_{t+k} - E_t[\pi_{t+k}]) = \sum_{\ell=0}^{K-1} Cov(R_{t-\ell}^e, \pi_t - E_{t-\ell-1}[\pi_t]) \quad (41)$$

Notice that the excess returns must be unpredictable from any past information, hence in the last term, $E_{t-\ell-1}[\pi_t]$ can be replaced by $E_{t-K}[\pi_t]$:

$$\sum_{\ell=0}^{K-1} Cov(R_{t-\ell}^e, \pi_t - E_{t-\ell-1}[\pi_t]) = \sum_{\ell=0}^{K-1} Cov(R_{t-\ell}^e, \pi_t - E_{t-K}[\pi_t]) \quad (42)$$

Thus, we obtain the K period conditional inflation hedge portfolio weight

$$x^{\pi,K} = Var(R_{t+1}^e)^{-1} Cov\left(\sum_{\ell=0}^{K-1} R_{t-\ell}^e, \pi_t - E_{t-K}[\pi_t]\right) \quad (43)$$

The inflation hedge portfolio weights can thus be estimated from the reverse regression

$$\pi_t - E_{t-K}[\pi_t] = \alpha + \gamma' \left(\sum_{\ell=0}^{K-1} R_{t-\ell}^e\right) + \epsilon_t \quad (44)$$

and correcting the estimator using the formula of Wei and Wright (2013):

$$x^{\pi,K} = Var(R_{t+1}^e)^{-1} Var\left(\sum_{\ell=0}^{K-1} R_{t-\ell}^e\right) \gamma \quad (45)$$

If excess returns are IID, the variance of the sum of excess returns is $K \cdot Var(R_{t+1}^e)$, and the formula simplifies to $x^{\pi,K} = K\gamma$, and $x_0^{\pi,K} = -l'x^{\pi,K}$.

5 Empirical results

5.1 Data and Portfolio Construction

The stock price data used in this study consists of monthly closing values of the BIST 100 Index, and monthly dividend-adjusted closing prices of 329 individual stocks trading in the Turkish stock market. BIST 100 data was obtained from Borsa Istanbul Datastore.

Data for individual stocks were obtained from Thomson Reuters Datastream. CPI data was obtained from the Turkish Central Bank. Yields on the Turkish lira denominated Treasury bills with a maturity of one month were obtained from Datastream. Finally, end-of-month values for the Turkish Lira-U.S. dollar exchange rate are obtained from the website of the Turkish Central Bank, and 3-month U.S. T-bill rates are collected from the St. Louis Fed FRED database, series TB3MS. The sample period is October 1998 until March 2022.

Our asset menu consists of the stock market index, the one month T-bill and five portfolios of stocks sorted on their inflation beta. These portfolios are constructed as follows. We selected all stocks trading in Borsa Istanbul that were (i) still actively trading as of March 2022 and (ii) had at least 5 years of trading history. Then, we estimate for each stock its Bekaert and Wang (2010) inflation beta using all the data available for that stock. The five portfolios are found by sorting all stocks into quintiles, where P1 is the quintile of stocks with the lowest inflation beta, and P5 the quintile with the highest inflation beta. Then, for each month we calculate the equally weighted return of the stocks within each quintile.

Finally, we take logarithms of all monthly portfolio returns and inflation. 12-month stock returns and inflation are formed by taking 12-month moving sums of the log returns and inflation.

5.2 Inflation hedge and replication portfolios

Table 3 shows the results on the unconditional inflation hedging and replication. The first column of the table contains the Bekaert-Wang inflation betas. At the one-month horizon (Panel A), the market index return has an inflation beta of 1.05, whereas the inflation-exposure sorted portfolios P1 through P5 have inflation betas ranging from 0.20 to 1.40, again on average significantly positive. These are remarkable results because the literature on U.S. stock returns typically finds that inflation betas are negative. For example, Boons et al. (2020) report inflation betas ranging from -3.02 to -0.02 for inflation-beta sorted decile portfolios. The T-bill return also has a positive exposure to inflation (the inflation beta is 0.36), indicating that the Turkish short term interest rates positively correlate with inflation. At the 12-month horizon (Panel B), the results are

very similar. The T-bill has an even stronger exposure to inflation with a beta of 0.75.

