Regulatory Model Secrecy and Bank Reporting Discretion

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Abstract

This paper studies how banking regulators should disclose the regulatory models they use to assess banks that have reporting discretion. In my setting, such assessments depend on both economic conditions and the fundamentals of banks' assets. The regulatory models provide signals about economic conditions, while banks report information about their asset fundamentals. On the one hand, disclosing the models helps banks understand how their assets perform in different economic environments. On the other hand, it induces banks with socially undesirable assets to manipulate reports in order to obtain favorable assessments. While regulators can partially deter manipulation by designing the assessment rule optimally, the disclosure decision of the regulatory models remains necessary. The optimal disclosure policy is to disclose the regulatory models when the assessment rule is more likely to induce manipulation and keep them secret otherwise. In this way, disclosure complements the assessment rule by reducing manipulation when it harms the regulators more. These analyses speak directly to supervisory stress tests and climate risk stress tests.

Keywords: Regulatory secrecy, Bank reporting discretion, Communication, Disclosure, Stress test

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1 Introduction

Regulators assess banks on a regular basis to ensure the stability and sustainability of the banking industry. In order to do this, regulators rely on models which capture various features of the economy and banks. These models are not always disclosed to banks, making the process of regulatory assessment opaque and its implications unclear. One important reason for not disclosing these models is to prevent banks from gaming the regulatory assessment (Flannery 2019; Clark and Li 2022). A common way for banks to game is to provide uninformative reports that do not represent their underlying risks (Huizinga and Laeven 2012; Bushman and C. D. Williams 2012; Bushman 2016). However, regulatory models contain valuable information which can help banks understand how their assets would perform under different economic conditions. By disclosing the models, regulators enable banks to make more informed decisions. In this paper, I study how regulators should disclose the models they use to assess banks, when banks have reporting discretion.

This study is especially relevant for supervisory stress tests and climate risk stress tests. Supervisory stress tests employ a batch of regulatory models to evaluate the resilience of large banks to adverse macroeconomic shocks. The regulatory models translate these shocks into parameters that affect the valuation of bank balance sheet components and banks' loss absorption capacity. Regulators in different countries vary in their approaches to disclosing regulatory models. For example, in the past, the Federal Reserve only provided the broad framework used in its supervisory stress tests. In recent years, it has moved towards more disclosure of its models, including key variables and certain equations. Whereas in Europe, comprehensive disclosure of stress test methodologies has become common practice.[1] While the regulators' disclosure of the models helps banks understand the impact of macroeconomic shocks on their business activities, it also facilitates banks to manipulate information, which compromises the reliability of stress test results. The severity of such manipulations, which is governed by banks' reporting discretion, crucially affects the regulators' tradeoff of disclosing the models. In the case of climate risk stress tests, disclosing models is even more valuable due to the long-term nature of climate risks.[2] However, substantial reporting discretion for banks on climate issues

[1] For example, the European Banking Authority (EBA) discloses the details of models used in stress tests (see https://www.eba.europa.eu/eba-launches-2023-eu-wide-stress-test-0), the Federal Reserve instead discloses high-level information about the models underlying Dodd-Frank Act Stress Test (DFAST) and demonstrates how these models work on hypothetical loan portfolios (See https://www.federalreserve.gov/publications/files/2023-june-supervisory-stress-test-methodology.pdf).

[2] To fully capture the climate risk factors, the time horizons for climate risk stress tests usually range between 30 and 50 years. Such long time horizons considerably increase the uncertainty about the implication of business activities. See Baudino and Svoronos (2021).

undermines the stress test results, limiting the scope of the disclosure of regulatory models.[3]

I develop a tractable model to study the optimal disclosure policy about regulatory assessment models. To fix idea, I describe the regulatory assessment in the context of stress test. My model features one bank and one regulator. The bank has an existing asset whose payoff depends on both economic conditions and the asset's fundamental, which can be either high or low. Specifically, the asset yields a higher payoff in good economic conditions or when its fundamental value is high. While the bank is better than the regulator at measuring its asset's fundamental, the regulator is more proficient in evaluating the economic conditions.[4]

To evaluate the asset, the regulator conducts a stress test which proceeds as follows. The regulator uses the regulatory models to obtain a signal about the economic conditions. The regulator then discloses the signal to the bank according to a predetermined disclosure policy (discussed later). In this paper, disclosing the regulatory models is equivalent to disclosing the signal generated by the models. Subsequently, the bank, which has reporting discretion, reports the fundamental of the asset. Based on the signal and the bank's report, the regulator makes a pass/fail decision. Passing the test allows the bank to retain the asset, while failing requires the bank to liquidate the asset.[5]

The bank obtains large private benefits when retaining the asset such that it prefers to hold the asset regardless of its payoff. But the regulator prefers that the bank keeps the asset only when its fundamental is high. This conflict of interest motivates the bank to engage in report manipulation. By manipulating the report, the bank aims to make the low fundamental asset and the high fundamental asset appear more similar, reducing the informativeness of the report. Manipulation is costly to the bank, and this cost is determined by the amount of reporting discretion that the bank has.

In my setting, the regulator wields two tools to mitigate the impact of the bank's manipulation: adjusting the pass/fail decision and designing the disclosure policy about the signal.

[3]Governments and market watchdogs have recently introduced new climate-related reporting rules. For example, in Europe, the Corporate Sustainability Reporting Directive (CSRD) entered into force on 5 January 2023, which requires large companies and listed SMEs to disclose social and environmental related information (see https://finance.ec.europa.eu/capital-markets-union-and-financial-markets/company-reporting-and-auditing/company-reporting/corporate-sustainability-reporting_en). Similarly, the U.S. Securities and Exchange Commission (SEC) has proposed rule changes which require registrants to include certain climate-related disclosures (see https://www.sec.gov/news/press-release/2022-46). Nevertheless, banks still have substantial reporting discretion regarding climate-related issues.

[4]This captures that the banking regulators, responsible for overseeing the entire banking industry, possess superior industry-wide knowledge compared to the focal bank. The economic conditions can be interpreted as indicators of the health of the financial system, including aggregate credit risk and aggregate non-performing loans.

[5]I use the term "liquidation" to represent all possible remedial actions after failing the stress test. Further discussions on this point are in Section 2.

The necessity for such disclosure policy arises endogenously from the interactions between the pass/fail decision and the bank's manipulation choice, which ultimately determine the structure of the disclosure policy.

I show that the regulator's pass/fail decision is characterized by a cutoff rule based on the bank's report. Specifically, a high-value report is indicative of a high fundamental asset, leading the regulator to pass the bank if the report exceeds a threshold, and to fail it otherwise. When choosing the passing threshold, the regulator trades off the cost of passing a low fundamental asset (i.e., inefficient continuation or type II error) against the cost of failing a high fundamental asset (i.e., inefficient liquidation or type I error). The cost of inefficient continuation is equivalent to retaining the payoff of the low fundamental asset, while the cost of inefficient liquidation corresponds to forgoing the payoff of the high fundamental asset.

However, even when the passing threshold is chosen optimally, I find that the regulator passes the low fundamental asset too often. This occurs because while the optimal passing threshold trades off inefficient liquidation against inefficient continuation, the bank's manipulation increases the likelihood of inefficient continuation unconditionally. As a result, from the ex ante perspective, the optimal passing threshold alone is insufficient in reducing the inefficient continuation. Therefore, the regulator requires additional tools to tackle this issue.

The disclosure of the regulator's signal complements the pass/fail decision as an ex ante approach to mitigate the issue of excessive inefficient continuation caused by the bank's manipulation. The bank's manipulation incentive is driven by the desire to have the low fundamental asset pass the test. The benefit of manipulation is then determined by two factors: the extent to which manipulation increases the probability of passing the test and the gain from passing the test. The former is determined by the passing threshold, while the latter depends on the payoff of the low fundamental asset. Since the economic conditions impact both the threshold and the asset's payoff, the disclosure of the regulator's signal affects how the bank perceives the benefit of manipulation. In the absence of such disclosure, the bank chooses manipulation based on the expected economic conditions, resulting in a constant level of manipulation across different economic conditions. In contrast, when the regulator's signal is disclosed, the bank observes the realized economic conditions.

The effect of the disclosure of the regulator's signal on the bank's manipulation is two-fold. First, it induces the bank's manipulation to be contingent on the economic conditions. I show that when the cost of inefficient continuation (i.e., retaining the payoff of the low fundamental asset) and that of inefficient liquidation (i.e., forgoing the payoff of the high fundamental asset) are similar, the bank's manipulation choice varies with the gain from passing the test with the low fundamental asset. When this gain is lower, or, equivalently, when the payoff of the low fundamental asset is lower, the bank manipulates less. Such manipulation choice occurs because, in this case, the regulator's choice of the passing threshold is not responsive to the bank's manipulation since any adjustment in the threshold would affect both inefficient liquidation and inefficient continuation which are equally costly. As a result, the bank can manipulate to increase the probability of passing the test for the low fundamental asset without triggering the regulator to adjust the passing threshold. Given that manipulation is effective in increasing the passing probability, the bank's manipulation choice is then driven by the gain from passing the test with the low fundamental asset, resulting in reduced manipulation as this gain decreases. However, when the cost of one inefficiency outweighs the other, I show that the bank's manipulation choice changes based on the extent to which manipulation can increase the probability of passing the test for the low fundamental asset. Because in this case, as the bank's report becomes less informative due to increased manipulation, the regulator adjusts the passing threshold to prevent the inefficiency that is more costly. This suggests that the regulator's choice of the passing threshold is sensitive to the bank's manipulation, restricting the extent to which manipulation can increase the passing probability for the low fundamental asset. Consequently, the bank manipulates more when manipulation increases the passing probability by a larger magnitude.

The second effect of disclosing the regulator's signal on the bank's manipulation is that it affects the bank's *expected* amount of manipulation across the economic conditions. When the bank's manipulation choice varies with the gain from passing the test with the low fundamental asset, disclosing the regulator's signal reduces the expected amount of manipulation. Conversely, if the bank increases its manipulation when it can raise the passing probability for the low fundamental asset by a larger amount, disclosing the regulator's signal increases the expected amount of manipulation. This second effect of disclosing the regulator's signal is caused by the bank's varying manipulation choice across different economic conditions. This variation triggers the regulator to further adjust the passing threshold, amplifying the first effect discussed above.

The optimal disclosure policy is to disclose the regulator's signal when the cost of inefficient continuation and that of inefficient liquidation are comparable and keep the signal secret otherwise. When the costs of the two inefficiencies are comparable and the regulator's signal is disclosed, the bank manipulates less when the payoff of the low fundamental asset is lower. This manipulation choice benefits the regulator as it reduces the bank's manipulation when passing the low fundamental asset results in higher losses. Hence, disclosing the signal diverts the bank's manipulation away from cases where the regulator suffers higher losses from manipulation. Moreover, disclosing the signal in this case reduces the expected amount of manipulation.

In contrast, when the cost of one inefficiency dominates, the regulator should keep the signal secret to hinder the bank from assessing how manipulation can affect the pass/fail decision. Specifically, this approach prevents the bank from manipulating more when manipulation results in larger increase in the passing probability for the low fundamental asset, reducing the regulator's expected losses from passing the low fundamental asset. In addition, keeping the signal secret reduces the expected amount of manipulation.

The optimal disclosure policy crucially depends on the bank's private benefit when retaining the asset and the reporting discretion. Lower private benefit and lower reporting discretion reduce the bank's manipulation incentive, mitigating the regulator's concerns. I show that when disclosing the signal results in increased (reduced) manipulation by the bank, the regulator is more (less) likely to disclose as the bank's private benefit and reporting discretion decrease.

I consider several extensions to the baseline model. First, in a situation where the regulator cannot commit to a predetermined disclosure policy, I show that the regulator will always disclose the signal and be worse off. This result resembles the "unraveling" of private information (e.g., Grossman and Hart (1980)). However, the rationale differs. In absence of commitment, the regulator discloses the signal to reduce the bank's manipulation for a given economic condition. However, such disclosure destroys the possibility to share the bank's manipulation incentive across different economic conditions. Consequently, the bank may increase its manipulation especially when it is more effective in influencing the regulator's pass/fail decision, leading to an increase in the expected level of manipulation and ultimately reducing the regulator's ex ante payoff. Second, I extend the model to incorporate the bank's investment choice. In this extension, the regulator's disclosure choice of the signal may not only affect the bank's reporting decision but also its investment decision. I demonstrate that the regulator's additional concern about the bank's investment may increase the likelihood of disclosing the signal.

The remainder of the paper is organized as follows. The rest of the introduction discusses the relevant literature. Section 2 presents the model. Section 3 studies the optimal pass/fail decision and the bank's manipulation response. Section 4 analyzes the optimal disclosure policy about the regulatory models. Section 5 conducts comparative statics and demonstrates how the bank's reporting discretion and private benefit affect the optimal disclosure policy. Section 6 discusses the model assumptions and possible extensions. Section 7 concludes. All proofs are included in Appendix A.

1.1 Related literature

This paper is related to the literature on the design of stress tests and their implications. Parlatore and Philippon (2022) study the design of stress test scenarios. Ding, Guembel, and Ozanne (2022) examine how the leniency of stress tests and the level of granularity in test results affect the incentives for financial markets to generate information about banks. Shapiro and Zeng (2023) study the leniency of stress tests in cases where the regulator trades off the reputational concern against the banks' lending efficiency. I contribute to the design of stress tests by considering the optimal disclosure policy about regulatory models, taking into account its implication on the banks' reporting incentive which further influences the accuracy and reliability of stress test results.

This paper is also related to the literature on the transparency of stress tests and regulatory information. Vast literature focuses on the disclosure of stress test results (e.g., Goldstein and Sapra (2014); Goldstein and Leitner (2018); Corona, Nan, and Zhang (2019); Goldstein and Leitner (2020); Quigley and Walter (2023); Parlasca (2023)), or, more generally, the disclosure of regulatory information obtained through supervisions (e.g., Prescott (2008); Bouvard, Chaigneau, and Motta (2015); Faria-e-Castro, Martinez, and Philippon (2017)). Instead, I focus on, before conducting the stress tests (and/or examinations), whether regulators should communicate with banks about the test models. Similar to my paper, Leitner and B. Williams (2023) also study the disclosure policy about the regulatory models. In their paper, revealing the regulatory models induces the bank to always invest in the risky asset even when the value is low, but not revealing them may lead to underinvestment. While their focus is on the riskiness of bank's investment, I examine the role of banks' information inputs in the stress tests and study how reporting discretion affects the disclosure policy about regulatory models.

In this paper, I study two-way communication between regulators and banks when both have information to share. Relatedly, this paper is connected to the communication literature with an informed receiver (e.g., Kolotilin et al. (2017); Guo and Shmaya (2019)). Different from the existing literature which focuses on the sender, I study the communication strategy of the informed receiver (i.e., the regulator), and, in particular, whether the informed receiver can gain from communicating first to the sender (i.e., the bank). Extending the cheap talk game to two-way communication, Chen (2009) shows that the informed receiver cannot credibly disclose her information when communicating first and benefits little from two-way communication. In this paper, I consider an informed receiver who can commit to a disclosure policy, and I show that in this case, the receiver may sometimes benefit from two-way communication.

I also study the banks' reporting incentive when reporting discretion exists. The banks' reporting discretion determines how much information banks communicate with regulators. Prior literature has analyzed the role of reporting discretion in other settings. For instance, Fischer and Verrecchia (2000) study how reporting bias affects the informativeness of the firms' financial reports, when the user of the reports (e.g., the capital market) is uncertain about the manager's reporting objective. Gao and Jiang (2018) study reporting discretion in the context of bank run. In their paper, the reporting discretion reduces the panic-based runs, but it may also reduce the fundamental-based runs. This paper also contributes to the literature on the determinants of reporting quality (e.g., Leuz, Nanda, and Wysocki (2003); Barth, Landsman, and Lang (2008); Holthausen (2009); Leuz and Wysocki (2016)). The consensus is that the reporting quality depends on various factors. Among others, the regulatory environment and the development of capital market are crucial. This paper shows that the design of stress tests can affect the reporting quality of banks.

More broadly, this paper contributes to the discussion about the interplay between prudential and accounting regulation. Bertomeu, Mahieux, and Sapra (2023) show that accounting measurement complements capital requirements to affect the level and efficiency of banks' credit decisions. Corona, Nan, and Zhang (2015) examine the impact of accounting information quality on banks' risk-taking incentives, taking into account the interbank competition. This paper shows that the design of stress tests should be coherent with the prevailing accounting regulation to achieve informative assessment.

2 The model

Consider a risk-neutral economy with no discounting. There is one regulator and one bank. The regulator conducts a stress test on the bank. I model the stress test as a four-period game.

At $t = 1$, the stressed scenarios are exogenously given and observed by everyone. The regulator uses regulatory models to predict the impact of the macroeconomic variables included in these stressed scenarios on the banking industry. The output of the regulatory models is summarized in a signal $s \in S = [\underline{s}, \bar{s}]$ with a cumulative distribution function F and density f. The density f has full support. The regulator privately observes s. Throughout the paper, I refer to the signal s as the economic condition. The signal s could represent the probability of a liquidity shock in the interbank market during a given macroeconomic stress, or the aggregate amount of deposit withdrawals from a specific industry due to supply chain disruptions.

The focus of this paper is to study the optimal disclosure policy about the signal s. At $t = 0$, the regulator commits to a disclosure policy before conducting the stress test. The disclosure policy is defined by the disclosure set $D \subseteq S$ and the no-disclosure sets $N_n \subseteq S$, where $n \in [1, +\infty)$ denotes the number of no-disclosure sets, and, for simplicity, the first no-disclosure set is denoted by $N \equiv N_1$. For any signal $s \in D$, the regulator communicates it truthfully to the bank. For any signal $s \in N_n$, the regulator informs the bank that the signal falls within N_n. The no-disclosure sets are assumed to be convex sets and $N_n \cap N_{n'} = \emptyset$ for $\forall$ $n \neq n'$ and $\bigcup\limits_n N_n = S \setminus D$.

