A theory of front-line management

Daniel Bird* Alexander Frug†

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Abstract

Mid- and low-level managers play a significant role within the organizational hierarchy, far beyond monitoring. It is often their responsibility to respond to opportunities and threats within their units by adjusting their subordinates' assignments. Most such managers, however, lack the authority to adapt their subordinates' wages. Instead, they rely on other, more restrictive incentive schemes. We study the interaction between a front-line manager and worker, and characterize the "managerial style" as a function of the players' relative patience and information. Our analysis also offers insights into broader organizational design problems such as selecting the managers' discretion level, promotion policies, etc.

Keywords: Front-line management, Perishable incentives, Asymmetric discounting.
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*Eitan Berglas School of Economics, Tel Aviv University (e-mail: dbird@tauex.tau.ac.il).
†Department of Economics and Business, Universitat Pompeu Fabra and Barcelona School of Economics (e-mail: alexander.frug@upf.edu).
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1 Introduction

Large organizations generally adopt a hierarchical structure. Indeed, due to limitations on managers' effective span of control and the need to execute parallel processes requiring different skills, such a structure becomes almost unavoidable. If labor contracts were complete (in the sense that they could address every possible contingency), the role of managers in the lower parts of the hierarchy would mainly consist of monitoring activities. However, complete contracts are merely a theoretical idealization rather than a practical benchmark.[1] First, the uncertainty organizations face is typically too complex to be accurately understood and fully described. Moreover, some events are not even ex-ante describable (e.g., creative ideas down the road). Finally, as the organizational knowledge is dispersed and changes over time, it is sometimes more practical and efficient to solve problems within the relevant unit as they arrive. In this respect, mid- and low-level managers fulfill a fundamental role within organizations beyond the supervisory function discussed in, for example, Williamson (1967), Calvo and Wellisz (1978,1979), Tirole (1986), and Laffont (1988)) in that such managers, if given sufficient discretion, can effectively adapt the assignments of their subordinates to provide timely responses to unexpected opportunities and threats. That is, they can become an effective means to *complement* workers' incomplete labor contracts.[2]

The managerial positions along the organizational hierarchy differ not only in the responsibilities and required expertise but also in the manager's ability to provide additional incentives that fall outside their subordinates' baseline contracts. While some senior managers may have the authority to adapt wages and promote workers, such flexibility is rarely granted to many mid-level managers and even less so to the front-line managers at the bottom of the managerial hierarchy.

Although front-line managers typically lack control over wages, they can influence workers' job satisfaction through their managerial practices and allocation of perks. First, they can improve the work atmosphere and work–life balance of their subordinates by reducing monitoring frequency, exempting workers from mundane tasks, and permitting remote work or flexible working hours.[3] Second, front-line managers can enhance

[1]Indeed, Williamson (1975) and Hart (1995) argue that incomplete contracting is necessary for a cogent theory of the firm.

[2]The importance of providing timely responses to such shocks has been noted, for example, by Williamson (1967, p. 125) who writes that "the firm is required to adapt to circumstances which are predictable in the sense that although they occur with stochastic regularity, precise advance knowledge of them is unavailable. ... Coordination in these circumstances is thus essential."

[3]According to a 2022 Gallup poll the second-most important factor, after wages, in deciding whether to accept a new job is "work–life balance and personal well-being," and the third-most important factor is "the ability to do what they do best."

their subordinates' prospects in the firm by providing them with training programs or increasing their visibility within the organization.[4] Third, front-line managers can convey to their subordinates that the organization values their contribution, cares about their well-being, and is ready to support their socioemotional needs.[5]

We argue that these incentivization tools have a substantially different structure compared to the traditional model of monetary incentives. In particular, the per-period incentive budget is typically *small* (e.g., allowing work from home for only one day may not be enough compensation for substantial effort), and the incentives available to the manager are usually *perishable* (e.g., the possibility to allow work from home on a given day is wasted if not utilized on that day).

In this paper, we study the interaction between a front-line manager and a worker, which is of key importance for organizational performance. We obtain a rich set of qualitative results regarding the dynamics at the bottom of the organizational hierarchy. Our results connect the players' relative patience and information structure to a variety of commonly observed managerial practices. Our analysis also offers insights into the broader organizational level (beyond the interaction between the front-line manager and the worker) and suggests potential links between organizational promotion policies, employee retention and productivity, and endogenous managerial practices.

We propose a tractable model in which the (front-line) manager can offer only small and perishable per-period compensation, which she uses to incentivize the worker's "extra" effort that is needed on random occasions. The interpretation is that, occasionally, there are peaks at work or events that require special attention and effort above and beyond the regular work requirements. For example, workers may sometimes need to stay past their usual work hours to prepare urgent presentations or comply with an unexpected demand from a major client, deal with unexpected breakdowns, or exert special effort to implement creative ideas that increase output. We refer to such events as "opportunities." In some cases, the manager observes the arrival of opportunities (e.g., when she receives an urgent task from her superiors), while in other cases, the worker observes the arrival of opportunities privately (e.g., his creative ideas).

As the manager's per-period compensation budget is small, there is structural asynchronicity between work and compensation in our model: the manager is unable to *instantaneously* compensate the worker for his effort. This asynchronicity, in turn, gives rise

[4]For a discussion on the importance of organizational visibility (e.g., by allowing workers to participate in meetings with important clients or higher-up managers) see, for example, Holmström (2017).

[5]The importance of such behavior is the basis of organizational support theory, which also highlights the importance of front-line managers in shaping workers' perceptions of organizational support. See Rhoades and Eisenberger (2002) for a review of this theory.

to a dynamic spillover between opportunities since new opportunities may arrive while the worker is still receiving compensation for his previous effort. Thus, the manager faces a rather complex dynamic problem where the optimal way to resolve tradeoffs depends on the players' relative patience.

While most papers focus on the case where the players discount the future by using the same discount rate, there is no compelling reason to assume that, within their specific interaction, the worker and the manager discount the future in the same way. For example, a recently promoted manager (or a newly hired worker) may wish to quickly create a positive impression on her superiors. Likewise, a manager or a worker who plans to hold her position for only a short while might be impatient (or a short-termist) compared to her counterpart who expects to stay in the same position for a very long time. Furthermore, (unmodeled) baseline contracts in the organization may lead to different intertemporal preferences within the manager–worker interaction. In general, each party can be more or less patient than the other and, to an extent, this can be endogenously determined by the organization's compensation, promotion, and layoff policies.

We characterize the optimal contracts for any pair of the players' discount rates, both for the case where the arrival of opportunities is observable and for the case where opportunities can be concealed by the worker.[6] The optimal contracts generate distinct qualitative features for different specifications of the model that naturally correspond to various "managerial styles." For example, what degree of flexibility do workers enjoy at work and what affects this flexibility? How do the workers' workloads and perks change over time? Do managers generate artificial ranks within their teams or do they maintain equal status among similar workers? If certain perks are granted, are they permanent or temporary?

We begin by considering the case in which the arrival of opportunities is observed by the manager. When the manager is *patient* relative to the worker, the manager treats workers identically, regardless of their past work or tenure. When an opportunity arrives, the manager requires the worker's effort and promises all perks within her discretion for a given amount of time. These promises, however, are "conditional" and do not accumulate: each promise will be nullified upon the arrival of the next opportunity. It is therefore a question not of *how much* but rather *whether* some work was done in the recent past.

When the manager is *impatient*, on the other hand, the interaction features completely different dynamics. Initially, the worker has a *junior* status and is expected to exert maxi-

[6]Our main solution concept is manager-optimal contract. We discuss below why, in our view, this is the appropriate solution concept. In Section 5.5 we argue that the main qualitative results would hold were we to use a relational contracting approach instead.

mal effort on every opportunity that arrives, without enjoying any perks. At some point, the worker moves up to an *intermediate* status, in which he still needs to work whenever an opportunity arrives, but now he also enjoys all the perks at the manager's discretion. Finally, he attains a *senior* status, in which he enjoys the maximal level of perks without exerting any effort (beyond his unmodeled baseline duties). The transition times between the different statuses are (essentially) fixed and do not depend on the actual amount of effort the worker exerted over time. Hence, the manager adopts a "*tenure-based seniority system*" – a substantially different managerial style from that of the patient manager.

Note that the above contracts (both for the patient and the impatient manager) feature very little correlation between work and compensation. In particular, the arrival of an opportunity is typically bad news for the worker. Hence, such contracts cannot be used to incentivize opportunities that the worker can conceal. For the worker to reveal that such an opportunity has arrived, the promise of future compensation must increase by at least the cost of the required effort (and it is clearly not in the manager's interest to over-compensate the worker). Thus, optimal contracts for the case of *concealable* opportunities have the *perfect bookkeeping* property: on every path of play, the discounted value (using the worker's discount rate) of granted perks is equal to the discounted cost of the effort exerted.

As in the case of observable opportunities, the relative patience of the players affects the managerial style when opportunities are concealable. However, the transition between the different managerial styles is more nuanced in the case of concealable opportunities. A seniority system still becomes more likely as the manager becomes more impatient. As a consequence of the perfect bookkeeping property, however, the seniority system in this case is *performance-based* rather than tenure-based. The *junior* and *senior* statuses are similar under both of these seniority systems: junior workers exert effort without receiving any perks, whereas senior workers receive all the perks without exerting any effort when opportunities arrive. The *intermediate* status, on the other hand, has a different structure: when opportunities are concealable, the worker enjoys a *partial* level of perks as a guaranteed baseline, and whenever he exerts effort he enjoys an immediate *temporary* increase in perks.[7] The seniority system is performance-based in that promotion is related directly to the realized amount of work over time.

Another substantial difference is that if opportunities are concealable, the seniority system is adopted only if the manager is *sufficiently* impatient relative to the worker. If the manager is only slightly impatient (or if she is patient), then the worker enjoys perks immediately after exerting effort. Due to the perfect bookkeeping property, these promises

[7] This intermediate status exists so long as the manager is not extremely impatient.

of future compensation accumulate. As for the effort dynamics, the slightly impatient manager will continue to incentivize maximum effort until she completely runs out of incentives. Thus, in this case, the worker eventually stops exerting effort on new opportunities. On the other hand, it is suboptimal for a manager who is more patient than the worker to increase the promises of future compensation without limit. As a result, the required effort on new opportunities is always positive, but it may change nonmonotonically over time.

Table 1 summarizes the qualitative dynamics of incentive provision for varying information structures and relative patience.

Opportunities	*Manager*		
	Patient	**Slightly Impatient**	**Very Impatient**
Observable	Conditional promises (Finite)	Tenure-based seniority system	
Concealable	Accumulating promises (Finite)	Accumulating promises (Infinite)	Performance-based seniority system

Table 1: Incentive Dynamics.

In addition to studying the interaction between the front-line manager and the worker in an organization, our analysis provides a tool for studying broader questions in organizational design. Specifically, it can be considered as an analysis of a "continuation game" in a more general interaction in which the organization first shapes certain organizational regulations. In Section 5, we offer several insights into important organizational problems such as determining managers' discretion, designing layoff and promotion policies, and choosing between professional and technical managers.