We now turn to the optimal inflation hedge portfolio weights. Table 3, Panel A reports the results for the one-month investment horizon. When using only the market excess return, the optimal weight on the stock market is small (0.019).⁴ The fraction of inflation hedged is also very low, as evidenced by the R-squared of 0.013. Using the five inflation-exposure sorted portfolios as inflation hedges substantially improves the inflation hedge. The R-squared increases to 0.208. The optimal weights have an intuitive pattern, with a negative weight on the low inflation beta portfolios and a positive weight on the high inflation beta portfolios. Finally, adding the market portfolio to P1–P5 gives a modest improvement of the inflation hedge to an R-squared of 0.225. Interestingly, the weight on the T-bill in these hedge portfolios is always small. Removing the T-bill from the asset menu lowers the inflation hedge only a little bit: the R-squared is 0.219.

Table 3, Panel B reports the results for a 12-month horizon buy-and-hold strategy. The hedge portfolio weight patterns are very similar to the one-month hedge, but with lower (negative) weights on the low inflation beta portfolio (P1) and higher (positive) weights on the high inflation beta portfolio (P5). The R-squared is also higher than for the 1-month hedge: it increases to 0.321 for the five inflation sorted portfolios, and to 0.487 when also the market portfolio is added. All in all, these results show that inflation-beta sorted stock portfolios can provide a good hedge against inflation.

The inflation replication portfolios are dominated by the T-bill. In all specifications where the T-bill is included, its weight is close to 1. Recall that the inflation replication portfolio weights are the sum of the inflation hedge portfolio and the minimum variance (MV) portfolio weights. The MV portfolio is dominated by the T-bill, which reflects the much lower return variance of the T-bill relative to the stock portfolios. The R-squared of the inflation replication regressions (calculated as the variance of the best replicating portfolio return divided by the variance of inflation) ranges from 0.309 for the T-bill alone (unreported in the table) to 0.487 when all stock returns and the T-bill are included. Excluding the T-bill from the asset menu gives a terrible inflation replication, with strongly negative R-squares. The results at the 12-month horizon are similar, with an R-squared of 0.821 for the portfolio with all assets included. Interestingly, this portfolio

⁴As the inflation hedge portfolio by definition is a zero net investment portfolio, the weight on the T-bill is -0.019 .

has a weight of almost exactly 1 on the T-bill, and hence weights on the stock portfolios add up to approximately zero.

5.3 Unexpected-inflation hedge portfolios

Table 4 presents the results of the monthly unexpected inflation hedge regressions. Unexpected inflation is calculated as the residual of an AR(1) model fitted on the monthly inflation series

$$\pi_{t+1} = a + \rho\pi_t + u_{t+1} \quad (46)$$

The 12-month version of that regression is (with $\pi_{t:t+12} = \sum_{k=1}^{12} \pi_{t+k}$)

$$\pi_{t:t+12} = a + \rho\pi_{t-12:t} + u_{t+12:t} \quad (47)$$

Estimates of these models are reported in Table 2. Inflation is fairly persistent, with an estimated auto-regressive coefficient ρ around 0.7.

The unexpected-inflation hedge portfolio weights follow the same patterns as the inflation hedge portfolio weights, but are typically a bit smaller (in absolute value). The R-squared of the unexpected-inflation hedge regression is also smaller, 0.129 for the portfolio with the inflation-beta sorted portfolios and the market excess returns. For the 12-month horizon, the R-squared is again somewhat higher (up to 0.428) than for the 1-month horizon. The hedge portfolio weights are similar the one month hedge, except that P3 gets a much lower weight and P4 a much higher weight.