The bank's asset has continuation value $X(s, \omega)$ and liquidation value $L(s, \omega)$, which depend on a state variable ω and the economic condition s. The variable ω represents the fundamental of the asset. It is either ω_h with probability q_h or ω_l with probability $q_l \equiv 1 - q_h$ and $\omega_h > \omega_l$. Let $x(s, \omega)$ denote the relative gains from continuing the asset. That is,

$$x(s, \omega) = X(s, \omega) - L(s, \omega).$$

For the remainder of the paper, the solutions are derived in terms of $x(s, \omega)$. I assume that $x(s, \omega)$ is increasing and weakly concave in s.[6]

To define efficient liquidation and efficient continuation, I assume that $x(s, \omega_l) \leq 0 \leq x(s, \omega_h)$. This implies that the asset should only continue if its fundamental value is high. Moreover, I assume that $\frac{x(s,\omega_l)}{x(s,\omega_h)}$ is weakly log-concave in s. This assumption ensures that the ratio is not too concave and that the relative gain $x(s, \omega_l)$ compared to $x(s, \omega_h)$ increases at a sufficiently high rate with respect to s. For example, $x(s, \omega) = s^q + \omega$ with $0 < q \leq 1$ satisfies all the assumptions for appropriate values of s and ω. I make the following further assumption about the bank's asset.

Assumption 1. *The bank's asset is ex ante worth continuing:* $\mathbb{E}_\omega \big[x(s, \omega)\big] \in [0, q_h x(\bar{s}, \omega_h)]$ *for* $s \in [\underline{s}, \bar{s}]$.

This assumption also suggests that the expected continuation value of the bank's asset exceeds its liquidation value for any signal s.[7] Consequently, in expectation, liquidating a high fundamental asset (inefficient liquidation) is more costly than continuing a low fundamental asset (inefficient continuation). This assumption helps to characterize the optimal disclosure policy, but my main results can be extended to cases where this condition is violated. I discuss this assumption in Section 6.1.

[6]As it will become clear in later sections, all analyses revolve around the low fundamental asset. Hence, I can alternatively assume $x(s, \omega_h)$ to be a constant for simplicity and the analyses will remain unchanged.

[7]Notice that $x(s, \omega_l)$ is assumed to be non-positive, hence, the maximum value of $\mathbb{E}_\omega \big[x(s, \omega)\big]$ is $q_h x(\bar{s}, \omega_h)$.

The fundamental of the bank's asset ω is not observable to anyone but it is measured by the bank's report. More specifically, the fundamental of the bank's asset determines the report distribution. If the fundamental is ω_i, then the report t follows a distribution with density $g^i(t)$ over $t \in [\underline{t}, \bar{t}]$, where $i = \{h, l\}$. The density functions have full support and satisfy the monotone likelihood ratio property (MLRP), i.e., $\frac{g^l(t)}{g^h(t)}$ is decreasing in t. This assumption implies that the report t is informative about the asset fundamental. Moreover, I assume that the ratio $\frac{g^l(t)}{g^h(t)}$ is weakly concave in t. I impose the regularity condition that the hazard rate $\frac{g^h(t)}{1-G^h(t)}$ and $\frac{g^l(t)}{1-G^l(t)}$ are decreasing on the support of t. This assumption means that when the bank receives a high-value report, it becomes more probable to receive a higher report.

At $t = 2$, the bank may engage in costly manipulation to affect the report distribution. I follow Gao and Jiang (2020) to model bank's manipulation as ex ante manipulation. That is, the bank chooses the manipulation level before observing the fundamental of the asset or the report.[8] Specifically, the bank chooses manipulation $m \in [0, 1]$ to change the report distribution from $g^i(t)$ to

$$g^i_m(t) = g^i(t) + m\big(g^h(t) - g^i(t)\big). \tag{1}$$

If $m = 0$, the report distribution is not affected by manipulation. If $m = 1$, then the report is always generated from the distribution of high fundamental asset $g^h(t)$. If $m \in (0, 1)$, then manipulation improves the distribution in the sense of first-order stochastic dominance. The cost of manipulation m is $kc(m)$ for the bank, where $k \in (0, +\infty)$ measures the degree of reporting discretion determined by rules and regulations and the cost function $c(m)$ is increasing and convex with $c(0) = c'(0) = 0$. I also assume that $\frac{c'(m)}{c''(m)}$ is weakly increasing in m, or, equivalently, that $c'(m)$ is weakly log-concave. The conditions are often used in the literature (see Gao and Jiang (2020)) and are satisfied for common convex functions, e.g., $c(m) = m^q$ for $q \geq 2$.

After observing the economic condition s and receiving the report t from the bank, the regulator makes a pass/fail decision a at $t = 3$. In particular, the regulator passes ($a = 1$) or fails ($a = 0$) the bank to maximize the payoff u given by

$$u \equiv ax(s, \omega). \tag{2}$$

The bank's payoff v is

$$v \equiv a\big(x(s, \omega) + B\big) - kc(m). \tag{3}$$

Where B is the bank's private benefit from continuing the asset. I assume $x(s, \omega) + B > 0$ for all s and ω, meaning that the private benefit B is sufficiently large for the bank to prefer

[8] All results continue to hold with ex post manipulation. See Section 6.2 for more details.

continuation regardless of the value of x. The private benefit introduces a conflict of interest between the regulator and the bank concerning the low fundamental asset.[9]

The timeline of the model is as follows,

At $t = 0$, the regulator commits to a disclosure policy about the signal s.

At $t = 1$, the regulatory models generate a signal s. The regulator privately observes s and discloses it to the bank according to the disclosure policy.

At $t = 2$, the bank chooses the level of manipulation m to affect the report distribution.

At $t = 3$, the state ω is realized, and the bank's report t is generated. Based on the signal s and report t, the regulator passes or fails the bank. And payoffs are realized.

The equilibrium is characterized by the regulator's disclosure policy about s, the pass/fail decision a, and the bank's manipulation m. I solve the model by backward induction. I first solve for the regulator's pass/fail decision a for given manipulation level m and disclosure policy about s. Anticipating the pass/fail decision rule, the bank then chooses the manipulation m for given disclosure policy about s. Lastly, the regulator chooses the disclosure policy about s, taking into account its impact on the bank's manipulation choice and, consequently, the pass/fail decision.

3 Manipulation and pass/fail decision

In this section, I discuss the bank's manipulation choice and the regulator's pass/fail decision, taking the regulator's disclosure policy about s as given.

At $t = 3$, the regulator forms expectation of the relative continuation value x based on the signal s and the bank's report t. The regulator passes the bank ($a = 1$) if and only if

$$\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] \geq 0. \tag{4}$$

Where $\hat{m}$ is the regulator's conjecture about the bank's manipulation.[10] Since $g^h(t)$ is a monotone likelihood ratio improvement of $g^l(t)$, the expected relative continuation value $\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}]$

[9]Continuation and liquidation should be interpreted broadly. While continuation represents cases where the bank operates as usual, liquidation refers to situations where regulatory interventions are imposed on the bank. In the context of stress tests, the common regulatory interventions include increasing capital buffers or restricting dividend distributions.

[10]The conditional expectation is

$$\begin{aligned}\mathbb{E}_\omega[x(s,\omega)|t,\hat{m}] &= x(s,\omega_h)\Pr(\omega=\omega_h|t,\hat{m}) + x(s,\omega_l)\Pr(\omega=\omega_l|t,\hat{m})\\ &= x(s,\omega_h)\frac{q_h g^h(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)} + x(s,\omega_l)\frac{q_l g^l_{\hat{m}}(t)}{q_h g^h(t)+q_l g^l_{\hat{m}}(t)}.\end{aligned}$$

is increasing in the report t. As a result, the pass/fail decision follows a cutoff rule.

Lemma 1. *For a given signal s, a bank's report t, and a conjecture about the bank's manipulation $\hat{m}$, the regulator passes the bank if and only if $t \geq t_p(s, \hat{m})$, where the passing threshold $t_p(s, \hat{m})$ solves*

$$\mathbb{E}_\omega[x(s, \omega)|t_p, \hat{m}] = 0.$$

All proofs are included in Appendix A. The passing threshold $t_p(s, \hat{m})$ is defined by the regulator's indifferent condition. That is, the regulator is indifferent between passing and failing the bank when the report is $t_p(s, \hat{m})$. This passing threshold is chosen to equalize the expected cost of failing a high fundamental asset (inefficient liquidation) and the expected cost of passing a low fundamental asset (inefficient continuation) *for a given signal s and a given conjecture about manipulation $\hat{m}$*. The following lemma characterizes the passing threshold $t_p(s, \hat{m})$.

Lemma 2. *For a given level of manipulation m, the passing threshold $t_p(s, m)$ is decreasing in s. For a given signal s, the passing threshold $t_p(s, m)$ is decreasing in m.*

The intuition of the first result follows from how the cost of failing a high fundamental asset and that of passing a low fundamental asset change with the signal s. For a given manipulation level m, the relative gain from continuing the asset $x(s, \omega)$ is increasing in s, implying that failing a high fundamental asset becomes more costly relative to passing a low fundamental asset. In response to the rising cost of inefficient liquidation, the regulator is willing to lower the passing threshold and pass the bank more often. The second result captures how manipulation affects the relative cost of failing a high fundamental asset and passing a low fundamental asset. Assumption 1 assumes that in absence of the report, the regulator's expectation of the relative gain from continuing the asset is non-negative. This implies that inefficient liquidation (failing a high fundamental asset) is more costly than inefficient continuation (passing a low fundamental asset) in expectation for all signal s. Manipulation makes the report distribution of low and high fundamental asset more similar, making it more difficult for the regulator to differentiate between the two types of assets. In order to preserve the high fundamental asset, the regulator needs to lower the passing threshold.

At $t = 2$, the bank anticipates the passing threshold $t_p(s, \hat{m})$ and chooses the manipulation m to maximize the expected payoff. The bank's expected payoff depends on the regulator's disclosure choice of s. If the bank does not observe the regulator's signal s, its expected payoff is

$$V(\hat{m}, m) = \mathbb{E}_s \left[q_h\big(x(s,\omega_h) + B\big) \int_{t \geq t_p(s,\hat{m})} g^h(t)dt + q_l\big(x(s,\omega_l) + B\big) \int_{t \geq t_p(s,\hat{m})} g^l_m(t)dt \,\middle|\, s \in N_n \right] - kc(m).$$

Where N_n is the no-disclosure set containing signals s that are not disclosed to the bank. For ease of exposition, I introduce the following definition.

$$\Delta\big(t_p(s,\hat{m})\big) \equiv \int_{t \geq t_p(s,\hat{m})} \big(g^h(t) - g^l(t)\big)dt. \tag{5}$$

This term is the difference in passing probability between high and low fundamental asset. It also measures the increases in passing probability for the low fundamental asset if the bank manipulates the report distribution. Taking derivative of $V(\hat{m}, m)$ with respect to m, I obtain the following first-order condition of the bank's manipulation m,

$$\mathbb{E}_s \left[q_l\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,\hat{m})\big) \,\middle|\, s \in N_n \right] - kc'(m) = 0.$$

In equilibrium, the regulator's conjecture about the manipulation $\hat{m}$ is consistent with the bank's choice. Hence, the equilibrium manipulation m_{N_n} solves

$$\mathbb{E}_s \left[q_l\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,m_{N_n})\big) \,\middle|\, s \in N_n \right] - kc'(m_{N_n}) = 0. \tag{6}$$

Proposition 1. *When s is not disclosed, the level of manipulation m_{N_n} is unique and it is a constant over s for $s \in N_n$.*

This result suggests that no disclosure of s forces the bank's manipulation m_{N_n} to be constant over the regulator's signal s.

If the bank observes the regulator's signal s, its expected payoff is

$$V(s, \hat{m}, m) = q_h\big(x(s,\omega_h) + B\big) \int_{t \geq t_p(s,\hat{m})} g^h(t)dt + q_l\big(x(s,\omega_l) + B\big) \int_{t \geq t_p(s,\hat{m})} g^l_m(t)dt - kc(m).$$

The first-order condition of the bank's manipulation response m is as follows,

$$q_l\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,\hat{m})\big) - kc'(m) = 0.$$

Similar to the no disclosure case, the regulator's conjecture about the manipulation is consistent with the bank's choice in equilibrium. Hence, the equilibrium manipulation $m_D(s)$ is determined by

$$q_l\big(x(s,\omega_l)+B\big)\Delta\Big(t_p\big(s,m_D(s)\big)\Big) - kc'\big(m_D(s)\big) = 0. \tag{7}$$

I make the following notation for ease of exposition.

$$MB_b\big(s,t_p(s,m)\big) \equiv q_l\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big). \tag{8}$$

Where "MB" stands for "marginal benefit" and "b" represents "bank". $MB_b\big(s,t_p(s,m)\big)$ is the bank's marginal benefit of manipulation for given regulator's signal s and manipulation level m. It consists of two components concerning the low fundamental asset. The first component, $q_l\big(x(s,\omega_l)+B\big)$, is the bank's gain after passing the test with the low fundamental asset. Since the relative gain from continuing the low fundamental asset $x(s,\omega_l)$ is increasing in the signal s, the bank's gain is also increasing in s. All else equal, the bank manipulates more when the signal s is high.

The second component, $\Delta\big(t_p(s,m)\big)$, represents the increases in the passing probability for the low fundamental asset when the bank changes the report distribution from $g^l(t)$ to $g^h(t)$. This term crucially depends on the passing threshold $t_p(s,m)$. According to Lemma 2, the passing threshold $t_p(s,m)$ is decreasing in s due to the rising relative cost of failing the high fundamental asset. As the passing threshold decreases, the test becomes more lenient in the sense that the low fundamental asset is more likely to pass the test without manipulation. In other words, the difference in the passing probability between $g^h(t)$ and $g^l(t)$ shrinks. Hence, manipulation is less effective in increasing the passing probability for the low fundamental asset as s increases. All else equal, the bank manipulates less when the signal s is high. The following lemma summarizes the impact of the signal s on $\Delta\big(t_p(s,m)\big)$.

Lemma 3. *For given manipulation level m, $\Delta\big(t_p(s,m)\big)$ is decreasing in s.*

When evaluating the bank's manipulation incentive MB_b, the increases in the passing probability for the low fundamental asset $\Delta\big(t_p(s,m)\big)$ acts as a counterforce to the gain from passing the test with the low fundamental asset $q_l\big(x(s,\omega_l)+B\big)$. The magnitude of the two forces then determines how the manipulation $m_D(s)$ responds to the signal s.

Proposition 2. *When s is disclosed, the level of manipulation $m_D(s)$ is unique and it is increasing in s for $s < s_D$ and it is decreasing in s for $s > s_D$, where $s_D \in (\underline{s},\bar{s})$ is the unique solution for* $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} = 0$.

This result identifies the force that determines the bank's manipulation $m_D(s)$ when s is disclosed, and it highlights the effect of the passing threshold $t_p\big(s,m_D(s)\big)$ on the bank's manipulation $m_D(s)$. When the signal is relatively low, i.e., $s < s_D$, the cost of inefficient liquidation and that of inefficient continuation are comparable. Hence, the regulator sets the passing threshold at a medium level to prevent both types of inefficiency. In this case, the regulator's choice of the passing threshold is also unresponsive to the bank's manipulation, as any adjustment would affect both types of inefficiency, which are similarly costly. This choice of the passing threshold results in a substantial difference in the passing probability between the high and low fundamental asset, i.e., $\Delta\Big(t_p\big(s, m_D(s)\big)\Big)$ is large, and it also implies that manipulation is very effective in increasing the likelihood of passing the test for the low fundamental asset. Since manipulation can always increase the passing probability considerably, the bank focuses on the gain from passing the test with the low fundamental asset, $q_l\big(x(s, \omega_l) + B\big)$, when choosing the level of manipulation. As a result, the bank's manipulation $m_D(s)$ follows the changes in such gain, and it is increasing in s. When $s > s_D$, the inefficient liquidation becomes more costly than the inefficient continuation. Hence, the regulator sets the passing threshold relatively low to prevent inefficient liquidation. Moreover, the regulator's choice of the passing threshold becomes responsive to the bank's manipulation to further mitigate inefficient liquidation. In this case, the bank manipulates more when manipulation still has incremental effect on increasing the passing probability. Hence, the bank's manipulation $m_D(s)$ changes with the difference in passing probability between high and low fundamental asset $\Delta\Big(t_p\big(s, m_D(s)\big)\Big)$, and it is increasing in s.

Disclosure of s affects how manipulation changes with s. When s is not disclosed, the bank's manipulation m_{N_n} is constant over the signals in the no-disclosure set N_n. When s is disclosed, the bank's manipulation $m_D(s)$ varies with both the gain from passing the test with the low fundamental asset $q_l(x(s, \omega_l) + B)$ and the increases in passing probability for the low fundamental asset after manipulation $\Delta\Big(t_p\big(s, m_D(s)\big)\Big)$. Such variation in $m_D(s)$ further affects the expected level of manipulation. The following proposition compares the expected level of manipulation when s is disclosed with the one when s is not disclosed.

