Relying on Intermittency: Clean Energy, Storage, and Innovation in a Macro Climate Model *

Claudia Gentile[†]

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November 13^{th}, 2023

Abstract

The transition to clean energy technologies is essential to reduce CO2 emissions. One significant challenge associated with renewable energy sources, such as solar and wind, is their intermittency. I study the intermittency problem by introducing a novel micro-founded energy sector with directed technical change in a macro climate model. I show that the aggregate elasticity of substitution between clean and dirty energy is not constant, and it crucially depends on the development of storage technologies. Without policies, the provision of storage technologies is inefficiently low, impeding the transition towards clean, intermittent technologies. In the optimal allocation, the clean energy transition is accelerated with an initial clean energy share increasing from 25% to 70% and a reallocation of all R&D resources away from dirty energy towards clean energy and, in particular, energy storage technologies. The introduction of clean energy subsidies under the US Inflation Reduction Act is successful at increasing the short-run clean energy share, but insufficient to solve the intermittency problem.

Keywords: Environmental Macroeconomics, Clean Energy, Directed Technical Change, Intermittency

*I thank David Hémous, Florian Scheuer, Michael Peters, Fabrizio Zilibotti, Alessandro Ferrari, Lint Barrage, Luca Citino, Steve Cicala, Tony Smith, Samuel Kortum, and Kenneth Gillingham for their helpful comments. I also thank seminar participants at the University of Zurich, Yale University, the Swiss Macro Workshop, the PSE-CEPR policy forum, the EEA 2023 conference, and the SERE 2023 workshop for their helpful comments. I gratefully acknowledge financial support from the UZH Doc Mobility grant and the STEG research grant.

[†]University of Zurich. Email: claudia.gentile@econ.uzh.ch

1 Introduction

Energy production and use alone accounted for a staggering 76% of global greenhouse gas (GHG) emissions in 2019 (Climate Watch, 2020). Hastening the transition to clean energy sources is imperative to mitigate our impact on the climate. Encouragingly, the past decade has witnessed tremendous progress in clean energy technologies. Notably, the production costs for solar and wind energy have plummeted by 89% and 70%, respectively.[1] The average costs of generating energy with new solar and wind technologies are now lower than the average generation costs of new coal and gas power plants.[2] Behind these promising figures, a significant obstacle remains: intermittency. Given sun and wind are intermittent natural resources, we can use them to produce energy only when they are available. The intermittency challenge inevitably inflates the costs of deploying clean energy, necessitating the maintenance of backup sources such as fossil fuel power plants or energy storage during periods of insufficient sun or wind availability. This paper aims to examine whether the market possesses the necessary incentives to foster the development of storage technologies as a solution to the intermittency problem, and what is the role of policy in the clean energy transition.

Macroeconomic climate studies typically incorporate the intermittency problem by employing a constant elasticity of substitution within a CES production function, reflecting the limited substitutability between clean and dirty energy. However, there is no guarantee that the aggregate elasticity of substitution between clean and dirty energy is constant. One might expect the elasticity of substitution to decrease as the clean energy share increases, as intermittency makes it increasingly difficult to substitute clean for dirty when dirty energy is used mostly in hours with insufficient sun and wind.

In this paper, I study the intermittency problem by introducing a novel micro-founded model of the energy sector explicitly capturing intermittency in a macro climate model. This paper makes three contributions. First, it microfounds the aggregate elasticity of substitution between clean and dirty energy sources. It shows that the elasticity is not constant, and it crucially depends on the development of storage technologies. If the storage technology is not developed, the economy is trapped in a scenario in which the elasticity of substitution eventually becomes zero. Second, it studies theoretically and quantitatively the optimal environmental policies in the presence of intermittency. In the decentralized economy, the use of storage and storage innovation is inefficiently low. This result calls for a crucial optimal policy instrument: R&D subsidies to energy storage technologies. An optimal policy mix of carbon taxes and R&D subsidies can avert the environmental disaster by pushing for stor-

[1]These cost drops were measured using LCOE data from the International Renewable Energy Agency. Levelized Cost of Energy (LCOE) is a standard measure of electricity generation costs. It represents the average revenue per unit of electricity generated that would be required to recover the costs of building and operating a generating plant during an assumed financial life and duty cycle.

[2]The average costs of generating energy using new solar and wind technologies are, respectively, \$28–\$54/MWh and \$32–\$42/MWh. The average costs of generating energy using new coal and gas combined cycle power plants are, respectively, \$66–\$152/MWh and \$44–\$68/MWh (Lazard, 2019).

age technological change, increasing the elasticity of substitution, and leading to a complete clean energy transition. Third, it analyzes the impact of clean energy subsidies introduced by the US Inflation Reduction Act. The implemented subsidies are insufficient to induce the development of storage technologies, resulting in low welfare effects relative to the optimal policies. In what follows, I first describe the key novelties of the model, and then I discuss the above contributions in detail.

The model features a neoclassical growth model and a climate cycle à la Golosov et al. (2014) (GHKT). Output is produced using a Cobb-Douglas production function, combining capital, labor, and energy. Dirty energy generates GHG emissions, which affect the climate cycle. Deterioration of the climate results in output damages.

At the heart of my theory is an explicit model of intermittency. Within an aggregate five-year period, there is a continuum of hours, which differ in the availability of clean energy. Clean and dirty energy producers decide how much capacity to build (i.e., solar panels, fossil fuel power plants) and how much energy to produce every hour. Storage firms can transfer energy across hours. Dirty energy producers can always decide to produce at their full capacity. Instead, clean energy producers cannot produce in some hours because of intermittency (i.e., when solar radiation and wind speeds are low). Energy producers make choices based on hourly energy prices, which they take as given. Equilibrium hourly energy prices depend on the marginal production technology, as in the IO electricity literature (Borenstein & Holland, 2005; Bushnell, 2011; Holland et al., 2022).

To study whether the market has incentives to develop clean and storage technologies, I model directed technical change in the energy sector. I augment the Acemoglu et al. (2012) framework with an explicit model of innovation in storage technologies. Researchers decide whether to improve clean energy technologies, dirty energy technologies, or energy storage technologies. The direction of innovation crucially depends on the relative size of the markets for clean, dirty, and energy storage technologies. Storage technologies are currently too expensive to be competitive on a large scale.[3] If the market for storage technologies is initially small, storage innovation might be less than optimal.

The first theoretical contribution of my paper is to provide a microfoundation for the aggregate elasticity of substitution between clean and dirty energy sources, which the existing literature has highlighted as a key determinant of optimal policies. In a model with a constant substitution elasticity ρ, Acemoglu et al. (2012) show that if the two energy inputs are gross complements ($\rho < 1$),[4] the only solution to avert an environmental disaster is to stop long-run growth. If the inputs are gross substitutes ($\rho > 1$), an optimal policy mix of carbon taxes and R&D subsidies can avert the environmental disaster. However, there is no guarantee that the

[3]The average costs of generating energy using new solar, wind, gas, and coal technologies are in the range of 28-152\$/Mwh (Lazard, 2019). The average cost of energy storage is in the range of 132-250\$/MWh (Lazard, 2020).

[4]In a CES production function, the two energy inputs are gross complements if the elasticity of substitution is smaller than one. The two inputs are gross substitutes if the elasticity of substitution is larger than one.

elasticity of substitution is constant.

I show analytically and numerically that the aggregate elasticity of substitution is not constant, and it crucially depends on the technological progress in storage. In my theory, the substitutability between clean and dirty energy depends on the degree of intermittency and technological development in the energy sector, which affect the optimal choices of energy producers. I find two key results on the elasticity of substitution. First, if storage costs are high, the elasticity of substitution between clean and dirty energy is high when the clean technology is less advanced. As the clean technology improves, the elasticity decreases, and it eventually becomes zero. The intuition behind this result is that when the clean technology is less advanced, the clean energy share is low, and one can easily substitute clean for dirty by increasing clean production in the hours with abundant sun and wind. As the clean technology improves, the clean energy share increases. Eventually, dirty energy is used mostly in hours with insufficient sun and wind, and it becomes increasingly difficult to substitute clean for dirty, absent cheap storage technology. The second result is that if storage costs are low, the elasticity of substitution drastically increases at current clean technology costs. Furthermore, as the clean costs decrease, the elasticity of substitution increases, and it eventually becomes infinite.

Endogenizing the elasticity of substitution has important implications for the role of policy and the optimal speed of the clean energy transition. First, if the storage technology is not developed, the economy is trapped in a scenario in which the elasticity of substitution eventually becomes zero, and the only way to avert an environmental disaster is to stop long-run growth. An optimal policy mix of carbon taxes and R&D subsidies can avert the environmental disaster by pushing for storage technological change, increasing the elasticity of substitution, and leading to a complete clean energy transition. I find that R&D subsidies should not redirect innovation equally towards clean and storage technologies, but in particular towards storage technologies. Secondly, endogenizing the elasticity of substitution results in a slower optimal speed of the clean energy transition relative to assuming a constant elasticity of substitution, which does not accurately reflect the problem of intermittency and the time needed to develop the storage technology.

In order to study optimal policies, I solve the problem of a social planner who internalizes the environmental externality of dirty energy use. I derive an analytical formula for the optimal carbon tax, which equals the marginal externality damage of emissions and is proportional to current GPD in line with GHKT.

I calibrate the model of the energy sector not to aggregate moments but rather using rich microdata from the US electricity sector. These data include capacity and hourly production costs for all US electric utilities with more than 10 GWh of annual sales as well as hourly electricity generation by fuel type (e.g., coal, gas, solar, wind) from the Energy Information Administration. Hourly electricity generation data inform the distribution of aggregate energy demand across hours. The actual hourly generation of wind and solar energy quantifies the degree of intermittency.

On the quantitative side, a second contribution of the paper is to assess the role of policy in the clean energy transition. I compare how the economy evolves in the decentralized equilibrium and the optimal allocation. I also investigate the impact of one of the major real-world policies implemented to accelerate the transition: the clean energy subsidies introduced by the US Inflation Reduction Act (IRA).

In the absence of policies, the market does not have incentives to develop storage technologies. Even if clean energy becomes the cheapest energy source, the market keeps using and developing dirty energy technologies because of intermittency. In the hours in which sun and wind are not available, the market uses dirty energy instead of storage because storage technologies are relatively more expensive. The clean energy share remains limited at around 25-30%. The optimal allocation would instead require an immediate increase of the clean energy share from 25 to 70%. The social planner moves all R&D resources away from dirty energy technologies towards clean energy and, in particular, energy storage technologies. The results highlight that subsidies should not redirect innovation equally towards all clean technologies, but in particular towards energy storage technologies. As a result, the economy achieves a complete clean energy transition by 2075 along the optimal path, resulting in welfare gains of 2.7% relative to the decentralized economy.

In contrast, clean energy subsidies introduced by the IRA increase the clean energy share in the short run but do not induce the necessary development of storage technologies. As a result, the long-run clean energy share is unchanged, and welfare increases by 0.1% relative to a world without the IRA.

Related Literature. This paper contributes to the growing macro-climate literature studying optimal environmental policies. Leading papers in this literature use a stylized representation of the energy sector, where clean and dirty energy sources are combined in a CES production function to produce energy (Acemoglu et al., 2012; Golosov et al., 2014; Hassler et al., 2020; Nordhaus, 1994, 2017). A range of estimates exist for the elasticity of substitution between clean and dirty energy sources (Papageorgiou et al., 2017; Stern, 2012). Macro climate papers typically report results for different assumptions on a constant value of this elasticity (Acemoglu et al., 2012; Hassler et al., 2020). I contribute to the literature by showing that the aggregate elasticity of substitution is not constant, and it crucially depends on storage technological change. If storage costs are high, the economy is trapped in an environment with a low aggregate elasticity of substitution. If the storage technology is substantially improved, the elasticity of substitution increases. Jo and Miftakhova (2022) investigate the impact of a variable elasticity of substitution on the optimal speed of the clean energy transition and optimal policies. The authors empirically find that the elasticity of substitution between clean and dirty energy is increasing in the clean energy share in the context of the French manufacturing sector. This pattern creates a virtuous cycle whereby the endogenous elasticity of substitution amplifies the effect of a climate policy. My paper highlights that in the electricity sector this virtuous cycle can be broken by the intermittency problem if the storage technology is not developed.

This project is part of an emerging strand of literature that considers important features of the energy sector and, in particular, the electricity sector, within macro models. A key paper in this emerging literature is Arkolakis and Walsh (2023), which integrates a model of the electrical grid into a standard spatial economy model. The authors find that the IRA accelerates the uptake of clean energy but does not change long-run outcomes, in line with my findings. I contribute to this literature by explicitly modeling how intermittency affects the optimal choices of energy producers. In addition, by modeling hourly energy prices, I can study the uptake of energy storage under laissez-faire and whether the intermittency problem is solved in the absence of policies.

This paper builds on the existing macro climate literature on computational general equilibrium (CGE) models, which has developed different methodologies to account for the variability of renewable energy sources in IAMs (Pietzcker et al., 2017). These methodologies go from the introduction of a flexibility constraint, which requires a minimum installation of dispatchable[5] generation capacity (e.g., fossil fuel power plants, energy storage) for every unit of variable renewable energy capacity (Carrara & Marangoni, 2017), to the consideration of varying residual load curves as the integration of renewable energy increases (Ueckerdt et al., 2015).[6] The approach used in this paper is most similar to the latter methodology, given I consider a detailed time resolution by modeling energy production at the hourly level. I develop a tractable microfounded model of the energy sector, which considers the optimal choices of energy producers and the intermittency problem. Differently from the CGE literature, the tractability of the model allows me to derive analytical solutions for optimal environmental policies.

In addition, this paper relates to the microeconomic literature on the energy sector. When building the micro-founded model of the energy sector, I build on the IO literature studying electricity markets (Borenstein & Holland, 2005; Bushnell, 2011; Fabra, 2021; Holland et al., 2022). This paper contributes to this literature by considering endogenous technological change. By taking into account directed technical change in the energy sector, I can study how innovation responds to policies, and how the clean energy share changes in response to policies and endogenous technological change. This paper also relates to the literature studying market incentives to install storage capacity and the complementarity between storage and clean energy adoption. This literature emphasizes that storage investment is likely to be suboptimal in a competitive market (Andrés-Cerezo & Fabra, 2023; Karaduman, 2020),[7]

[5]Dispatchable generation capacity is electricity generation capacity that can be turned on on-demand, not suffering from the intermittency problem.

[6]The residual load curve describes hourly energy demand minus renewables' generation, which needs to be satisfied by dispatchable generation capacity.

[7]Storage firms might make less investment than optimal in order to maximize their profits by limiting

and that large cost decreases are needed for storage to have a substantial effect on renewable capacity and emissions reduction (Holland et al., 2022). Butters et al. (2021) highlights that incentivizing renewables, for instance, through an ambitious renewable standard, is not enough to incentivize battery adoption. This paper contributes to the literature by studying the optimal deployment of the storage technology over time, market incentives to innovate on storage technologies, and the optimal R&D subsidies to storage innovation.

Finally, I build on the literature on directed technical change and the environment (Acemoglu et al., 2012; Acemoglu et al., 2023; Acemoglu et al., 2016; Casey, 2019; Fried, 2018; Lemoine, 2017). I contribute to this literature by explicitly modeling intermittency and storage innovation. This allows me to study the complementarity between clean energy and storage technologies and to assess whether policies should prioritize clean energy or storage R&D subsidies.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. It illustrates how I introduce intermittency and storage in a macro climate model. Section 3 characterizes the decentralized equilibrium. In Section 4, I describe the social planner problem and the optimal policies. I lay out the model calibration in Section 5, and I perform a model validation exercise in Section 6. Section 7 illustrates quantitative results on the role of policy. It shows how innovation, clean energy share, and welfare evolve under different policy scenarios. Section 8 presents results on the aggregate elasticity of substitution between clean and dirty energy. Finally, Section 9 presents a robustness analysis.

2 A Macro Climate Model with Intermittent Clean Energy

The description of the economy and the climate is articulated in different subsections. The first subsection describes the final good sector, the household problem, and the carbon cycle. This part of the model builds on Golosov et al. (2014); therefore, the description is briefer. The following subsections describe the main novelties of this paper: the micro-founded model of the energy sector and the model of directed technical change in the energy sector.

2.1 Growth Model, Climate Cycle, and Time Periods

Time is discrete, and there is a representative household whose utility is given by:

$$U_0 = \sum_{s=0}^{\infty} \beta^t log(C_t) \tag{1}$$

their impact on the hourly energy prices (Andrés-Cerezo & Fabra, 2023). Even when profits are negative for privately operated storage firms, storage value is positive for consumers because of its impact on the reduction of emissions and electricity generation costs (Karaduman, 2020).

β is the rate of time preference of the household, and C_t is consumption. Output in a period t is produced by means of a Cobb-Douglas production function combining three inputs: capital, labor, and energy.

$$Y_t = (1 - D(S_t))A_t(K_t^Y)^{\alpha}(L_t^Y)^{1-\alpha-v}E_t^v \tag{2}$$

Aggregate TFP A_t is exogenous. Aggregate energy E_t is produced in the energy sector by clean and dirty energy producers. The use of dirty energy generates GHG emissions affecting the climate cycle. Deterioration of the climate results in output damages, $D(S_t)$, which measures the fraction of output lost due to climate change. S_t is the carbon concentration in the atmosphere. Damages take the exponential form: $(1 - D(S_t)) = e^{-\epsilon_t(S_t - \bar{S})}$. The climate cycle and the damage function are the same as in Golosov et al. (2014).

$$S_t = \bar{S} + \sum_{s=0}^{t+T}(\phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s)\xi^d E_{t-s}^d \tag{3}$$

The economy starts at t=-T, which is the time of industrialization. $\bar{S}$ is carbon concentration in the pre-industrial time. E_t^d is dirty energy used in period t. ξ^d measures the carbon intensity of dirty energy. A fraction ϕ_L of carbon emitted stays in the atmosphere forever. Of the remaining emissions, a share $1 - \phi_0$ exits immediately, and a share ϕ_0 decays at a geometric rate ϕ.

There are two time dimensions in the model. Within an aggregate period t, which I take to be five years in my empirical application, there is a continuum of hours, h $\in [0, 1]$. I introduce hours to capture the intermittency of clean energy sources, which happens at the hourly level.

2.2 Aggregate Energy Production

The final good firm buys hourly energy e_{ht} from energy producers. It aggregates these intermediate energy inputs into aggregate energy E_t by means of a Leontief production function.

$$E_t = min_h\{\frac{e_{ht}}{q_h}\} \tag{4}$$

A Leontief production function implies the factors of production are used in fixed proportions, as there is no substitutability between factors. The Leontief function captures the essence of the intermittency problem: production drops to zero if there is any hour at which energy cannot be delivered. This results in the following hourly energy demand:

$$e_{ht} = q_h E_t \text{ for } \forall \text{ h} \tag{5}$$

Energy demanded by the final good firm in hour h is a fixed fraction of aggregate energy. This fixed fraction is determined by the exogenous parameter q_h, which I calibrate using data on hourly electricity generation. The final good firm can adjust its demand of aggregate energy E_t in response to changes in energy prices, but it cannot adjust its relative demand across the different hours. The sum of the q_h parameters equals one ($\int_0^1 q_h dh = 1$), which implies the following:

$$E_t = \int_0^1 e_{ht} dh \tag{6}$$

In the real world, there is some substitutability of energy inputs across the different hours; nonetheless, this substitutability is considered to be low[8]. Demand response is expected to play an important role in the clean energy transition, but the supply adjustment is expected to play a larger role[9]. As a starting point, I shut down the demand response channel by assuming a fixed relative hourly energy demand. In this project, I only focus on supply adjustment, and I aim to incorporate demand adjustment in future work.

2.3 Hourly Energy Production and Intermittency

There is a finite number of energy producers who are heterogeneous in their production costs. There are two producer types, clean and dirty, indexed by i=c or d. There are n_i producers of type i.