Related Literature

This paper contributes to the literature that has provided various explanations for the prevalence of hierarchical structures in large organizations. Williamson (1967) and Calvo and Wellisz (1978,1979) argue that hierarchies arise due to limitations on the number of employees that a manager can effectively control and monitor.[8] Rosen (1982) suggests that hierarchies enable highly talented senior managers to increase the productivity of

[8]Tirole (1986) and Laffont (1988) show that, in this case, hierarchical structures are susceptible to collusion between the workers and the low-level managers that monitor them. More recently, Letina, Liu and Netzer (2020) show that if low-level managers care about their workers' welfare, but do not collude with them outright, then the firm should induce a contest between the workers.

their subordinates. Garicano (2000) and Harris and Raviv (2002) propose the idea that hierarchies enable the efficient utilization of expert knowledge within the firm, whereas Rajan and Zingales (2001) argue that hierarchies can also prevent employees from stealing a firm's core knowledge. Hart and Moore (2005) show that hierarchies can be an efficient method for allocating resources within the firm. See Mookherjee (2013) for an extensive review of this literature.[9]

Two papers that, like ours, assume that the front-line manager is responsible for determining the responsibilities of her workers are McAfee and McMillan (1995) and Melumad, Mookherjee and Reichelstein (1995). These papers differ from ours in that they consider a static problem in which there is no stochasticity in the tasks that the worker must perform, and the manager can offer monetary incentives to the worker. Moreover, they focus on deriving conditions under which adding an intermediate level between the principal and the worker is beneficial.

Our paper also complements works that study the optimal timing of compensation (e.g., Lazear 1981; Carmichael 1983) and, in particular, those that analyze the mixture between short- and long-term incentives in settings where information changes over time (e.g., Sannikov 2008; Garrett and Pavan 2012, 2015). We contribute to this literature by analyzing the efficient use of limited and perishable incentives in a dynamically changing environment.

Our work also contributes to the recent literature on optimal contracting under different discount factors. Opp and Zhu (2015) study relational contracting in a repeated moral hazard setting, Frankel (2016) studies dynamic delegation, Hoffmann, Inderst and Opp (2021) study a one-shot moral hazard problem in which there is a delay in the arrival of information, and Krasikov, Lamba and Mettral (2023) analyze a canonical adverse selection problem. We contribute to this literature by studying a model in which per-period compensation is limited and perishable. These natural features lead to a tractable model of contracting with different discount rates that exhibits rich and realistic dynamics.

Finally, our work is related to the growing literature that studies principal–agent interactions with randomly arriving "projects" under symmetric discounting. Forand and Zápal (2020) and Bird and Frug (2021) consider optimal contracting under symmetric information: Forand and Zápal (2020) study a model with no transfers in which projects of different types arrive randomly over time, whereas Bird and Frug (2021) study a canonical employment model in which the agent's productivity of effort varies over time. Li, Matouschek and Powell (2017), Bird and Frug (2019), and Lipnowski and Ramos (2020)

[9]In the broader context of organizational design, Rantakari (2008) studies how the need to coordinate a firm's activities affects the optimal allocation of decision rights within a two-tier hierarchy.

consider transfer-free environments with asymmetric information. More specifically, Li, Matouschek and Powell (2017) derive the optimal relational contract when the agent has private information on project availability. Bird and Frug (2019) derive the optimal contract under full commitment in a setting where the agent privately observes the stochastic arrival of different types of projects as well as compensation opportunities. Lipnowski and Ramos (2020) characterize efficient equilibria when the agent has private information on project payoffs.

The paper proceeds as follows. In Section 2 we present the model. In Sections 3 and 4 we analyze the cases where opportunities are observable and concealable, respectively. In Section 5 we present some organizational implications of our analysis and discuss the role of selected modeling assumptions. All proofs are relegated to the Appendix.

2 Model

We consider an infinite-horizon continuous-time interaction between a manager (she) and a worker (he), in which opportunities arrive stochastically over time according to a Poisson process with arrival rate $\mu > 0$. The no-effort action, $\alpha = 0$, is always available to the worker. When an opportunity arrives, and only then, in addition to the no-effort action, the worker can exert effort $\alpha \in (0, 1]$.[10] The worker's effort $\alpha \in [0, 1]$ induces a benefit of $\alpha \cdot B$ to the manager and a cost of $\alpha \cdot A$ to the worker, where $B > A > 0$. At each instant, the manager chooses a flow compensation $\varphi \in [0, 1]$. We assume that both the worker's marginal utility from compensation and the manager's marginal cost of compensation are constant and equal to 1.

The players maximize expected discounted payoffs. We denote the worker's discount rate by $r_w > 0$ and focus on the case where there is no fundamental shortage of incentives. That is, we assume that the worker's discounted payoff from setting $\varphi = 1$ indefinitely exceeds his expected discounted cost of full-intensity work, $\alpha = 1$, on all opportunities that arrive, even if one is currently available.[11] Formally,

Assumption 1.

$$A + \frac{\mu A}{r_w} < \frac{1}{r_w}.$$

We denote the manager's discount rate by $r_m > 0$ and refer to the manager as *patient* if $r_m < r_w$ and as *impatient* if $r_m > r_w$.

[10] We discuss the case of storable opportunities in Section 5.4.

[11] Allowing for the opposite inequality would add trivial cases with corner solutions that would not add much of substance but would needlessly impede the exposition.

Solution Concept. Our objective is to capture formal as well as informal arrangements between a manager and worker in a large organization, and also the asymmetry in their commitment abilities. Our solution concept is the manager's optimal contract (assuming the manager has full commitment power and the worker has none). As the interaction between the manager and the worker takes place in an organization, in many cases, not only the manager's but also the organizational reputational concerns contribute to the credibility of promises for future allocation of perks to the worker. Thus, the manager gains credibility beyond that of the self-sustaining promises that never lead to negative continuation payoff (which is the only source of credibility under the relational contracting approach e.g., MacLeod and Malcomson, 1989; Ray, 2002; Levin, 2003).

For example, if as a result of the manager's support, the organization sponsors the worker's enrollment to a long-term training program (e.g., an MBA), it may be almost impossible for the manager to revoke this privilege. Moreover, such a promise is likely to be kept even if later on the manager learns that after completing the program the worker is likely to move to a new position within the organization, or were the manager to be replaced by a new manager.

It is worth emphasizing that due to the manager's limited discretion (i.e., the upper bound on flow compensation) the organizational cost of upholding her promises is likely to be low compared to the organizational reputational cost from breaking them, which makes full commitment on the part of the manager a reasonable approximation. Indeed, in hierarchical organizations, workers are typically allowed to approach higher-level managers in case of a disagreement with their immediate superior.[12]

Information. Throughout the paper we assume that the worker's effort is perfectly observed by the manager, but we vary our assumptions about whether or not she observes the arrival of opportunities. If the manager does observe the arrival of opportunities ("*observable opportunities*"), then a public history h_t specifies for every $s < t$ whether or not an opportunity was available and the worker's choice of effort.[13] On the other hand, if the manager does not observe the arrival of opportunities ("*concealable opportunities*"), then a public history h_t contains only the worker's effort choices. Given the Poisson arrival of opportunities, any private information that the worker has about the availability of opportunities in the past is irrelevant, and so there is no need to keep track of his private information. Hence, to reduce notation and terminology we refer to a public history as a history under both information structures we consider. We denote the set of all

[12]The nature of the compensation tools in our model are present in other applications in which a relational contracting approach may be applicable. In Section 5.5 we argue that the main qualitative features of our results would also hold under a relational contracting approach.

[13]In this case, whether or not the worker observes the arrival of opportunities is immaterial.

histories of length t by H_t and the set of all finite histories by $H = \cup_{t \in \mathbb{R}_+} H_t$.

Contracts. At the beginning of the interaction, the manager specifies a *work schedule*

$$\alpha : H \to [0,1],$$

which assigns a required effort to every history should an opportunity arrive at that history, and she commits to a *compensation policy*

$$\varphi : H \to [0,1],$$

which maps histories into a flow compensation. A pair $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is referred to as a *contract*. Without loss of generality, we assume that after the manager detects a deviation from the work schedule, she provides no compensation indefinitely.[14]

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is *measurable* if, at every history, the worker's continuation utility and the manager's continuation value are well defined. That is, the expectations

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)} \left(\varphi(h_s) - \mu\alpha(h_s)A\right) ds | h_t\right),$$
$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_m(s-t)} \left(\mu\alpha(h_s)B - \varphi(h_s)\right) ds | h_t\right)$$

exist for every $h_t \in H$.

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is *incentive compatible* if it is measurable and, for every $h_t \in H$, it is optimal for the worker to choose $\alpha = \alpha(h_t)$ (conditional on the availability of an opportunity), given the continuation of the contract. Note that if a deviation to a positive effort level is profitable at a given history, then so is a deviation to no effort.[15] Hence, a (measurable) contract is incentive compatible if and only if the worker (weakly) prefers to follow $\alpha(\cdot)$ than to deviate to $\alpha = 0$ at some history. Since the worker can guarantee himself a payoff of zero by never exerting effort, there is no need to impose an explicit individual rationality constraint. In what follows, we restrict attention to incentive-compatible contracts. We state the relevant incentive compatibility constraints formally in Sections 3 and 4.

[14]In what follows, the functions $\phi(\cdot)$ and $\alpha(\cdot)$ will be specified only for histories that are on the path of play.

[15]A deviation to a non-zero level of effort is detected by the manager under both information structures, and so following such a deviation the worker will receive a continuation utility of zero. Since exerting effort is costly for the worker, such a deviation provides the worker with a strictly lower discounted payoff than exerting no effort on the current opportunity and then receiving a nonnegative continuation payoff.

Timing of effort and compensation

In our analysis and discussion of the results, we often compare contracts in terms of the timing of effort/compensation. We use the following relations. A work schedule $\alpha(\cdot)$ *postpones* effort relative to $\alpha'(\cdot)$ at a history h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha'(h_s)|h_t\right) \tag{1}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ.[16] Similarly, a compensation policy $\varphi(\cdot)$ *postpones* compensation relative to $\varphi'(\cdot)$ at h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi'(h_s)|h_t\right) \tag{2}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ. Analogous definitions for *expediting* effort and compensation are obtained by reversing the inequalities in (1) and (2). Note that the above definitions use the worker's discount factor.

The manager-discounted marginal cost of providing the worker with a worker-discounted util t units of time from now is $e^{-r_m t}\frac{1}{e^{-r_w t}} = e^{(r_w - r_m)t}$ and, similarly, the manager-discounted marginal benefit from the worker exerting one worker-discounted util of effort t units of time from now is $\frac{B}{A}e^{(r_w-r_m)t}$. Whether these expressions are increasing or decreasing in t is fully pinned down by whether the manager is patient or impatient. The following observation is implied.

Observation 1.

1. *Expediting compensation and postponing effort are both profitable for a patient manager.*

2. *Postponing compensation and expediting effort are both profitable for an impatient manager.*

3 Observable Opportunities

We begin our analysis by studying the case where the manager observes the arrival of opportunities (e.g., assignments allocated to the front-line manager by her superiors). In this case, any deviation by the worker is detected. Since the worker can guarantee himself a continuation payoff of zero at any point in time, the incentive compatibility constraints

[16] The latter requirement implies that a work schedule does not postpone effort relative to itself.

are given by

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right) ds | (h_t, O, \alpha(h_t))\right) \geq 0 \quad \forall h_t \in H, \; (IC_{obs})$$

where $(h_t, O, \alpha(h_t))$ is the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$.

Patient manager: ***"Have you done anything for me lately?"***

When the manager is patient ($r_m < r_w$), increasing the lag between compensation and effort in a manner that keeps the worker indifferent is detrimental for the manager. It is therefore easy to see that to maximize the gain from the *first* opportunity that arrives, the manager would have to provide the worker with maximal compensation, $\varphi = 1$, immediately after his work, for an interval of time that is just long enough to compensate him for his cost of effort. However, were the manager to do so, it would not be possible for her to provide prompt compensation for any additional opportunities that arrive within that time interval. Hence, the cheapest form of compensation for the first opportunity reduces the potential gain from further opportunities. Thus, a patient manager faces a complex optimization problem where she endeavors to provide timely compensation for the worker's effort on each of the randomly arriving opportunities. The main result of this section shows that the solution to this problem is simple and qualitatively appealing: under the optimal contract, the worker exerts the same effort α^* on all opportunities, and receives a flow compensation of $\varphi = 1$ if he exerted effort in the last S^* units of time.