Proposition 3. *$\mathbb{E}_s\big[m_D(s)|s \in N\big] \leq m_N$ for any $N \subseteq [\underline{s}, s_D]$ and $\mathbb{E}_s\big[m_D(s)|s \in N\big] \geq m_N$ for any $N \subseteq [s_D, \bar{s}]$.*

This result shows the additional effect of disclosing s. When $s \leq s_D$, the bank's manipulation $m_D(s)$ is driven by the gain from passing the test with the low fundamental asset $q_l\big(x(s, \omega_l)+B\big)$ and it is increasing in the signal s. In response, the regulator lowers the passing threshold $t_p\big(s, m_D(s)\big)$, which makes the bank more likely to pass the test regardless of the fundamental value. Such endogenous response of the regulator's pass/fail decision then decreases the extent to which the manipulation can increase the passing probability for the low fundamental asset, i.e., $\Delta\Big(t_p\big(s, m_D(s)\big)\Big)$ is reduced, rendering manipulation less useful and decreasing the bank's manipulation incentive. Such endogenous response is absent if s is not disclosed, because the bank's manipulation remains constant. Hence, the expected level of manipulation is less if s is disclosed. However, when $s \geq s_D$, the bank manipulates to increase the passing probability and the manipulation level $m_D(s)$ is decreasing in s. In response, the regulator increases the passing threshold $t_p\big(s, m_D(s)\big)$ to make the test more difficult. Such endogenous response of the regulator's pass/fail decision then widens the difference in passing probability between the low and high fundamental asset, i.e., $\Delta\Big(t_p\big(s, m_D(s)\big)\Big)$ increases. More importantly, such response makes the manipulation effective in increasing the passing probability for the low fundamental asset, amplifying the bank's manipulation incentive. Hence, the expected level of manipulation when s is disclosed is larger compared to the case when s is not disclosed.

4 Disclosure policy

In this section, I discuss the optimal disclosure policy about the regulator's signal s, taking into account the bank's manipulation response and its impact on the regulator's pass/fail decision. I show that the disclosure choice of s and the pass/fail decision are complementary tools for the regulator to minimize the adverse consequence of the bank's manipulation.

For given signal s, the regulator's expected payoff at $t = 1$ is obtained by integrating across all report values that are higher than the passing threshold $t_p(s, m^*)$,

$$\begin{aligned} u(s, m^*) &= \int_{t \geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t, m^*] g_{m^*}(t) dt \\ &= \int_{t \geq t_p(s,m^*)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m^*}(t)\big) dt. \end{aligned} \tag{9}$$

Where $m^* = \{m_D(s), m_{N_n}\}$ is the equilibrium manipulation choice of the bank and $g_{m^*}(t)$ is the unconditional distribution of report t when the manipulation is m^*. That is,

$$g_{m^*}(t) = q_h g^h(t) + q_l g^l_{m^*}(t).$$

At $t = 0$, the regulator chooses disclosure policy D and N_n to maximize the ex ante payoff.

$$\begin{aligned} U &= \int_{s \in D} u\big(s, m_D(s)\big) dF(s) + \sum_n \left(\int_{s \in N_n} u(s, m_{N_n}) dF(s) \right) \\ &= \int_{s \in D} \left(\int_{t \geq t_p\left(s, m_D(s)\right)} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_D(s)}(t)\big) dt \right) dF(s) \\ &\quad + \sum_n \left(\int_{s \in N_n} \left(\int_{t \geq t_p(s, m_{N_n})} \big(q_h x(s,\omega_h) g^h(t) + q_l x(s,\omega_l) g^l_{m_{N_n}}(t)\big) dt \right) dF(s) \right). \end{aligned} \tag{10}$$

As briefly discussed in Lemma 2, manipulation increases the similarity between the report of the low fundamental asset and that of the high fundamental asset, making it more likely that the regulator fails the high fundamental asset (inefficient liquidation) and passes the low fundamental asset (inefficient continuation). The regulator is able to use the pass/fail decision to control this adverse consequence of manipulation, but only partially. To see this, consider the total derivative of $u(s,m)$ in (9) with respect to m. This total derivative represents the regulator's marginal loss due to the bank's manipulation,

$$\frac{du(s,m)}{dm} = \underbrace{\frac{\partial u(s,m)}{\partial t_p(s,m)}\frac{\partial t_p(s,m)}{\partial m}}_{\substack{\text{The impact of manipulation} \\ \text{through the pass/fail decision}}} + \frac{\partial u(s,m)}{\partial m}.$$

Since the regulator's pass/fail decision is optimal, the first term of the derivative is zero, suggesting that the bank's manipulation does not affect the regulator's ex ante payoff through the pass/fail decision. However, the second term of the derivative remains. This is because the manipulation, driven by the bank's preference to continue the asset regardless of its payoff, increases the ex ante likelihood of inefficient continuation. Such inefficient continuation cannot be prevented by using the optimal pass/fail rule. Formally, I make the following definition.

$$ML_r\big(s, t_p(s,m)\big) \equiv \frac{\partial u(s,m)}{\partial m} = q_l x(s,\omega_l)\Delta\big(t_p(s,m)\big). \tag{11}$$

Where "ML" stands for "marginal loss" and "r" represents "regulator". Given that the asset should be liquidated when the fundamental value is low, i.e., $x(s,\omega_l) \leq 0$ for all s, this term is non-positive. It captures the regulator's additional marginal losses from continuing the low fundamental asset due to the bank's manipulation.

The regulator's additional losses caused by the bank's manipulation $ML_r\big(s, t_p(s,m)\big)$ consists of two components. The first component, $q_l x(s,\omega_l)$, is the regulator's expected loss of passing the low fundamental asset. The second component, $\Delta\big(t_p(s,m)\big)$, is the increases in the probability of passing the low fundamental asset after the bank changes the report distribution from $g^l(t)$ from $g^h(t)$. This component captures the regulator's inability to distinguish the low fundamental asset and the high fundamental asset due to the bank's manipulation.

Lemma 4. *For any disclosure set D or no-disclosure set N_n, $ML_r\big(s, t_p(s,m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

This lemma suggests that, regardless of the disclosure choice of s, the regulator bears less additional losses from manipulation as the signal s increases. The intuition is as follows. Since the relative gain from continuing the low fundamental asset $x(s, \omega_l)$ is increasing in s, the regulator's loss from passing the low fundamental asset is ameliorated. In addition, the passing threshold $t_p(s, m^*)$ is decreasing in the signal s, shrinking the difference in passing probability between the low and high fundamental asset $\Delta\big(t_p(s, m^*)\big)$. This implies that the extent to which manipulation increases the passing probability for the low fundamental asset is declining as the signal increases. Consequently, the regulator is less likely to pass low fundamental asset, reducing the additional losses caused by manipulation.[11]

The regulator needs an additional tool to control the additional losses caused by manipulation $ML_r\big(s, t_p(s, m)\big)$. Lemma 4 shows that $ML_r\big(s, t_p(s, m)\big)$ is increasing in the signal s. To minimize the expected loss from manipulation, the regulator should distribute more manipulation to cases where the marginal loss $ML_r\big(s, t_p(s, m)\big)$ is small and reduce the overall level of manipulation. Recall that Proposition 2 and Proposition 3 state that the disclosure choice of s not only affects how manipulation distributes across the signal s but also affects the expected amount of manipulation. Hence, the regulator can leverage the disclosure choice of the regulatory signal s to minimize the expected loss from manipulation.

To pin down the optimal disclosure policy about the regulatory signal s, I first discuss the cost and benefit of disclosure for the regulator. For given expected amount of manipulation, the disclosure of the regulatory signal s affects how manipulation distributes across the regulator's marginal loss ML_r. Disclosing s reveals $\Delta\big(t_p(s, m)\big)$ which is the increases in the passing probability after the bank changes the report distribution from $g^l(t)$ to $g^h(t)$. As captured by $MB_b(s, t_p(s, m))$, all else equal, the bank's marginal gain from manipulation is higher when $\Delta\big(t_p(s, m)\big)$ is large. A large $\Delta\big(t_p(s, m)\big)$ also means that the regulator is more likely to be misled by the bank's manipulation and make wrong passing decisions, which in turn increases the regulator's marginal loss from the bank's manipulation $ML_r(s, t_p(s, m))$. Hence, the disclosure of s incurs cost for the regulator because it facilitates the bank to manipulate more when the regulator is more susceptible to manipulation. Disclosing s also gives benefit to the regulator. Because the payoff of the asset $x(s, \omega)$ depends both on the regulator's information about s and on the asset's fundamental value ω. Disclosing s reduces the bank's uncertainty about its asset's payoff. All else equal, the bank manipulates less when the gain from passing the test with the low fundamental asset is low, i.e., when $q_l(x(s, \omega_l) + B)$ is low. This manipulation choice is

[11] Notice that when the passing threshold is very low, the regulator is more likely to pass the low fundamental asset. However, such continuation is already considered when the regulator chooses the optimal passing threshold $t_p(s, m^*)$, which balances the tradeoff between inefficient liquidation and inefficient continuation. The term $ML_r\big(s, t_p(s, m^*)\big)$ does not capture such continuation and it only reflects the *additional* inefficient continuation caused by the bank's manipulation. And such additional inefficient continuation is reduced as the signal s increases.

beneficial to the regulator. Because when $x(s, \omega_l)$ is low, passing the bank incurs large loss for the regulator. In other words, the regulator demands more informative report when $x(s, \omega_l)$ is low. Disclosing s then makes the regulator's pass/fail decision more accurate. Given the result in Proposition 2, the benefit of disclosing the regulatory signal s outweighs the cost when the signal s is small.

In addition, Proposition 3 shows that the disclosure choice of the signal s changes the expected amount of manipulation across s. This additional layer strengthens the existing tradeoff of disclosure. As a result, the optimal disclosure policy follows a simple cutoff rule.

Proposition 4. *The optimal disclosure policy follows a cutoff rule where $D = [\underline{s}, s^*)$ and $N = [s^*, \bar{s}]$. That is, the regulator discloses the signal s when $s < s^*$ and does not disclose the signal s when $s > s^*$, where $s^* \in [\underline{s}, s_D]$ solves*

$$\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) f(s^*) = \frac{\partial m_N}{\partial s^*} \int_{s^*}^{\bar{s}} ML_r\big(s, t_p(s, m_N)\big) dF(s). \tag{12}$$

The intuition for a cutoff rule is embedded in the tradeoff of disclosure. It is beneficial for the regulator to disclose the signal s when the manipulation is driven by the gain from passing the test with the low fundamental asset $q_l\big(x(s, \omega_l) + B\big)$. In this case, the bank's manipulation is increasing in s which implies that the bank's manipulation is less (more) when it causes more (less) losses to the regulator as measured by $ML_r(s, t_p(s, m))$. In addition, the bank manipulates less in expectation when observing the regulator's signal s. Hence, disclosing the signal s improves the regulator's ex ante payoff. However, as the signal s increases, the bank's manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. If the signal is disclosed to the bank, then the bank would manipulate more when the regulator is more susceptible to manipulation. Hence, no disclosure complements the passing threshold to deter the bank's manipulation. The disclosure cutoff point s^* is characterized by equation (12). This equation captures the regulator's tradeoff between the utility gain (loss) from disclosing more information and the loss (gain) from increased (decreased) manipulation in the no-disclosure region. No disclosure at all can be optimal if it sufficiently reduces the expected level of manipulation. In sum, the regulator's disclosure choice of s complements the pass/fail decision to minimize the adverse consequence of the bank's manipulation.

5 Comparative statics

In this section, I analyze how the optimal disclosure policy about the regulator's signal s changes with the bank's private benefit B when passing the test and its cost of manipulation k.

All else equal, an increase in the private benefit B or a decrease in the manipulation cost k incentivizes the bank to manipulate more for any given regulatory signal s. Such increases in manipulation occur irrespective of whether the bank observes the signal s or not. As a result, the implications on the regulator's disclosure policy about s is unclear. The following lemma shows the effect of the bank's private benefit B and its cost of manipulation k on the regulator's disclosure choice of s.

Proposition 5. *The optimal disclosure policy is characterized in Proposition 4. If $m_D(s^*) \leq m_N$, then the disclosure cutoff point s^* is increasing in k and decreasing in B. Otherwise, the disclosure cutoff point s^* is decreasing in k and increasing in B.*

The intuition of this result follows the tradeoff underpinning the disclosure cutoff point s^*. Recall equation (12), the regulator determines the disclosure cutoff point by weighing the utility gain from disclosure against the loss resulting from manipulation in the no-disclosure region. When the optimal disclosure policy satisfies $m_D(s^*) \leq m_N$, it implies that the regulator can attain a non-negative utility by disclosing. However, increasing disclosure also increases manipulation in the no-disclosure region, exacerbating the regulator's loss from manipulation. Consequently, concerns regarding manipulation in the no-disclosure region discourage the regulator from realizing the benefits of disclosure. The cost of manipulation k addresses the regulator's concerns about manipulation in the no-disclosure region. As k increases, the regulator can disclose more information about s to recapture previously forgone gains. Therefore, an increase in the cost of manipulation incentivizes a higher level of disclosure of the regulatory signal. Conversely, an increase in the bank's private benefit B would have the opposite effect on the disclosure of s.

In the case where the optimal disclosure policy satisfies $m_D(s^*) > m_N$, it exhibits excessive disclosure at the disclosure cutoff point s^*. This excessive disclosure is utilized to mitigate the manipulation m_N and the resulting losses in the no-disclosure region. In other words, the regulator leverages disclosure as a mechanism to reduce manipulation in the no-disclosure region. When the cost of manipulation k increases, the regulator can rely less on disclosure as a means of deterring manipulation, thereby incurring less losses associated with disclosure. Conversely, as the bank's private benefit B increases, the regulator needs to increase disclosure to counteract the bank's stronger incentive for manipulation. In sum, the disclosure of the regulatory signal s acts as both a substitute for the cost of manipulation k and a counterforce for the bank's private benefit B in curbing manipulation in the no-disclosure region.

6 Discussions

In this section, I discuss some of the assumptions and possible extensions of the model.

6.1 Cost of inefficient liquidation and inefficient continuation

Assumption 1 assumes that the bank's asset is worth continuing ex ante. This assumption affects how the regulator chooses passing threshold $t_p(s, m)$ in response to the bank's manipulation m (Lemma 2) and how the bank's manipulation changes with the regulatory signal s when s is disclosed (Lemma 3 and Proposition 2). Nevertheless, the main insight for the disclosure of the regulator's signal s does not depend on this assumption. In Appendix B, I derive the results formally.

The main finding of the paper is that the regulator's ex post pass/fail decision alone is insufficient to fully prevent the adverse consequence of the bank's manipulation. Hence, the disclosure of the regulatory signal s is useful. When the signal s is disclosed, the bank's marginal benefit of manipulation is determined by two factors: the gain from passing the test with the low fundamental asset $q_l\big(x(s, \omega_l) + B\big)$ and the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. And the regulator's marginal loss from the bank's manipulation $ML_r(s, t_p(s, m))$ depends on the expected losses of inefficient continuation $q_l x(s, \omega_l)$ and the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. Disclosure is always beneficial to the regulator when both the bank's marginal benefit of manipulation and the regulator's marginal loss from manipulation are driven by the changes in the relative gain from continuing the low fundamental asset $x(s, \omega_l)$. Conversely, no disclosure is preferred when the changes in manipulation is driven by the increases in passing probability after manipulation $\Delta\big(t_p(s, m)\big)$. The former force is more likely to dominate when the cost of inefficient liquidation and the cost of inefficient continuation are comparable. Because in such case, the regulator's choice of passing threshold leads to large increases in passing probability if the bank manipulates, i.e., $\Delta\big(t_p(s, m)\big)$ is large. This then incentivizes the bank to care about the gain from passing the test when choosing the manipulation. Hence, the bank's manipulation is more likely to be driven by the gain from passing the test with the low fundamental asset $q_l\big(x(s, \omega_l) + B\big)$. As discussed above, such manipulation choice benefits the regulator. In any case, the regulator's disclosure choice of s still complements the pass/fail decision.

6.2 Ex post manipulation

In the baseline model, the bank chooses manipulation before observing the fundamental value of its asset. Although this manipulation choice fits in various contexts, it is pertinent to examine the case where the bank privately observes the asset's fundamental before engaging in report manipulation. In this section, I demonstrate that all results continue to hold with this manipulation choice.

In contrast to the baseline model, the bank's manipulation choice becomes dependent on the asset's fundamental. When its asset's fundamental is ω_h, the bank does not gain by manipulating the report distribution $g^h(t)$. Consequently, regardless of the disclosure of the regulator's signal s, the bank does not manipulate its report, i.e., $m_N(\omega_h) = m_D(s, \omega_h) = 0$ for all s. However, the bank still has incentive to manipulate its report when its asset's fundamental is ω_l. The marginal benefit of manipulation is $\big(x(s, \omega_l) + B\big)\Delta\big(t_p(s, m)\big)$ if the regulator discloses s, or $\mathbb{E}\left[\big(x(s, \omega_l) + B\big)\Delta\big(t_p(s, m)\big)|s \in N_n\right]$ if the regulator discloses that the signal s belongs to the set N_n. With these slight modifications in the bank's manipulation choice, all the results can be derived similarly as in the baseline model, and the insights and intuitions remain unchanged.

With the ex post manipulation, this paper is also related to the literature on two-sided incomplete information. In this alternative model, both the regulator and the bank have private information that matters for the regulator's pass/fail decision. This paper then provides analyses on whether and how the regulator can elicit more information from the bank by communicating to the bank first. The two-sided private information is prevalent in many different settings. Cramton (1984) studies bargaining game and shows that two-sided incomplete information causes costly delays in trade. Watson (1996) studies information transmission when the sender's and the receiver's private information are complementary. He shows that, in contrast to communication games with one-sided incomplete information, fully revealing equilibria often exist.

6.3 No commitment to disclosure policy

Suppose that the regulator cannot commit to any disclosure policy about the signal s. Instead, the regulator decides to disclose or not to disclose the signal s after observing the realization of it. In the following, I show that the only equilibrium in this case is full disclosure.