A type i producer makes two choices. She decides how much capacity K_t^i to build for period t (i.e., fossil fuel power plants, solar panels), and how much energy e_{ht}^i to produce in every hour h of period t. Capacity is built using labor. A unit of capacity for the type i producer (K_t^i) is built with a continuum of intermediate inputs (k_{jt}^i).

$$n_i K_t^i = exp(\int_0^1 ln(k_{jt}^i) dj) \tag{7}$$

$$k_{jt}^i = A_{jt}^i l_{jt}^i \tag{8}$$

k_{jt}^i is built using labor with a productivity level A_{jt}^i.

In order to produce energy in a given hour h, an energy producer pays the following

[8] It is interesting to note that the substitutability of energy inputs across hours is positive only between nearby hours. Firms and consumers might be willing to postpone their energy consumption by a few hours with the right price incentives, but they are not willing to postpone their energy consumption to hours far away in time.

[9] For instance, Junge et al. (2022) claim that given the estimated possible demand response by firms in the US, shifting energy consumption across hours in order to consume more in the hours in which we have more renewables can reduce the costs of the clean energy transition by 3%. On the other hand, technology development and choosing which technology to use to produce energy makes a bigger difference for the total costs of the energy transition. For instance, technological advances and the deployment of long-term duration storage (like redox flow batteries) can decrease the cost of the transition by 15%.

variable production costs in terms of the final good.

$$\begin{cases} \underbrace{(e^i_{ht})^\lambda}_{\text{O\&M}} + \underbrace{e^i_{ht} z^i}_{\text{fuel}} & \text{if } e^i_{ht} \leq \xi^i_h K^i_t \\ \infty & \text{if } e^i_{ht} > \xi^i_h K^i_t \end{cases} \tag{9}$$

Variable production costs have two components: a type-specific fuel cost z^i per unit of energy, and operation and maintenance (O&M) costs $(e^i_{ht})^\lambda$, which are increasing in the amount of hourly energy produced. Capacity limits the amount of hourly energy that can be produced. Production costs become infinite, if a producer produces more than capacity. ξ^i_h is the intermittency parameter. It takes value 1 for dirty producers ($\xi^d_h = 1$). For clean producers $\xi^c_h \in [0, 1]$. Dirty producers can always produce energy up to their full capacity, while clean producers, in some hours, can produce energy only up to a fraction of their capacity because of intermittency. This fraction is determined by the exogenous intermittency parameter ξ^c_h, which I calibrate using microdata from the electricity sector. ξ^c_h is low in hours with low wind speeds and solar radiation.

2.4 Storage

Storage can be used to transfer energy across hours. e^s_{ht} is the energy stored/released in hour h. $e^s_{ht} > 0$ when energy is stored (i.e., energy produced by energy producers in hour h is stored into batteries), while $e^s_{ht} < 0$ when energy is released (i.e., batteries are discharged, and the final good sector uses the discharged energy). All the energy stored in period t must be released in the same period. [10]

$$\int_h e^s_{ht} dh = 0 \tag{10}$$

I define E^s_t as the total quantity of energy stored in period t [11]. The total energy stored

[10] Constraint (10) describes a simplified model of storage relative to what is used in the micro IO literature on electricity markets (see for instance Holland et al. (2022)). Typically a storage firm can perform many storage cycles within a year, that is, it can charge and discharge its batteries multiple times. In order to maximise its profits, it decides when to store and discharge energy considering the (expected) temporal sequence of prices in the hours at which it buys and sells energy. In the current calibration, I do not consider the temporal sequence of hours with different levels of demand and intermittency (and consequently different prices). Therefore, I cannot study the optimal choice of storage cycles. I have instead an aggregate constraint, which says that the total quantity of energy stored in an aggregate period t must equal the total quantity of energy released in period t. I calibrate the cost of storage capacity using the levelized cost of storage estimates. These estimates incorporate assumptions on the lifetime use of storage capacity and the patterns of storage cycles. I do not solve for the optimal storage cycle choices of storage firms, but I rely on the storage cycle patterns that are embedded in the studies producing LCOS estimates.

[11] $E^s_t = \int_h e^s_{ht} \mathbb{1}(e^s_{ht} > 0) dh$.

in period t is limited by the installed storage capacity K^s_t, built using labor.

$$K^s_t = exp(\int_0^1 ln(k^s_{jt})dj) \tag{11}$$

$$k^s_{jt} = A^s_{jt} l^s_{jt} \tag{12}$$

$$E^s_t \leq K^s_t \tag{13}$$

A^s_{jt} is the productivity of labor in building the intermediate input j to storage capacity.

2.5 Directed Technical Change in the Energy Sector

Scientists can direct their innovation toward clean energy, dirty energy, or energy storage technologies. There are two types of energy costs towards which R&D can be directed: capacity costs, which are the costs of building solar panels and fossil fuel power plants, and variable costs, which include fossil fuels and O&M costs. O&M costs represent a very small share of energy production costs (EIA, 2018), so technological progress in this area is disregarded. Given fossil fuels are used not only by the energy sector but also by the transport sector and the industrial sector, the technological evolution of fossil fuel costs cannot be determined exclusively by R&D in the energy sector. Therefore, I will model technological progress in fuel costs as an exogenous trend. Technological progress in fuel costs evolves at the same rate as the exogenous technological progress in the final good sector. Innovation in the energy sector increases the productivity of labor in building energy capacity.

There is a mass 1 of scientists who decide to direct their innovation towards one of three sectors: clean energy (c), dirty energy (d), or storage (s).

$$s_{ct} + s_{dt} + s_{st} = 1 \tag{14}$$

Every scientist has a probability of success of $\eta_i s^{\psi}_{it}$ for i=c,d,s. η_i is a sector-specific productivity, and $\psi < 1$ is a stepping-on-the-toe externality. A successful scientist improves the quality of a random intermediate used in the capacity production of the sector towards which she chose to direct her innovation by a factor γ.

In what follows the index i describes not only clean and dirty energy producers, but also energy storage firms (for $i = c, d, s$). I define the average technological level of capacity of type i energy producer as follows (for $i = c, d, s$):

$$ln(A^i_t) = \int_0^1 ln(A^i_{jt})dj \tag{15}$$

The average technological level evolves as follows:

$$A^i_t = \gamma^{\eta_i s_i^{1-\psi}} A^i_{t-1} \tag{16}$$

2.6 Market Clearing and the Economy Resource Constraint

The representative household inelastically supplies 1 unit of labor. Labor is used to produce the final good (L_t^Y) and to build energy sector capacity (l_t^i for i=c,d,s). Labor market clearing requires

$$L_t^Y + n_c l_t^c + n_d l_t^d + l_t^s = 1 \tag{17}$$

Energy market clearing requires that the hourly energy produced by energy producers equals the hourly energy consumption by the final good firm.

$$e_{ht} = n_d e_{ht}^d + n_c e_{ht}^c + e_{ht}^s \ \forall \text{ h} \tag{18}$$

Output is used for consumption, to build capital, and to pay variable energy production costs. The capital fully depreciates every period. Therefore, the economy resource constraint is as follows:

$$Y_t = C_t + K_{t+1} + \sum_{i=c,d} n_i \int_h (e_{ht}^i)^\lambda + e_{ht}^i z^i dh \tag{19}$$

3 Decentralized Economy

3.1 Household and Final Good Firm

The household maximizes the discounted sum of per period utility (1) subject to a per period budget constraint $w_t L_t + R_t K_t + \pi_t = C_t + K_{t+1}$. The HH inelastically supplies L_t units of labor every period t. She receives profits as a lump sum transfer from energy and capacity producers.

The final good firm decides how much capital, labor, and energy to buy in order to maximize profits. The final good firm buys hourly energy e_{ht} from energy producers at the hourly price p_{ht}^e.

$$\max_{K_t, L_t, e_{ht}} Y_t - R_t K_t - w_t L_t^Y - \int_h e_{ht} p_{ht}^e dh \tag{20}$$

subject to the Leonthief production function (4). Replacing in demand (5), I obtain

$$\max_{K_t, L_t, e_{ht}} Y_t - R_t K_t - w_t L_t^Y - E_t p_{E_t} \tag{21}$$

where p_{E_t} is the average hourly energy price, defined as follows:

$$p_{E_t} = \int_0^1 q_h p_{ht}^e dh \tag{22}$$

As described in section 2.2, the relative hourly energy demand is fixed and determined by the exogenous parameter q_h.The aggregate energy demand from the final good sector responds to changes in the average hourly price, but the relative hourly energy demand does not respond to changes in the hourly prices. This choice is motivated by empirical results that electricity consumers respond to changes in average prices and not to changes in real time prices (Fabra et al., 2021; Ito, 2014).

3.2 Hourly Energy Production and Prices

Energy producers sell energy at the hourly energy price p^e_{ht}. Hourly energy prices are an endogenous object that is determined in equilibrium. Energy producers make capacity and energy production choices taking the distribution of hourly energy prices as given. The equilibrium price distribution must be such that, given the hourly energy prices, optimal hourly energy supply by energy producers equals optimal hourly energy consumption by the final good firm. A competitive firm produces capacity and sells a unit of capacity to type i energy producer at price p^i_t. I will describe how this price is determined in the coming subsections.

The decision problem of type i energy producer can be divided into two steps. In the first step, for a given capacity K^i, she chooses e^{i*}_h that maximizes $\pi^i_h(K^i) = p^e_h e^i_h - (e^i_h)^\lambda - e^i_h z^i$. In the second step, given maximized hourly profits $\pi^{i*}_h(K^i)$ for every possible capacity value, she chooses the capacity level K^{*i} that maximizes total profits for period t $\pi^i = \int \pi^*_h(K^i)dh - K^i p^i$. This *two-step-decision problem* results in the following optimal choices.

Result 1 *The optimal hourly production choice of type i energy producer for a given distribution of hourly prices is:*

$$\underbrace{e^{i*}_{ht}}_{\text{hourly production choice}} = \begin{cases} 0 & \text{if } p^e_{ht} \leq z^i \\ \left(\frac{p^e_{ht} - z^i}{\lambda}\right)^{\frac{1}{\lambda-1}} & \text{if } z^i < p^e_{ht} \leq \underbrace{\lambda(\xi^i_h K^i_t)^{\lambda-1} + z^i}_{\text{capacity constrained price}} \\ \xi^i_h K^i_t & \text{if } p^e_{ht} > \underbrace{\lambda(\xi^i_h K^i_t)^{\lambda-1} + z^i}_{\text{capacity constrained price}} \end{cases} \tag{23}$$

Proof: This expression can be derived by maximizing the hourly profit $\pi_h(K^i) = p^e_h e^i_h - (e^i_h)^\lambda - e^i_h z^i$.

The optimal hourly production choice function is consistent with empirical facts on energy producers' participation in the electricity market. Low variable costs producers tend to be always active, in hours with both high and low prices, while high variable costs producers only enter the market when prices are high.

Result 2 *The optimal capacity choice of energy producers, who take hourly energy prices as given, is implicitly defined by the following equation:*

$$\underbrace{K_t^{i*}}_{\text{capacity choice}} \; : \; \underbrace{\int_{h^*(K_t^{i*})}^{h_{max}} [p_h^e \xi_h^i - \lambda(\xi_h^i K_t^{i*})^{\lambda-1} \xi_h - z\xi_h^i] dh}_{\text{capacity MB=increase in profits in constrained hours}} \; = \; \underbrace{p_t^i}_{\text{capacity MC}} \tag{24}$$

Proof: see Appendix.

I order the hours by increasing hourly prices. $h^*(K_t^i)$ is the first hour where producer i becomes capacity constrained. At $h^*(K^i)$ and at all hours between $h^*(K^i)$ and h_{max}, producer type i optimally produces at capacity. The optimal capacity choice does not affect the producer's profit level or optimal choices when she is unconstrained. Therefore, the optimal capacity choice only depends on its effect on the profits when the producer is constrained.

3.3 Storage

A representative price taker storage firm chooses how much energy to store/release every hour h (e_{ht}^s). Similarly to the capacity of energy producers, a competitive firm produces storage capacity and sells it to the storage firm at a constant per unit price p_t^s.

The *profit maximization problem* of the storage firm is the following:

$$\max_{e_{ht}^s} (p_{ht}^{e,max} - p_{ht}^{e,min}) E_t^s - K_t^s p_t^s \tag{25}$$

subject to conditions (10) and (13). The first term in their profit maximization problem is revenues, given by the price differential between the hours at which the firm buys and sells energy times the total quantity of energy stored (E_t^s). To maximize profits, the storage firm buys energy in the hours with the minimum price ($p_{ht}^{e,min}$) and sells it during the hours with the maximum price ($p_{ht}^{e,max}$). The second term in expression (25) is the cost of building storage capacity, given by the storage capacity K_t^s times the capacity price p_t^s. The following Lemma characterizes when storage is used and the impact of storage on prices in equilibrium. The proof is provided in the Appendix.

Result 3 *If the quantity of energy stored is positive, the difference between the maximum and the minimum hourly energy price ($p_{ht}^{e,max} - p_{ht}^{e,min}$) equals the cost of storing one additional unit of energy (p_t^s). If the price difference is smaller than the storage cost (p_t^s), the quantity of energy stored is zero.*

$$p_{ht}^{e,max} - p_{ht}^{e,min} \leq p_t^s \text{ with equality if } E_t^s > 0$$

If the maximum-minimum hourly price differential is lower than the cost of storing energy, the storage firm does not build capacity and does not enter the market. The revenues the storage firm would make by buying in hours with low prices and selling in hours with high prices would not be enough to recover the costs of building storage capacity. If the price differential is large, the storage firm enters the market. It drives down the price differential by buying in hours with low prices and selling in hours with high prices. In an equilibrium with storage, the price differential equals the marginal cost of building capacity to store a marginal unit of energy. The storage firm makes zero profits in equilibrium.

3.4 Static Analysis of the Energy Market

Before turning to the dynamic part of the model, I further characterize the static equilibrium in the energy market. I perform a comparative static analysis to study how different levels of clean energy and energy storage technologies affect the clean energy share. The following results hold assuming a fixed level of aggregate energy demand E_t. These results rely on the assumption that the intermittency parameter takes on either a positive value equal to ξ or a zero value, $\xi_h^c \in \{0, \xi\}$ with $0 < \xi < 1$. As explained in Section 5, ξ_h^c will satisfy this property in the main model calibration. Proofs can be found at Appendix Section 11.4.

Result 4 *p^c and p^s are, respectively, the costs of the clean energy and energy storage technologies. Given a high p^c, if p^s decreases, the clean energy share can increase or decrease. Given a low p^c, if p^s decreases, the clean energy share increases.*

Result 4 says that when the clean technology is less advanced, an improvement in the storage technology can have a positive or a negative effect on the clean energy share. When clean technology is advanced, an improvement in the storage technology has an unambiguously positive effect on clean energy use. When p^c is high, the clean energy share is low. At this stage, prices are mostly determined by the marginal costs of the dirty energy technology. Clean energy supplies energy in the market also at hours with high demand and high prices, which are the most profitable. Increased use of storage pushes down high prices, which decreases the profitability of clean producers, therefore possibly decreasing the clean energy share. Instead, when p^c is low, the clean energy share is high. Prices tend to be low in hours with high clean availability and high in hours with no availability of clean energy. In this case, increased use of storage drives down the prices in the hours in which clean is not available, and it drives up the prices in the hours in which clean is plentiful, therefore increasing the clean relative to dirty profitability. Result 4 is in line with findings from the IO literature on electricity markets. Holland et al. (2022) show that if renewables generate electricity in high price periods, then increased use of storage can decrease their profitability.

Result 5 *p^c and p^s are, respectively, the costs of the clean energy and energy storage technologies. Given a high p^s, if p^c decreases, the clean energy share can increase or stay constant. It exists a level of p^c, $\bar{p}^c > 0$, such that for $p^c < \bar{p}^c$, as p^c decreases, the clean energy share remains constant at a level strictly smaller than 100%. Given a low p^s, if p^c decreases, the* *clean energy share increases or stays constant. If p^s is sufficiently low, it exists a level of p^c, $\bar{\bar{p}}^c > 0$, such that for $p^c \leq \bar{\bar{p}}^c$ the clean energy share gets to 100%.*

Intuitively, if the storage costs are high, the clean energy share remains limited to a level well below 100% because of the intermittency problem. In this case, the clean energy share equals the share of aggregate energy demand realized in hours with a positive intermittency parameter ($\xi_h^c > 0$).

3.5 Direction of Innovation in the Energy Sector

A successful scientist improves the quality of a random intermediate input used to build energy capacity. This will cause the price of the capacity of a given sector to decrease. The successful scientist becomes the monopolist producing the intermediate input. A competitive fringe can use the previous technology to produce intermediates, which limits the monopolist's markup. The monopolist sells the intermediate at the price

$$p_{jt}^i = \frac{\gamma w}{A_{jt}^i}$$

A competitive firm buys intermediates, produces capacity, and sells capacity to energy producers. By aggregating across intermediates and solving the problem of the competitive capacity producer, I can derive the price per unit of capacity p_t^i:

$$p_t^i = \frac{\gamma w}{A_t^i} \tag{26}$$

The larger the level of technology in sector i, the lower the costs of building capacity for energy producers in sector i.

The scientist directs her innovation toward the sector with the highest expected profit. The expected profits in the three different sectors are:

$$\Pi_{ct} = \eta_c s_{ct}^{-\psi}(1 - \frac{1}{\gamma})n_c p_t^c K_t^c \tag{27}$$

$$\Pi_{dt} = \eta_d s_{dt}^{-\psi}(1 - \frac{1}{\gamma})n_d p_t^d K_t^d \tag{28}$$

$$\Pi_{st} = \eta_s s_{st}^{-\psi}(1 - \frac{1}{\gamma})p_t^s K_t^s \tag{29}$$

Expected profits depend on the size of the three different sectors. If, for instance, more dirty capacity is built relative to clean capacity, a scientist will expect to make more profits by innovating on dirty capacity, because it can sell to a larger market. At the same time, innovators expect to make more profits by innovating on less advanced technologies, given they sell at a higher price. Price effect and market size effect play crucial roles in determining the direction of innovation (Acemoglu et al., 2012).

At an interior equilibrium, the expected profits in the three sectors are equalized, giving the following innovation conditions.

$$\eta_c s_{ct}^{-\psi} n_c p_t^c K_t^c = \eta_d s_{dt}^{-\psi} n_d p_t^d K_t^d = \eta_s s_{st}^{-\psi} p_t^s K_t^s \tag{30}$$

4 The Planning Problem and Optimal Policies

This section describes the optimal allocation and optimal policies. First, I define the social planner problem (SPP). Secondly, I characterize some key relationships that hold in the optimal allocation. Finally, I compare how the optimal allocation differs from the allocation in the decentralized economy and I identify the optimal policies. Further details on the SPP solution can be found in Appendix section 11.3.

4.1 The Problem, Optimal Energy Use, and Innovation

The SP maximizes the discounted sum of utility

$$\max \sum_{t=0}^{\infty} \beta^t log(C_t) \tag{31}$$

subject to equations (19), (18), (5), (10), (17) and (14). In addition, the SP must satisfy the following constraints:

$$e_{ht}^c \leq \xi_h^c \gamma^{\eta_c s_{ct}^{1-\psi}} A_{t-1}^c l_t^c \tag{32}$$

$$e_{ht}^d \leq \gamma^{\eta_d s_{dt}^{1-\psi}} A_{t-1}^d l_t^d \tag{33}$$

$$\int_{h:e_{ht}^s>0} e_{ht}^s dh \leq \gamma^{\eta_s s_{st}^{1-\psi}} A_{t-1}^s l_t^s \tag{34}$$

where e_{ht}^i is energy produced by one type i energy producer. l_t^i is labor used to produce the capacity of one type i energy producer. The right-hand side of equations (32)-(34) represents, respectively, clean energy, dirty energy, and energy storage capacity. These constraints indicate that dirty and clean capacity limits hourly energy production by energy producers, and storage capacity limits total energy stored in period t.