This form of compensation can be understood as "conditional promises"; following the worker's effort on a given opportunity, the manager promises a fixed periodic compensation for a given time interval, but this promise is nullified upon the arrival of the next opportunity. The randomness in the arrival of the next opportunity implies that the time lag between effort and compensation on a given opportunity varies across histories, which, in turn, implies that the marginal cost of compensation is higher in some histories than in others. However, note that under the compensation policy above, all of the compensation budget (across all histories) within S^* units of time from the arrival of an opportunity is guaranteed to be fully utilized (to provide compensation for effort on one opportunity or another). Hence, it is impossible to reduce the cost of compensation for a given opportunity without increasing, by at least as much, the cost of compensation for other opportunities. The complete nullification of the manager's obligations to the worker upon the arrival of a new opportunity frees incentivization resources precisely when they are needed, and enables the manager to incentivize effort on every opportunity via a new conditional promise. In essence, this compensation method "shifts" compensation budget from histories with few opportunities to those with many opportunities.

To complement the above intuition, one needs to argue that it is optimal for the manager to incentivize the same level of effort on all opportunities, which we establish formally in the proof. Roughly speaking this is a consequence of the fact that a patient manager does not want to accumulate debt to the worker between opportunities.

Assumption 1 implies that it is possible to induce full effort on all opportunities. However, doing so need not be optimal for the manager. To see why this is the case, recall that the manager's cost of providing the worker with one worker-discounted util t units of time in the future is $e^{(r_w - r_m)t}$. As the manager's profit from a util worth of effort exerted by the worker is $\frac{B}{A}$, it follows that the *maximal profitable lag* between compensation and work is T^*, where T^* is defined implicitly by

$$e^{T^*(r_w - r_m)} = \frac{B}{A}$$

if $r_m < r_w$, and is given by $T^* = \infty$ if $r_m \geq r_w$. In optimum, the manager will require the worker to exert the maximal effort that can be incentivized via a conditional promise of length (at most) T^*.

To formally characterize the optimal contract, let σ denote the random amount of time that will pass until the arrival of the next opportunity. Then, we can denote the maximal effort that the worker is willing to exert on an opportunity in exchange for a conditional promise of length T^* (i.e., setting $\varphi = 1$ until either T^* units of time have passed or an opportunity arrives) by

$$\alpha^* \equiv \min\{\frac{1}{A}\mathbb{E}\left(\int_0^{\min\{T^*,\sigma\}} e^{-r_w t} dt\right), 1\}.$$

If $\alpha^* = 1$, providing the worker with a conditional promise of length T^* overcompensates him for his effort. Hence, we will denote by S^* the length of the conditional promise needed to exactly compensate the worker for incurring an effort cost of $\alpha^* A$, i.e.,

$$\alpha^* A = \mathbb{E}\left(\int_0^{\min\{S^*,\sigma\}} e^{-r_w t} dt\right).$$

Note that if $S^* < T^*$ then $\alpha^* = 1$, whereas $\alpha^* < 1$ implies $S^* = T^*$. Finally, let $\sigma_{-1}(h_t)$ denote the supremum of opportunity arrival times along h_t. In all subsequent results, equalities and uniqueness statements should be interpreted as holding almost surely.

Proposition 1. *Assume that the manager observes the arrival of opportunities. If $r_m \leq r_w$, then*

$$\alpha(h_t) = \alpha^* \quad ; \quad \varphi(h_t) = \begin{cases} 1 & \text{if } t - \sigma_{-1}(h_t) \leq S^* \\ 0 & \text{if } t - \sigma_{-1}(h_t) > S^* \end{cases}$$

is an optimal contract. Moreover, this is the unique optimal contract if $r_m < r_w$.

When $r_w = r_m$ there are multiple optimal contracts. Intuitively, in this case, postponing compensation does not alter the manager's value or violate any of the worker's incentive compatibility constraints. Thus, any contract that results from postponing compensation relative to the optimal contract characterized in Proposition 1 is also optimal. Assumption 1 implies that if $r_w = r_m$, then under the optimal contract described in Proposition 1, $\alpha^* = 1$ and $S^* < \infty$. Hence, postponing compensation is feasible.

Impatient manager: *"Tenure-based seniority"*

By Observation 1, postponing compensation and expediting effort (according to the worker's discount factor) are both profitable for an impatient manager. The first observation underlying the characterization in this section is that neither postponing compensation nor expediting effort violates incentive compatibility. A direct implication of this is that *within* each history, the worker will exert full effort on opportunities that arrive before some (history-dependent) date and enjoy compensation from some other date onward. However, it turns out that the manager is able to postpone compensation and expedite effort *across* histories as well.

We now build an intuition for why an impatient manager uses a tenure-based seniority system in which the clock begins at the arrival time of the first opportunity. Assume for a moment that an opportunity is available at the beginning of interaction. Clearly, it is optimal for the impatient manager (and feasible by Assumption 1) to incentivize maximal effort on that opportunity. Consider an arbitrary incentive-compatible contract $\mathcal{C}$ that does so, and denote by X and Y the worker's future discounted expected effort and compensation, respectively. Incentive compatibility of $\mathcal{C}$ implies that $Y \geq X + A$. Define τ_X^O implicitly by

$$\int_0^{\tau_X^O} \mu A e^{-r_w t} dt = X,$$

and similarly define τ_Y^C by

$$\int_{\tau_Y^C}^{\infty} e^{-r_w t} dt = Y.$$

Note that, after working on the first opportunity, the worker is indifferent between the continuation of $\mathcal{C}$ and the modified continuation contract where: i) he is required to exert full effort on all opportunities that arrive before time τ_X^O and no effort afterwards, and ii) he will receive no compensation until τ_Y^C and maximal compensation thereafter. Moreover, the modified continuation contract is incentive compatible due to the Poisson arrival of opportunities and the fact that $Y \geq X + A$.

Since this modification of the contract expedites effort and postpones compensation relative to $\mathcal{C}$, it is profitable for the manager. Hence, if the interaction begins with an available opportunity, the optimal contract must have a threshold structure as described above.

The relation between the two thresholds under the optimal contract, τ^O and τ^C, can be identified by two simple optimality conditions. First, because compensation is postponed and effort is expedited, the only relevant incentive compatibility constraint is the one at $t = 0$ (the arrival time of the first opportunity). As compensation is costly to the manager, in optimum, this constraint will be binding

$$A + \int_0^{\tau^O} \mu A e^{-r_w t} dt = \int_{\tau^C}^{\infty} e^{-r_w t} dt. \tag{3}$$

Second, note that $\tau^O > \tau^C$ as otherwise the manager can require the worker to exert effort at τ^O and provide compensation at the (weakly) later time of τ^C. This modification is profitable for an impatient manager. Indeed, in optimum, $\tau^O - \tau^C$ is such that the net surplus generated by effort is offset by the manager's relative impatience over $\tau^O - \tau^C$ units of time,

$$\frac{B}{A} = e^{(r_m - r_w)(\tau^O - \tau^C)}. \tag{4}$$

Returning to the original interaction (which does not begin with an opportunity), it is straightforward that providing compensation to the worker before he exerts effort for the first time is suboptimal. Hence, in the optimal contract, "the clock is set to zero" at the arrival of the first opportunity. To characterize the optimal contract formally, let $\sigma_1(h_t)$ denote the infimum of the arrival times of opportunities along the history h_t.

Proposition 2. *Assume that the manager observes the arrival of opportunities. If $r_m > r_w$, then the unique optimal contract is*

$$\alpha(h_t) = \begin{cases} 1 \textit{ if } t \leq \sigma_1(h_t) + \tau^O \\ 0 \textit{ if } t > \sigma_1(h_t) + \tau^O \end{cases} \quad \textit{and} \quad \varphi(h_t) = \begin{cases} 0 \textit{ if } t \leq \sigma_1(h_t) + \tau^C \\ 1 \textit{ if } t > \sigma_1(h_t) + \tau^C \end{cases},$$

where τ^C, τ^O are the unique solution to (3) *and* (4).

Proposition 2 suggests that tenure-based seniority systems arise naturally if the front-line manager is impatient. Under the contract characterized in the proposition, initially, the worker exerts effort on all opportunities but does not receive compensation; at some point ($\sigma_1 + \tau^C$), he begins to receive compensation but still has to work whenever an opportunity arrives; and finally (at $\sigma_1 + \tau^O$), he receives compensation without being required to exert further effort.

Furthermore, Proposition 2 shows that as the manager becomes more impatient relative to the worker, seniority is attained more quickly and the worker's expected effort decreases. To see this, note that the optimality condition (4) can be written as

$$\frac{B}{A} e^{(r_w - r_m)\tau^O} = e^{(r_w - r_m)\tau^C}, \tag{5}$$

and since $\tau^O > \tau^C$, the derivative, with respect to r_m, of the LHS of (5) is less than that of the RHS. As (3) implies that τ^C is decreasing in τ^O, it follows that increasing r_m will lead to a decrease in τ^O (and an increase in τ^C).

Propositions 1 and 2 are visualized in Figure 1 for a typical sequence of opportunity arrivals. The upper panel depicts the worker's effort (in red) and compensation (in blue) when the manager is patient, and the lower panel depicts the same outcomes when the manager is impatient.

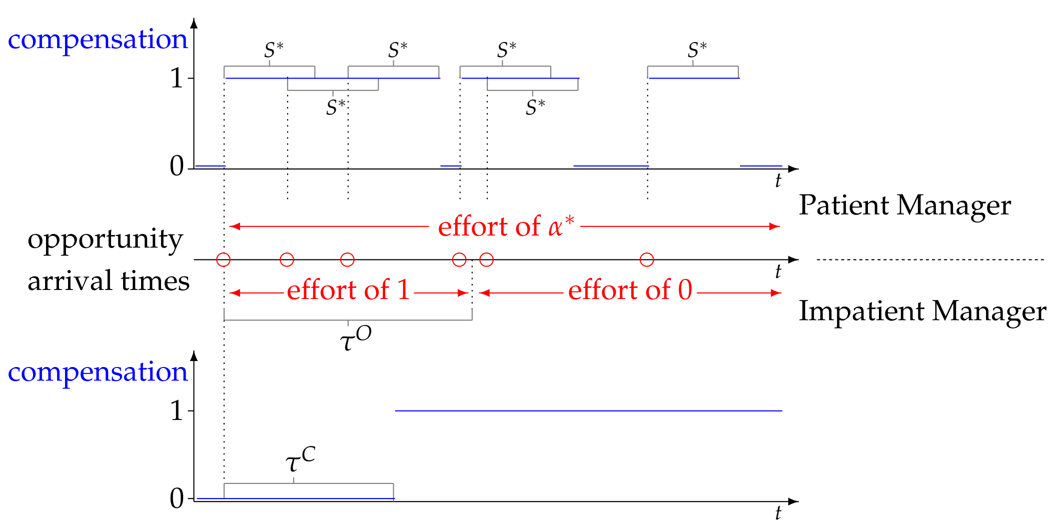

Figure 1: Qualitative dynamics of effort and compensation for patient and impatient managers for a representative sequence of opportunity arrivals.