The intuition is as follows. Consider the no-disclosure set $N = [s_1, s_2]$ with $s_1 < s_2$. Denote the bank's manipulation response as m_N. Upon observing the signal s, the regulator would disclose s if the bank's manipulation $m_D(s)$ is less than m_N. This implies that the no-disclosure set must consist of signals s such that $m_N \leq m_D(s)$ holds, implying that $\mathbb{E}\Big[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_1, s_2]\Big] \leq MB_b\big(s, t_p(s, m_D(s))\big)$ for $s \in [s_1, s_2]$. Since $MB_b\big(s, t_p(s, m)\big)$ is a continuous function of s, the regulator must be indifferent between disclosing and not disclosing the signals at the boundary of the no-disclosure set, i.e., $m_N = m_D(s_1) = m_D(s_2)$. Hence, the following condition must hold

$$\mathbb{E}_s\Big[\,MB_b\big(s, t_p(s, m_N)\big)\big|\, s \in [s_1, s_2]\Big] = MB_b\big(s_1, t_p(s_1, m_N)\big) = MB_b\big(s_2, t_p(s_2, m_N)\big).$$

However, given that $MB_b(s, t_p(s, m))$ is first increasing and then decreasing in s for any given manipulation m, this condition cannot hold if $s_1 < s_2$. Hence, $s_1 = s_2$ and full disclosure is the equilibrium.

As indicated in Proposition 4, full disclosure policy is suboptimal. Hence, lack of commitment to the disclosure policy makes the regulator worse off. The intuition is that commitment enables the regulator to use a no-disclosure set (i.e., partial disclosure policy) to share the bank's manipulation incentive across different signals s. Such manipulation sharing increases the regulator's ex ante payoff.

6.4 Real activity

In the baseline model, the bank engages in costly manipulation to affect the report distribution. The manipulation improves the bank's report in the sense of first-order stochastic dominance, but it does not affect the bank's asset payoff. Hence, the disclosure of the regulator's private information only has informational consequences for the bank. It informs the bank about the gain from manipulation and the probability of obtaining the gain.

In Appendix C, I extend the analysis to a case where the disclosure of the regulator's private information not only affects the bank's reporting choice but also affects the bank's investment decision. More specifically, the bank exerts costly effort to improve the asset's fundamental and such effort manifests itself in the report. The effort still increases the similarity between the report of the low and high fundamental asset, but the similarity arises from the actual improvement in the asset's fundamental. As a result, the disclosure of the regulator's private information affects the bank's real activity, i.e., effort choice. I show that, in this case, the regulator may benefit from disclosing more of the private information. In particular, full disclosure policy may be optimal under some conditions.

7 Conclusion

This paper presents a tractable model to analyze the optimal disclosure policy about the regulatory assessment models in the presence of concerns about the banks' manipulation. While disclosing the regulatory models helps banks understand how their assets perform in different economic environments, it also creates opportunities for banks to game the regulatory assessments. The main message of this paper is that the disclosure policy about the regulatory models should complement the assessment rules. Additionally, the paper highlights that the banks' internal governance and the rules and regulations concerning their reporting discretion complement the design and improve the effectiveness of regulatory assessments. The implications of this paper extend to regulatory practices such as supervisory stress test and climate risk stress test. By understanding the interactions between performing stress tests and reporting incentives of banks, regulators can improve the design of stress tests and enhance the effectiveness of regulatory assessments.

A Proofs

For ease of exposition, I define the following ratio.

$$r(t) \equiv \frac{g^l(t)}{g^h(t)}. \tag{13}$$

Due to the assumption that the density functions of report t satisfy MLRP, the ratio $r(t)$ is decreasing in t.

Proof. Lemma 1

All the necessary steps for the cutoff rule are explained in the text. □

Proof. Lemma 2

The regulator chooses the passing threshold based on the signal s and the conjecture about the bank's manipulation $\hat{m}$. I drop the $\hat{}$ for simplicity.

The passing threshold is determined by

$$\mathbb{E}_\omega[x(s,\omega)|t_p, m] = 0.$$

This condition is equivalent to

$$x(s,\omega_h)\frac{q_h g^h(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} + x(s,\omega_l)\frac{q_l g^l_m(t_p)}{q_h g^h(t_p) + q_l g^l_m(t_p)} = 0,$$

Since the density function $g^l(t)$ and $g^h(t)$ have full support, the condition reduces to

$$x(s,\omega_h)q_h g^h(t_p) + x(s,\omega_l)q_l g^l_m(t_p) = 0.$$

This is equivalent to

$$x(s,\omega_h)q_h + x(s,\omega_l)q_l - x(s,\omega_l)q_l(1-m)\big(1-r(t_p)\big) = 0. \tag{14}$$

Apply the implicit function theorem, I derive the following two partial derivatives.

$$\frac{\partial t_p}{\partial s} = -\frac{q_h\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)q_l\big(x(s,\omega_l)\big)^2 r'(t_p)}. \tag{15}$$

Where $r'(t_p)$ is the derivative of $r(t_p)$ with respect to t_p and it is negative. Given that the relative gain from continuing the asset $x(s,\omega_l)$ and $x(s,\omega_h)$ are increasing in s, this derivative is negative.

And the following is the partial derivative of t_p with respect to m,

$$\frac{\partial t_p}{\partial m} = -\frac{q_h x(s,\omega_h) + q_l x(s,\omega_l)}{(1-m)^2 q_l x(s,\omega_l) r'(t_p)}. \tag{16}$$

Given Assumption 1, the unconditional expected relative gain from continuing the asset is non-negative. Hence, this derivative is non-positive and it equals to zero only when $s = \underline{s}$. □

Proof. Proposition 1

The first-order condition of m_{N_n} is given in equation (6). I repeat the condition here,

$$\mathbb{E}_s \left[q_l \big(x(s,\omega_l) + B \big) \Delta \big(t_p(s, m_{N_n}) \big) \middle| s \in N_n \right] - kc'(m_{N_n}) = 0.$$

This condition implies that m_{N_n} is a constant over the signal space N_n. The uniqueness of m_{N_n} can be proved by the monotonicity of $\mathbb{E}_s \left[q_l \big(x(s,\omega_l) + B \big) \Delta \big(t_p(s, m_{N_n}) \big) \middle| s \in N_n \right] - kc'(m_{N_n})$ in m_N.

$$\begin{aligned} &\frac{\partial \, \mathbb{E}_s \left[q_l \big(x(s,\omega_l) + B \big) \Delta \big(t_p(s, m_{N_n}) \big) \middle| s \in N_n \right] - kc'(m_{N_n})}{\partial m_{N_n}} \\ &= \mathbb{E}_s \left[q_l \big(x(s,\omega_l) + B \big) \frac{\partial \Delta \big(t_p(s, m_{N_n}) \big)}{\partial m_{N_n}} \middle| s \in N_n \right] - kc''(m_{N_n}) \end{aligned}$$

This derivative is negative since $c(m)$ is convex and the derivative $\frac{\partial \Delta \big(t_p(s,m) \big)}{\partial m}$ is non-positive for a given m. I show the latter holds in the following. I repeat the definition of $\Delta\big(t_p(s,m)\big)$ here,

$$\Delta\big(t_p(s,m)\big) \equiv \int_{t \geq t_p(s,m)} \big(g^h(t) - g^l(t) \big) dt.$$

Taking derivative with respect to m, I obtain the following,

$$\begin{aligned} \frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m} &= \frac{d \Delta\big(t_p(s,m)\big)}{d t_p(s,m)} \frac{\partial t_p(s,m)}{\partial m} \\ &= \Big(g^l\big(t_p(s,m)\big) - g^h\big(t_p(s,m)\big) \Big) \frac{\partial t_p(s,m)}{\partial m} \\ &\propto \Big(r\big(t_p(s,m)\big) - 1 \Big) \frac{\partial t_p(s,m)}{\partial m}. \end{aligned} \tag{17}$$

Recall that equation (14) pins down the passing threshold $t_p(s,m)$, and the ratio $r\big(t_p(s,m)\big)$ solves

$$r\big(t_p(s,m)\big) = \frac{m q_l x(s,\omega_l) + q_h x(s,\omega_h)}{m q_l x(s,\omega_l) - q_l x(s,\omega_l)} \geq \frac{m q_l x(s,\omega_l) - q_l x(s,\omega_l)}{m q_l x(s,\omega_l) - q_l x(s,\omega_l)} = 1. \tag{18}$$

The inequality holds because Assumption 1 implies that $q_h x(s,\omega_h) \geq -q_l x(s,\omega_l)$ for all s and equality holds only when $s = \underline{s}$. As a result, the derivative $\frac{d \Delta\big(t_p(s,m)\big)}{d t_p(s,m)}$ is non-negative. Given the result of Lemma 2 that $\frac{\partial t_p(s,m)}{\partial m}$ is non-positive, the derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m} \leq 0$ and equality holds only when $s = \underline{s}$. $\square$

Proof. Lemma 3

For given passing threshold $t_p(s,m)$, the difference in passing probability between g^l and g^h is $\Delta\big(t_p(s,m)\big)$. Taking derivative with respect to s, I obtain the following

$$\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial s} = \frac{d\Delta\big(t_p(s,m)\big)}{dt_p(s,m)} \frac{\partial t_p(s,m)}{\partial s}.$$

The proof of Proposition 1 shows that the derivative $\frac{d\Delta\big(t_p(s,m)\big)}{dt_p(s,m)}$ is non-negative. Given the result of Lemma 2 that $\frac{\partial t_p(s,m)}{\partial s}$ is negative, the derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial s} \leq 0$ and equality holds only when $s = \underline{s}$. $\square$

Proof. Proposition 2

When s is disclosed, the manipulation level is determined by the first-order condition in equation (7). I repeat the first-order condition here,

$$q_l\big(x(s,\omega_l) + B\big)\Delta\big(t_p(s,m_D)\big) - kc'(m_D) = 0.$$

The first term of the left-hand side is $MB_b\big(s, t_p(s,m_D)\big)$. The uniqueness of m_D is proved by the monotonicity of $MB_b\big(s, t_p(s,m_D)\big) - kc'(m_D)$ in m_D. I omit the proof of the uniqueness because the proof is similar to the proof of Proposition 1.

Apply the implicit function theorem to the first-order condition, I derive the derivative of m_D with respect to s,

$$\frac{\partial m_D}{\partial s} = \frac{\frac{\partial MB_b\big(s,t_p(s,m_D)\big)}{\partial s}}{kc''(m_D) - \frac{\partial MB_b\big(s,t_p(s,m_D)\big)}{\partial m_D}} = \frac{q_l\left(\Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial \Delta\big(t_p(s,m_D)\big)}{\partial s}\right)}{kc''(m_D) - q_l\big(x(s,\omega_l)+B\big)\frac{\partial \Delta\big(t_p(s,m_D)\big)}{\partial m_D}}. \tag{19}$$

The derivative $\frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}$ is non-positive as shown in the proof of Proposition 1. Consequently, the following holds

$$\frac{\partial m_D}{\partial s} \propto \frac{\partial MB_b\big(s,t_p(s,m_D)\big)}{\partial s} \propto \Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial \Delta\big(t_p(s,m_D)\big)}{\partial s}.$$

For ease of exposition, I introduce the following notation

$$F \equiv \Delta\big(t_p(s,m_D)\big)\frac{dx(s,\omega_l)}{ds} + \big(x(s,\omega_l)+B\big)\frac{\partial \Delta\big(t_p(s,m_D)\big)}{\partial s}$$

In the following, I first show that $F = 0$ holds at some $s \in (\underline{s}, \bar{s})$ and then I prove that $F = 0$ is unique at $s = s_D$.

When $s = \underline{s}$, Assumption 1 assumes that $x(\underline{s}, \omega_h)q_h + x(\underline{s}, \omega_l)q_l = 0$. According to equation (14), the passing threshold satisfies $r\big(t_p(\underline{s}, m)\big) = 1$ which implies that $\frac{\partial \Delta\left(t_p(s,m)\right)}{\partial s} = 0$, hence, the function F is

$$F|_{s=\underline{s}} = \Delta\big(t_p(\underline{s}, m_D)\big) \left.\frac{dx(s, \omega_l)}{ds}\right|_{s=\underline{s}} > 0.$$

When $s = \bar{s}$, Assumption 1 implies that $x(\bar{s}, \omega_l) = 0$. Hence, the passing threshold is $t_p(\bar{s}, m_D) = \underline{t}$ and $\Delta(\underline{t}) = 0$. Hence, the function F is

$$F|_{s=\bar{s}} = B \left.\frac{\partial \Delta\big(t_p(s, m_D)\big)}{\partial s}\right|_{s=\bar{s}} < 0.$$

By the intermediate value theorem, $F = 0$ must hold at some value of $s \in (\underline{s}, \bar{s})$.

Next, I show that $F = 0$ is unique at $s = s_D$. When $F = 0$, the following equation holds,

$$\Delta\big(t_p(s, m_D)\big) \frac{\frac{dx(s,\omega_l)}{ds}}{x(s, \omega_l) + B} = -\frac{\partial \Delta\big(t_p(s, m_D)\big)}{\partial s}.$$

I drop the indicator D for the manipulation m_D. Then $F = 0$ is equivalent to

$$\frac{\Delta\big(t_p(s, m)\big)}{-\frac{d\Delta\left(t_p(s,m)\right)}{dt_p(s,m)}} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s, \omega_l) + B} = \frac{\partial t_p(s, m)}{\partial s}. \tag{20}$$

I first show that the left-hand side is increasing in s. I drop the arguments for t_p when no confusion caused. The left-hand side is equivalent to

$$LHS \equiv -\frac{\Delta(t_p)}{\Delta'(t_p)} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s, \omega_l) + B}.$$

Where $\Delta'(t_p) \equiv \frac{d\Delta(t_p)}{dt_p}$. The derivative of LHS with respect to s is

$$\frac{\partial LHS}{\partial s} = -\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} \frac{\partial t_p}{\partial s} \frac{\frac{dx(s,\omega_l)}{ds}}{x(s, \omega_l) + B} - \frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} \frac{\Delta(t_p)}{\Delta'(t_p)}.$$

The derivative $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p}$ is

$$\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} = \frac{\big(\Delta'(t_p)\big)^2 - \Delta(t_p)\Delta''(t_p)}{\big(\Delta'(t_p)\big)^2}.$$

By assumption, the decreasing hazard rate $\frac{g^i(t)}{1-G^i(t)}$ implies that $g^i(t)$ is decreasing in t. Moreover, the MLRP assumption implies that $r(t)$ is decreasing in t, that is

$$\frac{dr(t)}{dt} = \frac{\frac{dg^l(t)}{dt}g^h(t) - \frac{dg^h(t)}{dt}g^l(t)}{\big(g^h(t)\big)^2} < 0.$$

Recall that at the passing threshold t_p, it holds that $r(t_p) > 1$ which is equivalent to $g^l(t_p) > g^h(t_p)$. Hence, it also holds that $\frac{dg^l(t_p)}{dt_p} < \frac{dg^h(t_p)}{dt_p}$. That is, $\Delta''(t_p) < 0$, which in turn implies that $\frac{d\left(\frac{\Delta(t_p)}{\Delta'(t_p)}\right)}{dt_p} > 0$. The derivative $\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds}$ is

$$\frac{d\left(\frac{\frac{dx(s,\omega_l)}{ds}}{x(s,\omega_l)+B}\right)}{ds} = \frac{-\left(\frac{dx(s,\omega_l)}{ds}\right)^2 + \big(x(s,\omega_l)+B\big)\frac{d^2x(s,\omega_l)}{ds^2}}{\big(x(s,\omega_l)+B\big)^2}.$$

Since $x(s,\omega_l)$ is increasing and weakly concave in s, this derivative is negative. As a result, $\frac{\partial LHS}{\partial s} > 0$.

Now consider the right-hand side of equation (20),

$$RHS \equiv \frac{\partial t_p(s,m)}{\partial s}.$$

And the derivative of the RHS with respect to s is

$$\frac{\partial RHS}{\partial s} = \frac{\partial^2 t_p(s,m)}{\partial s^2}.$$

Recall that equation (14) determines the passing threshold $t_p(s,m)$. I repeat the equation here,

$$x(s,\omega_h)q_h + x(s,\omega_l)q_l - x(s,\omega_l)q_l(1-m)\big(1-r(t_p)\big) = 0.$$

For ease of exposition, I define

$$\tilde{x}(s,\omega_l) \equiv \frac{x(s,\omega_l)}{x(s,\omega_h)}.$$

With this notation, equation (14) is equivalent to,

$$q_h + \tilde{x}(s,\omega_l)q_l - \tilde{x}(s,\omega_l)q_l(1-m)\big(1-r(t_p)\big) = 0.$$

Hence, by the implicit function theorem, the derivative $\frac{\partial t_p(s,m)}{\partial s}$ is,

$$\frac{\partial t_p(s,m)}{\partial s} = \frac{q_h\frac{d\tilde{x}(s,\omega_l)}{ds}}{(1-m)q_l\big(\tilde{x}(s,\omega_l)\big)^2 r'(t_p)}.$$

Where $\frac{d\tilde{x}(s,\omega_l)}{ds}$ is

$$\frac{d\tilde{x}(s,\omega_l)}{ds} = \frac{-x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} + x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}}{\big(x(s,\omega_h)\big)^2} > 0.$$

By the chain rule, the second derivative $\frac{\partial^2 t_p(s,m)}{\partial s^2}$ is,

$$\begin{aligned}\frac{\partial^2 t_p(s,m)}{\partial s^2} =& \frac{\partial \frac{\partial t_p(s,m)}{\partial s}}{\partial s} + \frac{\partial \frac{\partial t_p(s,m)}{\partial s}}{\partial t_p}\frac{\partial t_p(s,m)}{\partial s} \\ =& q_h \frac{q_l(1-m)\big(r'(t_p)\big)^2 \tilde{x}(s,\omega_l)\left(\tilde{x}(s,\omega_l)\frac{d^2\tilde{x}(s,\omega_l)}{ds^2} - \left(\frac{d\tilde{x}(s,\omega_l)}{ds}\right)^2\right)}{(1-m)^2 q_l^2\big(\tilde{x}(s,\omega_l)\big)^4\big(r'(t_p)\big)^3} \\ &+ q_h \frac{\left(\frac{d\tilde{x}(s,\omega_l)}{ds}\right)^2\Big(-(1-m)q_l\tilde{x}(s,\omega_l)\big(r'(t_p)\big)^2 - q_h r''(t_p)\Big)}{(1-m)^2 q_l^2\big(\tilde{x}(s,\omega_l)\big)^4\big(r'(t_p)\big)^3} < 0.\end{aligned} \tag{21}$$

Given the assumption that $\tilde{x}(s,\omega_l)$ is weakly log-concave and $r(t_p)$ is decreasing in t_p (i.e., MLRP assumption), the first term is non-positive. Also, the second term is negative since $r(t_p)$ is weakly concave in t_p. Hence, $\frac{\partial RHS}{\partial s} < 0$.