I define λ_{2t}^h as the multiplier on constraint (5), and μ_{1t}^{hi} as the multiplier on constraint (32)-(33) for i=c,d. The optimal choice of hourly energy production by clean and dirty energy producers satisfy the following equations:

$$\frac{\lambda_{2t}^h}{q_h} = \beta^t u'(C_t)\lambda(e_{ht}^c)^{\lambda-1} + \mu_{1t}^{h,c} \tag{35}$$

$$\frac{\lambda_{2t}^h}{q_h} = \beta^t u'(C_t)\tau_t + \beta^t u'(C_t)\Big(\lambda(e_{ht}^d)^{\lambda-1} + z^d\Big) + \mu_{1t}^{h,d} \tag{36}$$

$$\tau_t = \frac{\sum_{s=0}^{\infty}\beta^{t+s}u'(C_{t+s})Y_{t+s}\Big[\epsilon_{t+s}(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s)\xi_d\Big]}{\beta^t u'(C_t)} \tag{37}$$

Equation (35) is the FOC of the SPP with respect to hourly energy production by a clean energy producer, e_{ht}^c. Equation (35) summarizes the costs and benefits of producing a unit of clean energy. The marginal benefit, on the LHS, is the benefit of relaxing constraint (5) in hour h and therefore producing more aggregate energy and output. The RHS equals the marginal cost, which is composed of two terms. The first term is the marginal hourly production cost paid in terms of the final good. The second term is the marginal capacity cost[12], which is positive if the producer is capacity constrained in hour h and zero otherwise. Equation (36) is the FOC with respect to hourly energy production by a dirty energy producer, e_{ht}^d. The marginal cost of producing dirty energy has a third component. This new component is the first term on the right hand side of the equation, $\beta^t u'(C_t)\tau_t$, where τ_t is given by equation (37). It represents the marginal environmental cost of dirty energy, which is the present discounted value of current and future output damages resulting from the GHG emissions of a marginal unit of dirty energy.

The optimal innovation allocation between clean, dirty, and energy storage technologies in the SPP satisfies the following conditions:

$$\eta_d s_{dt}^{-\psi}\Big[n_d\bar{p}_t^d K_t^d + \sum_{s=1}^{\infty} n_d\bar{p}_{t+s}^d K_{t+s}^d\Big] = \eta_c s_{ct}^{-\psi}\Big[n_c\bar{p}_t^c K_t^c + \sum_{s=1}^{\infty} n_c\bar{p}_{t+s}^c K_{t+s}^c\Big]$$
$$= \eta_s s_{st}^{-\psi}\Big[\bar{p}_t^s K_t^s + \sum_{s=1}^{\infty} \bar{p}_{t+s}^s K_{t+s}^s\Big] \tag{38}$$

where $\bar{p}_t^i = \beta^t u'(C_t)\frac{\bar{w}_t}{\gamma^{\eta_i s_{it}^{1-\psi}} A_{t-1}^i}$ is the shadow price of one unit of capacity in the SPP and $\bar{w}_t$ is the shadow price of labor. The marginal benefit of employing a scientist in clean energy research must equal the marginal benefit of employing a scientist in dirty energy research or in energy storage research. The marginal benefit of employing a scientist in research in a given sector is given by the probability that the scientist makes an innovation, $\eta_i s_{it}^{-\psi}$, times the value of additional capacity that can be built thanks to improved labor productivity. The SP considers that innovating on today's technology has an impact on today's labor productivity as well as on labor's productivity in all future periods. The condition must be satisfied with

[12] The multiplier $\mu_{1t}^{h,i}$ measures the cost of relaxing the constraint (32)-(33), which is the cost of taking away labor from other uses and moving it to the production of type i producer capacity.

equality only in case of an interior solution. Proofs and additional results are in Appendix section 11.3.

4.2 Optimal Policies

I now compare the SPP solution with the allocation in the decentralized economy. First of all, the optimal allocation differs from the decentralized economy solution because the SP internalizes the environmental externality of dirty energy. The environmental cost of using dirty energy is captured by the first term appearing in the marginal cost of dirty energy as shown in equation (36). The optimal hourly production choice by energy producers in the decentralized economy, which is summarized in Result 1, does not consider this externality.

Secondly, the optimal allocation differs from the competitive equilibrium in the innovation allocation. The difference between the two allocations becomes clear by comparing the SP innovation allocation in equation (38) with the decentralized equilibrium innovation equation (30). The innovation conditions are different because the SP also considers the impact of current innovation on future productivity. Scientists do not internalize the impact of their research on future productivity in the decentralized economy because of a building on the shoulder of the giants' externality and because of limited patent protection. Only the leading technology is used to produce. Once a new scientist builds on the innovation of an old scientist, she will be the only one selling to the market. She will not compensate the old scientist for having built on her innovation (building on the shoulder of the giants' externality). The present-oriented incentives of scientist in the decentralized economy is motivated by the present-profit maximizing nature of innovative firms and by the limited duration of patent protection. It is important to note that also if long-lasting patents were introduced in the model, the innovation allocations would not be the same. As shown by Hémous and Olsen (2021), even if patents were long-lasting, future innovators would compensate earlier innovators based on the current levels of productivity disregarding the impact on future productivity levels, which would make the decentralized innovation allocation differ from the optimal one.

The last difference between the SPP solution and the decentralized economy allocation stands in the capacity levels. The capacity of energy producers is built using a continuum of intermediate inputs produced by monopolists. In the decentralized economy, monopolists charge a markup over the production costs, while the SP only considers the actual production costs of building new capacity. The markup charged by monopolists results in underproduction of capacity in the decentralized economy.

Theorem 1. *The optimal allocation can be implemented in the decentralized economy using a carbon tax, research subsidies to clean energy and energy storage innovation, and a capacity intermediate inputs subsidy.*

Proof: see Appendix section 11.3. The optimal carbon tax τ_t is defined by equation (37).

The introduction of the optimal carbon tax in the decentralized economy increases the costs of dirty energy, which will result in a larger clean energy share. The market size effect, that is the relative size of the clean and dirty energy markets, affects innovation incentives. Therefore, the carbon tax itself will push for more clean innovation in the decentralized economy relative to the laissez-faire outcome. Whether, in addition, R&D subsidies to clean or energy storage technologies are needed depends on how large are the current benefits to innovation, captured by condition (30), relative to the future benefits to innovation, characterized by condition (38).

If the optimal clean energy transition is gradual, the dirty energy share will initially be large and then gradually decrease. If the optimal allocation delivers a complete clean energy transition, at some future period, the economy will produce energy using only the clean energy and energy storage technologies, and the dirty energy share will get to zero. In this case, the current benefits from innovation are relatively larger for dirty technologies than the sum of current and future benefits. A positive current dirty energy share will push innovators to innovate on the dirty technology, as there is a market they can sell their innovation to in the present. The SP internalizes larger future benefits of current innovation on clean and storage technologies than on dirty energy technologies, because dirty technologies will keep being used only for a limited period of time. In this case, the optimal allocation entails a larger clean and/or storage innovation and lower dirty energy innovation than in the decentralized economy. This allocation can be implemented in the decentralized economy by subsidizing R&D on clean and/or storage technologies.

If the storage technology is relatively more expensive than the clean and dirty energy technologies, as indeed is the case in the data (see Sections 5 and 6), the decentralized economy could be characterized by a corner solution. If storage technologies are relatively more expensive than clean and dirty, only dirty and clean energy are used in equilibrium in the decentralized economy. If storage is not used, it has a zero market size, which means no scientist will direct her innovation to storage. The storage technologies are not improved and not used in the decentralized economy. If the social planner optimally decides to pursue the clean energy transition, she considers that in the future storage technologies will be needed to complete the clean energy transition because of the intermittency problem. By considering the future benefits of developing the storage technology, the social planner wants to do a higher level of innovation on storage technologies relative to the decentralized equilibrium. This innovation allocation can be implemented in the decentralized economy with R&D subsidies on energy storage technologies.

5 Calibration

The model is calibrated to the US economy in 2019. A model period corresponds to five years. I consider an economy with a 500-year horizon. I first describe the calibration of the energy market using detailed microdata from the US electricity sector, which is a key novelty of this project. Secondly, I present the calibration of the innovation model. Finally, I outline the calibration of the aggregate economy and the climate cycle.

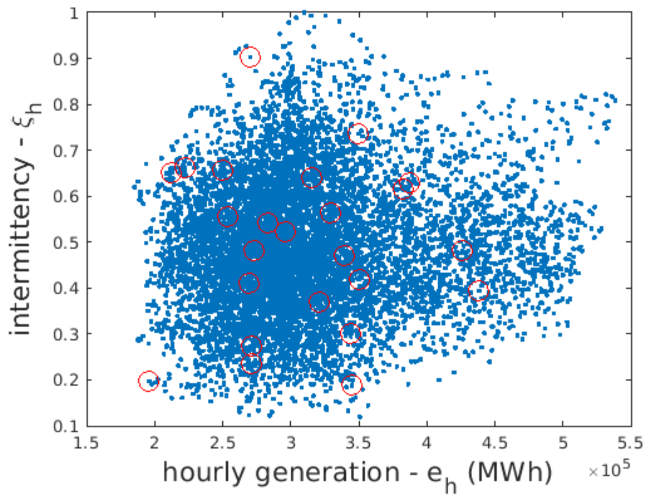
Energy Sector. I calibrate capacity costs and variable costs of energy producers using microdata from the US Federal Energy Regulatory Commission (FERC). FERC provides data on capacity costs and fuel costs for all utilities having more than 10 GWh of annual sales. Clean (dirty) producers costs are estimated as the capacity-weighted average costs for all solar and wind (coal and gas) producers in the FERC microdata. Capacity costs reported by producers are adjusted to the five-year lifetime of capacity in the model.[13] The capacity costs of storage are taken from the levelized cost of storage (LCOS) estimates from Lazard (2020). The starting levels of capacity costs (p_{c0}, p_{d0}, p_{s0}) are reported in Table 1, together with the fuel cost parameter z^d. The starting levels of technology (A_0^c, A_0^d, A_0^s) are computed as the ratio of the starting level of wage w_0 over the initial capacity cost ($A_0^i = \frac{w_0}{p_{i0}}$ for $i = c, d, s$). The calibration of the starting level of wage is described below in the aggregate economy paragraph.

I assume a total of 1,000 energy producers ($n_c + n_d = 1000$). Each producer can be clean or dirty. The percentage of producers who use dirty and clean technologies are taken to match the same percentage of producers active in the US in 2018 (EIA (2018), Table 4.3). The parameter λ determines the importance of operation and maintenance (O&M) costs relative to fuel costs for all producers. This parameter is calibrated to match the average share of O&M costs over fuel costs per unit of energy produced for gas producers (EIA (2018), Table 8.4). I solve for the equilibrium in the energy sector given initial costs and the US aggregate energy demand for 2019, and I select the level of λ such that the model average ratio of O&M costs over fuel costs per unit of energy produced for gas producers matches the one from the data. This calibration ensures that the choice of the finite number of producers does not affect the average production costs.

Hourly energy demand q_h and the intermittency parameter ξ_h^c are calibrated using hourly energy production data from the Energy Information Administration (EIA). The EIA 930 data provide information on hourly generation by fuel type (coal, gas, solar, wind) for the US. Total hourly electricity generation is used to measure the parameter q_h, which measures the share

[13] Given the data on capacity cost per MW from the FERC data and data on the lifetime of different plants from Lazard estimates, I compute the annualized capacity cost (the annual payment such that the producer is indifferent between paying the upfront investment cost or the annual payment). Given the annualized capacity cost, I can then compute the model capacity cost as the upfront payment that makes the producer indifferent between paying this upfront payment or paying the annualized capacity cost for five years (given a period t in the model is five years).

Figure 1. Intermittency and Hourly Energy Demand



Notes: This figure shows the data used to calibrate the intermittency parameter ξ_h^c and the hourly energy demand parameter q_h. A blue dot represents an hour of 2019. The figure reports the estimated level of intermittency, which can be read on the y-axis, and the total hourly electricity generation, which can be read on the x-axis, for the US. The estimated intermittency is computed as the fraction of the total installed clean capacity in the US in 2019 that is used to generate electricity in a given hour. The red circles indicate the 24 representative hours selected for the calibration. Data source: 930 data from the US EIA.

of aggregate energy (energy produced over a whole five-year period) demanded/generated in a single hour h. The actual generation of wind and solar energy infers the availability of sun and wind. The intermittency parameter ξ_h^c is measured as the fraction of installed clean capacity used to produce energy in a given hour. This derivation of the parameter ξ_h^c from actual generation data assumes that clean energy is always used when available.[14] Figure 1 shows total hourly electricity generation and the intermittency parameter ξ_h^c for 2019 in the US. 2019 is the first year for which data are available. Hours in which a large fraction of the installed clean capacity is used to generate energy are hours with a high ξ_h^c. These are hours with high wind speeds and solar radiation. The current calibration of the model assumes that ξ_h^c can take on only two values: $\xi_h^c \in \{0, \xi\}$. There are two types of hours. A set of hours have $\xi_h^c > 0$, during which all installed clean energy capacity can be used to produce energy. In the remaining hours, $\xi_h^c = 0$, during which only dirty energy sources or energy storage can be used to produce energy. Assuming ξ_h takes only the values 0 and 1 is a simplification which helps in the numerical solution. In reality, ξ_h can take on any value between 0 and 1.[15]

[14]This a realistic assumption given that clean energy sources have lower marginal costs than dirty energy sources, and so they tend to place first in the merit order curve, which determines which source of energy is to be used first to satisfy demand in the electricity market. In addition, given the relatively low clean energy share in the US in 2019 (13%), the overall level of curtailment can be assumed to be low.

[15]In hours with high solar radiation/wind speed, installed wind and solar capacity can be used to produce at the capacity nameplate potential. There are hours without wind or solar radiation in which $\xi_h = 0$. There is also a range of medium solar radiation/wind speeds where the clean energy generated is just a fraction of the installed nameplate capacity of solar panels and wind turbines.

From the hours presented in Figure 1, I select 24 representative hours. The selected hours are representative in terms of the degree of intermittency, of the hourly energy demand, and of the correlation between intermittency and demand. I classify hours as having $\xi_h^c = 0$ when the clean generation is lower than the 20th percentile of clean generation in all hours. The remaining hours have $\xi_h^c = \xi$. ξ is calibrated such that the average degree of intermittency in the model is the same as in the data.

Innovation. The parameter γ, which measures the step size of innovation and also the markup charged by capacity producers; and the parameter ψ, which measures the stepping-on-the-toe externality in the innovation sector, are taken from Acemoglu et al. (2023). The authors calibrate their model of directed technical change in the energy sector using micro-level data from the US energy sector. I assume the productivity of research η_i is the same in the clean, dirty, and energy storage sectors ($\eta_c = \eta_d = \eta_s$). The parameter η is calibrated to obtain the decrease in clean producers capacity costs in the decade 2009–2019 observed in the FERC microdata. The assumed clean innovation share in this period is taken to match the observed share of clean electricity over overall electricity patents of approximately 30% in the Patstat data for the same period.

Aggregate Economy. The GDP for the initial five-year period (2015–2019) Y_0 is obtained from the World World Bank (2023). The initial aggregate level of energy E_0 is based on total US electricity generation from the EIA 930 data. The share of capital in the economy α is 0.3, and the discount factor is 0.985 as standard in the literature. The share of energy υ equals 0.0188, which is the share of electricity sector revenues in the US in 2019. I assume a constant exogenous growth of capital and of aggregate TFP A_t equal to 2%. I normalize the labor supply L to 100. The initial labor used in the final good production is $L_0^Y = 96.6$, to match the energy sector employment share in North America of 3.4% reported from the IEA (2022). K_0 is calibrated to match a yearly 5% return. Once Y_0, E_0, L_0^Y, and K_0 together with the shares of the Cobb-Douglas production function are known, we can use the production function to derive A_0 and then compute the marginal product of labor to derive w_0.

Climate Cycle. The parameters of the climate cycle ϕ, ϕ_L, ϕ_0 are based on GHKT and the adaptation to a five-year period by AABH. I take an emission factor of 0.00063 GtC/thousand GWh, to match the emissions per unit of dirty energy in the US EIA (2020). I base the initial stock of carbon $S0_0$ on data from the NOAA observatory.[16] GHKT reports an initial stock of carbon $S0$ of 802Gtc, which is approximately equal to the observation from 2004 from the NOAA observatory. As permanent stock $S1_0$ I take the permanent stock from GHKT ($S1 = 684$Gtc) + 20% of accumulated emissions between 2004 and my calibration year, which is 2019. Accumulated emissions are always taken from the NOAA observatory. The initial decaying emissions $S2_0$ are the difference between $S0_0$ and $S1_0$.

In addition to the parameters reported above for the climate cycle, I assume an exogenous emissions path for emissions other than the US electricity sector. I use data on projected

[16] https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_annmean_mlo.txt

Table 1: Calibrated Parameters

Parameter	Value	Source
I. Energy Producers' Costs		
p_{c0}	637'680 \$/MW	Capacity-weighted average cost from FERC microdata
p_{s0}	191 \$/MWh	LCOS estimate from Lazard (2020)
p_{d0}	331'550 \$/MW	Capacity-weighted average cost from FERC microdata
z^d	27 \$/MWh	Capacity-weighted average cost from FERC microdata
λ	1.34	Match average share of O&M cost/fuel cost from EIA (2018)
ξ	0.62	Match the average degree of intermittency in the EIA 930 data
II. Innovation		
γ	1.07	Literature (e.g., AABH, 2023)
η	2.48	match observed decrease in clean energy costs over 2009–2019
ψ	0.5	Literature (e.g., AABH, 2023)
III. Aggregate Economy		
Y_0	106.9 trillion \$	World Bank Data
E_0	14348 thousand GWh	EIA 930 data
α	0.3	Literature (e.g., GHKT, 2014)
v	0.0188	Share of electricity sector revenues (US)
β	0.985	Literature (e.g., GHKT, 2014)
L	100	Normalization
L_0^Y	96.6	Match energy sector employment share of 3.4% from IEA report
K_0	19.68 trillion \$	Match a 5% yearly return in the initial period
g_A, g_K	2%	Consistent with 2% long-run growth
IV. Climate Cycle		
ϕ	0.0115	Literature (e.g., AABH, 2023)
ϕ_L	0.2	Literature (e.g., GHKT, 2014)
ϕ_0	0.2	Literature (e.g., AABH, 2023)
ξ^d	0.00063 GtC/thousand GWh	EIA (2020)
ϵ	5.3e-5	Literature (e.g., GHKT, 2014)
$S0_0$	877 GtC	NOAA gml data
$S1_0$	698 GtC	Literature (e.g., GHKT, 2014) and NOAA gml data
$S2_0$	179 GtC	Literature (e.g., GHKT, 2014) and NOAA gml data

emissions from DICE 2023 at the global level. I compute the emissions associated with US electricity production in 2020–2025 (assuming 2020 emission level stay constant for five years). I compute how large these emissions are relative to total world emissions between 2020 and 2025 projected by DICE 2023. I assume this share (9%), that is the share of global emissions coming from the US electricity sector, stays constant over time in the business-as-usual scenario. I subtract electricity emissions from the global emissions path and use this global emissions path from DICE 2023 as exogenous emissions (coming from other sectors in the US and from the ROW).

Further details about the numerical methods can be found in the Appendix section 11.4.

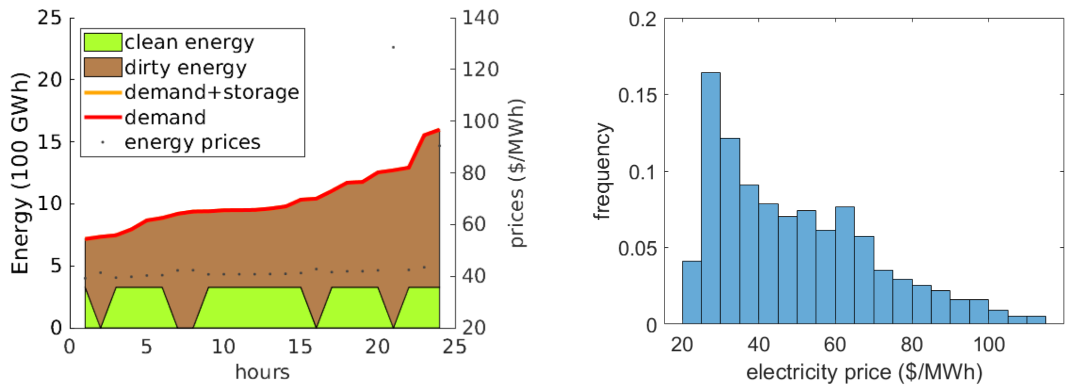
6 Validation: Model vs. Data

I now show that my model matches two key non-targeted moments in the data: the clean energy share and the level of hourly energy prices in the US electricity market.

