Mathematics for finance

Aim

The course motivates and illustrates the role and limitations of mathematics in quantitative finance, with a focus on linear algebra and optimization, which are also the foundation of modern machine learning methods. We take for granted basic concepts of deterministic calculus (limits and derivatives; gradients and directional derivatives; Taylor expansions; integrals). We also illustrate and generalize basic ideas in normed and inner product spaces, which are essential in both financial theory and numerical methods. Everything is illustrated from three related points of view: theory, applications, and numerical implementation in MATLAB.

Content

- *Motivations*. Basic problems in quantitative finance: portfolio optimization, risk management, asset pricing.
- *Linear algebra*. Linear spaces. Matrices, eigenvalues, quadratic forms. Semidefinite quadratic forms.
- *Norms, distances, and inner products*. Defining norms and inner products for vector and matrices. Projection on subspaces. Applications: linear regression, regularized linear regression.
- *Convex analysis*: convex sets and functions; concave functions and risk aversion; convex cones; separation theorems; Farkas lemma.
- *Optimization theory*: unconstrained optimization (gradient, subgradient, black box); classical Lagrange methods; KKT conditions.
- *Applications*:
    - Mean-variance portfolio optimization.
    - Foundations of mathematical finance: pricing by no arbitrage; state prices; complete and incomplete markets.

Bibliography

- P. Brandimarte. *Numerical Methods in Finance and Economics: A MATLAB-Based Introduction (2nd ed)*. Wiley, 2006.
- P. Brandimarte. *Quantitative Methods: An Introduction for Business Management*. Wiley, 2011.
- P. Brandimarte. *An Introduction to Financial Markets: A Quantitative Approach*. Wiley, 2018.
- C.P. Simon, L.E. Blume. *Mathematics for Economists*. W.W. Norton & Company, 1994.