I have shown that when $F = 0$, the left-hand side of equation (20) is increasing in s whereas the right-hand side of equation (20) is decreasing in s, which implies that $F = 0$ has a unique solution s_D. And $F > 0$ for $s < s_D$ and $F < 0$ for $s > s_D$. Recall that $\frac{\partial m_D(s)}{\partial s}$ is proportionate to F, hence, $m_D(s)$ is increasing in s for $s < s_D$ and is decreasing in s for $s > s_D$. Since $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$ is proportionate to F, s_D also solves $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s} = 0$. □

Proof. Proposition 3

I prove this proposition by contradiction.

Suppose that $\mathbb{E}_s\left[m_D(s)|s\in N\right] > m_N$ for $N = [\underline{s}, s_D]$.[12] Denote $\mathbb{E}_s\left[m_D(s)|s\in[\underline{s},s_D]\right]$ by $\overline{m_D}$,

$$\begin{aligned}\overline{m_D} - m_N &\propto kc'(\overline{m_D}) - kc'(m_N) \\ &\le \mathbb{E}_s\left[\,kc'\big(m_D(s)\big)\,\big|\, s\in[\underline{s},s_D]\right] - kc'(m_N) \\ &= \frac{\int_{\underline{s}}^{s_D} kc'\big(m_D(s)\big)dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - kc'(m_N) \\ &\propto \int_{\underline{s}}^{s_D}\Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big)dF(s).\end{aligned}$$

The inequality is due to the assumption that $kc'(m)$ is weakly convex in m.

The first-order condition for $m_D(s)$ is

$$MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) = kc'\big(m_D(s)\big). \tag{22}$$

[12]The proof also applies to cases where $N \subset [\underline{s}, s_D]$.

And the first-order condition for m_N when $N = [\underline{s}, s_D]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right] = kc'(m_N).$$

I can simplify the difference between $\overline{m_D}$ and m_N further,

$$\begin{aligned}
\overline{m_D} - m_N \leq & \int_{\underline{s}}^{s_D} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [\underline{s}, s_D]\right]\right) dF(s) \\
\leq & \int_{\underline{s}}^{s_D} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right]\right) dF(s) \\
= & \int_{\underline{s}}^{s_D} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\right) dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [\underline{s}, s_D]\right] \int_{\underline{s}}^{s_D} dF(s) \\
= & \int_{\underline{s}}^{s_D} \left(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\right) dF(s) \\
= & \int_{\underline{s}}^{s_D} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big) dF(s) \\
\leq & 0.
\end{aligned}$$

The first line is obtained by using the first-order conditions of m_N and $m_D(s)$. The second line is due to the fact that $MB_b\big(s, t_p(s, m)\big)$ is decreasing in m. This is verified by the following derivative

$$\frac{\partial MB_b\big(s, t_p(s, m)\big)}{\partial m} = q_l\big(x(s, \omega_l) + B\big)\frac{\partial \Delta\big(t_p(s, m)\big)}{\partial m} \leq 0. \tag{23}$$

The derivative $\frac{\partial \Delta\left(t_p(s,m)\right)}{\partial m}$ is non-positive as shown in the proof of Proposition 1. Then the assumption that $m_N < \overline{m_D}$ implies the second line. The third and fourth line follow from the definition of conditional expectation. The last inequality is obtained by applying FKG inequality, which I now explain in details. The manipulation level $m_D(s)$ is increasing in s when $s < s_D$. And the proof of Proposition 1 shows that $\frac{\partial \Delta\left(t_p(s,m)\right)}{\partial m} \leq 0$. This means that the term $\Delta\big(t_p(s, m_D(s))\big)$ is decreasing in s through $m_D(s)$. The term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s. By FKG inequality, the following holds

$$\mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p\big(s, m_D(s)\big)\Big)\right] \leq \mathbb{E}_{s \leq s_D}\left[q_l\big(x(s, \omega_l) + B\big)\Delta\Big(t_p(s, \mathbb{E}_{s \leq s_D}[m_D(s)])\Big)\right].$$

Where $\mathbb{E}_{s \leq s_D}$ denotes expectation over s conditional on $s \leq s_D$. This implies that the last inequality holds and it contradicts to $m_N < \overline{m_D}$.

Next I prove by contradiction that $\overline{m_D} \geq m_N$ for $N = [s_D, \bar{s}]$.[13] Suppose that the opposite

[13] The proof also applies to cases where $N \subset [s_D, \bar{s}]$.

holds, that is, $\overline{m_D} < m_N$ for $N = [s_D, \bar{s}]$. Then the following holds,

$$\begin{aligned}
\overline{m_D} - m_N &\propto \log\big(kc'(\overline{m_D})\big) - \log\big(kc'(m_N)\big) \\
&\geq \mathbb{E}_s\Big[\log\Big(kc'\big(m_D(s)\big)\Big)\Big| s \in [s_D, \bar{s}]\Big] - \log\big(kc'(m_N)\big) \\
&= \frac{\int_{s_D}^{\bar{s}} \log\Big(kc'\big(m_D(s)\big)\Big) dF(s)}{\int_{\underline{s}}^{s_D} dF(s)} - \log\big(kc'(m_N)\big) \\
&\propto \int_{s_D}^{\bar{s}} \bigg(\log\Big(kc'\big(m_D(s)\big)\Big) - \log\big(kc'(m_N)\big)\bigg) dF(s) \\
&= \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} dF(s) \\
&\geq \int_{s_D}^{\bar{s}} \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} dF(s) \\
&\propto \int_{s_D}^{\bar{s}} \Big(kc'\big(m_D(s)\big) - kc'(m_N)\Big) dF(s)
\end{aligned}$$

The first inequality holds because $c'(m)$ is weakly log-concave. By the definition of conditional expectation, I obtain the first equality. I derive the second equality by using the mean value theorem, where $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$ or $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$ depending on the relation between $kc'(m_N)$ and $kc'\big(m_D(s)\big)$. I now explain the second inequality.

- If $kc'\big(m_D(s)\big) < kc'(m_N)$, then $kc'(m_s) \in \Big(kc'\big(m_D(s)\big), kc'(m_N)\Big)$. Hence, the following holds
$$\frac{1}{\frac{1}{kc'(m_N)}} \geq \frac{1}{\frac{1}{kc'(m_s)}} \geq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$
Which implies that
$$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

- If $kc'\big(m_D(s)\big) > kc'(m_N)$, then $kc'(m_s) \in \Big(kc'(m_N), kc'\big(m_D(s)\big)\Big)$. Hence, the following holds
$$\frac{1}{\frac{1}{kc'(m_N)}} \leq \frac{1}{\frac{1}{kc'(m_s)}} \leq \frac{1}{\frac{1}{kc'(m_D(s))}}.$$
Which implies that
$$\frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_N)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_s)}} \leq \frac{kc'\big(m_D(s)\big) - kc'(m_N)}{\frac{1}{kc'(m_D(s))}}.$$

Hence, regardless of the difference between $kc'\big(m_D(s)\big)$ and $kc'(m_N)$, the second inequality holds.

The first-order condition for $m_D(s)$ is the same as in equation (22). And the first-order condition for m_N when $N = [s_D, \bar{s}]$ is

$$\mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right] = kc'(m_N).$$

I further simplify the difference between $\overline{m_D}$ and m_N,

$$\begin{aligned}
\overline{m_D} - m_N \geq & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, m_N)\big)|s \in [s_D, \bar{s}]\right]\Big)\, dF(s) \\
\geq & \int_{\underline{s}}^{s_D} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right]\Big)\, dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big)\Big)\, dF(s) - \mathbb{E}\left[MB_b\big(s, t_p(s, \overline{m_D})\big)|s \in [s_D, \bar{s}]\right] \int_{\underline{s}}^{s_D} dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(MB_b\Big(s, t_p\big(s, m_D(s)\big)\Big) - MB_b\big(s, t_p(s, \overline{m_D})\big)\Big)\, dF(s) \\
= & \int_{s_D}^{\bar{s}} \Big(q_l\big(x(s, \omega_l) + B\big)\Big(\Delta\big(t_p(s, m_D(s))\big) - \Delta\big(t_p(s, \overline{m_D})\big)\Big)\Big)\, dF(s) \\
\geq & 0.
\end{aligned}$$

The second inequality uses the assumption that $m_N > \overline{m_D}$. The last inequality is derived by using FKG inequality. The manipulation level $m_D(s)$ is decreasing in s when $s > s_D$. Hence $\Delta\big(t_p(s, m_D(s))\big)$ is increasing in s through $m_D(s)$. Given that the term $q_l\big(x(s, \omega_l) + B\big)$ is increasing in s, FKG inequality implies the last inequality. □

Proof. Lemma 4

I first show that $ML_r\big(s, t_p(s, m)\big)$ is increasing in s for any given m.

$$\frac{\partial ML_r\big(s, t_p(s, m)\big)}{\partial s} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m)\big) + q_l x(s, \omega_l) \frac{\partial \Delta\big(t_p(s, m)\big)}{\partial s}.$$

Lemma 3 shows that for any given m, $\Delta\big(t_p(s, m)\big)$ is decreasing in s, i.e., $\frac{\partial \Delta\left(t_p(s,m)\right)}{\partial s} < 0$. Since the low fundamental asset has non-positive value, i.e., $x(s, \omega_l) \leq 0$, the derivative $\frac{\partial ML_r\left(s, t_p(s,m)\right)}{\partial s} > 0$. This result implies that the derivative $\frac{\partial ML_r\left(s, t_p(s, m_{N_n})\right)}{\partial s} > 0$ holds for any no-disclosure set N_n.

Next, consider $ML_r\big(s, t_p(s, m_D(s))\big)$.

$$\begin{aligned}
&\frac{dML_r\big(s, t_p(s, m_D(s))\big)}{ds} = q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{ds} \\
&= q_l \frac{dx(s, \omega_l)}{ds} \Delta\big(t_p(s, m_D(s))\big) + q_l x(s, \omega_l) \frac{d\Delta\big(t_p(s, m_D(s))\big)}{dt_p} \left(\frac{\partial t_p(s, m_D(s))}{\partial s} + \frac{\partial t_p(s, m_D(s))}{\partial m_D(s)} \frac{\partial m_D(s)}{\partial s}\right).
\end{aligned}$$

Lemma 2 shows that $\frac{\partial t_p}{\partial m}$ and $\frac{\partial t_p}{\partial s}$ are non-positive. Therefore, when $\frac{\partial m_D(s)}{\partial s}$ is non-negative, the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is positive.

In the following, I show that the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is non-negative even when $\frac{\partial m_D(s)}{\partial s}$ is negative. When $m_D(s)$ is decreasing in s, it suggests that the bank's marginal benefit of manipulation $MB_b\big(s,t_p(s,m_D)\big)$ is also decreasing in s for given m_D. Taking into account the changes in $m_D(s)$, the following shows that the total derivative of $\frac{dMB_b\left(s,t_p(s,m_D(s))\right)}{ds}$ is proportionate to $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$. I drop the arguments when no confusion caused.

$$
\begin{aligned}
&\frac{dMB_b\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\left(\frac{\partial t_p(s,m_D)}{\partial s} + \frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\frac{\partial m_D(s)}{\partial s}\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right) + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}\left(\frac{\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\left(1 + \frac{\frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\right)\\
&= \left(\frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}\right)\frac{kc''(m_D(s))}{kc''(m_D(s)) - \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}\\
&\propto \frac{\partial MB}{\partial s} + \frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial s}.
\end{aligned}
$$

Since $\frac{kc''(m_D(s))}{kc''(m_D(s))-\frac{\partial MB}{\partial t_p}\frac{\partial t_p(s,m_D)}{\partial m_D}}$ is positive, the total derivative of $MB_b\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s is proportionate to the partial derivative of $MB_b\Big(s,t_p\big(s,m_D(s)\big)\Big)$ with respect to s taking $m_D(s)$ as given.

When $\frac{\partial m_D(s)}{\partial s}$ is negative, the following holds

$$
\frac{\partial m_D(s)}{\partial s} \propto \frac{\partial MB_b\Big(s,t_p\big(s,m_D\big)\Big)}{\partial s} \propto \frac{dMB_b\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} < 0.
$$

The total derivative $\frac{dMB_b\left(s,t_p\left(s,m_D(s)\right)\right)}{ds}$ equals to

$$
\frac{dMB_b\Big(s,t_p\big(s,m_D(s)\big)\Big)}{ds} = q_l\frac{dx(s,\omega_l)}{ds}\Delta\big(t_p(s,m_D(s))\big) + q_l\big(x(s,\omega_l)+B\big)\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds}.
$$

Hence, $\frac{\partial m_D(s)}{\partial s} < 0$ implies

$$
\frac{d\Delta\big(t_p(s,m_D(s))\big)}{ds} < -\frac{\Delta\big(t_p(s,m_D(s))\big)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}. \tag{24}
$$

Then the total derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}$ is

$$\begin{aligned}\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds} &= q_l\frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+q_l x(s,\omega_l)\frac{d\Delta\left(t_p(s,m_D(s))\right)}{ds}\\ &\propto \frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+x(s,\omega_l)\frac{d\Delta\left(t_p(s,m_D(s))\right)}{ds}\\ &\geq \frac{dx(s,\omega_l)}{ds}\Delta\left(t_p(s,m_D(s))\right)+x(s,\omega_l)\left(-\frac{\Delta\left(t_p(s,m_D(s))\right)}{x(s,\omega_l)+B}\frac{dx(s,\omega_l)}{ds}\right)\\ &= \frac{B}{x(s,\omega_l)+B}\Delta\left(t_p(s,m_D(s))\right)\frac{dx(s,\omega_l)}{ds}\geq 0.\end{aligned}$$

The first inequality uses the results in equation (24) and the assumption that $x(s,\omega_l)\leq 0$. Hence, the derivative $\frac{dML_r\left(s,t_p(s,m_D(s))\right)}{ds}\geq 0$ always holds. □

Proof. Proposition 4

I complete the proof in two steps. I first show that a cutoff disclosure rule dominates all other forms of disclosure. Next, I solve for the optimal cutoff point s^* and show that $s^*\leq s_D$.