Finally, we construct unexpected-inflation hedge portfolios for 12 months ahead for an investor who rebalances each month. The unexpected inflation in that case is given from a regression of 12 months ahead unexpected inflation on the sum of returns as in equation (44). For the 12 months ahead inflation expectations we use a regression of monthly inflation on its value 12 months ago, plus a moving average term

$$\pi_t = a + \rho\pi_{t-12} + u_t \quad (48)$$

results of which are reported in Table 2. The conditional inflation betas and hedge portfolio weights are reported in Table 5. The results in this case are rather weak:

the betas are never significantly different from zero, and the R-squared of the hedge regressions are also quite small. This hedging approach therefore seems unsuccessful.

5.4 Foreign investments

In countries with volatile inflation, investment in foreign assets is often considered a good inflation hedge, as the exchange rate depreciates in times of high inflation. We add foreign investments to our asset menu in the form of an investment in 3-month US treasury bills. The monthly return on this investment is calculated as the sum of the change in the logarithm of the Turkish Lira-U.S. dollar exchange rate plus the US T-bill rate (divided by 12 in order to convert the annualized rate to a monthly return).

Table 6 presents the results of the various hedge portfolios that include foreign investments. At the one month horizon, the foreign investment has a modest weight, between 4% and 6%, in the hedge portfolios. The R-squared is somewhat bigger than in the hedge regressions without the foreign investment. At the 12 month horizon, the foreign investment gains a much larger weight, between 17% and 28%, with the exception of the inflation hedge regression with all portfolios included. The R-squared of the replication regressions increases substantially, to a highest value of 0.898 for the portfolio with all assets included. These results confirm that in a country with high and volatile inflation, foreign investments can provide a good inflation hedge.

6 Conclusion

We have shown that inflation hedge and replication portfolios can be found from regressions of inflation on a vector of asset returns, with adding-up restrictions on the slope coefficients. We apply the methods to the case of Turkey and create inflation-beta sorted quintile portfolios. A zero-net investment strategy based on these inflation-beta sorted portfolios, the stock market index and treasury bills provides a good inflation hedge, and hedges between 13% (monthly) and 43% (annually) of unexpected inflation. A portfolio designed to track inflation as closely as possible replicates between 46% (monthly) and 82% (annually) of the variation in inflation.

Appendix

This Appendix derives the unconditional inflation hedge and replication portfolios by constrained optimization. For notational convenience, we omit all time subscripts.

Inflation hedge and replication portfolio

Following Solnik (1978) and Bekaert and Wang (2010, Appendix C), we first derive the inflation hedge and replication portfolios by an explicit constrained variance minimization. The problem is

$$\min_{\bar{w}} \text{Var}(\pi - \bar{w}'\bar{R}) \quad \text{s.t. } \iota'\bar{w} = a$$

where $a = 0$ gives the inflation hedge portfolio and $a = 1$ the inflation replication portfolio. Note that this variance minimization problem is equivalent to a least squares regression of inflation on the vector of returns,

$$\pi = \alpha + \bar{w}'\bar{R} + \epsilon$$

with the restriction $\iota'\bar{w} = a$. The Lagrangian of this problem is

$$L = \bar{w}'\bar{\Sigma}\bar{w} - 2\bar{w}'\text{Cov}(\bar{R}, \pi) - 2\lambda(\iota'\bar{w} - a)$$

with $\bar{\Sigma} = \text{Var}(\bar{R})$. Setting the derivative with respect to \bar{w} to zero gives

$$\bar{w} = \bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \pi) + \lambda\bar{\Sigma}^{-1}\iota$$

Pre-multiplying by ι and substituting the constraint $\iota'\bar{w} = a$ gives the solution

$$\lambda = \frac{a - \iota'\bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \pi)}{\iota'\bar{\Sigma}^{-1}\iota}$$