Figure 2a shows how energy is produced across hours in the first five-year period of the model (2020–2025) in the decentralized economy. The x-axis shows the 24 representative hours, repeated for 365 days within a year. These hours are representative in terms of energy demand, intermittency, and correlation between energy demand and intermittency. Hours in the figure are ordered by hourly energy demand, represented by the red line. The green shaded area stands for clean energy production, while the brown area stands for dirty energy production. When solar panels and wind turbines cannot be used because of intermittency ($\xi_h^c = 0$), clean energy production drops to zero. The orange line represents energy demand plus storage. In this case, we do not see any orange line, because storage is not used (a more in-depth discussion of why storage is not used in the absence of policies is deferred to Section 7.2). Even in the absence of policies, clean energy is competitive in the market thanks to the rapid technological change observed in the last decades on clean energy sources. The clean energy share is 25%.

As explained in the previous section, the model is calibrated to the US economy in 2019. I use microdata from the US electricity sector to calibrate the degree of intermittency, hourly energy demand, and energy producers' costs. I do not target the clean energy share or the level of hourly energy prices. The model approximately replicates the clean energy share in the US. Clean energy accounts for 21.5% of total utility-scale electricity generation in the United States in 2022 (EIA, 2023). Figure 2a reports the equilibrium hourly energy prices, whose level can be read on the right y-axis. The level of hourly energy prices are in line with data on US hourly electricity prices supplied by the EIA, which are reported in Figure 2b. In most of the hours, prices are in the 30-60\$/MWh range. In a few hours, prices can peak to levels above 100\$/MWh.

Figure 2. Model Validation



(a) Energy Market in the Short Run (2020–2025)

(b) US Electricity Prices

Notes: Figure 2a displays the model equilibrium in the energy market in the decentralized economy in the first period (2020-2025). The x-axis shows the 24 representative hours that are repeated for 365 days within every year. The red line shows the total hourly energy demand (generation), which can be read on the y-axis. The shaded brown (green) area is dirty (clean) energy generation. The difference between the red and the orange line is energy storage generation, which is none in this case. The grey dots are hourly energy prices. The clean energy share is 25%, in line with the untargeted 21.5% clean electricity share in the US in 2022. Figure 2b shows hourly energy prices in US price data from the Energy Information Administration.

7 Results: The Role of Policy

In this section, I quantify the role of policy in the clean energy transition. First, I compare how the economy evolves in the decentralized equilibrium and the optimal allocation. I focus on describing the equilibrium in the energy market, the direction of R&D, and the optimal policies. Results on other outcomes can be found in the Appendix Section 11.7. Secondly, I investigate the impact of one of the major real-world policies implemented to accelerate the transition to clean energy: the clean energy subsidies introduced by the US Inflation Reduction Act. Finally, I compare welfare in the different policy scenarios.

7.1 Decentralized Economy

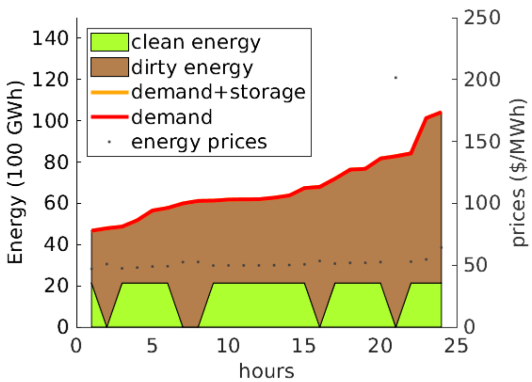
7.2 Clean and Dirty Energy Use

Figure 2a, previously discussed in section 6, shows how energy is produced in the short run in the decentralized economy. Even in the absence of policies, clean energy is competitive in the market thanks to the rapid development of clean technologies observed in the last decades. The clean energy share reaches 25%.[17]

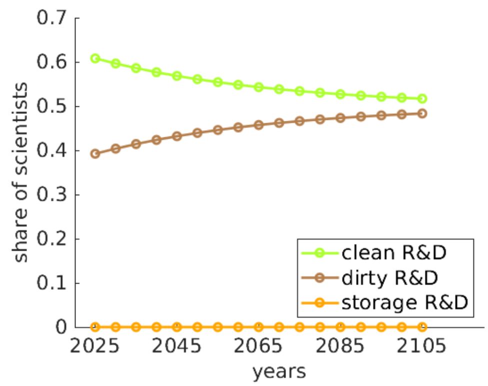
However, the long-run clean energy share remains low. Figure 3a shows how energy is

[17] The clean energy share is the share of the green shaded area (clean energy production) relative to the green plus the brown shaded area (total energy production).

Figure 3. Decentralized Economy



(a) Energy Market in the Long Run (2100–2105)



(b) Innovation

Notes: Figure 3a displays the equilibrium in the energy market in the decentralized economy in the long run. The x-axis shows the 24 representative hours that are repeated for 365 days within every year. The red line shows the total hourly energy demand (generation), which can be read on the y-axis. The shaded brown (green) area is dirty (clean) energy generation. The difference between the red and the orange line is energy storage generation, which is none in this case. The grey dots are hourly energy prices. Figure 3b shows the innovation allocation in the decentralized economy. The green line, brown line, and orange line describe, respectively, the share of scientists innovating on clean energy, dirty energy, and energy storage technologies.

produced in the long run. The clean energy share remains at a level between 25 and 35%. In the hours in which clean energy is not available, the market uses dirty energy instead of storage, because storage is relatively more expensive. Given that the dirty capacity is built for these hours, it is then cheaper to also use dirty energy to produce some of the energy in the hours in which clean is available rather than also building a large clean capacity. Intermittency creates an important market for dirty energy, and it limits the clean energy share in the long run. Because of intermittency, the market keeps developing and using dirty energy technologies.

Notably, storage is not competitive and is not used in the market. Storage firms make revenues by buying at hours with low prices and selling at hours with high prices. The difference between the maximum and the minimum hourly energy price (89$/MWh) is smaller than the cost of building storage capacity (205$/MWh). Therefore, storage firms decide not to build any capacity.

Additional figures on output, labor, consumption, and other outcomes can be found in Appendix section 11.7.

7.3 Innovation

Figure 3b illustrates the share of scientists doing clean, dirty, and storage innovation. Scientists innovate on clean and dirty energy technologies, and they have no incentives to develop storage technologies. Initially, scientists innovate more on clean than on dirty energy technologies. In the long run, the clean and dirty technologies progress at the same rate. There is no innovation in storage technologies. I will now explain further these results.

Initially, the market innovates more on clean energy technologies than on dirty energy technologies. The market size effect pushes scientists to innovate on the technology with a larger market, while the price effect pushes scientists to innovate on the least advanced technology. More dirty capacity is built than clean capacity. Therefore, the market size effect is larger for dirty technologies. Nonetheless, the price effect is larger for the clean technologies, which are initially less advanced.[18] Overall, the price effect prevails, and scientists innovate more on clean energy technologies. Faster technological progress in clean energy technologies closes the price gap between clean and dirty technologies. As this gap closes, scientists redirect their innovation toward dirty technologies. In the long run, the two technologies progress at the same rate.

Storage is relatively more expensive than the other energy sources. Therefore, it is not competitive, and it is not used in the energy market. Scientists do not have incentives to innovate on storage technologies as it is not used in the energy market. As explained in the previous section, storage is not used in the short run, because the equilibrium maximum price differential is smaller than the cost of building storage capacity. Whether storage is used in the following periods depends on how the price of the storage technology (p_t^s) and the difference between the minimum and the maximum hourly energy price (${p_{ht}^{e}}^{max} - {p_{ht}^{e}}^{min}$) evolve. As no innovation is done on the storage technology, storage capacity becomes relatively more expensive over time.[19] On the one hand, the continued innovation of clean and dirty energy technologies tends to decrease the equilibrium prices of hourly energy. On the other hand, the positive growth of aggregate energy demand E_t tends to increase equilibrium hourly energy prices. Figure 3a shows the first force prevails. The difference between the maximum and minimum hourly energy prices decreases over time. At the same time, the price of storage increases. Therefore, in the medium and long runs, storage firms do not build any capacity, and scientists do not innovate on storage technologies that have a zero market size.

7.4 Optimal Allocation

Figure 4a illustrates the clean energy share in the optimal allocation. The social planner (SP) who internalizes the environmental externality of dirty energy produces a much higher share of energy with clean energy sources relative to the decentralized economy. The clean energy share starts at 70% in the 2020–2025 period, and it reaches 80% by 2060 and 100% by 2075.

[18] It is important to notice that the model focuses on incentives to innovate on energy capacity technologies. The clean capacity is less advanced and more expensive than the dirty energy capacity. If we look instead at average production costs, the clean energy costs are lower than the dirty energy costs.

[19] The storage productivity A_s^t remains constant due to no innovation. The labor cost to build storage capacity increases because of positive wage growth. This second force results in the price of storage increasing over time. See Figures in Appendix section 11.7.

Figures 4b and 4c show the equilibrium in the energy market in the short and long run. The orange line shows storage use. When the orange line is above the red line, energy production by clean and dirty energy producers is larger than energy demand by the final good firm (red line). The difference between energy production and energy demand is energy stored. Storage firms are storing energy produced by energy producers (i.e., they are charging the batteries). When the orange line is below the red line, storage firms release (supply) energy that the final good firm uses. In hours with sun and wind availability ($\xi_h^c > 0$), the SP decides to produce a level of clean energy larger than the energy demand by the final good sector. This energy is stored and released to satisfy energy demand in the high demand hours with no sun or wind availability ($\xi_h^c = 0$). The SP internalizes the environmental costs of using dirty energy resulting in output damages. Even though the storage technology is initially substantially less advanced than the dirty technology, the SP uses more clean energy and storage once the environmental externality is internalized. The cost of the environmental externality increases over time, which leads the SP to increase the clean energy share over time. Figure 4c shows the optimal energy use in the long run when the clean energy share is 100%.

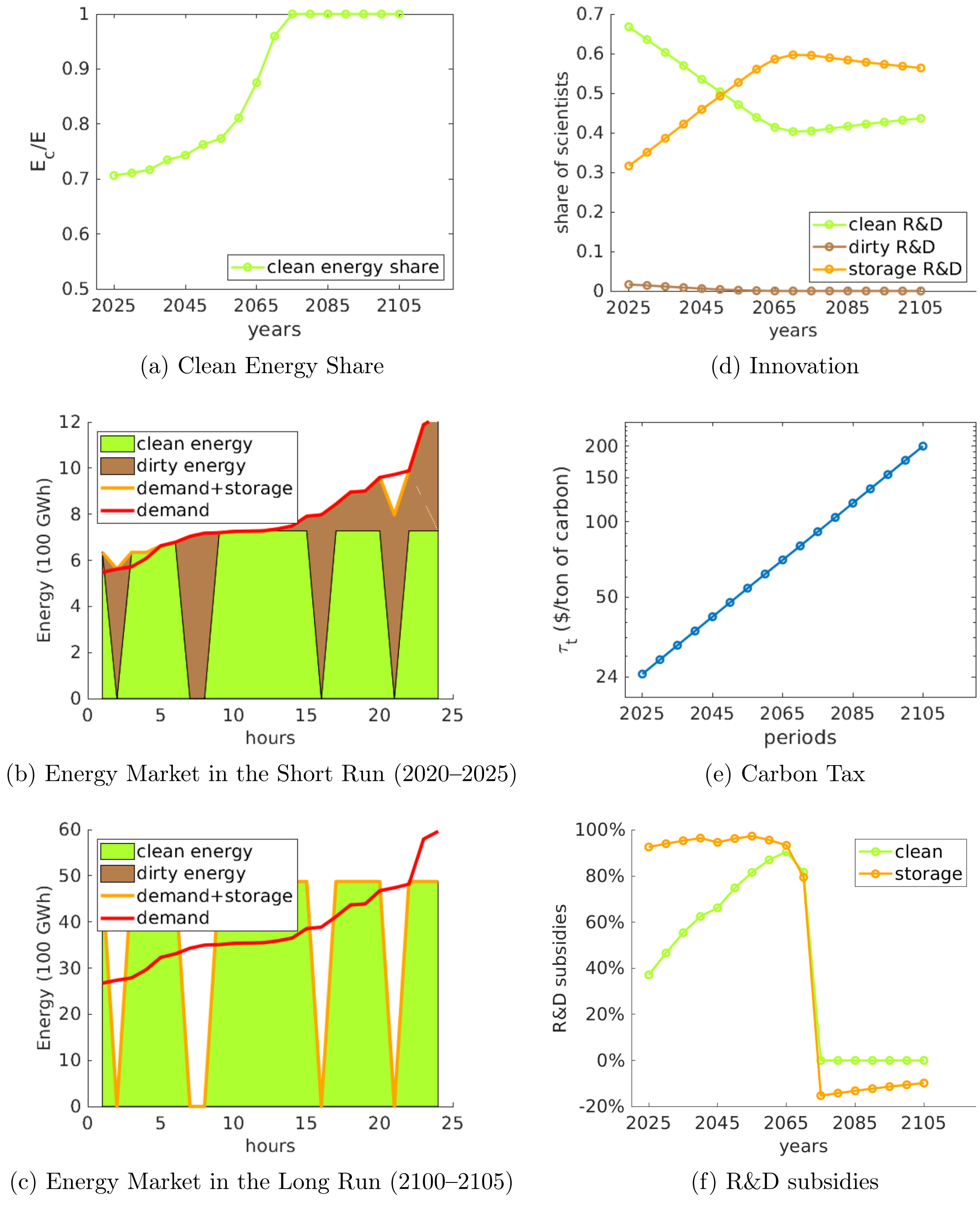
Figure 4d illustrates the allocation of R&D resources in the optimal allocation. The SP redirects all innovation away from dirty energy technologies towards storage technologies. Given from 2075 the SP uses only clean energy and energy storage, the benefit of innovating on clean and storage technologies is substantially higher than the benefit of innovating on dirty technologies, which are used for a limited number of periods. The SP innovates almost exclusively on clean and storage technologies (Figure 4d). The share of R&D resources devoted to dirty energy technologies starts at 1.6% and it decreases to zero. Dirty technology is no longer developed.

Figure 4e and 4f illustrate the optimal environmental policies. The value of the carbon (figure 4e) tax starts at 24\$/ton of carbon and then it increases over time. The values are comparable to other estimates in the literature (Acemoglu et al., 2023). The carbon tax is computed according to formula (37). The carbon tax that makes the competitive equilibrium equal to the optimal allocation is not uniquely determined in this setting after 2075. For instance, given dirty energy is not used at all in the optimal allocation from 2075, setting a carbon tax in the decentralized economy equal to the one determined by the formula (37) or higher after 2075 will result in the same optimal allocation.[20] It is still informative to compute it as it shows the value of the marginal damage of pollution. Even though the social planner reduces the emissions from the US electricity sector to zero, the model calibration assumes other sectors and countries continue to emit. This exogenous emissions path drives the continued increase in the carbon tax.

Figure 4f shows the optimal R&D subsidies. R&D subsidies are much higher for energy storage

[20] Given no innovation is done on the dirty energy technology, the dirty technology becomes relatively more expensive over time. Therefore, at some period after 2075, also a carbon tax lower than the one shown in Figure 4e, and eventually a zero carbon tax, will result in the same allocation.

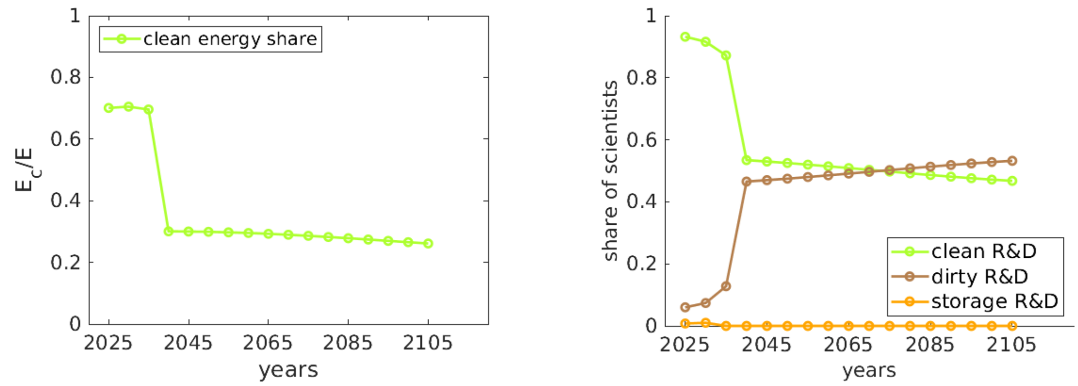
Figure 4. Optimal Allocation



(a) Clean Energy Share
(d) Innovation
(b) Energy Market in the Short Run (2020–2025)
(e) Carbon Tax
(c) Energy Market in the Long Run (2100–2105)
(f) R&D subsidies

Notes: These figures characterize the optimal allocation. Figure 4a displays the evolution of the clean energy share, which starts at 70%, and reaches 100% by 2075. Figures 4b and 4c characterize respectively the equilibrium in the energy market in the short and long run. The x-axis displays the 24 representative hours. The green (brown) shaded area is clean (dirty) energy generation. The difference between the orange and the red line is energy storage. Energy storage is positive (negative) when the orange line is above (below) the red line (i.e., batteries are being charged (discharged)). Figure 4d shows the share of scientists innovating in clean energy, dirty energy, and energy storage technologies. Figure 4e and 4f display respectively the optimal carbon tax and R&D subsidies.

Figure 5. Clean Energy Subsidies - IRA



(a) Clean Energy Share

(b) Innovation

Notes: These figures show the impact of the clean energy subsidies introduced in the US by the Inflation Reduction Act. Figure 5a displays the evolution of the clean energy share. Figure 5b describes the allocation of scientists to innovation on clean, dirty, and energy storage technologies.

technologies than for clean energy technologies. The market lacks incentives to innovate on energy storage technologies, whose capacity will be substantially increased only in the future.[21]

7.5 Suboptimal Policies: The Inflation Reduction Act

This section evaluates the impact of clean energy measures in the US IRA. One of the main measures introduced by the IRA to accelerate the clean energy transition is a production tax credit on clean energy. The value of the tax credit is 26\$/MWh for clean energy producers that satisfy some standards on inputs sourcing from the US and reduced to 5\$/MWh otherwise (Arkolakis & Walsh, 2023). I estimate the impact of this measure by introducing a subsidy of 26\$/MWh on clean energy and energy storage production up to 2035. The subsidy is financed through a labor income tax. If many producers do not satisfy the requirements to get the full level of the subsidy, the following figures will provide an upward-biased estimate of the IRA impact on the clean energy share.

Figures 5a and 5b show the clean energy share and innovation direction after the subsidy. The clean energy share increases from 25% in a scenario with no policy to 70%, which equals the optimal clean energy share. A positive level of storage is used (see figures on the equilibrium in the energy market in the appendix section 11.7). The share of R&D resources devoted to clean energy innovation increases substantially. Nonetheless, the subsidy is not

[21]The clean energy subsidies are computed as the subsidy that equalizes the clean over dirty ratio of scientists in the DE with policies to the optimal allocation. Given the presence of the stepping-on-the-toe externality in the innovation production function, as less and less scientists innovate on dirty, the returns of innovating on dirty becomes very large. This mechanism explains the increase in the clean energy subsidies over time, which is needed to enforce no innovation on dirty technologies.