Suppose that $D=[\underline{s},s_D)$ and $N=[s_D,\bar{s}]$. The regulator's ex ante expected utility with this disclosure policy is denoted as U

$$U=\int_{\underline{s}}^{s_D}u\left(s,m_D(s)\right)dF(s)+\int_{s_D}^{\bar{s}}u(s,m_N)dF(s).$$

In the following, I show that adding more cutoff points to partition the signal space does not improve the regulator's ex ante expected utility. First, I show that adding cutoff point in D does not improve the regulator's ex ante utility. Without loss of generality, consider a disclosure policy which partitions the signal space into $N_2=[\underline{s},s_1]$, $D=(s_1,s_D)$ and $N_1\equiv N=[s_D,\bar{s}]$. The regulator's ex ante expected payoff with such disclosure policy is

$$U'=\int_{\underline{s}}^{s_1}u\left(s,m_{N_2}\right)dF(s)+\int_{s_1}^{s_D}u\left(s,m_D(s)\right)dF(s)+\int_{s_D}^{\bar{s}}u(s,m_N)dF(s).$$

The difference in the regulator's expected utility is

$$
\begin{aligned}
U - U' &= \int_{\underline{s}}^{s_D} u\big(s, m_D(s)\big)dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big)dF(s) - \int_{s_1}^{s_D} u\big(s, m_D(s)\big)dF(s) \\
&= \int_{\underline{s}}^{s_1} u\big(s, m_D(s)\big)dF(s) - \int_{\underline{s}}^{s_1} u\big(s, m_{N_2}\big)dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{du(s,m)}{dm}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left(\left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} + \frac{\partial u(s,m)}{\partial t_p(s,m)} \left.\frac{\partial t_p(s,m)}{\partial m}\right|_{m=m(s)} \right) dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m(s)} dF(s) \\
&= \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m(s))\big)dF(s) \\
&\geq \int_{\underline{s}}^{s_1} \big(m_D(s) - m_{N_2}\big) ML_r\big(s, t_p(s, m_{N_2})\big)dF(s) \\
&\propto \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big)\Big] - m_{N_2}\, \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] \\
&\geq \mathbb{E}_{s \leq s_1} \Big[m_D(s) ML_r\big(s, t_p(s, m_{N_2})\big)\Big] - \mathbb{E}_{s \leq s_1} \big[m_D(s)\big]\, \mathbb{E}_{s \leq s_1} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] \\
&\geq 0.
\end{aligned}
$$

The first two lines are derived from simplifications of the differences in expected utility. Apply the mean value theorem to the second line gives the third line, where $m(s) \in (m_D(s), m_{N_2})$ if $m_D(s) < m_{N_2}$ or $m(s) \in (m_{N_2}, m_D(s))$ if $m_{N_2} < m_D(s)$. The fourth line shows the total derivative of $u(s,m)$ with respect to m, and it reduces to the fifth line because the passing threshold $t_p(s,m)$ maximizes the regulator's utility $u(s,m)$ for given signal s and given manipulation m, hence, $\frac{\partial u(s,m)}{\partial t_p(s,m)} = 0$. Equation (11) defines $ML_r\big(s, t_p(s,m)\big)$. I now explain the first inequality in details. The following derivative shows that $ML_r\big(s, t_p(s,m)\big)$ is weakly increasing in m,

$$
\frac{\partial ML_r\big(s, t_p(s,m)\big)}{\partial m} = q_l x(s, \omega_l) \frac{\partial \Delta\big(t_p(s,m)\big)}{\partial m}.
$$

The derivative $\frac{\partial \Delta\left(t_p(s,m)\right)}{\partial m}$ is non-positive as shown in the proof of Proposition 1, hence, this derivative is non-negative. The following proves the first inequality

- If $m_D(s) < m_{N_2}$, then $m(s) \in (m_D(s), m_{N_2})$. Hence, the following holds

$$
ML_r\big(s, t_p(s, m_D(s))\big) \leq ML_r\big(s, t_p(s, m(s))\big) \leq ML_r\big(s, t_p(s, m_{N_2})\big),
$$

which implies that

$$\begin{aligned}\big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m_D(s))\big) &\geq \big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m(s))\big)\\ &\geq \big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m_{N_2})\big).\end{aligned}$$

- If $m_D(s) > m_{N_2}$, then $m(s) \in (m_{N_2}, m_D(s))$. Hence, the following holds

$$ML_r\big(s,t_p(s,m_D(s))\big) \geq ML_r\big(s,t_p(s,m(s))\big) \geq ML_r\big(s,t_p(s,m_{N_2})\big),$$

which implies that

$$\begin{aligned}\big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m_D(s))\big) &\geq \big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m(s))\big)\\ &\geq \big(m_D(s)-m_{N_2}\big)ML_r\big(s,t_p(s,m_{N_2})\big).\end{aligned}$$

Hence, regardless of the difference between $m_D(s)$ and m_{N_2}, the first inequality holds. The second inequality makes use of the result from Proposition 3, hence, it holds that $\mathbb{E}_{s\leq s_1}\big[m_D(s)\big] \leq m_{N_2}$. Since $m_D(s)$ is increasing in s for $s \leq s_1$ and Lemma 4 shows that $ML_r\big(s,t_p(s,m_D(s))\big)$ is also increasing in s, the last inequality is obtained by FKG inequality. This proof can be generalized to cases where more than one cutoff points are added on D.

Using the same approach, I show that adding cutoff point in N does not improve the regulator's ex ante utility. Consider a disclosure policy which partitions the signal space into $D = [\underline{s}, s_D)$, $N_2 = [s_D, s_2)$ and $N_3 = [s_2, \bar{s}]$. The regulator's ex ante expected payoff with such disclosure policy is

$$U' = \int_{\underline{s}}^{s_D} u\big(s, m_D(s)\big)dF(s) + \int_{s_D}^{s_2} u(s, m_{N_2})dF(s) + \int_{s_2}^{\bar{s}} u\big(s, m_{N_3}\big)dF(s).$$

For ease of exposition, I make the following definition in this proof.

$$p_2 \equiv \Pr\big(s \in [s_D, s_2)|s \geq s_D\big) = \frac{\int_{s_D}^{s_2} dF(s)}{\int_{s_D}^{\bar{s}} dF(s)}.$$

And

$$p_3 \equiv \Pr\big(s \geq s_2|s \geq s_D\big) = \frac{\int_{s_2}^{\bar{s}} dF(s)}{\int_{s_D}^{\bar{s}} dF(s)}.$$

The difference in the regulator's expected utility is

$$
\begin{aligned}
U - U' &= \int_{s_D}^{\bar{s}} u\big(s, m_N\big) dF(s) - \int_{s_D}^{s_2} u(s, m_{N_2}) dF(s) - \int_{s_2}^{\bar{s}} u\big(s, m_{N_3}\big) dF(s) \\
&= \int_{s_D}^{s_2} \Big(u\big(s, m_N\big) - u\big(s, m_{N_2}\big)\Big) dF(s) + \int_{s_2}^{\bar{s}} \Big(u\big(s, m_N\big) - u\big(s, m_{N_3}\big)\Big) dF(s) \\
&= \int_{s_D}^{s_2} \big(m_N - m_{N_2}\big) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m_2} dF(s) + \int_{s_2}^{\bar{s}} \big(m_N - m_{N_3}\big) \left.\frac{\partial u(s,m)}{\partial m}\right|_{m=m_3} dF(s) \\
&= \int_{s_D}^{s_2} \big(m_N - m_{N_2}\big) ML_r\big(s, t_p(s, m_2)\big) dF(s) + \int_{s_2}^{\bar{s}} \big(m_N - m_{N_3}\big) ML_r\big(s, t_p(s, m_3)\big) dF(s) \\
&\geq \int_{s_D}^{s_2} \big(m_N - m_{N_2}\big) ML_r\big(s, t_p(s, m_{N_2})\big) dF(s) + \int_{s_2}^{\bar{s}} \big(m_N - m_{N_3}\big) ML_r\big(s, t_p(s, m_{N_3})\big) dF(s) \\
&\propto m_N \left(p_2 \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] + p_3 \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big)\Big] \right) \\
&\quad - \left(p_2 m_{N_2} \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] + p_3 m_{N_3} \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big)\Big] \right) \\
&\geq \big(p_2 m_{N_2} + p_3 m_{N_3}\big) \left(p_2 \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] + p_3 \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big)\Big] \right) \\
&\quad - \left(p_2 m_{N_2} \, \mathbb{E}_{s \in [s_D, s_2)} \Big[ML_r\big(s, t_p(s, m_{N_2})\big)\Big] + p_3 m_{N_3} \, \mathbb{E}_{s \geq s_2} \Big[ML_r\big(s, t_p(s, m_{N_3})\big)\Big] \right) \\
&\geq 0.
\end{aligned}
$$

The derivations follow the previous proof. Notice that the second inequality is obtained by generalizing the result from Proposition 3. And the last inequality is derived by applying FKG inequality. This proof can be applied to cases where more than cutoff points are added on N.

I have shown that the disclosure policy with $D = [\underline{s}, s_D]$ and $N = [s_D, \bar{s}]$ dominates all other forms of disclosure. Next, I solve for the optimal cutoff point. Denote the regulator's ex ante utility with the optimal disclosure policy by U^*,

$$
U^* = \int_{\underline{s}}^{s^*} u\big(s, m_D(s)\big) dF(s) + \int_{s^*}^{\bar{s}} u(s, m_N) dF(s).
$$

First, the optimal cutoff point $s^* \leq s_D$ must hold. Otherwise, by the previous proof, the regulator can gain by not disclosing the signals $s \in [s_D, s^*]$. But such disclosure policy features two no-disclosure sets which is dominated by the disclosure policy with single no-disclosure set $[s_D, \bar{s}]$. Hence, $s^* \leq s_D$ must hold.

Take derivative of U^* with respect to the cutoff point s^*, the first-order condition determines the optimal cutoff point,

$$\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) f(s^*) + \frac{\partial m_N}{\partial s^*} \int_{s^*}^{\bar{s}} ML_r\big(s, t_p(s, m_N)\big) dF(s) = 0. \tag{25}$$

This condition is equivalent to equation (12) in the proposition.

□

Proof. Proposition 5

The no-disclosure set N is $[s^*, \bar{s}]$. The bank's manipulation m_N for $s \in N$ is then determined by the following first-order condition

$$F_N \equiv \mathbb{E}_s\left[MB_b\big(s, t_p(s, m_N)\big) | s \in [s^*, \bar{s}]\right] - kc'(m_N) = 0.$$

By the implicit function theorem,

$$\frac{\partial s^*}{\partial k} = -\frac{\frac{\partial F_N}{\partial k}}{\frac{\partial F_N}{\partial s^*}} = \frac{c'(m_N) \int_{s^*}^{\bar{s}} dF(s)}{f(s^*)\Big(\mathbb{E}_{s \geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)\Big)}.$$

Also, by the implicit function theorem, I derive the derivative $\frac{\partial m_N}{\partial s^*}$,

$$\frac{\partial m_N}{\partial s^*} = \frac{f(s^*)}{\int_{s^*}^{\bar{s}} dF(s)} \frac{\mathbb{E}_{s \geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)}{kc''(m_N) - \mathbb{E}_{s \geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]}.$$

With this derivative, I further reduce the first-order condition in equation (25) to the following

$$\begin{aligned}&\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) \\ &+ \Big(\mathbb{E}_{s \geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big)\Big) \frac{\mathbb{E}_{s \geq s^*}\left[ML_r\big(s, t_p(s, m_N)\big)\right]}{kc''(m_N) - \mathbb{E}_{s \geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]} = 0.\end{aligned} \tag{26}$$

Which is equivalent to,

$$\begin{aligned}&\mathbb{E}_{s \geq s^*}\left[MB_b\big(s, t_p(s, m_N)\big)\right] - MB_b\big(s^*, t_p(s^*, m_N)\big) \\ &= \Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) \frac{kc''(m_N) - \mathbb{E}_{s \geq s^*}\left[\frac{\partial MB_b\left(s, t_p(s, m_N)\right)}{\partial m_N}\right]}{-\mathbb{E}_{s \geq s^*}\left[ML_r\big(s, t_p(s, m_N)\big)\right]}.\end{aligned} \tag{27}$$

With this equation, I reduce the derivative $\frac{\partial s^*}{\partial k}$ to the following,

$$\begin{aligned}\frac{\partial s^*}{\partial k} &= \frac{-\mathbb{E}_{s\geq s^*}\left[ML_r\big(s,t_p(s,m_N)\big)\right]c'(m_N)\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(kc''(m_N)-\mathbb{E}_{s\geq s^*}\left[\frac{\partial MB_b\left(s,t_p(s,m_N)\right)}{\partial m_N}\right]\Big)}\frac{1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}\\ &\propto \frac{1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}.\end{aligned}$$

The derivative is positive if $m_D(s^*)\leq m_N$ and it is negative if $m_D(s^*)>m_N$.

Similarly, I derive the following derivative by the implicit function theorem,

$$\frac{\partial s^*}{\partial B} = -\frac{\frac{\partial F_N}{\partial B}}{\frac{\partial F_N}{\partial s^*}} = \frac{-\mathbb{E}_{s\geq s^*}\left[q_l\Delta\big(t_p(s,m_N)\big)\right]\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(\mathbb{E}_{s\geq s^*}\left[MB_b\big(s,t_p(s,m_N)\big)\right]-MB_b\big(s^*,t_p(s^*,m_N)\big)\Big)}.$$

Apply the result in equation (27), the above partial derivative becomes

$$\begin{aligned}\frac{\partial s^*}{\partial B} &= \frac{-\mathbb{E}_{s\geq s^*}\left[ML_r\big(s,t_p(s,m_N)\big)\right]\mathbb{E}_{s\geq s^*}\left[q_l\Delta\big(t_p(s,m_N)\big)\right]\int_{s^*}^{\bar{s}}dF(s)}{f(s^*)\Big(kc''(m_N)-\mathbb{E}_{s\geq s^*}\left[\frac{\partial MB_b\left(s,t_p(s,m_N)\right)}{\partial m_N}\right]\Big)}\frac{-1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}\\ &\propto \frac{-1}{\Big(u\big(s^*,m_D(s^*)\big)-u(s^*,m_N)\Big)}.\end{aligned}$$

The derivative is negative if $m_D(s^*)\leq m_N$ and it is positive if $m_D(s^*)>m_N$. □

B Cost of inefficiencies

In this appendix, I derive the regulator's optimal disclosure policy assuming that the bank's asset should be liquidated ex ante. That is, I replace Assumption 1 by the following

Assumption B.1. *The bank's asset is ex ante worth liquidating:* $\mathbb{E}_\omega\left[x(s,\omega)\right] \in [q_l x(\underline{s},\omega_l),0]$ *for* $s \in [\underline{s},\bar{s}]$.

For given bank's report t with conjectured manipulation level $\hat{m}$ and the signal s, the regulator's pass/fail decision still follows equation (4). As in Lemma 1, the regulator's pass/fail decision is characterized by a cutoff rule on the bank's report t. That is, the regulator passes the bank if and only if the bank's report t is higher than the threshold $t_p(s,\hat{m})$.

Lemma B.1. *For given level of manipulation* m*, the passing threshold* $t_p(s,m)$ *is decreasing in* s*. For given signal* s*, the passing threshold is increasing in* m*.*

Proof. The proof follows the proof of Lemma 2. The only difference is that Assumption B.1 assumes that $q_h x(s,\omega_h) + q_l x(s,\omega_l) < 0$. This assumption implies that $\frac{\partial t_p}{\partial m}$ in equation (16) is positive. □

Compared to Lemma 2, the effect of the signal s on the regulator's choice of the passing threshold remains unchanged. However, the effect of manipulation is the opposite due to Assumption B.1, which assumes that inefficient continuation is more costly than inefficient liquidation. Despite the bank's manipulation still increasing the report similarity between low and high fundamental asset, the regulator is more concerned about the inefficient continuation. Hence, when the regulator is facing a report that is less informative, the regulator would raise the passing threshold in order to avoid inefficient continuation.

The regulator's choice of passing threshold determines the difference in passing probability between low and high fundamental asset $\Delta\big(t_p(s,m)\big)$. The following lemma shows that this difference in passing probability becomes larger as the signal increases.

Lemma B.2. *For given manipulation level* m*,* $\Delta\big(t_p(s,m)\big)$ *is increasing in* s*.*

Proof. The proof is similar to the proof of Lemma 3. The only difference is that Assumption B.1 implies that $q_h x(s,\omega_h) \leq -q_l x(s,\omega_l)$ for all s. As a result, the ratio $r\big(t_p(s,m)\big)$ is less than 1, which then implies that $\Delta\big(t_p(s,m)\big)$ is increasing in s. □

Anticipating the regulator's pass/fail decision, the bank chooses the manipulation level. The bank's manipulation choice depends on whether the regulator discloses the regulatory signal s.

Proposition B.1. *When s is disclosed, the level of manipulation $m_D(s)$ is unique and it is increasing in s for all s. When s is not disclosed, the level of manipulation m_{N_n} is unique and it is a constant over s for $s \in N_n$.*

Recall that when the bank observes the signal s, its manipulation incentive is determined by the increases in the passing probability after manipulation $\Delta\big(t_p(s,m)\big)$ and the gain from passing the test with the low fundamental asset $q_l\big(x(s,\omega_l)+B\big)$. As the signal s increases, both forces become stronger, leading the bank to manipulate more. When the bank does not observe the signal s, its manipulation choice is a constant over the signals in N_n.

Similar to Proposition 3, the following proposition compares the expected manipulation level under different disclosure policies.

Proposition B.2. $\mathbb{E}_s\left[m_D(s)|s \in N\right] \leq m_N$ *for any* $N \subseteq S$.

Proof. The proof follows the proof of Proposition 3 when $m_D(s)$ is increasing in s. □

This result states that the disclosure of regulatory signal reduces the expected manipulation level across signals. The rationale for this result is rooted in the interactions between the bank's manipulation choice and the regulator's passing threshold choice when the signal s is disclosed. As manipulation increases with the signal s, the regulator increases the passing threshold $t_p(s,m)$ according to Lemma B.1. This response of the passing threshold reduces the bank's passing probability, and, more importantly, narrows the difference in passing probability between the high and low fundamental asset, decreasing the bank's manipulation incentive. In the absence of the disclosure of the signal s, such interactions between the bank's manipulation choice and the regulator's passing threshold choice are muted. Hence, the expected manipulation level is lower when s is disclosed. The same rationale also forms the basis for Proposition 3.

I now analyze the regulator's disclosure policy about the regulatory signal s. Similar to Lemma 4, I derive how the regulator's marginal loss caused by the bank's manipulation, $ML_r\big(s, t_p(s,m^*)\big)$, changes with the signal s.

Lemma B.3. *If the following condition holds,*

$$\frac{d}{ds}\left(\frac{\frac{d\Delta\left(s,t_p(s,m^*)\right)}{ds}}{\Delta\big(s,t_p(s,m^*)\big)}\right) \leq 0, \tag{28}$$

then $ML_r\big(s,t_p(s,m^*)\big)$ is decreasing in s for $s < s_r$ and increasing in s for $s > s_r$, where $m^* = \{m_D(s), m_{N_n}\}$ and $s_r \in (\underline{s}, \bar{s})$ is the unique solution for $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$.