4 Concealable Opportunities

In the optimal contracts derived in the previous section, the arrival of opportunities typically leads to an immediate decrease in the worker's continuation utility. In settings where the arrival of an opportunity is observed only by the worker, such contracts are not incentive compatible as the worker prefers to conceal opportunities from the manager. To provide incentives for the worker to reveal when opportunities become available, the arrival of opportunities must never be "bad news" for the worker. The incentive compatibility constraints for the case of concealable opportunities are

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq$$

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, N)\right) \quad \forall h_t \in H, \qquad (IC_{conc})$$

where (h_t, N) denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity does not arrive; and, as before, $(h_t, O, \alpha(h_t))$ denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$ on that opportunity.

The need to make the arrival of opportunities not bad news for the worker suggests that when opportunities are concealable, the manager must keep track of both the worker's compensation and his effort. This, in turn, suggests that the optimal contract should depend on the worker's continuation utility. It is well known in the dynamic contracting literature that if the environment is stationary, then the agent's continuation utility can be used as a state variable for deriving the optimal contract (see Spear and Srivastava 1987 and Thomas and Worrall 1990). The argument behind this useful result relies on the property that the continuation payoffs of an efficient contract must always lie on the constrained Pareto frontier. This, in turn, follows from two simple observations: first, if the agent receives a continuation utility u via an inefficient continuation contract, then the principal can increase her value by replacing that continuation contract with a different one that provides the agent with u utils; second, since the agent is indifferent between the original and modified continuations of the contract, this change has no impact on earlier incentive compatibility constraints. Notice that these observations do not depend on the assumption that the players share the same discount rate and, thus, they are valid in our model where the players use different discount factors.

We denote, respectively, by $\alpha(u)$, $\varphi(u)$, and $V(u)$ the Markovian work schedule, the Markovian compensation policy, and the manager's value as a function of the worker's continuation utility, u. Notice that $u \in [0, \frac{1}{r_w}]$ as, from any point in time onward, the worker can guarantee himself a nonnegative payoff by exerting no effort, and his payoff from receiving the maximal compensation indefinitely is $\int_0^\infty 1 \cdot e^{-r_w t} dt = \frac{1}{r_w}$. The Markovain value function satisfies two properties that help derive the optimal contract:

Lemma 1. *$V(u)$ is strictly decreasing and weakly concave.*

Lemma 1 has two important consequences. First, it directly implies that the worker's expected utility from an optimal contract is zero. Second, it implies that if opportunities are concealable, then the incentive compatibility constraint at every history is binding regardless of the relative patience of the players.

Corollary 1. *Assume that opportunities are concealable. Under an optimal contract:*

1. *The worker's expected utility is zero.*

2. *All the incentive compatibility constraints are binding.*

Corollary 1 reveals a property of the managerial style of a front-line manager that does not observe the arrival of opportunities: she engages in *perfect bookkeeping* wherein, path by path, compensation and effort discounted according to the worker's discount rate are equal. This is in contrast to the low correlation between work and compensation when opportunities are observable.

When opportunities are concealable, it is convenient to describe the optimal contract via its Markovian representation. By Corollary 1, the worker's continuation utility at the beginning of the interaction is zero and after he exerts effort $\alpha(u)$ his continuation utility increases by $\alpha(u)A$. Moreover, the drift in the worker's continuation utility while no opportunities arrive is

$$du = r_w u - \varphi(u). \tag{6}$$

Hence, the optimal contract is characterized by the solution of the HJB equation:

$$\sup_{\varphi(u), \alpha(u) \in [0,1]} \{ -r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ + \mu \left(\alpha(u)B + V\left(u + \alpha(u)A\right) - V(u) \right) \} = 0, \quad (HJB)$$

subject to (6), where $V'(u)$ exists almost everywhere since $V(\cdot)$ is concave (Lemma 1). The following is the main result of this section.

Proposition 3. *If opportunities are concealable, then the optimal contract is generically unique. Moreover, there exist thresholds $u^C, u^O \in [0, \frac{1}{r_w}]$, such that the optimal contract is given by*

$$\alpha(u) = \min\{1, \frac{u^O - u}{A}\} \quad ; \quad \varphi(u) = \begin{cases} 1 & \text{if } u > u^C \\ r_w u & \text{if } u = u^C \\ 0 & \text{if } u < u^C \end{cases}.$$

The threshold u^O dictates the dynamics of work. The threshold value $u^O = \frac{1}{r_w}$ corresponds to a work schedule in which the manager instructs the worker to fully exploit every opportunity that arrives until all of her compensation budget is exhausted. For lower values of u^O, the manager will sometimes forgo opportunities even though her compensation budget is not exhausted.

The threshold u^C dictates the dynamics of compensation. In particular, it captures the degree of back/front-loading of compensation. So long as the worker's continuation utility is below u^C, compensation is deferred to the future. Setting the compensation threshold at the maximal possible value, $u^C = \frac{1}{r_w}$, corresponds to full back-loading: when the worker's continuation utility reaches that level, it is necessary to set $\varphi = 1$ indefinitely. At the other extreme, the compensation threshold $u^C = 0$ corresponds to full front-loading because, in this case, the manager provides the maximal compensation whenever the worker's promised continuation utility is positive.

For $u^C \in (0, \frac{1}{r_w})$, the compensation dynamics consists of two phases. In the beginning, the *back-loading phase* takes place. So long as $u < u^C$, the worker exerts effort and accumulates promises of future compensation but does not receive any compensation. When his continuation utility attains (or exceeds) u^C, the *front-loading phase* begins. In this phase, the worker receives a permanent base compensation of $r_w u^C$ – which can be thought of as compensation for effort exerted in the earlier back-loading phase – and a temporary additional compensation of $1 - r_w u^C$ whenever $u > u^C$. Observe that if $u = u^C$, then the base compensation of $r_w u^C$ maintains the worker's continuation utility constant at that level. Hence, once the worker's continuation utility reaches u^C, it never drops below this level again, which, in turn, establishes that the optimal arrangement of compensation begins with the back-loading of compensation and transitions to the front-loading of remaining compensation budget.

The Effect of Relative Patience

The relative patience of the players determines the values of the thresholds u^C and u^O. For the next result denote by $u(T^*) = \int_0^{T^*} e^{-r_w t} dt$ the maximal compensation that can be promised without exceeding the maximal profitable lag between effort and compensation.

Proposition 4. *Assume that opportunities are concealable. If $r_m < r_w$, then the thresholds of an optimal contract defined in Proposition 3 satisfy $u^O \in (0, u(T^*))$ and $u^C = 0$.*

Proposition 4 implies that compensation is provided in the form of "accumulating promises": following the worker's effort on a given opportunity, the manager promises the maximal compensation for a given time interval (which starts at the present moment); to compensate the worker for additional effort, the length of the existing promise is extended. Hence, after the worker has performed a lot of work in a short time interval, requiring additional effort necessitates providing compensation in the distant future. The patient manager will clearly avoid promises greater than $u(T^*)$ as those create negative marginal value. To see why it is indeed optiaml to set $u^O < u(T^*)$, note that increasing the promise all the way to $u(T^*)$ corresponds to a marginal profit of zero. Hence, if u is near enough to $u(T^*)$ it is profitable for the manager to reserve her compensation budget for future opportunities at which time the lag between effort and compensation will be shorter and the marginal profit will be strictly positive.

This feature differentiates between the observable and concealable opportunities cases. In the former, the manager requires the worker to exert a constant effort on all opportunities, whereas in the latter case the required effort fluctuates over time.

Next, we consider an impatient manager. The threshold u^C is determined by the manager's benefit from postponing compensation. Postponing compensation has two effects. First, it reduces the manager's discounted cost of providing compensation, and second, it reduces her ability to incentivize future effort. Intuitively, the threshold u^C balances the cost and benefit from postponing compensation. The next result shows that a slightly impatient manager will fully front-load compensation, a moderately impatient manager will create a seniority system that exhibits a combination of front- and back-loading of compensation, and full back-loading occurs only if the manager is extremely impatient.

Proposition 5. *Assume that opportunities are concealable. Fix A, B, μ, and r_w and suppose that $r_m \geq r_w$. The thresholds of an optimal contract defined in Proposition 3 satisfy*

- $u^O = \frac{1}{r_w}$.
- *There exists $r''_m > r'_m > r_w$ such that* $u^C \in \begin{cases} \{0\} & \text{if } r_m < r'_m \\ (0, \frac{1}{r_w} - A] & \text{if } r_m \in (r'_m, r''_m) \\ \{\frac{1}{r_w}\} & \text{if } r_m > r''_m \end{cases}$.

The optimal contract offered by an impatient manager requires the worker to exert effort for as long as it is possible to incentivize him to do so. Hence, due to perfect book-keeping, the worker will eventually stop exerting effort.

A slightly impatient manager ($r_w < r_m < r'_m$) could reduce the direct cost of compensation by deferring it to the future. However, due to the perishability of her compensation budget, she prefers to absorb the higher direct cost of providing compensation immediately in order to free up future compensation resources.

A sufficiently impatient manager ($r_m > r'_m$) will establish a system of seniority, albeit a more nuanced one than the tenure-based seniority system described in Section 3. If $r_m \in (r'_m, r''_m)$, the manager will institute a seniority system with three levels: juniors that work and don't receive compensation, intermediates that work and receive compensation, and seniors the receive compensation but don't work.[17] However, this seniority system is performance-based in the sense that "promotion" times depend on the entire history. In particular, the worker attains intermediate status when his continuation utility reaches u^C for the first time, and he reaches senior status when his continuation utility reaches the absorbing state of $\frac{1}{r_w}$. Moreover, the worker's compensation while he is of intermediate status oscillates between $r_w w^C$ and 1 according to the random arrival of opportunities.

Propositions 4 and 5 are visualized in Figure 2. As can be seen in Figure 2 (and Proposition 5) there is a discontinuity in the optimal compensation threshold at $r_m = r''_m$. This discontinuity, as well as the kinks and abrupt changes in curvature of u^c (as a function of r_m), is due to the discrete arrival of opportunities. This effect of discreteness becomes more dominant as the number of opportunities needed to reach the absorbing state of $u = \frac{1}{r_w}$ from u^C becomes small.

[17] If $r_m > r''_m$, the intermediate status does not exist.

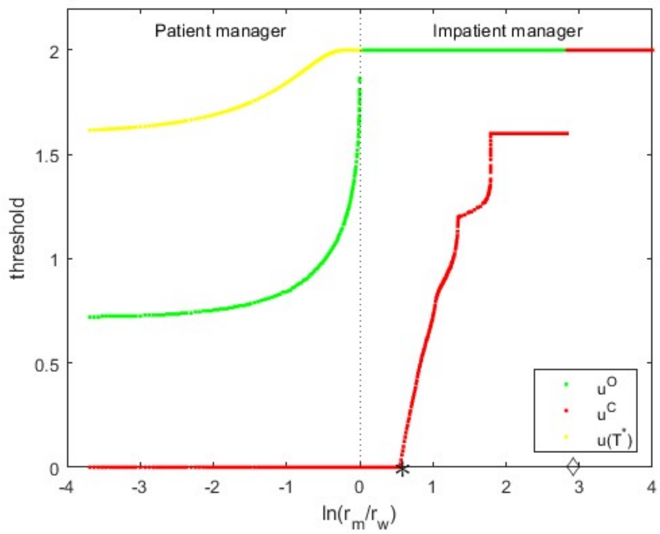

Figure 2: u^O, u^C, and $u(T^*)$ as a function of $ln(\frac{r_m}{r_w})$, for $A = \frac{2}{5}, B = 2, r_w = \frac{1}{2}$, and $\mu = 2$. The middle of the x-axis corresponds to $r_m = r_w$. The $*$ symbol corresponds to the value of r'_m, and the $\diamondsuit$ symbol corresponds to the value of r''_m.[18]

5 Discussion and Organizational Implications

5.1 Interdependence of Contracts within an Organization

Our analysis shows that evaluating individual contracts in isolation may be insufficient as the relative properties of workers' and managers' respective contracts affect the dynamics of their interaction. For example, if these contracts lead to different intertemporal incentives, then a manager who is relatively more patient than the worker would increase the worker's long-run effort, whereas a relatively impatient manager would improve the worker's retention rate through the adoption of a seniority system.