Table 2: Welfare Effects

	Optimal Allocation (1)	IRA Clean Energy Subsidies (2)
% change	2.7	0.1

This table shows the % increase in consumption required in the decentralized economy allocation to achieve the same level of utility as in the optimal allocation (column 1) and in the IRA clean energy subsidies allocation (column 2).

large enough to incentivize the necessary storage innovation. When the subsidy is phased out, the clean energy share reverts to pre-subsidy levels, and clean innovation slows down. In the long run, the storage technology is not developed, and the intermittency problem still limits the clean energy share. The clean energy share, in the long run, is the same as in the case with no policy (25%-35%).

7.6 Welfare Effects

Table 2 displays the welfare effects of moving from the decentralized allocation to the optimal allocation or to the suboptimal allocation induced by the IRA clean energy subsidies. To compute the welfare effects, I find the permanent percentage change in consumption that applied to every period of the decentralized economy (DE) gives the same utility as the two other scenarios. The welfare effect of moving from the DE to the SP allocation is a 2.7% welfare increase. In comparison, the welfare effect of moving from the DE to the IRA allocation is 0.1%. The IRA accelerates the uptake of clean energy, but it does not induce the necessary development of storage technology. The long run clean energy share is unchanged, remaining limited by the intermittency problem. Therefore, the IRA welfare effects are substantially lower relative to the optimal allocation.

8 Results: Aggregate Substitutability between Clean and Dirty Energy

8.1 The Aggregate Elasticity of Substitution and the Storage Technology

In this section, I estimate the aggregate elasticity of substitution between clean and dirty energy sources. I show the elasticity of substitution changes over time, and it crucially depends on the development of the storage technology.

The macro climate literature typically uses a CES production function to represent the energy sector. Energy is produced with both clean and dirty energy inputs, which are combined with a constant and exogenous elasticity of substitution ρ.

$$E_t = (E_{ct}^{\frac{\rho-1}{\rho}} + E_{dt}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} \tag{39}$$

ρ measures how substitutable clean and dirty energy sources are. Due to endogeneity problems, empirically identifying the parameter ρ is challenging (Papageorgiou et al., 2017). Macro climate papers often show a range of results for different values of the elasticity ρ.

The value of the elasticity is key for optimal policies. Hassler et al. (2020) show that using an IAM with an elasticity of 0.95, following the metastudy Stern (2012), implies that subsidizing research on clean energy technologies results in larger emissions than absent the policy. As clean technology develops, energy becomes cheaper. Cheaper energy results in a larger energy demand. If the estimated substitutability between clean and dirty sources is moderately low, larger demand increases the consumption of both clean and dirty energy, resulting in larger emissions. If the elasticity was five times larger, subsidizing clean R&D would reduce emissions. Acemoglu et al. (2012) show that if the two energy inputs are gross complements ($\rho < 1$), the only solution to avert an environmental disaster is to stop long-run growth. If the inputs are gross substitutes, the optimal policy mix includes carbon taxes and R&D subsidies. Whether the implemented policies need to be permanent depends on the level of elasticity.

In addition, there is no guarantee that the elasticity of substitution is constant. It would be reasonable to expect the elasticity decreases with the further integration of renewables. It might be easier to substitute away from dirty to clean energy when the clean energy share is low. When we are already producing most of the energy with renewables in the hours with high wind speeds and solar radiation, it will become increasingly difficult to substitute clean for dirty.

My theoretical model does not rely on an assumed value for the elasticity of substitution. The substitutability between clean and dirty energy sources depends on the degree of intermittency and technological development in the energy sector, which affect the optimal choices of energy producers and, thus, the clean energy share.

I now compute the implied aggregate elasticity of substitution at different levels of clean energy technologies. For every level of clean energy technology, I compute the percentage change in the clean over dirty energy ratio for a 1% decrease in the average costs of clean energy technologies.[22] I perform this exercise for a constant level of aggregate energy demand, which is fixed at the level of energy use in the first period of the decentralized economy allocation. First, I compute the elasticity of substitution for a high cost of storage fixed at current technology levels. Secondly, I lower the storage cost by 90%[23] and recompute the

[22] I compute the average cost of clean energy as the LCOE. I focus on the LCOE as a measure of average costs because it is the energy cost measure typically used in the macro climate literature.

[23] To put in perspective such a decline, the LCOE of clean energy technologies declined by 87% between 2009 and 2019.

elasticity of substitution.

Figure 6a shows the elasticity of substitution ρ for a high storage cost. The x-axis shows the different levels of the clean energy technology at which ρ is computed, in terms of percentage decline relative to current costs. As expected, the substitutability between clean and dirty energy technologies is, on average, higher when the clean technology is less advanced.[24] When clean costs are high, the clean energy share is low. In the hours in which clean energy is available, clean energy is used to produce only a low share of energy. If clean technology improves, it will be easy to substitute clean for dirty. The share of clean energy increases in the hours in which clean is available. When clean technology is very advanced, the elasticity of substitution decreases because of intermittency. When we are already producing all of the energy with clean in the hours in which clean is available, it is difficult to substitute clean for dirty if storage is expensive. The elasticity of substitution decreases from an average level of 5 when clean costs are high, to an average level below 1 when costs are low.

Figure 6*b* shows the elasticity of substitution is drastically increased when storage costs are low. The estimated levels of ρ are higher for high clean energy costs. Most importantly, as the clean technology improves, the elasticity increases instead of decreasing. The elasticity becomes infinite at a 70% decrease in clean technology costs.

This exercise, together with the quantitative results on the decentralized economy, highlights the role of policy. An optimal policy mix of carbon taxes and R&D subsidies can avert the environmental disaster by pushing for storage technological change, increasing the elasticity of substitution, and leading to a complete clean energy transition.

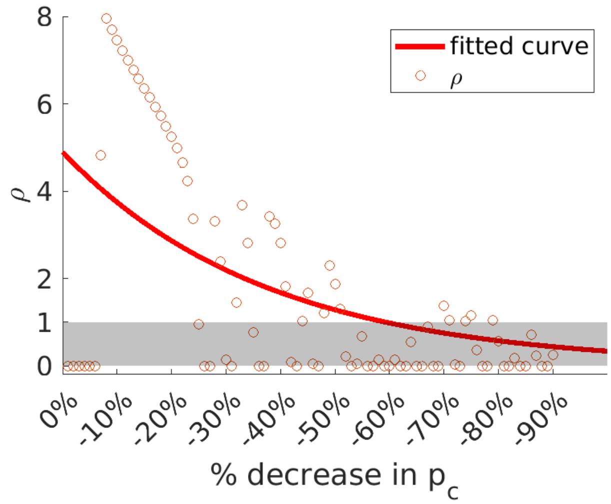
8.2 A Comparative Analysis: Microfounded Energy Sector vs. CES Energy Production Function

This section compares the CES energy sector model typically used in the macro climate literature with the microfounded energy sector model I develop in this paper explicitly considering the intermittency problem.[25] I first investigate how the clean energy share evolves in the

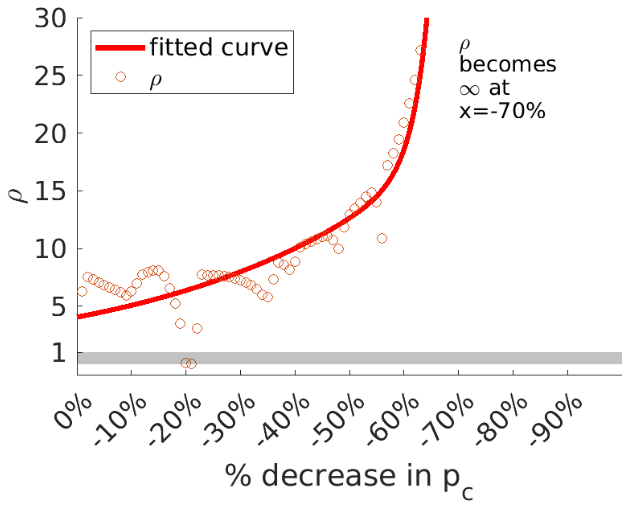
[24]Figure 6a shows that for low percentage decreases in the clean technology costs relative to current costs, the elasticity is zero. This is a feature of the baseline model calibration, where the intermittency parameter ξ_h^c can take only two values: zero or a positive value ξ. Appendix section 11.8 shows that if we relax this assumption and assume a continuous ξ_h^c taking on values between 0 and 1, the estimated values of the elasticity are also more continuous. Appendix section 11.4 further elaborates on why the elasticity takes on many zero values when clean technology costs are high in the baseline calibration. It shows there is an interval of values for the clean technology cost p_t^c such that the clean energy share is constant, and the change in technology costs only affects the hourly energy prices but not the clean energy share.

[25]The CES energy sector model considers the following aggregate energy production function: $E_t = (\kappa_c E_{ct}^{\frac{\rho-1}{\rho}} + \kappa_d E_{dt}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}$. Both the models are calibrated to the US economy in 2019. The calibration of the aggregate economy and the climate cycle is the same. The main differences are the calibration for the energy production costs and the share parameters κ_c and κ_d. The energy production costs in the CES model are calibrated such that in the calibration period, they equal the average costs of clean and dirty energy in the microfounded model. I measure the average costs in the microfounded model solution by computing the levelized cost of energy. The share parameters κ_c and κ_d are calibrated to match the same clean energy share as in the microfounded model in the calibration period of 25%. Both models are simulated with endogenous

Figure 6. Aggregate Elasticity of Substitution



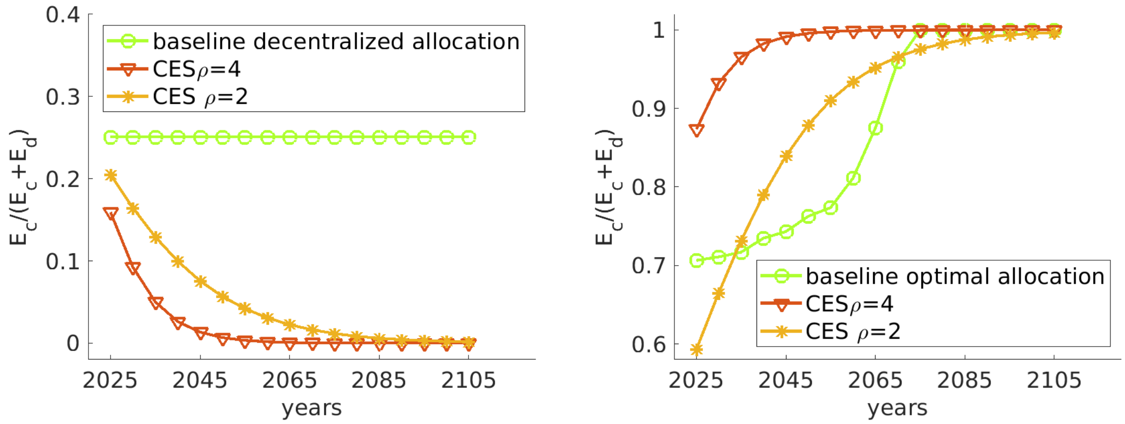
(a) High Storage Cost



(b) Low Storage Cost

Notes: These figures show the aggregate elasticity of substitution between clean and dirty energy sources implied by the microfounded energy sector model calibrated to the US electricity sector in 2019. The elasticity is computed at different levels of clean energy technologies. The level of the clean technology is shown on the x-axis, in terms of percentage decrease from current levels. Every dot shows the elasticity of substitution computed at a different level of clean energy technology. Figure 6a shows estimates of the elasticity of substitution ρ for a high cost of storage, at current levels. Figure 6b displays estimates for a low cost of storage, which is decreased by 90%. The shaded gray area represents values of the elasticity between 0 and 1.

Figure 7. CES vs. Microfounded Energy Sector



(a) Decentralized Economy

(b) Optimal Allocation

Notes: These figures compare the evolution of the clean energy share in a model with a CES energy sector, and in the model with a microfounded energy sector I develop in this paper. The CES model is solved for with two different values of the elasticity of substitution ρ. A higher value of 4, which is a reasonable value at current technology levels according to the estimates I provide in section 8.1, and a lower value of 2. Figure 7a shows the results for the decentralized economy. Figure 7b displays the optimal speed of the transition with optimal policies.

decentralized economy in the two models, and secondly, I compare the optimal speed of the clean energy transition.

Figure 7a shows how the clean energy share evolves in the decentralized economy in the CES model and in the model with a microfounded energy sector. The first remarkable difference between the two models is that the CES model delivers a more pessimistic prediction of the evolution of the clean energy share in the absence of policies. The clean energy share starts at a level of around 20%, and it gradually declines towards zero. Instead, a microfounded energy sector shows a clean energy share that remains positive but is limited to around 25% by the intermittency problem. There are mainly two mechanisms that explain this stark contrast. First of all, the microfounded energy sector distinguishes two energy costs: hourly variable costs and capacity costs, while the CES model only looks at average costs. Secondly, the microfounded model explicitly considers the intermittency problem. I now explain how these two mechanisms lead to the observed results.

The dirty energy share is initially large (75-83%) in all models. In the CES model, the dirty energy price is initially larger than the clean energy price, as the average cost of dirty energy is larger than the average cost of clean energy at current technology levels. Both the price effect and market size effect push researchers to innovate on the dirty technology. As

technological change. In the CES world, innovators can choose only between the clean and the dirty sectors, and there is no storage technology.

the dirty relative price decreases thanks to innovation, the dirty energy share increases. This mechanism continues until the clean energy share eventually gets to zero. This mechanism is the path dependence channel, emphasized by the existing literature (Acemoglu et al., 2012). The initial higher dirty energy share pushes for more innovation on dirty, which leads to an even higher dirty energy share in the future.

In the microfounded energy sector model, the initial conditions are the same. The clean energy share and the average cost of clean energy are relatively low. A key difference is that innovation in my model depends on the relative price of capacity, not on the relative average cost. The clean capacity price is initially relatively high.[26] The price effect pushes innovators to innovate relatively more on clean energy technologies. The clean energy source becomes relatively cheaper than the dirty over time, in terms of average cost. Clean energy has lower variable costs, which means within an hour, clean energy is always used first, before dirty energy. Yet, the clean energy share is limited by the problem of intermittency. In the long run, we use both clean energy, which is the cheaper energy source in terms of average costs and is always used first within an hour, and dirty energy, given dirty is the only energy source that can be used in some hours (hours with $\xi_h^c = 0$ during which clean energy cannot be used because of intermittency). Innovation improves both clean and dirty energy technologies.

This analysis shows that considering a microfounded energy sector model, and in particular the interplay between variable and capacity costs, is important to get more realistic predictions of how the clean energy share evolves in the absence of policies.

Figure 7b compares the optimal allocation in the CES model and in the microfounded energy sector model. The optimal speed of the clean energy transition in a CES model is different from the optimal speed in a microfounded energy sector model. Notably, if we assume an elasticity of substitution of 4, which is, as shown in Section 8.1, a reasonable value for the elasticity of substitution at current technology levels, a CES model delivers a much faster optimal transition than the one advocated for by a microfounded energy sector model. The former advocates an increase in the clean energy share to 87% by 2025, and a complete transition to 100% clean by 2050. The latter characterizes the optimal speed with an increase in the clean energy share to 70% by 2025, 81% by 2060, and 100% by 2075. A microfounded model delivers a slower optimal transition as it explicitly considers the intermittency problem and the high costs of using an energy storage technology that is initially less advanced. The social planner optimally decides to gradually increase the clean energy share from 70% to 100%, while it pushes for the development of energy storage technologies through R&D subsidies. Only once the storage technology is developed further is the clean energy transition accelerated.

Assuming a lower CES elasticity of substitution of 2 delivers an optimal speed of the

[26] The clean capacity price is much larger than the dirty capacity price at current technology levels. Nonetheless, the clean energy average costs are lower than the dirty energy average costs, because clean energy producers do not pay fuel costs and therefore have much lower variable costs.

transition closer to the one obtained with a microfounded model. Some differences still persist. The CES model initially undershoots the optimal clean energy share by 10 percentage points. The two models deliver the same optimal clean energy share of approximately 72% by 2035, and afterward, the CES model predicts a faster transition than optimal. The CES model does not capture the time needed to develop the storage technology to solve the intermittency problem.

9 Robustness Analysis

9.1 Alternative Cost of Storage

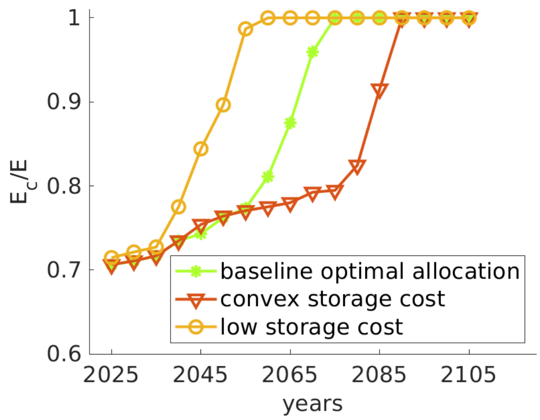
This section shows how the optimal speed of the clean energy transition changes for alternative calibrations of the starting level of the storage technology.

The range of costs for different energy storage technologies is large. Lazard (2020) shows the levelized cost of storage (LCOS) ranges from 132 \$/MWh to 250 \$/MWh. The EIA reports LCOS for new energy storage facilities entering service in 2027 of 124\$/MWh. In the baseline results, I took a LCOS of 191 \$/MWh, which is the midpoint of the range of LCOS reported by Lazard (2020). Lazard (2020) considers multiple uses of utility standalone storage. They consider large-scale energy storage systems designed to meet varying system needs (i.e., short-duration frequency regulation, longer-duration energy arbitrage, capacity). These cost estimates also consider the use of storage to meet system needs emerging with increased use of renewables, which are not accounted for in the 124\$/MWh storage cost reported by the EIA.

In this section, I compare the baseline results with results obtained under two different calibrations. First, I consider the much lower LCOS estimate provided by the EIA for storage planned to enter service in 2027, which is an LCOS of 124\$/MWh (EIA, 2022). Secondly, I consider a convex storage cost. The optimal transition requires an increase in energy storage generation by 2.5 times relative to current generation in the US. If we focus only on current energy storage generation with batteries in the US, the optimal allocation requires an increase in energy storage generation by 21 times.[27] The baseline model considers a constant marginal cost of storage. If there are bottlenecks in the storage batteries supply chain or if the marginal cost of storage increases substantially for a large demand of storage capacity, the optimal speed of the transition might decrease. In the second counterfactual, I consider how the speed of the transition changes with a convex cost of storage. I consider a degree of convexity such that the marginal cost of storage equals 132\$/MWh when the installed storage capacity K_s is zero, and 250\$/MWh at the optimal level of storage capacity K_s in the first period of the baseline results. 132\$/MWh and 250\$/MWh are respectively the lower bound and the upper

[27] The EIA reports a generation of 25.37 thousand GWh of energy storage in the US in 2022, of which 2.91 thousand GWh are generated with batteries. The optimal allocation requires a yearly generation of 63 thousand GWh of energy storage in the first five-year period.

Figure 8. Alternative Storage Costs



(a) Clean Energy Share

Notes: Figure 8a displays the optimal speed of the clean energy transition for different assumptions on the cost of the storage technology. The baseline estimates (green line) assume a cost of storage of 191\$/MWh, which is the midpoint of the LCOS estimates from Lazard (2020). The low storage cost (orange line) assumes a lower cost of storage of 124\$/MWh, following EIA (2022). The red line instead depicts the evolution with a convex storage cost, where the degree of convexity is calibrated using the lower and upper bounds of the Lazard (2020) estimates.

bound of the LCOS estimates by Lazard (2020).[28]

Figure 8a shows how the clean energy share changes along the optimal transition considering these new calibrations. When I lower the starting level of storage technology to an LCOS of 124\$/MWh, the optimal transition is faster. The SP decides to use approximately the same clean energy share of 70% in the first five-year period, and the clean energy transition is completed by 2055, which is 20 years earlier than in the baseline results. When I consider a convex cost of storage, the speed of the transition is slower. The initial clean energy share is always around 70%, which is only one percentage point lower than in the baseline optimal results. However, the complete decarbonization of the energy sector takes longer. The economy will reach a 100% clean energy share by 2090.