Proof. The derivative of $ML_r\big(s, t_p(s, m^*)\big)$ with respect to s is given by the following,

$$\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds} = q_l \left(x(s, \omega_l) \frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds} + \Delta\big(s, t_p(s, m^*)\big) \frac{dx(s, \omega_l)}{ds} \right).$$

When $s = \underline{s}$, Assumption B.1 assumes that $x(\underline{s}, \omega_h) = 0$. According to equation (14), the passing threshold is $t_p(\underline{s}, m^*)) = \bar{t}$ which implies that $\Delta\big(t_p(\underline{s}, m^*)\big) = 0$, hence, the derivative $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds}$ is

$$\left.\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\underline{s}} = q_l x(\underline{s}, \omega_l) \left.\frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\underline{s}} < 0.$$

When $s = \bar{s}$, Assumption B.1 implies that $x(\bar{s}, \omega_h)q_h + x(\bar{s}, \omega_l)q_l = 0$. According to equation (14), the passing threshold satisfies $r\big(t_p(\bar{s}, m^*\big) = 1$, which implies that $\left.\frac{d\Delta\left(s,t_p(s,m^*)\right)}{ds}\right|_{s=\bar{s}} = 0$. Hence, the derivative $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds}$ is

$$\left.\frac{dML_r\big(s, t_p(s, m^*)\big)}{ds}\right|_{s=\bar{s}} = q_l \Delta\big(\bar{s}, t_p(\bar{s}, m^*)\big) \left.\frac{dx(s, \omega_l)}{ds}\right|_{s=\bar{s}} > 0.$$

By the intermediate value theorem, $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$ must hold at some value of $s \in (\underline{s}, \bar{s})$.

Next, I show that $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$ is unique at $s = s_r$. When $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$, the following equation holds,

$$x(s, \omega_l) \frac{d\Delta\big(s, t_p(s, m^*)\big)}{ds} + \Delta\big(s, t_p(s, m^*)\big) \frac{dx(s, \omega_l)}{ds} = 0.$$

This is equivalent to

$$\frac{\frac{d\Delta\left(s,t_p(s,m^*)\right)}{ds}}{\Delta\big(s, t_p(s, m^*)\big)} = -\frac{\frac{dx(s,\omega_l)}{ds}}{x(s, \omega_l)}.$$

The left-hand side is positive for $s \in (\underline{s}, \bar{s})$. In addition, the condition in equation (28) ensures that it is weakly decreasing in s. The right-hand side is also positive for $s \in (\underline{s}, \bar{s})$. And the assumption that $\frac{x(s,\omega_l)}{x(s,\omega_h)}$ is weakly log-concave in s implies that $x(s, \omega_l)$ is also weakly log-concave.[14] Hence, the right-hand side is weakly increasing in s. As a result, $\frac{dML_r\left(s,t_p(s,m^*)\right)}{ds} = 0$ is unique and the solution is denoted as s_r. □

This result shows that under certain condition, the regulator's marginal loss from manipulation has U-shape. That is, the marginal loss $ML_r\big(s, t_p(s, m^*)\big)$ first decreases in s and then

[14] I provide the proof of this statement in the following. The weakly log-concavity of $\frac{x(s,\omega_l)}{x(s,\omega_h)}$ implies

increases in s. The condition in equation (28) means that $\Delta\big(s, t_p(s,m)\big)$ is weakly log-concave in s which ensures that $\Delta\big(s, t_p(s,m)\big)$ is not too convex in s.

Following the intuition of Proposition 4, the optimal disclosure policy should minimize the part of the regulator's loss that cannot be controlled by the optimal pass/fail decision. For given level of manipulation, the disclosure policy should allocate less manipulation to cases where the regulator is more susceptible to it. In addition, the optimal disclosure policy should minimize the expected level of manipulation. To fix the idea, I decompose the regulator's ex ante utility difference between disclosure U_D and no disclosure U_N for a given set of signals S',

$$
\begin{aligned}
U_D - U_N =& \int_{S'} \Big(u\big(s, m_D(s)\big) - u(s, m_N)\Big) dF(s) \\
=& \int_{S'} \big(m_D(s) - m_N\big) ML_r\big(s, t_p\big(s, m(s)\big)\big) dF(s) \\
=& \mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
=& \mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
&+ \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - m_N\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] \\
=& \underbrace{\mathbb{E}_{s\in S'}\Big[m_D(s) ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big] - \mathbb{E}_{s\in S'}\big[m_D(s)\big]\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Distribution effect}} \\
&+ \underbrace{\Big(\mathbb{E}_{s\in S'}\big[m_D(s)\big] - m_N\Big)\, \mathbb{E}_{s\in S'}\Big[ML_r\big(s, t_p\big(s, m(s)\big)\big)\Big]}_{\text{Expected level effect}}
\end{aligned}
$$

Where m_N is the bank's manipulation response when $N = S'$ and $m(s)$ is the manipulation level that satisfies the mean value theorem. This decomposition shows the two effects of the disclosure of regulatory signal s on the regulator's ex ante payoff. The first two terms capture the "distribution effect", which measures the extent to which disclosure can allocate more manipulation to cases where the regulator suffers less from it. The last two terms are the "expected

that $\left(\frac{x(s,\omega_l)}{x(s,\omega_h)}\right) \Big/ \frac{d\frac{x(s,\omega_l)}{x(s,\omega_h)}}{ds}$ is weakly increasing in s.

$$
d\left(\frac{x(s,\omega_l)}{x(s,\omega_h)} \Big/ \frac{d\frac{x(s,\omega_l)}{x(s,\omega_h)}}{ds}\right) \Big/ ds
$$
$$
= \frac{\big(x(s,\omega_h)\big)^2 \left(\left(\frac{dx(s,\omega_l)}{ds}\right)^2 - x(s,\omega_l)\frac{d^2x(s,\omega_l)}{ds^2}\right) - \big(x(s,\omega_l)\big)^2 \left(\left(\frac{dx(s,\omega_h)}{ds}\right)^2 - x(s,\omega_h)\frac{d^2x(s,\omega_h)}{ds^2}\right)}{\left(x(s,\omega_l)\frac{dx(s,\omega_h)}{ds} - x(s,\omega_h)\frac{dx(s,\omega_l)}{ds}\right)^2} \geq 0.
$$

Given that $x(s,\omega_h) > 0$ for all s and it is weakly concave in s, it holds that $x(s,\omega_h)$ is log-concave in s. Hence, the term $\left(\frac{dx(s,\omega_h)}{ds}\right)^2 - x(s,\omega_h)\frac{d^2x(s,\omega_h)}{ds^2}$ is positive. Consequently, the non-negative derivative above implies that $\left(\frac{dx(s,\omega_l)}{ds}\right)^2 - x(s,\omega_l)\frac{d^2x(s,\omega_l)}{ds^2}$ must be non-negative, which proves that $x(s,\omega_l)$ is weakly log-concave in s.

level effect", which captures the impact of disclosure on the expected level of manipulation. The following proposition characterizes the optimal disclosure policy. It shows that the optimal disclosure policy still follows a single cutoff rule.

Proposition B.3. *Suppose that condition (28) holds. The optimal disclosure policy follows a cutoff rule where $D = (s^*, \bar{s}]$ and $N = [\underline{s}, s^*]$. That is, the regulator discloses the signal s when $s > s^*$ and does not disclose the signal s when $s < s^*$, where $s^* \in [s_r, \bar{s}]$ solves*

$$\Big(u\big(s^*, m_D(s^*)\big) - u(s^*, m_N)\Big) f(s^*) = \frac{\partial m_N}{\partial s^*} \int_{\underline{s}}^{s^*} ML_r\big(s, t_p(s, m_N)\big) dF(s). \tag{29}$$

Proof. A single cutoff disclosure policy is optimal because the regulator's marginal loss caused by the bank's manipulation $ML_r\big(s, t_p(s, m)\big)$ has U-shape across s. The proof of the optimality of a cutoff disclosure policy is similar to the proof of Proposition 4, hence, omitted. In what follows, I solve the optimal cutoff point s^*. The regulator's ex ante utility with the optimal disclosure policy is U^*,

$$U^* = \int_{\underline{s}}^{s^*} u(s, m_N) dF(s) + \int_{s^*}^{\bar{s}} u\big(s, m_D(s)\big) dF(s).$$

Where the optimal cutoff point s^* solves the following,

$$\Big(u(s^*, m_N) - u\big(s^*, m_D(s^*)\big)\Big) f(s^*) + \frac{\partial m_N}{\partial s^*} \int_{\underline{s}}^{s^*} ML_r\big(s, t_p(s, m_N)\big) dF(s) = 0.$$

□

The rationale for the optimality of a cutoff disclosure policy is similar to that of Proposition 4. Disclosing the signal s benefits the regulator when the bank's manipulation strategy features reducing manipulation in cases where the regulator is more vulnerable to it. Proposition B.1 shows that as s increases, the bank manipulates more. Additionally, Lemma B.3 indicates that for relatively high values of s, the regulator is less susceptible to manipulation as s increases. Consequently, disclosing s may benefit the regulator when s exceeds a certain threshold, i.e, $s \geq s_r$. No disclosure at all may be optimal, provided that the expected level of manipulation in this case is sufficiently low.

C Real activity

In this appendix, I discuss an extension in which the bank exerts costly effort to improve the fundamental of the asset, and such effort is reflected in the bank's report. Although, similar to the baseline model, the bank is able to improve the report in the sense of first-order stochastic dominance, such improvement in report arises endogenously from the improvement in the asset's fundamental. As a result, the disclosure of the regulatory information now affects the real activity of the bank, i.e., effort choice. The purpose of this extension is to show that full disclosure may be optimal when the bank's effort improves the asset's fundamental value.

The model is as follows. Suppose that the bank can exert effort m to improve the fundamental of the asset. To keep the notation consistent, I continue to use m to denote the bank's effort. The effort m determines the probability that the asset's fundamental value is ω_h. That is, when the bank exerts effort m and the regulator's signal is s, the relative gain from continuing the asset is $x(s, \omega_h)$ with probability m and $x(s, \omega_l)$ with probability $1 - m$.

The fundamental of the asset determines the report distribution in the same way as in the baseline model. That is, the report t is drawn from a distribution with density $g^i(t)$ when the fundamental is ω_i, where $i = \{h, l\}$. Since the bank's effort determines the asset fundamental, it also determines the report distribution. With effort m, the bank's report distribution is $g^h(t)$ with probability m and $g^l(t)$ with probability $1 - m$. Notice that Assumption 1 no longer holds, since the expected relative gain from continuing the asset for given s is now endogenously determined by the bank's effort. I assume that $x(s, \omega_l) \leq x(\bar{s}, \omega_l) \equiv 0$ and $x(s, \omega_h) \geq x(\underline{s}, \omega_h) \equiv 0$. All other elements of the model remain the same as in Section 2. The following analysis focuses on how the regulator should disclose the regulatory signal to affect the bank's effort choice. I solve the model backwards.

After observing the signal s and the bank's report t, the regulator's pass/fail decision remains the same as in the baseline model. The regulator passes the bank if and only if the expected gain from passing the bank exceeds that from failing the bank. That is,

$$\mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] \geq 0.$$

Where $\hat{m}$ is the regulator's conjecture about the bank's effort. The conditional expectation is

$$\begin{aligned} \mathbb{E}_\omega[x(s, \omega)|t, \hat{m}] &= x(s, \omega_h) \Pr(\omega = \omega_h|t, \hat{m}) + x_{\hat{m}}(s, \omega_l) \Pr(\omega = \omega_l|t, \hat{m}) \\ &= x(s, \omega_h) \frac{\hat{m} g^h(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)} + x(s, \omega_l) \frac{(1 - \hat{m}) g^l(t)}{\hat{m} g^h(t) + (1 - \hat{m}) g^l(t)}. \end{aligned}$$

The pass/fail decision still features a threshold $t_p(s, \hat{m})$ on the bank's report. The bank passes the test if and only if the report t satisfies $t \geq t_p(s, \hat{m})$. The passing threshold $t_p(s, \hat{m})$ solves $\mathbb{E}_\omega[x(s, \omega)|t_p, \hat{m}] = 0$, which indicates that the regulator is indifferent between passing and failing the bank when the bank's report is $t_p(s, \hat{m})$.

Lemma C.1. *For given level of effort m, the passing threshold $t_p(s, m)$ is decreasing in s. For given signal s, the passing threshold $t_p(s, m)$ is decreasing in m.*

Proof. The proof is similar to the proof of Lemma 2. The regulator chooses the passing threshold based on the signal s and the conjecture about the bank's manipulation $\hat{m}$. I drop the $\hat{\cdot}$ for simplicity. The passing threshold is determined by

$$\mathbb{E}_\omega[x(s, \omega)|t_p, m] = x(s, \omega_h)\frac{mg^h(t_p)}{mg^h(t_p) + (1-m)g^l(t_p)} + x(s, \omega_l)\frac{(1-m)g^l(t_p)}{mg^h(t_p) + (1-m)g^l(t_p)} = 0.$$

Since the density function $g^l(t)$ and $g^h(t)$ have full support, the condition reduces to

$$x(s, \omega_h)mg^h(t_p) + x(s, \omega_l)(1-m)g^l(t_p) = 0.$$

This is equivalent to

$$x(s, \omega_h)m + x(s, \omega_l)(1-m)r(t_p) = 0.$$

Apply implicit function theorem, I derive the following two partial derivatives.

$$\frac{\partial t_p}{\partial s} = -\frac{m\left(x(s, \omega_l)\frac{dx(s,\omega_h)}{ds} - x(s, \omega_h)\frac{dx(s,\omega_l)}{ds}\right)}{(1-m)\big(x(s, \omega_l)\big)^2 r'(t_p)}.$$

Where $r'(t_p)$ is the derivative of $r(t_p)$ with respect to t_p and it is negative. Given that the relative gain from continuing the asset $x(s, \omega_l)$ and $x(s, \omega_h)$ are increasing in s, this derivative is negative.

And the following is the partial derivative of t_p with respect to m,

$$\frac{\partial t_p}{\partial m} = -\frac{x(s, \omega_h)}{(1-m)^2 x(s, \omega_l) r'(t_p)}.$$

This derivative is non-positive and it equals to zero only when $s = \underline{s}$. □

This result echoes to Lemma 2. The explanation for the first result is the same as in Lemma 2. However, the effect of effort is different from that of manipulation. Effort improves the relative gain from continuing the asset $x(s, \omega)$, which endogenously increases relative cost of inefficient liquidation. Consequently, the regulator decreases the passing threshold to pass the bank more often.

Anticipating the regulator's pass/fail decision, the bank chooses the effort level. Suppose that the bank observes the regulator's private signal s. The bank's payoff is

$$V(s,\hat{m},m) = m\big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,\hat{m})} g^h(t)dt+(1-m)\big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,\hat{m})} g^l(t)dt-kc(m).$$

The first-order condition with respect to m determines the bank's effort choice. In equilibrium, the regulator's conjecture about the effort is consistent with the bank's choice. Hence, the equilibrium effort $m_D(s)$ is determined by

$$\big(x(s,\omega_h)+B\big)\int_{t\geq t_p\left(s,m_D(s)\right)} g^h(t)dt-\big(x(s,\omega_l)+B\big)\int_{t\geq t_p\left(s,m_D(s)\right)} g^l(t)dt-kc'\big(m_D(s)\big)=0. \quad (30)$$

The first two terms are the bank's marginal benefit of exerting effort. I modify the definition of MB_b in equation (8) to the following

$$\begin{aligned} MB_b\big(s,t_p(s,m)\big) &\equiv \big(x(s,\omega_h)+B\big)\int_{t\geq t_p(s,m)} g^h(t)dt - \big(x(s,\omega_l)+B\big)\int_{t\geq t_p(s,m)} g^l(t)dt \\ &= \big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt + \big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big). \end{aligned} \quad (31)$$

Where $\Delta\big(t_p(s,m)\big)$ is defined in equation (5) and it captures the difference in the passing probability between the low and high fundamental asset. The first term of MB_b, $\big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt$, is the bank's gain from improving the asset's fundamental from low to high, provided that the bank passes the test. This term captures *the effect of effort on the relative gain from holding the asset.* The second term, $\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big)$, is identical to equation (8) and it captures the bank's gain from having the low fundamental asset pass the test after the effort makes its report resemble that of the high fundamental asset. This term captures *the effect of effort on the bank's report.* This effect is identical to the effect of manipulation in the baseline model.

Lemma C.2. *For given signal s, if*

$$\big(x(s,\omega_h)-x(s,\omega_l)\big)g^h(\underline{t}) \geq \big(x(s,\omega_l)+B\big)\big(g^l(\underline{t})-g^h(\underline{t})\big), \quad (32)$$

then $MB_b\big(s,t_p(s,m)\big)$ is increasing in m. Otherwise, $MB_b\big(s,t_p(s,m)\big)$ is increasing in m for $m<m_r(s)$ and it is decreasing in m for $m>m_r(s)$, where $m_r(s)$ solves $\frac{\partial MB_b\left(s,t_p(s,m)\right)}{\partial m}=0$.

Proof. Take partial derivative of $MB_b\big(s, t_p(s,m)\big)$ with respect to m, I obtain the following

$$\begin{aligned}
&\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m} \\
&= \Big(- \big(x(s,\omega_h) - x(s,\omega_l)\big) g^h(t_p) + \big(x(s,\omega_l) + B\big)\big(g^l(t_p) - g^h(t_p)\big)\Big) \frac{\partial t_p(s,m)}{\partial m} \\
&= \Big(- \big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(t_p) - 1\big)\Big) g^h(t_p) \frac{\partial t_p(s,m)}{\partial m}.
\end{aligned}$$

If condition (32) holds, then the following also holds

$$-\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(\underline{t}) - 1\big) \leq 0.$$

Since the ratio $r(t) \equiv \frac{g^l(t)}{g^h(t)}$ is decreasing in t, the following also holds for all $t_p \geq \underline{t}$,

$$\begin{aligned}
&- \big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(t_p) - 1\big) \\
&\leq -\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(\underline{t}) - 1\big) \\
&\leq 0.
\end{aligned}$$

Together with the result that $\frac{\partial t_p(s,m)}{\partial m}$ is negative, this implies that $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m}$ is positive, suggesting that $MB_b\big(s, t_p(s,m)\big)$ is increasing in m in this case.