In addition, a significant part of employees' incentives is often provided through promotion opportunities rather than salary, as such compensation may have a lower direct cost for the organization (see, e.g., Holmström 2017). Such promotions affect the relative patience within the manager–worker interaction. Consequently, our model highlights novel indirect costs and benefits of such multidimensional compensation packages through their effect on how managers complement the (incomplete) contracts of their workers.

[18]The optimal thresholds were derived via Monte Carlo simulations. Note that on the extreme right of the figure the green dots are obscured by the red ones as $u^C = u^O = \frac{1}{r_w}$ when r_m is sufficiently large.

5.2 Volatility of Opportunity Arrival and Managerial Discretion

The allocation of authority within organizations is of prime importance in organizational design. Managerial discretion may vary across organizations due to variation in production processes and business environments. In our model, the manager's level of discretion is represented by a fixed cap on the periodic compensation (normalized to one). In a more general model, this cap could be selected optimally by the organization based on the characteristics of the manager–worker interaction, and the cost of providing such discretion.

The interests of the manager and the organization are typically not identical. However, since the organization designs the incentives of the front-line managers to represent its own interests, it is plausible to assume that, in some cases, the interests of the manager and the organization are sufficiently close. The difference, in such cases, between the manager and the organization is that the manager takes her level of discretion as given, whereas from the perspective of the organization this level is a choice variable, which may be associated with some organizational costs that are not taken into account by the manager.

It is intuitive that as opportunities become larger (while their arrival rate is kept constant) or more frequent (while their size is kept constant), the manager should receive more discretion. It is less clear how the *volatility* of the opportunity arrival process impacts the desired level of discretion.

An important step in answering this question is to analyze the effect of volatility on the manager's ability to effectively utilize a *given* level of managerial discretion. We compare (using the manager's value) settings where the frequency and magnitude of opportunities vary, while the total value of expected opportunities stays constant. Formally, we say that the opportunities represented by (A_1, B_1, μ_1) are *lumpier* than those represented by (A_0, B_0, μ_0) if there is $\lambda > 1$ such that $A_1 = \lambda A_0, B_1 = \lambda B_0$, and $\mu_1 = \frac{\mu_0}{\lambda}$. That is, opportunities are lumpier if their arrival process is more volatile (holding the expectation fixed).

In the special case where $r_w = r_m$ and opportunities are observable, the lumpiness of opportunities does not impact the manager's value, so long as Assumption 1 continues to hold. This follows from three observations. First, since $r_w = r_m$, Assumption 1 implies that, under the contract characterized in Proposition 1, the worker exerts full effort on all opportunities. Second, recall that the worker's expected utility from that contract is zero. Finally, $r_w = r_m$ also implies that the timing of compensation does not affect the manager's cost of providing compensation.

However, even though common discounting is the standard assumption in the literature, the irrelevance of the degree of lumpiness in this case is a knife-edge result. In general, lumpiness is detrimental to the manager.

Proposition 6. *The manager's value strictly decreases when opportunities become lumpier, unless opportunities are observable and* $r_m = r_w$.

Proposition 6 shows that it is harder for a manager who operates in a more volatile environment to complement the worker's contract. This insight may have implications on the allocation of managerial discretion in organizations.

5.3 Value of Information Frictions

Assume that both the front-line manager and the worker are less patient than the organization. If the manager is more patient than the worker, the intertemporal preferences of the manager and the organization are generally aligned: they both prefer to front-load the worker's compensation and value his future effort.

Consider what happens as the manager becomes less patient. If the arrival of opportunities is observable, the managerial style changes discontinuously around the focal point where the worker and manager discount the future identically: once the manager becomes even slightly impatient relative to the worker, she fully back-loads compensation and front-loads effort, which is exactly the opposite of the organization's preferences. By contrast, if opportunities are concealable, the managerial style of a slightly patient manager is similar to that of a slightly impatient one.

Example 1 demonstrates that if the manager is slightly impatient, then information frictions can be beneficial for the organization.

Example 1. *Assume that* $A = 1, B = 1.2, \mu = 0.5, r_m = 0.11, r_w = 0.1$, *and that the organization discounts future payoffs at a rate of* $r_o = 0.09$. *The value for the organization is calculated by solving the manager's problem, and evaluating the resulting streams of effort and compensation according to the discount rate* r_o. *For these parameter values, the organization value is higher under concealable opportunities: if the arrival of opportunities is observable, the organization's value is* 0.477, *while if opportunities are concealable, the organization's value is* 1.031.

When the organization is the most patient player, there is a discrete drop in its profits when the manager becomes slightly less patient than the worker and opportunities are observable. To understand the discontinuity, recall that, in this case, there is a discrete change in the induced streams of effort and compensation around $r_m = r_w$. As the manager selects the worker's contract optimally, Berge's theorem of the maximum implies that *her* value is continuous in r_m, despite the discrete change in managerial style. This is not true for the organization that evaluates the induced streams of effort and compensation using a different discount rate. Indeed, as the organization is strictly more patient than the manager, its value drops discontinuously due to the change in managerial style. By contrast, when opportunities are concealable, the managerial style itself, and hence the payoffs of both the manager and the organization, are continuous around $r_m = r_w$.

Example 1 also demonstrates how significant the drop in the organizational value around $r_m = r_w$ can be. Specifically, it shows that the magnitude of this drop may be greater than the loss of value due to information frictions between the manager and worker.

One interpretation of the example is that, under some circumstances, the organization may prefer to hire professional managers that lack the technical expertise needed to observe the arrival of opportunities, even if they do not possess superior managerial skills.[19] In addition, this result may suggest that organizations may benefit from imposing certain constraints on the manager's ability to "punish" workers for not exerting extra effort. Such restrictions, in essence, turn observable opportunities into concealable ones.

5.4 Storable Opportunities

The focus of our analysis was on applications in which opportunities are wasted if they are not acted upon immediately. However, our analysis is also relevant for some applications where it is possible to store opportunities.

If the manager is impatient, expediting effort is profitable. Hence, storing opportunities is suboptimal for an impatient manager.

By contrast, if the manager is patient, it is profitable for her to postpone effort and reduce compensation accordingly since it reduces the time lag between effort and compensation. Observe that requiring effort in t units of time in exchange for a util of compensation provided at the same time generates a current benefit of $e^{-r_m t}\frac{B}{A}$ for the manager. Whereas requiring immediate effort in exchange for a util of compensation provided in t units of time generates a current benefit of $e^{-r_w t}\frac{B}{A} < e^{-r_m t}\frac{B}{A}$. Intuitively, by requiring immediate effort (rather than storing the opportunity) the manager is, in essence, using the worker's higher discount rate to discount future benefits.

This suggests that if opportunities are perfectly strorable a patient manager would want to slice arriving opportunities into infinitesimal bits, and have the worker exert a

[19] In fact, it is easy to construct examples in which the front-line manager will generate profit for the organization only if she does not observe the arrival of opportunities.

constant flow effort in return for immediate compensation. However, as opportunities oftentimes represent random events that require immediate action, they may depreciate or become obsolete over time. Indeed, if stored opportunities depreciate at a rate greater than $r_w - r_m$, then the cost of depreciation outweighs the manager's gain from reducing the time lag between effort and compensation. Hence, our analysis holds also for applications in which storing opportunities is sufficiently costly.

5.5 Relational Contracting

In our main analysis, the manager gains credibility from the discipline imposed by the organization within which the worker–manager interaction takes place. Even outside the context of large organizations, e.g., in simple two-layer principal-agent hierarchies, a principal may utilize compensation tools similar to the ones studied above due to efficiency considerations. In such settings, it may be reasonable to restrict attention to self-sustaining promises. The main qualitative insights from our full contracting approach would remain valid under such a relational contracting approach. That is, they would remain valid if we were to add a restriction that the manager's continuation value must be nonnegative.

If the manager is patient, then, under the optimal contracts characterized in Propositions 1 and 4, she requires effort on all opportunities and refrains from accumulating a large debt to the worker. Hence, in most cases of interest, i.e., when players are not too myopic and the profit from opportunities is not too low, the optimal contracts characterized above are also relational contracts. Even if these conditions do not hold, the optimal relational contracts are identical to the optimal contracts, apart from inducing slightly lower effort requirements.

If the manager is impatient, she is unable to use the seniority systems that arise under the optimal contracts characterized in Propositions 2 and 5, as they induce a senior status in which workers enjoy compensation without exerting effort. Under observable opportunities, the optimal relational contract still consists of a three-tier tenure-based promotion system, albeit one in which senior workers are required to exert a (low) constant level of effort. Under concealable opportunities, the optimal relational contract generates a two-tier performance-based promotion system, in which juniors exert full effort and do not receive compensation, and non-juniors receive compensation and exert some effort on all opportunities.

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A Appendix: Proofs

Proof of Proposition 1.

Providing the worker with compensation before he exerted effort is suboptimal. In addition, the worker will not receive compensation if he has not exerted effort in the last T^* units of time since providing compensation outside the maximal profitable lag is suboptimal. Hence, under an optimal contract $\varphi(h_t) = 0$ if $t - \sigma_{-1}(h_t) \geq T^*$.

The first step in the proof establishes that it is suboptimal for the manager to require effort on a given opportunity that will necessitate providing compensation after the next opportunity arrives.

Lemma A.1. *Under an optimal contract $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$.*

Proof. Assume by way of contradiction that under an optimal contract $\alpha(h_t) > \alpha^*$ for a set of histories with positive measure. Since $\alpha^* = 1$ if $r_w = r_m$, within this lemma we consider only the case where $r_m < r_w$.[20] Denote by ν the set of finite histories (of various lengths) such that for each $h \in \nu$, and every h' that is a (proper) prefix of h, $\alpha(h) > \alpha^*$ and $\alpha(h') \leq \alpha^*$. Note that the contract reaches a history in ν with positive probability and that if $h \in \nu$ and $\tilde{h} \in \nu$, then neither history is a prefix of the other. Thus, it is sufficient to construct a profitable modification of the continuation contract conditional on an opportunity arriving at every $h \in \nu$.

Fix $h_t \in \nu$ and assume that an opportunity is available. A conditional promise of length T^* is not enough to compensate the worker for exerting an effort of more than α^* at h_t. Hence, with positive probability some of the compensation for the effort exerted at h_t must be provided after the arrival of the next opportunity. Formally, there exists a set

[20] If $r_w = r_m$, then $T^* = \infty$, and so it is both profitable and feasible (Assumption 1) to incentivize full effort on all opportunities.

$\nu_1(h_t)$ of continuation histories of h_t (of various lengths) with positive measure such that for every $h_s \in \nu_1(h_t)$ opportunities do not arrive in (t, s), and the worker's continuation utility conditional on an opportunity arriving at h_s is strictly greater than $A \cdot \alpha(h_s)$.

If there exists $\tilde{\nu} \subset \nu_1(h_t)$ with positive measure (conditional on reaching h_t) such that $\alpha(h_s) < 1$ for every $h_s \in \tilde{\nu}$, then postponing effort (according to the worker's discount factor) from h_t to the histories in $\tilde{\nu}$ is strictly profitable for the (patient) manager (Observation 1) and does not violate incentive compatibility.