This section shows that alternative assumptions on the initial cost of storage affect the optimal speed of the transition to a completely decarbonized energy market. Nonetheless, all the scenarios support an immediate transition to a minimum 70% clean energy share.

[28]The marginal cost of storage now depends on the level of storage capacity. The total cost of installing K_s units of storage capacity is $p_s K_s^{\mu}$, with $\mu > 1$. p_s is 132\$/MWh. The marginal cost of storage capacity when the level of capacity $K_s = 63 * 5$ thousand GWh increases to 250\$/MWh.

10 Conclusion

Human-induced climate change is expected to have significant adverse impacts on human welfare, especially in developing nations and among vulnerable populations. Expediting the transition to cleaner energy sources is a key objective to reduce greenhouse gas emissions.

This paper develops a macro climate model explicitly considering the intermittency problem of clean energy sources. I embed a micro-founded model of the energy sector into an otherwise standard IAM à la Golosov et al. (2014). This model delivers an endogenous elasticity of substitution between clean and dirty energy sources, which is determined by the degree of renewables' intermittency and by technological progress in the energy sector.

I show that the aggregate elasticity of substitution is not constant, and it crucially depends on changes in storage technology. In the absence of policies, storage costs are high, and the economy is trapped in an environment with a low aggregate elasticity of substitution. If the storage technology is substantially improved, the elasticity of substitution increases. An optimal policy mix of carbon taxes and R&D subsidies can avert the environmental disaster by pushing for storage technological change, increasing the elasticity of substitution, and leading to a complete clean energy transition.

My theoretical framework allows the calibration of the model using detailed microdata from the energy sector. In particular, I use microdata on the hourly energy demand and hourly availability of sun and wind to measure the degree of intermittency, which determines the substitutability between clean and dirty energy.

I examine the role of policy in the clean energy transition. In the absence of policies, clean energy is competitive in the market thanks to the rapid change in clean technologies observed in the last decades. However, the clean energy share is limited by the intermittency problem to a level of around 30%. The market uses dirty energy instead of storage when there is no sun or wind because storage technologies are relatively more expensive. Intermittency creates a market for dirty energy. Scientists innovate on clean and dirty energy technologies, and they have no incentives to develop storage technologies.

In the optimal allocation, the clean energy transition is accelerated. The social planner (SP), who internalizes the environmental externality of dirty energy, immediately increases the clean energy share to 70%. The SP moves all the R&D resources away from dirty energy towards clean and, in particular, towards energy storage technologies. When the storage technology is more advanced, the clean energy transition is completed. The clean energy share gets to 100% by 2075.

Clean energy production subsidies introduced by the US IRA are successful in increasing the short-run clean energy share, but insufficient to push for the necessary development of the storage technology. Welfare in the optimal allocation increases by 2.7% relative to the no policy scenario. In comparison, the welfare effect to the IRA subsidy allocation is a 0.1% increase, given that the long-run outcomes are close to unchanged.

The theoretical model makes several simplifications, and much space is left for future work. Some of the main issues that deserve further attention are demand response, grid infrastructure investments to accommodate a large increase in variable renewable energy, and dirty energy producers' shutdown and start-up costs. As a starting point, I have abstracted from these features of the electricity market, which can have a substantial impact on the costs of the clean energy transition.

This project is part of an emerging strand of literature that considers important features of the energy sector and, in particular, the electricity sector, using macro models (e.g., Arkolakis and Walsh (2023)). Energy production and use account for a substantial share of global GHG emissions.[29] Macro climate models can provide a more accurate analysis of the role of policy by considering key constraints and features of the energy market interacting with policies.

[29] Energy production and use account for 76% of global GHG emissions in 2019 (Climate Watch, 2020).

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11 Appendix

11.1 Proof of Result 2

The energy producers' decision problem can be divided into two steps. In the first step, for a given K, she chooses e_h^{i*} that maximizes $\pi_h^i = p_h e_h^i - ((e_h^i)^\lambda + e_h^i z^i)$. In the second step, for a given $\pi_h^{i*}(K)$, she chooses K^{i*} that maximizes $\pi^i = \int \pi_h^{i*}(K^i)dh - K^i p^i$. e_h^{i*} is given by lemma (1). By plugging e_h^{i*} into π_h^i, summing across the different hours and subtracting capacity costs, we can write the capacity choice problem of energy producer of type i. I drop i in what follows to simplify notation.

$$\pi_c = \max_{K_t} \underbrace{\int_{h_{min}}^{h^*(K_t)} (p_h - z)^{\frac{\lambda}{\lambda-1}} (\frac{1}{\lambda})^{\frac{1}{\lambda-1}} (\frac{\lambda-1}{\lambda}) dh}_{\text{profits when unconstrained}} + \underbrace{\int_{h^*(K_t)}^{h_{max}} [(p_h - z)\xi_h K_t - (\xi_h K_t)^\lambda] dh}_{\text{profits when constrained}} - \underbrace{K_t p_t}_{\text{capacity costs}} \tag{40}$$

This expression orders the hours such as to distinguish between the hours where producer i is capacity constrained, from the hours when producer i is unconstrained. Define the hour $h^*(K_t^i)$ as the hour s.t. at h^*: $p_h = \lambda(\xi_h K_t^i)^{\lambda-1} + z$. Or equivalently, at h^*: $(\frac{p_h - z}{\lambda})^{\frac{1}{\lambda-1}} \frac{1}{\xi_h} = K_t^i$. We can define $(\frac{p_h - z}{\lambda})^{\frac{1}{\lambda-1}} \frac{1}{\xi_h}$ as a sufficient statistic that captures in which hours producer i is constrained. We can order the hours by $(\frac{p_h - z}{\lambda})^{\frac{1}{\lambda-1}} \frac{1}{\xi_h}$. For $h > h^*(K_t^i)$, producer i is constrained, while for $h \leq h^*(K_t^i)$ producer i is unconstrained.

$$\frac{\partial \pi}{\partial K_t} = \frac{\partial h^*(K_t)}{\partial K_t} \lambda^{\frac{\lambda}{\lambda-1}} (\xi_{h^*} K_t)^\lambda (\frac{1}{\lambda})^{\frac{1}{\lambda-1}} (\frac{\lambda-1}{\lambda}) \tag{41}$$

$$+ \int_{h^*(K_t)}^{h_{max}} [(p_h - z)\xi_h - \lambda(\xi_h K_t)^{\lambda-1}\xi_h] dh - \frac{\partial h^*(K_t)}{\partial K_t} [\lambda(\xi_h K_t)^\lambda - (\xi_h K_t)^\lambda] - p_t = 0$$

$$\underbrace{\int_{h^*(K_t)}^{h_{max}} [(p_h - z)\xi_h - \lambda(\xi_h K_t)^{\lambda-1}\xi_h] dh}_{\text{MB of capacity}} = \underbrace{p_t}_{\text{MC of Capacity}} \tag{42}$$

11.2 Proof of Result 3

The FOC of problem ((25)) gives:

$$[e_{th}^s]: \; p_{ht}^{e\ max} - p_{ht}^{e\ min} = p_t^s \tag{43}$$

If the LHS is smaller than the RHS, $e_{th}^{s*} = 0 \; \forall h$. If the LHS is larger than the RHS, $e_{th}^{s*} = \infty$ $\forall h$. Therefore in equilibrium, the LHS cannot be larger than the RHS. The LHS is smaller or equal to the RHS, with equality if $e_{th}^{s*} > 0$ for at least some h.

11.3 Social Planner Problem: Analytical Solution

11.3.1 The Problem

$$\max_{\substack{Y_t, K^Y_{t+1}, L^Y_t, E_t, e^i_{th} \\ e^s_{th}, s_{ct}, s_{dt}, s_{st}, l^i_t, l^s_t}} E_0 \sum_{t=0}^{\infty} \beta^t log\Big(\underbrace{Y_t - K^Y_{t+1} - \sum_{i=c,d} [\int_h ((e^i_{ht})^\lambda + e^i_{ht} z^i) dh)]}_{C_t}\Big) \tag{44}$$

subject to (λ are the multipliers for the equality constraints, μ for inequality constraints)

$$(\lambda_{1t})\ Y_t = e^{-\epsilon_t (\sum_{s=0}^{t+T} (\varphi_L + (1-\varphi_L)\varphi_0 (1-\varphi)^s) \xi_d n_d \int_h e^d_{t-s,h} dh)} A_t (K^Y_t)^\alpha (L^Y_t)^{1-\alpha-v} E^v_t \tag{45}$$

$$(\lambda^h_{2t})\quad E_t = \frac{n_c e^c_{ht} + n_d e^d_{ht} + e^s_{th}}{q_h} \tag{46}$$

$$(\mu^{hd}_{1t})\quad e^d_{th} \leq \gamma^{\eta s^{1-\psi}_{dt}} A^d_{t-1} l^d_t \tag{47}$$

$$e^c_{th} \leq \xi^c_t \gamma^{\eta s^{1-\psi}_{ct}} A^c_{t-1} l^c_t \tag{48}$$

$$(\lambda_{3t})\quad \int_h e^s_{th} dh = 0 \tag{49}$$

$$(\mu_{2t})\quad \int_{h:e_s>0} e^s_{th} dh \leq \gamma^{\eta s^{1-\psi}_{st}} A^s_{t-1} l^s_t \tag{50}$$

$$(\lambda_{4t})\quad L^Y_t + n_c l^c_t + n_d l^d_t + l^s_t = L_t \tag{51}$$

$$(\lambda_{5t})\quad s_{ct} + s_{dt} + s_{st} = 1 \tag{52}$$

11.3.2 FOCs

$$[K^Y_{t+1}]: \quad -\beta^t u'(C_t) + \lambda_{1,t+1}\alpha \frac{Y_{t+1}}{K^Y_{t+1}} = 0 \tag{53}$$

$$[Y_t]: \quad \beta^t u'(C_t) - \lambda_{1t} = 0 \tag{54}$$

$$[L^Y_t]: \quad \beta^t u'(C_t)(1-\alpha-v)\frac{Y_t}{L^Y_t} = \lambda_{4t} \tag{55}$$

$$[E_t]: \quad \underbrace{\beta^t u'(C_t) v \frac{Y_t}{E_t}}_{\text{MB of } E_t \text{ in terms of additional output}} = \underbrace{\int_h \lambda^h_{2t} dh}_{\substack{\text{MC = cost of producing} \\ \text{energy in different hours}}} \tag{56}$$

$$[e^c_{th}]: \quad \underbrace{\frac{\lambda^h_{2t}}{q_h}}_{\text{MB hourly production}} = \underbrace{\beta^t u'(C_t)\lambda(e^c_{ht})^{\lambda-1} + \mu^{h,c}_{1t}}_{\substack{\text{MC=VC + FC:fraction of capacity cost accruing to hour h} \\ \text{(FC=0 for non capacity constrained hours)}}} \tag{57}$$

$$[e^d_{th}]: \quad \underbrace{\frac{\lambda^h_{2t}}{q_h}}_{\substack{\text{MB hourly} \\ \text{production}}} = \underbrace{\sum_{s=0}^{\infty} \lambda_{1,t+s} Y_{t+s}(\epsilon_{t+s}(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s)\xi_d) +}_{\text{MC=pollution cost (output damages)}} \tag{58}$$

$$\underbrace{\beta^t u'(C_t)(\lambda(e^d_{ht})^{\lambda-1} + z^d) + \mu^{h,d}_{1t}}_{\substack{\text{VC + FC} \\ \text{FC = fraction of capacity cost accruing to hour h} \\ \text{FC=0 for non capacity constrained hours} \\ \text{FC = capacity cost if h is the only hour in which constrained}}}$$

$$[e^s_{th}]: \quad \underbrace{\frac{\lambda^h_{2t}}{q_h}}_{\substack{\text{MB of producing} \\ e_{th}\text{/discharging} \\ \text{(equalized across hours)}}} = \underbrace{\lambda_{3t}}_{\text{cost of charging}} + \underbrace{\mu_{2t}}_{\substack{\text{cost of} \\ \text{building capacity}}} \quad \text{for h s.t. } e_s > 0 \tag{59}$$

$$\underbrace{\frac{\lambda^h_{2t}}{q_h}}_{\substack{\text{MC of producing} \\ e_{th}\text{/charging} \\ \text{(equalized across hours)}}} = \underbrace{\lambda_{3t}}_{\substack{\text{MB of relaxing} \\ \text{the constraint (49)} \\ \text{= value of discharging} \\ \text{one more unit of energy}}} \quad \text{for h s.t. } e_s < 0 \tag{60}$$

$$[s_{dt}]: \quad \underbrace{ln(\gamma)\gamma^{\eta s_{dt}^{1-\psi}}\eta(1-\psi)s_{dt}^{-\psi}}_{\substack{\text{MP of a scientist} \\ \text{in terms of increase in A}}} \quad \underbrace{\Big[n_d(\int_h \mu^{h,d}_{1t}\xi_h dh)A^d_{t-1}l^d_t}_{\text{MV of additional capacity installed today}} \tag{61}$$

$$+\underbrace{\sum_{s=1}^{\infty} n_d(\int_h \mu_{1,t+s}^{h,d} dh) \prod_{i=1}^{s} \gamma^{\eta s_{dt+i}^{1-\psi}} A_{t-1}^d l_{t+s}^d\Big]}_{\text{MV of additional capacity installed tomorrow}} = \underbrace{\lambda_{5t}}_{\substack{\text{MC= less resources}\\ \text{to do research}\\ \text{in other sectors}}}$$

$$[s_{ct}]: \underbrace{ln(\gamma)\gamma^{\eta s_{ct}^{1-\psi}} \eta(1-\psi) s_{ct}^{-\psi}}_{\substack{\text{MP of a scientist}\\ \text{in terms of increase in A}}} \underbrace{\Big[n_c(\int_h \mu_{1t}^{h,c} \xi_h dh) A_{t-1}^c l_t^c}_{\text{MV of additional capacity installed today}} \tag{62}$$

$$+\underbrace{\sum_{s=1}^{\infty} n_c(\int_h \mu_{1,t+s}^{h,c} \xi_h dh) \prod_{i=1}^{s} \gamma^{\eta s_{ct+i}^{1-\psi}} A_{t-1}^c l_{t+s}^c\Big]}_{\text{MV of additional capacity installed tomorrow}} = \underbrace{\lambda_{5t}}_{\substack{\text{MC = less resources}\\ \text{to do research}\\ \text{in other sectors}}}$$

$$[s_{st}]: \underbrace{ln(\gamma)\gamma^{\eta s_{st}^{1-\psi}} \eta(1-\psi) s_{st}^{-\psi} A_{t-1}^s}_{\substack{\text{MP of a scientist}\\ \text{in terms of increase in A}}} \underbrace{\Big[\mu_{2t} l_t^s}_{\text{MV of additional capacity installed today}} \tag{63}$$

$$+\underbrace{\sum_{s=1}^{\infty} \mu_{2,t+s} \prod_{i=1}^{s} \gamma^{\eta s_{st+i}^{1-\psi}} l_{t+s}^s\Big]}_{\text{MV of additional capacity installed tomorrow}} = \underbrace{\lambda_{5t}}_{\substack{\text{MC= less resources}\\ \text{to do research}\\ \text{in other sectors}}}$$

$$[l_t^d]: \underbrace{\int_h \Big(\mu_{1t}^{h,d} \gamma^{\eta s_{dt}^{1-\psi}} A_{t-1}^d\Big) dh}_{\substack{\text{MB=build additional capacity}\\ \text{to use in constrained hours}}} = \underbrace{\lambda_{4t}}_{\substack{\text{less labour resources}\\ \text{for other capacities}\\ \text{or final output}}}$$

$$[l_t^c]: \int_h \Big(\mu_{1t}^{h,c} \xi_h^c \gamma^{\eta s_{ct}^{1-\psi}} A_{t-1}^c\Big) dh = \lambda_{4t}$$

$$[l_t^s)]: \mu_{2t}\gamma^{\eta s_{st}^{1-\psi}} A_{t-1}^s = \lambda_{4t} \tag{64}$$

11.3.3 Insights from combining FOCs

- Combining $[K_{t+1}^Y]$ and $[Y_t]$, Euler Equation:

$$u'(C_t) = \beta u'(C_{t+1}) \alpha \frac{Y_{t+1}}{K_{t+1}}$$

  The Euler equation governs the consumption/savings decisions. At the optimum, the consumer must be indifferent between getting one additional unit of consumption today or forgoing it and increasing K_{t+1} by 1, therefore getting $\alpha \frac{Y_{t+1}}{K_{t+1}} = MPK_{t+1}$ additional units of consumption tomorrow.

- Combining $[E_t]$ and $[e_{th}^i]$: the MCs of producing energy in a given hour by different sources (producers) must be equalized. The MB of producing aggregate energy in terms of the final good must equal the MC, which is given by the sum of the MCs of producing energy in the different hours weighed by the share of energy produced in every hour (q_h). Assuming some clean is produced in every hour (FOC does not hold at a corner solution

$e^c_{th} = 0$):

$$\beta^t u'(C_t) v \frac{Y_t}{E_t} = \int_h q_h(\beta^t u'(C_t)\lambda(e^c_{ht})^{\lambda-1} + \mu^{h,c}_{1t})dh$$

- Combining [L^Y_t], [l^i_t] and [l^s_t]:

$$\beta^t u'(C_t)(1-\alpha-v)\frac{Y_t}{L^Y_t} = \int_h \left(\mu^{h,d}_{1t}\gamma^{\eta s^{1-\psi}_{dt}} A^d_{t-1}\right)dh = \int_h \left(\mu^{h,c}_{1t}\xi^c_h \gamma^{\eta s^{1-\psi}_{ct}} A^c_{t-1}\right)dh = \mu_{2t}\gamma^{\eta s^{1-\psi}_{st}} A^s_{t-1}$$

  The MB of labor across the different sectors must be equalized.

  - The MB of L^Y_t: additional final good.
  - 1 marginal unit of labor produces $\gamma^{\eta s^{1-\psi}_{jt}} A^i_{t-1}$ additional units of capacity, which can relax the capacity constraints by ξ_h in the hours in which producer i is capacity constrained ($\mu^{hi}_{1t} > 0$). The value of extra capacity is $\mu^{hi}_{1t} = \frac{\lambda^h_{2t}}{q_h} - VC-$emissions costs for dirty producers = MB of producing energy in a given hour.

- Combining [e^s_{th}], [e^i_{th}] we can learn: MC of producing one additional e_{th} must be equalized across hours with $e_s > 0$, and equalized across hours with $e_s < 0$. The difference between these two MCs is given by $\mu_{2t} = MC$ of K_s.

$$\underbrace{\frac{\lambda^h_{2t}}{q_h}}_{\text{for h s.t. } e_s > 0} = \mu_{2t} + \underbrace{\frac{\lambda^{h\,\prime}_{2t}}{q(h')}}_{\text{for h' s.t. } e_s < 0}$$

  Value of discharging energy in hour h = MC of producing energy in hour h' + MC of building K_s. $e_s > 0$ in hours with high $\frac{\lambda^h_{2t}}{q_h}$, and $e_s < 0$ in hours with low $\frac{\lambda^h_{2t}}{q_h}$. Storage not used if $\frac{\lambda^h_{2t}}{q_h} < \mu_{2t} + \frac{\lambda^{h\,\prime}_{2t}}{q(h')}$ for any h,h'. If there are hours with $\frac{\lambda^h_{2t}}{q_h} > \mu_{2t} + \frac{\lambda^{h\,\prime}_{2t}}{q(h')}$, it's optimal to use storage in these hours (increase e_{sh}), until $\frac{\lambda^h_{2t}}{q_h} = \mu_{2t} + \frac{\lambda^{h\,\prime}_{2t}}{q(h')}$ in some hours, and $<$ in the remaining hours.