If condition (32) does not hold, then I show that $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m} = 0$ has a unique solution. Since $r(t_p)$ is decreasing in t_p and $t_p(s,m)$ is decreasing in m, the first term in $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m}$ is monotonically increasing in m, i.e., $-\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(t_p(s,m)) - 1\big)$ is monotonically increasing in m. When $m = 1$, the passing threshold is $t_p(s,1) = \underline{t}$. As a result, $-\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(\underline{t}) - 1\big) > 0$ when $m = 1$. When $m = 0$, the passing threshold is $t_p(s,0) = \bar{t}$. Since $r(\bar{t}) < 1$ holds, it implies that $-\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(\bar{t}) - 1\big) < 0$. Hence, by the intermediate value theorem, the solution for $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m} = 0$ exists. Moreover, since $-\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l) + B\big)\big(r(t_p(s,m)) - 1\big)$ is monotonically increasing in m, the solution is unique. I denote the unique solution for $\frac{\partial MB_b\big(s, t_p(s,m)\big)}{\partial m} = 0$ as $m_r(s)$. For $m < m_r(s)$, the marginal benefit $MB_b\big(s, t_p(s,m)\big)$ is increasing in m. And for $m > m_r(s)$, the marginal benefit $MB_b\big(s, t_p(s,m)\big)$ is decreasing in m. □

This result shows that the bank's marginal benefit of exerting effort is nonmonotonic in the level of effort m. The intuition is as follows. The first component of $MB_b\big(s, t_p(s,m)\big)$ is $\big(x(s,\omega_h) - x(s,\omega_l)\big) \int_{t \geq t_p(s,m)} g^h(t)dt$, and it represents the bank's gain from improving the asset fundamentals. As the effort m increases, the bank is more likely to have the high fundamental asset. In response, the regulator passes the bank more often by lowering the passing threshold.

Consequently, the first term $\big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt$ is increasing in the amount of effort, incentivizing the bank to exert more effort. This term captures the bank's incentive to exert effort to improve the fundamental of the asset.

However, the second component $\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big)$ may decrease with the effort m, depending on how $\Delta\big(t_p(s,m)\big)$ changes with m. Lemma C.1 shows that the passing threshold $t_p(s,m)$ is decreasing in effort m, meaning that the bank is more likely to pass the test when exerting more effort. When the level of effort is very low (high), the regulator will set the passing threshold very high (low) which makes the bank less (more) likely to pass the test regardless of the fundamental of the asset. In such cases, the difference in the passing probability between the low and high fundamental asset is small. When the level of effort is intermediate, the regulator will set the passing threshold at an intermediate level which makes the passing probability depend crucially on the fundamental of the asset. Hence, the *difference* in the passing probability between the low and high fundamental asset $\Delta\big(t_p(s,m)\big)$ is first increasing and then decreasing in the effort level m, which then makes the term $\big(x(s,\omega_l)+B\big)\Delta\big(t_p(s,m)\big)$ follow the same pattern. This term captures the bank's incentive to exert effort only when it leads to higher probability of passing the test for the low fundamental asset.

The condition (32) captures the cases when the bank's effort choice is solely driven by the first component. This condition depends on the bank's private benefit of passing the test. When the private benefit B is relatively small, the first effect ("improving the asset fundamental") always dominates the second effect ("improving the report") and determines the bank's effort choice.

One implications of Lemma C.2 is that the first-order condition in equation (30) may be nonmonotonic in m, hence, the interior solution of $m_D(s)$ may not exist and the solution of $m_D(s)$ may not be unique. One sufficient condition for the interior solution of $m_D(s)$ to exist is

$$x(s,\omega_h)-x(s,\omega_l)<kc'(1),\ \forall s. \tag{33}$$

This condition means that the effort is costly such that the bank does not have incentive to improve the fundamental to $x(s,\omega_h)$. In the following analyses, I assume this condition holds. To avoid having multiple equilibria for the effort $m_D(s)$, I also assume that the bank chooses the highest effort level when it is indifferent. The solution $m_D(s)$ must satisfy the second order condition.

For the purpose of this extension, I focus on the case when condition (32) is satisfied for all s.

Proposition C.1. *When s is disclosed, the effort level $m_D(s)$ is increasing in s if condition*

(32) *holds for all* s.

Proof. Apply the implicit function theorem to the first-order condition of $m_D(s)$, I obtain the following

$$\frac{\partial m_D}{\partial s} = -\frac{\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}}{\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial m_D} - kc''(m_D)} \propto \frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}.$$

Where the derivative $\frac{\partial MB_b\left(s,t_p(s,m_D)\right)}{\partial s}$ is as follows. I drop the arguments for t_p when no confusion is caused.

$$\begin{aligned}\frac{\partial MB_b(s,t_p)}{\partial s} =& \frac{dx(s,\omega_h)}{ds}(1-G^h(t_p)) - \frac{dx(s,\omega_l)}{ds}(1-G^l(t_p)) \\ &+ \Big(- \big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l)+B\big)\big(r(t_p)-1\big)\Big) g^h(t_p)\frac{\partial t_p(s,m)}{\partial s}.\end{aligned}$$

When condition (32) holds for all s, the following also holds for all $t_p \geq \underline{t}$,

$$\begin{aligned}&- \big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l)+B\big)\big(r(t_p)-1\big) \\ &\leq -\big(x(s,\omega_h) - x(s,\omega_l)\big) + \big(x(s,\omega_l)+B\big)\big(r(\underline{t})-1\big) \\ &\leq 0.\end{aligned}$$

Since $\frac{dx(s,\omega_h)}{ds} \geq \frac{dx(s,\omega_l)}{ds}$ and $\frac{\partial t_p(s,m)}{\partial s} < 0$, the derivative $\frac{\partial MB_b(s,t_p)}{\partial s}$ is positive for all s. Hence, $\frac{\partial m_D(s)}{\partial s}$ is positive for all s. □

Now consider the effort choice when the bank does not observe the regulator's signal s. The equilibrium effort m_N solves,

$$\mathbb{E}_s\left[MB_b\big(s,t_p(s,m_{N_n})\big)\,\big|\, s\in N_n\right] - kc'(m_{N_n}) = 0. \tag{34}$$

The effort m_N is unique and it is a constant over the regulator's signal s.

Proposition C.2. *When s is not disclosed, the effort level m_{N_n} is unique and it is a constant over s for $s \in N_n$.*

Proof. The proof is shown in equation (34). □

The following proposition compares the expected effort level when the bank observes the signal with the expected effort level when the bank does not observe the signal.

Proposition C.3. *If condition (32) holds for all s, then $\mathbb{E}_s\left[m_D(s)|s\in N\right] \geq m_N$ for any $N \subseteq S$.*

Proof. I omit the proof because it is similar to the proof of Proposition 3. □

The intuition is as follows. When condition (32) is satisfied, the effort $m_D(s)$ is increasing in s if s is disclosed. In response, the regulator lowers the passing threshold t_p, which makes the test more lenient regardless of the asset fundamental. Such endogenous response of the passing threshold has two opposite effects on the bank's incentive to exert effort. One the one hand, an easier test allows the bank to pass even without exerting effort, which then decreases the bank's incentive to exert effort. On the other hand, an easier test increases the likelihood that the bank's effort is realized, i.e., the bank passes the test after increasing the asset fundamental. This effect increases the bank's incentive to exert effort. Notice that this second effect is missing in the baseline model. Depending on the magnitude of the two forces, the bank may increase or decrease effort. When condition (32) holds, the second effect dominates. As a result, the interactions between the regulator's pass/fail decision and the bank's effort choice increase the expected level of effort, compared to the case when such interactions are absent, i.e., when s is not disclosed.

Consider the regulator's disclosure policy. For given signal s and equilibrium effort m^*, the regulator's payoff is

$$\begin{aligned} u(s,m^*) &= \int_{t\geq t_p(s,m^*)} \mathbb{E}_\omega[x(s,\omega)|t,m^*]g_{m^*}(t)dt \\ &= m^* x(s,\omega_h)\int_{t\geq t_p(s,m^*)} g^h(t)dt + (1-m^*)x(s,\omega_l)\int_{t\geq t_p(s,m^*)} g^l(t)dt. \end{aligned}$$

Where $g_{m^*}(t)$ is the unconditional distribution of report t when the bank's effort is m^*. That is,

$$g_{m^*}(t) = m^* g^h(t) + (1-m^*)g^l(t).$$

Taking derivative of $u(s,m)$ with respect to m, I obtain the marginal effect of the bank's effort on the regulator. I modify the definition of ML_r in equation (11) to the following,

$$\begin{aligned} ML_r\big(s,t_p(s,m)\big) &\equiv x(s,\omega_h)\int_{t\geq t_p(s,m)} g^h(t)dt - x(s,\omega_l)\int_{t\geq t_p(s,m)} g^l(t)dt \\ &= \big(x(s,\omega_h)-x(s,\omega_l)\big)\int_{t\geq t_p(s,m)} g^h(t)dt + x(s,\omega_l)\Delta\big(t_p(s,m)\big). \end{aligned}$$

Lemma C.3. *If condition (32) holds for all s, then for any disclosure set D or no-disclosure set N_n, $ML_r\big(s,t_p(s,m^*)\big)$ is increasing in s for $m^* = \{m_D(s), m_{N_n}\}$.*

Proof. For given m, the derivative of $ML_r\big(s, t_p(s,m)\big)$ with respect to s is

$$\begin{aligned}
&\frac{\partial ML_r\big(s, t_p(s,m)\big)}{\partial s} \\
&= \frac{dx(s,\omega_h)}{ds}\big(1 - G^h(t_p)\big) - \frac{dx(s,\omega_l)}{ds}\big(1 - G^l(t_p)\big) - \Big(x(s,\omega_h)g^h(t_p) - x(s,\omega_l)g^l(t_p)\Big)\frac{\partial t_p(s,m)}{\partial s} \\
&> 0.
\end{aligned}$$

This derivative is positive because $\frac{dx(s,\omega_h)}{ds} > \frac{dx(s,\omega_l)}{ds}$ by assumption. This result implies that $\frac{\partial ML_r\big(s, t_p(s, m_{N_n})\big)}{\partial s} > 0$ for any no-disclosure set N_n.

Next, consider $ML_r\big(s, t_p(s, m_D(s))\big)$

$$\begin{aligned}
&\frac{dML_r\Big(s, t_p\big(s, m_D(s)\big)\Big)}{ds} \\
&= \frac{dx(s,\omega_h)}{ds}\big(1 - G^h(t_p)\big) - \frac{dx(s,\omega_l)}{ds}\big(1 - G^l(t_p)\big) - \Big(x(s,\omega_h)g^h(t_p) - x(s,\omega_l)g^l(t_p)\Big)\frac{dt_p\big(s, m_D(s)\big)}{ds}.
\end{aligned}$$

Where the total derivative $\frac{dt_p\big(s, m_D(s)\big)}{ds}$ is

$$\frac{dt_p\big(s, m_D(s)\big)}{ds} = \frac{\partial t_p\big(s, m_D(s)\big)}{\partial s} + \frac{\partial t_p\big(s, m_D(s)\big)}{\partial m}\frac{\partial m_D(s)}{\partial s}.$$

When condition (32) holds for all s, the derivative $\frac{\partial m_D(s)}{\partial s}$ is positive. Hence, the total derivative $\frac{dt_p\big(s, m_D(s)\big)}{ds}$ remains negative. As a result, $\frac{dML_r\Big(s, t_p\big(s, m_D(s)\big)\Big)}{ds} > 0$. □

Different from the baseline model, the term ML_r is now positive, capturing the regulator's marginal benefit from the bank's effort. Nevertheless, the intuition for the disclosure of s remains the same. As argued in the baseline model, disclosure is less likely to be optimal when the changes in effort m and changes in the regulator's marginal utility change $ML_r\big(s, t_p(s, m^*)\big)$ are driven by the the difference in the passing probability between the low and high fundamental asset $\Delta(t_p(s,m))$. This continues to be the case in this extension with effort choice. However, the presence of effort exertion increases the likelihood of disclosure. This is because the regulator's disclosure about the signal s not only informs the bank about the gain from passing the test but also provides information about the extent to which exerting effort can improve such gain. Hence, disclosure is more useful to align the interests of the regulator and the bank regarding when effort is more desirable.

Proposition C.4. *If condition (32) holds for all s, the optimal disclosure policy is full disclosure policy, i.e., $D = [\underline{s}, \bar{s}]$.*

Proof. The proof follows the proof of Proposition 4. □

References

Barth, Mary E, Wayne R Landsman, and Mark H Lang (2008). “International accounting standards and accounting quality”. *Journal of Accounting Research* 46.3, pp. 467–498.

Baudino, Patrizia and Jean-Philippe Svoronos (2021). *Stress-Testing Banks for Climate Change: a Comparison of Practices*. Bank for International Settlements, Financial Stability Institute.

Bertomeu, Jeremy, Lucas Mahieux, and Haresh Sapra (2023). “Interplay between accounting and prudential regulation”. *The Accounting Review* 98.1, pp. 29–53.

Bouvard, Matthieu, Pierre Chaigneau, and Adolfo De Motta (2015). “Transparency in the financial system: Rollover risk and crises”. *The Journal of Finance* 70.4, pp. 1805–1837.

Bushman, Robert M (2016). “Transparency, accounting discretion, and bank stability”. *Economic Policy Review, Issue Aug*, pp. 129–149.

Bushman, Robert M and Christopher D Williams (2012). “Accounting discretion, loan loss provisioning, and discipline of banks’ risk-taking”. *Journal of Accounting and Economics* 54.1, pp. 1–18.

Chen, Ying (2009). “Communication with two-sided asymmetric information”. *Available at SSRN 1344818*.

Clark, Robert and Shaoteng Li (2022). “The strategic response of banks to macroprudential policies: Evidence from mortgage stress tests in Canada”. *Review of Finance* 26.1, pp. 187–216.

Corona, Carlos, Lin Nan, and Gaoqing Zhang (2015). “Accounting information quality, interbank competition, and bank risk-taking”. *The Accounting Review* 90.3, pp. 967–985.

— (2019). “The Coordination Role of Stress Tests in Bank Risk-Taking”. *Journal of Accounting Research* 57.5, pp. 1161–1200.

Cramton, Peter C (1984). “Bargaining with incomplete information: An infinite-horizon model with two-sided uncertainty”. *The Review of Economic Studies* 51.4, pp. 579–593.

Ding, Haina, Alexander Guembel, and Alessio Ozanne (2022). “Market information in banking supervision: the role of stress test design”. *Available at SSRN 4237535*.

Faria-e-Castro, Miguel, Joseba Martinez, and Thomas Philippon (2017). "Runs versus lemons: information disclosure and fiscal capacity". *The Review of Economic Studies* 84.4, pp. 1683–1707.

Fischer, Paul E and Robert E Verrecchia (2000). "Reporting bias". *The Accounting Review* 75.2, pp. 229–245.

Flannery, Mark J (2019). "Transparency and Model Evolution in Stress Testing". *Available at SSRN 3431679*.

Gao, Pingyang and Xu Jiang (2018). "Reporting choices in the shadow of bank runs". *Journal of Accounting and Economics* 65.1, pp. 85–108.

— (2020). "The economic consequences of discrete recognition and continuous measurement". *Journal of Accounting and Economics* 69.1, p. 101250.

Goldstein, Itay and Yaron Leitner (2018). "Stress tests and information disclosure". *Journal of Economic Theory* 177, pp. 34–69.

— (2020). "Stress tests disclosure: Theory, practice, and new perspectives". *Handbook of Financial Stress Testing Edited by Farmer, Kleinnijenhuis, Wetzer, and Schuermann.*

Goldstein, Itay and Haresh Sapra (2014). "Should banks' stress test results be disclosed? An analysis of the costs and benefits". *Foundations and Trends® in Finance* 8.1, pp. 1–54.

Grossman, Sanford J and Oliver D Hart (1980). "Disclosure laws and takeover bids". *The Journal of Finance* 35.2, pp. 323–334.

Guo, Yingni and Eran Shmaya (2019). "The interval structure of optimal disclosure". *Econometrica* 87.2, pp. 653–675.

Holthausen, Robert W (2009). "Accounting standards, financial reporting outcomes, and enforcement". *Journal of Accounting Research* 47.2, pp. 447–458.

Huizinga, Harry and Luc Laeven (2012). "Bank valuation and accounting discretion during a financial crisis". *Journal of Financial Economics* 106.3, pp. 614–634.

Kolotilin, Anton et al. (2017). "Persuasion of a privately informed receiver". *Econometrica* 85.6, pp. 1949–1964.

Leitner, Yaron and Basil Williams (2023). "Model secrecy and stress tests". *The Journal of Finance* 78.2, pp. 1055–1095.

Leuz, Christian, Dhananjay Nanda, and Peter D Wysocki (2003). "Earnings management and investor protection: an international comparison". *Journal of financial economics* 69.3, pp. 505–527.

Leuz, Christian and Peter D Wysocki (2016). "The economics of disclosure and financial reporting regulation: Evidence and suggestions for future research". *Journal of Accounting Research* 54.2, pp. 525–622.

Parlasca, Markus (2023). "Stress Tests and Strategic Disclosure". *Available at SSRN 3803876*.

Parlatore, Cecilia and Thomas Philippon (2022). *Designing Stress Scenarios*. Working Paper 29901. National Bureau of Economic Research.

Prescott, Edward S (2008). "Should bank supervisors disclose information about their banks?" *FRB Richmond Economic Quarterly* 94.1, pp. 1–16.

Quigley, Daniel and A Walter (2023). "Inside and outside information". *Journal of Finance*.

Shapiro, Joel and Jing Zeng (2023). "Stress Testing and Bank Lending". *The Review of Financial Studies*.

Watson, Joel (1996). "Information transmission when the informed party is confused". *Games and Economic Behavior* 12.1, pp. 143–161.