Substituting the formula for λ in the previous expression gives the optimal portfolio weight vector:

$$\bar{w}^* = \bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \pi) + [a - \iota'\bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \pi)]\bar{w}^{MV}$$

where \bar{w}^{MV} are the weights of the minimum variance portfolio,

$$\bar{w}^{MV} = \frac{\bar{\Sigma}^{-1}\iota}{\iota'\bar{\Sigma}^{-1}\iota}$$

Setting $a = 1$ gives the inflation replication portfolio, and $a = 0$ gives the inflation hedge portfolio. The difference between these portfolios is exactly the minimum variance portfolio. The return on the inflation hedge portfolio is uncorrelated with the return on the minimum variance portfolio, because $\text{Cov}(R^\pi, R^{MV}) = (\bar{w}^\pi)'\bar{\Sigma}\bar{w}^{MV} = (\bar{w}^\pi)'\iota/(\iota'\bar{\Sigma}^{-1}\iota) = 0$.

Alternative derivation

Chipman and Rao (1964) write the constrained regression as $y = X\beta + \epsilon$ with restrictions $\Psi\beta = a$, where y and X are a vector and matrix with number of rows equal to the number of observations N . In our case, after de-meaning the data, $y = \boldsymbol{\pi}$, $X = \bar{\mathbf{R}} = [\mathbf{R} \ \mathbf{R}_0]$, where $\boldsymbol{\pi}$ is a vector with each row an observation on the inflation and $\bar{\mathbf{R}}$ is a matrix with each row an observation on the $(1 \times k)$ vector of returns. The restriction $\iota'\beta = a$ implies $\Psi = \iota'$, with $a = 0$ or $a = 1$. Define the matrix

$$T = \begin{bmatrix} \tilde{F} \\ \Psi \end{bmatrix} = \begin{bmatrix} I & 0 \\ \iota' & 1 \end{bmatrix}$$

with I and ι' of size $(k-1) \times (k-1)$ and $1 \times (k-1)$. The inverse of this matrix is

$$T^{-1} = \begin{bmatrix} \Phi & \tilde{G} \end{bmatrix} = \begin{bmatrix} I & 0 \\ -\iota' & 1 \end{bmatrix}$$

Theorem 1 of Chipman and Rao (1964) shows that the restricted estimator for β can be written as

$$\bar{\beta} = \Phi b^* + \tilde{G}a, \quad b^* = (X^*X^*)^{-1}X^{*'}y^*$$

with $X^* = X\Phi$ and $y^* = y - X\tilde{G}a$. In our setting, this amounts to $X^* = \mathbf{R} - \mathbf{R}_0\iota'$ and $y^* = \boldsymbol{\pi} - a\mathbf{R}_0$ and hence

$$b^* = [(\mathbf{R} - \mathbf{R}_0\iota')'(\mathbf{R} - \mathbf{R}_0\iota')]^{-1}(\mathbf{R} - \mathbf{R}_0\iota')'(\boldsymbol{\pi} - a\mathbf{R}_0) = \text{Var}(R - \iota R_0)^{-1} \text{Cov}(R - \iota R_0, \boldsymbol{\pi} - a R_0)$$

The estimator for the full vector of portfolio weights with adding-up constraints then is

$$\bar{\beta} = \Phi b^* + \tilde{G}a = \begin{bmatrix} I \\ -\iota' \end{bmatrix} b^* + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a = \begin{bmatrix} b^* \\ a - \iota' b^* \end{bmatrix}$$

which is exactly how we defined the inflation hedge and replication portfolio weights.