- Combining [s_{ct}], [s_{dt}] and [s_{st}]:

$$\underbrace{ln(\gamma)\gamma^{\eta s^{1-\psi}_{dt}}\eta(1-\psi)s^{-\psi}_{dt}}_{\substack{\text{MP of a scientist}\\ \text{in terms of increase in A}}} \quad \underbrace{\Big[n_d(\int_h \mu^{h,d}_{1t}dh)A^d_{t-1}l^d_t}_{\text{MV of additional capacity installed today}} \quad \underbrace{+\sum_{s=1}^{\infty} n_d(\int_h \mu^{h,d}_{1,t+s}dh)\prod_{i=1}^{s}\gamma^{\eta s^{1-\psi}_{dt+i}}A^d_{t-1}l^i_{t+s}\Big]}_{\text{MV of additional capacity installed tomorrow}} =$$

$$\underbrace{ln(\gamma)\gamma^{\eta s^{1-\psi}_{ct}}\eta(1-\psi)s^{-\psi}_{ct}}_{\substack{\text{MP of a scientist}\\ \text{in terms of increase in A}}} \quad \underbrace{\Big[n_c(\int_h \mu^{h,c}_{1t}\xi^c_h dh)A^c_{t-1}l^c_t}_{\text{MV of additional capacity installed today}} \quad \underbrace{+\sum_{s=1}^{\infty} n_c(\int_h \mu^{h,c}_{1,t+s}\xi^c_h dh)\prod_{i=1}^{s}\gamma^{\eta s^{1-\psi}_{ct+i}}A^c_{t-1}l^c_{t+s}\Big]}_{\text{MV of additional capacity installed tomorrow}} =$$

$$\underbrace{ln(\gamma)\gamma^{\eta s^{1-\psi}_{st}}\eta(1-\psi)s^{-\psi}_{st}A^s_{t-1}}_{\substack{\text{MP of a scientist}\\ \text{in terms of increase in A}}} \quad \underbrace{\Big[\mu_{2t}l^s_t}_{\text{MV of additional capacity installed today}} \quad \underbrace{+\sum_{s=1}^{\infty}\mu_{2,t+s}\prod_{i=1}^{s}\gamma^{\eta s^{1-\psi}_{st+i}}l^s_{t+s}\Big]}_{\text{MV of additional capacity installed tomorrow}}$$

  MB of research across the different sectors must be equalized. MB is given by increased productivity/capacity that can be produced with a given l^i_t/l^s_t in constrained hours today and in the future.

11.3.4 Optimal Policies

Hourly Energy Production and the Carbon Tax: in both the DE and the SPP the MC of producing energy in a given hour from different active[30] producers is equalized. In hours in which producers are not capacity constrained respectively in the SPP and the CE:

$$\underbrace{\frac{\lambda_{2t}^h}{q_h}}_{\substack{\text{MB hourly}\\\text{production}}} = \underbrace{\sum_{s=0}^{\infty} \beta^{t+s} u'(C_{t+s}) Y_{t+s}(\epsilon_{t+s}(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s)\xi_d)}_{\text{MC=pollution cost (output damages)}} + \underbrace{\beta^t u'(C_t)(\lambda(e_{ht}^d)^{\lambda-1} + z^d)}_{\text{VC}}$$

$$p_{ht}^e = \lambda(e_{ht}^d)^{\lambda-1} + z^d$$

The first difference between DE and SPP is that the SP considers the pollution cost of dirty energy into its marginal cost, while energy producers do not internalize this externality of dirty energy in the CE. (In hours in which some producers are capacity constrained, the MC also includes some capacity costs in both the DE and the SPP. See the next paragraph).
If a carbon tax on dirty energy production

$$\tau_t = \frac{\sum_{s=0}^{\infty} \beta^{t+s} u'(C_{t+s}) Y_{t+s}(\epsilon_{t+s}(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s)\xi_d)}{\beta^t u'(C_t)}$$

is imposed in the CE, the FOC in the DE becomes:

$$p_{ht}^e = \lambda(e_{ht}^d)^{\lambda-1} + z^d + \tau_t$$

If we replace in this FOC the value of τ_t and $p_{ht}^e = \frac{\lambda_{2t}^h}{q_h}\frac{1}{\beta^t u'(C_t)}$, the FOC in the DE will equal the FOC in the SPP.

Innovation Conditions and Optimal R&D subsidies: The innovation conditions are different because the SP also considers the impact of current innovation on future productivity. Only the leading technology is used to produce, so once a new scientist builds on the innovation of an old scientist, he will be the only one selling to the market. He will not compensate the old scientist for having built on his innovation (building on the shoulder of the giants' effect).

[30]FOCs do not hold at corner solutions (that is when $e_{ht}^i = 0$).

- Innovation allocation in the SPP:

$$\underbrace{\eta_d s_{dt}^{-\psi}}_{\text{MP of a scientist}} \underbrace{\left[n_d \bar{p}_t^d K_t^d\right.}_{\substack{\text{MV of additional}\\ \text{capacity}\\ \text{installed today}}} \underbrace{\left. + \sum_{s=1}^{\infty} n_d \bar{p}_{t+s}^d K_{t+s}^d\right]}_{\substack{\text{MV of additional}\\ \text{capacity}\\ \text{installed tomorrow}}} = \underbrace{\eta_c s_{ct}^{-\psi}}_{\text{MP of a scientist}} \underbrace{\left[n_c \bar{p}_t^c K_t^c\right.}_{\substack{\text{MV of additional}\\ \text{capacity}\\ \text{installed today}}} \underbrace{\left. + \sum_{s=1}^{\infty} n_c \bar{p}_{t+s}^c K_{t+s}^c\right]}_{\substack{\text{MV of additional}\\ \text{capacity}\\ \text{installed tomorrow}}}$$

$$= \eta_s s_{st}^{-\psi} \left[\bar{p}_t^s K_t^s + \sum_{s=1}^{\infty} \bar{p}_{t+s}^s K_{t+s}^s\right]$$

- Innovation allocation in the CE:

$$\underbrace{\eta_d s_{dt}^{-\psi}}_{\text{MP of a scientist}} \underbrace{\left[n_d p_t^d K_t^d\right]}_{\substack{\text{MV of additional}\\ \text{capacity}\\ \text{installed today}}} = \eta_c s_{ct}^{-\psi}\left[n_c p_t^c K_t^c\right] = \eta_s s_{st}^{-\psi}\left[p_t^s K_t^s\right]$$

where $\bar{p}_t^i = \beta^t u'(C_t) \underbrace{\bar{w}_t}_{\substack{\text{shadow}\\ \text{cost of}\\ \text{labor}}} \underbrace{\frac{1}{\gamma^{\eta s_{it}^{1-\psi}} A_{t-1}^i}}_{\substack{\text{units of labor}\\ \text{per unit of}\\ \text{capacity}}}$ is the shadow price of one unit of capacity in the SPP.

I can also show $\bar{p}_t^i = \int_0^1 \mu_{1t}^{hi} dh$, which is the MV of additional capacity in terms of additional energy and output production.

The optimal allocation can be implemented in the decentralized economy by using some R&D subsidies to clean (q_t^c) and energy storage technologies (q_t^s). The optimal subsidy is such that the innovation allocation ($S^c t, s_t^d, s_t^s$) in the decentralized economy with the subsidies, determined by the following expression, is the same as in the optimal allocation.

$$\eta_d s_{dt}^{-\psi}\left[n_d p_t^d K_t^d\right] = \frac{\eta_c s_{ct}^{-\psi}\left[n_c p_t^c K_t^c\right]}{1 - q_t^c} = \frac{\eta_s s_{st}^{-\psi}\left[p_t^s K_t^s\right]}{1 - q_t^s}$$

Capacity level and Subsidies to Intermediate Inputs: the capacity choices will differ because of the excessively high capacity prices in the DE due to the monopolistic distortion (monopolists charge a markup of γ on production costs) and because of the pollution externality, which decreases the MB of dirty capacity (already taken into account by the previous paragraph). The capacity conditions for a dirty producer are respectively in the DE and in the SPP:

$$\int_{h^*(K_t^d)}^{h_{max}} [p_{th}^e - \lambda (K_t^d)^{\lambda - 1} - z^d] dh = p_t^d$$

$$\int_{h \in H^*} \mu_{1t}^{hd} dh = \beta^t u'(C_t) \underbrace{\frac{(1-\alpha-v) Y_t}{\gamma^{\eta s_{dt}^{1-\psi}} A_{t-1}^d \frac{Y_t}{L_t^Y}}}_{\frac{p_t^d}{\gamma}} = \int_{h \in H^*} \frac{\lambda_{2t}^h}{q_h} - \sum_{s=0}^{\infty} \beta^{t+s} u'(C_{t+s}) Y_{t+s} (\epsilon_{t+s}(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi)^s)\xi_d)$$

$$-\beta^t u'(C_t)(\lambda K_t^{d\lambda-1} + z^d)$$

H^* is the set of hours s.t. we are not at a corner solution ($e_{th}^d > 0$), and $\mu_{1t}^{hd} > 0$ (the producer is capacity constrained). The first equality of the second equation follows from condition 64 together with $p^d_t = \frac{\gamma(1-\alpha-v)Y_t}{\gamma^{\eta s_{dt}^{1-\psi}} A^d_{t-1}\frac{Y_t}{L^Y_t}}$ which holds in the DE.

If a tax τ_t on dirty energy production is specified as detailed above, a subsidy on capacity is introduced such that the price of capacity becomes p^d_t/γ, the new FOC of the DE becomes

$$\int_{h^*(C_t)}^{h_{max}} [p^e_{th} - \lambda(K^d_t)^{\lambda-1} - z^d - \tau_t]dh = \frac{p^d_t}{\gamma}$$

By replacing in the expressions for τ_t, p^d_t and $p^e_{th} = \frac{\lambda^h_{2t}}{q_h}\frac{1}{\beta^t u'(C_t)}$[31], then we can see the FOC in the DE is the same as the FOC in the SPP.

11.4 Numerical Methods

11.4.1 Equilibrium in the Energy Market

This section explains how to solve for the equilibrium in the energy market given an aggregate energy level E_t and capacity costs p_{ct}, p_{dt} and p_{st}. In order to simplify the notation, I drop the aggregate time index t. I restrict the attention to an economy with: $\xi^c_h \in \{0, \xi\}$ with $0 < \xi < 1$, as in the main model calibration. For a given level of hourly energy demand ($e_h = q_h E$) and capacity costs (p_c, p_d and p_s), I will describe how to find the equilibrium hourly energy price p_h and the optimal capacity choices (K^c, K^d, K^s) by the energy producers. Given the level of capacity and the hourly price, it is straightforward to compute the hourly energy production by every energy producer using the optimal choice (23). I am first showing how to solve for the equilibrium in the case with no storage, and then I solve for the equilibrium with storage. I will call day the hours with $\xi_h > 0$, and night the hours with $\xi_h = 0$. From now onwards p_c stands for the effective capacity price, which I define as the capacity price to produce one unit of energy in the hours in which $\xi_h > 0$ [32].

No Storage

I define h_d and h_n as, respectively, day and night hours. h^{max}_d is the day hour with maximum energy demand, and h^{max}_n is the night hour with maximum energy demand. I assume $e_{h^{max}_d} > e_{h^{max}_n}$, in line with the data. Given hourly energy demand (e_h) and capacity costs p_c and p_d, the solution will be characterized by one of four cases. The clean producer might be active

[31] Correspondence between p^e_{ht} and λ^h_{2t}: In the CE, $v\frac{Y_t}{E_t} = p_{E_t} = \int_0^1 q_h p^e_{ht} dh$. In the SPP, $\beta^t u'(C_t) v \frac{Y_t}{E_t} = \int_0^1 \lambda^h_{2t} dh$. p^e_{ht} represents the marginal value of an additional unit of e_{ht} in the CE. λ^h_{2t} represents the marginal value of an additional unit of e_{ht} in the SPP. There is a correspondence between p^e_{ht} and $\lambda_2 t^h$. In particular, if the energy and output choices are the same in the SPP and the CE, then $\lambda^h_{2t} = \beta^t u'(C_t) p^e_{ht}$.

[32] For every unit of installed capacity, the producer can produce only 0.7 units of energy in the hours with $\xi_h > 0$. Therefore, the effective capacity price is equal to the unit capacity price p_c divided by 0.7. It is useful to note that the solution in a model with $\xi_h \in \{0, 0.7\}$ and the capacity price equal to p_c, is equivalent to the solution to a model with an alternative calibration with $\xi'_h \in \{0, 1\}$ and the capacity price equal to the effective capacity price in the first model (that is $p'_c = p_c/0.7$).

or inactive. If the clean producer is inactive, the dirty producer will be capacity constrained at day given $e_{h_d^{max}} > e_{h_n^{max}}$ (case A). If the clean producer is active, the dirty producer could be constrained at day, at night or both. The following conditions allow to identify which case characterizes the equilibrium:

- CASE A: The dirty producer type is constrained only at day and $K^{c*} = 0$.
  The solution is characterized by case A if:

$$p_d + \lambda K_d^{\lambda-1} + z^d + \sum_{h^d \backslash \{h_d^{max}\}} \left[\lambda (\frac{e_{h^d}}{n_d})^{\lambda-1} + z^d\right] \leq p_c \quad (65)$$

$$\text{where } K_d = \frac{e_{h_d^{max}}}{n_d}$$

  If the dirty producer is the only active producer, she must be capacity constrained only at day given $e_{h_d^{max}} > e_{h_n^{max}}$. She will be constrained in h_d^{max}, which implies the price in this hour equals the marginal capacity cost plus the marginal hourly production cost ($p_d + \lambda K_d^{\lambda-1} + z^d$). The LHS of (65) is the sum of the prices in the day hours, which are determined by the marginal production cost of the dirty producer. The clean producer optimally chooses to build zero capacity, if the sum of the prices during the day hours is smaller than the cost of building one unit of capacity, which is exactly what condition (65) states.

- CASE B: The dirty producer type is constrained at day and $K^{c*} > 0$.
  The solution is characterized by case B if condition B1 or condition B2 are satisfied.
  Condition B1:

$$\exists K^d > \frac{e_{h_n^{max}}}{n_d} \text{ s.t. given } K^c = \frac{e_{h_d^{max}} - K^d n_d}{n_c}$$

$$p_d + \lambda K_d^{\lambda-1} + z^d - \lambda (K^c)^{\lambda-1} + \sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c \\ \backslash \{h_d^{max}\}}} \left[\lambda \left(\frac{e_{h^d} - n_c K^c}{n_d}\right)^{\lambda-1} + z^d - \lambda (K^c)^{\lambda-1}\right] = p_c \quad (66)$$

  The total dirty capacity ($n_d K^d$) must be larger than $e_{h_n^{max}}$, given dirty is the only supplier at night. The price in hour h_d^{max} will be the maximum hourly price given supply increases in the hourly price. Therefore, if dirty is constrained at day, she will be constrained in h_d^{max}. The clean producer must also be constrained in this hour. It follows that the clean plus dirty capacity must equal the $e_{h_d^{max}}$. (67) states that the clean optimal capacity condition must be satisfied. The LHS represents the sum of the prices during the day hours in which clean is constrained, minus the clean hourly production cost in these hours. I have ensured the dirty capacity condition is satisfied

because I inserted in (67) that $p(h_d^{max}) = p_d + \lambda K_d^{\lambda-1} + z^d$, which is the dirty optimal capacity condition.
Condition B2:

$$\exists K^d > \frac{e_{h_n^{max}}}{n_d} \text{ and } \exists p(h_d^{mp}) : z^d > p(h_d^{mp}) > \lambda(K^c)^{\lambda-1} \text{ s.t. given } K^c = \frac{e_{h_d^{max}} - K^d n_d}{n_c}$$
$$p_d + \lambda K_d^{\lambda-1} + z^d + p(h_d^{mp}) - 2\lambda(K^c)^{\lambda-1} \tag{67}$$
$$+ \sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c \\ \setminus\{h_d^{max}, h_d^{mp}\}}} \left[\lambda\Big(\frac{e_{h^d} - n_c K^c}{n_d}\Big)^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1}\right] = p_c$$

This condition is equivalent to the previous one, but it also considers the possibility that the clean producer is marginal producer and capacity constrained in a day hour defined as h_d^{mp}. If this is the case the clean producer makes some revenues also in this hour that are now included in the LHS. $p(h_d^{mp})$ must be lower than z^d, to ensure clean is the marginal producer in this hour and dirty is not active. This price must also be greater or equal than the clean capacity constrained price ($\lambda(K^c)^{\lambda-1}$) given clean is producing at capacity.

- CASE C: The dirty producer type is constrained at both day and night and $K^{c*} > 0$. The solution is characterized by case C if given $K^d = \frac{e_{h_n^{max}}}{n_d}$ and $K^c = \frac{e_{h_d^{max}} - e_{h_n^{max}}}{n_c}$ (which follow from the fact that the dirty producer is constrained at night, and demand must be satisfied in h_d^{max} by clean plus dirty), the following conditions are satisfied:

$$\sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c}} \left[\lambda(\frac{e_{h^d} - n_c K^c}{n_d})^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1}\right] \leq p_c \tag{68}$$

$$p_d + \lambda K_d^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1} + \sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c \\ \setminus\{h_d^{max}\}}} \left[\lambda(\frac{e_{h^d} - n_c K^c}{n_d})^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1}\right] \geq p_c \tag{69}$$

The LHS of the two conditions represent the maximum and minimum possible sum of prices during the day. Given K^c and K^d, the prices in the day hours other than h_d^{max} are uniquely determined by the marginal hourly production cost of the uncontrained producer. $p(h_d^{max})$ can take on a value between the capacity constrained price of the dirty producer ($\lambda K_d^{\lambda-1} + z^d$) and the price that would make the dirty producer recover all of her capacity costs ($p_d + \lambda K_d^{\lambda-1} + z^d$) (considering the dirty producer can recover the reminder of her capacity cost in the night hours). Equations (68) and (69) state that as we vary $p(h_d^{max})$ between its maximum and minimum value, there is a value s.t. the clean capacity condition is satisfied.

- CASE D: The dirty producer type is constrained only at night and $K^{c*} > 0$. Given $K^d = \frac{e_{h_n^{max}}}{n_d}$, we can define a minimum value for K^c: $\bar{K}^c = \frac{e_{h_d^{max}} - e_{h_n^{max}}}{n_c}$, such that supply is large enough to satisfy demand at day. The equilibrium is characterized by case D if the following condition is satisfied:

$$\sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c}} \left[\lambda \left(\frac{e_{h^d} - n_c \bar{K}^c}{n_d} \right)^{\lambda-1} + z^d - \lambda (K^c)^{\lambda-1} \right] > p_c \tag{70}$$

  This condition states that at the minimum value of K^c, the marginal benefit of one additional unit of capacity (LHS) is larger than the marginal cost (RHS). By increasing capacity, the LHS will decrease until it equals 0 when $K^c = \frac{e_{h_d^{max}}}{n_c}$. This condition ensures that there is a value of K^c larger than the minimum required value to ensure supply is enough to satisfy demand in any hour, such that the clean capacity condition is satisfied. Night prices will ensure that the dirty capacity condition is satisfied.

The conditions determine some threshold values for p_c that can be used to determine which case characterizes the equilibrium. I define $\bar{p_c}$ as the LHS of equation (65), $\bar{\bar{p_c}}$ as the LHS of equation (69), and $\bar{\bar{\bar{p_c}}}$ as the LHS of equation (70). Note that these threshold values are a function of p_d. The following rule determines which case characterizes the equilibrium:

- If $p_c \geq \bar{p_c}$, we are in case A.
- If $\bar{p_c} > p_c > \bar{\bar{p_c}}$, we are in case B.
- If $\bar{\bar{p_c}} \geq p_c \geq \bar{\bar{\bar{p_c}}}$, we are in case C.
- If $p_c < \bar{\bar{\bar{p_c}}}$, we are in case D.