If, on the other hand, $\alpha(h_s) = 1$ for almost all $h_s \in \nu_1(h_t)$, then for every such h_s there exists a set of continuation histories (of various lengths) with positive measure (conditional on reaching h_s) $\nu_2(h_s)$ such that for every $h_{s'} \in \nu_2(h_s)$: 1) no opportunities arrive in (s, s'), and 2) the worker's continuation utility if an opportunity arrives at $h_{s'}$ is greater than his continuation utility at h_s by at least $(1 - \alpha^*)A$. To see why such histories exist, recall that $\varphi = 0$ if no opportunity arrived in the last T^* units of time, and so the maximal worker-discounted expected compensation that can be provided between two successive opportunities is $\alpha^* A$. Hence, to compensate the worker for exerting an effort of 1 on the opportunity at h_s, at least $(1 - \alpha^*)A$ of this compensation is provided after the next opportunity arrives with positive probability.

If there exists $\tilde{\nu} \subset \nu_2(h_s)$ with positive measure (conditional on reaching h_s) such that $\alpha(h_{s'}) < 1$ for every $h_{s'} \in \tilde{\nu}$, then postponing effort from h_s to the histories in $\tilde{\nu}$ does not violate incentive compatibility. Moreover, if effort can be postponed in this manner from a subset of $\nu_1(h_t)$ with positive measure, then doing so increases the manager's value at h_t. Otherwise, for almost all $h_s \in \nu_1(h_t)$, it holds that $\alpha(h_{s'}) = 1$ for almost all $h_{s'} \in \nu_2(h_s)$. In this case, with strictly positive probability the worker's continuation utility at $h_{s'}$ is greater than his continuation utility at h_t by at least $(1 - \alpha^*)2A$. Continuing in an iterative manner shows that the manager's value can be increased by postponing effort, as otherwise the worker's continuation utility increases without bound with positive probability (which cannot be the case as it is bounded from above by $\frac{1}{r_w}$). ■

The next part of the proof establishes that, under an optimal contract, the manager will require the maximal effort that can be required without accumulating debts or exceeding the maximal profitable lag.

Lemma A.2. *Under an optimal contract $\alpha(h_t) = \alpha^*$ for almost all $h_t \in H$.*

Proof. By Lemma A.1, $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$. Assume by way of contradiction that under an optimal contract $\alpha(h_t) < \alpha^*$ on a set of histories with positive measure. Let n be the minimal element of $\mathbb{N}$ for which the worker's effort on the n^{th} opportunity to arrive is st rictly less than α^* with positive probability. Let ν denote the set of histories (of various lengths) that end at the arrival of the $n+1-$th opportunity such that the required effort on the n-th opportunity is strictly less than α^*. Finally, for $k \in \{1, \ldots, n\}$, define $NOC(k)$ to be the set of prefixes of histories in ν along which exactly k opportunities have arrived and the $k-$th opportunity arrived at most S^* units of time ago. We say that maximal compensation is provided in $NOC(k)$ if $\varphi(h) = 1$ for almost all $h \in NOC(k)$.

By construction, maximal compensation in $NOC(k)$ for all $k \in \{1, \ldots, n\}$ is sufficient to incentivize effort α^* on the first n opportunities to arrive. Hence, if maximal compensation is provided in $NOC(k)$ for all $k \in \{1, \ldots, n\}$, compensation can be decreased in $NOC(n)$ without violating any of the incentive-compatibility constraints. Otherwise, there exists a maximal $\bar{k} \leq n$ such that maximal compensation is not provided in $NOC(\bar{k})$. If $\bar{k} = n$, an improvement can be reached by increasing the required effort on the n-th opportunity and increasing the compensation within $NOC(n)$, without affecting any of the earlier or later IC constraints. Finally, if $\bar{k} < n$, since the required effort on opportunity $\bar{k}$ is α^*, some compensation for effort on that opportunity must be postponed until after future opportunities have arrived. Hence, the IC constraint at the arrival of the $\bar{k}+1$ opportunity is not binding with positive probability. Since maximal compensation is provided within $NOC(\bar{k}+1)$, the principal can reach an improvement by expediting compensation from $NOC(\bar{k}+1)$ to $NOC(\bar{k})$. ■

To conclude the proof, note that the contract described in the proposition is the only incentive compatible contract under which $\alpha(\cdot) \equiv \alpha^*$ and the worker does not receive compensation if he has not exerted effort in the last S^* units of time. ■

Proof of Proposition 2. When opportunities are observable, the manager's problem can be solved separately for each possible arrival time of the first opportunity. This is because the manager will not provide compensation prior to the first opportunity, and the worker must have a nonnegative continuation utility at all times.

Consider an arbitrary first arrival time σ_1. By Assumption 1 the manager can incentivize the worker to exert maximal effort on the opportunity at σ_1. As the manager is impatient, she will do so in an optimal contract. Moreover, as we established in the main text, the manager will use a threshold structure. Hence, the manager's objective function (conditional on σ_1) is

$$\max_{\tau^O(\sigma_1), \tau^C(\sigma_1)} e^{-r_m \sigma_1} B + \int_{\sigma_1}^{\sigma_1 + \tau^O(\sigma_1)} \mu B e^{-r_m t} dt - \int_{\sigma_1 + \tau^C(\sigma_1)}^{\infty} e^{-r_m t} dt$$

$$\text{s.t. } A + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu A e^{-r_w t} dt = \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_w t} dt. \quad (7)$$

Assumption 1 implies that, in optimum, both $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are interior. To see this, note that the constraint (7) is violated if $\tau^C(\sigma_1) = \infty$ or $\tau^C(\sigma_1) = 0$. Furthermore, setting $\tau^O(\sigma_1) = 0$ implies that $\tau^C(\sigma_1) > \tau^O(\sigma_1)$, and so by slightly increasing $\tau^O(\sigma_1)$ (and decreasing $\tau^C(\sigma_1)$ to maintain incentive compatibility) the worker will exert more effort on opportunities for which he will receive compensation after he has exerted effort. As the manager is impatient, this change is profitable. Finally, setting $\tau^O(\sigma_1) = \infty$ implies that the worker continues exerting effort for an arbitrarily long period of time after he begins receiving compensation. Because the manager is impatient, slightly increasing $\tau^C(\sigma_1)$ (and decreasing $\tau^O(\sigma_1)$ to maintain incentive compatibility) is profitable.

The above discussion implies that the optimal thresholds are given by the FOC of the Lagrangian that corresponds to the above (concave) maximization problem. The first-order conditions with respect to $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are, respectively,

$$\mu B e^{-r_m \tau^O(\sigma_1)} - \gamma(\sigma_1) e^{-r_w \tau^O(\sigma_1)} \mu A = 0$$
$$e^{-r_m \tau^C(\sigma_1)} - \gamma(\sigma_1) e^{-r_w \tau^C(\sigma_1)} = 0,$$

where γ is the Lagrange multiplier. It follows that $\frac{B}{A} = e^{(r_m - r_w)(\tau^O(\sigma_1) - \tau^C(\sigma_1))}$. Hence, $\tau^O(\sigma_1) = \tau^C(\sigma_1) + K$, where $K > 0$ is a constant that does not depend on σ_1. This relation implies that the LHS of (7) is increasing in $\tau^O(\sigma_1)$ while the RHS is decreasing in $\tau^O(\sigma_1)$. Hence, there is a unique solution that does not depend on σ_1. ■

In the analysis that follows we use a technical lemma that states that for every incentive-compatible contract for which $u > 0$, there exists another incentive-compatible contract that implements the same work schedule via a compensation policy that is (pointwise) weakly lower, and strictly lower on a set of histories with strictly positive measure.

Lemma A.3. *Assume that opportunities are concealable. Moreover, assume that under an incentive-compatible contract the continuation contract at h_t, $\langle \alpha(\cdot), \varphi(\cdot) \rangle$, is such that the worker's continuation utility is $u > 0$. There exists $\tilde{u} < u$ such that for every $u' \in (\tilde{u}, u)$ there exists an incentive-compatible contract $\langle \alpha'(\cdot), \varphi'(\cdot) \rangle$ that provides the worker with a continuation value of u', and for which $\varphi'(h_s) \leq \varphi(h_s)$ and $\alpha'(h_s) = \alpha(h_s)$ at every h_s that is a continuation of h_t.*

Proof of Lemma A.3. Consider an incentive-compatible contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ under which the worker's continuation utility is $u > 0$ and normalize the current time to zero. If the worker's expected discounted compensation along the histories in which there are no binding incentive compatibility constraints is positive, then the worker's continuation utility at time zero can be decreased by reducing his compensation along those histories. If this is not the case, then the worker's compensation is almost surely zero prior to a binding incentive compatibility constraint. Hence, concealing all opportunities is a best response for the worker. However, as $\varphi = 0$ before the worker exerts effort, this best response provides a payoff of $0 < u$. ■

Proof of Lemma 1. Let $\langle \hat{\alpha}(\cdot), \hat{\varphi}(\cdot) \rangle$ be an incentive-compatible (continuation) contract under which the worker's expected discounted payoff is $u > 0$. From Lemma A.3 it follows that there exists $\tilde{u} < u$ such that if the worker's continuation utility is in $(\tilde{u}, u)$, then the manager can induce the same work schedule for a lower compensation. Thus, there is an open neighborhood to the left of u for which the manager can obtain a value strictly greater than $V(u)$. The strict monotonicity of $V(\cdot)$ follows from the fact that the choice of u is arbitrary.

Next, we show that $V(u)$ is weakly concave. Let $u_1 < u_2$ such that $u_1, u_2 \in [0, \frac{1}{r_w}]$. One (unnatural) way the manager can deliver a promise of $\frac{u_1+u_2}{2}$ is to fictitiously split all opportunities and compensation in half and create two (perfectly correlated) fictitious worlds, each of which contains half of the compensation flow and half of each opportunity. Observe that scaling all payoffs by $\frac{1}{2}$ multiplies the players' discounted payoffs by half in any contract, and so any optimal contract in the original non-scaled world is also an optimal contract in each fictitious world. The manager can then use the continuation contract that supports $V(u_1)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_1}{2}$ in fictitious world 1, and the continuation contract that supports $V(u_2)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_2}{2}$ in fictitious world 2. Since using these continuation contracts cannot increase the manager's payoff, $V(\frac{1}{2}(u_1 + u_2)) \geq \frac{1}{2}V(u_1) + \frac{1}{2}V(u_2)$, which establishes the concavity of $V(\cdot)$. ■

Proof of Proposition 3. We establish this proposition separately for the case where the manager is weakly patient and the case where she is impatient. In each case, we first derive one part of the optimal contract (the work schedule when the manager is impatient, and the compensation policy when she is impatient), and then use the HJB equation to derive the optimal contract and show that it is, generically, unique.

Case 1: impatient manager ($r_m > r_w$). The first step of the proof is to show that under any optimal contract the work schedule is $\overline{\alpha}(u) = \min\{1, \frac{1/r_w - u}{A}\}$.

Assume by way of contradiction that $\alpha(\hat{u}) < \min\{1, \frac{1/r_w - \hat{u}}{A}\}$ for some $\hat{u} \in [0, \frac{1}{r_w}]$. Suppose that the current state is $\hat{u}$ and that an opportunity is currently available. If the worker's expected discounted future effort is zero, then it is both possible and profitable to increase α and increase the worker's compensation in the future without changing his continuation utility. If, on the other hand, the worker's expected discounted future effort is positive, then the manager can expedite effort (in the non-Markovian representation of the contract) without altering the compensation policy. By Observation 1 it is profitable for the manager to expedite effort, and, since she does so according to the worker's discount factor, it also relaxes all incentive-compatibility constraints.