Chipman and Rao (1964) also show that the constrained estimator is identical to

$$\bar{\beta} = b - (X'X)^{-1}\Psi'[\Psi(X'X)^{-1}\Psi']^{-1}(\Psi b - a)$$

with $b = (X'X)^{-1}X'y$ the unrestricted OLS estimator for β . In our case, $X'X = \bar{\Sigma}$, $X'y = \text{Cov}(\bar{R}, \boldsymbol{\pi})$ and $\Psi' = \iota$, so we get

$$\bar{\beta} = \bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \boldsymbol{\pi}) - \bar{\Sigma}^{-1}\iota(\iota'\bar{\Sigma}^{-1}\iota)^{-1}[\iota'\bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \boldsymbol{\pi}) - a] = \bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \boldsymbol{\pi}) + [a - \iota'\bar{\Sigma}^{-1}\text{Cov}(\bar{R}, \boldsymbol{\pi})]\bar{w}^{MV}$$

with $\bar{w}^{MV} = \bar{\Sigma}^{-1}\iota/\iota'\bar{\Sigma}^{-1}\iota$. This is the same formula that we derived before by the method of Lagrange.

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Table 1. Descriptive statistics.

This table shows the mean, standard deviation and correlation with inflation of the monthly asset returns. The assets included are five inflation-beta sorted portfolios P1–P5, the BIS100 index return (MKT), the one-month T-bill return (TBILL) and monthly inflation (INFL). The sample period is October 1998 to March 2022.

	mean	st.dev.	$Corr(R, \pi)$
P1	2.45%	11.74%	0.029
P2	2.79%	10.57%	0.111
P3	2.46%	10.44%	0.126
P4	2.65%	10.62%	0.185
P5	2.71%	10.79%	0.226
MKT	1.63%	10.70%	0.171
TBILL	1.48%	1.11%	0.561
INFL	1.41%	1.74%	1

Table 2. Time-series models for inflation.

This table report the results of time-series models for inflation. The dependent variable is monthly inflation (columns 1 and 3) or 12-month annual inflation (column 2). Numbers in parentheses are Newey-West t-statistics with $K - 1$ lags ($K = 1$ for the regressions in columns 1 and 3, $K = 12$ for the regression in column 2). The sample period is October 1998 to March 2022.

	π_t	$\pi_{t:t-11}$	π_t
constant	0.004 (4.34)	0.034 (1.93)	0.006 (5.38)
π_{t-1}	0.710 (17.09)		
$\pi_{t-12:t-23}$		0.722 (14.20)	
π_{t-12}			0.535 (10.03)
R^2	0.510	0.745	0.265

Table 3. Inflation hedging and replication regressions.

This table shows results of the unconditional inflation hedging and replication portfolios for the Turkish stock market. The assets included are the BIS100 index return (MKT), the five inflation-beta sorted portfolios P1–P5 and the one-month T-bill return. The column shows the betas of the returns with respect to inflation (π). The second set of columns shows the weights of the unconditional inflation hedge portfolio, w^π , obtained by regressing inflation on a vector of returns, with the restriction that the coefficients add up to zero. The third set of columns shows the weights of the inflation replication portfolio, w^{rep} , obtained by regressing inflation on returns with the restriction that the coefficients add up to one. Numbers in parentheses are Newey-West t-statistics with $K - 1$ lags ($K = 1$ for the 1-month horizon and $K = 12$ for the 12-month horizon). The last line shows the R^2 , defined as the fraction of variance of inflation explained by the return of the replication or hedge portfolio. The regression uses monthly data from October 1998 to March 2022.