With Storage

I first solve for the equilibrium in the energy market in the case no storage is used. Then, I check whether the difference between the maximum and the minimum hourly price is larger or smaller than the storage cost p_s. If the difference is smaller than p_s, $K^s* = 0$. If the difference is larger than p_s, storage firms can make positive profits by building K^s, buying energy in hours with low prices, and selling it in hours with high prices. Note that for the way I set up the model, K^s represents both the total capacity and the total quantity of energy

stored ($\int_h e^s_{ht} \mathbb{1}(e^s_{ht} > 0)dh = K_s$). In what follows I will refer to it as the total energy stored, to simplify the interpretation.
I now first describe how for a given value of total energy stored K_s, I determine the equilibrium K^c, K^d, and prices. After, I will describe how to determine the equilibrium value of K^s. The day hours with low demand are the hours with minimum prices. In these hours clean producers are active, and demand is low. Therefore equilibrium prices are low. Storage firms will always buy electricity during hours with minimum prices, therefore during day hours. They can instead sell electricity both at day and at night. More precisely, they will always sell electricity at the day and night hours with maximum demand, given prices are increasing in the supply. I define the residual demand e^{res}_h as hourly energy demand plus storage ($e^{res}_h = e_h + e^s_h$), that is the actual hourly energy clean and dirty producers must supply to the market after storage demand/supply is taken into account. Given prices are strictly increasing in supply within day and night, supply must be constant at the day hours with $e^s_h < 0$ (storage buys) and at the day or night hours with $e^s_h > 0$ (storage sells)[33]. Considering that i) storage buys at day hours with minimum demand, ii) supply at these hours must be constant, for a given total energy stored K_s, it is straightforward to determine in which hours $e^s_h > 0$ and what is the level of e^s_h and e^{res}_h in these hours. Storage starts buying in the hour with minimum demand, therefore increasing the residual demand e^{res}_h (and therefore supply by energy producers) in this hour. Once e^{res}_h becomes equal to e_h in the next hour with minimum e_h, storage starts buying also in this other hour. Storage buys in both hours such that e^{res}_h is the same in the two hours. When e^{res}_h equals e_h in the next hour with minimum demand, then storage starts to buy also in a third hour. This process goes on until $\int_h e^s_{ht} \mathbb{1}(e^s_{ht} > 0)dh = K_s$. The same process can be applied to determine hours with $e^s_h < 0$ and the level of e^s_h and e^{res}_h in these hours, once we know what fraction of stored energy ($\int_h e^s_{ht} \mathbb{1}(e^s_{ht} < 0)dh = K_s$) is released at night and what fraction at day. I will explain in the coming paragraphs how this fraction is determined. For a given total energy stored and fraction of energy stored released at day and night, I can determine the residual demand e^{res}_h. Given the residual demand, the equilibrium in the energy market can be determined in the same way as in the no storage case replacing e^{res}_h for e_h.

I now describe how the level of total energy stored K^s is solved for. I distinguish between the case in which total energy stored is greater or smaller than total energy supply at night (I define total energy supply at night as the sum of hourly energy demand at night hours $\sum_{h_n} e_h$). This distinction is useful because when $K^s < \sum_{h_n} e_h$, then the dirty producer must active at night. When $K^s \geq \sum_{h_n} e_h$, the dirty producer need not be active at night and we might have an equilibrium in which only the clean producer is active.

[33]Supply must be constant at day and night hours with $e^s_h < 0$. Note supply need not take the same value in the day and night hours with $e^s_h < 0$. It could take a value for day hours with $e^s_h < 0$ and a different value for night hours with $e^s_h < 0$. Prices, though, must be identical in all hours (day and night) with $e^s_h < 0$.

In order to solve for the equilibrium level of K^s, I use the following numerical procedure:

1. I start by considering levels of $\mathbf{K^s} < \sum_{\mathbf{h_n}} \mathbf{e_h}$. I make a grid for possible levels of K^s between 0 and $\sum_{h_n} e_h$. Start from $K^s = 0$ that is the lowest point of the grid. Compute e_h^{res} and the equilibrium prices using the rules from the no storage case, replacing e_h^{res} for e_h. Check if $p_h^{max} - p_h^{min} > p_s$. If it is, then move to the next point of the K^s grid. Else, stop and look for the value of storage between 0 and the last grid point tried that makes $p_h^{max} - p_h^{min} = p_s$. Note, it could be there is no K^s that makes $p_h^{max} - p_h^{min} = p_s$ (because there can be some jumps in $p_h^{max} - p_h^{min}$), in this case pick the minimum K^s that makes $p_h^{max} - p_h^{min} \geq p_s$ (such that storage profits are positive).
If at $S = \sum e_h(\xi_h = 0)$, $pmax - pmin > p_s$ go to the next section.

    - How to determine for every point of the grid what is the fraction of total energy stored K^s that is sold at day and at night:

2. Now consider levels of $\mathbf{K^s} > \sum_{\mathbf{h_n}} \mathbf{e_h}$. I compute pmin as the equilibrium price in the hour with minimum e_h, given the residual demand e_h^{res} associated with $K^s = \sum_{h_n} e_h$ if clean is the only active unconstrained producer in this hour.

    - If $pmin + p_s < z^d$: clean is the only active producer.
    Find the value of $K^s > \sum e_h(\xi_h = 0)$ s.t. clean capacity condition is satisfied and $p_h^{max} - p_h^{min} = p_s$
    - If $pmin + p_s > z^d$: the active producers could be clean and dirty or only clean.
    Make a grid for K^s between $\sum_{h_n} e_h$ and Smax. Slowly add storage and check if $p_h^{max} - p_h^{min} > p_s$. If it is, then keep adding storage. Else, stop and look for the value of storage between $\sum_{h_n} e_h$ and the last grid point tried that makes $p_h^{max} - p_h^{min} = p_s$. Note, it could be there is no S that makes $p_h^{max} - p_h^{min} = p_s$ (because there can be some jumps in $p_h^{max} - p_h^{min}$), in this case pick the minimum S that makes $p_h^{max} - p_h^{min} \geq p_s$ (s.t. storage profits are positive).
        - Find which fraction of S should be sold at night and which fraction at day: s.t. clean and dirty capacity conditions are satified. Note that C =supply at night, and C_c = highest day supply minus C. Mechanism: as more of S is sold during the day, C increases and pmax (which is determined by dirty as long as $C > 0$) increases, which makes clean condition more or less likely to be satisfied. If we get to the point $C = 0$, implement code from the previous subsection that is: find the value of $S > \sum e_h(\xi_h = 0)$ s.t. clean capacity condition is satisfied and $pmax - pmin = p_s$.

    If I get to $S = Smax$, go to next step.
    - If $pmin + ps > z$ and $S = Smax$:
    Only clean is producing.

Proof: When S=Smax storage sells all at night and supply during the day is constant. If supply during the day is constant it means all of the built capacity is used in every day hour. If also dirty capacity is used during day hours it determines the pmax during day hours. Given pmax during day hours happens in day hours with $e_s < 0$, this would also equal the pmax during night hours. This means dirty would also produce positive quantity (at capacity) during night hours, which violates the starting condition that everything at night is produced by storage.

It can be that even if only clean is producing $K^{d*} > 0$.

In this case, I can usually choose between an equilibrium with $K^{d*} > 0$, and $pmax - pmin > p_s$, that is all conditions are satisfied for clean and dirty but not for storage. Or, in alternative, I can choose an equilibrium where $K^d = 0$ and $pmax - pmin = p_s$, but $K^{d*} > 0$. For now, when this happens, K^{d*} is very small, like $1.e-4$, so I select the second solution as the equilibrium solution.

11.5 Proof of Result 4

Proof of the first part of Result 4: Given a high p_c, if p_s decreases, the clean energy share can increase or decrease.

Consider a level of $\bar{p_c} > p_c > \bar{\bar{p_c}}$, where $\bar{\bar{p_c}}$ and $\bar{p_c}$ are defined in section 11.4. Given we are interested in the case with storage, one must replace the residual demand e_h^{res} characterized by the previous section for demand e_h in the definition of p_c. When $\bar{p_c} > p_c > \bar{\bar{p_c}}$, the equilibrium is characterized by the case B described in Section 11.4. The dirty producer is capacity-constrained at day and $K^{c*} > 0$. The equilibrium is characterized by condition B1:

$$K^c = \frac{e_{h_d^{max}} - K^d n_d}{n_c}$$

$$p_d + \lambda K_d^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1} + \sum_{\substack{\text{h}^d \text{ s.t.} \\ \text{e}_{h^d} > n_c K^c \\ \backslash\{h_d^{max}\}}} \left[\lambda\Big(\frac{e_{h^d} - n_c K^c}{n_d}\Big)^{\lambda-1} + z^d - \lambda(K^c)^{\lambda-1}\right] = p_c \tag{71}$$

In this case, the maximum hourly price will be at day hours. Storage will sell only at day hours. As p_s decreases, the storage firm decides to store more energy. It sells the additional energy at the day hours with the maximum price, driving down the price in these hours. Given both dirty and clean producers are active in these hours and they are both capacity constrained in these hours, K^{c*} and K^{d*} decrease. How does E_c/E_d change? It is ambiguous. Let's understand why by analyzing what happens in the hours in which storage buys energy. Storage can buy energy at hours in which:

- clean is unconstrained: in this case, more clean energy will be used to satisfy demand in the hours at which storage sells. Whether E_c/E_d increases or decreases is ambiguous.

On one hand, more clean energy is used to satisfy demand in the hours in which storage sells, which pushes E_c/E_d to increase. On the other hand, K^{c*} decreased, which means less clean energy is produced in all the hours in which clean is capacity-constrained. Which effect prevails is ambiguous, and it depends on the parameters' values.

- clean is constrained and dirty is active: this will increase the revenue of clean in the hours in which it is capacity constrained. K^{c*} increases. Whether E_c/E_d increases or decreases depends on whether this increase in K^{c*} is strong enough to counteract the decrease in K^{c*} driven by lower maximum prices. E_c/E_d increases if overall K^{c*} increases, which is ambiguous and depends on the parameters' values.

Proof of the second part of Result 4: Given a low p_c, if p_s decreases, the clean energy share increases.

Consider a level of $p_c < \bar{\bar{\bar{p}}}_c$, where $\bar{\bar{\bar{p}}}_c$ is defined in section 11.4. Given we are interested in the case with storage, one must replace the residual demand e_h^{res} characterized by the previous section for demand e_h in the definition of p_c. When $p_c < \bar{\bar{\bar{p}}}_c$, the equilibrium is characterized by the case D described in Section 11.4. The dirty producer is constrained at night. In this case the maximum hourly price will be at night hours. Storage will sell only at night hours. As p_s decreases, the storage firm decides to store more energy. It sells at the night hours with the maximum price. This will push down the maximum price, which will decrease K^{d*}. K^{c*} is not affected by night prices. Therefore it does not change in response to a change in the price in the hours at which storage sells. K^{c*} might instead be affected by the change in prices in the hours at which storage buys. Storage buys at hours during the day with the minimum price. The change in K^{c*} and E_c/E_d will depend on whether storage buys at hours in which:

- Clean is unconstrained: in this case, storage will store clean energy at day and sell it during the night. K^{c*} will not change, as clean does not make revenues in the hours at which it is unconstrained. E_c/E_d increases, as more clean energy is produced at day and less dirty energy is produced at night.

- Clean is constrained: storage demand drives up prices in an hour in which clean is constrained, therefore increasing clean profits. K^{c*} increases, which results in more clean energy production in the hours in which storage buys, but also in all the other hours in which storage is constrained. As a consequence, E_c/E_d increases.

Note I do not consider the case in which storage buys at hours in which only dirty is active because I have restricted the attention to values of $p_c < \bar{\bar{\bar{p}}}_c$ and therefore an equilibrium characterized by case D. In this case $K^{c*} > 0$, and given clean hourly production costs are lower than the dirty ones clean will always be active during the day.

In the case in which no dirty energy is produced already at the initial point, a decrease in p_s will result in no change in the clean energy share, which remains fixed at 100%.

11.6 Proof of Result 5

Proof of the first part of Result 5: Given a high p_s, if p_c decreases, the clean energy share can increase or stay constant. It exists a level of p_c, $\bar{p_c} > 0$, such that for $p_c < \bar{p_c}$, as p_c decreases, the clean energy share remains constant at a level strictly smaller than 100%. The clean energy share will equal the share of aggregate energy demand realized in hours with $\xi_h^c = \xi$.

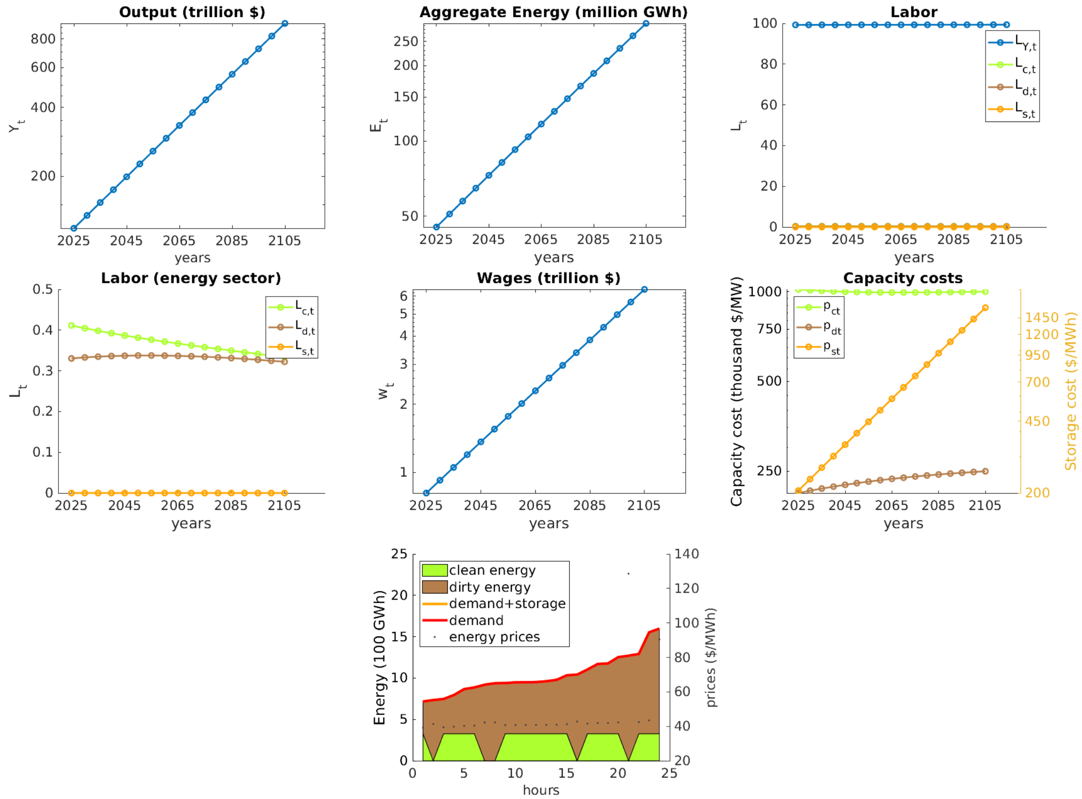
As p_c decreases we transit from case A, to case B, then C, and finally D as described in Section 11.4. Once the equilibrium is characterized by case D, as p_c decreases a larger and larger fraction of energy during the day will be produced by clean. Until at a certain point, all of energy during the day will be produced by clean. I define p_c^* as the clean capacity costs such that all of the energy during the day is produced by clean. In this case, p_h^{max} is determined by the dirty marginal capacity and hourly production cost at h_n^{max}. p_h^{min} will be determined by the marginal hourly production cost of clean during the day. Take a value of $p_s : p_s^* > p_h^{max} - p_h^{min}$ given the p_h^{max} and p_h^{min} I just defined. When $p_c < p_c^*$ and $p_s \geq p_s^*$, as p_c decreases p_h^{max} and p_h^{min} do not change. Storage keeps being not used, and the clean energy share remains equal to the fraction of energy demand that realizes during day hours.

Proof of the second part of Result 5: Given a low p_s, if p_c decreases, the clean energy share increases or stays constant. If p_s is sufficiently low, it exists a level of p_c, $\bar{\bar{p_c}} > 0$, such that for $p_c \leq \bar{\bar{p_c}}$ the clean energy share gets to 100%.

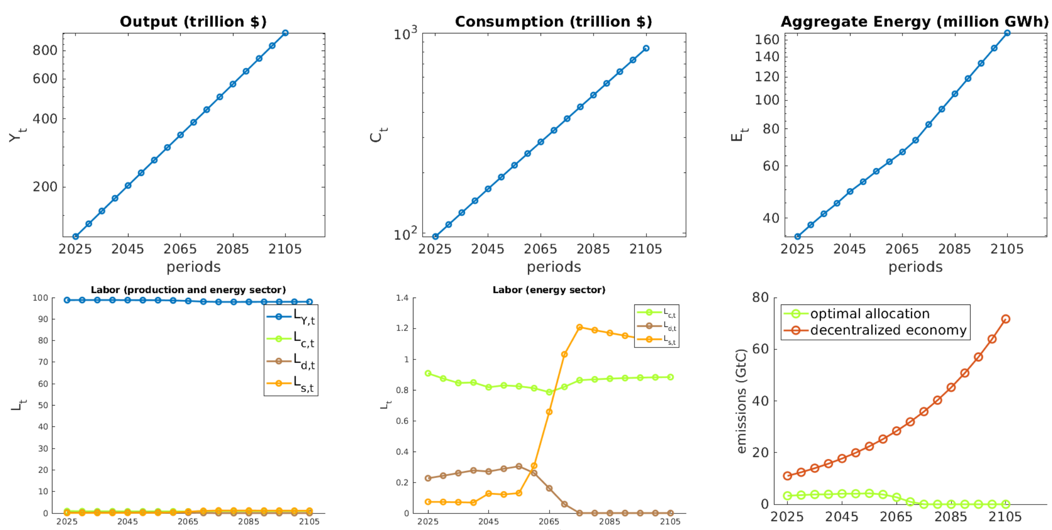
Define $pmin$ as the marginal hourly production cost of clean during the day hours with minimum demand if clean is the only producer in this hour. Assume $p_s + pmin < p_d + z^d$, which is satisfied in the data. Then $\bar{\bar{p_c}} = p_s + pmin - \lambda * K^{c\lambda - 1}$ where $K^c = \frac{e_{h_d^{max}}}{n_c}$.

11.7 Simulations: Additional Figures

11.7.1 Decentralized Economy

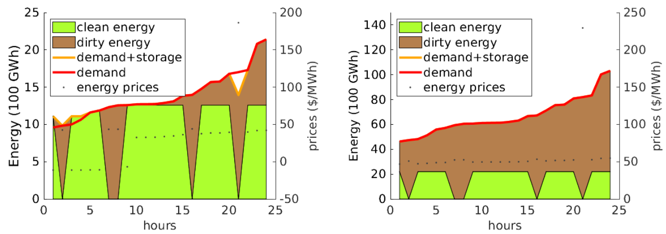


11.7.2 Optimal Allocation



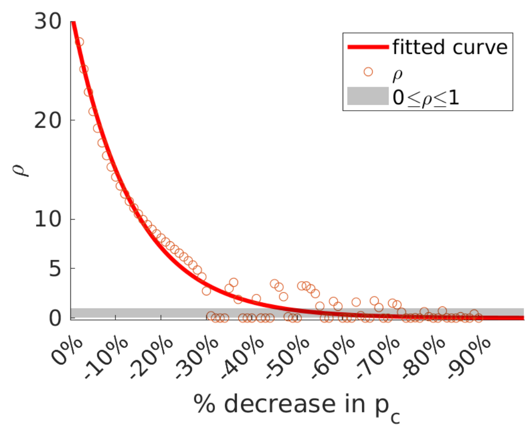
11.7.3 IRA Allocation

(a) IRA: Energy Market Short-Run (b) IRA: Energy Market Long-Run



11.8 Aggregate Elasticity of Substitution with a continuous ξ_h^c

Figure 10. High storage cost



Notes: This figure shows estimates of the elasticity of substitution for a model with a continuous ξ_h^c. The calibration is a sample calibration, so the exact values of the elasticity should not be interpreted as the values of the elasticity estimated for the US. The figure shows that with a model with a continuous ξ_h^c that takes on positive values between zero and one, we get more continuous estimates of the elasticity.