The above claim enables us to simplify the HJB equation to

$$(HJB_{imp}) \qquad \sup_{\varphi(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ +\mu\left(\overline{\alpha}(u)B + V\left(u + \overline{\alpha}(u)A\right) - V(u)\right)\} = 0.$$

From the FOC of (HJB_{imp}) it follows that $\varphi(u) = 1$ if $V'(u) < -1$ and that $\varphi(u) = 0$ if $V'(u) > -1$. Since $V(\cdot)$ is weakly concave (Lemma 1), there is a (possibly degenerate) interval $I \subset [0, \frac{1}{r_w}]$ over which $V'(u) = -1$. Note that for any $u^\dagger \in I$ the compensation policy given by

$$\varphi_{u^\dagger}(u) = \begin{cases} 1 & \text{if } u > u^\dagger \\ r_w u^\dagger & \text{if } u = u^\dagger \\ 0 & \text{if } u < u^\dagger \end{cases}$$

is an optimal compensation policy.

Next, we show that, generically, I is degenerate. Fix A, B, μ, and r_w, and let $I(r_m)$ denote the interval (or point) for which $V'(\cdot) = 1$ for a manager with discount rate r_m. To establish the generic uniqueness of optimal contracts we will show that if there exist $\tilde{r}_m < \hat{r}_m$ such that both $I(\tilde{r}_m)$ and $I(\hat{r}_m)$ have a positive measure, then these intervals have a disjoint interior. The result then follows from a standard argument about the density of rational numbers.

Assume by way of contradiction that for some $\tilde{r}_m < \hat{r}_m$, the set $I^* \equiv I(\tilde{r}_m) \cap I(\hat{r}_m)$ has a nonempty interior. Select u^* and $\epsilon > 0$ such that $u^*, u^* - \epsilon \in int(I^*)$.

Fix the optimal compensation policy $\varphi_{u^*}(\cdot)$, and let $\Delta\varphi_s = \mathbb{E}(\varphi_s | u_0 = u^* - \epsilon) - \mathbb{E}(\varphi_s | u_0 = u^*)$ and $\Delta\alpha_s = E(\alpha_s | u_0 = u^* - \epsilon) - \mathbb{E}(\alpha_s | u_0 = u^*)$. Since the chosen compensation policy, $\varphi_{u^*}(\cdot)$, is optimal, we have

$$V(u^* - \epsilon) - V(u^*) = \int_0^\infty e^{-r_m s}(\mu B \Delta\alpha_s - \Delta\varphi_s) ds. \tag{8}$$

As path by path u_s is monotone in u_0, and the work schedule and compensation policies are threshold policies, it follows that $\mu B \Delta\alpha_s - \Delta\varphi_s \geq 0$ for all s, with a strict inequality on a set of times with strictly positive measure. Hence, differentiating the RHS of (8) with respect to r_m shows that the RHS of (8) is decreasing in r_m. However, as $V'(u) = -1$ for all $u \in I^*$ it holds that $V(u^* - \epsilon) - V(u^*) = \epsilon$. Hence, (8) can be satisfied for at most one r_m and so the interior of I^* is empty.

It follows that if the interior of $I(r_m)$ is nonempty, then the compensation policies corresponding to elements of $int(I(r_m))$ are suboptimal for any $r'_m \neq r_m$. Thus, we can index every r_m for which the optimal contract is not unique by a rational number from the interior of $I(r_m)$. Hence, the set of manager-discount factors for which the optimal contract is not unique is (at most) countable.

Case 2: weakly patient manager ($r_m \leq r_w$). We begin by showing that the optimal compensation policy is

$$\overline{\varphi}(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u = 0. \end{cases}$$

To do so, we show that if $u(h_t) > 0$ then in an optimal contract $\varphi(h_t) = 1$ in the next dt units of time conditional on no opportunity arriving in that interval. If $u(h_t) = \frac{1}{r_w}$, this is immediate. Assume by way of contradiction that $u(h_t) \in (0, \frac{1}{r_w})$, and that the worker does not receive the maximal compensation with probability 1 in the next dt units of time conditional on no opportunity arriving in that interval. By arguments analogous to those used in the proof of Lemma A.3, it is possible to expedite compensation into the interval $[t, t+dt]$ (conditional on no opportunity arriving) without violating the incentive compatibility constraints in any history that is a continuation of h_t. If $r_m < r_w$, then expediting compensation is profitable (Observation 1). If, on the other hand, $r_m = r_w$, expediting compensation is profitable as it enables the manager to require more effort in the future.

The above claim enables us to simplify the HJB equation to

$$(HJB_p) \quad V(u) = \sup_{\alpha(u) \in [0, \min\{1, \frac{\frac{1}{r_w} - u}{A}\}]} \{-r_m V(u) + V'(u)[r_w u - \overline{\varphi}(u)] - \overline{\varphi}(u) + \mu\left(\alpha(u) B + V\left(u + \alpha(u) A\right) - V(u)\right)\} = 0.$$

The FOC of HJB_p with respect to $\alpha(u)$ is $B + V'\left(u + \alpha(u) A\right) A = 0$. Thus, to show that there is a unique optimal contract, it is sufficient to show that $V(\cdot)$ is strictly concave.

To do so, we return to the construction used to establish the weak concavity in Lemma 1 and further the analysis by utilizing the structure of $\overline{\varphi}(\cdot)$.

In the event with strictly positive probably where no opportunity arrives for T units of time, where T solves $u_1 = \frac{1-e^{-r_w T}}{r_w}$, the worker's continuation utility in fictitious world 1 is zero while his continuation utility in fictitious world 2 is strictly positive. At this point, the manager can temporarily merge the two fictitious worlds and expedite compensation in world 2 by using the compensation from world 1. By Observation 1 this modification is profitable for a strictly patient manager and, hence, $V(u)$ is strictly concave if $r_m < r_w$. If $r_w = r_m$, then merging the fictitious worlds increases the discounted effort the worker can be incentivized to exert in the future, which also strictly increases the manager's profit. ■

Proof of Proposition 4.

In Proposition 3 we established that if $r_m \leq r_w$, then $u^C = 0$ and there is a unique optimal contract. Setting $u^O = 0$ implies that the worker never exerts effort, which, in turn, implies that the manager's value is zero; an outcome that is clearly suboptimal. If the manager is patient, the setting $u^O > u(T^*)$ is suboptimal as when the worker exerts effort that increases his continuation utility to above $u(T^*)$, the manager will have promised compensation more than T^* units of time in the future. By the definition of T^*, this reduces the manager's value. Thus, to establish the proposition it remains to show that setting $u^O = u(T^*)$ is also suboptimal.

Assume towards a contradiction that under the optimal contract $u^O = u(T^*)$, and let τ be the first time at which the worker's continuation utility reaches u^O. Since $u^C = 0$, an opportunity must be available at τ. If the first opportunity to arrive after τ arrives sufficiently quickly, then the worker will not exert full effort on that opportunity. Choose $s \in (0, T^*)$ such that if the first opportunity to arrive after τ arrives at $\tau + s$, then the worker's effort on that opportunity is strictly less than 1, and denote the probability that the first opportunity after τ arrives in $[\tau + s/2, \tau + s]$ by $p > 0$. For $\epsilon > 0$ define the following (non-Makrovian) modification of the contract at time τ: reduce the worker's required effort at τ by $u(\epsilon) = \left(\int_{T^*-\epsilon}^{T^*} e^{-r_w t} dt\right) / A$ and refrain from promising compensation in $[\tau + T^* - \epsilon, \tau + T^*]$. Then, use the freed compensation in $[\tau + T^* - \epsilon, \tau + T^*]$ to increase the worker's effort on the first opportunity to arrive after τ, if it arrives in $[\tau + s/2, \tau + s]$. If it does not arrive in this interval, never provide compensation in $[\tau + T^* - \epsilon, \tau + T^*]$.

The manager's loss of value at time τ from this deviation is bounded from above by the manager's value from the worker exerting an effort of $u(\epsilon)$ and receiving compensation in $T^* - \epsilon$ units of time. This upper bound is given by

$$\frac{e^{-r_wT^*}\left(e^{r_w\epsilon}-1\right)\left(\frac{B}{A}-e^{(r_w-r_m)(T^*-\epsilon)}\right)}{r_w}=e^{-r_wT^*}\left(\frac{B}{A}-e^{(r_w-r_m)T^*}\right)\epsilon+O\left(\epsilon^2\right)=O\left(\epsilon^2\right),$$

since $\left(\frac{B}{A}-e^{(r_w-r_m)T^*}\right)=0$ by the definition of T^*.

If ϵ is small enough, then the worker will not exert full effort on the first opportunity in $[\tau+s/2,\tau+s]$ after this deviation. In this case, the gain from the additional effort on that opportunity is bounded from below by the product of p and the manager's time τ discounted gain should the the next opportunity arrive at $\tau+s/2$ (Observation 1). This bound is given by

$$p\left(\frac{B\left(e^{r_w\epsilon}-1\right)e^{r_w(s/2-T^*)-r_ms/2}}{Ar_w}-\frac{e^{-r_mT^*}\left(e^{r_m\epsilon}-1\right)}{r_m}\right)$$
$$=\epsilon p\left(e^{(r_w-r_m)s/2}-1\right)\left(\frac{B}{A}\right)^{\frac{r_m}{r_m-r_w}}+O\left(\epsilon^2\right).$$

Since $r_w>r_m$ and $s>0$, the first-order term in the approximation is strictly positive. Hence, for sufficiently small ϵ, the modification is profitable. ■

Proof of Proposition 5. In Proposition 3 we established that $u^O=\frac{1}{r_w}$ if $r_m>r_w$. Furthermore, when $r_m=r_w$ the manager will use the same threshold, to avoid wasting her limited capacity to compensate the worker. To establish the second part of the proposition we begin by showing that $u^C=\frac{1}{r_w}$ is an optimal threshold if and only if $r_m\geq r''_m\equiv r_w+(\frac{B}{A}-1)\mu$. An upper bound on the manager's marginal net gain from providing the worker with a util at present is attained by the worker exerting a worker-discounted util on the first opportunity to arrive. Note that if $u^C>\frac{1}{r_w}-A$ and $u=u^C$, then this upper bound is attained. The value of this upper bound is given by

$$\int_0^\infty \mu e^{-\mu t}\frac{B}{A}e^{(r_w-r_m)t}dt-1=\frac{\mu}{\mu+r_m-r_w}\frac{B}{A}-1.$$

It is straightforward to show that this expression is positive if and only if $r_m\leq r''_m$. It follows that if $r_m>r''_m$, then providing compensation while $u<\frac{1}{r_w}$ is suboptimal. On the other hand, if $r_m<r''_m$, then it is strictly suboptimal to set $u^C=\frac{1}{r_w}$ since for such discount rates it is profitable to compensate the worker when $u>\frac{1}{r_w}-A$.

Next, we establish that there exists $r'_m>r_w$ for which $u^C=0$. Let τ denote the random arrival time of the first opportunity on which the worker will not exert full effort under the contract with $u^C = 0$. Note that under any other contract, the worker will not exert full effort (weakly) earlier. It follows that the marginal value from decreasing the worker's continuation utility is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$. If $r_w = r_m$ the timing of compensation does not affect the manager's cost of providing compensation. This, in turn, implies that when $r_w = r_m$ the manager's marginal gain from providing compensation is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$ for all u. By the continuity of the manager's payoff in r_m, it follows that there exists $r'_m > r_w$ such that the manager strictly benefits from full front-loading of compensation if $r_m < r'_m$.