Panel A: 1 month horizon

	β^π		w^π	w^π	w^π	w^π	w^{rep}	w^{rep}	w^{rep}	w^{rep}
P1	0.20	(0.49)		-0.105	-0.104	-0.099		-0.068	-0.066	-0.403
P2	0.67	(1.86)		-0.060	-0.068	-0.075		-0.057	-0.067	0.388
P3	0.75	(2.13)		-0.049	-0.063	-0.074		-0.033	-0.048	0.757
P4	1.12	(3.14)		0.049	0.079	0.081		0.020	0.054	-0.109
P5	1.40	(3.88)		0.185	0.222	0.227		0.159	0.201	-0.137
MKT	1.05	(2.90)	0.019		-0.052	-0.060	0.017		-0.060	0.504
TBILL	0.36	(11.34)	-0.019	-0.019	-0.013		0.983	0.979	0.986	
R^2			0.013	0.208	0.225	0.219	0.320	0.437	0.460	-30.1

Panel B: 12 months horizon

	β^π		w^π	w^π	w^π	w^π	w^{rep}	w^{rep}	w^{rep}	w^{rep}
P1	0.06	(0.74)		-0.457	-0.517	-0.512		0.074	0.041	-0.243
P2	0.31	(1.73)		-0.038	-0.066	-0.058		-0.127	-0.142	-0.575
P3	0.35	(2.00)		0.114	-0.009	-0.023		-0.309	-0.378	0.449
P4	0.57	(3.06)		0.222	0.452	0.460		0.254	0.384	-0.076
P5	0.61	(3.59)		0.235	0.490	0.481		0.131	0.275	0.821
MKT	0.69	(2.66)	-0.008		-0.334	-0.348	-0.016		-0.188	0.623
TBILL	0.75	(2.54)	0.008	-0.076	-0.016		0.984	0.975	1.008	
R^2			0.000	0.321	0.487	0.485	0.705	0.769	0.821	-6.1

Table 4. Unexpected inflation hedging regressions.

This table shows results of the unexpected inflation hedging portfolios for the Turkish stock market. The first column shows the unexpected-inflation betas of the returns on the BIS100 index (MKT), the one month treasury bill (TBILL) and the returns on five inflation-beta sorted portfolios P1–P5. The second set of columns shows the weights of the conditional inflation hedge portfolio, w^u , obtained by regressing unexpected inflation on a vector of returns, with the constraint that the coefficients add up to zero. Numbers in parentheses are Newey-West t-statistics with $K - 1$ lags. ($K = 1$ for the 1-month horizon and $K = 12$ for the 12-month horizon). The last line shows the R^2 , defined as the fraction of variance of unexpected inflation explained by the returns. The regression uses monthly data from October 1998 to March 2022.

Panel A: 1 month horizon

	β^u	w^u	w^u	w^u	w^u
P1	0.12 (0.21)		-0.054	-0.054	-0.051
P2	0.70 (1.36)		-0.011	-0.016	-0.019
P3	0.67 (1.31)		-0.043	-0.051	-0.058
P4	1.08 (2.09)		0.019	0.037	0.038
P5	1.40 (2.68)		0.101	0.123	0.126
MKT	0.94 (1.81)	0.010		-0.032	-0.036
TBILL	0.19 (3.60)	-0.010	-0.012	-0.007	
R^2		0.007	0.116	0.129	0.125

Panel B: 12 months horizon

	β^u	w^u	w^u	w^u	w^u
P1	-0.23 (-0.45)		-0.058	-0.081	-0.075
P2	0.21 (0.27)		-0.076	-0.087	-0.078
P3	0.08 (0.14)		-0.296	-0.344	-0.361
P4	0.80 (1.29)		0.315	0.404	0.414
P5	0.85 (1.37)		0.159	0.257	0.246
MKT	0.65 (1.07)	0.020		-0.129	-0.146
TBILL	0.22 (1.24)	-0.020	-0.044	-0.020	
R^2		0.009	0.353	0.428	0.419

Table 5. Conditional inflation hedging regressions.

This table shows results of the conditional inflation hedging portfolios for the Turkish stock market. The regression estimated is equation (44), in which unexpected inflation is defined as $\pi_t - E_{t-12}[\pi_t]$ and the regressors are 12-month moving sums of returns. The first column of the table shows the unexpected-inflation betas of the returns on the BIS100 index (MKT), the one month treasury bill (TBILL) and the returns on five inflation-beta sorted portfolios P1–P5. The second set of columns shows the weights of the conditional inflation hedge portfolio, $w^{\pi,12}$, obtained by regressing unexpected inflation on a vector of returns, with the constraint that the slope coefficients add up to zero. Numbers in parentheses are Newey-West t-statistics with 11 lags. The last line shows the R^2 , defined as the fraction of variance of unexpected inflation explained by the excess returns. The regression uses monthly data from October 1998 to March 2022.

	β		w	w	w	w
P1	-0.17	(-0.74)		-0.098	-0.116	-0.117
P2	-0.05	(-0.22)		0.039	0.031	0.030
P3	-0.10	(-0.40)		-0.339	-0.376	-0.374
P4	-0.02	(-0.09)		0.165	0.235	0.234
P5	-0.07	(-0.29)		0.248	0.325	0.326
MKT	0.08	(-0.30)	-0.002		-0.101	-0.099
TBILL	0.08	(2.09)	0.002	-0.015	0.002	
R^2			0.000	0.068	0.076	0.076

Table 6. Inflation hedging regressions with foreign investments.

This table shows results of the unconditional inflation hedging and replication portfolios for the Turkish stock market. The assets included are the BIS100 index return (MKT), US T-bill investment (FX), the five inflation-beta sorted portfolios P1–P5 and the one-month T-bill return. The column shows the betas of the returns with respect to inflation (π). The second set of columns shows the weights of the unconditional inflation hedge portfolio, w^π , obtained by regressing inflation on a vector of returns, with the restriction that the coefficients add up to zero. The third set of columns shows the weights of the inflation replication portfolio, w^{rep} , obtained by regressing inflation on returns with the restriction that the coefficients add up to one. The final columns show the unexpected inflation hedge portfolio weights, obtained by regressing unexpected inflation on a vector of returns, with the restriction that the coefficients add up to zero. Numbers in parentheses are Newey-West t-statistics with $K - 1$ lags ($K = 1$ for the 1-month horizon and $K = 12$ for the 12-month horizon). The last line shows the R^2 , defined as the fraction of variance of inflation explained by the return of the replication or hedge portfolio. The regression uses monthly data from October 1998 to March 2022.

Panel A: 1 month horizon

	β^π		w^π		w^{rep}		w^u	
P1	0.20	(0.49)		-0.111		-0.076		-0.061
P2	0.67	(1.86)		-0.063		-0.060		-0.010
P3	0.75	(2.13)		-0.047		-0.027		-0.035
P4	1.12	(3.14)		0.067		0.039		0.025
P5	1.40	(3.88)		0.218		0.197		0.120
MKT	1.05	(2.90)	0.024	-0.045	0.025	-0.050	0.016	-0.024
FX	0.65	(3.18)	0.046	0.043	0.060	0.057	0.044	0.044
TBILL	0.36	(11.34)	-0.070	-0.062	0.915	0.920	-0.060	-0.059
R^2			0.029	0.239	0.347	0.483	0.038	0.158

Panel B: 12 months horizon

	β^π		w^π		w^{rep}		w^u	
P1	0.06	(0.74)		-0.518		0.009		-0.104
P2	0.31	(1.73)		-0.064		-0.108		-0.062
P3	0.35	(2.00)		0.006		-0.126		-0.160
P4	0.57	(3.06)		0.438		0.141		0.226
P5	0.61	(3.59)		0.484		0.165		0.177
MKT	0.69	(2.66)	0.018	-0.326	0.033	-0.053	0.056	-0.030
FX	1.21	(11.12)	0.179	0.014	0.287	0.231	0.242	0.169
TBILL	0.75	(2.54)	-0.197	-0.034	0.680	0.741	-0.298	-0.222
R^2			0.068	0.487	0.879	0.898	0.387	0.553

Figure 1. Year-on-year inflation in Turkey from October 1998 to March 2022.