To conclude the proof we use the following lemma that states that u^C is increasing in r_m (proof below) to show that $r'_m < r''_m$

Lemma A.4. *Fix B, C, μ, and r_w and assume that the manager is impatient. If u^C is an optimal threshold for r_m and $\tilde{u}^C$ is an optimal threshold for $\tilde{r}_m > r_m$, then $\tilde{u}^C \geq u^C$.*

Proof. Consider two contracts, $\mathcal{C}_1$ and $\mathcal{C}_2$, that differ in their compensation threshold, $u_2^C > u_1^C$. Denote by $u_{t,i}$ the worker's continuation utility at time t under contract $\mathcal{C}_i$ and let $\overline{\tau} = \sup\{t : u_{t,2} \leq u_2^C\}$. That is, $\overline{\tau}$ is the latest time at which the worker's continuation utility is lower than u_2^C under $\mathcal{C}_2$. Note that $\overline{\tau}$ is finite (almost surely) by the Borel–Cantelli lemma.

Observe that since $u_2^C > u_1^C$, it holds that $u_{t,1} \leq u_{t,2}$ for all t. This implies that the worker exerts the same effort on every opportunity that arrives before $\overline{\tau}$ under both $\mathcal{C}_1$ and $\mathcal{C}_2$. In addition, it implies that the compensation for effort exerted on those opportunities is postponed under $\mathcal{C}_2$ relative to $\mathcal{C}_1$. Denote by $g(r_m)$ the gain from this postponement as a function of r_m. Moreover, from $\overline{\tau}$ onwards, under $\mathcal{C}_2$ the worker exerts weakly less effort than he does under $\mathcal{C}_1$, and he receives a compensation of $\varphi_t = 1$ at all times. Let $d(r_m)$ denote the difference in the time-zero discounted continuation value from $\overline{\tau}$ onward between $\mathcal{C}_1$ and $\mathcal{C}_2$. The net gain from replacing $\mathcal{C}_1$ with $\mathcal{C}_2$ is $g(r_m) - d(r_m)$. Note that $g(\cdot)$ is increasing and $d(\cdot)$ is decreasing. Hence, whenever $g(r_m) \geq d(r_m)$, we also have $g(r'_m) > d(r'_m)$ for all $r'_m > r_m$, which establishes the monotonicity of u^C. ■

To see why the lemma implies that $r'_m < r''_m$, observe that when $r_m = r''_m$ there exist histories for which under the optimal contract the worker exerts effort on only the first opportunity, whereas, when $r_m = r'_m$ in every history the worker's discounted cost of effort must equal his discounted compensation, which, by Assumption 1, is greater than the cost of effort on a single opportunity. ■

Proof of Proposition 6. We prove the proposition in two parts.

Part A: concealable opportunities. To establish this part of the proposition, it is convenient to think of each opportunity as being composed of many "small opportunities." We will show that making opportunities lumpier in the actual model is equivalent to a certain change in the correlation structure of these small opportunities.

First, we consider the case where opportunities become lumpier by a rational factor. Assume that opportunities become lumpier by $\frac{N}{M} > 1$, where $N, M \in \mathbb{N}$. We analyze this change by considering an auxiliary representation of the model in which there are $M \times N$ Poisson processes, each with an arrival rate of $\frac{\mu}{N}$, that govern the arrival of the small opportunities. Moreover, we assume that the payoff from exerting full effort on each small opportunity is $(-\frac{A}{M}, \frac{B}{M})$. Both the original and the lumpy versions of the model correspond to appropriately defined correlation structures of the arrival processes in the auxiliary representation.

To map the auxiliary representation to the original model, divide the Poisson processes into N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. To see why this correlation structure represents the original model, note that when a group of opportunities is available the payoff vector from exerting full effort on all opportunities in the group is $M \times (-\frac{C}{M}, \frac{B}{M}) = (-A, B)$, which is exactly the payoff vector from exerting full effort on a single opportunity in the original model. Moreover, the probability that a given group arrives in an (infinitesimal) interval dt is $\frac{\mu}{N}dt$, and since the groups are independent, the probability that some group arrives in that interval is $\sum_{i=1}^{N} \frac{\mu}{N}dt = \mu dt$.

To map the auxiliary representation to the lumpy model, divide the processes into M groups of N processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. For this correlation structure, the payoff from exerting full effort on all opportunities in a group is $N \times (-\frac{A}{M}, \frac{B}{M}) = (-\frac{N}{M}A, \frac{N}{M}B)$ and the probability that some group of opportunities is available in an interval dt is $\frac{\mu}{N/M}dt$.

Next, we construct a sequence of modifications that begins with the lumpy representation and ends with the original one, such that the first two modifications do not impact the manager's value, and the third modification strictly increases it.

Consider the lumpy representation. The first modification utilizes the idea of splitting the interaction into fictitious worlds introduced in Lemma 1. In particular, we consider N fictitious worlds, denoted by $(1, \ldots, N)$, that each contain $\frac{1}{N}$ of the flow compensation and M arrival processes, one from each group. We denote the processes in fictitious world n by $(P_1^n, \ldots, P_M^n)$. Note that the arrival processes within each fictitious world are independent of one another, and so each fictitious world is a scaled version of the lumpy representation. Hence, by the argument used in Lemma 1, the sum of the manager's values across all fictitious worlds is equal to her value in the lumpy representation.

The second modification is to the correlation structure of the processes across fictitious worlds. Changing the correlation structure of two arrival processes that are assigned to *different* fictitious worlds does no impact the manager's value in either fictitious world. Hence, so long as the processes within each fictitious world are independent of one another, the correlation across fictitious worlds is immaterial. Thus, we can replace the original correlation structure with the following correlation structure: P_m^n and $P_{m'}^{n'}$ are perfectly correlated if $m - n \overset{(\text{mod} N)}{=} m' - n'$, and independent otherwise. This modification maintains the independence of the processes within each fictitious world. To see this note that for any $n \leq N$ and $m, m' \leq M$, such that $m' \neq m$, the fact that $M < N$ implies that $m - n \overset{(\text{mod} N)}{\neq} m' - n$.

The third modification is to re-merge the fictitious worlds. Note that under the correlation structure created in the second modification, there are N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. Thus, merging these fictitious worlds creates the auxiliary representation of the original model. Regardless of the relative patience, there are instances in which the manager benefits from merging two fictitious worlds: if $r_m < r_w$ this occurs when in fictitious world i the worker's continuation is positive while in fictitious world j it is zero, whereas if $r_m \geq r_w$ this occurs when in fictitious world i an opportunity is (partially) forgone while in fictitious world j the worker's continuation utility is below its maximal level. It follows that the sum of the manager's values across all fictitious worlds is strictly less than her value in the original model.

Finally, consider the case where $\lambda \notin \mathbb{Q}$. The manager's value is continuous in λ as i) the distribution of arrival times is continuous in λ, and ii) if opportunities are made slightly lumpier then the manager can instruct the worker to incur the same *cost* of effort on every opportunity that arrives by using the same compensation policy. As the set of rational numbers is dense, this establishes the proposition. ||

Part B: observable opportunities. We establish this part of the proposition separately for the case where the manager is patient and the case for which she is impatient.

Case 1: patient manager. We start this proof by establishing the comparative statics of α^* with respect to λ. If the worker's expected utility from a conditional promise of length T^* is strictly greater than A, then the worker exerts full effort on all opportunities. Moreover, this will remain the case if opportunities become marginally lumpier. Thus, we focus on the case where the worker's expected utility from such a promise is at most A.

The worker's expected utility from a conditional promise of length T^*, as a function of the parameter λ, is $\frac{1}{r_w+\mu/\lambda}(1-e^{-T^*(r_w+\mu/\lambda)})$. The marginal increase in the value of such a promise from making opportunities lumpier (i.e., the derivative of this value with regard to λ, evaluated at $\lambda = 1$) is $\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w))$. Thus, to establish that α^* is decreasing in λ it is enough to show that making opportunities marginally lumpier has a larger impact on the cost of exerting the required effort than on the value of a conditional promise of length T^*.

The worker's cost of exerting effort is $\lambda\alpha^* A$, and so the marginal effect of making opportunities lumpier (i.e., the derivative of this cost with regard to λ, evaluated at $\lambda = 1$) is $\alpha^* A$. Under the assumption that a conditional promise of length T^* does not provide excess compensation, it holds that $A\alpha^* = \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}$. Hence, it is sufficient to show that

$$\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w)) < \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}.$$

When $\mu + r_w$ is kept constant, this inequality is harder to satisfy for higher values of μ. Thus, it is sufficient to show that it holds for $r_w = 0$, i.e., to show that

$$1-e^{-\mu T^*}(1+\mu T^*) < 1-e^{-\mu T^*},$$

which holds for any $\mu T^* > 0$. Note that the above calculation does not depend on the value of T^*. Hence, the same calculation shows that when it is possible to induce full effort, S^* increases when opportunities become marginally lumpier.

Next, we show that lumpier opportunities are detrimental for the manager. If $\alpha^* = 1$, this is an immediate consequence of S^* being increasing in λ. Assume that $\alpha^* < 1$ and let $f(r,\lambda) = \frac{1-e^{-T^*(r+\frac{\mu}{\lambda})}}{r\lambda+\mu}$ denote the $r-$discounted compensation that is provided via a conditional promise of length T^* as a function of λ. Note that the average cost of providing a util of compensation is $\frac{f(r_m,\lambda)}{f(r_w,\lambda)}$. The cross-derivative of $f(r,\lambda)$ evaluated at $\lambda = 1$ is

$$\frac{\partial^2 f(r,\lambda)}{\partial r \partial \lambda}|_{\lambda=1} = \frac{\mu e^{-T(\mu+r)}\left(T^*(\mu+r)(T^*(\mu+r)+2)-2e^{T^*(\mu+r)}+2\right)}{(\mu+r)^3}.$$

The sign of this cross-derivative is the sign of $x(x+2)+2-2e^x$, where $x = T^*(r+\mu)$. As this sign is negative, the cross-derivative is negative. As $f(r,\lambda)$ is positive and decreasing in λ, it follows that the average cost of compensating the worker for his effort is increasing in λ (recall that $r_w > r_m$). As the worker's total effort is also decreasing in λ, we can conclude that making opportunities lumpier reduces the manager's value.

Case 2: Impatient manager. Solving the optimal thresholds for the contract characterized in Proposition 2, τ^O, τ^W, as a function of λ gives

$$\tau^O(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + \lambda r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w},$$
$$\tau^C(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + \lambda r_w))}{r_w}.$$

Recall that the manager's value is

$$\mathbb{E}\left(e^{-r_m\sigma_1}\left(B\lambda + \frac{B\mu(1 - e^{-r_m\tau^O(\lambda)})}{r_m} - \frac{e^{-r_m\tau^C(\lambda)}}{r_m}\right)\right).$$

Plugging in the expressions for the optimal thresholds, differentiating with respect to λ, and evaluating at $\lambda = 1$ shows that making opportunities marginally lumpier changes the manager's value by

$$(r_w - r_m)\frac{\mu\left(\frac{B}{A}\right)^{-\frac{r_m}{r_w - r_m}}\left(B\mu\left(\frac{B}{A}\right)^{\frac{r_m}{r_w - r_m}} + 1\right)e^{-r_m\left(\frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w - r_m}} + 1\right) - \ln(A(\mu + r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w}\right)}}{(\mu + r_w)(\mu + r_m)^2}.$$

This expression is negative as $(r_w - r_m) < 0$ and the fraction is clearly positive. ■